Soutenabilité environnementale et préservation des capacités. Le cas de la capacité d’assimilation des écosystèmes dans l’analyse économique de la pollution optimale.

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Le cas de la capacité d'assimilation des écosystèmes dans l'analyse économique de la pollution optimale

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"We are coming to realize, in part through the process of losing them, that environmental assets are key determinants of the quality of life in most societies.

These assets –forests, clean water, clean air, species, rivers, seas, and many more– are not like physical or financial assets : they are alive and have dynamics, requirements, imperatives of their own.

Recognizing this and recognizing that they provide for the essential infrastructure for human existence is a key step on the road to building an economic framework that can contribute to the development of sustainable policies."

Geoffrey Heal, *Valuing the Future*, 2000.

Introduction Générale

Toute recherche provient d'un étonnement. Ce travail trouve son inspiration première dans la remise en question de l'analyse coût-bénéfice[1] de la pollution, opérée par Pearce (1976) et prolongée trente ans plus tard par Godard (2006). A partir de l'observation des mécanismes de dégradation de la capacité d'assimilation de la pollution de l'environnement[2] explicitement formalisés dans son modèle stylisé, Pearce a mis en garde contre des politiques économiques de régulation de la pollution qui, selon lui, mènent inévitablement à la destruction totale de cette capacité d'assimilation. Godard (2006) a ensuite mobilisé ce modèle dans une réflexion élargie aux exigences du nouveau paradigme de développement qui a émergé depuis la contribution de Pearce : la soutenabilité. L'intuition derrière ce modèle met en évidence un défaut majeur des modèles de contrôle économique de la pollution. Leurs optima statiques et dynamiques de pollution reposent en effet sur un postulat implicite de reproduction à l'identique des conditions environnementales. Or ces conditions déterminent en partie les dommages socio-environnementaux associés à un niveau d'émissions et influencent donc les régimes optimaux d'accumulation de la pollution. Si l'on veut évaluer le bien-fondé d'une politique environnementale calibrée sur ce type d'optimum, voire en évaluer la soutenabilité, comme le propose Godard[3], il est essentiel de prendre en compte dans les modèles économiques les dynamiques propres aux écosystèmes qui subissent ces émissions "optimales". Telle est la ligne directrice qui sous-tend les analyses et les investigations de la présente thèse.

[1]Nous reviendrons plus loin sur le contenu de la notion d'analyse coût-bénéfice mais elle doit s'entendre ici comme la détermination d'un optimum de pollution ou d'une trajectoire optimale de pollution, par internalisation des effets externes dans un cadre utilitariste actualisé sur un horizon infini.

[2]Pearce la définit ailleurs (Pearce et Turner, 1990) comme la capacité à *"recevoir un niveau déterminé de résidus, à les dégrader et à les convertir en produits non dommageables et même bénéfiques"*.

[3]Nous emploierons le terme français de soutenabilité pour traduire *sustainability* et de développement durable, l'expression consacrée pour *sustainable development.*

La soutenabilité de la pollution optimale en question

Une intuition commune de la soutenabilité derrière la multiplicité des propositions

L'économie de l'environnement est traversée depuis deux décennies par la polarisation entre la soutenabilité faible (weak sustainability) et la soutenabilité forte (strong sustainability[1]). La soutenabilité faible demande la non-décroissance d'un stock de capital total quand la soutenabilité forte exige la non-décroissance de chaque stock de capital pris séparément. La première témoigne d'un optimisme généralisé en matière de substitution entre capital physique et capital naturel qui invite à la plus grande des prudences, voire à la méfiance. La seconde fait preuve d'une intransigeance en termes de conservation du capital naturel qui pourrait trouver des justifications dans d'autres champs disciplinaires mais qui s'auto-exclut ainsi par construction du champ de l'économie, domaine des arbitrages par excellence. Depuis le rapport fondateur de la Commission Brundtland (1988), vingt ans de controverses sur le contenu de la notion de soutenabilité, d'un point de vue théorique comme d'un point de vue concret, n'ont pas débouché sur une conception opératoire consensuelle de la soutenabilité ni sur un accord quant au rôle que peut jouer la discipline économique dans l'élaboration de politiques "soutenables"[2]. Au-delà du débat sur la substituabilité des formes de capital, le concept de soutenabilité a donné lieu à un vaste questionnement éthique et philosophique sur la justice et l'équité intergénérationnelles (Asheim et al., 2001), qui revêt toute son acuité dans le contexte du changement climatique. Ces interrogations soulèvent des problématiques fondamentales sur les bases philosophiques sous-jacentes à la science économique mais elles n'ont pas pour autant fait beaucoup avancer la capacité des modèles théoriques et de leurs transcriptions pratiques à opérationnaliser la soutenabilité.

Face au foisonnement des définitions, qui tourne parfois à la chasse au slogan sur le "marché" académique, il n'a pas été possible de parvenir à un consensus sur un critère de soutenabilité d'une économie ou d'une trajectoire. Il ne faudrait pas pour autant

[1]Voir Pearce et Atkinson (1995) ou Ayres et al. (1998) pour une présentation des deux "écoles" de soutenabilité.

[2]A travers leur réflexion en termes d'invariance, Rotillon et Martinet (2007) questionnent la capacité ontologique des modèles de croissance optimale avec ressource épuisable à "*conserver quelque chose*".

que l'absence de critère de soutenabilité universellement reconnu, qui semble encore bien utopique aujourd'hui, exonère l'analyse économique standard de tout "test" de soutenabilité, même imparfait. Partant, on peut arguer qu'une intuition commune sous-tend la majeure partie de ces définitions : le souci de préserver les capacités d'une société à créer de la richesse, ou du moins à assurer la survie à long terme de ses citoyens contre des risques majeurs, à satisfaire leurs besoins de base et à transmettre un patrimoine humain, culturel et physique, de générations en générations, le tout de façon compatible avec le maintien des équilibres naturels essentiels dont dépend la vie humaine.

Selon nous, la divergence entre les définitions qui s'affrontent provient finalement plus de la volonté de leurs auteurs de produire un cadre d'analyse "unificateur" que d'un désaccord majeur sur le fond. Dès lors que l'on cesse de vouloir à tout prix intégrer toutes les dimensions du développement durable dans un seul critère et que l'on s'intéresse à une situation concrète, on peut s'accorder sur la nécessité de préserver certaines capacités humaines, physiques et environnementales menacées de disparition. Sur cette base il devient faisable, dans une situation donnée, non pas de prétendre détenir *la* solution durable, mais de mettre en garde contre une politique résolument "insoutenable". Proposer une définition contingente et négative de la soutenabilité semble en effet plus constructif et plus sage que de prétendre à un concept universel et systématique. On peut affirmer sans trop s'avancer qu'un pays ou une région dont l'activité économique dépend en grande partie d'une ressource renouvelable et qui exploite cette ressource à un rythme "insoutenable" ne mène pas une politique de gestion durable, même s'il en réinvestit une partie dans du capital physique. Il en va de même pour une exploitation trop intensive des sols agricoles qui provoque progressivement un épuisement irrémédiable de ces derniers ou pour l'absence de réaction des autorités face à une épidémie qui décime les habitants ou qui laisse partir à la dérive son système d'éducation.

La préservation des capacités, naturelles ou humaines, voire physiques quand celles-ci sont en péril semble intuitivement indissociable du développement durable. Les appels de Sen (2003) à la conservation des "capabilities", les exhortations d'Hartwick (1977) à réinvestir les rentes de ressources épuisables dans la formation de capital ou les recommandations par Holling (1973) de protéger la résilience des écosystèmes peuvent être rapprochés, malgré leurs inscriptions dans des champs totalement différents, dans une volonté commune de préserver des capacités, inscrite en filigrane dans la fameuse définition du rapport Brundltand[1] (CMED, 1988). Etant donné l'urgence écologique du XXI$^{\text{ème}}$ siècle, nous pensons que ce souci de préservation des capacités doit prendre partiellement le pas sur l'attribution d'une valeur normative aux prix de marché et sur le fondement des préférences individuelles[2] dont la discipline économique a fait ses principaux repères, avant même l'obligation de survie de l'espèce.

Une condition de soutenabilité *a minima* : la non-extinction locale d'un capital naturel critique

L'approche de la soutenabilité que nous souhaitons privilégier ici consiste à choisir pragmatiquement de préserver des capacités humaines économiques ou environnementales à un certain niveau, ou du moins d'éviter leur dégradation totale et irréversible. Notre analyse se consacrera principalement à la dimension environnementale de la soutenabilité et sur sa compatibilité avec un certain niveau d'activité économique. Comme souvent en économie de l'environnement, la dimension sociale de la soutenabilité, notamment la question des inégalités intragénérationnelles, ne sera pas explicitement abordée. Loin de vouloir minimiser l'importance de cet aspect du développement économique, nous nous concentrons ici sur la viabilité environnementale des activités économiques, qui constitue un impératif premier en termes de "survie". Cependant à travers notre réflexion sur les cercles vicieux de dégradation environnementale et notre application de la grille de lecture de la viabilité (Chapitre 4), nous observerons les effets du respect d'un "minimum social" imposé appliqué au nombre d'employés d'un secteur (polluant) ou au revenu individuel dans ce secteur. Circonscrite à la préservation des capacités environnementales, notre approche peut s'apparenter à la conception de la soutenabilité comme conservation d'un capital naturel critique (voir par exemple Brand, 2009 qui fournit des services environnementaux cruciaux[3]).

[1] Le développement durable doit *"répondre aux besoins du présent sans compromettre la capacité des générations futures de satisfaire les leurs"*.

[2] On peut arguer que de toute façon l'horizon économique limité des agents ne permet pas d'inclure la préférence pour la soutenabilité. Attendre que les comportements individuels révèlent une telle préférence revient à choisir délibérément de ne pas agir ou d'attendre qu'il soit trop tard pour le faire.

[3] Nous qualifierons indistinctement de fonction environnementale ou de service environnemental le concept d' "ecosystem service" largement diffusé par des auteurs comme Daily (1997), et que l'on retrouve au coeur de certaines démarches comme le Millennium Ecosystem Assessment de l'Organisation des Nations Unies (2005). Cette notion recouvre des services aussi divers que la fertilité des sols, l'assimilation de la pollution, la conservation de la biodiversité, etc. (voir Daily, 1997).

Notre perspective rejoint également la conception de la soutenabilité développée par Hueting and Reijnders (1998) qui consiste à utiliser les fonctions vitales de notre environnement biophysique de manière à ce qu'elles restent indéfiniment disponibles. Nous reviendrons sur cette définition lorsque nous aborderons explicitement le capital naturel dans le Chapitre 5. Il n'est pas question ici de proposer une conservation intégrale de la nature sous toutes ses formes, des populations de poissons aux paysages en passant par l'air pur, mais d'identifier certaines situations (et non pas une catégorie immuable d'actifs) dans lesquelles la destruction totale et irréversible d'un actif naturel doit être évitée. Pour s'inscrire dans la soutenabilité, l'actif en question peut subir une dégradation "optimale" d'un point de vue économique, mais ne peut être mené à l'extinction en temps fini.

La définition standard de la soutenabilité en termes de capital naturel critique soulève un scepticisme légitime quand il s'agit de déterminer une fois pour toutes et à une échelle agrégée quel capital naturel est critique, à partir de quel seuil et qui en décide. Néanmoins, nous pensons qu'elle offre un cadre d'action raisonnable dès lors qu'elle est appliquée à une échelle locale ou sur un problème global aux contours écologiques bien définis comme l'accumulation des gaz à effet de serre, susceptible de provoquer un changement climatique majeur à l'échelle de la planète. Ainsi, d'un point de vue prosaïque, sans se laisser perturber par les débats formels sur la substituabilité des facteurs de production ni par les anticipations optimistes ou pessimistes sur le progrès technique, il nous semble possible d'affirmer qu'une trajectoire économique qui mène, en temps fini, à l'extinction totale de la capacité d'assimilation naturelle en CO_2 n'est pas soutenable.

Ce caractère critique est bien évidemment contingent des conditions environnementales et économiques locales. Cette dimension critique dépend en effet largement du degré de développement de l'économie. Si cette dernière dispose d'un capital technologique conséquent, elle pourra se passer plus facilement d'une fonction environnementale comme la purification de l'eau ou la lutte contre l'érosion. En revanche pour une économie peu dotée en capital physique et humain, la disparition irréversible d'un service environnemental de ce type est beaucoup plus problématique. Elle peut avoir de graves conséquences sur le bien-être humain et piéger l'économie dans une trappe à pauvreté. Dans de telles conditions, un service environnemental comme l'assimilation de la pollution peut être qualifié de critique. Pour un certain nombre d'économies peu développées, en particulier à une échelle infranationale, les questions de substitution du capital naturel par du capital technique dans une optique de soutenabilité faible se posent avec moins d'acuité que la prévention d'une extinction irréversible d'une capacité environnementale dont dépend toute une communauté. Dans ce cas, la préservation de cette capacité permet d'éviter les cercles vicieux de dégradation environnementale analysés plus loin dans cette introduction. Sur cette base, il nous semble ainsi légitime de travailler d'abord en supposant l'absence d'un progrès technique qui viendrait peu à peu diminuer la pression sur les ressources naturelles, car les contextes qui demandent de manière critique des politiques de gestion soutenable sont généralement les moins susceptibles d'en bénéficier[1] rapidement.

Par cette approche *ad hoc* plus incarnée nous retrouvons la dimension "développement" du développement durable dont les priorités critiques ne sont pas forcément les mêmes dans toutes les nations. La marge de manoeuvre laissée dans la détermination du caractère critique d'un actif naturel demande bien sûr une certaine confiance dans le mode de gouvernance local mais nous ne voyons pas pourquoi il incomberait à la science économique et à elle seule d'en juger. Nous reviendrons tout au long de notre analyse sur le bien-fondé d'inscrire les raisonnements économiques dans un jeu de contraintes. En conclusion le critère de soutenabilité *a minima* que nous défendons gagne en application opérationnelle ce qu'il perd en universalité et en exhaustivité, deux objectifs qui nous semblent souvent trop ambitieux par rapport à l'urgence de soutenabilité.

Pollution optimale et préservation de la capacité d'assimilation

Parmi les fonctions environnementales dont le caractère critique peut être raisonnablement établi dans certaines situations, nous retiendrons exclusivement la capacité d'assimilation de la pollution par les écosystèmes qui est l'une des plus notables. C'est à elle que ce travail de thèse est consacré. Intrigué par les conclusions pessimistes de Pearce (1976) et de Godard (2005) sur la capacité de l'analyse coût-bénéfice à prendre en compte ou respecter implicitement une condition de soutenabilité *a minima*, nous avons exploré le panorama de la théorie du contrôle optimal de la

[1] Il faut noter qu'un progrès technique mal orienté ou inadapté aux conditions locales peut aussi précipiter la détérioration des capacités environnementales. Ainsi la modernisation des flottes de pêche ou l'instauration de pratiques agricoles intensives ont contribué à accélérer respectivement la dégradation des réserves halieutiques et l'épuisement des sols dans des pays en développement.

pollution et d'autres cadres d'analyse économique. Certains auteurs partisans d'une soutenabilité forte comme Daly (1990) fixent comme niveau de pollution soutenable une quantité d'émissions qui ne dépasse pas le potentiel d'assimilation naturelle. Sans déflorer nos résultats, nous pouvons dès maintenant opposer notre condition de soutenabilité *a minima* à la contrainte *ex ante* de Daly qui fait abstraction de tout arbitrage économique et vide de son sens l'analyse économique de la pollution. Afin d'éviter l'extinction de la capacité d'assimilation, limiter les émissions polluantes au niveau assimilable par l'environnement devient nécessaire à un point d'équilibre stationnaire mais ne doit pas pour autant être imposé initialement. Une période d'excès de pollution, accompagnée de la dégradation de la capacité d'assimilation correspondante, peut précéder un équilibre respectant durablement la capacité d'assimilation restante. Symétriquement, il peut être économiquement efficace de mener une politique, forcément coûteuse, de restauration de la capacité d'assimilation afin d'augmenter le potentiel d'émissions futures.

Un des apports de notre travail consiste à accorder à la capacité d'assimilation le statut d'une variable autonome, qui suit une dynamique propre. Ce faisant, la modélisation que nous avons élaborée a dû en passer par certaines simplifications. Ainsi, cette capacité d'assimilation, qui est avant tout une fonction environnementale, a été traitée comme un "stock". De plus ce stock a été supposé se dégrader ou se restaurer de manière continue et déterministe. Si ces simplifications sont indispensables pour mener à bien notre analyse, et rejoignent en cela la plupart des modèles stylisés de ce type, elles ne sont toutefois pas anodines. La représentation sous forme de stock demeure largement discutable dans la mesure où la fonction d'assimilation n'est pas une quantité donnée existant en soi mais le résultat d'un équilibre écosystémique complexe. Il convient de souligner également que l'hypothèse de dégradation continue ignore des effets de seuil qui sont susceptibles de se produire dans ce genre de dynamiques écologiques (Crépin, 2007), mais nous verrons que nos illustrations empiriques peuvent échapper dans une certaine mesure à ces phénomènes. Enfin raisonner dans un cadre déterministe dès lors que des dynamiques écologiques encore très mal connues sont en jeu ne saurait être entièrement satisfaisant. Notre travail trouvera donc une extension future particulièrement importante dans l'introduction de dynamiques stochastiques plus à même de rendre compte de leur incertitude intrinsèque. Ce raccourci de modélisation ne devrait que nous inciter à prendre encore plus de précautions dans l'interprétation des résultats et leur transcription en politique économique.

Evaluer la soutenabilité des recommandations économiques

Les trajectoires optimales déterminées par les modèles économiques ont vocation à calibrer des instruments de politiques économiques capables de décentraliser l'optimum social. Cependant ce n'est pas le propos de cette thèse que d'entrer dans le débat nourri sur le choix de ces instruments.

Contribution de notre modélisation économique

Si l'objectif de ce travail est avant tout de participer à la vaste réflexion sur l'introduction de critères de soutenabilité dans l'analyse économique et de tester la compatibilité des recommandations économiques avec les exigences de soutenabilité, la contribution originale des modèles proposés réside d'abord dans la représentation explicite de la capacité d'assimilation de la pollution et de son évolution. Afin de mettre en évidence cet apport, il est utile de rappeler la manière dont le rôle de la capacité d'assimilation est habituellement traité dans les problèmes d'accumulation de la pollution. Si l'on note $Z(t)$, $y(t)$ et $\alpha(t)$ respectivement le stock de pollution accumulée, le niveau brut d'émissions polluantes et le taux d'assimilation à l'instant t, on peut écrire :

$$\dot{Z}(t) = y(t) - \alpha(t)Z(t) \tag{1}$$

Le stock de pollution $Z(t)$ a un impact négatif sur le bien-être de la société par le biais d'une fonction de dommage socio-environnemental $D(Z(t)$, alors que le niveau des émissions polluantes $y(t)$, expression d'un certain niveau de production, contribue indirectement à un bénéfice privé $f(y(t))$. Les modèles de contrôle optimal de pollution cherchent à établir des trajectoires qui maximisent le bénéfice social net du dommage environnemental tout en tenant compte de la dynamique environnementale (1). Dans le cas d'un problème de flux, le bénéfice est toujours lié au niveau d'émission $y(t)$ et le dommage dépend également de $y(t)$. En revanche, aucune capacité d'assimilation n'est prise en compte dans les modèles de flux standard.

Nous nous attachons dans cette section à mettre en évidence l'originalité de notre modélisation et son double apport à l'égard de la branche de la littérature dans laquelle elle s'inscrit. Il ne s'agit pas ici d'effectuer une revue de littérature car nous reviendrons dans chaque chapitre en détail sur le positionnement de nos modèles vis-à-vis des analyses antérieures, tant pour ce qui concerne les options de modélisation retenues que pour les résultats obtenus.

En premier lieu, notre analyse permet un découplage entre niveau de pollution et niveau de capacité d'assimilation, ce qui permet d'aller au-delà de la relation bijective simpliste qui prédomine dans les autres modèles et donne une dimension dynamique au contrôle de la pollution de flux. En second lieu, elle offre la possibilité d'introduire la restauration de la capacité d'assimilation comme variable d'ajustement supplémentaire. Enfin elle propose une vision élargie et surtout dynamique des problèmes de pollution de flux systématiquement traités dans un cadre statique. Pour des raisons essentiellement techniques, une large partie de notre analyse sera fondée sur le modèle de flux mais nous proposerons néanmoins de partager des intuitions en termes de stock. L'idée directrice d'une capacité d'assimilation autonome demeurera ainsi au coeur de notre analyse.

Le découplage niveau de capacité d'assimilation-niveau de pollution

Les limites de la modélisation standard

Les modèles les plus standards de contrôle de pollution font systématiquement l'hypothèse d'un taux d'assimilation constant α de la pollution par l'environnement. Cela se traduit formellement par l'équation de mouvement du stock de pollution (1) comme le montre la Figure (i) dans laquelle le stock de pollution est noté S.

Cette simplification, rapidement critiquée dans la littérature (Forster, 1975), n'en demeure pas moins, pour des raisons d'habitude et de "confort technique", la formalisation de référence dans les modèles théoriques depuis les contributions séminales de Keeler et al. (1972) jusqu'aux publications les plus récentes (par exemple Schubert, 2008). Elle ne reflète pourtant qu'une catégorie très spécifique de pollution de stock qui se décompose à un taux constant, principalement la pollution radioactive. En présence d'un taux d'assimilation constant α, une augmentation du stock de pollution $Z(t)$ se traduit par une augmentation de la capacité d'assimilation "totale" que nous noterons A telle que $A(t) = \alpha Z(t)$. Cette amélioration de la capacité d'un écosystème à absorber des émissions polluantes suite à une accumulation de ce polluant va à l'encontre de l'intuition et des observations écologiques empiriques sur la majorité des problèmes liés à l'accumulation de pollution.

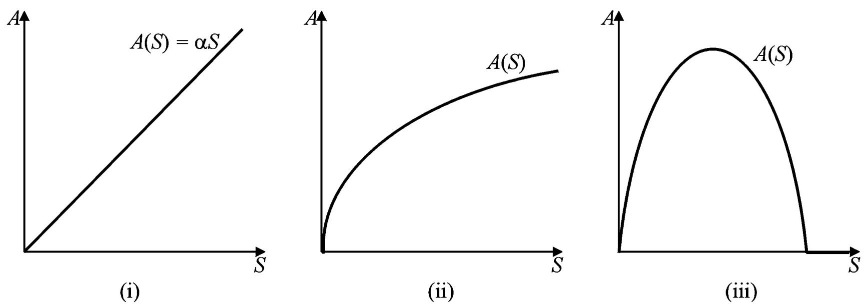

FIG. 1 – Exemples de capacité d'assimilation (Hediger, 2009)

Les fonctions d'assimilation alternatives

Face aux limites de la représentation linéaire de la capacité d'assimilation exposées ci-dessus, des propositions de modélisation alternatives ont été avancées. Afin de refléter plus fidèlement les processus écologiques à l'oeuvre, le taux d'assimilation $\alpha(t)$ n'est plus constant mais dépend directement (et exclusivement) du niveau de pollution $Z(t)$. Deux types de fonction d'assimilation plus sophistiqués ont été élaborés[1].

Fonction décroissante

Une première alternative consiste à faire décroître le taux d'assimilation quand le stock de pollution accumulée croît. Ainsi on a

$$\alpha(t) = \alpha(Z(t))$$
$$\frac{d\alpha(Z)}{dZ} < 0$$

Dans une certaine mesure cette modélisation, illustrée dans la Figure (ii), peut refléter la dissipation atmosphérique des gaz à effet de serre présents dans l'atmosphère, qui se fait à un taux décroissant.

[1]Nous récapitulons en détail dans le Chapitre 2 les contributions significatives sur ce point.

Fonction en U-inversé

La deuxième proposition alternative consiste en une fonction en U inversé qui permet de rendre compte du comportement particulier de certains écosystèmes dont la capacité d'assimilation est d'abord stimulée par l'accumulation de pollution avant de se réduire jusqu'à être irréversiblement détruite. Cette spécification illustre particulièrement bien la réaction des lacs menacés d'eutrophisation. Dans ce cas là il existe un seuil de pollution $\bar{Z}$ tel que

$$\begin{aligned}
&\alpha(t) = \alpha(Z(t)) \\
&\frac{d^2\alpha(Z)}{d^2Z} \leq 0 \;\; \forall Z \in [0, \bar{z}] \\
&\exists\, \hat{Z} \text{ s.t. } \frac{d\alpha(\hat{Z})}{Z} = 0 \\
&\alpha(Z) = 0 \;\; \forall Z \in [\bar{Z}, \infty[
\end{aligned}$$

Cette fonction d'assimilation est représentée sur la Figure (iii).

Les conséquences sur les trajectoires optimales de pollution de ces amendements de la fonction standard d'assimilation sont détaillées dans le Chapitre 2. D'une part les niveaux optimaux d'émissions doivent être plus bas, comme cela se comprend intuitivement dès lors que la société ne bénéficie plus d'une augmentation "gratuite" de sa capacité d'assimilation. D'autre part, selon la forme des fonctions de bénéfice et de dommage et le taux d'actualisation, de multiples situations d'équilibre peuvent survenir. Dans ces équilibres il convient alors de distinguer des états "soutenables" et des états non soutenables selon que la capacité d'assimilation ait été au moins partiellement préservée ou complètement détruite.

Pallier les défauts de la modélisation préexistante avec un modèle original de découplage de la capacité d'assimilation

Ces fonctions d'assimilation alternatives, utilisées à la fois dans des modèles de contrôle de pollution (Tahvonen et Salo, 1996 ; Tahvonen et Withagen, 1996) mais aussi dans des modèles de croissance avec accumulation de capital (Forster, 1975 ; Chevé, 2000 ; Hediger, 2009), ont le mérite de mieux refléter l'évolution de la capacité d'assimilation en évitant notamment une évolution trop "optimiste" de cette dernière.

Néanmoins, la modélisation qui consiste à faire dépendre directement la capacité d'assimilation courante du niveau courant de pollution n'est pas sans poser des problèmes conceptuels que notre modèle se propose justement de surmonter. En effet, cette "bijection" systématique entre le taux d'assimilation et le stock de pollution peut donner lieu à des situations paradoxales.

Un postulat de réversibilité partielle

En premier lieu, elle suppose, du moins tant que le stock de pollution $Z(t)$ est inférieur au seuil de dégradation irréversible $\bar{Z}$, qu'il est toujours possible de retrouver un taux d'assimilation élevé si l'on réduit le stock de pollution d'une manière ou d'une autre. Ce premier postulat de réversibilité partielle de la dégradation de la capacité d'assimilation peut être largement contesté d'un point de vue écologique, notamment en ce qui concerne l'accumulation des gaz à effet de serre et l'inertie des phénomènes qu'elle déclenche.

Un taux d'assimilation "anhistorique"

En second lieu, cette formalisation associe systématiquement à un niveau de pollution donné Z_d le même niveau d'assimilation $\alpha(Z_d)$. Cela signifie que ce taux d'assimilation sera identique que l'on arrive au stock Z_d "par le bas" ou "par le haut". Une économie ayant conservé pendant une longue période L un stock de pollution faible qui atteindrait Z_d suite à une hausse très récente de ses émissions se voit donc dotée de la même capacité d'assimilation qu'une économie qui est caractérisée tout au long de cette longue période de temps L par un stock élevé du même polluant et qui le réduit soudain jusqu'à Z_d. Il apparaît clairement que pour une durée L significative, cette identification n'est pas tenable. L' "historique" de pollution a nécessairement des conséquences sur le niveau d'assimilation disponible[1], pour un même niveau courant de pollution accumulée Z_d.

Des conditions initiales "couplées"

Enfin cette association systématique se traduit en termes de conditions initiales par un couplage systématique d'un niveau de pollution Z_0 avec le même niveau d'assimilation α_0. Cette propriété est bien entendu problématique dans la mesure où elle ne permet pas d'envisager des divergences locales selon les écosystèmes concernés qui

[1]Tahvonen (2000) introduit cette historicité dans la fonction de dommage environnemental, qui ne dépend plus uniquement du stock de pollution courant mais aussi de la vitesse d'accumulation de ce stock mais ne discute pas d'éventuels effets sur la capacité d'assimilation.

peuvent offrir des niveaux d'assimilation différenciés pour un même stock de pollution "initiale"[1].

La capacité d'assimilation comme variable autonome

La proposition de modélisation qui anime notre travail permet de pallier efficacement ces insuffisances. Directement inspiré des intuitions de Pearce (1976), notre modèle se distingue de la littérature préexistante en donnant à la capacité d'assimilation le statut d'une variable d'état à part entière telle que :

$$\alpha = \alpha(t)$$
$$\dot{\alpha}(t) = -g(Z(t))$$

Différentes formes spécifiques de la fonction g seront explorées dans la suite de ce travail mais l'idée directrice est que cette fonction est strictement positive au-delà d'un certain seuil de pollution accumulée.

Dans notre modèle, le taux d'assimilation suit sa propre dynamique et évolue sur une trajectoire découplée de celle du niveau de pollution. Cette dynamique est bien entendu déterminée par le niveau de pollution ambiant, mais le niveau absolu d'assimilation à un instant t, $\alpha(t)$ dépend de sa propre trajectoire. En particulier, une réduction du stock de pollution qui ferait suite à une longue période de hauts niveaux de pollution peut ne pas suffire à éviter la disparition irréversible de cette capacité d'assimilation. De plus, cette spécification permet d'envisager une grande variété de conditions initiales car les couples (α_0, Z_0) ne sont plus contraints par la bijection entre α et Z.

Nous qualifierons ce modèle de modèle Pezzey-Pearce dans la mesure où Pezzey (1996) a été le premier, et le seul à ce jour, à chercher à formaliser les intuitions de Pearce dans un cadre d'optimisation dynamique rigoureux. L'analyse de Pezzey, limitée à un *working paper*, n'a cependant pas été menée jusqu'au bout comme nous l'expliquons dans le Chapitre 2. Par le statut de variable d'état autonome qu'il attribue à la capacité d'assimilation, notre modèle permet donc une étude des

[1]Cette propriété n'est en fait qu'une extension de l'anhistoricité du taux d'assimilation soulignée plus haut dans la mesure où le concept de "condition initiale" dans les modèles économiques dépend du moment où l'on démarre le processus d'optimisation, et n'a donc rien en commun avec des conditions écologiques initiales.

problèmes de pollution dans lesquels cette capacité d'assimilation est découplée, mais pas indépendante, du niveau de pollution accumulée. Cette configuration se révèle particulièrement adaptée à l'étude de l'assimilation de CO_2 par la biosphère (océans, forêts) dans le contexte du changement climatique. Si les modèles précédents pouvaient rendre compte de la dégradation atmosphérique d'une partie des gaz à effet de serre accumulés, notre modèle est le plus à même de mettre en évidence le rôle et l'évolution de l'assimilation de la biosphère qui est dégradée par des *feedbacks* climatiques ou écologiques initiés par un excès de CO_2.

La restauration de la capacité d'assimilation

Le deuxième apport de notre modèle tient à l'autonomie de la capacité d'assimilation traitée comme une variable d'état à part entière. Celle-ci permet en effet d'ajouter la restauration de la capacité d'assimilation à la palette de leviers d'action à la disposition de la société. Celle-ci n'est plus limitée à la quantité de pollution/production comme variable de contrôle mais peut “investir” dans cet actif dont le flux de service environnemental périodique sera augmenté de manière durable. Comme nous le détaillons dans la première partie, il existe en effet de nombreuses formes de restauration *naturelle* de la capacité d'assimilation qui peut être augmentée, ou dont la dégradation suite à des excès de pollution peut être compensée. L'afforestation et la reforestation constituent des exemples de restauration de la capacité d'assimilation dans le cas du CO_2. Etant donné la grande incertitude qui entoure ces “suggestions” nous avons choisi de ne pas inclure dans notre analyse les formes de restauration *artificielle* de la capacité d'assimilation qui sont rassemblées sous le qualificatif de “geo-engineering”. Ainsi, les récentes propositions de fertilisation des océans avec du sulfate de fer, censées stimuler le développement du phytoplancton, un important puits de CO_2, suscitent aujourd'hui moins d'enthousiasme que d'inquiétudes par rapport aux perturbations de l'équilibre des écosystèmes marins[1].

D'une manière plus générale, cette introduction de la restauration enrichit l'analyse des problèmes de pollution optimale en introduisant des arbitrages économiques intertemporels qui se présentent à une société entre le bénéfice immédiat des activités polluantes et les retombées futures d'une capacité d'assimilation restaurée. Au-delà

[1]Les premiers résultats de l'expérience de fertilisation menée en janvier 2009 par le consortium LOHAFEX indiquent l'échec de ce procédé.

des seuls problèmes de pollution, elle ouvre la voie à une réflexion formelle sur les opportunités d'investir dans le capital naturel quand ce dernier se dégrade proportionnellement à l'intensité de son utilisation. Nous approfondirons ainsi la question de la maintenance du capital naturel dans le Chapitre 2 et le Chapitre 5 de ce travail.

Penser la pollution de flux dans un cadre dynamique

Le dernier apport majeur de notre spécification originale de la capacité d'assimilation comme une variable d'état autonome réside dans son application aux problèmes de flux. En effet, les problèmes de pollution de flux sont systématiquement traités dans un cadre d'analyse statique (voir la revue de littérature du Chapitre 1). L'optimum constant qui émerge d'une analyse coût-bénéfice statique, souvent utilisé dans les manuels pour illustrer le mécanisme d'internalisation des effets externes, repose sur le postulat non trivial que les conditions environnementales qui déterminent à chaque période l'ampleur des dommages environnementaux (la forme de la fonction $D(p)$) sont maintenues intactes d'une période sur l'autre. La reconduction du même niveau optimal de pollution suppose implicitement la reproduction des conditions environnementales (Godard, 2006). Or c'est justement tout l'objet de notre étude d'expliciter les mécanismes de dégradation environnementale sous-jacents à un problème d'internalisation d'externalités de ce type. C'est pourquoi notre première mise en pratique des intuitions de Pearce concerne les problèmes de pollution de flux, certes moins cruciaux à l'échelle planétaire que les problèmes de pollution de stock, mais qui peuvent gravement entraver le développement soutenable à l'échelle locale. D'ailleurs, bien qu'il prétendent couvrir aussi bien les problèmes de flux que de stock, le modèle statique répété de Pearce lui-même (1976) traite en fait implicitement des problèmes de flux car sa fonction de dommage dépend du niveau d'émissions courant et non d'un stock de pollution. En développant cet argument dans un cadre d'optimisation dynamique rigoureux, notre modèle nous permet d'analyser les conséquences écologiques intertemporelles de la pollution de flux quand celle-ci est en partie absorbée par un écosystème local. Notre formalisation dans le contexte de pollution de flux repose sur une capacité d'assimilation totale (et non plus sur un taux d'assimilation), qui suit sa propre dynamique selon les excès de pollution que reçoit l'écosystème. Ainsi le dommage environnemental (de flux) ne dépend plus exclusivement du niveau brut d'émissions mais de l'éventuel excès d'émissions par rapport à la capacité d'assimilation. Comme dans le cas de la pollution de stock, cette formalisation permet également d'introduire la possibilité de restaurer la capacité d'assimilation à l'oeuvre. Comme nous le montrons dans le Chapitre 1, elle se révèle tout à fait adéquate pour étudier de manière plus exhaustive certains problèmes concrets de pollution de flux comme la contamination des cours d'eau par les nitrates d'origine agricole.

Les cercles vicieux de dégradation environnementale

La mise en évidence par Pearce (1976) d'un mécanisme de dégradation écologique que nous qualifions de "cercles vicieux de dégradation environnementale"[1] a été determinante dans la genèse de notre travail. L'intuition de son modèle séquentiel peut être utilement étendue à un modèle dynamique ainsi qu'à d'autres fonctions environnementales pour montrer la nécessité d'intégrer exhaustivement les dynamiques écologiques aux modèles économiques.

L'illustration de Pearce : la dégradation de la capacité d'assimilation

Comme le montrent Pearce (1976) et Godard (2006), une politique de régulation environnementale myope fondée sur l'internalisation statique des effets externes de la pollution, qu'elle soit traduite opérationnellement par une taxe pigouvienne ou un quota, donne lieu à un processus de dégradation de la capacité d'assimilation qui s'accélère jusqu'à sa disparition totale. Ce cycle de dégradation est illustré dans la Figure 2 et peut se décrire de manière très simple. Supposons qu'un niveau "optimal" de pollution soit fixé par optimisation statique et reconduit à chaque période. Si ce niveau optimal est en deçà de la capacité d'assimilation, aucune dégradation écologique ne se produit et la configuration économique et écologique peut effectivement se reproduire indéfiniment à chaque période dans les mêmes conditions. En revanche, si cet optimum de pollution constant excède la capacité d'assimilation[2] alors cette dernière sera dégradée. A la période suivante, la différence entre le niveau d'émissions, toujours identique, et la capacité d'assimilation disponible, réduite, sera plus grand encore et la capacité d'assimilation sera à nouveau dégradée, mais dans des proportions plus importantes. On retrouve cette dégradation de la capacité d'assimi-

[1] Nous utiliserons le terme de "environmental degradation cycles" dans le reste de ce travail.
[2] Ce qui, par construction, est systématiquement le cas dans le modèle de Pearce.

lation dans le déplacement vers le bas des lignes horizontales A_1, A_2, etc. de la partie supérieure du diagramme (2) suite à un niveau de pollution en excès X_1^*, X_2^*, etc. dans la partie inférieure. Ce processus provoque donc une dégradation exponentielle de la capacité d'assimilation qui mène rapidement à sa destruction totale. Conformément à notre définition d'une soutenabilité environnementale *a minima*, une telle trajectoire est bien entendu non durable.

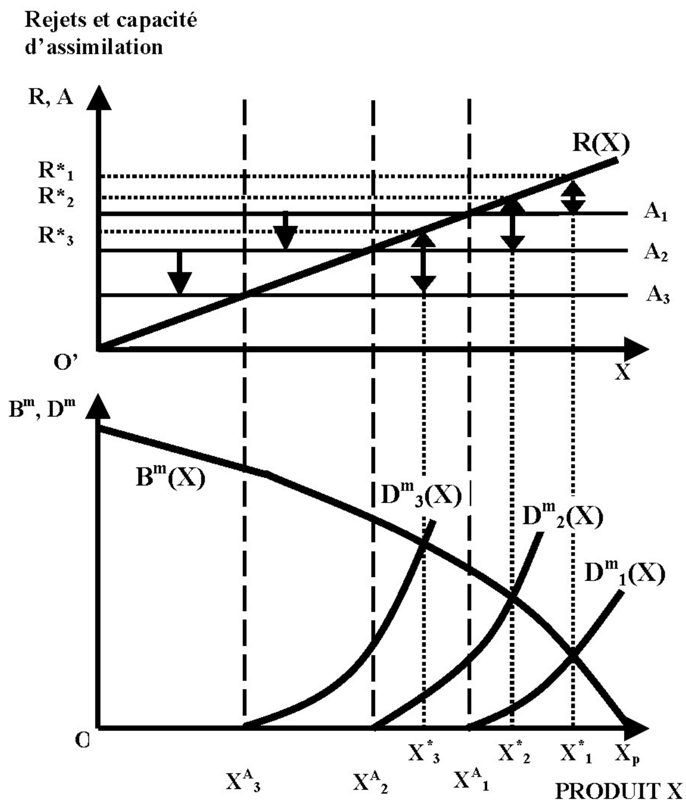

Fig. 2 – Cercle vicieux de dégradation environnementale (Godard, 2006)

Contraintes sociales, développement et dégradation de la capacité d'assimilation

Le résultat "insoutenable" dégagé par Pearce et étendu par Godard n'est plus systématique dès lors que l'on se place dans un cadre d'optimisation véritablement dynamique qui prend en compte les dynamiques écologiques en jeu, comme nous le ferons dans la première partie de cette thèse. Cependant, hors d'un cadre d'optimisation, il convient de garder à l'esprit qu'un niveau constant d'émissions peut être imposé par les pouvoirs publics pour satisfaire des contraintes sociales, comme un niveau minimum d'activité sectorielle ou de revenu assuré aux producteurs-pollueurs[1]. Si ce niveau minimum de production, donc de pollution, autorisé sur la base d'exigences sociales ou même électoralistes, est supérieur à la capacité d'assimilation, le mécanisme de dégradation accélérée explicité plus haut se produira également. Le péril d'un tel cercle vicieux de dégradation de la capacité d'assimilation prend toute son acuité si l'on considère les problématiques des pays en développement. En effet, quand les conditions de production économique (accumulation de capital, progrès technique) ne permettent pas de garantir la survie des citoyens en respectant la capacité d'assimilation de la pollution, ce type de cycle de dégradation est susceptible non seulement de mener rapidement à la destruction de ce service environnemental mais, plus grave encore, d'installer la société dans une trappe environnementale qui combine faibles revenus et dommages environnementaux importants, notamment en termes de santé.

Extension à l'ensemble des fonctions environnementales : le capital naturel en péril

A l'aune de notre critère de soutenabilité *a minima*, la destruction accélérée de la capacité d'assimilation peut être interprétée comme une trajectoire véritablement non soutenable en tant que disparition de capacités environnementales difficilement substituables. Mais ce mécanisme de dégradation accélérée n'est pas restreint à la seule capacité d'assimilation de la pollution. Il concerne en effet de nombreuses fonctions environnementales comme l'usage des sols. De trop nombreuses catastrophes environnementales et humaines illustrent les conséquences d'une exploitation trop intensive de la capacité de charge. C'est le cas par exemple de la désertification du Sahel qui a fait suite à un changement de régime d'exploitation des sols entre agriculture et élevage. On trouve notamment chez Hardin (1977) une étude détaillée de ce mécanisme de dégradation environnementale et de ses impacts sur la situation économique des populations locales. De nombreux exemples de ce type[2] émaillent la littérature de l'économie de l'environnement et de l'économie du développement. Dans la mesure où les ressources naturelles et environnementales peuvent jouer un rôle majeur dans l'activité productive d'un pays, leur dégradation sous l'effet d'une exploitation trop

[1]La compatibilité entre des objectifs sociaux, économiques et environnementaux sera étudiée plus en détail dans le Chapitre 4 à l'aide du cadre d'analyse de la viabilité.

[2]Barbier (1989) analyse les effets pervers d'une agriculture trop intensive en Indonésie.

intensive peut mettre en péril son développement. Ainsi si l'on raisonne en termes de formes de capital, ces cycles de dégradation environnementale sont déclenchés par une utilisation abusive du capital naturel sous ses différentes formes (assimilation de la pollution, fertilité des sols, etc.). Cette surexploitation du capital naturel est elle-même due aux insuffisances du capital physique et du capital humain. La réduction du capital naturel qui s'ensuit ne fait qu'aggraver la situation générale du pays ou de la région concernés. Ce phénomène a été représenté de manière très illustrative par Giraud et Loyer (2006) sous la forme d'un modèle de développement *aux élastiques*. Ce modèle, décrit dans la Figure 3, permet ainsi de distinguer les enchaînements qui mènent à la perte irréversible de certaines fonctions environnementales et à l'installation durable du pays dans une trappe à pauvreté. La problématique de la soutenabilité dépasse ici clairement le strict cadre de la soutenabilité environnementale : c'est de la survie même des populations qu'il s'agit. A travers le prisme des enjeux du développement de pays encore peu industrialisés, notre raisonnement en termes de préservation des capacités prend donc un sens plus ample. Nous reviendrons sur le statut particulier du capital naturel et sur la soutenabilité des stratégies de développement des pays peu industrialisés dans le Chapitre 5.

L'exploration de trajectoires soutenables à travers le prisme théorique des modèles économiques : les moments de la réflexion

L'analyse développée dans la présente thèse s'articule en trois parties complémentaires. La première partie propose d'étudier la modification des trajectoires optimales de pollution dans un cadre utilitariste actualisé suite à une introduction explicite d'une capacité d'assimilation dynamique et de discuter leur compatibilité avec la définition *a minima* de la soutenabilité présentée précédemment. Elle se compose de deux chapitres traitant respectivement de la pollution de flux et de la pollution de stock. Partant des insuffisances de l'optimisation dynamique actualisée standard mises en lumière dans la première partie, la deuxième partie s'attache à explorer d'autres critères économiques et d'autres méthodes d'analyse. Ainsi le Chapitre 3 examine la capacité de critères d'optimisation alternatifs comme la Règle d'Or Verte ou le Maximin à produire des trajectoires de pollution qui respectent une soutenabilité minimum. Des méthodes de substitution à l'analyse coût-bénéfice standard comme

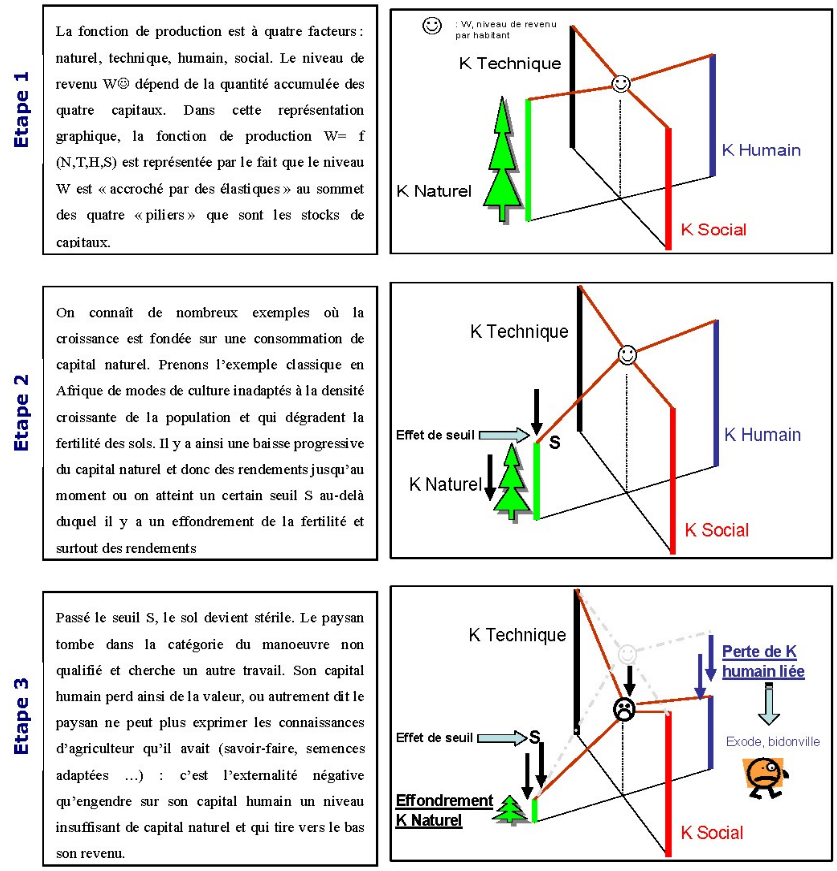

FIG. 3 – Modèle *aux élastiques* (Giraud et Loyer, 2006)

l'analyse coût-efficacité ou l'analyse séquentielle sont également considérées. Le Chapitre 4 prolonge cette exploration en approfondissant l'analogie entre les problèmes de pollution et les problèmes de gestion de ressources renouvelables. Ce changement de perspective permet notamment l'application originale du critère de viabilité aux problèmes de pollution. Enfin la troisième partie consiste en un chapitre conclusif qui cristallise les réflexions soulevées sur l'exploitation soutenable des actifs naturels dans un modèle de croissance avec capital physique et capital naturel.

Chapitre 1 : contrôle optimal de la pollution de flux avec capacité d'assimilation

Dans un premier temps, nous amendons le modèle standard de contrôle optimal de la pollution de flux (analyse coût-bénéfice actualisée) en intégrant les variations de la capacité d'assimilation induites par des excès de pollution, conformément aux intuitions de Pearce (1976) développées par Godard (2006). A l'optimum de pollution statique standard des problèmes de flux, dit optimum de Turvey, nous substituons une trajectoire optimale de pollution plus stricte le long de laquelle l'économie peut être guidée grâce à un système de prix implicites adaptés. Concrètement ces prix implicites peuvent s'incarner dans une taxe pigouvienne dynamique pour réguler les problèmes de pollution locale qui correspondent à cette configuration théorique, comme la contamination des cours d'eau par les nitrates d'origine agricole et le rôle de la capacité d'assimilation des écosystèmes ripariens. Cette proposition originale de modélisation dynamique des problèmes de pollution de flux permet de mettre en évidence l'évolution des conditions environnementales qui sont encore trop souvent considérées comme immuables dans les modèles économiques. Nous discutons de la soutenabilité de ces trajectoires optimales à l'aune de notre critère de préservation des capacités et mettons en évidence le rôle bien connu du taux d'actualisation mais aussi des conditions environnementales initiales. Enfin notre modèle offre la possibilité à la société de restaurer la capacité d'assimilation à l'oeuvre, ce qui permet à la fois d'élargir les conditions initiales soutenables pour des taux d'actualisation suffisamment bas, mais aussi d'étudier les arbitrages intertemporels entre les coûts de restauration, qui correspondent à une forme d'investissement dans le capital naturel, et les bénéfices nets liés à l'activité polluante.

Chapitre 2 : contrôle optimal de la pollution de stock avec capacité d'assimilation

Dans ce chapitre, nous prolongeons notre démarche d'inclusion des variations des conditions environnementales sous-jacentes aux problèmes plus préoccupants de pollution de stock liés à un certain type de capacité d'assimilation[1], bien représentés par l'accumulation des gaz à effet de serre. Nous introduisons dans le modèle standard de contrôle de pollution de stock un seuil de pollution au-delà duquel le taux d'assimilation de la pollution se dégrade. C'est donc l'évolution de la capacité d'assimilation, et non son niveau courant, qui dépend du stock de pollution accumulé. Ce recours à deux variables d'état distinctes distingue notre modèle des spécifications proposées jusque-là dans la littérature en permettant un découplage partiel du stock de pollution et de la capacité d'assimilation. Ce cadre d'analyse est plus à même d'englober

[1] Notre modélisation formelle des dynamiques de la capacité d'assimilation à l'oeuvre ne permet pas de rendre compte de certains problèmes non linéaires avec effets de seuil comme l'eutrophisation des lacs.

un vaste spectre de situations écologiques comme nous l'avons expliqué plus haut. Il rend également possible l'étude de l'arbitrage entre restauration de la capacité d'assimilation et bénéfices économiques polluants qui viendra compléter notre discussion de la soutenabilité des nouvelles trajectoires optimales et des états stationnaires.

Conclusion de la Partie I

Suite à l'amendement des modèles standard de contrôle optimal de la pollution, nous mettons en évidence deux résultats fondamentaux. Le premier, raisonnablement intuitif, veut que lorsque l'on prend en compte la dégradation de la capacité d'assimilation induite par le dépassement de seuils d'émissions ou de seuils de pollution accumulée, les trajectoires optimales de pollution doivent être plus strictes que dans le cas standard. Le prix implicite de la capacité d'assimilation qui émerge de nos processus d'optimisation dynamique permet de calibrer des instruments économiques plus contraignants. Le second résultat concerne la compatibilité de ces trajectoires avec les exigences minimales du développement durable. Nous montrons qu'en plus de l'effet connu du taux d'actualisation social sur le degré de soutenabilité des trajectoires, le niveau initial des conditions environnementales détermine pour une large part l'occurrence d'extinction optimale de la capacité d'assimilation. C'est donc dans la perspective d'explorer des trajectoires dont la soutenabilité est moins contingente à ces conditions initiales que s'inscrit la deuxième partie de notre réflexion.

Chapitre 3 : la soutenabilité hors du cadre de l'analyse coût-bénéfice actualisée

Afin de dépasser l'horizon de l'analyse coût-bénéfice actualisée qui ne satisfait pas systématiquement notre critère *a minima* de soutenabilité en termes de préservation des capacités (environnementales), nous explorons dans ce chapitre des critères alternatifs d'optimisation, tels que la Règle d'Or Verte, le Maximin ou l'Overtaking criterion, ainsi que des méthodes alternatives comme l'analyse coût-efficacité et l'analyse séquentielle. Après avoir rappelé les enjeux de l'équité intertemporelle posés par le recours au taux d'actualisation, nous comparons donc les trajectoires obtenues dans ces différents cadres d'analyse, et testons leur soutenabilité.

Chapitre 4 : la capacité d'assimilation comme ressource renouvelable, application de l'analyse de viabilité

Au-delà des résultats formels, notre travail de recherche vise à contribuer à une meilleure prise en compte des services environnementaux sous-jacents dans l'analyse économique, et en particulier, du service d'assimilation de la pollution fourni par les écosystèmes. Les efforts déployés pour expliciter non seulement le rôle crucial mais également le caractère fini de la ressource environnementale que constitue la capacité d'assimilation incitent ainsi à modifier le prisme de perception économique des problèmes de pollution. A une approche fondée sur l'internalisation d'effets externes supposés réversibles, nous souhaitons substituer un mode de gestion similaire au *management* des ressources renouvelables. L'analogie entre capacité d'assimilation et ressource renouvelable que nous construisons tout au long de notre réflexion nous amène à proposer dans ce chapitre l'application d'outils d'analyse réservés jusqu'ici à l'exploitation des ressources naturelles comme l'analyse de viabilité. En reformulant le problème de pollution dans les termes d'un problème d'exploitation de ressources renouvelables, nous parvenons à étendre notre analyse de soutenabilité à des critères sociaux et économiques.

Conclusion de la Partie II

Les critères explorés dans le Chapitre 3 pour pallier les insuffisances de l'analyse coût-bénéfice actualisée n'offrent pas d'alternative convaincante au cadre standard. Leurs recommandations se rapprochent plus d'une conception "intuitive" d'une situation finale soutenable mais ils ne fournissent pas d'indications assez tranchées sur les trajectoires ou l'ensemble de trajectoires qui peuvent y mener, ni sur les arbitrages intertemporels à opérer. A l'aune de notre analogie approfondie entre capacité d'assimilation et ressource naturelle, nous ouvrons dans le Chapitre 4 des horizons prometteurs pour une gestion soutenable des activités économiques sources de pollution, notamment grâce aux apports de notre modèle de viabilité appliqué à la pollution. Cet instrument a le double mérite de proposer une variété de trajectoires viables au lieu d'une unique solution et, surtout, de libérer l'économie de la contrainte des conditions initiales qui prédéterminent la possibilité d'une conservation des capacités environnementales dans l'analyse coût-bénéfice traditionnelle.

Chapitre 5 : dépréciation endogène du capital naturel et stratégies de développement

Nourri des intuitions et de la vision élargie des problèmes de pollution qui émergent des deux premières parties, le dernier chapitre propose une double contribution sur la notion de capital naturel. D'une part nous redéfinissons à partir des intuitions réunies au long de notre réflexion l'approche "fundist" du concept de capital naturel que nous définissons comme un artefact virtuel dont les variations peuvent s'inférer à partir du niveau des services environnementaux fournis mais qu'il n'y a pas lieu d'observer directement. D'autre part nous étendons notre réflexion sur les capacités aux capacités *économiques* en étudiant un modèle agrégé de croissance optimale avec capital physique et capital naturel. L'introduction du capital physique permet de compléter l'analyse menée en première partie qui n'incluait pas l'accumulation de capital et met en lumière des arbitrages entre les deux facteurs de production. Notre modèle autorise en outre une dépréciation endogène du capital naturel qui dépend du degré d'utilisation de ce capital. Cette spécification originale, inspirée d'une sous-branche de la littérature sur la théorie du capital, permet d'assimiler dans la même dynamique des composantes variées du capital naturel : fertilité des sols, ressources halieutiques, assimilation de la pollution, etc. A travers ce modèle nous observons différentes stratégies *ad hoc* de développement pour les pays à faible capital manufacturé et nous établissons des conditions nécessaires pour que les trajectoires optimales soient également durables.

Part I

Optimal flow and stock pollution control with dynamic assimilative capacity

Chapter 1

Optimal degradation and restoration of assimilative capacity in flow pollution control

1.1 Introduction

The assimilative capacity of an ecosystem receiving pollution can be defined as the ability *"to receive a determined level of residues, to degrade them and to convert them in non-damaging and even beneficial products"* (Pearce and Turner, 1990, p.38)[1]. This environmental sink function is at work in both stock and flow pollution. The assimilation of CO_2 by oceans and forests and the protection of watercourses from lixiviated nutrient flows[2] by riparian buffer zones (Correll, 1996) illustrate these respective cases[3]. The level of assimilative capacity is not constant over time and depends either on the current stock of pollution (the concentration of greenhouse gases in the atmosphere) or on the "history" of pollution flows (periodic emissions of nitrates originating from fertilizers).

[1]A shortened version of this chapter has been published in February 2009 under the title "The shadow price of assimilative capacity in optimal flow pollution control" in *Ecological Economics*, 68(4): 1020-1031.

[2]In this setting, flow damages consist in increased costs of artificial purification for drinking water, health problems, temporary loss of recreational amenities and commercial benefits due to the temporary clogging of estuaries by seaweed.

[3]Noise can be considered as another example of flow pollution involving assimilative capacity. In that case the assimilative capacity at work is the human ability to cope with noise without suffering from stress. This special case is described in Appendix F.

These dynamics are all the more important in flow pollution problems as the level of assimilative capacity reflects the maximum amount of pollution that does not cause any social damage and that does not trigger any permanent alteration of the ecosystem functions. For instance, as long as the flows of lixiviated nitrates remain below the assimilative capacity threshold of riparian buffer ecosystems, no social damage is sustained and this capacity remains unaffected for future use. If the emissions exceed this threshold, not only will there be contamination of the watercourses but the riparian buffers' assimilative capacity will be impaired by temporary nitrogen saturation (Hanson et al., 1994; Fromm, 2000). Therefore a merely static economic analysis of optimal flow pollution control will prove inappropriate when assimilative capacity is involved. Since the pollution optima serve as theoretical landmarks for environmental regulation, an economic instrument such as a pigouvian tax can fail to prevent the extinction of the assimilative capacity if it is not calibrated properly. Indeed, if the static optimal level of pollution exceeds the assimilative capacity, it will cause damage and lower the threshold at which this social damage occurs in the future. At the next period, the same constant amount of pollution will thus be even more in excess of the assimilative capacity and will cause even more social damage and more degradation of assimilative capacity. This vicious cycle, first highlighted by Pearce (1976), can continue until the assimilative capacity is extinguished[1]. That is why it is crucial to carry out the economic analysis of flow pollution with assimilative capacity in an adequate dynamic framework.

The flow pollution control models found in the economic literature are either set in a static framework (Perman et al., 2003, p.171) or they do not allow for actual ecological dynamics (Schou, 2002). Meanwhile, the seminal articles on optimal stock pollution acknowledge the role played by assimilative capacity and its evolution over time (Forster, 1975). A survey of the different representations of assimilative capacity in the literature can be found in Pezzey (1996). Recently some authors such as Cesar and de Zeeuw (1994), Tahvonen and Salo (1996), Tahvonen and Withagen (1996), Toman and Withagen (2000), Chevé (2000), Hediger (2009) and Prieur (2009) have improved the specification of the assimilative capacity in various models of stock pollution control. However these contributions neither address the case of flow pollution nor allow for assimilative capacity restoration. The latter can provide a useful tool to

[1] A similar cycle degrades soil productivity when farmers fail to consider the intertemporal impact of their activity on soil quality (Barbier, 1990).

a society that wishes to offset the degradation of assimilative capacity. For instance, CO_2 assimilation can be increased by afforestation while the assimilative capacity of riparian ecosystems can be restored through expansion and revegetation of buffer strips (Anderson and Ohmart, 1985; Hubbard et al., 1995). Although there exists significant work on environmental quality restoration (Phillips and Zechkauser, 1998; Keohane et al., 2007) little attention has been paid specifically to the restoration of assimilative capacity (d'Arge, 1971; Pearce and Common, 1973) and to our knowledge this policy option has never been represented in a stylized model.

We therefore propose to build an optimal flow pollution control model, based on an intuition of Pearce (1976) extended later by Pezzey (1996) and Godard (2006), that takes into account the role and dynamics of assimilative capacity. We treat this assimilative capacity as an autonomous state variable that follows its own dynamics. This dynamic flow pollution model allows for a more comprehensive view of the economy-ecology interactions at stake and enables us to consider explicitly the option of restoring the assimilative capacity. After specifying in Section 2 our original pollution control model, we characterize in Section 3 the optimal pollution path and compare it to the static optimum. We introduce in Section 4 the possibility of restoring the assimilative capacity and we determine the new optimal path corresponding to this enhanced version of the model. In Section 5 we discuss the policy applications of our set of results. Section 6 provides an illustration of the model with the empirical case of riparian buffer zone protection from agricultural nitrates. Section 7 concludes and points out potential extensions of our model. Our discussion of the optimal paths obtained in a discounted utilitarian framework is stimulated by the increasing contributions from mainstream economics to environmental policy but it is important to remember that other frameworks of decision making can propose a consistent alternative to economic optimization. The policy interpretations of our results are of course bound by this initial postulate on the "necessary optimality" of public policies.

1.2 The modified flow pollution model

1.2.1 Assimilative capacity in flow pollution problems

Let $p(t)$ be the level of emissions of a space-invariant homogeneous pollutant. We note $A(t)$ the assimilative capacity of a local ecosystem at instant t namely the amount of pollution the ecosystem is able to absorb at each moment without suffering any permanent alteration of its internal functioning or causing social damage. This assimilative capacity sets a double threshold in flow pollution problems (see Pearce and Turner, 1990, pp.38-40, for a general exposition of the explicit introduction of assimilative capacity into pollution control models). First, it determines the maximum level of "damage-free" pollution as the assimilative capacity will neutralize a given amount of polluting emissions before they cause harm. The occurrence of socially valued environmental damage, defined afterwards, will thus depend on both $p(t)$ and $A(t)$. Second, it represents the threshold above which the intensity of polluting emissions will alter the internal equilibrium of the ecosystem and thus impair the services it provides. The identification between this two thresholds need not be systematic as the degradation threshold could be a fixed threshold while the damage threshold is necessarily the current assimilative capacity. However it is of significant interest from a methodological point of view to study this "dynamic threshold" mechanism.

By definition we have

$$\begin{cases} A(t) \geq 0 \quad \forall t \geq 0 \\ \\ A(0) = A_0 \end{cases}$$

where A_0 is the initial level of assimilative capacity, supposedly known.

1.2.2 Pollution and private benefit

We adopt a simplified pollution control model without capital accumulation similar to Ulph and Ulph (1994) and Farzin (1996). The polluting activity yields an instantaneous private benefit accounted for by the function f depending on the current level of pollution $p(t)$ with $p(t) \geq 0$ $\forall t \geq 0$. The rationale for this is that pollution can be interpreted as an input in production, and the polluting firm must necessarily increase its polluting emissions if it wants to increase its profit, through either a greater production of goods or a reduction of its pollution control costs.

We work with a very general function characterized by the standard properties of the literature: f positive, non-decreasing, concave, defined over $\mathbb{R}^+$, $f_p \geq 0$, $f_{pp} \leq 0$.

There is no particular need to give an essential dimension to this production, the benefit function should thus not impose an "infinite penalty" on a zero level of production, and therefore we shall reject the Inada conditions (see Heal, 2000, p.37). In particular, if the environmental conditions are such that any strictly positive level of emissions will have negative welfare effects, then the economy will switch to any backstop production solution yielding positive welfare effects.

$$\lim_{p \to 0} f_p(p) < +\infty$$
$$f(0) = 0$$

As we suppose that the polluting firm ignores the externality it imposes on society, its private pollution optimum x_p is such that

$$f_p(x_p) = 0$$

Finally we assume that $A_0 < x_p$. If this condition were not established, then the polluting producer would immediately choose a production-pollution level below the assimilative capacity and no external damage would ever occur.

Our framework does not include the possibility for an exogenous or an endogenous technological change that could modify the private benefit function over time[1]. This somewhat restrictive assumption nevertheless fits empirical applications of the model, as we shall see later.

[1]Technological change would allow the same level of private benefit for a lower volume of pollution.

1.2.3 Environmental damage as a function of pollution and assimilative capacity

Flow damages formally distinguish themselves from stock damages in the sense that if the flow of pollution is nil at time t, the corresponding environmental damage will be nil as well. When there is a neutralizing assimilative capacity at work in the ecosystem, the environmental damage is also nil for any level of pollution below the current assimilative capacity level. Conversely, the higher the excess of pollution vis à vis the assimilative capacity, the higher the environmental damage sustained by society. This mechanism thus implies a damage function depending not only on the level of emissions $p(t)$ but also on the level of assimilative capacity $A(t)$. Since we have stressed that the latter is not constant but can evolve over time, a given amount of pollution that was harmless (harmful) before can become harmful (harmless) as it becomes higher (lower) than the assimilation threshold of the ecosystem. In this configuration, the damage function still reflects flow damages but the ecological features determining the shape of this damage function are not static. Formally this is reflected by a slight modification with respect to the standard models: the damage function depends now on the excess of pollution vis à vis the assimilative capacity and not only on the absolute amount of pollution.

Assumption 1.1. *The occurrence of socioeconomic externalities at a given time depends on the excess of pollution vis à vis the current assimilative capacity.*

If the pollution level is below the assimilative capacity, no negative social externality will occur in the sense that no additional cost is borne by other agents. Assumption 1.1 fits concrete flow pollution problems such as noise[1] or the contamination of surface water by lixiviated nitrates. In that case, the riparian ecosystems provide an assimilative buffer through denitrification processes (Correll, 1996) and the flow damage[2] occurs when this assimilative capacity is exceeded.

Based on these assumptions we can specify the following environmental damage function, measuring the social damage suffered by society when pollution exceeds the

[1]In this case the assimilative capacity at work is the human capacity to cope with noise without suffering from stress.

[2]In this setting, flow damages can consist in increased costs of artificial water-purification for drinking water, health problems and loss of recreational amenities and sea-related commercial benefits due to “green tides” of seaweed temporarily clogging estuaries.

assimilative capacity. We work with a very general damage function D displaying the standard properties found in the literature: D is increasing and convex with respect to the excess of pollution $(p-A)$, $D(p,A)=0\ \forall p \leq A$ and $D(p,A)>0\ \forall p > A$. Given the mechanisms described above, the following properties[1] are straightforward[2]:

$$\begin{aligned} &\forall\ p < A \quad D_p(p,A) = -D_A(p,A) = 0 \\ &\forall\ p \geq A \quad D_p(p,A) = -D_A(p,A) > 0 \end{aligned} \tag{1.1}$$

and

$$\begin{aligned} &\forall\ p \geq A \quad D_{pA}(p,A) = D_{Ap}(p,A) < 0 \\ &\forall\ p \geq A \quad D_{AA}(p,A) = D_{pp}(p,A) > 0 \end{aligned}$$

Relation (1.1) reflects the fact that when the assimilative capacity is strictly respected, an incremental change in the pollution level or in the assimilative capacity level does not trigger an ecological reaction. The function D displays a continuity problem in the neighborhood of $p = A$ but this problem will be dealt with later on. We also make the following reasonable assumption on the behavior of D:

$$\forall (p,A) \ \lim_{A\to 0} D_p(p,A) = \lim_{A \to 0} -D_A(p,A) = L > 0 \tag{1.2}$$

L can be either finite or infinite. This assumption is quite realistic as the assimilative capacity tends towards extinction, the marginal damage imposed by an additional unit of pollution is equal to a high enough positive value L.

To clarify the formal expressions, we will later use a function $U(p(t), A(t))$ combining the benefits f and the damages D such that $U(p,A) = f(p) - D(p,A)$. It is straightforward that

$$U_{pp} < 0 \text{ and } U_{pA} > 0 \tag{1.3}$$

It must be noted that despite this specification under the form $U(p,A)$, the social

[1]For any function $f(x,y)$ we note f_x the derivative with respect to x and so forth.

[2]They can be easily be verified by looking at a very simple special functional form of D such that $D(p,A) = D(p-A)$. This special functional form immediately yields $D_p = \frac{dD}{dp} = D'(p-A)$, $D_A = \frac{dD}{dA} = -D'(p-A) = -D_p$ and so on.

welfare function is not similar to the models that explicitly value the existence of an environmental stock (see Heal, 2000, p.36). In our model the assimilative capacity provides an environmental function that reduces the potential damage of pollution but has no existence value whatsoever. Indeed, if pollution is nil, social utility is also nil, no matter what the current level of assimilative capacity. We do not acknowledge the other services and amenities provided by a healthy ecosystem.

1.2.4 Assimilative capacity degradation and restoration

The waste assimilation properties of an ecosystem are obviously subject to change if the ecosystem is disrupted by external stress. It is asserted ecologically that above a certain threshold of flow pollution, the ecosystem's equilibrium is affected and functions such as the assimilative capacity are altered. We will consider here, based on Pearce's arguments (1976), that this threshold can be identified in some cases with the level of assimilative capacity itself, although, as noted by Pezzey (1996), there is no systematic evidence to back up this assumption. Indeed, according to Pearce (1988, p.61) "the act of excessive pollution produces a negative feedback, making the ecosystem even less capable of dealing with waste". This is also stated by Pethig (1994, p.218): "the environment has a limited capacity of assimilating pollutants. As long as the flow of released pollutants exceeds that capacity, the environmental quality is reduced until eventually nature's assimilative services are exhausted". This phenomenon is quite clear in the case of microbial assimilation as the biological organisms degrading the pollutant can be harmed by the excess of pollution they are unable to digest immediately (Pezzey, 1996). Since it is this "moving" threshold effect that is of interest here and the economic model would hardly be tractable with an additional "degradation threshold" variable, we will consider that the thresholds for assimilative capacity degradation and the occurrence of environmental damage are identical and measured by the current level of assimilative capacity. We can therefore state the following assumptions:

Assumption 1.2. *An excess of pollution in comparison to the assimilative capacity reduces the assimilative capacity available in the future.*

Assumption 1.3. *If the assimilative capacity is respected, e.g., not exceeded, the ecosystem remains unharmed and the assimilative capacity stays constant or can be increased if restoration is available.*

We will work here in a deterministic framework, assuming that the effects of pollution excess on assimilative capacity are not characterized by uncertainty. This is clearly unsatisfactory from an empirical standpoint, since neither the initial assimilative capacity nor the exact degradation mechanism are easy to know and monitor, whether it concerns the overall earth CO_2 assimilative capacity or a riparian buffer strip. We shall extend the model to a stochastic framework in future work.

1.2.4.1 Degradation function

The biological degradation resulting from excesses of pollution can be determined by a degradation function h with both $A(t)$ and $p(t)$ as arguments

$$\dot{A}(t) = -h(p(t), A(t)) \tag{1.4}$$

According to the mechanisms described in Assumption (1.2) we have

$$h(p, A) = 0 \ \forall \ p \leq A$$
$$h(p, A) > 0 \ \forall \ p > A$$

As with the damage function D, the following properties of h can be established[1]

$$\begin{aligned} &\forall \ p < A \quad h_p(p, A) = -h_A(p, A) = 0 \\ &\forall \ p \geq A \quad h_p(p, A) = -h_A(p, A) > 0 \\ &\forall \ p \geq A \quad h_{pA}(p, A) = h_{Ap}(p, A) < 0 \\ &\forall \ p \geq A \quad h_{AA}(p, A) = h_{pp}(p, A) > 0 \end{aligned} \tag{1.5}$$

It must be noted that h displays a continuity problem similar to the one affecting the damage function D in the neighborhood of $p = A$. We must also establish an upper

[1] They can be easily be verified by looking at a very simple special functional form of h such that $h(p, A) = h(p - A)$. This special functional form immediately yields $h_p = \frac{dh}{dp} = h'(h - A)$, $h_A = \frac{dh}{dA} = -h'(h - A) = -h_p$ and so on.

bound equal to 0 on the marginal degradation of A in order to avoid negative values of A.

$$\lim_{A\to 0} h_A = \lim_{A\to 0} h_p = 0$$

Figures 1.1 and 1.2 provide a graphical illustration of the properties of the degradation function, respectively for $p = \tilde{p}$ given and for $A = \tilde{A}$ given:

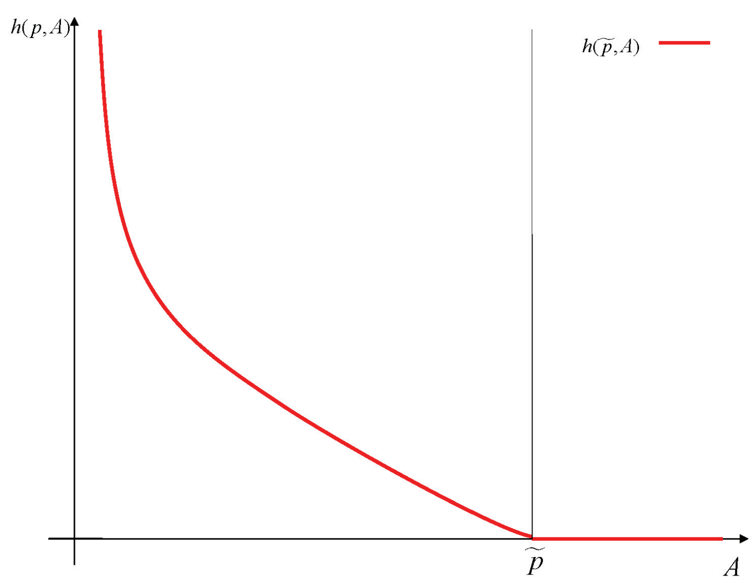

Figure 1.1: Degradation function for a given $\tilde{p}$

1.2.4.2 Restoration

Restoration process

We will now consider that when the social planner decides to allow the emission of an amount of pollution strictly lower than its assimilative capacity, the assimilative capacity increases proportionally to the "rest" granted[1]. The restoration process is thus symmetrical to the depletion process. We may thus proceed to the following

[1]Here we assume that the level of restoration obtained can be determined in a continuous way, with no thresholds effects. However, works on restoration of the environment's quality such as Keohane et al. (2007) consider restoration as a destination-driven threshold process.

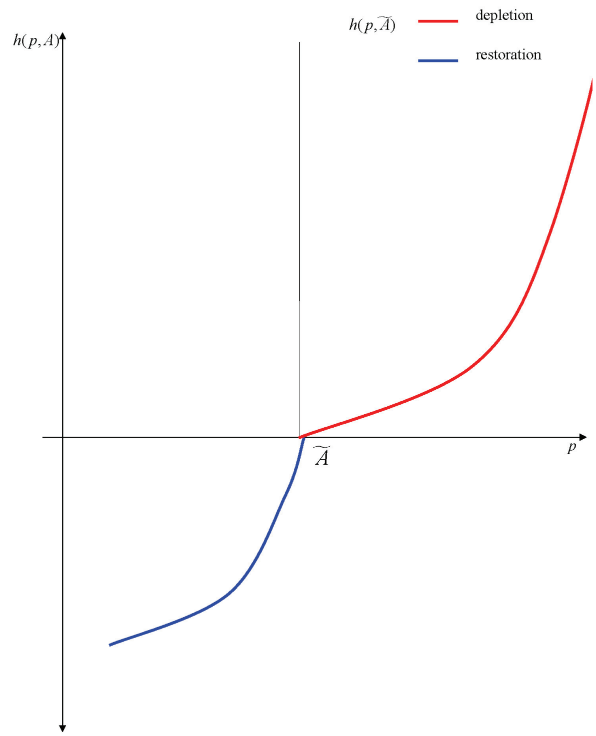

Figure 1.2: Degradation function for a given $\tilde{A}$

modification of the function h in the restoration setting:

$$\begin{aligned} &\forall\ p < A \quad h(p, A) < 0 \Rightarrow \dot{A} > 0 \\ &\forall\ p > A \quad h(p, A) > 0 \Rightarrow \dot{A} < 0 \\ &p = A \Rightarrow h(p, A) = 0 \Rightarrow \dot{A} = 0 \end{aligned} \tag{1.6}$$

Equation (1.6) states that if the assimilation capacity is strictly respected, it increases by an amount $-h(p, A)$. Given the convexity of h, the restoration efforts display decreasing returns, which seems like the most plausible assumption in such a context. Each additional unit of pollution "given up" yields a smaller restoration effect than the previous one. Now we must also have

$$\begin{aligned} &\forall\ p\ \ h_p(p, A) > 0 \\ &\forall\ p\ \ h_A(p, A) < 0 \end{aligned} \tag{1.7}$$

This new specification (strict inequalities) frees us from the continuity problem that we pointed out above for h in the neighborhood of $p = A$ in the previous case.

The other assumptions on h remain valid. Figure 1.3 provides an illustration of the degradation-restoration function for a given $\tilde{A}$.

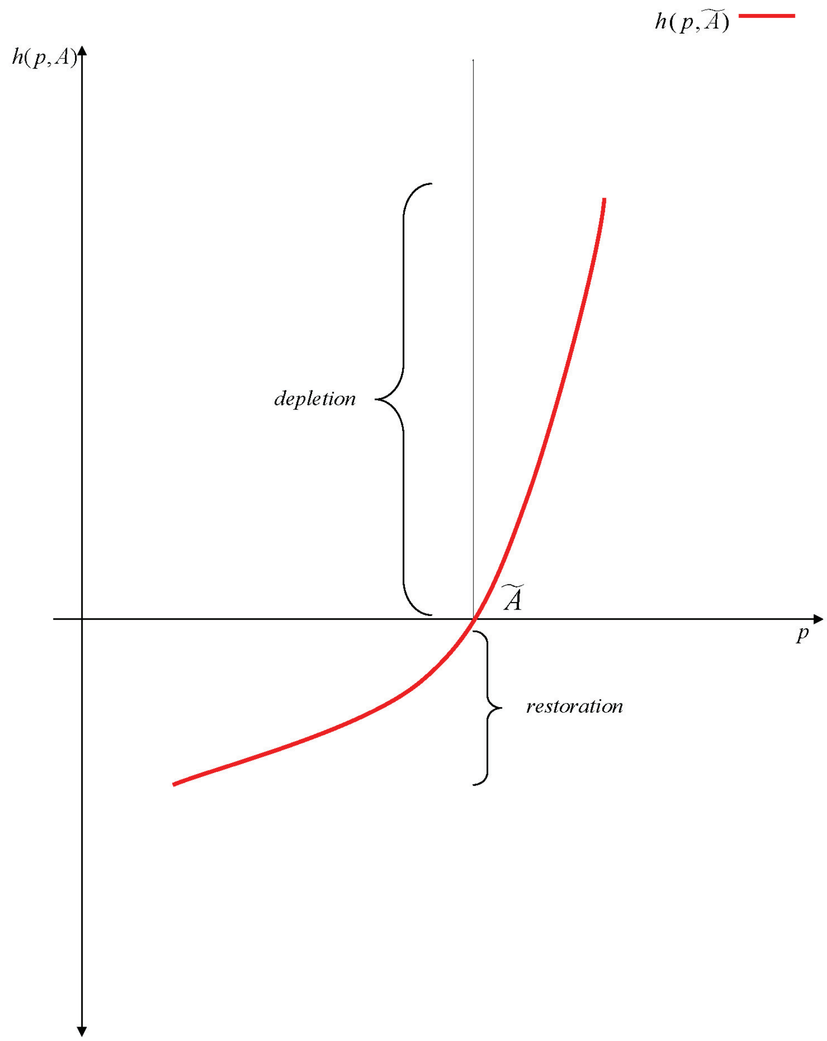

Figure 1.3: Degradation-restoration function for a given $\tilde{A}$

It seems natural to assume that the artificial restoration of the assimilative capacity cannot be infinite. There must indeed be an upper bound $\bar{A}$ that cannot be exceeded: $A(t) \leq \bar{A}\,\forall t$. We shall assume here that this upper limit $\bar{A}$ is at least equal to the initial assimilative capacity A_0, e.g., $A_0 \leq \bar{A}$. This implies that it is always possible to restore the ecosystem's assimilative capacity at least up to its initial level. This assumption might be challenged empirically in some local cases but is robust for the wide range of problems where the assimilative capacity can increased globally without restoring the exact spot where it had been depleted previously[1]. We also need to discuss the

[1]This is the case when deforestation in Brazil is offset by afforestation in Europe to increase

relation between $\bar{A}$ and x_p. From an ecological standpoint, there is obviously no *ex ante* relation between the maximum restoration threshold of a given ecosystem and the private pollution optimum of a firm. However, since our model will not attribute an additional social value[1], or to the assimilative capacity -considered here exclusively as a source of sink service- there is no economic justification for restoring beyond the private pollution optimum x_p as it would yield no additional social benefit compared to the situation where $A = x_p$. We shall thus restrict our analysis to the case where $\bar{A} \leq x_p$.

Restoration costs

We shall specify the cost of restoration in a negative way, treating it as a foregone benefit. We have will see in Section 3 that in the no-restoration case it is always optimal to pollute at least as much as the level of assimilative capacity. Polluting less than this level to let the assimilative capacity "rest" means giving up an economic benefit. We shall use this forgone benefit to account for the cost of restoration without introducing an additional cost function in the model. There are two ways to interpret a level of pollution $p(t) < A(t)$. If the social planner resorts to "natural restoration" it imposes on the polluters a pollution level strictly below the assimilative capacity and prevents them (and society as a whole) from enjoying the highest "damage-free" feasible benefit for a given $A(t)$, $f(A(t))$. In that case, the cost of restoring the assimilative capacity by an amount $|h(p(t), A(t))|$ is equal to the difference between the maximum "damage-free" benefit $f(A(t))$ that could have been achieved and the actual benefit $f(p(t))$. Given the concavity of f, the cost of restoration $f(A(t)) - f(p(t))$ is a convex function of p which is coherent. In that case the social planner's objective function writes simply

$$f(p(t)) - D(p(t), A(t)) \tag{1.8}$$

If the social planner resorts to "artificial restoration", it will let the polluters enjoy the maximal damage-free level of pollution[2] $p(t) = A(t)$ yielding a private benefit $f(A(t))$ and spend an amount $f(A(t)) - f(p(t))$ (the cost of restoration assessed above) in

the planet's carbon assimilative capacity or when a degraded buffer strip protecting a stream from nutrient runoffs is extended or widened.

[1]Such an additional social value could be justified for forests or oceans providing at the same time assimilative services and amenities but here we restrict our analysis to the less favorable scenario.

[2]Given the decreasing returns of restoration and the convexity of the degradation function, it would be inconsistent to let the polluter degrade the assimilative capacity while restoring it through costly investments at the same time.

order to restore the assimilative capacity by an amount $|h(p(t), A(t))|$. In that case the social planner's objective function writes

$$f(A(t)) - D(p(t), A(t)) - (f(A(t)) - f(p(t))) = f(p(t)) - D(p(t), A(t)) \tag{1.9}$$

The two interpretations amount to equivalent relations (1.8) and (1.9) that can fit various cases of restoration depending on local conditions.

1.2.4.3 General properties and additional ecological assumption

Given the properties described above, A displays the following dynamics

$$\dot{A}(t) \leq 0 \ \forall \ t \tag{1.10}$$

$$A(t) \leq A(0) < x_p \quad \forall \ t \geq 0 \tag{1.11}$$

and in the restoration case

$$\begin{aligned} &\dot{A}(t) \gtreqless 0 \ \forall \ t \\ &A(t) \leq x_p \ \ \forall \ t \geq 0 \end{aligned} \tag{1.12}$$

In order to overcome some mathematical complexities that would be beyond the scope of this paper, we need to make the following assumption, valid for both the irreversible and the reversible case, where ρ is the social rate of discount introduced in the next section:

$$\forall (p, A) \ \ - h_A(p, A) < \rho \tag{1.13}$$

Although the degradation processes are far from thoroughly understood by ecological science, this assumption does not seem counter-intuitive. Indeed, for standard "low values" of ρ such as $\rho < 1$, (1.13) implies that a unit of pollution in excess reduces the assimilative capacity by an amount less than a unit (or in the restoration case, that a unit of pollution given up produces less than one unit of new assimilative capacity). In any event, this assumption does not affect the general range of our results and fits most empirical problems, especially for a high discount rate which may put a risk the preservation of environmental assets.

1.2.4.4 Continuous degradation, ecosystems' flip and threshold effects

The degradation/restoration process we have modeled above displays a continuous behavior that can be legitimately questioned. Indeed, the literature on shallow lake eutrophication (see Mäler et al., 2003) and resources with thresholds (Crépin, 2007) on the one hand and the environmental restoration economic literature on the other hand (Keohane et al., 2007) point out the crucial role played by ecological thresholds in both the degradation and the restoration of environmental functions such as assimilative capacity. Shallow lakes can thus "flip" overnight from a "healthy" state to a completely "eutrophicated" state because a vital threshold of accumulated nutrients has been reached. Such a sudden flip is not allowed in our model. It is thus necessary to justify the modeling choice that lead our analysis. The rationale behind our model is twofold.

From an empirical point of view, the continuous degradation process we describe fits adequately the two kinds of concrete problem that we wish to address: flows of nutrient emissions through riparian buffer ecosystems and carbon dioxide emissions (in Chapter 2). Considering that an important part of the assimilative capacity at stake in those problems depends on vegetal systems (riparian plants, carbon sequestrating forests), both their degradation and their restoration can be represented in a simplified way with a continuous function. As far as these two case studies are concerned, it seems reasonable to discard the possibility of a sudden "flip" of the ecosystem overnight.

From a theoretical point of view, we can argue that although there are no *ex ante* thresholds in our model, the level of assimilative capacity itself consists in a varying threshold. As we explain in the next subsection, when pollution exceeds the assimilative capacity, the latter decreases and is even more likely to be exceeded at the next period. This "moving threshold" mechanism allows us to shed some light on possible environmental degradation cycles that have been observed with other environmental functions and that can be especially destructive in developing countries (Barbier, 1990). We shall develop this issue in Chapter 5.

1.2.5 Sustainability and environmental degradation cycle

A minima *sustainability*

Considering the essential service provided by the environment's assimilative capacity, its degradation due to pollution excesses raises serious sustainability concerns. In terms of strict "environmental sustainability" or strong sustainability[1], it is obvious that any pollution path degrading, even slightly, the assimilative capacity of an ecosystem must be discarded as unsustainable.

A weaker definition of sustainability[2] could be to maintain environmental and economic capacities such that a minimum level of intergenerational equity is guaranteed through a "use of environmental services at rates which can hold on for very long time periods, and in theory, indefinitely" (Pearce, 1988, p.58). In the absence of technological change, this minimal intergenerational equity cannot be maintained unless the socially useful environmental functions are at least partially preserved. Therefore no pollution path destroying the assimilative capacity can be considered sustainable. Hence the following necessary *a minima* sustainability condition:

Condition 1. *In the absence of technological change, any pollution path reducing to nil the assimilative capacity is incompatible with a sustainable development policy as it deprives future generations of an essential environmental function.*

If a pollution path does not respect this condition, it will not respect, *a fortiori*, the requirements of sustainable development. Our sustainability condition is quite similar to the main sustainability concern expressed by Batabyal et al. (2002, p.343), who claim that "the most critical factor in sustainability is likely to be the maintenance of adequate stocks of environmental resources to ensure an adequate flow of ecosystem services". It must be made clear that the previous condition is mostly a non-sustainability condition and that boils down to a partial definition of intergenerational equity. Indeed, even if the assimilative capacity is only partially degraded, it still implies a reduction of the damage-free pollution potential of future generations.

[1]The "strong sustainability" concept aims to conserve integrally the level of all forms of natural capital. See Ayres et al. (1998) for a review of the different conceptions of sustainability. It must be noted that sometimes the interpretation of "strong sustainability" consists in the conservation of critical stocks of natural capital that we address in Chapter 5.

[2]In our model with only one category of capital, the standard definition of weak sustainability cannot be tested.

As it accounts only for one ecosystem service, our approach gives a lower bound estimation of the actual intensity of irreversible damages caused by excessive pollution. It is straightforward, considering the dynamics of the assimilative capacity, that a sustainable path must at some point respect the assimilative capacity (such that $p = A$) before the latter is entirely depleted. This is tantamount to the third principle of sustainable development advocated by Daly (1990). Cesar and de Zeeuw (1994) apply this third principle, which they define as "generating waste and pollution at rates less than or equal to the rates at which they can be absorbed by the assimilative capacity of the environment" (p.26), to climate change and assert that the respect of the CO_2 assimilative capacity is a necessary condition of sustainability.

The environmental degradation cycle

On a local scale, our approach highlights a crucial vicious cycle in economy-environment interactions that needs to be acknowledged in order to implement sustainable policies. The salient feature of our model is that an excess of pollution today not only causes social damage but also degrades the assimilative capacity, thus lowering the threshold at which social damage will occur in the future. At the next period, the same flow of pollution will be even more in excess of the assimilative capacity and consequently will cause more social damage and degradation of assimilative capacity. This vicious cycle can go on until the assimilative capacity is extinguished. That is why the static optimum that prevails in flow pollution literature jeopardizes the natural capital that should be passed down to future generations. Such an overshoot cycle is not specific to assimilative capacity. It affects various other environmental functions such as soil productivity when farmers fail to acknowledge the intertemporal impact of their activity on soil quality. As shown in many models (see Barbier, 1990), intensive use of agricultural lands will provide extra short-term benefits but degrade soil productivity in such a way that if the same production intensity is maintained at the next period, soil degradation will be even greater. Myopic behavior can thus lead to an overshoot cycle that quickly and irreversibly depletes the environmental assets available. Preventing such cycles from taking place while guaranteeing survival of the economic agents who depend on a threatened ecosystem is a major challenge for sustainable development policies. We shall analyze this phenomenon on an aggregate level with a model of natural capital utilization in the last chapter of this dissertation.

1.3 Optimal pollution path without restoration

As in most social optimization problems, we use a discounted utilitarian framework with a social welfare function including both the private benefit and the environmental damage with ρ the social discount rate, supposed constant.

$$1 > \rho > 0$$

The social planner problem amounts to

$$\max_p W = \int_0^{+\infty} U\left(p(t), A(t)\right) e^{-\rho t} dt = \int_0^{+\infty} \left(f\left(p(t)\right) - D(p(t), A(t))\right) e^{-\rho t} dt \quad (1.14)$$

subject to $\dot{A}(t) = -h(p(t), A(t))$ and $A(0) = A_0$.

We can state right away a useful result in the no-restoration case, denoting $p^*(t)$ and $A^*(t)$ the value of p and A along the optimal pollution path at time t.

Claim 1. $p^*(t) \geq A^*(t)$ *along the optimal path*

Indeed, with no-restoration allowed, we have already noted that $p = A$ yields a higher private benefit $f(p)$ and the same amount of damage and assimilative-capacity depletion (both equal to zero) than any p such that $p < A$. Thanks to this restriction of the definition set of p on the optimal path, we can avoid the difficulties caused by the continuity problem of h and D in the neighborhood of $p = A$.

The current value Hamiltonian H of our problem, according to equations (1.14) and (1.10) and under the assumption of no technological change, is

$$H = f(p(t)) - D(p(t), A(t)) - \lambda(t)h(p(t), A(t))$$

where $\lambda(t)$ is the co-state variable representing the shadow price of the assimilative capacity. Given that the contribution of the latter to the social welfare function is obviously positive, this price λ will necessarily be positive along the optimal path:

$$\lambda(t) \geq 0 \;\; \forall \;\; t \geq 0$$

1.3.1 First-order conditions

Let us establish the first-order conditions[1] determining this optimal path. These conditions are

$$\frac{dH}{dp} = 0$$
$$\frac{dH}{dA} = \rho\lambda - \dot{\lambda}$$
$$\frac{dH}{d\lambda} = \dot{A}$$

hence

$$f_p(p) - D_p(p, A) - \lambda h_p(p, A) = 0 \quad (1.15)$$
$$\rho\lambda - \dot{\lambda} = -D_A(p, A) - \lambda h_A(p, A)$$

which yields

$$f_p(p) = D_p(p, A) + \lambda h_p(p, A) \quad (1.16)$$
$$\dot{\lambda} = \lambda(\rho + h_A(p, A)) + D_A(p, A) \quad (1.17)$$

In addition there is the transversality condition:

$$\lim_{t \to +\infty} e^{-\rho t}\lambda(t)A(t) = 0 \quad (1.18)$$

Equation (1.16) establishes a very intuitive result: the net private benefit from an additional unit of pollution must be equal to the total marginal damage caused by this unit. This marginal damage includes the standard flow marginal damage D_p as well as the marginal loss of assimilative capacity, valued by the product of the shadow price of assimilative capacity, λ, and the amount of assimilative capacity depleted by this incremental unit of pollution $h_p(p, A)$. The shadow price reflects *"the most one would would be willing to pay to relax the constraint"* along the optimal path (Kamien and Schwartz, 1991) and can be interpreted here as the loss of current and future flows of welfare associated with the loss of one unit of assimilative capacity today.

[1]For notational ease, the time index of variables will be omitted whenever no ambiguity can arise.

Rewriting equation (1.17) and using (1.16) we get:

$$\frac{\dot{\lambda}}{\lambda} = \rho + \left(h_A + \frac{D_A h_p}{D_p - f_p}\right) \tag{1.19}$$

The rate of change in the shadow price of the undepleted assimilative capacity is determined not only by the depletion-adjusted social discount rate[1] $(\rho + h_A)$ but also by an additional factor indicating the social value of *in situ* assimilative capacity.

Equation (1.19) reads as a modified version of the Hotelling rule. If we treat the assimilative capacity as an exhaustible resource its productivity, which in a perfectly competitive market must be equal to its price at the equilibrium, must grow at a rate given by the right-hand side of equation (1.19). This term needs to be compared with the standard discount rate ρ but unfortunately the sign of the additional term $h_A + \frac{D_A h_p}{D_p - f_p}$ cannot be determined unambiguously. However, it can be shown that when A is low enough, the additional term is negative and the shadow price of assimilative capacity must grow at a rate lower than the value of the discount rate, which implies a slower depletion rate of the resource itself.

1.3.2 Comparison with the standard Turvey optimum

From now on we shall denote $(p^*(t), A^*(t))$ the set of optimal values of p and A on the optimal path at time t. Relation (1.15) can be written for A and λ given with the function $\pi_{A,\lambda}$ such that

$$\pi_{A,\lambda}(p) = f_p(p) - D_p(p, A) - \lambda h_p(p, A)$$

$\pi_{A,\lambda}$ is decreasing in p according to the properties of f, D and h for a given A and λ. For $p^*(t)$ the pollution optimum at any time t we have, according to (1.15)

$$\pi_{A,\lambda}(p^*) = f_p(p^*) - D_p(p^*, A) - \lambda h_p(p^*, A) = 0 \tag{1.20}$$

Let us define $\bar{p}(A)$ as the static Turvey optimum for a given assimilative capacity A. $\bar{p}(A)$ is determined by standard static internalization of external effects such that

[1] This adjusted discount rate is lower than the initial discount rate and similar to the *pollution-adjusted discount rate* found in the literature, see Hediger (2009).

$f_p(\bar{p}(A)) = D_p(\bar{p}(A), A)$. This optimum fails to consider any dynamic evolution of the problem since it statically equalizes the marginal benefit and the marginal damage for a given level of A. We assume for simplicity of exposition and to focus on the most interesting case that f and D are such that

$$\forall \quad A \geq 0 \quad \exists \quad \bar{p}(A) > 0 \quad \text{s.t.} \quad f_p(\bar{p}(A)) = D_p(\bar{p}(A), A) \tag{1.21}$$

We can now compare this Turvey optimum with the dynamic optima by using our function $\pi_{A,\lambda}$. We have thus

$$\begin{aligned} \pi_{A,\lambda}(\bar{p}(A)) &= f_p(\bar{p}(A)) - D_p(\bar{p}(A), A) - \lambda h_p(\bar{p}(A), A) \\ \pi_{A,\lambda}(\bar{p}(A)) &= -\lambda h_p(\bar{p}(A), A) \leq 0 \end{aligned} \tag{1.22}$$

hence, given the decreasing nature of $\pi_{A,\lambda}$, (1.20) and (1.22) yield, for any given A and any t:

$$p^*(t) \leq \bar{p}(A(t)) \tag{1.23}$$

Proposition 1.1. *In the no restoration case, the level of optimal pollution must be lower than the static Turvey optimum at all times.*

We establish here a very intuitive result since it is natural that the dynamic optimum p^* accounting for intertemporal externalities should be lower than the static optimum $\bar{p}(A)$.

1.3.3 Existence and phase-diagram analysis

The existence of a steady state is guaranteed by the joint concavity of our Hamiltonian (Theorem 13, Seierstad and Sydsaeter, 1987, p.234), proven in Appendix A. Along the optimal path, the level of emissions must be adjusted continuously to satisfy the first-order condition. The optimal level of pollution can thus be represented as an implicit function of λ and A where

$$p = p(\lambda, A)$$

Rewriting equation (1.16) with the utility function U we get

$$U_p(p(\lambda, A), A) = \lambda h_p(p(\lambda, A), A) \tag{1.24}$$

Differentiating each side with respect to λ yields (after simplifications)

$$\frac{dp(\lambda, A)}{d\lambda} = \frac{h_p}{U_{pp} - \lambda h_{pp}}$$

According to the properties of f, D, h and λ we have

$$\frac{dp(\lambda, A)}{d\lambda} < 0 \tag{1.25}$$

Similarly, differentiation equation (1.24) with respect to A gives us after simplifications

$$\frac{dp(\lambda, A)}{dA} = \frac{\lambda h_{pA} - U_{pA}}{U_{pp} - \lambda h_{pp}}$$

According to (1.3) we know that $U_{pA} > 0$. Combining with the properties of h and λ we have $\lambda h_{pA} - U_{pA} < 0$. Hence

$$\frac{dp(\lambda, A)}{dA} > 0 \tag{1.26}$$

It is straightforward from equations (1.4) and (1.17) that the behavior of the system from any initial point (A_0, λ_0) is governed by

$$\dot{\lambda} \gtreqless 0 \quad \text{as} \qquad \lambda(\rho + h_A(p, A)) \gtreqless -D_A(p, A) \tag{1.27}$$

$$\dot{A} \gtreqless 0 \quad \text{as} \qquad - h(p, A) \gtreqless 0 \tag{1.28}$$

Note that the absence of restoration rules out the case $\dot{A} > 0$. In addition, according to the properties of h, the space where $h(p, A) = 0$ should not be a curve (the standard isocline) but a plane since $h(p, A) = 0 \;\; \forall (p, A)$ s.t. $A \geq p$. However, we have proven previously that $p < A$ is never optimal, so the candidate steady states on the optimal path are necessarily on the $[A = p]$-isocline. The slopes of the stationary loci satisfying (1.27) and (1.28) with equality are given by (see Appendix B for calculation details)

$$\frac{d\lambda}{dA} \mid_{\dot{\lambda}=0} = -\frac{\frac{dp(\lambda,A)}{dA}(D_{Ap} + \lambda h_{Ap}) + \lambda h_{AA} + D_{AA}}{\rho + h_A + \frac{dp(\lambda,A)}{d\lambda}(\lambda h_{Ap} + D_{Ap})} \tag{1.29}$$

and

$$\frac{d\lambda}{dA}\mid_{\dot{A}=0} = -\frac{\frac{dp(\lambda,A)}{dA}h_p + h_A}{\frac{dp(\lambda,A)}{d\lambda}h_p} \tag{1.30}$$

Inequalities (1.25) and (1.26), equations (1.29) and (1.30) respectively yield (see Appendix B)

$$sgn\left(\frac{d\lambda}{dA}\mid_{\dot{\lambda}=0}\right) = -sgn(\rho + h_A) \tag{1.31}$$

$$\frac{d\lambda}{dA}\mid_{\dot{A}=0} < 0 \tag{1.32}$$

Since according to (1.13) we have $\rho + h_A > 0$, we can write

$$\frac{d\lambda}{dA}\mid_{\dot{\lambda}=0} < 0 \tag{1.33}$$

Let us now characterize the optimal paths in the $A - \lambda$ plane. In this plane, A is bounded below by 0 and above by x_p while λ is only bounded below by 0. According to (1.33), the isocline I_λ, where $\dot{\lambda} = 0$, is monotonically decreasing in the $A - \lambda$ plane. Based on equation (1.17) and the properties of h_A and D_A[1] we can easily show that $\dot{\lambda} > 0$ on the right of I_λ and $\dot{\lambda} < 0$ on the left of I_λ. In addition, we know from (1.32) that the isocline I_A, where $\dot{A} = 0$, is decreasing. We obviously have $\dot{A} < 0$ below I_A and must exclude the gray area where $A > p$ from our phase analysis.

The geometrical properties of the two isoclines, established in Appendix C, imply the following graphical representation in the diagram (note that we arbitrarily draw the I_A curve in a linear manner and set L to infinity for simplicity of exposition).

For the sake of clarity, we show in Figure 1.5 the phase diagram analysis in the (A, p)-plane. This representation can be easily derived using relation (1.25) and the previous system dynamics of A. This graph points out the excess of pollution vis à vis the assimilative capacity along the optimal depletion path. The $\dot{A} = 0$ isocline is naturally represented by the first bisectrix of the (A, p)-plane such that above this line we have $\dot{A} < 0$. We also have $\dot{A} > 0$ below but in the no-restoration case a path in this zone is not feasible. The optimal path, above the first bisectrix, degrades the assimilative capacity until it reaches the steady state where by definition we have

[1]An increase in A *ceteris paribus* induces an increase in $\dot{\lambda}$.

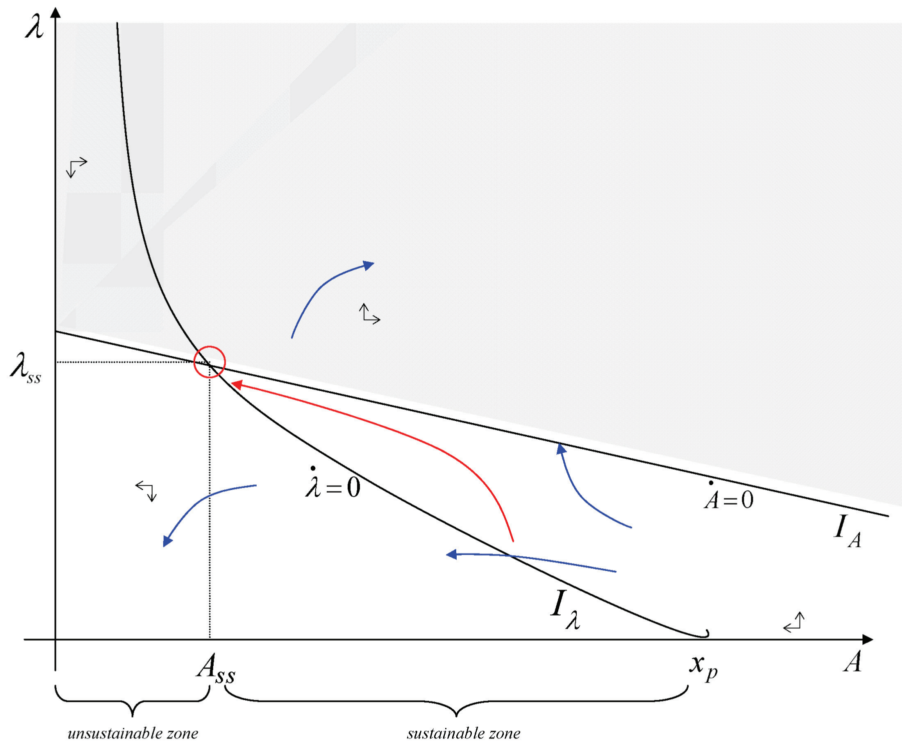

Figure 1.4: Optimal pollution path

$p_{ss} = A_{ss}$. In the no-restoration setting no path can reach the steady state starting from $A_0 < A_{ss}$ as it would belong to the unfeasible zone defined above.

1.3.4 Sustainability analysis

Considering the geometrical properties characterized in Appendix C, it is straightforward that there is a unique interior solution for the steady state. The two isoclines intersect at a unique equilibrium candidate (A_{ss}, λ_{ss}) that is a saddle point (see Figure 1.4). The *a minima* sustainability of the optimal path depends on the initial level of assimilative capacity A_0. The different cases are discussed in Proposition (1.2).

Proposition 1.2. *Case (1): If $A_0 \geq A_{ss}$, the optimal policy is to select λ_0 so as to place the economy on a path that ends at the stable equilibrium (A_{ss}, λ_{ss}). As $\dot{A} < 0$ along this path, the level of polluting emissions exceeds the assimilative capacity until it reaches the equilibrium and stabilizes with $p^* = A_{ss}$. We call this set $[A_{ss}, x_p[$ the sustainable zone, keeping in mind that the sustained level of assimilative capacity A_{ss} might be low. The shadow price of the assimilative capacity rises along this path while*

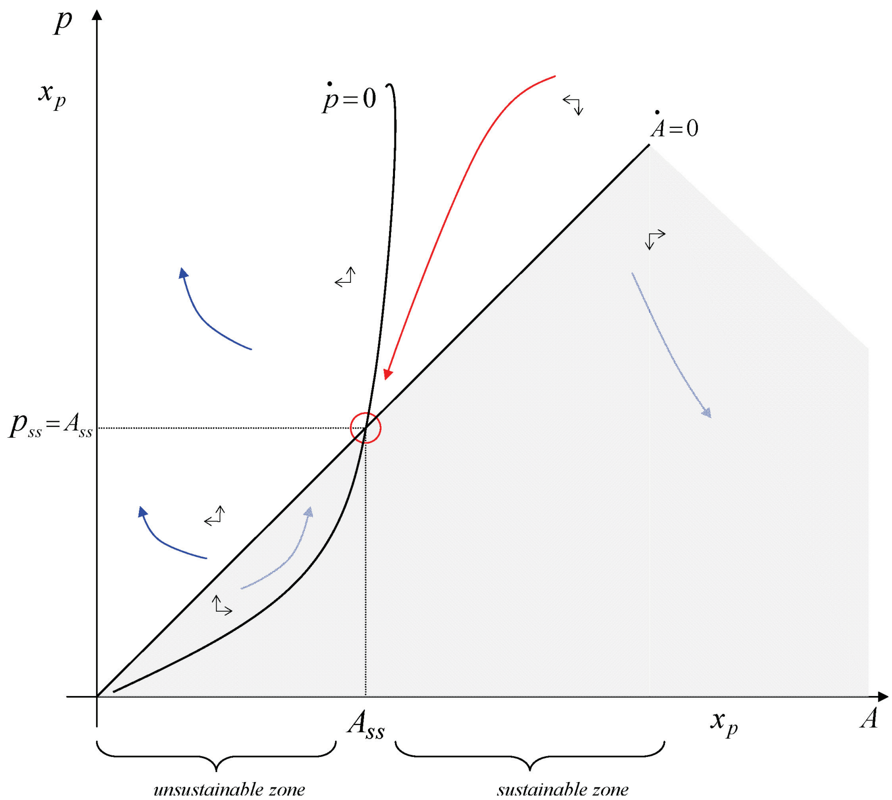

Figure 1.5: Optimal pollution path in the (A,p)-plane

the pollution level decreases according to the properties of $p(\lambda, A)$.

Case (2): If $A_0 < A_{ss}$*, then the optimal path will never reach the steady state as the assimilative capacity cannot be increased. The optimal path in this quadrant will lead to extinction of the resource (see details in Appendix D). We call the* $[0, A_{ss}[$ *set the unsustainable zone.*

A_{ss} is thus the minimum initial level of assimilative capacity required to ensure a sustainable optimal pollution path. As A necessarily decreases along the optimal path, we can conclude from (1.26) that p also decreases along this path.

1.3.5 Comparative statics

The qualitative conclusions drawn in the previous section do not provide us with a quantitative definition of the equilibrium A_{ss}. However, comparative statics can help us describe how the exogenous parameters of the model affect this equilibrium level. In equation (1.17) an increase in the social rate of discount ρ for a constant λ must be compensated by a lower value for h_A and/or D_A, which implies a lower value for A. Graphically this means that the I_λ isocline will shift to the left for a higher ρ. Figure 1.6 shows the twofold ambiguous effect of a higher discount rate on the sustainability of the optimal path. On the one hand, it diminishes the level of A_{ss}, leaving future generations with less natural capital. On the other hand, it widens the sustainable zone, increasing the range of initial ecological conditions that are compatible with a sustainable path.

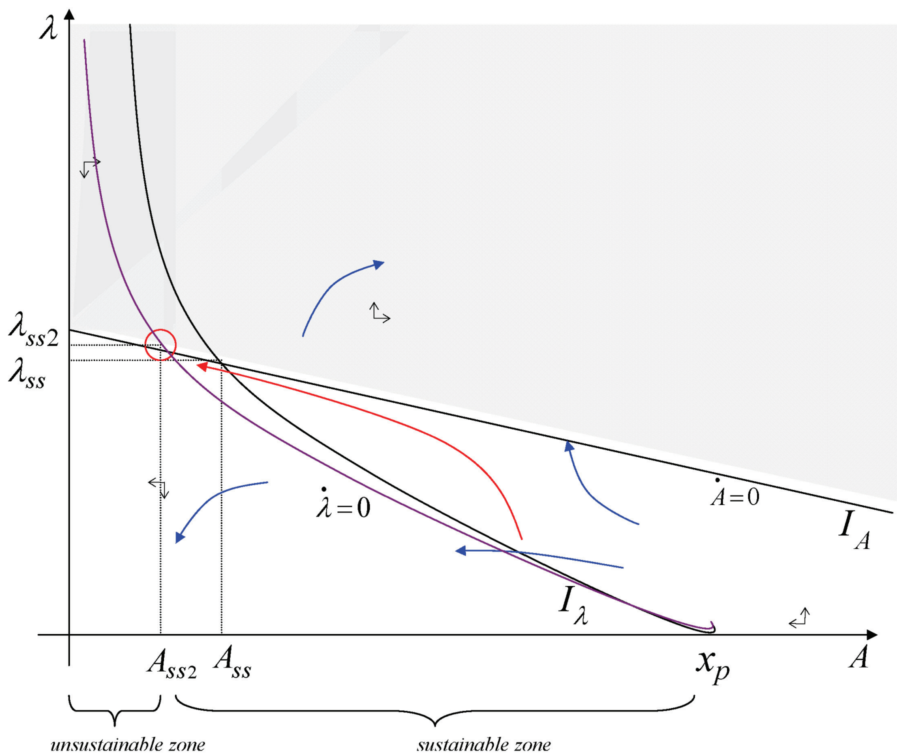

Figure 1.6: The effect of a higher discount rate on the optimal path

1.4 Optimal pollution path with restoration

1.4.1 Optimal restoration

We will now determine the optimal path of pollution when restoration is an option available to the social planner. It is straightforward that this new problem is very similar to the irreversible case addressed previously. Indeed, the only modifications to the formal specification is the extension of the degradation function h which can now take negative values and the introduction of an upperbound $\bar{A}$ on A.

$$\max_p W = \int_0^{+\infty} \left(f\left(p(t)\right) - D(p(t), A(t))\right) e^{-\delta t} dt$$

subject to $\dot{A}(t) = -h(p(t), A(t))\ A(0) = A_0,\ A(t) \leq \bar{A}\ \forall t$

Adding a multiplier ω to ensure that the constraint on A holds at all times[1], we obtain the following Lagrangian:

$$L(t) = f(p(t)) - D(p(t), A(t)) - \lambda(t)h(p(t), A(t)) + \omega(\bar{A} - A(t))$$

The transversality condition (1.18) still holds and we can extend the interpretation of the first order conditions.

$$\begin{aligned} f_p(p) - D_p(p, A) - \lambda h_p(p, A) &= 0 \\ \dot{\lambda} &= \lambda(\delta + h_A(p, A)) + D_A(p, A) + \omega \\ \omega(\bar{A} - A) = 0,\ \omega \geq 0,&\ \ \bar{A} - A \geq 0 \end{aligned} \tag{1.34}$$

Equation (1.34) can be reinterpreted in a very interesting way in the light of the restoration possibility. If $p < A$, the marginal damage is nil (equation (1.1)) and we have $f_p(p) = \lambda h_p(p, A)$. If at any time restoration is the optimal choice along the optimal path, it must be carried out until the marginal cost of the restoration effort $f_p(p)$ equals the value of an additional unit of *in situ* assimilative capacity. This value corresponds to the product of the marginal increase of assimilative capacity $h_p(p, A)$ and the shadow price of assimilative capacity λ.

[1]This multiplier will not play a determinant role here as we focus on the situations where this constraint is not binding

Regarding the comparison with the Turvey optimum, it is easy to verify that in the case with restoration, equation (1.22) and Proposition 1.1 still hold.

1.4.2 Sustainability analysis

Since $A(t) \leq x_p$ in the restoration setting as well, x_p remains the upper bound of A but in a weaker way. The phase analysis remains very similar to the irreversible case (see Appendix C). The introduction of the multiplier ω affects the optimal path when the constraint bites but we shall focus here on the most interesting case where $\bar{A}$ is large enough. The isoclines display the same properties and this time also there is a unique stable equilibrium (A_{ss}, λ_{ss}) that is a saddle point (see Figure 1.7).

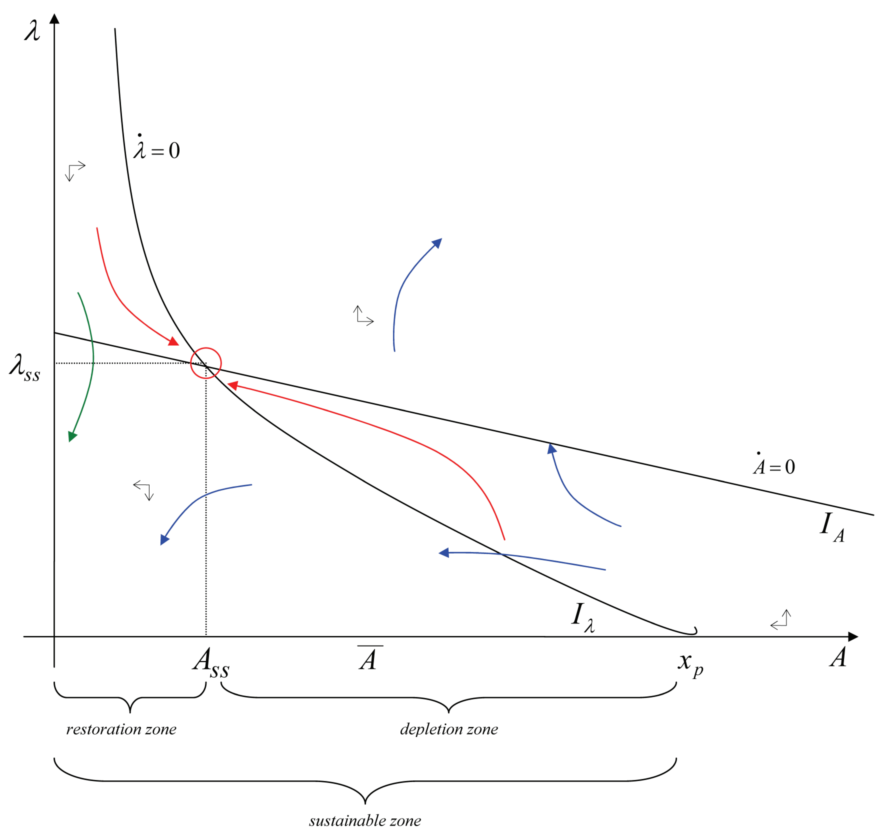

Figure 1.7: Optimal pollution path with restoration ($\bar{A} > A_{ss}$)

It is now possible to move from left to right on the optimal path (e.g., increasing the assimilative capacity). This possibility grants access to the area on the diagram that was off-limits in the irreversible case and changes the "extinction zone" into a "restoration" zone. However, this restoration of the assimilative capacity up to the steady state (A_{ss}, λ_{ss}) is obviously feasible only if $\bar{A} \geq A_{ss}$. Figure 1.7 illustrates this case and the general conclusions are drawn in Proposition (1.3).

The phase diagram in the (A,p)-plane reflects even more clearly the restoration effect that allows to reach the steady-state "from below". As shown in Figure 1.8, the restoration path starts below the first bisectrix, *ie* for levels of pollution strictly inferior to the current assimilative capacity, which will thus increase along the path. Indeed, a path within this zone is now feasible thanks to the restoration process.

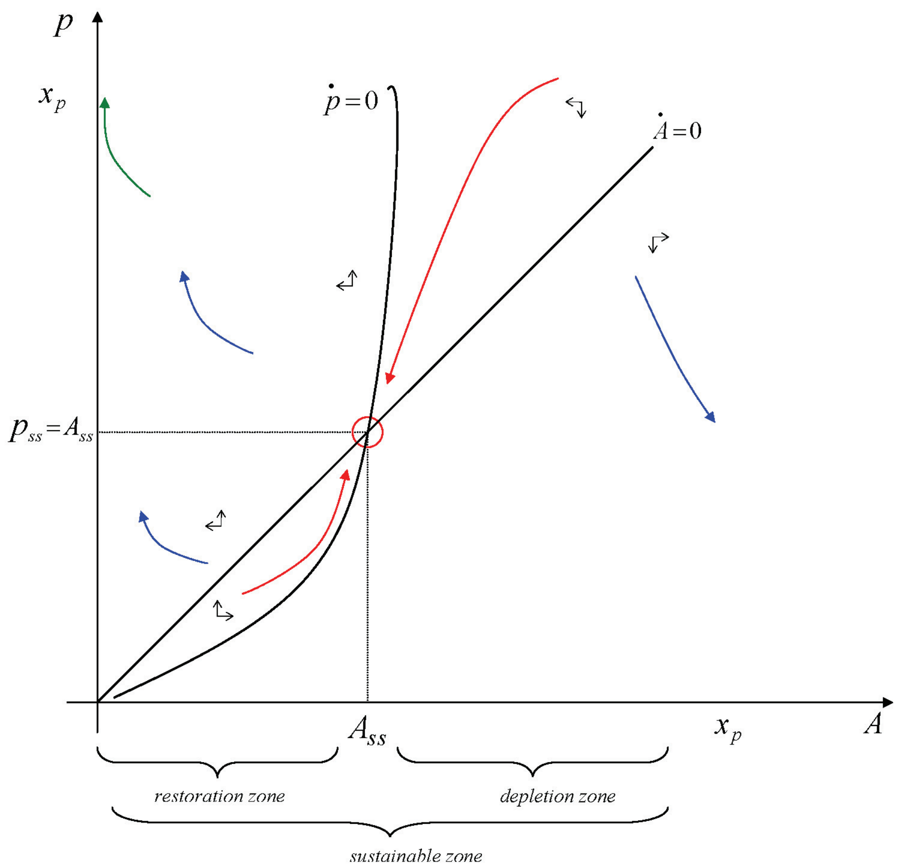

Figure 1.8: Optimal pollution path with restoration in the (A,p)-plane ($\bar{A} > A_{ss}$)

1.4.3 Comparative statics

A variation of the discount rate has the same graphical effects as in the reversible case but has different implications since restoration is allowed. As shown in Figure 1.9, a higher δ will reduce the level of A_{ss}, leaving future generations with less natural capital. In doing so, the discount rate increases the chance that the optimal path be a depletion path driving the assimilative capacity to zero instead of restoring it, as shown by the green trajectory on Figure 1.9 and on Figure 1.8.

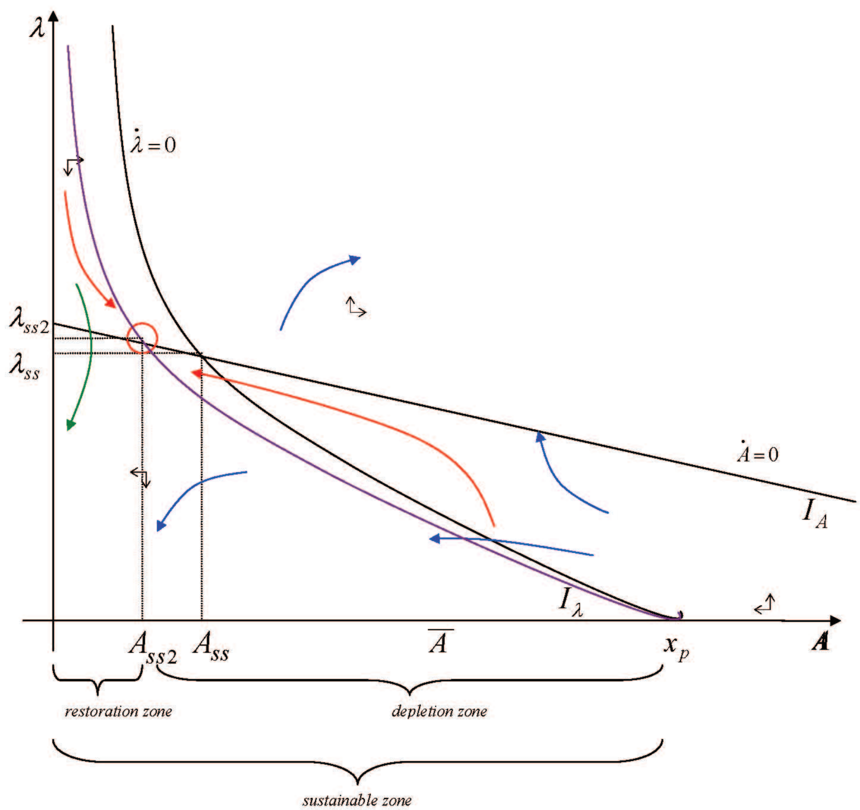

Figure 1.9: The effect of the discount rate on the optimal path with restoration

A lower δ will raise the steady state level A_{ss} (with an upper limit of $A_{ss} = x_p$ if condition (1.37) is met) and thus increase the size of the restoration zone and the length of the optimal restoration path.

Proposition 1.3. *If $A_0 \geq A_{ss}$, the optimal policy is to select λ_0 so as to place the economy on a path that ends at the stable equilibrium (A_{ss}, λ_{ss}). As $\dot{A} < 0$ along this path, the level of polluting emissions exceeds the assimilative capacity until it reaches the equilibrium and stabilizes with $p^* = A_{ss}$. The shadow price of the assimilative capacity rises along this path while the pollution level decreases, just like in the no-restoration case.*

If $A_0 < A_{ss}$ and if the discount rate is low enough, then the optimal path will increase the assimilative capacity up to A_{ss} if $\bar{A} \geq A_{ss}$. It is thus optimal for the *social planner to restore the assimilative capacity up to A_{ss}. Along such a restoration path, the shadow price of assimilative capacity decreases while the optimal level of pollution, initially strictly lower than the assimilative capacity level, increases.*

If $A_0 \leq \bar{A} < A_{ss}$ or if $A_0 < A_{ss}$ and the discount rate is too high, then it is then optimal to immediately deplete the assimilative capacity until extinction in order to get the maximum social benefit. The effect of the discount rate is similar to the no-restoration case. The effect of the assimilative capacity upper-bound reflects the fact that in a discounted framework it cannot be optimal to restore A up to $\bar{A}$ and then start depleting it again since the steady state cannot been reached[1].

The restoration option, when the maximum restoration threshold is sufficiently high and the discount rate sufficiently low, can free the economy from the depletion path imposed by low initial conditions.

1.5 Illustration of the model: riparian buffer ecosystems and lixiviated nitrates

The framework of dynamic externality is particularly fitted for an aspect of nitrate contamination problems incompletely addressed by static flow externality models. We shall focus exclusively on the flow-pollution aspect of nitrate contamination of rivers and streams and ignore the stock externality of accumulative pollution in groundwater[2]. This contamination leads to a concentration of nitrates above the acceptable thresholds and triggers significant damage to society. This damage consists of increased costs of artificial water-purification for drinkable water (or health problems if this purification is not achieved), a negative impact on soil fertility and tree health (Vitoussek et al, 1997), and the loss of recreational amenities and sea-related commercial benefits due to "green tides" of seaweed clogging estuaries[3]. To provide an order of magnitude, it is interesting to note that the French Ministry of Environment estimated in 1996 the annual damages of surface water contamination (increase in treatment costs, production loss, health costs) to approximate 3 billion euros and

[1] This would be reflected by a strictly positive Lagrangian multiplier ω.

[2] This phenomenon falls into the category of stock externality that will be dealt with in future works.

[3] This phenomenon is well known on the coast of Brittany in western France.

little has been done since.

1.5.1 The role of riparian buffer-ecosystems in the assimilation of lixiviated nitrates

The mechanisms leading to the contamination of surface water by lixiviated nitrates from chemical fertilizers and animal manure are well known[1], and an abundant literature, in both the fields of economics and ecological science, has dealt with this issue. However, little attention has been paid by economists to the evolution of the assimilative capacity of the ecosystems involved and especially to the crucial role played by riparian buffer zones[2]. These riparian ecosystems, defined as *"the narrow ecotones between aquatic and terrestrial ecosystems that consist of several fluvial surfaces, including channel islands and bars, channel banks, floodplains, and lower terraces"* (Goodwin et al., 1997), offer a fundamental ecosystem service that reduces the socio-economic impact of nitrogen-based fertilizers and intensive stockbreeding by absorbing a portion of the nitrates[3] on their way from agricultural sources to water courses (see Figure 1.10). If their assimilative capacity is exceeded at any given time, a greater flow of nitrates reaches the surface water directly[4], causing more damage. Furthermore, these excesses of nitrates lead to nitrogen saturation and degrade the assimilative capacity (Hanson et al., 1994; Fromm, 2000) available for the future. Thus after a period when the past flows of lixiviated nitrates have significantly exceeded the assimilative capacity, a given volume of current nitrate flows will cause a greater contamination of the surface waters[5].

1.5.2 A standard configuration of dynamic externalities

Our model accurately reflects this mechanism of dynamic agricultural externalities caused by the degradation of the riparian ecosystem's assimilative capacity. The

[1]See for example Vitousek et al. (1997).

[2]See Correll (1996) for a comprehensive survey of the abundant ecological literature on this topic.

[3]Their filtering activity also targets sediments, pesticides and other nutrients such as phosphorus. Empirical studies, such as Peterjohn and Correll (1984), are able to compute estimates of the removal rate of nitrates by a given riparian zone.

[4]Vitousek et al. (1997) explain that "*in theory when an ecosystem is fully nitrogen saturated and its soils, plants, and microbes cannot use or retain any more, all new nitrogen deposits will be dispersed to streams, groundwater and the atmosphere*".

[5]See Hansjürgen (2004, p.250): "*[...] agricultural soil's absorption ability is at risk of being depleted owing to the high input of nutrients*".

socio-economic damage described above is a true flow externality[1], depending on the current level of the riparian zone's assimilative capacity. As the private short-term benefit of farmers stems directly from the use of fertilizers or on the quantity of cattle-stock, it amounts to a benefit function similar to ours. Given the unavoidable need for fertilizers in farming and the direct link between cattle-stock and animal manure, no major technological change is to be expected in those fields to reduce the pollution[2]. Hence the relevancy of our framework without technological change.

The restoration of riparian buffer-zones

Given the filtering functions they offer[3], riparian buffer zones are a very useful tool of proactive environmental management. A vast body of ecological literature advocates a proactive use of these ecosystems that may be extended (adding "riparian strips") or introduced from scratch to improve the overall assimilative capacity of the buffer zone. Although many specific factors such as hydrology, wind, temperature, plant size and type, play a part in the determination of the intensity of the filtering service through denitrification and other biochemical processes, in some areas the buffer strips offer a somehow flexible management tool that renders reversible a previous depletion of the overall assimilative capacity[4] according to the mechanisms described previously. There are several example of environmental programs, especially in the United States [5] that focus on the protection and on the artificial restoration of riparian buffer zones. The restoration of the assimilative capacity of these ecosystems demands a preliminary thorough analysis of the riparian zone features (plant species portfolio, soil acidity levels, etc.) and potential threats as well as a careful monitoring of its retaining capacities. The actual restoration consists in revegetating (Anderson and Ohmart, 1985; Hubbard et al., 1995), increasing the buffer area through the delineation of riparian corridors[6], stabilizing the river banks. An extensive overview

[1]We ignore the specific contexts where the damage can be time-lagged depending on the properties of the receiving ecosystem. See for example Yadav (1997).

[2]Behavior changes such as the prohibition against ploughing at specific periods or the use of nitrate-trap cultures like mustard (see Mollard, 1997; Viavattene and Monget, 2004) can nonetheless help reduce lixiviation.

[3]We will focus on the sink function they provide and ignore the additional benefit they yield such as recreational amenities and biodiversity conservation.

[4]We can consider that if a buffer strip's assimilative capacity has been impaired in one place, another strip can be introduced or extended in order to restore the global capacity.

[5]Among others, the famous Watershed and Clean Water Grants Program protecting New York City's water supply through the use of riparian forest buffers in the Catskills region.

[6]The Catskills Watershed Program includes financial incentives for land-owner to expand the riparian area on their property.

of riparian zones restoration methods and case studies can be found in Goodwin et al. (1997). As the actual restoration processes necessarily may imply some thresholds effects, our continuous restoration function cannot systematically reflect the empirical restoration mechanisms but this necessary simplification does not affect the general scope of our results nor the policy message they convey. The costs of these restorative tools can be known and are estimated for specific sites (Anderson and Ohmart, 1979). Our convexity assumption on the cost function seems very reasonable given the type of costs incurred.

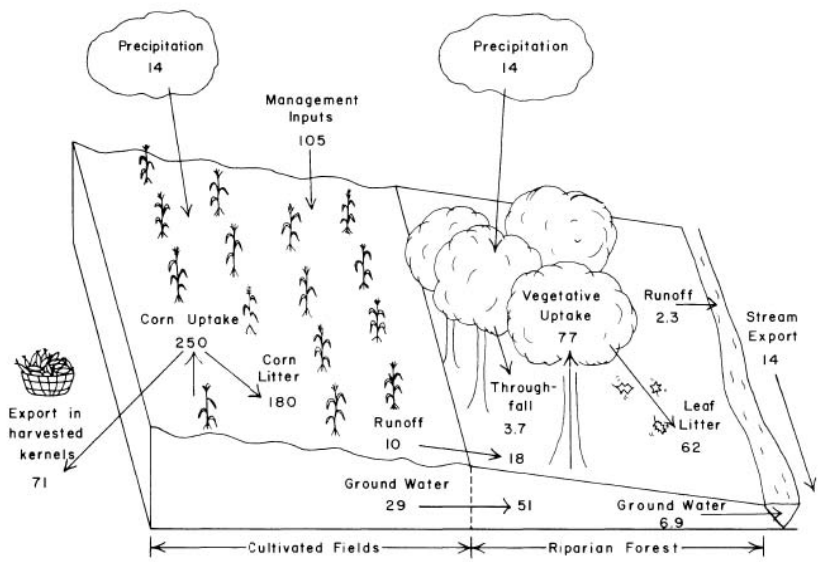

Figure 1.10: The protection of watersheds by riparian buffer ecosystems (from Peterjohn and Correll, 1984)

1.5.3 Operational interpretation of the theoretical results

The application of our formal results to this concrete problem leads us to consider with caution the somehow short-sighted use of the static internalization of external effects, whether it is implemented by way of quotas or taxes[1]. We note that the truly optimal level of pollution (determining the level of taxation in the case of an eco-tax for example) must take into consideration the dynamic impact of nitrates on the ecological conditions of the buffer ecosystems, thereby imposing a stricter

[1]It must be remembered, however, that as a prime example of non-point pollution, nitrate regulation remains difficult to operationalize with standard instruments. See Segerson (1988).

constraint on the use of nitrates. An environmental policy based on cost-benefit analysis should dedicate the necessary means to the identification and the monitoring of the assimilative capacity of the buffer zones, so as to guarantee that a sustainable optimal pollution path can be followed. Despite the operational difficulties arising from our imperfect knowledge of the denitrification mechanisms at stake, our analysis advocates an increased focus on the assimilative properties of the ecosystems involved and provide for a shadow pricing for these properties that may be used to implement an efficient tax policy and to back up a restoration program.

Attempts to implement the European water standards in France have included the use of economic instruments such as an input tax on fertilizers, based on a theoretical optimal level of pollution per hectare. Implementing a dynamic tax such as the one we theoretically designed in the previous sections requires an efficient monitoring of the assimilative capacity that can be challenging but is not impossible[1]. What's more, the policy message of our results on the optimal restoration path should be interpreted as a rationale to design and implement active riparian zone management programs and to extend the basic conservation goals to restoration objectives.

1.6 Interpretation and illustration of the results

Our formal analysis sets forth two significant theoretical results that can be illustrated with a concrete example.

1.6.1 Stricter regulation to prevent overshoot cycles

We have shown with Proposition (1.1) that the dynamic pollution optima in the two settings of dynamic externalities ought to be stricter than those obtained under classic static optimization. Since the intertemporal ecological effect of pollution as well as the marginal flow damage must be internalized, the economic instruments of environmental regulation based on internalization of external effects (such as eco-taxes and emission permits) should be calibrated in a stricter way. This result is valid for both the restoration and the no-restoration cases and must therefore be considered

[1]Such a monitoring of the different assimilative capacity of the soils is already part of the French Plan for Nitrates, waiting to be implemented since 2002.

in addressing any actual flow-pollution problem involving assimilative capacity. It is all the more imperative to question static optima as they can be responsible for an environmental overshoot cycle leading to extinction of the assimilative capacity and depriving future generations of a significant natural asset. Although social and/or political factors might incline the policymaker to guarantee a steady level of production/pollution, if this level is in significant excess of the ecosystem's assimilative capacity it will trigger an unsustainable cycle that will affect society as a whole, including the agents who overexploited the resource in the first place as a result of myopic pollution standards.

1.6.2 Shadow price and optimal pigouvian tax

Propositions (1.2) and (1.3) state that the sustainability of the optimal pollution path depends on the initial level of assimilative capacity and the rate of discount. This result is very similar to the fundamental result found in the seminal literature on optimal environmental quality (Barbier and Markandya, 1990) and optimal resource extinction (Cropper et al., 1979).

Assuming the initial conditions for sustainability are met, our analysis allows the determination of the shadow price of the assimilative capacity λ as a means ensuring that the economy will follow the optimal path leading to a sustainable steady state. As such, this shadow price can play a complementary role in setting the pigouvian tax that could implement the optimal pollution policy. Whereas a pigouvian tax in a standard flow pollution problem would be based exclusively on the marginal flow damage corresponding to the static pollution optimum, in our dynamic framework this tax needs to internalize the detrimental effect of assimilative capacity depletion. The new shadow price must thus be incorporated into the tax, which will follow its variations (see Appendix E).

Another interesting result of our model is that when the restoration option is available it can be optimal, under certain initial conditions, to start on the optimal path with net restoration. When $\bar{A}$ is high enough, this restoration transforms the "unsustainable zone" into a "restoration zone". Our model thus outlines the fact that the optimality of restoring the assimilative capacity instead of depleting it is tantamount to the optimality of investing in (natural) capital instead of depleting it.

In this case, future flows of services from this capital stock more than offset the short-term benefit of consuming it beyond its regeneration threshold. From this standpoint, we suggest that our model helps to restore symmetry between natural and physical capital in the standard economic analysis framework that tends to treat them in an asymmetrical manner (Godard, 2006).

Finally, the explicit determination of the shadow price of this function often left out of economic analysis and policymaking is a step in the overall contribution of economics to reach the Millennium Ecosystem Assessment goals set up by the United Nations (2005). In this perspective, economists are expected to "estimat[e] shadow prices of such vital assets as local and global ecosystems and the services they offer" (Dasgupta, 2009) and this is precisely what we tried to do here.

1.7 Conclusion: towards stricter environmental standards

The main contribution of this work is to build a stylized model that more precisely accounts for the ecological processes at work in flow pollution problems and to draw some policy conclusions in terms of environmental regulation. Following Pearce (1976) and Godard (2006), our model draws attention to the necessary inclusion of the environment's regenerative conditions in economic frameworks that tend to ignore them. The focus on the assimilative capacity as an autonomous variable contributes to the identification of the implicit ecological function at work in some flow pollution problems. Furthermore, we show that the introduction of the restoration process of the assimilative capacity can allow the economy to avoid the unsustainable paths linked to a low initial capacity. We thus join Cesar and de Zeeuw (1994, p.44) in their call for future research to "get a better grip on the assimilation function".

The bottom line conclusion of this work it to call for increased caution in the management of ecosystems threatened by pollution, on a local and global scale. Through our original model we have shed some light on the possible "vicious cycle" initiated by an excess of pollution which causes social damage and decreases the assimilative capacity threshold, thus making it more likely that future pollution will be in excess as well. Our results recommend stricter environmental regulation when assimilative capacity is at stake. We have thus demonstrated that optimal pollution levels ought to be stricter than the standard static level once assimilative capacity dynamics are taken into consideration. But more importantly, our framework of analysis has shown the partial validity of Pearce's results (1976) on the risk of assimilative capacity depletion. Under certain typical conditions (high discount rate), and even when restoration is feasible, amended cost-benefit analysis will recommend the optimal extinction of assimilative capacity unfavorable initial environmental conditions (a low level of assimilative capacity). In terms of *a minima* sustainability, this tendency of the optimal pollution control is truly problematic. Our conclusive stance on the issue, in terms of policy recommendation, is thus to support the use of cost-benefit analysis when the initial environmental conditions are solid and to discard it for a safer approach when they are already fragile. We shall see in the next chapter if this conclusion can be extended to stock pollution problems as well. It must be reminded that as was noted previously determining *ex ante* if the initial level of assimilative capacity is "solid enough" to sustain a cost-benefit analysis remains tricky as it is often the case, with riparian buffers for instance, that the assimilative capacity's actual level is known only when it has been exceeded and is hard to monitor.

An interesting extension of this model is to introduce uncertainty into the dynamics of the assimilative capacity. It must be noted that though there is a vast body of literature addressing optimal pollution within a stochastic framework (uncertainty on the intensity of damages or on the evolution of abatement technologies), only Heal (1984) regards the assimilative capacity itself as uncertain. Our original specification of assimilative capacity as a state variable provides a comfortable framework in which to explore this issue. Identifying the assimilative capacity with a particular type of renewable resource will indeed allow us to apply uncertainty to both the initial level of assimilative capacity available A_0 and the degradation function h. With a little adaptation effort, the numerous results of the literature on the management of renewable resources under uncertainty could thus be usefully applied to the pollution control configuration.

Appendix Chapter 1

Appendix A: concavity of the Hamiltonian

We assume that we have $U_{pp}U_{AA} - U_{pA}^2 \geq 0$ to ensure the concavity of the Hamiltonian, e.g., $(f_{pp} - D_{pp})D_{AA} - D_{pA}^2 \geq 0$. This property can be proved easily for a simple functional form such as $D(p, A) = D(p - A)$. Under this specification we can show, after simplifications, that

$$U_{pp}U_{AA} - U_{pA}^2 = -D"f" + D"^2 - D"^2 = -D"f" \geq 0$$

Appendix B: determination of the isoclines

According to (1.27):

$$\dot{\lambda} = 0 \Rightarrow \lambda(\delta + h_A) + D_A = 0$$

Let us have the function $M(A, \lambda)$ such that $M(A, \lambda) = \lambda(\delta + h_A) + D_A$. The theorem of implicit functions gives us

$$\frac{d\lambda}{dA} |_{\dot{\lambda}=0} = -\frac{M_A}{M_\lambda} \tag{1.35}$$

$$= -\frac{\frac{dp(\lambda,A)}{dA}(D_{Ap} + \lambda h_{Ap}) + \lambda h_{AA} + D_{AA}}{\delta + h_A + \frac{dp(\lambda,A)}{d\lambda}(\lambda h_{Ap} + D_{Ap})} \tag{1.36}$$

From equation (1.15) and the properties of h and D, we can easily show that when $p > A$, $\lambda h_{Ap} + D_{Ap} = \lambda h_{pA} + D_{pA} f_{pA} = 0$. When $p \leq A$, according to the properties of h and D, we know that $h_{Ap} = D_{Ap} = 0$. Hence for any p and A along the optimal path we have

$$\frac{dp(\lambda, A)}{d\lambda}(\lambda h_{Ap} + D_{Ap}) = 0$$

As the denominator of the right-hand side of expression (1.36) is positive, we can write

$$sgn\left(\frac{d\lambda}{dA} |_{\dot{\lambda}=0}\right) = -sgn(\delta + h_A)$$

Applying the same method to (1.28) gives us

$$\frac{d\lambda}{dA} |_{\dot{A}=0} = -\frac{\frac{dp(\lambda,A)}{dA}h_p + h_A}{\frac{dp(\lambda,A)}{d\lambda}h_p}$$

and since $h_p = -h_A$ for any (A, p), we can write

$$\frac{d\lambda}{dA}\mid_{\dot{A}=0} = \frac{(1 - \frac{dp(\lambda,A)}{dA})h_p}{\frac{dp(\lambda,A)}{d\lambda}h_p}$$

We can easily show that $\frac{dp(\lambda,A)}{dA} < 1$ for any A as equation (1.26) leads to a contradiction with the initial assumptions if $\frac{dp(\lambda,A)}{dA} \geq 1$.

Hence

$$\frac{d\lambda}{dA}\mid_{\dot{A}=0} < 0$$

Appendix C: steady state analysis

The No-restoration case

Behavior of I_λ

Let us determine the y-axis and x-axis intercepts of I_λ. To estimate the y-axis intercept of I_λ, we must study the behavior of equation (1.17) when A tends towards 0:

$$\lim_{A\to 0} \lambda \mid_{\dot{\lambda}=0}$$

From the properties of h_A and D_A applied to equation (1.17) when A tends towards 0, we get

$$\lim_{A\to 0} \lambda \mid_{\dot{\lambda}=0} = \lim_{A\to 0} -D_A = L$$

For the x-axis intercept, we define $p_\lambda(\lambda, A)^{-1}$ as the inverse function of $p(\lambda, A)$ for A given. Let us choose $\lambda = p_\lambda(\lambda, A)^{-1}(x_p)$. Equation (1.17) with $p = p(\lambda, A)$ yields

$$\begin{aligned}
\dot{\lambda} &= p_\lambda(\lambda, A)^{-1}(x_p)(\delta + h_A(p_\lambda(\lambda, A)^{-1}(x_p)), A)) + D_A(p((p_\lambda(\lambda, A)^{-1}, A)(x_p)) \\
&= p_\lambda(\lambda, A)^{-1}(x_p)\delta - (h_p(x_p, A)) + D_A(x_p, A)) \\
&= p_\lambda(\lambda, A)^{-1}(x_p)\delta - f_p(xp) = p_\lambda(\lambda, A)^{-1}(x_p)\delta
\end{aligned}$$

On the isocline, this writes

$$\dot{\lambda} = p_\lambda(\lambda, A)^{-1}(x_p)\delta = 0$$

so

$$p_\lambda(\lambda, A)^{-1}(x_p) = \lambda = 0$$

and

$$\lambda \mid_{\{\dot{\lambda}=0\}\cap\{p((\lambda=x_p,A)\}} = 0$$

The x-axis intercept of I_λ is x_p.

Behavior of I_A

Given the economic meaning of λ, $\lambda = 0$ on the optimal path is equivalent to a situation where a marginal variation of A has no effect on the welfare. Taken at $A = p = x_p$, this would mean (see equation (1.16)) that

$$D_p(x_p, x_p) = -D_A(x_p, x_p) = 0 \tag{1.37}$$

The interpretation of this condition is that for a sufficiently high level of assimilative capacity ($A \geq x_p$ for example), a marginal variation of A or p, Δ_A or Δ_p will not trigger additional damage and since it will not modify the benefit function either ($f_p(x_p) = 0$ by definition), it will have no effect on the welfare and therefore its shadow value $\lambda\Delta_A$ will be nil.

- If (1.37) is not respected, I_A does not cross the x-axis at x_p, and in our restricted definition set $[0, x_p[$ it is always above the x-axis.
- If (1.37) is respected, then I_A crosses the x-axis at x_p and the two isoclines intersect at two equilibrium candidates (A_{ss}, λ_{ss}) and x_p. But since x_p can never be reached in the irreversible configuration, it boils down to the first case.

Regarding the y-axis intercept, since the limit of D_A when A tends towards 0 is finite (equation (1.2)), there is no reason for $\lambda \mid_{\dot{A}=0}$ to tend towards infinity when A tends towards 0, contrary to what happens with the I_λ curve. Therefore the y-axis intercept of I_A is finite.

The Restoration case

The geometrical properties are identical to those in the previous case. If condition (1.37) is not met, the two isoclines intersect at a unique equilibrium candidate

(A_{ss}, λ_{ss}) that is a saddle point. If condition (1.37) is met the two isoclines intersect at two equilibrium candidates (A_{ss}, λ_{ss}) and x_p. In the reversible configuration, it is not impossible *a priori* to have $A(t) = x_p$. However the phase-diagram shows that given the motion vectors defined by the isoclines around x_p, x_p cannot be on an optimal path and it boils down to the first case again.

Appendix D: conditions of optimal extinction

No-restoration case

Let us shed some light on the extinction trajectory. According to our phase diagram in Figure 1.4, we know that an optimal path initiated for $A_0 < A_{ss}$ cannot lead to a "sustainable" steady state. We can nevertheless characterize such a path more precisely under reasonable assumptions.

In particular, such a path can lead to the complete depletion of the assimilative capacity if $p^*(t) > A^*(t)$ for all $A^*(t) \geq 0$ and especially if $p^*(t) > 0$ when A tends towards 0. This is the case if and only if the marginal degradation of the assimilative capacity triggered by a strictly positive level of pollution has a positive welfare effect for all $A \geq 0$. According to the first-order condition (1.16), this implies

$$f_p(p) \geq D_p(p, A) + \lambda h_p(p, A) \quad \forall \ A \geq 0$$

We know from equation (1.6) that $h_p(p, A)$ tends towards 0 when A tends towards 0. The depletion path leads to extinction if and only if

$$\lim_{A \to 0} (f_p(p) - D_p(p, A)) \geq 0 \tag{1.38}$$

Let us focus on the most interesting case where Assumption (1.21) holds. In that case we know from equation (1.23) that $p^*(t) \leq \bar{p}(t)$ for all t. Given the concavity of U we have $U_p(p^*(t), A^*(t)) > U_p(\bar{p}(t), A^*(t))$. Still assuming that Assumption (1.21) holds, this yields

$$\lim_{A \to 0} (f_p(p^*(t)) - D_p(p^*(t), A^*(t))) \geq (f_p(\bar{p}(t)) - D_p(\bar{p}(t), A^*(t))) = 0$$

Condition (1.38) is thus verified for all $A \in [0, A_0[$. The optimal path originating in the extinction zone leads systematically to the extinction of the assimilative capacity. It will depend on the functional forms of f, D and h to determine if this extinction takes place in a finite time or not. A possible economic interpretation of this behavior is that if a strictly positive pollution level has always a positive marginal welfare effect, it is not worthwhile to preserve a low initial stock of assimilative capacity. It is more efficient, on pure economic grounds, to totally deplete the assimilative capacity in order to reap the benefits of pollution today rather than allowing for a higher damage-free pollution in the future.

Appendix E: optimal emission tax with assimilative capacity

Let us describe synthetically the polluter initial private's program:

$$\max_p W^p = \int_0^{+\infty} f(p(t))e^{-\rho t} \tag{1.39}$$

The polluter is not concerned by the assimilative capacity level, hence the absence of the state variable $A(t)$. The trivial solution to this problem is to emit $p^{**}(t)$ such that

$$\forall t \quad p^{**}(t) = x_p \tag{1.40}$$

If the social planner imposes a dynamic tax $\tau(t)$ on the polluting emissions[1], it modifies the polluter's social problem into W_2^p

$$\max_p W_2^p = \int_0^{+\infty} (p(t) - \tau(t)p)\, e^{-\rho t} \tag{1.41}$$

Taking the tax into account, the solution for this new private problem is

$$\forall t \quad f'(p^{***}(t)) = \tau(t) \tag{1.42}$$

Therefore the social planner must calibrate τ such that it puts the economy on the optimal path. Given the initial level of assimilative capacity, the economy will either follow a restoration path or a depletion path and the value and the behavior of τ will be different on these paths. In order to avoid significant mathematical complexities

[1]We have already mentioned the informational problems linked with the target of the tax (fertilizing inputs, actual nutrients leakage...?).

and to focus on the policy-oriented conclusions, we shall work here with a linear form of $h(p, A)$ that preserves the necessary properties of the problem:

$$h(p, a) = \gamma(p - A)$$
$$\gamma > 0$$

Let us distinguish the two cases: the restoration tax and the depletion tax.

Restoration Tax

We know from (1.34) that along a restoration path we must have[1]

$$f_p(p) = \lambda h_p(p, A) = \lambda\gamma$$

Since we can determine $\lambda(t)$, the shadow price of assimilative capacity, at any time t, we simply need the following emission tax to put the economy on the optimal path:

$$\tau(t) = \lambda(t)\gamma \quad \forall t$$

According to (1.42) this tax will force the polluter to follow the optimal restoration path.

Depletion Tax

We know from (1.15) that along a depletion path we must have

$$f_p(p) = Dp(p, A) + \lambda\gamma$$

Since we can determine $p^*(t)$ from $\lambda(t)$ and $A^*(t)$, we know the value $V(t)$ of $D_p(p^*(t), A^*(t))$ at any time t and we can design a tax such that:

$$\tau(t) = V(t) + \lambda(t)\gamma \quad \forall t$$

This tax, according to (1.42) will force the polluter to follow the optimal depletion path.

[1] We ignore here the Lagrangian multiplier as we focus on a path where the maximum level of restoration is not reached.

Appendix F: Noise-induced stress and human assimilative capacity

The response of ecosystems to some sources of external stress, such as anthropogenic pollution, is somehow similar to the response of the human organism to stress. Comparing the mechanisms at work when the human organism tries to cope with stress can prove useful for the understanding of the specificity of our model. Let us suppose that an individual with no mobility (in his office or in his house) has a tolerance level for noise (the loud music played by a neighbor for example) θ, meaning that any sound level below θ has absolutely no psychological effect on him/her but that a sound above θ will bother him. The higher the sound compared to his tolerance level, the higher the disturbance. It seems reasonable to assume that the tolerance level θ itself will not be affected across time if all the noise sustained are below θ. In this case, no stress is accumulated by the organism. However, a succession of noise excesses, ie $p(t) > \theta(t)$ for several t, will cause, in additional to the punctual stress $D(p-\theta)$, a reduction in the tolerance level θ over time. To put it crudely, a repetition of noise excesses makes the individual more "touchy", psychologically exhausted and more sensitive to noise, and a level of noise that he/she could tolerate before will be annoying if it comes after numerous occurrences of bothering noise. This process can be seen as the opposite of an adaptation process through which an individual can get accustomed to a common noise in his environment such as trains passing by. We can therefore refer to it as a "des-adaptation" process.

The extension of this analogy to the restoration process introduced in the pollution problem is less straightforward. On the one hand, it could be argued that if the individual is given a rest, that is to say that not only he/she is no longer exposed to noise excesses, but that the noise he/she is surrounded with is significantly below his current tolerance threshold θ, then he/she will "restore" his/her tolerance level up to a higher threshold, enabling him to cope in the future with higher noise levels than before. On the other hand, the reverse phenomenon can also be considered: an individual enjoying a very quiet environment for a given period might be bothered by noise levels that left him/her unaffected when he/she was under more external stress. No unambiguous conclusion can be drawn on this side of the mechanism but our analogy can nonetheless contribute to a better understanding of the assumptions made on the response of living organisms to external stress.

Chapter 2

Assimilative capacity degradation and optimal stock pollution control

2.1 Introduction

Ecological evidence asserting climate change related feedback loops has accumulated at a concerning rate over the last decade. These feedback loops are estimated to be responsible for more than a third of the global temperature increase caused by an increase in the radiative forcing of greenhouse gases (Cleveland et al., 2000). Regarding CO_2 alone, these positive feedbacks[1] can take the form of changes in albedo associated with a reduction in snow cover, decline in cloud cover, reduced photosynthesis productivity and, more importantly, reduction of the ocean CO_2 uptake due to changes in the thermohaline currents and in the wind patterns (Le Quéré et al., 2007). According to the IPCC Report (2005, Chapter 10) there is "unanimous agreement among the coupled climate-carbon cycle models driven by emission scenarios run so far that future climate change would reduce the efficiency of the Earth system (land and ocean) to absorb anthropogenic CO_2".

The scientific evidence gathered so far is characterized by a very strong uncertainty on the magnitude of these feedbacks but the most conservative results remain a source

[1] The feedback is "positive" in terms of the correlation with climate change, although it is clearly "negative" for our planet and its inhabitants. The negative feedback of climate change, due to the increased rate of photosynthesis among terrestrial and oceanic vegetation, may no longer increase as the CO_2 concentration increases according to the IPCC.

of significant concern (Friedlingstein et al., 2003). The recent study by Raupach et al. (2007) estimates that due to the positive feedbacks, global warming will increase by an extra 15% to 78% on a one century scale with respect to the baseline scenario. Most studies agree that these feedback mechanisms, although expected, are occurring much sooner than it had been foreseen and represent a significant part of the total increase in the atmospheric concentration (18% according to Raupach et al., 2007). Another study by Knutti et al. (2003) show that these climate-carbon cycle feedbacks are responsible for an increase of about 0.6 C° in the "average" scenario and of about 1.5 C° when the upper bound of the uncertainty range is considered.

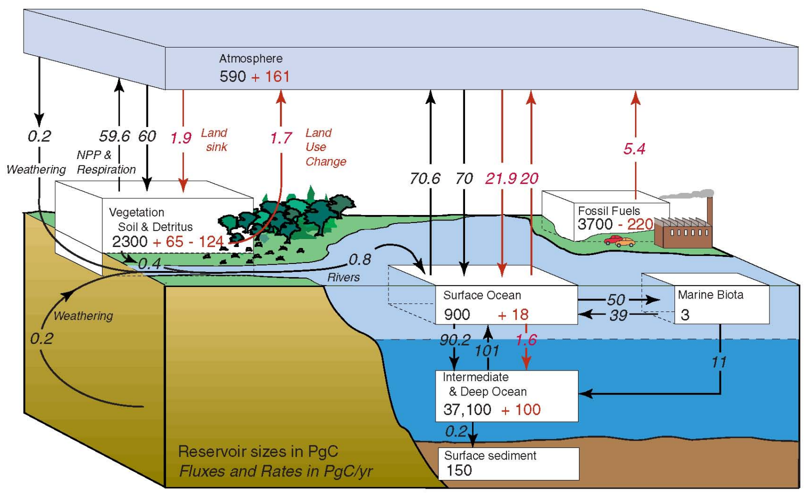

Figure 2.1: The global carbon cycle (Sarmiento and Gruber, 2002)

From an economic perspective, this alteration of the global carbon cycle (see Figure 2.1) might have a tremendous impact on the mitigation strategies as they demand increased emission reductions or sequestration in order to achieve the same stabilization objective (set around 350 ppm, 450 ppm or as high as 550 ppm depending on the experts). In its general summary, the Stern report on climate change (2006) estimates the damages of climate change to amount to 5% of the world yearly net product on the "business as usual" scenario[1]. The report insists on the uncertainty characterizing the potential feedbacks and acknowledges that the yearly damage could be as high as 20% in the worst case scenario including feedbacks[2].

[1]In comparison, an active policy aiming to stabilize the atmospheric concentration at 550 ppm would require only 1% of the net product according to this report.

[2]Although the magnitude of its estimated damages from climate change and its related policy recommendations have been much debated (Nordhaus, 2006; Sterner and Pearson, 2007; Godard, 2008), the relative impact of feedback mechanism highlighted by the Stern report has not been

This concerning issue makes it all the more urgent to adapt the pollution control economic models to this ecological feature. The standard linear representation of the natural decay activity with a constant rate of assimilation has already been discussed by different authors such as Forster (1975), Tahvonen and Salo (1996), Tahvonen and Withagen (1996), Toman and Withagen (2000) or Chevé (2000). These contributions have tried to introduce more realistic decay function, such as the concave-convex function. These attempts to describe more accurately the ecological mechanisms at stake imply the introduction of non-convexity in the dynamic optimization, which brings up heavy mathematical complications. The main conclusion from these contributions is the existence of multiple equilibria associated with either a positive or an irreversibly depleted assimilative capacity and the impossibility for the affected ecosystem to return to its initial state when it has reached an irreversible basin of attraction.

However as we have stressed in the General Introduction these natural decay functions display, like the standard assumption of a constant rate of decay, an exclusive dependency on the pollution stock variable Z that does not fit well the ecological reality. Indeed, according to empirical ecological evidence it seems reasonable in a wide range of cases to assume that this assimilative capacity will not be only impacted by the absolute stock level, but also by the accumulation path that leads to this specific amount of pollution. Consequently this assimilative capacity cannot simply increase back to a higher level if the pollution stock decreases as these models assume. The irreversibility of the degradation of the assimilative capacity should thus be reflected in its own autonomous dynamics. The assimilative capacity should not depend directly on the total accumulated stock of pollution but should follow dynamics on its own right that are determined by the stock of pollution relatively to a degradation threshold.

Our approach is inspired by the intuitions developed by Pearce (1976) on the environmental degradation cycles triggered by excessive pollution. In a simple graphic model with a myopic social planner, Pearce highlights the degradation of assimilative capacity that runs in parallel with social-environmental damage and questions the capacity of standard discounted cost-benefit analysis to determine optimal pollution paths that do not result in the complete depletion of the assimilative capacity of the

questioned.

environment. Despite the questionable validity of its results from a purely technical point of view, Pearce's assumption on the evolution of assimilative capacity of an ecosystem receiving polluting emissions proves quite appropriate to build a dynamic model that fits the requirements mentioned just above. Following the extensions of Pearce's work carried out by Pezzey (1996) and Godard (2006), our model will thus partially build on Pearce's propositions to account for the autonomous dynamics of the assimilative capacity. It must be noted that although the ecological feedbacks described above are characterized by heavy uncertainty we shall work here in a deterministic framework. Introducing formally this uncertainty is beyond the scope of this paper but it is a very interesting lead for further inquiry.

The formal transcription of Pearce's argument implies that any degradation of assimilative capacity is irreversible. In the other models mentioned above, if the pollution stock has not reached the irreversible threshold, the assimilative capacity can be increased back to a higher level simply through a decrease in this stock of pollution since it is a direct function $A(Z)$ of this stock. Consequently, none of these models allow for a deliberate effort of restoration of assimilative capacity that would be decided by the social planner. Similarly to what we have seen in Chapter 1 for flow pollution, there are various cases of stock pollution configuration where the assimilative capacity can be artificially or naturally regenerated. The particular case of climate change offers vocal illustrations of this lever at the disposal of society to restore or maintain the CO_2 assimilative capacity. "Natural" options such as afforestation or reforestation can increase, or at least partially offset the loss of carbon assimilative capacity due to climate feedbacks. In addition, the serious advances of carbon capture and sequestration technologies (Lackner, 2002) can provide an artificial answer[1] to the question of CO_2 assimilation. It seems thus of true interest to introduce this restoration option in a stock pollution control model in order to analyze the possible trade-offs between consumption and assimilative capacity restoration. As far as we know there has not been yet any explicit contribution considering this policy option but our modeling proposition of the assimilative capacity as a state variable offers a promising framework for this inquiry. Here we propose to extend

[1]The recent projects of geo-engineering strategies (Blain et al., 2007), such as increasing the ocean's CO_2 uptake through iron fertilization, could provide solutions to compensate the climate feedbacks according to their advocates, but as noted in the General Introduction their effectiveness if far for being asserted yet.

the argument on assimilative capacity degradation and to build on the Pezzey-Pearce model to shed some light on these trade-offs and stress the importance of natural capital maintenance.

Our analysis will follow four steps. First we recall in Section 2 the benchmark model of pollution control with constant invariant assimilative capacity rate and study more precisely the sensitivity of the resulting optimal paths to variations in the assimilative capacity level. In doing so we derive some preliminary insights on the impact that would have the introduction of ecological feedbacks into a pollution control model. In a second phase (Section 3) we present the basic results obtained by Pezzey (1996) in his attempt to provide a formal version of Pearce's intuitions. In Section 4 we extend this analysis through a comparative approach, highlighting the major consequences of assimilative capacity degradation feedbacks on optimal pollution paths. An exploration of the broader perspectives opened by the introduction of restoration efforts in an enriched version of the Pezzey-Pearce model is initiated in Section 5. Section 6 concludes with the policy-oriented interpretation of our results and points out potential extensions.

2.2 Stock pollution benchmark model

In order to draw some qualitative results from the models studied afterwards that are not very tractable, we will need to compare them to the benchmark model of pollution control without capital accumulation. Interestingly enough, the seminal stock pollution control model without capital accumulation is not described thoroughly in its simplest version in any academic contribution that we know of[1], besides its use in textbook demonstrations and introductory classes to optimal control.

We believe that it is thus necessary to recompute this model and review its main properties in order to base our analysis on a set of benchmark results.

A preliminary remark on the notations

In the previous chapter we used $p(t)$ as the control variable representing the flow

[1]With the exception of a prototype model given as an example in Kamien and Schwartz (1984). The model developed by Ulph and Ulph (1994) is more complicated because it introduces an additional state variable: global temperature.

of *pollution* emitted at time t by a productive activity. We hereby assumed that the production/pollution ratio was equal to one with the appropriate units. In this chapter we shall use a variable $y(t)$ to denote the level of *production* at time t to focus on the trade-offs that will arise between production and environmental maintenance expenditures and ensure that we combine variables that have the same physical nature. We will nevertheless assume once again a production/pollution ratio equal to one and y will subsequently represent also the flow of emissions.

As in most social optimization problems, we work in a discounted utilitarian framework with a social welfare function including the utility of consumption and the environmental damage, with ρ the social discount rate,$1 > \rho > 0$, supposed constant. This standard model writes, with our notations:

$$\max_{y(t)} \int_0^\infty [f(y(t)) - D(Z(t))]\, e^{-\rho t} dt$$

$$\text{s.t. } \dot{Z}(t) = y(t) - \alpha(t)Z(t))$$

$$Z(0) = Z_0$$

Here the assimilative factor is assumed to be constant such that $\alpha(t) = \alpha > 0 \; \forall t$, as it is the case in most optimal pollution control models in the literature.

The most standard assumptions of the literature are used. The utility/profit provided by a given level of pollution (e.g., the maximized profit of a polluting firm given its technology) is given by a function f similar to the one used in Chapter 1 such that:

$$f(y)\ ,\ f'(y) > 0\ ,\ f"(y) \leq 0$$

Contrary to the assumption made in Chapter 1 that fits local problems, we shall adopt here the Inada condition that is more relevant when dealing with a global problem such as climate change.

$$\lim_{y \to 0} f'(y) = \infty$$

We have a "natural" upper-bound $\hat{y}$ on y such that

$$0 \le y \le \hat{y} \tag{2.1}$$
$$f(\hat{y}) = 0$$

This restriction allows us to keep the control region bounded.

The damage function displays the usual properties:

$$D(Z)\ ,\ D'(Z) \ge 0\ ,\ D'(0) = 0$$
$$D"(Z) \ge 0\ ,\ \lim_{z\to\infty} D(Z) = \infty$$

The Hamiltonian of this problem is

$$H(t) = f(y(t)) - D(Z(t)) + \lambda(y(t) - \alpha Z(t))$$

where $\lambda(t)$ is the shadow price of the accumulated pollution stock.

2.2.1 First Order Conditions

The first order conditions are[1]

$$f'(y) = -\lambda$$
$$\rho\lambda - \dot{\lambda} = -D'(Z) - \lambda\alpha$$

which can be written as

$$f'(y) = -\lambda \tag{2.2}$$
$$\dot{\lambda} = (\rho + \alpha)\lambda + D'(Z) \tag{2.3}$$

and in addition there is the transversality condition

$$\lim_{t\ \to\ \infty} e^{-\rho t}\lambda(t)Z(t) = 0$$

[1]From now on we shall drop the time index of the variable whenever no ambiguity can arise.

The interpretation of equation (2.2) is quite straightforward: at any time along the optimal path, the marginal benefit of an additional unit of pollution emitted must be equal to the shadow price of the pollution stock, e.g., this benefit must offset the "cost" (it must be noted that λ is negative given the contribution of Z to the welfare function) of this unit added to the current stock. This mechanism is better understood through the analysis of the phase diagram in the next subsection.

Equation (2.3) is more ambiguous. For higher pollution stocks, the absolute value of λ, e.g., the intensity of the shadow "cost", will decrease or at least increase at a slower rate. This can only be interpreted as a consequence of the fact that for high pollution levels, the shadow cost is already very high and it will either start decreasing or, more probably considering the role of a high λ in equation (2.3), increase more slowly. In addition, a higher rate of discount or a higher assimilative factor contribute to accelerating the increase in the shadow cost (in the absolute value of λ). This trend results from the fact that a high discount rate ρ induces a higher level of emissions y because more importance is granted to present benefits than to future pollution stocks. In addition, a higher assimilative factor α makes it possible to emit more while accumulating the same amount of pollution Z. According to equation (2.2) a higher level of emissions corresponds to a lower shadow cost, hence the higher growth rate for low levels of ρ or α.

Derivating equation (2.2) with respect to time yields

$$\dot{y} = -\frac{\dot{\lambda}}{f"(y)}$$

Given the negativity of $f"$, we have

$$sgn(\dot{y}) = sgn(\dot{\lambda})$$

which confirms that an increase in the shadow cost in absolute value triggers a decrease in the level of emissions on the optimal path.

2.2.2 Phase Diagram Analysis

The standard properties of the Hamiltonian guarantee that the steady state exists. We know that we have

$$\dot{Z} = 0 \Rightarrow y = \alpha Z$$
$$\dot{\lambda} = 0 \Rightarrow (\rho + \alpha)\lambda = -D'(Z)$$

Using (2.2), this yields

$$y = \alpha Z$$
$$(\rho + \alpha) f'(\alpha Z) = D'(Z)$$

There exists a unique stead state Z_{ss} if and only if f,D, ρ and α are such that there exists Z_{ss} the unique solution of

$$(\rho + \alpha) f'(\alpha Z) - D'(Z) = 0$$

It is easy to verify that a unique strictly positive Z_{ss} exists for any functional forms of f and D respecting the standard properties of the literature and for any ρ and α positive, as long as we have $(\rho + \alpha) f'(0) > D'(0)$, which is guaranteed by the Inada condition. Along the optimal path, equation (2.2) enables us to define y as a function of λ such that

$$\frac{dy(\lambda)}{d\lambda} > 0$$

We can now use the implicit function theorem to determine the behavior of the isoclines in the (Z,λ) plane. After simple calculations we get

$$\frac{d\lambda}{dZ} \mid_{\dot{\lambda}=0} = \frac{-D"(Z)}{(\rho + \alpha)} < 0 \tag{2.4}$$

The [$\dot{\lambda} = 0$]-isocline is decreasing in the (Z,λ) plane.

Similarly we get

$$\frac{d\lambda}{dZ} \mid_{\dot{Z}=0} = \frac{\alpha}{\frac{dy(\lambda)}{d\lambda}} > 0 \tag{2.5}$$

The $[\dot{Z} = 0]$-isocline is increasing in the (Z,λ) plane.

Finally we find that

$$\lim_{Z \to \infty} \lambda \mid_{\dot{Z}=0} = 0$$

$$\lim_{Z \to \infty} \lambda \mid_{\dot{\lambda}=0} = -\infty$$

Given that $f'(0)$ is assumed to be strictly positive and thus strictly greater that $D'(0) = 0$ there exists a unique steady state which is a saddle point as it appears on the phase diagram in Figure 2.2. We thus characterize the optimal path depending on the initial stock of pollution Z_0.

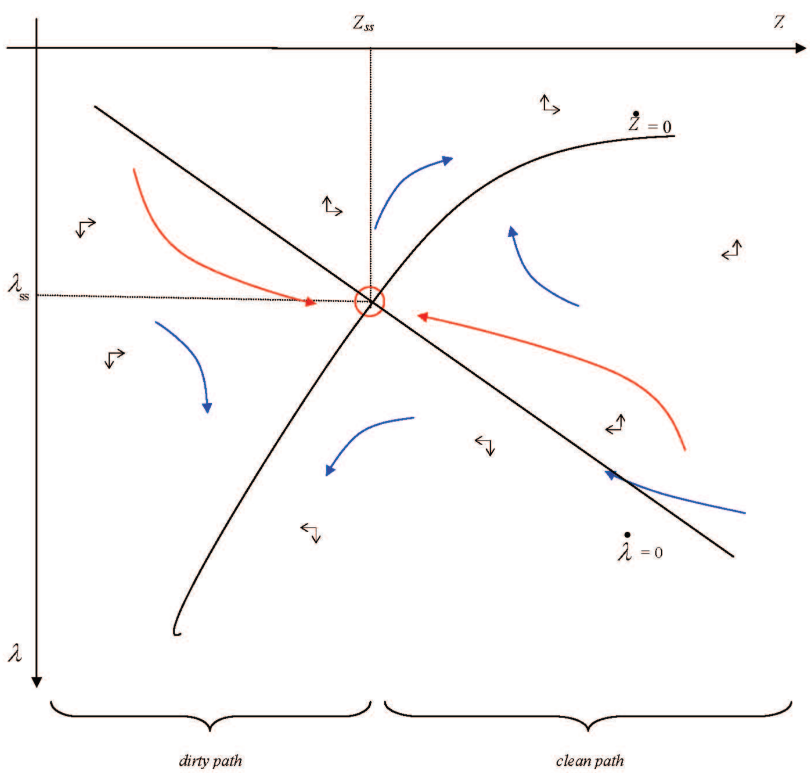

Figure 2.2: Optimal pollution path

Proposition 2.1. *If $Z_0 < Z_{ss}$, the optimal policy is to select λ_0 so as to place the economy on a path that ends at the stable equilibrium Z_{ss}. As $\dot{Z} > 0$ along this path, the level of polluting emissions y exceeds the assimilative capacity αZ all along. The shadow price $|\lambda|$, which can be seen as the level of the optimal dynamic pigovian tax, rises along this path until it is high enough to force the economy to settle at the steady state. We shall denote this path as the "dirty" path.*

If $Z_0 > Z_{ss}$, the optimal policy is to select λ_0 so as to place the economy on a path that ends at the stable equilibrium Z_{ss}. As $\dot{Z} < 0$ along this path, the level of polluting emissions y must be below the assimilative capacity αZ all along. The shadow price $|\lambda|$, which can be seen as the level of the optimal dynamic pigovian tax, decreases along this path until it is low enough to force the economy to settle at the steady state. We shall denote this path as the "clean" path.

This simple model can be applied to the problem of CO_2 emissions and climate change to get some insight on the optimal price of CO_2, and on the evolution of this price in time, a much debated issue (Schubert, 2008; Ulph and Ulph, 1994). According to Proposition (2.1) the trend of variation of the price of the stock pollutant depends on the relative level of the initial stock of pollution Z_0. If this level is higher than the optimal level (supposedly computable) then the carbon price will decrease from a high value until it reaches λ_{ss}. On the contrary, if Z_0 is lower than Z_{ss} then the price of carbon will increase.

2.2.3 Comparative Statics

Discount rate sensitivity

First it is quite clear that a higher discount rate will shift the $[\dot{\lambda} = 0]$-isocline in such a way that the steady state level of pollution will be higher (blue isocline in Figure 2.3). This reflects the standard effect of high discount rates in pollution problems: present benefits from the polluting activity are much more valued than the future damage incurred by accumulated pollution. We shall discuss in more details the role of the discount rate in discounted cost-benefit analysis in the next chapter.

Assimilation rate sensitivity

Of greater interest to our analysis is the sensitivity of the optimal path to the assimilative factor. A variation in α, whether it is exogenous, or endogenous as we will try to specify it later on, could have a significant impact on the optimal trajectory and on the steady state. In this case, a lower constant assimilative factor has a twofold effect. On the one hand, according to (2.4), a lower α will increase the absolute value of the (negative) slope of the $[\dot{\lambda} = 0]$-isocline. On the other hand it will increase the (positive) slope of the $[\dot{Z} = 0]$-isocline according to (2.5). Under this double impact, as shown in Figure 2.3 with the red isoclines, the steady state level of pollution will shift to the left to a much lower level, while the variation of the shadow price cannot be unambiguously determined[1]. Consequently, there are much more chances for the initial level of pollution Z_0 to be greater than Z_{ss} which makes the "pollution reduction" strategies (the clean path) more likely to be optimal.

On the contrary, a higher assimilative factor results in a higher steady state level of accumulated pollution and this increase is steeper that the one incurred by a similar variation in the discount rate. This reaction of the steady state level is not completely intuitive, as we could have expected that a higher assimilative capacity allowed for higher emissions (and thus productive benefits) for a given level of stock pollution. Instead it seems to be optimal to keep on increasing the stock of pollution through excessive emissions.

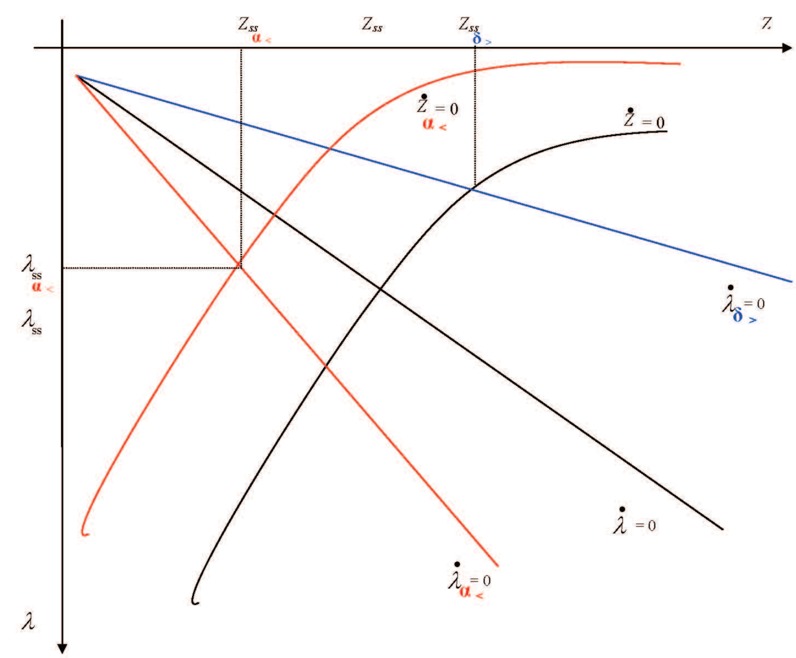

Figure 2.3: Comparative statics

[1]The greater slope of the $[\dot{\lambda} = 0]$-isocline induces a higher shadow price while the shift of the $[\dot{Z} = 0]$-isocline tends to reduce this shadow price.

2.2.4 Insights on the role of assimilative capacity for sustainable pollution policies

This analysis of the most basic pollution control model provides some insights on the role of assimilative capacity in pollution control. First, we confirm the intuition that if the assimilative factor is low in real life, a pollution reduction strategy is more likely to be optimal. Considering the uncertainty surrounding many pollution assimilation mechanisms, it seems only reasonable to assume that this assimilative capacity is low and to adopt a pollution reductions strategy. More importantly, we have identified that the impact of a variation of this assimilative factor is roughly twice as big as the impact of a variation of the discount rate (assimilative factor variations affect both isoclines in the same concurring way). This assessment should bring to light the urgency to take into consideration in a more comprehensive way the assimilative capacity at stake in stock pollution problems, as it has already been advocated by Cesar and de Zeeuw (1994) for example.

In addition to that, we can interpret the effect of a higher assimilative capacity in the perspective of assimilative capacity restoration that we shall try to take into consideration later. The possibility to increase the assimilative capacity could result in an optimal policy that actually leads to a higher steady state level of pollution. This would mean that the option of restoring this capacity would not necessarily enable the regulation policy to achieve a higher overall environmental quality but it would allow a higher levels off consumption in the long run thanks to the enhanced assimilative capacity.

2.3 A first attempt at a formal version of Pearce's intuitions

Since the intuition underlying our approach is based on the environmental degradation cycle identified by Pearce (1976) and explored by Godard (2006), it is quite useful for our purpose to review the model built by Pezzey (1996) to apply Pearce's approach to stock pollution.

The main contribution of this model is to disentangle the level of available assimilative capacity from the level of accumulated pollution. As we reviewed in the General Introduction, pollution control models acknowledging assimilative capacity dynamics have made so far this assimilative capacity depend on the current level of pollution $Z(t)$[1]. With such a formulation, even when it includes sophisticated dynamics, the same level of assimilative capacity will always correspond to the same level of accumulated pollution, regardless of the pollution path that leads to this situation (unless a critical level has been reached and assimilative capacity is forever gone). These specifications, although useful to compare different forms of assimilative capacity as we explained in the General Introduction, thus fail to acknowledge the "history" of pollution, a crucial factor in the determination of the current state of the assimilative capacity. Taking this "history" into account allows to contemplate a much wider variety of situations, whether the given pollution stock has just been reached for the first time or if it is the result of a reduction of pollution from a higher stock level that had been borne for a long time before. As we pointed out various times earlier, an assimilative capacity consistent with the ecological dynamics at stake must follow an evolution in its own right and not be a simple functional image of the stock of pollution.

In this perspective, the most satisfying model found in the stock pollution literature so far is the one suggested by Pezzey (1996) as a formal version of Pearce's (1976) intuition. Pezzey attempts to build a tractable model that translates in a stock pollution framework the main idea of Pearce that we adapted to flow pollution framework in Chapter 1, namely that assimilative capacity is degraded if the pollution stock exceeds a threshold [2]. The major innovation of this model is to define the following dynamics of the assimilative factor while keeping in line with the standard dynamics

[1]This can be said of both the absolute level $A(Z(t))$ of assimilative capacity (Tahvonen and Salo, 1996; Chevé, 2000) and of the assimilative factor form $\alpha(Z(t))$, such that $A(t) = \alpha(t)Z(t)$.

[2]Pezzey also assumes that the threshold for assimilative capacity degradation, $\bar{Z}$, is the same as the threshold for the occurrence of damages, Z_D, although he acknowledges himself that there is no *ex ante* reason for this identification. We already noted in Chapter 1 that Pearce assumes that the threshold at which environmental damage starts is the same as the threshold at which assimilative degradation starts. We believe this assumption makes sense in the flow pollution case discussed earlier ($p > A \Rightarrow D > 0 \bigcap \dot{A} < 0$) but is less obvious in the stock pollution case. There is indeed no ecological proof assessing such an identification. The stock of phosphorus in a shallow lake triggering damage (bad smell or fish population disturbance) is not necessarily the same as the threshold at which the lake's phosphorus assimilative capacity starts decreasing. The same applies to the accumulation of greenhouse gases.

of pollution stock.

$$\dot{Z}(t) = y(t) - \alpha(t)Z(t)$$
$$\dot{\alpha}(t) = -h(Z(t)) \tag{2.6}$$

Here it is not the level of the assimilative factor α that depends on the accumulated stock, but its variation $\dot{\alpha}$. This allows for an autonomous evolution of this assimilative factor across time. This evolution is driven by a degradation function that we note h. It must be acknowledged however that this specification cannot prevent the assimilative capacity rate α from taking negative values, we shall thus restrict our analysis to the realistic case where $\alpha \geq 0$.

This function is such that[1]

$$h(Z) = 0 \;\forall\; Z \leq \bar{Z}$$
$$h(Z) = k(Z - \bar{Z}) \;\forall\; Z > \bar{Z} \;\; k > 0$$

In the end, the total assimilative capacity A still depends partly on the stock of pollution as $A = \alpha Z$ but the "history"' of pollution matters most. As Pezzey acknowledges, the simplifications operated in the degradation function (a linear degradation function independent of the current level of assimilative capacity) are necessary to "make the algebra even remotely tractable".

Irreversible assimilative capacity degradation

The significant contribution of this model, in line with the flow model application we developed in Chapter 1, is that contrary to all the models in the related literature, it is the evolution of the assimilative factor α and not its current level that depends on the stock of pollution Z. In doing so, it discards the rather unrealistic assumption that if the pollution returns to lower levels, the assimilative capacity will increase back also. Here if the pollution stock diminishes after having reached high levels, the assimilative factor will stop decreasing ($\dot{\alpha} = 0$ if $Z \leq \bar{Z}$) but it will not rise back to its initial level. This specification thus addresses the shortcoming of most pollution control models

[1]We resort to a linear degradation process for the sake of clarity but in fact the degradation of assimilative capacity incurred by pollution levels is more likely to be convex as the marginal degradation imposed by an additional unit of pollution is higher for high pollution stocks and we should have thus $h_Z \geq 0$ and $h_{ZZ} \geq 0$.

that do not allow for irreversible assimilative capacity degradation. Here $\bar{Z}$ is the threshold above which assimilative capacity degradation begins[1]. Such a formulation prevents the occurrence of an unrealistic recuperation of assimilative capacity after several periods spent with high pollution stocks. Given the determinant role played by the discount factor in this kind of problem that we shall discuss in Chapter 3, it is straightforward that such an unrealistic possibility to get the assimilative capacity back to a high level as if "nothing happened" will favor pollution strategies drawing important benefits from polluting activities in the short term and worrying about restoring the assimilative capacity only in the long run. That is why the models mentioned previously could produce growth paths that are considered as sustainable because they settle at a steady state with a positive level of assimilative capacity whereas the latter should really have been depleted a long time ago under more realistic specifications. It must be noted that Pezzey, in the wake of Pearce, assumes that any degradation of the assimilative capacity is completely irreversible. We shall explore in Section 6 the possibility of restoring this assimilative capacity.

Continuous degradation vs. threshold effects

In line with the cautious warning we issued in the General Introduction and in Chapter 1, we must recall here that our specification of the degradation mechanisms at stake is considerably simplifying inasmuch as it assumes a smooth continuous degradation of assimilative capacity under excessive pollution. In doing so, we overlook the crucial threshold effects that are known to make an entire ecosystem "flip" overnight, as it has been observed for shallow lakes. These fascinating non-convex problems of stock pollution with threshold reactions to excesses of pollution have been rigorously dealt with in the literature (Dasgupta and Mäler, 2004 (Chapter 1, Section 7.2); Mäler et al., 2003) and we must acknowledge that the introduction of non-convexity in the dynamics of assimilative capacity would bring up technicalities that are beyond the scope of this work. The ecological evidence reviewed in the Introduction of this chapter on the degradation of the biosphere's CO_2 assimilative capacity nevertheless points out to rather continuous phenomena so far. From this perspective our assumption of a smooth degradation function thus seems fitting. However there are some contributions from ecological science that start to draw attention to potential "tipping points"

[1] It must be noted that the absolute level of assimilative capacity does not necessarily decline immediately since $A(t) = \alpha(t)Z(t)$ and when the stock of pollution exceeds $\bar{Z}$, its increase can compensate temporarily for the decrease in α.

in the Earth's climate system (Lenton et al., 2008).

Environmental degradation cycle

Finally it must be noted that this model, as Pearce's argument, is particularly relevant to illustrate a manifestation of an environmental degradation cycle similar to the one highlighted in Chapter 1. The assimilative factor decreases when the stock of pollution exceeds the threshold $\bar{Z}$ making it all the more difficult to maintain the same level of consumption without adding even more pollution to the stock, and subsequently inducing more assimilative capacity degradation. This thus increases the chances to have another increase in the pollution stock and so on until the assimilative capacity is completely destroyed. If a society is more or less bounded by a minimum utility level that must be enjoyed regardless of the ecological feedbacks, it can trigger such an environmental vicious degradation cycle that will lead to the destruction of its environmental asset.

Partial resolution of the maximization problem

The social planner problem now writes

$$\max_{y(t)} \int_0^{\infty} \left[f(y(t)) - D(Z(t)\right] e^{-\rho t} dt$$

$$\text{s.t. } \dot{Z}(t) = y(t) - \alpha(t)Z(t))$$

$$\alpha(0) = \alpha_0$$

$$\dot{\alpha}(t) = -h(Z(t))$$

An immediate conclusion that can be drawn from this model is that the bliss level of pollution stock corresponding to the undiscounted indefinite maximization of social welfare is equal to $\bar{Z}$. Any level of pollution exceeding $\bar{Z}$, at any time along a pollution path, will trigger an irreversible deviation from this bliss level. This "bliss level" emission level, equal to $\bar{z}\alpha(t)$ at any time t, is analogous to the "damage-less" pollution level of Chapter 1: the maximum level of production/pollution that does not reduce the assimilative capacity level. The major difference between these two cases is that in the stock configuration, social-environmental damage takes place even if the assimilative capacity is respected since this damage depends on the current stock.

Pezzey acknowledges that given the complexity of the mathematics involved, his analysis can unfortunately not go beyond these assessments without using numerical simulations requiring special functional forms. His attempts at a partial resolution of the problem are reviewed in Appendix A. He nevertheless establishes three qualitatively results depending on the level of discounting. It must be noted that due to his particular mode of demonstration Pezzey "chooses" the initial level of pollution stock in relation with the other parameters such as the discount rate.

Case a: For a very small ρ the maximum sustainable level $\bar{Z}$ is the optimal solution from the start. Along this path, the assimilative capacity remains unharmed at a constant level $\alpha(0) = \alpha_0$.

Case b: If ρ is higher, then the initial pollution stock (and thus the pollution flow) is high but it decreases quickly and reaches a steady state at $\bar{Z}$ with $\dot{\alpha} > 0$.

Case c: For yet higher levels of ρ, the initial levels of pollution are so high that the assimilative capacity is destroyed before a steady state is reached.

These results highlight once more the crucial role played by the discount rate, vividly illustrated by the recent Stern Review controversy.

2.4 An alternative approach of the Pezzey-Pearce model

As noted in Appendix A, the resolution of this optimal control problem suggested by Pezzey is not entirely satisfactory. Therefore in this subsection, we approach the Pezzey-Pearce model from a comparative perspective to stress the major differences displayed by the resulting optimal path in comparison to the benchmark optimal path[1]. We assume once more that the pollution stock $Z(t)$ accumulates according to the standard law of motion.

$$\dot{Z}(t) = y(t) - \alpha(t)Z(t) \tag{2.7}$$

Let us note immediately that this law of motion assumes away a steady state for $Z = 0$, which we assume would be unrealistic.

[1] In order to work as rigorously as possible, we do not use the variable substitution $y = \alpha Z$ operated by Pezzey and discussed in Appendix A.

The assimilative factor at play $\alpha(t)$ follows its own dynamics. We have thus[1]:

$$\dot{\alpha}(t) = -h(Z(t))$$

For the resolution of this problem, we shall work with the following functional form of h inspired from Pezzey's specification seen previously. Once again there is no degradation effect of pollution as long as the stock of accumulated pollutant remains below a threshold $\bar{Z}$.

$$h(Z(t)) = 0 \;\forall\; Z(t) \leq \bar{Z}$$
$$h(Z(t)) = k(Z(t) - \bar{Z}) \;\forall\; Z(t) > \bar{Z} \;\; k > 0$$

We do not retain the assumption of Pearce and used by Pezzey according to which $Z_D = \bar{Z}$, we work with the standard damage function described in Section 2.

The maximization problem can now be written:

$$\max_{y(t)} \int_0^\infty \left[f(y(t)) - D(Z(t)\right] e^{-\rho t} dt$$
$$\text{s.t. } \dot{Z}(t) = y(t) - \alpha(t)Z(t)$$
$$\dot{\alpha}(t) = -h(Z(t))$$
$$Z(0) = Z_0\,,\;\; \alpha(0) = \alpha_0$$

The current value Hamiltonian for this problem is

$$f(y(t)) - D(Z(t)) + \lambda(y(t) - \alpha(t)Z(t)) - \mu h(Z(t))$$

where λ and μ are the shadow prices of the pollution stock Z and the assimilative factor α. Given the contribution of each state variable to social welfare λ is negative and μ positive.

[1]As mentioned in the previous chapter, this feedback should in fact be delayed in time but to avoid additional mathematical complexity we will assume that the impact is immediate.

2.4.1 First order conditions

Along the optimal path we must have the following first order conditions:

$$f'(y(t)) = -\lambda(t) \tag{2.8}$$

$$\dot{\lambda}(t) = (\rho + \alpha(t))\lambda(t) + \mu(t)h'(Z(t)) + D'(Z(t)) \tag{2.9}$$

$$\dot{\mu}(t) = \rho\mu(t) + \lambda(t)Z(t) \tag{2.10}$$

and in addition the transversality conditions

$$\lim_{t \to \infty} e^{-\rho t}\lambda(t)Z(t) = 0 \tag{2.11}$$

$$\lim_{t \to \infty} e^{-\rho t}\mu(t)\alpha(t) = 0 \tag{2.12}$$

2.4.2 Economic interpretation of the shadow prices

Thanks to a method analogous to the one used by Farzin (1996, p.37), we can provide a very convincing economic interpretation of the shadow prices by solving the differential equations (2.9) and (2.10) and using the transversality conditions (2.11) and (2.12) to yield (integration by parts)

$$-\lambda(t) = \int_t^\infty e^{-(\rho+\alpha(s))(s-t)} D'(Z(s))ds + \int_t^\infty e^{-(\rho+\alpha(s))(s-t)}\mu(s)h'(Z(s))ds \tag{2.13}$$

$$\mu(t)Z(t) = -\int_t^\infty e^{-\rho(s-t)}\lambda(s)ds \tag{2.14}$$

It is clear that if the degradation threshold is never reached, equation (2.13) boils down to the standard expression of the shadow price of pollution (Schubert, 2008, p.199):

$$-\lambda(t) = \int_t^\infty e^{-(\rho+\alpha(s))(s-t)} D'(Z(s))ds$$

It is important to keep in mind that in a standard framework, $|\lambda|$ can be seen as the tax level necessary to put the economy on the optimal path.

Equation (2.13) indicates that the shadow externality cost of accumulated pollution at time t is the discounted stream of marginal costs that a unit of pollutant accumulation spills over into the future. If the degradation threshold has not been reached, $h'(Z(s)) = 0$ and these marginal costs consist exclusively in the first term on the right hand side of (2.13), namely the marginal environmental damage measured by $D'(Z(s))$. However, if the degradation threshold has been exceeded, these costs will also include the second term that reflects the cost of marginal degradation of assimilative capacity valued at its own shadow price $\mu(s)$. An interesting distinctive feature of this expression compared to the standard literature is that the pollution-adjusted discount rate $(\rho + \alpha(t))$ decreases over time once the degradation threshold is reached as α starts to decrease. From this angle, it can be suggested that the introduction of assimilative capacity reduction could partly offset the impact of a high discount rate, the latter usually favoring high levels of pollution in the long term future.

Equation (2.14) gives an explicit valuation of the shadow price of the "absolute" assimilative capacity $\alpha(t)Z(t)$. At any time t, this shadow price is equal to the discounted stream of marginal costs that would be caused by the depletion of one unit of "absolute" assimilative capacity. These costs thus correspond to the costs incurred by an additional unit of "non-assimilated" pollution, and are therefore equal to the current shadow price of pollution $\lambda(t)$.

2.4.3 Preliminary observations

As acknowledged by Pezzey himself, the presence of two interconnected state variables makes this optimal control problem hardly tractable. We shall thus base our analysis on a comparative study with the benchmark case, using mostly geometrical properties. The scope of this approach is of course limited as we will not be able to establish the specific properties of the new optimal path, but the comparison with the benchmark case will nonetheless provide interesting insights on the modifications involved by the introduction of ecological feedbacks. The consequences of introducing feedback mechanisms are quite clear and confirm intuition: the optimal level of emissions must be lower and the tax rate to internalize the externality must be higher. The most concerning issue regards the "survival" of the assimilative capacity and the conditions under which it can be optimal to conserve a strictly positive level of assimilative factor. We show that the discount rate and the initial conditions (Z_0) play a crucial role to determine whether the optimal path will also be a "sustainable" one. Our analysis compares the optimal paths and the steady state for the same Z_0 assuming $\alpha(0)$ in the Pezzey-Pearce model is equal to the constant α in the benchmark model.

A preliminary observation on the optimal emission level

Equation (2.8) tells us that along the optimal trajectory the marginal utility gained from an additional unit of polluting emission must compensate the (absolute value of the) shadow cost of *in situ* pollution. This condition is exactly identical to the one found in the benchmark model in equation (2.2). But as we will see, this time this shadow cost reflects not only the social damage caused by the pollution stock but also the reduction of welfare it entails through assimilative capacity degradation. This shadow cost should thus be higher in absolute value and according to (2.8) it will determine a lower level of emission along the optimal path if assimilative capacity degradation occurs.

2.4.4 Steady state analysis

Let us assume that there is an optimal solution and that the system has at least one steady state[1]. We shall call a steady state associated with a strictly positive level of assimilative capacity a "sustainable" steady state, and a steady state associated with a level of assimilative capacity equal to zero an "unsustainable" steady state, although as we discussed before this notation does not presume to reflect the "exhaustive" concept of sustainability. At the steady state we must have

$$\begin{aligned}
\dot{Z} &= y - \alpha Z = 0 \\
\dot{\alpha} &= -h(Z) = 0 \\
\dot{\lambda} &= (\rho + \alpha)\lambda + \mu h'(Z) + D'(Z) = 0 \\
\dot{\mu} &= \rho\mu + \lambda Z = 0
\end{aligned}$$

hence

$$\begin{aligned}
& y = \alpha Z \\
& h(Z) = 0 \\
& (\rho + \alpha)\lambda + \mu h'(Z) + D'(Z) = 0 \\
& \rho\mu + \lambda Z = 0
\end{aligned}$$

By definition we have $h(Z) = 0$ implies either $Z < \bar{Z}$ or $Z = \bar{Z}$. Two kinds of steady state levels of pollution can thus arise depending on the value of $\bar{Z}$.

[1] The standard existence theorems cannot be applied to guarantee the existence of an optimal solution because the concavity conditions are not necessarily been met.

2.4.5 Case 1: $Z_{ss} < \bar{Z}$

Preliminary remark on the property of a steady state resulting from a "degrading" path

A preliminary remark on the occurrence of a steady state solution implying $Z_{ss} < \bar{Z}$ is to be expected. We can assert that if $Z_0 < \bar{Z}$, there is very little economic rationale to justify the occurrence of such a steady state pollution on a "degrading" optimal path, e.g., a path along which the stock of pollution exceeds $\bar{Z}$ at some point. Indeed let us observe a path leading the economy from a "sustainable" level of pollution Z_0 to "unsustainable" levels higher than $\bar{Z}$ because of high social preferences for consumption relatively to the sensibility to environmental damage[1]. There is no economic justification for this path to "come back" to levels of pollution strictly lower that $\bar{Z}$, especially in a discounted future. We shall thus focus on a steady state stemming from a "clean" path and associated with a steady state level assimilative capacity rate equal to its initial level α_0.

For $Z_{ss} < \bar{Z}$ we have the system [H1] defined by

$$\begin{aligned} & Z = \frac{y}{\alpha_0} \\ & Z < \bar{Z} \\ & (\rho + \alpha_0)\lambda + D'(Z) = 0 \\ & \rho\mu = -\lambda Z \end{aligned}$$

According to (2.8) we have $f'(y) = -\lambda$ along the optimal path. We can focus on the variable y to solve the system.

[1] We shall ignore the case where an economy tries to recover from a very high initial stock of pollution back to a very low one as the assimilative capacity is likely to be destroyed on such a trajectory.

$$
\begin{aligned}
&Z = \frac{y}{\alpha} \\
&Z < \bar{Z} \\
&(\rho + \alpha) f'(y) = D'(\frac{y}{\alpha}) \qquad (2.15) \\
&\rho\mu = f'(y)\frac{y}{\alpha}
\end{aligned}
$$

This system has a solution if and only if equation (2.15) has a positive solution that respects $Z < \bar{Z}$.

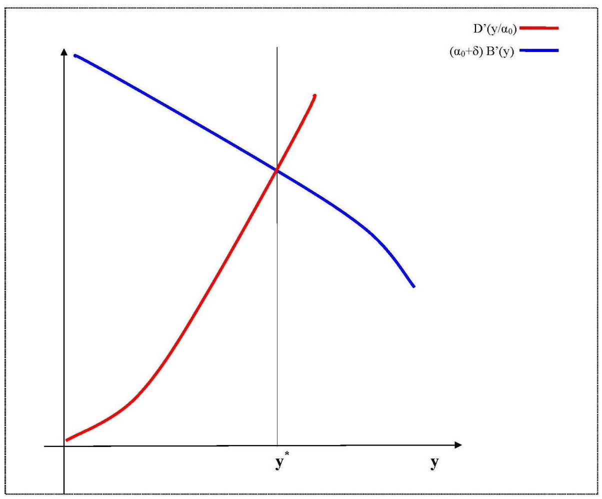

Figure 2.4: Solution to system [H1]

Given the properties of f and D, there exists a unique positive solution y^* to this equation as shown on Figure 2.4. We can thus deduce the other corresponding steady state values:

$$
\begin{aligned}
\lambda_{ss} &= -f'(y^*) \\
Z_{ss} &= \frac{y^*}{\alpha_0} \\
\mu_{ss} &= \frac{f'(y^*)y^*}{\rho\alpha_0}
\end{aligned}
$$

In order for the solution y^* to be acceptable we need to have $Z_{ss} < \bar{Z}$ which amounts to $\frac{y^*}{\alpha_0} < \bar{Z}$.

Trivial comparative statics show us that as intuition demands, a higher discount rate yields a higher stock of accumulated pollution at the steady-state. Conversely a higher initial assimilative capacity yields a lower steady state level of pollution.

2.4.6 Case 2: $Z_{ss} = \bar{Z}$

For $Z_{ss} = \bar{Z}$ we have the system [H2] defined as

$$Z = \bar{Z}$$
$$\alpha = \frac{y}{\bar{Z}}$$
$$(\rho + \alpha)\lambda + \mu h'(Z) + D'(\bar{Z}) = 0$$
$$\rho\mu = -\lambda\bar{Z}$$

We know that along the optimal path $f'(y) = -\lambda$ and $h'(Z) = k$. We can express the equations in terms of the variable y.

$$Z = \bar{Z}$$
$$\alpha = \frac{y}{\bar{Z}}$$
$$(\rho + \frac{y}{\bar{Z}})f'(y) + \frac{kf'(y)\bar{Z}}{\rho} = D'(\bar{Z}) \tag{2.16}$$
$$\mu = \frac{f'(y)\bar{Z}}{\rho}$$

This system has a solution if and only if equation (2.16) admits a positive solution. We show in Appendix B that if the marginal elasticity of the utility function is strictly lower than one, this equation admits a unique solution. Given our concern to work with very general functions, we can only determine a sufficient condition which corresponds to the most realistic economic setting, but we are aware that this condition is not a necessary one. Assuming that the benefit function admits this property regarding the marginal elasticity, we can deduce that equation (2.16) has a unique positive solution y^{**}. The corresponding steady state values of the other variables in the system are thus

$$Z_{ss} = \bar{Z}$$
$$\alpha_{ss} = \frac{y^{**}}{\bar{Z}}$$
$$\lambda = -f'(y^{**})$$
$$\mu_{ss} = \frac{f'(y)\bar{Z}}{\rho}$$

This time the concern on the feasibility of the steady state value of α is quite straightforward. We need to have $\alpha_{ss} = \frac{y^{**}}{\bar{Z}}$. Considering that restoration is not available, this induces the following loose feasibility conditions:

On a "clean" optimal path, where $\alpha(t) = \alpha_0 \ \forall t$, we need $\alpha_0 = \alpha_{ss} = \frac{y^{**}}{\bar{Z}}$.

On a "dirty" optimal path, where $\exists \ T > 0$ s.t. $\alpha(T) < \alpha_0$, we need $\alpha_0 > \frac{y^{**}}{\bar{Z}}$.

Once again it is clear that the lower the degradation threshold $\bar{Z}$, the higher the chances to reach a sustainable steady state.

2.4.7 Intermediary conclusions

Optimal extinction of assimilative capacity

Formally this steady state analysis has shown, under an acceptable assumption on the form of function f, that a sustainable steady state was not reachable for too low values of α_0 and low values of the degradation threshold. Given the dynamics of $\alpha(t)$, we can conclude that when such a sustainable steady state is not reached, the assimilative capacity is driven to extinction.

Cases (1) and (2) thus jointly reinstate the results obtained in Chapter 1 on the crucial role of initial conditions. When the available assimilative capacity is already very low in an economy and no restoration options are at hand, the optimal economic trajectory in a discounted utilitarian framework will lead this resource to extinction, even if the dynamics of this resource are acknowledged explicitly

Robustness of the results

We have shown in Case 1 that when we limit our analysis to the optimal paths reaching $Z_s s < \bar{Z}$ "from below", there always exists a unique steady state associated with an assimilative capacity equal to its initial level α_0. However if we were to extend our analysis to the paths reaching a steady state “from above”, the possibility of multiple sustainable steady states associated with different levels of assimilative capacity cannot be discarded. Although it is rather unlikely from an economic point of view, it could be the case that an economy tries to recover from a very high initial level of pollution because of a sudden change in environmental preferences and engages in an active emission-restriction policy. In doing so it will nevertheless remain for a period within the “degradation zone” ($Z > \bar{Z}$) and the assimilative capacity will be degraded far below α_0 as the economy reaches the “safe” zone. Such a situation could give way to many combinations of steady state levels of assimilative capacity and pollution stock but it is beyond the technical scope of this work to analyze them any further.

Our results on the steady state properties are not easy to compare with the few prominent results of the literature mentioned in the Introduction as they are heavily constrained by our original specification. Given the dynamics chosen for α, our model in its version without restoration calls *a priori* for a steady state level of pollution below the degradation threshold. The only other “equilibrium” situation is the case where the assimilative capacity is destroyed ($\alpha = 0$) which is tantamount to a steady state also. As noted below, such an “optimal extinction” is intrinsically linked with a high discount rate, unfavorable initial conditions and/or low degradation threshold $\bar{Z}$. In a way, this main finding is in line with the first result established by Tahvonen and Withagen (1996) who show in a setting similar to ours that there exists a single sustainable steady state if and only if $D'(\bar{Z}) - \rho U'(0) \geq 0$ and insist on the role of the initial stock of pollution. If the latter is too high, then the optimal path will not be the one that leads to this sustainable steady state. We can find in the mathematical condition the same effect of the degradation threshold and of the discount rate that our own results have highlighted. Moreover our model displays the same sensitivity towards the initial environmental conditions. Our results are also compatible with the conclusions of Chevé (2000) who introduces the assimilative capacity dynamics of the previous authors in a model of endogenous growth with capital accumulation. Assuming a balanced growth path she shows that the occurrence of a single steady state of stock pollution associated with a positive assimilative capacity (”sustainable” according to our notation) depends in fine on the level of the discount rate and on the initial stock of pollution (which determines automatically the initial level of assimilative capacity as we discussed in the General Introduction).

2.4.8 The modified dynamics

Now that we have studied the different steady state configurations and the conditions leading to these outcomes, we can attempt to characterize qualitatively the dynamics leading either to these steady state or to limit cases. In order to do so, we carry out a comparative geometrical analysis consisting in identifying the modifications sustained by the benchmark optimal trajectories under the introduction of assimilative capacity dynamics. This rather strenuous analysis involving many different cases and subcases is detailed in Appendix C but we sum up and compile the results in this present section.

As shown in equation (2.9), the dynamics of the shadow price of pollution are complexified by the introduction of assimilative capacity variations. Comparing equation (2.9) with its equivalent in the benchmark model, equation (2.3), we distinguish two cases depending on the relative position of $\bar{Z}$ compared to the benchmark steady state Z_{ss}.

We work with the (Z, λ) phase diagram but we must keep in mind one crucial feature of the steady state level of the economy. As shown above, in order to have $\dot{\alpha} = 0$, the economy must imperatively settle at a level of pollution lower than $\bar{Z}$, unless the assimilative capacity has been completely depleted. Denoting Z_{ss2} a potential steady state in the Pezzey-Pearce model, Z_{ss2} is compatible with a strictly positive level of assimilative capacity if

$$Z_{ss2} \leq \bar{Z} \tag{2.17}$$

In this section we will continue to call a steady state associated with a strictly positive level of assimilative capacity a “sustainable” steady state, and a steady state associated with a level of assimilative capacity equal to zero a “unsustainable” steady state. The stability of α is thus guaranteed by the stability of Z at a “sustainable” level. Considering this sufficient condition of stability for α, we can conduct a comparative phase diagram analysis based on the dynamics of Z.

2.4.9 Conclusions

To sum up these results we must first acknowledge once again that the strenuous tractability of the model does not allow for straightforward conclusions regarding the optimal path in our version of Pezzey-Pearce model. However we have been able to show three significant properties of this new optimal path (here we do not include case A.1 as it boils down to the benchmark case) involving a varying degree of assimilative capacity degradation.

1) If a sustainable steady-state stock of pollution can be reached, it will be lower than the benchmark steady state Z_{ss}, as shown in cases A.2.1 and B.1.1.1. This confirms the obvious intuition that in presence of irreversible assimilative capacity degradation, the economy must settle at a lower pollution level and that this level must be below the degradation threshold.

2) There are several chances, enhanced by a higher discount rate, for the economy to completely degrade the assimilative capacity before reaching a sustainable steady state (cases A.2.2, B.1.1.2, B.1.2.2 and B.2.2.). This result is in accordance with Pezzey's and shows that under certain intuitive conditions, namely a high discount rate or a high initial level of accumulated pollution, the extinction of assimilative capacity in finite time is optimal.

3) Non-monotonous optimal paths cannot be excluded, especially for high rates of discount, as shown in cases B.1.1.2 and B.2. In those cases, the maximization of social welfare will lead the economy away from the "bliss level" of pollution $\bar{Z}$ in a first phase and attempt to decrease back down to $Z_{ss2} < \bar{Z}$ in a second phase.

2.5 Natural capital maintenance: an enriched version of the model

Pezzey's model assumed away all possibilities of restoring the assimilative capacity, thus reflecting Pearce's argument. Equation (2.6) reflects the assumption that any degradation of this assimilative factor is irreversible. However we have seen in the Introduction that there exist various options available to restore or even increase natural assimilative capacity whether it applies to CO_2 assimilation or to other stock pollution problems. We work here with a rather optimistic assumption on the actual feasibility of such a restoration, the actual margin of restoration must indeed be very thin in many concrete cases.

We thus propose an enriched version of Pezzey's model in order to highlight the possible trade-offs between consumption and assimilative capacity restoration. In doing so, our model extends the analogy between physical and natural capital as it allows to sacrifice current consumption to invest in the capital, or at least to offset its depreciation. The particular depreciation process of natural capital, verified for environmental services such as assimilative capacity but also for renewable resources and land productivity will be discussed thoroughly in Chapter 5.

2.5.1 Presentation of the modified problem

The idea of investment in assimilative capacity has been only marginally discussed in the pollution control literature (see van der Ploeg and Withagen (1991) on the investment in artificial abatement capital) and our model could be reinterpreted by substituting physical abatement capital to our natural assimilative capacity and allowing for investment in this abatement capital. However, to our knowledge it has never addressed conjointly with the phenomenon of endogenous depreciation of (natural) capital, namely in our case the degradation of assimilative capacity.

In the previous analysis we assumed that all the output y was dedicated to consumption. Now the social planner's maximization problem remains the same except that the output level $y(t)$ is allocated between utility-yielding consumption $c(t)$ and spending in assimilative capacity restoration activities $r(t)$ such that

$$c(t) = y(t) - r(t)$$

The motivation behind this model is to assume candidly that if optimal growth models usually advocate economic trajectories with maintenance or even investment in physical capital to reach equilibrium levels with a higher flow of services, there is no reason that this should not be the case when we deal with natural capital. The total extinction of natural capital under one form or the other is thus rather counter-intuitive and might result from incomplete specification of the options available to the policy maker. That is why we attempt to reinstate a balance between the way physical and natural capital are dealt with in optimization models by introducing a restoration variable that plays a maintenance role for natural capital.

Since this problem involves two control variables and two state variables, it is even less tractable than the basic Pezzey-Pearce model dealt with before and we will have to resort once again to a comparative analysis with the previous models to yield some interesting qualitative results.

The social planner program now writes

$$\max_{y(t)\,,\,r(t)} \int_0^\infty \left[f(c(t)) - D(Z(t))\right] e^{-\rho t} dt = \int_0^\infty \left[f(y(t) - r(t)) - D(Z(t))\right] e^{-\rho t} dt$$

The pollution stock $Z(t)$ accumulates according to the standard law of motion.

$$\dot{Z}(t) = y(t) - \alpha(t) Z(t) \tag{2.18}$$

The assimilative factor at play α follows its own dynamics as in the Pezzey model, except that it can now be increased or maintained through restoration spending $r(t)$. We have thus

$$\dot{\alpha}(t) = -h(Z(t), r(t)) \tag{2.19}$$

For the sake of simplicity we shall work with a separable function $h(Z, r)$ assuming that there is no interaction between the pollution degradation effect and the restoration effect, e.g., that the cross derivatives of h are nil[1].

From an ecological perspective[2] it is quite straightforward that the assimilation rate cannot be increased indefinitely. We thus need to introduce an upperbound α_{max} such that $\alpha(t) \leq \alpha_{max}$ $\forall t$ just as we did for the "absolute" assimilative capacity $A < \bar{A}$ in Chapter 1.

[1]This assumption seems quite reasonable for the h_{rZ} derivative, however it could be argued that restoration is less efficient (or more efficient in certain cases) when the pollution level is high, which would translate as $h_{Zr} > 0$.

[2]The limited area available for reforestation or afforestation on the planet, as well as the oceans' acidification thresholds illustrate such an upperbound in the CO_2 assimilation at the planet's scale.

There is no degradation effect of pollution as long as the stock of accumulated pollutant remains below a threshold $\bar{Z}$ but restoration can be implemented. Beyond the threshold $\bar{Z}$, assimilative capacity degradation begins but it can be offset by restoration/maintenance.

$$h(Z(t), r(t)) = -g(r(t)) \ \forall \ Z(t) \leq \bar{Z} \tag{2.20}$$

$$h(Z(t), r(t)) = k(Z(t) - \bar{Z}) - g(r(t)) \ \forall \ Z(t) > \bar{Z} \ \ k > 0 \tag{2.21}$$

We are thus working with a linear functional form of $h(Z)$ and a linear restoration function $g(r)$ despite the simplifications implied[1].

As it would be highly unrealistic in this setting to make the simplification usually adopted when it comes to physical capital, we cannot pretend that a unit of restoration spending increases the natural capital by a unit and we must introduce a positive restoration function g that is analogous to the adjustment costs that are usually ignored in capital theory (Lucas, 1967). We will work here first with a linear form of g such that $g(r) = mr$ with $m > 0$. We have in particular $g(0) = 0$.

Also our acceptance of the Inada conditions mechanically guarantees that r is upper bounded along an optimal path as we must always have $y(t) - r(t) > 0$ and we know from (2.1) that $y(t) < \hat{y}$ at any time t. Subsequently $g(r)$ will also be strictly upper-bounded by $g(\hat{y})$.

$$g(r(t)) < g(\hat{y}) \ \ \forall \ t$$

It is also important, for the sake of plausibility, that once the pollution stock has reached a very high level, offsetting the assimilative capacity degradation is no longer feasible and restoration is impossible. This property would be taken care of by the combination of a strictly concave restoration function combined with a strictly convex degradation function but here we must settle for linear forms and keep this limit in mind.

Finally, before proceeding to the optimal control analysis we need to introduce a

[1] In fact the degradation of assimilative capacity incurred by pollution levels should be convex as the marginal degradation imposed by an additional unit of pollution is higher for high pollution stocks and we should have thus $h_Z \geq 0$ and $h_{ZZ} \geq 0$. Reversely, it seems only natural to have decreasing yields in the restoration activity, we should thus have the following properties: $h_r \leq 0$ and $h_{rr} \geq 0$.

preliminary condition to ensure the coherence of the model, we need indeed that

$$-h_r(Z,r) = g'(r) = m < 1 \tag{2.22}$$

Condition (2.22) is necessary to avoid an unrealistic arbitrage possibility between pollution and restoration. If the restoration factor is more efficient that one, it is obviously optimal for the social planner to produce an additional unit and to allocate a sufficient part of this extra production to increase the assimilative capacity in a way that completely offsets the extra pollution induced while the rest of this extra production is enjoyed as consumption. Such a behavior, at least until α reaches α_{max} and can no longer increase, does not make much economic sense and must thus be prevented through Condition (2.22). With our linear function form Condition (2.22) simply implies $m < 1$.

The maximization problem can now be written:

$$\max_{y(t)\,,\,r(t)} \int_0^\infty \left[f(c(t)) - D(Z(t))\right] e^{-\rho t} dt = \int_0^\infty \left[f(y(t) - r(t)) - D(Z(t))\right] e^{-\rho t} dt$$

$$\text{s.t. } \dot{Z}(t) = y(t) - \alpha(t)Z(t))$$

$$\dot{\alpha}(t) = -h(Z(t), r(t))$$

$$Z(0) = Z_0\,,\;\; \alpha(0) = \alpha_0$$

The current value Hamiltonian for this problem is

$$f(y(t) - r(t)) - D(Z(t)) + \lambda(y(t) - \alpha(t)Z(t)) - \mu h(Z(t), r(t))$$

where λ and μ are the shadow prices of the pollution stock Z and of the assimilation factor α and given the contribution of each state variable to social welfare λ is negative and μ positive.

2.5.2 First order conditions

The first order conditions are the following[1]:

Along the optimal path we must have

$$f'(y-r) = -\lambda \tag{2.23}$$

$$f'(y-r) = -h_r(Z,r)\mu = \mu m \tag{2.24}$$

$$\dot{\lambda} = (\rho+\alpha)\lambda + \mu h_Z(Z,r) + D'(Z) = (\rho+\alpha)\lambda + \mu k + D'(Z) \tag{2.25}$$

$$\dot{\mu} = \rho\mu + \lambda Z \tag{2.26}$$

and in addition the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)Z(t) = 0$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)\alpha(t) = 0$$

Except for equation (2.24) resulting from the introduction of the restoration variable, these first order conditions are exactly identical to those obtained in the standard Pezzey-Pearce model. This similarity is due to the fact that the degradation-restoration function h is separable in Z and r.

Output vs. allocation effect

The economic interpretation of equations (2.23) and (2.24) is quite straightforward if we distinguish two effects at play. It must be reminded that in our model social utility varies in two directions. First the absolute level of production (identified to the level of pollution) can be increased (lowered). This increase (reduction) has a positive (negative) effect on the stock of pollution through the motion equation (2.18). In a second phase, this increased (reduced) output must be allocated optimally between consumption and restoration. If all output goes to consumption, social utility will be at its highest. If a part of this output is devoted to the restoration of assimilative capacity, the latter will increase through equation (2.19) and partially offset the additional emissions accumulating into the pollution stock.

In relation with the first effect (absolute output), equation (2.23) tells us that

[1]From now on we shall drop the time index whenever no ambiguity arises.

along the optimal trajectory the marginal utility gained from an additional unit of polluting emission must compensate the (absolute value of the) shadow cost of *in situ* pollution. As we will see, this shadow cost reflects not only the social damage caused by the pollution stock but also the reduction of welfare it entails through assimilative capacity degradation.

In relation to the second effect (allocation), equation (2.24) can be read as follows. The marginal utility forgone through the sacrifice of a unit of consumption for a unit assimilative capacity restoration must be offset, at all time along the optimal path, by the marginal restoration effect ($-h_z$) of this unit of restoration valued at the shadow price of assimilative capacity μ. A high μ, caused by the increased scarcity of assimilative factor or by high needs for assimilation, results in a higher allocation of output to restoration efforts at the expense of consumption. This relation sheds some light on the trade-offs between consumption and restoration once the absolute level of output has been determined in function of stock pollution shadow price. This "anteriority" of the choice of absolute output level over the allocation between restoration and consumption can be established unambiguously since the restoration variable plays no part in the dynamics of the two shadow prices.

As shown in equation (2.25), the dynamics of the shadow price of pollution remain similar to the results commented previously in the Pezzey-Pearce model. Compared to the benchmark case, the additional term affecting these dynamics in this case is μh_Z. This positive term measures the marginal impact, economically valued at the shadow price of assimilative capacity μ, of an additional unit of pollution accumulated *in situ*. This term is nil as long as Z remains below the threshold $\bar{Z}$ and has thus no effect. However once the threshold is reached, h_Z becomes strictly positive and contributes to increasing $\dot{\lambda}$. Regarding its influence on the shadow price of pollution, the shape of the degradation function is thus similar to the damage function: the higher the marginal degradation/damage, the higher the increase in λ[1]. Once again this rather counter-intuitive feature can be explained by reminding that a very high initial absolute shadow cost is set when the economy faces rough environmental conditions and particularly severe degradation and damage functions. Since we have assumed away the possibility of a cross impact of restoration efforts on the marginal

[1]Let us remind that since $\lambda < 0$, this increase is in fact a decrease in the absolute value of λ, which makes the economic interpretation in terms of "shadow cost of pollution" clearer.

degradation of pollution h_Z, equation (2.25) is exactly tantamount to equation (2.9).

The equation driving the dynamics of μ is exactly the same as the one determined in the Pezzey-Pearce model, equation (2.10). The same interpretation applies.

It can be noted that at no point does the restoration option affect the dynamics of the shadow prices. The level of restoration only appears in (2.25), which confirms our argument developed above on the strictly "allocative" effect of the restoration variable in the model.

Shadow prices ratio

Combining equations (2.23) and (2.24) we get

$$g'(r)\mu = -\lambda$$

and thus, working with the linear form of g

$$\frac{-\lambda}{\mu} = m \tag{2.27}$$

Along the optimal path, the ratio between the shadow prices of pollution and assimilative capacity is equal to marginal restoration, which is constant and equal to m. Both shadow prices thus evolve jointly along this optimal path and we must have at all times

$$\dot{\lambda} = m\dot{\mu} \tag{2.28}$$

This relation asserts a clear link between the shadow price of pollution and the shadow price of assimilative capacity, especially regarding steady state features. Indeed it is straightforward that $\dot{\lambda} = 0 \Leftrightarrow \dot{\mu} = 0$. Since in our model there is no existence value associated with assimilative capacity, the impact of the latter on social welfare consists exclusively in reducing the stock of pollution, or at least slowing down it accumulation. It is thus natural that the shadow price of this asset, namely the present value of the flows of services it will offer from now on, be determined by the shadow price of the pollution it prevents. If the stock of pollution is such that its shadow price is nil, it is logical that the shadow value of the assimilative capacity be nil as well since there is no added value in assimilating pollution that causes no social harm. Symmetrically, a very high (negative) shadow price of pollution reflects a situation where each unit of added pollution has very serious detrimental impact on social welfare. In that case preventing the accumulation of a unit of pollutant has a great social value, hence the very high shadow price of assimilative capacity determined by relation (2.27).

It is of interest to note that the higher the marginal restoration (m) the lower the shadow price of assimilative capacity relatively to the shadow price of pollution. This can be interpreted economically as the fact that if the technological or natural conditions for assimilative capacity restoration are such that the maintenance of assimilative capacity is "cheap", that is to say that the investment in restoration displays very high yields, then the *in situ* stock of assimilative capacity is not "worth" so much since it is easy to restore and thus to increase the natural abatement potential of the economy. Thanks to equation (2.27) our steady state analysis can focus on the dynamics of λ and the dynamics of μ along the optimal path towards the steady state will be deduced *ex post*.

Shadow prices dynamics

We can use equation (2.27) to simplify the dynamics of the shadow prices. Substituting (2.27) respectively in (2.25) and (2.26) we get

$$\dot{\lambda} = (\rho + \alpha)\lambda + \frac{-\lambda}{m} h_Z(Z, r) + D'(Z) \tag{2.29}$$

$$\dot{\mu} = \rho\mu - m\mu Z \tag{2.30}$$

and thus

$$\dot{\lambda} = (\rho + \alpha(t) - \frac{h_Z(Z(t), r(t))}{m})\lambda + D'(Z(t)) \tag{2.31}$$

$$\dot{\mu} = (\rho - mZ(t))\mu \tag{2.32}$$

Once again, if we are in the no-degradation zone, ie if $Z < \bar{Z}$, equation (2.31) boils down to the benchmark model Condition (2.3). However we must keep in mind that now that restoration is available, the steady state level of pollution needs not anymore to be in this no-degradation zone to ensure that $\dot{\alpha} = 0$. In the degradation zone (2.31) writes

$$\dot{\lambda} = (\rho + \alpha(t) - \frac{k}{m})\lambda + D'(Z(t)) \tag{2.33}$$

2.5.3 Steady state analysis with maintenance

Given the complexity of this two-state variable problem that cannot "collapse" into a single dimension one, a detailed analysis of the steady state and a qualitative characterization of the optimal trajectories are beyond the scope of this work. We can nevertheless establish the basic existence conditions of the steady states under certain simplifying assumptions. Afterwards we shall discuss the consumption/maintenance allocation at these equilibrium situations. Let us now address the existence and the characterization of steady states in this setting with assimilative capacity maintenance available.

The major difference with the previous model without restorations is that now it is possible to consider steady state situations with a pollution stock higher than the degradation threshold. In terms of natural capital, this translates as the opportunity to utilize capital (assimilative capacity here) at an intensive rate and to offset periodically its degradation with maintenance spending. It thus opens a broader range of pollution stock levels compatible with a steady state.

We shall initiate this analysis with a general study of the steady state conditions and then characterize in more details the different cases arising from the value of the steady state level of pollution Z_{ss} relatively to the degradation threshold $\bar{Z}$. In this section, as in the previous model, we assume that there exists at least one steady state that we denote Z_r. The other variables at the steady state shall also be denoted with the index $_r$: α_r, y_r, etc. At the steady state we have, using in particular (2.31), (2.32):

$$\begin{aligned}
&\dot{Z} = y - \alpha Z = 0 \\
&\dot{\alpha} = -h(Z, r) = 0 \\
&\dot{\lambda} = (\rho + \alpha(t) - \frac{h_Z(Z(t), r(t)))}{m})\lambda + D'(Z(t)) = 0 \\
&\dot{\mu} = (\rho - mZ(t))\mu = 0
\end{aligned} \tag{2.34}$$

We can thus deduce immediately from (2.34) the value of Z_r, the level of accumulated pollution at the steady state:

$$Z_r = \frac{\rho}{m} \tag{2.35}$$

Condition (2.35) gives us some rather intuitive insights on the value of this unique steady state. The higher the discount rate, the higher the stock of accumulated pollution at the equilibrium. And, conversely, the higher the maintenance productivity factor m, the lower the stock of pollution. The terminal level of pollution stocked in this economy thus results from an interesting ratio comparing the discount rate and the maintenance productivity. If the rate at which the future "depreciates" is relatively much higher than the rate at which a forgone unit of consumption can offset natural capital degradation, then the economy will sustain a significant amount of accumulated pollution in the long term. In other words if the rentability of investing in natural capital does not compensate enough the impatience for the present, the economy will settle at unsustainable levels of pollution.

The value obtained for Z_r with (2.35) also tells us about the "sustainability" of the steady state situation in terms of assimilative capacity use.

If $\frac{\rho}{m} \leq \bar{Z}$, the economy will settle in the "non degradation zone" and the pollution stock will be associated with a positive level of assimilative capacity.

If $\frac{\rho}{m} > \bar{Z}$, the economy will settle in the "degradation zone", thus provoking the extinction of the assimilative capacity.

Let us discuss both cases separately.

2.5.3.1 Case 1: $Z_r \leq \bar{Z}$

In that case it is straightforward that no maintenance needs to be carried out, hence $r_r = 0$. As with the previous model, we assume here that there is no economic rationale for a "dirty" optimal path to settle at a steady state lower than the degradation threshold. We shall thus assume that the assimilative capacity has been unharmed when the steady state is reached and thus $\alpha_r = \alpha_0$. However it must be reminded that along such a clean path, the assimilative capacity might have been increased through restoration in order to enjoy a higher level of "non pollution-augmenting" emissions in the long run. Such an investment in natural capital would result in $\alpha_r > \alpha_0$. For the sake of simplicity we solve the following system for $\alpha_r = \alpha_0$. We have thus the following values at the steady state:

$$
\begin{aligned}
Z_r &= \frac{\rho}{m} \\
y_r &= \frac{\rho\alpha_0}{m} \\
r_r &= 0 \\
\alpha_r &= \alpha_0 \\
\lambda_r &= -f'(\frac{\rho\alpha_0}{m}) \\
\mu_r &= \frac{f'(\frac{\rho\alpha_0}{m})}{m}
\end{aligned}
$$

In Case 1, the environmental asset is thus preserved (or even increased through investments in order to enjoy higher "bliss" level) as $\alpha_r = \alpha_0 > 0$. We can guess that when the discount rate is not too high it can be optimal to increase the assimilative capacity up to a higher level than its initial amount and to settle at a low enough stock of pollution so that there is little social damage.

This environmental damage can be easily offset by consumption driven utility, and the latter will be all the more important in that there will be no restoration effort to finance.

2.5.3.2 Case 2: $Z_r > \bar{Z}$

In that case, the economy will settle at a situation such that the accumulated stock of pollution is above the degradation threshold. This means that the degradation of assimilative capacity will occur at each period indefinitely and that this degradation must be offset at each period and forever through restoration. The economy is in a situation where it "overuses" its natural capital thus entailing the endogenous depreciation of this capital, and this depreciation must be offset periodically by natural capital maintenance which in our case consists in the restoration of assimilative capacity.

In this setting, the degradation borne by the assimilative capacity at each period amounts to $k(Z_r - \bar{Z})$ and the restoration effort r must be such that it compensates this depreciation, ie $g(r) = mr = k(Z_r - \bar{Z})$, which yields $r_r = \frac{k(Z_r - \bar{Z})}{m}$. The level of consumption enjoyed at the steady state is thus $c_r = y_r - r_r = y_r - \frac{k(Z_r - \bar{Z})}{m}$. In addition we know that at the steady state $y_r = \alpha_r Z_r$ and that $Z_r = \frac{\rho}{m}$. Hence

$$
\begin{aligned}
Z_r &= \frac{\rho}{m} \\
y_r &= \frac{\rho\alpha_r}{m} \\
r_r &= \frac{k(\frac{\rho}{m} - \bar{Z})}{m} \\
\alpha_r &= \alpha_0 - \int_0^\infty h(Z(s), r(s))ds \\
\lambda_r &= -f'(\frac{\rho\alpha_r}{m}) \\
\mu_r &= \frac{f'(\frac{\rho\alpha_r}{m})}{m} \\
c_r &= y_r - r_r = Z_r(\alpha_r - \frac{k}{m}) + \frac{k\bar{Z}}{m}
\end{aligned}
\tag{2.36}
$$

We can thus determine the relation between the level of consumption enjoyed at the steady state and the stock of pollution, depending on the level of assimilative capacity left α_r.

Case 2a: $\alpha_r > \frac{k}{m}$

In that case, according to (2.36) $c > 0$ and c increases with Z_r. The higher the stock of accumulated pollution, the higher the consumption level. Consumption must indeed be high enough to compensate the high damages imposed on society by the high pollution stock. Despite the high level of pollution, a society in this situation can nevertheless enjoy a significant level of well-being thanks to its important assimilative capacity that enables it to produce/emit enough to maintain the assimilative capacity itself without sacrificing consumption. This configuration corresponds to an optimistic version of the Murky age equilibrium analyzed by Keeler et al. (1972): a high pollution stock has been accumulated but a high level of consumption is still enjoyed and a precious ecosystem service has been preserved and is still being preserved periodically by maintenance.

Case 2b: $\alpha_r = \frac{k}{m}$

In this special case, according to (2.36) we have $c > 0$ and c is constant and equal to $\frac{k\bar{Z}}{m}$, whatever the level of $\bar{Z}$.

Case 2c: $\alpha_r < \frac{k}{m}$

In that case according to (2.36), c is decreasing with Z_r and can be nil if Z_r is high enough (negative values of c excluded). This setting reflects the situation where the assimilative capacity is so degraded that the output must be strictly limited to ensure the equilibrium or the pollution stock demands such high maintenance efforts that consumption is sacrificed. It can result from either a low initial level of assimilative capacity or a very high degradation factor k. The steady state situation in that case offers very dim prospects for society as it corresponds to a very polluted environment causing significant social damage and a low level of consumption. Such a situation is very unlikely to be optimal as the utility derived from a low consumption level can hardly compensate the damages triggered by a high pollution stock. This configuration corresponds to a pessimistic version of the Murky age by Keeler et al. (1972): significant social damage suffered from a high level of pollution, little consumption to gain utility from and the definitive loss of a precious environmental asset: the assimilative capacity.

Case 3: limit cases

As we mentioned in Section 2, a shortcoming of this model is that theoretically the assimilative capacity could take negative values and be depleted indefinitely by a fast growing stock of pollution. The case where α reaches 0 must thus be considered as a limit case. Whatsoever, an ever-growing pollution stock would quickly generate social damage that even great levels of consumption could not offset. Soon enough Z would reach Z_{lim} such that

$$D(Z_{lim}) = \lim_{c \to \infty} f(c)$$

and the economy would either settle at $Z = Z_{lim}$ or stop producing to ensure a decrease in Z. As for the case $Z = 0$, we ignore this case as too unrealistic to begin with.

2.5.4 Preliminary conclusions

Two interesting properties have been highlighted despite the impossibility to characterize more explicitly the optimal solutions.

First we have seen that under favorable technological (high maintenance productivity) and specific economic conditions (low discount rate), the economy will settle at a low initial stock of pollution and it can be optimal to invest in natural capital by increasing through restoration the assimilative capacity. This investment will prove optimal if the discount rate is not so high that it minimizes the impact on intertemporal welfare of the bliss level enjoyed at the steady state. Just like it can be optimal for a society to invest in physical capital to increase its production potential for the future, investing in natural capital can found an optimal policy for a social planner eager to maximize the long term welfare of society.

Second, we have discussed the potential cases where an equilibrium is feasible at a pollution level above the degradation threshold, which was not possible in the standard Pezzey-Pearce model. This configuration can be interpreted as a case of endogenous depreciation of natural capital as the excessive stock of pollution causes, in addition to the environmental damage D, the degradation of the ecological asset. The restoration of this assimilative capacity inasmuch to offset this depreciation can thus be considered as a clear case of natural capital maintenance quite similar to the maintenance of physical capital in growth models with capital accumulation. If we replace this result in a "sustainable development" perspective, it is clear that this last situation requires effective maintenance technologies and could not be obtained in a developing country.

2.6 Conclusion: an interpretation in terms of climate change policy

The interpretation of our results in terms of economic policy in a society concerned by climate change is threefold. First our model provides analytical grounds for a stricter optimal carbon tax compared to the benchmark case. Second, it draws attention to the threat of an optimal depletion of the assimilative capacity that will lead the economy to a situation that can hardly be considered as sustainable. Finally it provides a sound illustration of the necessity to maintain natural capital along on optimal economic path, regardless of *ad hoc* sustainability concerns.

2.6.1 Optimal carbon tax in presence of assimilative capacity feedbacks

The question of the optimal time path of a carbon tax has been much debated in the literature (Ulph and Ulph, 1994; Nordhaus, 1992). In their contribution, Ulph and Ulph show that in the climate change framework, it is not so much the absolute level of the carbon tax that matters but the time path of this tax. Although there is no empirical observation available to confirm the theoretical results since the climate change regulation preferred economic tool is the tradable permits market (in the European Union, in the Kyoto protocol and the post-Kyoto schemes) there is a consensus on one specific situation from an analytical point of view. Ko et al. (1992) or van der Ploeg and Withagen (1991) conclude that if the CO_2 stock is below its steady state level then the carbon tax should rise in time. Accordingly, Ulph and Ulph also establish that under "plausible conditions" the optimal tax should rise and then fall (once the economy has adapted technologically to a high carbon price).

Our model shows (cases A.1, B.2 in Appendix C) that if the net initial stock of pollutant Z is below its steady state level the carbon tax ($|\lambda|$) must rise until the steady state is reached, but at a highest rate than in the benchmark case.

We have also shown that when the initial stock of pollution is higher than the benchmark steady state level (cases A.2 and B.1), a higher (in absolute value) shadow price must be selected to place the economy on the optimal path and that the decrease in the shadow price will be slower compared to the benchmark case standard "clean path".

We can thus conclude that in order to account correctly for the feedbacks on assimilative capacity, the optimal dynamic tax must follow the same standard pattern as in the benchmark case but it must increase at a higher rate or decrease at a slower rate. As intuition indicates, a higher environmental concern is thus necessary when these feedbacks are acknowledged.

2.6.2 Natural capital maintenance and endogenous depreciation

Although our modified model is not tractable enough to characterize explicitly the optimal pollution path and the joint evolution of the assimilative capacity, we have shed some light in Section 5 on the properties that would be displayed by the potential equilibria. Further work on the characterization of the optimal trajectories is still needed, in particular to explore the impact of restoration options on the optimal dynamic tax and see how a tax or a subvention could provide an adequate incentive for assimilative capacity restoration when the latter is optimal.

The most important conclusion that arises brings forward serious considerations on the status of natural capital in economic models. If the model is specified appropriately, the need to maintain natural capital appears spontaneously in a situation where there is repeated depreciation of the asset. It must be noted that the restoration option is not tantamount to the standard "abatement costs" generally found in the literature. Indeed, a unit of "abatement" reduces by one unit the total amount of emissions that is added to the stock but this is a punctual impact with no implication beyond the current period while one unit of restored assimilative capacity has a long-term impact on the level of damage-less emissions that can be enjoyed indefinitely. As such our model enables an explicit recognition of the flow of ecosystem service yielded by the assimilative capacity "stock". Policywise, our results recommend thus an increased monitoring of the planet's assimilative capacity (and this is valid not only for the case of global warming but for all kind of pollution problems involving depletable assimilative capacity) and an active management policy inspired from the optimal management of renewable resources. Our framework makes a case of an economically sound restoration policy that, depending on its productivity, is followed by emission reductions or not.

Moreover, our focus on restoration hints to an original perspective on the mechanisms of depreciation of natural capital. Although this issue is addressed in the literature on natural capital theory and environmental valuation (Azqueta and Sotelsek, 2007), to our knowledge the degradation of environmental assets has never been explicitly addressed as a form of endogenous depreciation of capital. A branch of capital theory has developed models (see references in Chapter 5) that reflect more realistically the phenomenon of physical capital depreciation instead of relying on the standard constant linear depreciation rate. The sound assessment that capital depreciates more or less according to how intensively it is used has been embodied in economic models with endogenous depreciation. One of the main contributions of the models we have worked with in the first two chapters is that they acknowledge explicitly such an endogenous depreciation of one form of natural capital, namely the assimilative capacity. But it can be easily shown that other environmental assets constitutive of natural capital actually follow similar dynamics. Indeed other environmental functions such as soil fertility are affected in the same way by "overuse" and must be maintained (through fallow periods or specific regenerating cultures). But this endogenous depreciation applies to natural resources as well. As we shall see in Chapter 4, if the rates of harvest[1] exceed the maximum sustainable yield, the ecological productivity of the resource will go down and remain low unless restoration takes place in the form of "rest" periods granted through low harvest rates. In that case also the depreciation of natural capital endogenously depends on the (over)use of this capital. The concept of endogenous depreciation thus covers a large part of the various forms of natural capital and provides a common feature that can help unifying[2] the way those forms of capital are dealt with in economic analysis. Therefore we believe that there is a strong case for further developments of the analogy between the works of seminal capital theory on endogenous depreciation and the status of natural capital.

2.6.3 Assimilative capacity degradation and uncertainty

As noted earlier, there are massive scientific uncertainties surrounding the evolution of assimilative capacity and considering the major impact of this assimilative capacity on a climate change outcome (Cesar and de Zeeuw, 1994) it should be addressed very seriously. It must be noted that although there is a vast body of literature addressing optimal pollution within a stochastic framework (uncertainty on the intensity of damages or on the evolution of abatement technologies, see Gollier and Baumstark,

[1]With the standard logistic natural growth function it can also be the case that a harvest rate exceeding the maximum sustainable yield ends up increasing productivity if the stock or resource was already very high, but this is, unfortunately, not often the case in real life resource management.

[2]There are of course other dimensions of natural capital that do not fit this category, such as biodiversity.

2008), only Heal (1984) regards the assimilative capacity itself as uncertain. The first form of uncertainty concerns the assessment and the monitoring of the assimilative capacity itself, as illustrated by the case of CO_2 absorption where the source of a fourth of the biosphere's absorption is not identified ("the missing sink", Sarmiento and Gruber, 2002). The second form of uncertainty characterizes the feedback effect, reflected by the degradation function in our model. The original specification of the assimilative capacity as a state variable developed here from the propositions of Pearce and Pezzey provides a promising framework in which to explore this issue.

2.6.4 Assimilative capacity degradation and Sustainability

Considering the essential service provided by the environment's assimilative capacity, its degradation induced by feedback loops raises serious sustainability concerns. In terms of strict "environmental sustainability" or strong sustainability[1], it is quite obvious that any pollution path leading to the degradation of the assimilative capacity of an ecosystem must be discarded as unsustainable. If the conception of sustainability is softened to a weaker definition and extended to the transmission to future generations of environmental and economic conditions such that a minimum level of intergenerational equity is guaranteed, we can apply to the assimilative capacity one principle of the definition of sustainability by Pearce (1988) as *"the use of environmental services at rates which can hold on for very long time periods, and in theory, indefinitely"*. In the case of the biosphere assimilative capacity, it seems reasonable to consider that despite the efficiency gains due to technological change that might occur (our model does not allow for a dynamic approach of technological change) this intergenerational equity cannot be maintained unless the socially useful environmental functions are at least partially preserved, or artificially restored when it is allowed. Therefore no pollution path destroying entirely the assimilative capacity "stock" can be considered as sustainable. This sustainability condition is quite similar to the main sustainability concern expressed by Batabyal et al. (2002) who claim that the most critical factor in sustainability is likely to be the maintenance of adequate stocks of environmental resources to ensure an adequate flow of ecosystem services. It must be made clear that this condition requiring at least the partial preservation of an envi-

[1]The "strong sustainability" concept aims to prevent the depreciation of any form of capital. See Ayres et al. (1998) for a review of the different conceptions of sustainability.

ronmental asset is mostly a non-sustainability condition and that it only deals with a minimal part of sustainable management of ecosystems and a partial definition of intergenerational equity. As it only accounts for one ecosystem service, our approach gives a lower bound estimation of the actual intensity of irreversible damages caused by excessive pollution.

As we have shown in the previous section, under a set of conditions that are easily met in reality (a high discount rate, a high initial stock of pollution or a low degradation threshold) the optimal path determined in a discounted utilitarian framework will lead to the complete depletion of the assimilative capacity, thus making the stock of accumulated pollution irreversible.

Appendix Chapter 2

Appendix A

In order to solve in a simpler manner the model with two state variables, Pezzey substitutes the pollution stock as the control variable in replacement of the emission level. This substitution is based on the following argument.

Given that

$$\dot{Z}(t) = y(t) - \alpha(t)Z(t) \tag{2.37}$$

for a constant y and α, equation (2.37) can be resolved to give

$$Z(t) = \frac{y(t)}{\alpha(t)} + [Z(0) - \frac{y(t)}{\alpha(t)}]e^{-\rho t} \tag{2.38}$$

For high enough values of α, the transient $e^{-\rho t}$-term can be ignored and relation (2.38) boils down to

$$Z(t) = \frac{y}{\alpha}$$

This formulation means that at the steady state, the stock of accumulated pollution is equal to the ratio between the constant emission level and the assimilation factor both supposed constant. The lower the assimilation factor, the higher the stock of accumulated pollution.

This substitution proves quite useful to collapse the optimal control problem to a one state variable problem but it fails to be totally convincing. It is indeed quite disturbing as it determines beforehand that the emission level is equal to its steady state value from the start of the optimal path. That is why we felt the need to propose a new approach to the same driving model intuition (see Section 4).

Exploring further the problem, Pezzey formalizes it as

$$\max_{y(t)} \int_0^{\infty} [f(\alpha(t)Z(t)) - D(Z(t))]\, e^{-\rho t} dt$$
$$\text{s.t. } \dot{Z}(t) = y(t) - \alpha(t)Z(t)$$
$$\dot{\alpha}(t) = -h(Z(t))$$

The Hamiltonian thus writes

$$H = f(\alpha Z) - D(Z) - \mu h(Z)$$

where μ is the shadow price of the assimilative capacity.

The first order conditions write:

$$\alpha f'(\alpha Z) - D'(Z) - \mu h'(Z) = 0 \tag{2.39}$$

$$ZU'(\alpha Z) = \rho\mu - \dot{\mu} \tag{2.40}$$

Let us call $Z^*(t)$ the level of Z along the optimal path. Pezzey notes that we must have $Z^*(t) \geq \bar{Z}$ as a lower level of pollution would reduce the utility from economic activity without any difference in the damage or in the assimilative capacity (it has been assumed that $Z_D = \bar{Z}$).

We can thus restrict the analysis on the $[\bar{Z}, \infty[$ set and determine if the maximum sustainable level of pollution $Z = \bar{Z}$ can be reached along an optimal path.

On the $[\bar{Z}, \infty[$ set, we have

$$h(Z) = k(Z - \bar{Z})$$

Equation (2.39) can thus be written as

$$\alpha f'(\alpha Z) - D'(Z) - \mu k = 0 \tag{2.41}$$

so that

$$\mu = \frac{\alpha f'(\alpha Z) - D'(Z)}{k} \tag{2.42}$$

which can be substituted in (2.40) to give

$$\dot{\mu} = \frac{\rho(\alpha f' - D')}{k} - Zf' \tag{2.43}$$

Pezzey attempts to shed some light on the behaviour of $\dot{Z}^*$ which he determines, after a few calculations (p.26), as

$$\dot{Z} = \frac{[k\bar{Z}f' + \alpha kZ(Z - \bar{Z})(-f") - \rho(\alpha f' - D')]}{[D" + \alpha^2(-f")]} \tag{2.44}$$

This complicated expression does not allow for more detailed analysis but two main conclusions can still be drawn.

For a very small discount rate ρ, we have always $\dot{Z} > 0$ which means that the assimilative capacity will be destroyed in finite time. On the contrary, for a very large discount rate $\dot{Z}$ will be negative. This pair of results is not as counter-intuitive as it may look at first sight since they describe growth rates and not absolute levels. As a smaller discount rate does play in favor of environmental conservation, we can deduce that the initial level of pollution (supposedly controllable!) on the optimal path will be much lower in that case[1].

Hence the three cases distinguished in Section 3.

Appendix B

Let us denote J the function such that

$$J(y) = (\rho + \frac{k\bar{Z}}{\rho} + \frac{y}{\bar{Z}})f'(y) - D'(\bar{Z})$$

The system [H2] admits a solution if and only if there exists y^{**} such that $J(y^{**}) = 0$. Given the properties of f and D we already know that $\lim_{y\to 0} J(y) = \infty$.

If we assume that the marginal elasticity of f is strictly lower than one then we have

$$\frac{-yf"(y)}{f'(y)} < 1$$

and thus

$$-yf"(y) < f'(y)$$

[1]It must be noted that once again the choice of the stock of pollution as a control variable made by Pezzey poses some conceptual problems. Since this stock evolves, by definition, according to the law of accumulation, it is rather hard to see how a “high"’ or a “low"’ initial stock of pollution can be chosen on an optimal path.

Let us now derive function J:

$$\begin{aligned} J'(y) &= (\rho + \frac{k\bar{Z}}{\rho} + \frac{y}{\bar{Z}})f"(y) + \frac{f'(y)}{\bar{Z}} \\ &< (\rho + \frac{k\bar{Z}}{\rho} + \frac{y}{\bar{Z}})f"(y) + \frac{-yf"(y)}{\bar{Z}} \\ &< f"(y)(\rho + \frac{k\bar{Z}}{\rho}) \end{aligned}$$

Given the properties of f, we have $J'(y) < 0$ if the marginal elasticity of f is strictly higher than one. Moreover it can be easily verified that with such a marginal elasticity $\lim_{y\to\infty} J(y) = -D'(\bar{Z}) < 0$. Consequently there exists a unique positive solution y^{**} to equation .

Appendix C

Our comparative analysis relies on the initial distinction between two cases depending on the position of the degradation threshold level. We consider the geometrical variations of the isoclines due to the introduction of assimilative capacity dynamics and discuss the implications for the steady state.

Case A: $\bar{Z} \geq Z_{ss}$

Case A.1: $Z_0 \leq \bar{Z}$

In that case we have $Z(t) \leq \bar{Z}$ for all t and $h'(Z(t)) = 0$. Condition (2.9) writes

$$\dot{\lambda}(t) = (\rho + \alpha(t))\lambda(t) + D'(Z(t) \tag{2.45}$$

which is equivalent, since $\dot{\alpha}(t) = 0$ all along the optimal path in that case, to relation (2.3).

Consequently, the shadow price of pollution follows exactly the same dynamics as long as the pollution stock has not exceeded the degradation threshold $\bar{Z}$. Both isoclines are identical to the benchmark case and the steady state remains the same, whether Z_0 is higher or lower than Z_{ss}. Intuitively enough, this shows that if the degradation threshold does not interfere in any way with the optimal benchmark path, the introduction of ecological feedbacks does not modify this optimal path.

Case A.2: $Z_0 > \bar{Z}$

In that case, as long as $Z(t) > \bar{Z}$ we have $h'(Z(t)) = k > 0$ and (2.9) writes

$$\dot{\lambda}(t) = (\rho + \alpha(t))\lambda(t) + \mu(t)k + D'(Z(t)) \tag{2.46}$$

All things equal, the introduction of the positive term $\mu(t)k$ will increase $\dot{\lambda}$. This impact should be differentiated according to the two possible situations of the economy on the optimal path.

If the economy is moving from right to left on the optimal path, according to Figure (2.2) the shadow price of pollution λ is increasing over time (the "shadow cost" is decreasing in absolute value) until it stabilizes at its steady state level. The introduction of the additional term $\mu(t)k$ will thus accelerate this increase. This will in turn be reflected in a shift in the λ-isocline. Indeed, with the addition of μk, $\dot{\lambda} = 0$ for a given λ will be reached for lower levels of Z. In addition, the reduction of α that is entailed by pollution levels above the degradation threshold will diminish the $(\rho + \alpha)$ factor and even lower levels of Z will be needed along the isocline. This will translate geometrically, on the $[\bar{Z}, Z_0]$ interval, in a shift to the "southwest" of the λ-isocline and in a steeper decreasing slope (the increase in λ is accelerated by the new factors).

Meanwhile, according to the law of motion of Z, a reduced α will request a lower Z to maintain $\dot{Z} = 0$ on the Z-isocline for a given λ and thus for a given y (y being a direct function of λ on the optimal path). This will result in a shift to the "northwest" of this isocline. Since the assimilative factor keeps on decreasing as long as $Z > \bar{Z}$, this shift will progressively accelerate, thus making the slope of the increasing isocline curve less and less steep, as illustrated on Figure 2.2.

Two subcases must be finally distinguished depending on the survival of the assimilative factor.

Case A.2.1: $\alpha(t) > 0 \ \forall t$ ***along the optimal path***

If the degradation caused by excessive levels of pollution is not too intense and if the initial stock of pollution Z_0 is not too high, the assimilative capacity will be kept at a positive level long enough so as to reach the new steady state Z_{ss2}. It is indeed necessary to have a strictly positive assimilative factor in order to reduce the stock of pollution to a point lower than $\bar{Z}$.

Geometrically the combined shifts of isoclines described just above will result, all things being equal, in a steady state characterized by a lower level of accumulated pollution Z_{ss2}. It must be noted that the effect on the shadow price remains ambiguous.

Case A.2.2.2: $\exists T_e$ such that $\alpha(T_e) = 0 \; \forall t > T_e$ If the degradation effect of accumulated pollution and/or the initial level of pollution are such that the degradation of assimilative capacity is too fast[1], then the latter will be completely extinguished before the new steady state can be reached. This will transform the law of motion of the pollution stock into the following relation:

$$\dot{Z} = y(t)$$

This law of motion does not allow for a reduction of the pollution stock and it will thus be impossible to reduce it back to a steady state level below $\bar{Z}$. The pollution stock will then settle at the level $Z(T_e)$ with $Z(T_e) > \bar{Z} > Z_{ss}$. The terminal situation of the economy is thus not *a minima* sustainable ($\alpha = 0$). This first part of this result is particularly interesting because beyond any considerations on sustainability, it shows that if the feedback mechanisms are non negligible, they can deviate the economy from what would have been the optimal path in the benchmark case.

<u>Case B:</u> $\bar{Z} < Z_{ss}$

According to (2.17), we know beforehand that if the steady state corresponds to a positive α, it will necessarily be at a level lower than $\bar{Z}$ and consequently lower than the benchmark steady state Z_{ss}, since in Case B we have $Z_{ss} > \bar{Z}$.

[1]This translates formally into $\int_0^{T_e} k(Z(s) - \bar{Z})ds = \alpha_0$.

Case B.1: $Z_0 > \bar{Z}$

Two additional subcases must be distinguished depending on the situation of Z_0 relatively to Z_{ss}.

Case B.1.1: $Z_0 > Z_{ss} > \bar{Z}$

In this case the modified optimal path is tantamount to Case A.2 for the most part. Indeed, the economy finds itself in a "degradation zone" from the start and the dynamics of λ and α are thus the same as described in Case A.1. Our geometrical analysis thus determines a steady state Z_{ss2} that will be pushed farther and farther to the left as the degradation of α goes on. Two outcomes are then possible depending on the initial conditions and on the degradation function.

Case B.1.1.1: $\alpha(t) > 0 \ \forall t$ **along the optimal path** If the degradation function is not too "strong" and/or if the initial level of pollution is not too high there is a possibility, although slight, that the economy can actually reach, after a "clean path", the new steady state Z_{ss2} respecting Condition (2.17). At this point, the economy will indefinitely enjoy a utility level based on emissions y_{ss2} such that $y_{ss2} = \alpha_{ss} Z_{ss2}$. Simple comparative statics show quite intuitively that the lower the discount rate, the higher the chances to reach such a "sustainable" equilibrium.

Case B.1.1.2: $\exists \ T_e$ **such that** $\alpha(T_e) = 0 \ \forall t > T_e$ If the degradation function is too "strong", and/or if the initial level of pollution is too high, the assimilative capacity will be completely depleted along the "clean path", and the economy will settle at a new steady state Z_{ss2}, the point at which $\alpha = 0$, thus preventing any further decrease in the pollution stock. This new steady state might be situated either to the left or to the right of the benchmark steady state Z_{ss} so no conclusion can be drawn on the optimality of this new steady state level. However its most important feature is that it will be associated with a completely exhausted assimilative capacity and will not respect Condition (2.17).

Case B.1.2: $Z_{ss} > Z_0 > \bar{Z}$

In that case the economy is also immediately in a degradation zone but since the benchmark optimal steady state can be attained via a "dirty" path, the new optimal path might be non-monotonous and consists in two phases. In a first phase it is likely to take the initial form of a dirty path but as the assimilative capacity degrades, the new steady state level of pollution diminishes. Since it can never be a steady state as long as it is above the degradation threshold $\bar{Z}$, the optimal path direction might shift and lead the pollution stock back down to a steady state Z_{ss2} respecting Condition (2.17). This second phase (a clean path trying to reach a sustainable steady state) is exactly tantamount to case B.1.1 and the same distinction between cases B.1.1.1 and B.1.1.2 applies.

Case B.1.2.1: The sustainable steady state $Z_{ss2} < Zss$ will actually be reached.

Case B.1.2.2: The economy will be forced to settle at an unsustainable steady state with a depleted assimilative capacity.

Case B.2: $Z_0 \leq \bar{Z} < Z_{ss}$

In this case, two phases must be distinguished. In a first phase, the dynamics driving the pollution stock and the shadow price will be analogous to the ones of the benchmark case and the assimilative capacity will be invariant. Similarly to case A.1, the optimal path will be exactly the same as the benchmark until the degradation threshold is reached. In a second phase we find ourselves in a similar situation to Case B.1.2 and the same conclusions apply, in particular the possibility of a non-monotonous optimal path.

Case B.2.1: The sustainable steady state $Z_{ss2} < Zss$ will actually be reached.

Case B.2.2: The economy will be forced to settle at an unsustainable steady state.

Conclusion of Part I: Initial conditions and natural capital utilization

Let us recap here briefly the main findings of the analysis we carried out in Chapters 1 and 2. Beyond the formal and rigorous confirmation of intuitive results on the need for stricter pollution standards when assimilative capacity dynamics are acknowledged, whether in flow of stock pollution frameworks, the analysis carried out in this first part of our discussion has led to two significant contributions.

Initial conditions and unsustainable paths

On the one hand we have extended a major result of renewable resource economics (Cropper, 1976) to the field of pollution economics concerning the crucial role of initial environmental conditions. Our analysis of optimal pollution paths in both flow and stock pollution settings has highlighted the crucial role played by the initial conditions. We have shown, respectively in Chapter 1 and Chapter 2 that if these initial endowments are very low in a no-restoration setting or if they are very low and combined with a high discount rate in a restoration setting, then it is "optimal" to drive the assimilative capacity to extinction. Based on this important assessment, we can conclude that our amendment of optimal pollution control models including assimilative capacity give way to efficient landmarks (through shadow prices) for environmental regulation when the initial environmental conditions are not too degraded.

In such a situation, we believe that dynamic optimal taxes (or equivalent optimal quotas) can provide a sound[1] and efficient solution to both flow and stock pollution externalities and guarantee an *a minima* sustainability. However, when the initial environmental conditions, in particular the initial state of the assimilative capacity, are already a subject of concern, environmental policies should not rely on the landmarks provided by discounted optimization framework as the latter are likely to be "unsustainable" ones in the sense that they lead to the extinction of the assimilative capacity. A more cautious approach should thus be adopted, especially as we have not acknowledged the uncertainty nor the possible threshold effects characterizing the ecological mechanisms at work. This conclusion naturally leads us to the exploration of alternative criteria and methods that are less likely to deprive future generations of an essential ecosystem service when the environment is in a poor "initial" state. We shall thus explore, in Chapters 3 and 4 such alternative frameworks.

Endogenous depreciation and maintenance of natural capital

Consequently our model should raise concerns on the compatibility, in some cases which might be the most realistic ones[2] between optimality and sustainability. We shall explore this potential incompatibility in Chapter 3. And since the massive uncertainty characterizing these phenomena is not acknowledged in our model, we should be all the more cautious when designing pollution control policies that aim at sustainability.

[1] Assuming the discount rate is set at a "reasonable" level, which is quite controversial to decide.

[2] The accumulation of greenhouse gases in the atmosphere has already exceeded the degradation threshold as shown by the various climate feedbacks already observed in the last decade.

Part II

From the Green Golden Rule to Viable Control: exploring alternative frameworks for sustainable environmental policies

Chapter 3

Beyond standard discounted optimization: alternative frameworks and methods for a sustainable economic analysis of pollution?

3.1 Introduction

As announced in the general introduction, our attempt to account more explicitly for the ecological dynamics characterizing the assimilative capacity of the environment in pollution control models goes hand in hand with a constant concern on the actual sustainability of the resulting optimal pollution paths. We have worked so far with a "minimal" sustainability condition inspired by Pearce (1988), Barbier and Markandya (1990) and Hodren et al. (1995)[1]. This condition is based on the idea that "*environmental degradation [destroying] the natural clean-up and regenerative processes in the environment [...] is tantamount to an environmental "collapse", and economic growth leading to such a collapse can be said to be environmentally unsustainable*" (Barbier

[1]For these authors, *"a sustainable process or condition is one that can be maintained indefinitely without progressive diminution of valued qualities inside or outside the system in which the process operates or the conditions prevails"*.

and Markandya, 1990). This "survivalist" definition of sustainability is one among many others in the economic field. Surveys like those of Pezzey (1992) and Pezzey and Toman (2005) count a great number of economic definitions of sustainability and many more have emerged since then. It is not the objective of the present work to impose another definitive definition but it is nonetheless important to assess the actual "content" of an alleged sustainable path. Our standpoint so far has been to suggest that a common feature of most of the definitions of sustainable development is the "preservation of capacities". Whether it concerns social capabilities, economic potential or environmental assets, a path that does not preserve at least partially those capacities can hardly claim to be sustainable.

In Chapters 1 and 2, our discussion on the actual sustainability of the optimal pollution paths stressed two crucial aspects. On the one hand, since the positive "sustainable" steady state level of preserved assimilative capacity resulted from the intertemporal trade-offs between current utility and the long-term use of environmental asset, its absolute level was very sensitive to the discount rate applied and this level could be very "low" with respect to biophysical thresholds. On the other hand, we have highlighted that when environmental irreversibility is introduced, the initial stock of environmental asset and the discount rate determine *ex ante* the existence of a sustainable path. These assertions are valid for both flow (Chapter 1) and stock (Chapter 2) pollution.

Although this second observation is in line with similar analysis found in the literature (Barbier and Markandya, 1990; Cropper, 1979), it is quite disturbing from a policy-oriented point of view as they suppose that when the assimilative capacity is already too low, it is optimal to deplete it completely. This is the most questionable discrepancy between an optimal and a sustainable solution. A naive view of pollution control would indeed assume that it is always possible to adopt a sustainable emission path (such that $p(t) = A(t)$ in Chapter 1), or at least a path that leads to a more sustainable situation (such that $Z(t) \leq \bar{Z}$ in Chapter 2). For instance, regulating nitrate emissions so that they do not exceed the assimilative capacity even if the latter is already very low. It is intuitive that it is always possible to adopt a pollution program respecting the assimilative capacity from day one and thus maintaining indefinitely the level of environmental asset at its initial/maximum level, even if this is not economically efficient in the short-term. We shall see later on under which setting such a strategy might be "optimal".

Considering these two points, it seems legitimate to engage in a broader discussion on the compatibility between the requirements of sustainability and the conceptual framework of dynamic cost-benefit analysis. There are numerous authors, especially in the ecological economics literature, who contest the ability of the latter to encompass the challenges of sustainability (Ekins, 2000; Beder, 1996; Godard, 2006). Based on these previous contributions, we shall thus dedicate this chapter to highlighting some of the gaps between sustainability and standard discounted cost-benefit analysis, and, more importantly, to explore alternative economic criteria. First we will recall the main conceptual obstacles between sustainability and the discounted utilitarian criterion that generally frames cost-benefit analysis (Section 2). Second we will explore the alternative optimality criteria (Green Golden Rule, Maximin, Overtaking Criterion) that have been developed to address the much-debated intertemporal equity dilemma. We shall apply analytically these criteria to our basic model in order to compare the sustainability of the paths they determine (sections 3, 4, and 5), building on Heal's approach (Heal, 2000). In a second phase we step aside from cost-benefit analysis *per se* to explore different methods such as cost-efficiency analysis (Section 6) and sequential approach (Section 7). We will once again submit our basic model to the cost-efficiency criterion in an attempt at constraint optimization. Finally we will question in our conclusion the conceptual postulates underlying the way economics address environmental problems and we will raise some doubts on the relevancy of a basic external effects internalization approach when sustainability is an explicit objective.

3.2 Intertemporal equity and alternative criteria

As we have shown with our two dynamic models in the previous chapters, standard cost-benefit analysis does not necessarily lead to the destruction of assimilative capacity, which is sustained at an "optimal level" A_{ss} or α_{ss}. In this sense, and according to the *a minima* definition of sustainability used in our analysis, the standard cost-benefit analysis can yield "sustainable" economic/pollution paths. However, we have underpinned that our analytical results do not guarantee that A_{ss} and α_{ss} will not be ridiculously small when the discount rate is high, thus leaving future generations with a very degraded natural capital. It is clear (and it is regularly pointed out in the ecological economics literature) that since A_{ss} depends *in fine* on the discount rate and the functional forms chosen to measure private benefit and social damage, no "critical natural capital"[1] consideration comes into play. We will thus analyze more deeply the impact (and the legitimacy) of the discount rate in this section. The most crucial issue arising when discussing the sustainability of optimal economic trajectories is indeed the role of the discount rate. As we verified in our previous chapters, a high discount rate reduces the concern for long term welfare and subsequently lead to lower environmental preservation[2]. That is why it is important to assess the specificities of the discounted utilitarian framework in which optimization problems are solved.

The standard net present value approach of optimal economic paths (usually noted as discounted utilitarianism) has been challenged on its ability to handle the intertemporal trade-offs between generations. The relevancy of the discount rate in consumption/investment trajectories by individuals with a finite lifespan is unanimously acknowledged in economics. However the extension of this discounting method to social investments involving many (an infinite number if the human race does not face extinction) generations proved to be much more problematic. From Ramsey's (1928) radical formula[3] to the current *Nordhaus vs. Stern* heated debate on the economics of climate change following the Stern Review (Stern, 2006)[4], the discount rate has always been a subject of controversy when it comes to intertemporal social choices. Whether it is in empirical estimations or in normative debates, the social discount rate is one of the most questioned concept in environmental economics and beyond. We shall shed some light on this on-going debate in the following subsection. In order to get around this dilemma, other criteria have been developed. We will review them briefly before applying them to our pollution control model in sections 3, 4 and 5.

[1]The concept of critical natural capital is defined by Ekins et al. (2003) as *"that part of the natural environment that performs important and irreplaceable functions"*.

[2]It is quite telling to see that in mathematical control theory the factor that economists interpret as a discount factor is called a forget factor.

[3]"*Discounting future utilities is ethically indefensible and arises purely from a weakness of the imagination*".

[4]See Heal (2009) for a clear meta-analysis of the academic debate generated by the Stern report and a sound presentation of the discounting argument in the context of climate change.

3.2.1 Discounting and Sustainability

3.2.1.1 A fundamental theoretical controversy

It is obviously beyond the scope of this paper to discuss the various answers that address the issue of discounting but it is helpful to remind briefly the consequences of the discounting option selected on the optimization process; especially as we will explore different optimality criteria that were built in reaction to these consequences. We wish in particular to draw attention on the "Malinvaud" argument that had been overlooked in the literature and that is brought up by Henry (2007, 2000), Godard (2007) and Heal (2009). The fundamental question about whether or not the initial concept of "intragenerational discount rate" (for private investment) can be extended to public investment implying intergenerational trade offs is far from gathering unanimity in the economic field. What's more, even if there were a conceptual agreement on the use of a discount rate in intergenerational welfare analysis, the problem of its value would arise. There is currently no robust methodology to assess positively or normatively a "correct" value for this discount rate. Many economists (van den Bergh, 2004; Azar and Sterner, 1996) are thus all the more careful with the arbitrary discount rates imputed in economic models, especially since the slightest change in its value can imply tremendous changes in the net present value. This mechanism is particularly true for large scale environmental problems and underlies the Stern report controversial conclusions.

The consumption discount rate consists in the combination of two parameters: the pure rate of time preference (generally noted δ[1] and the rate at which the marginal utility of consumption is falling ($\eta(c_t)R(c_t)$).

$$\varsigma_t = \delta + \eta(c_t)R(c_t) \tag{3.1}$$

In an intergenerational framework, δ is interpretable as the discrimination rate that is applied against future generations on the ground that they are more distant in time (Heal, 2009). As such, the only defendable ethical position from a utilitarian point of view is to choose a zero discount rate, or at most, a very low value integrating the probability of the disappearance of the human race (Stern, 2006). On the opposite,

[1] This δ must not be confused with the rate of capital depreciation used in Chapter 5 and generally noted δ in growth model.

instead of being decided *a priori* on ethical grounds, the higher values of δ advocated by Nordhaus (2006) are deduced in order to make the consumption discount rate equal to the marginal productivity of capital[1].

The debate on the second factor is much more complex but it seems important to mention, with great simplification, the argument of Malinvaud (1953) that has been summoned into the Stern Review debate and that applies to a wider class of public investment problems. Considering a multiple goods economy, the interdependency of marginal utilities with respect to each good, expressed through the crossed elasticities, should imply different discount rates for each good. One of this good can be an "environmental good", yielding less and less consumption flows as time and environmental degradation go by. Consequently, the marginal utility of this good will increase and this will reduce the discount rate for this good, through a decrease in $\eta(c_t)R(c_t)$, while increasing it for the complementary goods. In the end, discounting future consumption on the ground that the marginal utility brought by this consumption will have decreased is receivable only if there is a perfect substitutability between "normal" and "environmental" goods. If complementarity is acknowledged (which seems like a sound assumption), then a uniform consumption discount rate cannot be justified, and environmental goods should be discounted at a different rate, that can even be negative. In a similar line of reasoning, Sterner and Persson (2008) show that an explicit consideration of the changes in relative prices induced by economic growth will justify scenarios that are even stricter than the Stern Review's recommendation. Their results, backed up by simulations on Nordhaus DICE model, are grounded on the idea that the relative prices of environmental goods and services will increase with scarcity thus increasing the economic damage of climate change.
The other standard arguments, in favor of high, low or nil discount rates, can be found in many discussion papers such as Weitzman (1998), Howarth (1998) and Goulder and Stavins (2002).

[1]It must be reminded that this theoretical equality holds only under a very limited set of perfect-markets assumptions that are far from being met in the real economic sphere, especially in presence of a massive externality such as climate change.

3.2.1.2 Empirical evidence and hyperbolic discount rate

If the conceptual transfer of discounting of private finite projects to infinite social investments is accepted, it seems necessary to study the observed behavior of individuals towards "futurity". Although the "real" discount factor is impossible to observe at society scale, the most robust empirical studies of individual discounting tend to show that the human response to "futurity" follows a similar pattern as the Weber-Fechner law of natural science (Heal, 2001). According to this law, *"human responses to a change in stimulus are inversely proportional to the existing level of the stimulus"*[1]. We tend to be a lot more affected by a postponement of one year in one year than by a postponement of one year in twenty years (from twenty to twenty one years). Concretely, the discount rate we will apply intuitively to a postponement of one year in twenty years will be drastically lower than the one we used to discount the additional year in one year. Moreover, Gollier (2005) shows that given the uncertainty on growth in the distant future, the discount rate should decrease in the very long term[2].

In formal terms, this range of response to futurity can be expressed through hyperbolic discounting displaying attractive properties (Ainslie, 1991; Heal, 2001). The discount factor can be written as a logarithmic discount factor:

$$\Delta(t) = e^{-Klogt}$$
$$\text{with } K > 1$$

Despite its solid empirical foundations, such a discount rate is rarely used in the environmental and resource literature, mostly because of the complexity of the mathematical problems involved but also because the optimal paths based on such a discount rate are not "time consistent"[3] (analyzed at time t+k, the path that was optimal "forever" at time t is not optimal anymore). Some climate change models using such a

[1] This phenomenon is particularly notable in our responses to changes in the intensity of a sound.

[2] As noted by Godard (2007) the Lebègue comission in France (Baumstark, 2005) has recommended that the discount rate for public policy be set at 4% for the first 30 years and decrease progressively until it reaches a floor-rate of 2% in a 500 year-horizon.

[3] Turner (2007) argues that at the social planner scale this time inconsistency is not necessarily a problem: *"While policy inconsistency at a given period of time is an institutional failure that should be corrected, policy switching over longer periods of time are surely inevitable and "correct" if uncertainties and surprises are unavoidable"*. In a decision framework characterized by uncertainty (such as climate change policies), adaptive learning about the actual scenarios, or at least about the probability distribution of the scenarios, is crucial.

discount rate can be found nonetheless (Nordhaus and Boyer, 2000; Weitzman, 2001) and their conclusions call, as intuition indicates, for a stricter decrease in greenhouse gases emissions.

Another position based one empirical observation is that of Knetsch (2005). According to this author, individuals discount future losses at a lower rate than the rate at which they discount future benefits. If this pattern can be extended to social discounting, then future environmental losses, generally minimized by standard discounting, would take on more weight in the cost-benefit analysis.

3.2.2 Alternative criteria

Considering the arguments against the ability of discounted utilitarianism to take into account the long-term, especially when environmental issues are at stake, some authors have developed new criteria more suitable to intergenerational cost-benefit analysis. In doing so, they have paved the way for a possible reconciliation between cost-benefit analysis and sustainability (Heal, 2001), at least as far as intergenerational equity is concerned. Heal (2000, Chapter 5) proposes a very thorough account of these alternative criteria in relation to the challenge of encompassing sustainability within the cost-benefit analysis framework. We will review briefly these more "future-oriented" criteria and focus especially on the Green Golden Rule, the Maximin and the Overtaking criterion that seem the most promising to us in terms of operational sustainable frameworks. It is essential to keep in mind that although the criteria are modified, the method itself, ie cost-benefit analysis, remains the same. Therefore the results obtained with these criteria are not spared by the criticism laid on cost-benefit analysis itself (we will review the other limitations of cost-benefit analysis later).

3.2.2.1 Green Golden Rule, Maximin and Overtaking Criterion

In this chapter we will present and exploit three alternative intertemporal welfare functionals: the Green Golden Rule, the Maximin and the Overtaking criterion. They draw attention on crucial dimensions of sustainability: the long-run horizon and the intergenerational equity. We shall devote a specific section to each of them and take the time to apply these criteria to our own pollution control problem. In particular we will see if in terms of sustainable policies the resulting paths offer a robust alternative to the optimal paths determined in the previous chapters.

Considering the significant mathematical complications that arise in the stock configuration of our pollution model with assimilative capacity (see Chapter 2), we will use the flow pollution framework in this discussion of the alternative criteria. This restriction will allow us to highlight more easily the outstanding properties of the criteria. The extension of the following analysis to our stock pollution configuration will be tackled exhaustively in future works but we shall nonetheless try to provide an intuitive interpretation of the impact of these alternative criteria on the stock pollution problem.

3.2.2.2 The Chichilnisky criterion

The well-known Chichilnisky criterion can be presented as a consistent synthesis of the diverging requirements of sustainability as it suggests a weighted combination of the discounted cost-benefit analysis and the Green Golden Rule criteria. Throughout a very elegant axiomatic demonstration, Chichilnisky (1996) designs a criterion that is supposed to avoid both the discounted utilitarian bias towards the present and the Green Golden Rule bias towards the future (see next section). The respective "weight" of the present and the future are set by the parameter θ such that:

$$\max_{c_t} W = \theta \int_0^{+\infty} u(c_t, S_t)\Delta(t)dt + (1-\theta) \lim_{t\to\infty} u(c_t, S_t)$$

where $\Delta(t)$ is a measure[1] such that

$$\int_0^{+\infty} \Delta(t)dt = 1$$

Albeit its formal elegance and the intuitive "equilibrium" it introduces between short-term and long term, this criterion can hardly found a theoretical guideline for policy. It is clear that the value of the θ parameter is crucial to the final results and that since this parameter cannot be observed or estimated it is in the end a merely ethical and political decision. But as we will develop later, the role of such an exogenous constraint should not necessarily be seen as intrusive in economic policy making. However the

[1]In particular it can be a standard discount factor like $\Delta(t) = e^{-\delta t}$.

problem of choosing the “correct” rate of discount remains for the first term of the welfare functional so this criterion cannot spare the “discounting debate”. Finally, as noted by Heal (2000, p.101), this criterion can only work with declining discount rates. It involves thus already preliminary tampering with the standard discounting problem. Regarding technical obstacles, it must be noted that so far the optimization problem posed by this criterion has only been solved for a few very particular utility functions. Its operational implications seem therefore quite limited and we shall not investigate any further this criterion although it does embody a rather satisfying vision of intergenerational equity.

3.3 The Green Golden Rule

If we shift the prism of optimality from the sum of net present values to the long-term utility level, we can explore an alternative optimality criterion known as The Green Golden Rule. Inspired from Phelps' Golden Rule of economic growth (1961), this criterion has been formalized by Chichilnisky et al. (1995). It is defined by Heal (2000) as *“the path that of all feasible paths gives the highest value of the long run level of utility”* and formalized as the solution to a new maximization program. This program seeks the maximization of an intertemporal utility function that includes both consumption and the stock of natural resources as arguments. In the original formulation of the problem this utility function accounts for the standard satisfaction derived from consuming a produced good that demands natural resource input and the satisfaction yielded by environmental amenities linked with the stock of resources itself, as it can be the case with forests. This specification fits quite well our own pollution problem as we show below.

$$\max_{feasible\ paths} \lim_{t \to \infty} u(c_t, S_t)$$

where c_t is the level of consumption of a renewable resource S_t.

3.3.1 The maximum sustainable utility level

In order to achieve the maximum sustainable utility level, the economy must be set in a sustainable configuration, that is to say in our model that the polluting emissions must respect the assimilative capacity threshold. Formally in order to have the stock and the production reach a stationary level we must have

$$p(t) \leq A(t)$$

According to Proposition 1.1 the most efficient value of the unharmful subset of $p(t)$ is

$$p(t) = A(t)$$

Let us now determine the optimal pollution path under the Green Golden Rule criterion. We have

$$U(A, A) = f(A) - D(A, A)$$

and, since $D(A, A) = 0$ for all A

$$\max_{A} U(A, A) \Rightarrow f'(A) = 0$$
$$\Rightarrow A = x_p$$

In the case of irreversible degradation we have necessarily by assumption

$$A_{GGR} = \min\{x_p, A_0\} = A_0$$

The initial level of assimilative capacity should thus be conserved indefinitely.

However if restoration is available, the initial level of assimilative capacity A_0 can be increased to x_p if $A_{max} \geq x_p$. In that case $A*_{GGR} = \min\{x_p, A_{max}\}$. Let us note $\underline{A}$ such that $\underline{A} = \min\{x_p, A_{max}\}$.

3.3.2 Insights on the stock pollution interpretation

We can intuitively assert the type of solution that would arise from the application of the Green Golden Rule to our framework of stock pollution elaborated in Chapter

2. This criterion would request that the economy settles in a situation such that the stock of accumulated pollution Z is lower than $\bar{Z}$, the threshold level at which assimilative capacity degradation occurs. If the assimilative capacity is already too degraded and no restoration is available, this might not be feasible as even a complete stop of emissions would not be enough to bring back the pollution stock below $\bar{Z}$ before the assimilative capacity is exhausted. If restoration is available, then the assimilative capacity can be restored back to a higher level while the stock of pollution is reduced below $\bar{Z}$. Formally we would have the following solution:

1) without restoration[1]

If $Z_0 \leq \bar{Z}$: $\alpha^* = \alpha_0$ and $Z^* = \underline{Z}$

with $\underline{Z} = \min\{\bar{Z}, \tilde{Z}\}$ and $\tilde{Z}$ is such that $U'(\alpha_0 \tilde{Z}) - D'(\tilde{Z}) = 0$

If $Z_0 > \bar{Z}$: $\alpha^* = \max\{0, J(T)\}$ and $Z^* = \underline{Z}$

with $J(t) = \alpha_0 - \int_0^t h(Z(s))ds$.

$J(t)$ is the remaining assimilative capacity after the T periods necessary to reduce the stock of pollution below $\bar{Z}$.

2) with restoration

$\forall\ Z_0 \leq \bar{Z}$: $\alpha^* = \alpha_{max}$ and $Z^* = \underline{Z}_2$

with $\underline{Z}_2 = \min\{\bar{Z}, \tilde{Z}_2\}$ and $\tilde{Z}_2$ is such that $U'(\alpha_{max}\tilde{Z}_2) - D'(\tilde{Z}_2) = 0$

It is straightforward that if restoration is available the steady state level of assimilative capacity and pollution stock will be respectively higher and lower that in the first case. The transition paths leading to these situations of the stock problem will not be addressed here but in the next subsection we develop the transition paths for the flow configuration.

3.3.3 Intergenerational equity and the transition path

Let us now characterize the transition paths leading to this equilibrium. The Green Golden Rule determines the limiting behavior of the economy but it does not shed any light on the way this limit is approached (see Heal, 2000, p.53). This limitation raises

[1] Let us recall that at the equilibrium $y^* = \alpha^* Z^*$.

equity issues as it does not provide clear guidelines for the sharing of the economic or the ecological burden compared to the optimal paths that are determined explicitly at any time in the two first chapters thanks to the shadow price. In our framework, this acknowledged "weakness" of the criterion affects only the restoration case and raises intergenerational equity questions that can only be answered with another criterion, the Maximin criterion studied in the next section.

3.3.3.1 Irreversible case: a unique equitable transition path

Indeed, in the irreversible context, the only way for the economy to reach the corner optimal solution A_0 as an indefinitely sustained level of assimilative capacity is to have $p \leq A_0$ at all time. And since we must choose $p = A$ among this subset according to the most basic optimization argument[1], the trajectory leading to $A_{GGR} = A_0$ is unique and demands $p(t) = A(t) = A_0$ $\forall t$. Hence the following proposition:

Proposition 3.1. *If restoration is not available, then according to the Green Golden Rule,* $A(t) = A_{GGR} = A_0$ *and* $p(t) = A_{GGR} = A_0$ $\forall t$.

Figure 3.1: Green Golden Rule: irreversible case

Figure 3.1 represents this unique trajectory.

[1]Once again, this is true for the flow pollution control but we will not develop here the translation of the Green Golden Rule in the stock pollution framework.

3.3.3.2 Reversible case: distinctive transition paths

Nevertheless, in the restoration case, the Green Golden Rule does not provide a ready-made definite path to reach $A_{GGR} = \min\{x_p, A_{max}\}$. Given the flexibility offered by artificial restoration, there is an infinite set of decisions $\{p, A\}$ over time that end up at $\{p^*, A_{GGR}\}$ but it is not possible to point out immediately the one that "accumulates most utility along the way". It is necessary that the total net restoration of assimilative capacity along these paths be positive (strictly positive if $A_0 < A_{max}$) but this net restoration can be carried on either very early or later after a phase of degradation. These different paths imply inequalities of social welfare between different generations as one generation or more will have to support the burden of restoration costs *(ie* the investment in natural capital), while others can decide to overshoot the assimilative capacity to obtain more utility during their lifespan. Considering these discrepancies, it seems interesting to us to distinguish upon their intergenerational distributional characteristics some specific types of paths that lead to the Green Golden Rule maximum sustainable level of utility. This qualitative analysis will be far from exhausting all the possible solutions but it draws attention on the variety of paths that can achieve the Green Golden Rule optimum. We need to separate two cases depending on the value of $\underline{A}$, with $\underline{A} = \min\{x_p, A_{max}\}$:

Case 1: $\underline{A} = x_p$

In this case, it is straightforward that once $\underline{A} = x_p$ has been reached, it is always sub-optimal ("Pareto decreasing") to reduce the assimilative capacity again since any excess of pollution would contribute negatively to the benefit function, in addition to the environmental damages incurred. Consequently, since it is neither rational to increase it or to reduce it, the only solution left is to leave the assimilative capacity constant at $A_{GGR} = x_p$. The following proposition can be deduced from our previous arguments. Let us denote $\underline{T}$ the time at which $\underline{A}$ is reached.

Proposition 3.2. *Once the maximum sustainability level x_p has been reached at $\underline{T}$, the Green Golden Rule combined with the fundamental rationality assumption of microeconomics demand that the economy remains indefinitely at this level: $\forall t \geq \underline{T}$, $p = A_{GGR} = x_p$.*

This property translates graphically (see following Figures) by a constant straight trajectory as soon as A has reached $\underline{A}$. Unfortunately, we cannot infer anything more precise from the Green Golden Rule about the path before $\underline{T}$ as well as on the value of $\underline{T}$ itself. We can only be sure that it will never go above $\underline{A}$. It is not possible to say if those paths are monotonic or not. However, it is unambiguous that any generation coming after $\underline{T}$ will enjoy the maximum sustainable level of utility, no more no less, without having to support the costs of investment in natural capital that have lead to this situation. As such, it is undeniable that the Green Golden Rule, in this specific case, is biased in favor of future generations[1].

Let us now distinguish the different types of pollution path that can be followed on the transition path towards the Green Golden Rule equilibrium. As we explained previously, there are infinite possibilities for these paths, as long as they asymptotically converge towards $\underline{A}$, but we will point out three specific types to stress the equity issues at stake with the Green Golden Rule.

1) *The most rapid approach path*

This path minimizes the time needed to reach $\underline{A}$ by setting $p = 0$ until $A = \underline{A}$. It naturally deprives the first generations (until time $\underline{T}_1$) of any utility at all[2]. Obviously the larger the gap between A_0 and x_p is, the longer the utility-less phase lasts. In terms of intergenerational equity this path is of course quite problematic as it implies excessive sacrifices of the early generations. It could be argued that such sacrifices can still be imagined on a local scale but the social (and political) feasibility would be proportional to the average welfare level of the population concerned. As shown in Figure 3.2, the slope of $A(t)$ increases as the difference between the assimilative capacity available and the pollution level (always equal to 0) increases.

[1]This bias is explicitly acknowledged by Smulders (1999) for whom the Green Golden Rule implies that *"current generations are willing to sacrifice whatever is needed to attain the best for the future"* .

[2]By rejecting the Inada conditions in our working assumptions, we have permitted a nil level of private benefit.

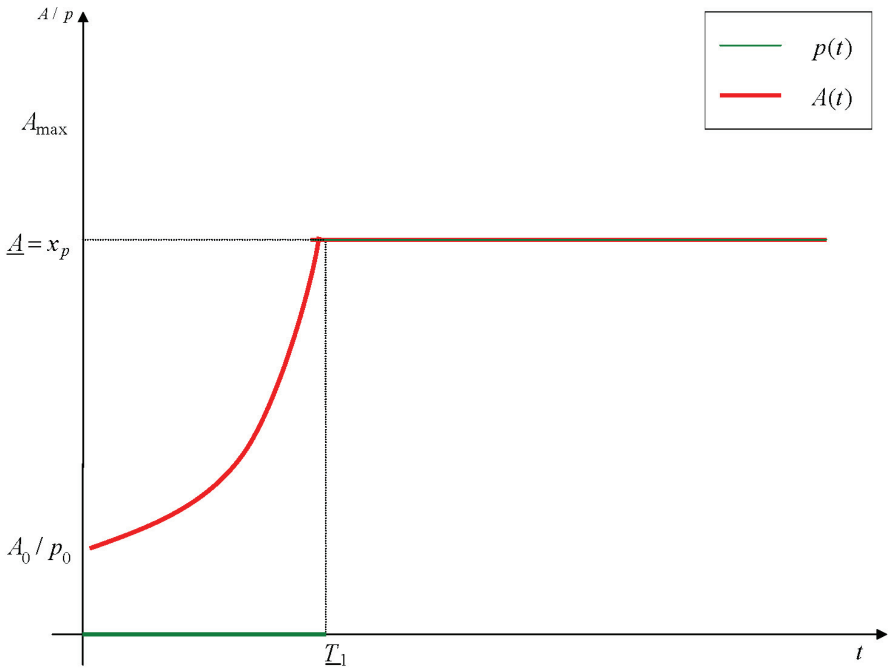

Figure 3.2: Green Golden Rule: Case 1. Path 1.

2) *The U-shaped*[1] *path*

This path starts with a degradation of the assimilative capacity under excessive levels of pollution and these emissions increase along the path to compensate the loss of environmental conditions. Before the assimilative capacity is completely depleted a restoration effort is launched that necessarily requires very low levels of pollution ($p(t) < A(t)$). This "authorized" pollution increases as the assimilative capacity is restored but meanwhile the "middle" generations have to bear low levels of welfare until time $\underline{T}_2$ when the steady state is reached. This path poses the same equity problem as the most rapid approach path although this time a different class of generations is underprivileged. It is not the current generations that are biased again but more distant ones in the mid-term future. This path is highlighted to show that although the Green Golden Rule objective maximizes the environmental asset level, it can allow for a serious degradation of this asset along the transition path. If we were to consider uncertainty in our model, we could fear that such a path might bring the assimilative capacity down to dangerous thresholds from which it might be impossible

[1]As shown in Figure 3.3, it is the assimilative capacity trajectory in time that has a U-shaped, the pollution follows a different pattern.

to restore it. In this case the environmental policy should thus take into account a set of safe minimum standards (Ciriacy-Wantrup, 1952; Bishop, 1978) so as to make sure that the dangerous zone that might lead to the extinction of the environmental asset is never attained (see Chapter 4).

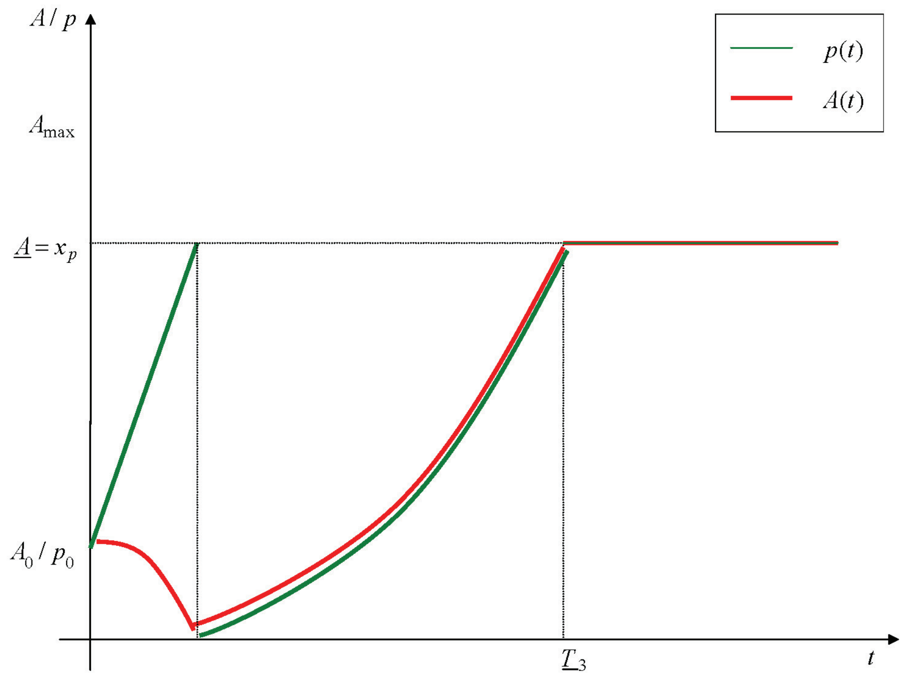

Figure 3.3: Green Golden Rule: Case 1. Path 2.

3) *The gradual path*

This path is an example of the path that can try to limit the bias against any generation by minimizing the differences in utility experienced by the various generations until time $\underline{T}_3$ when the steady state is reached. Let us design a monotonic path along which assimilative capacity is always respected. Given that an effort (in the sense of a restriction from polluting as much as the assimilative capacity could permit), is necessary to reach $A = x_p$, and that this effort will be borne exclusively by the first generations, a good guideline to allocate this effort among those first generations would be to share this effort as equitably as possible, by granting the same constant level of consumption/production to all generations contributing to the "effort". Let us denote p_0 a constant level of pollution such that

$$p(t) = p_0 \quad \forall \quad t < \underline{T}_3$$
$$p_0 < A_0$$

The level of utility experienced by the generations before $\underline{T}_3$ is constant and equal to $f(p_0)$. On such a path, the assimilative capacity dynamics are

$$\dot{A}(t) = -h(p_0, A(t)) > 0$$

The date when the steady state is reached, $\underline{T}_3$ depends on the initial gap between x_p and A_0 and on the constant level of pollution p_0. We have

$$x_p - A_0 = \int_0^{\underline{T}_3} -h(p_0, A(t))dt$$

In terms of forgone benefit, it is clear that the last generations of the $[0, \underline{T}_3]$ period will provide more effort (they still pollute only p_0 whereas their available assimilative capacity has increased). But since this effort is not a direct cost levied upon them, it can be seen as bearable. Moreover, the concavity of the restoration function, demands a higher effort to achieve the same restoration level as the total level of assimilative capacity increases. The absolute level of restoration carried on will increase along the path but its rate of increase will diminish. Our "gradual" path will thus provide a constant level of utility and a concavely increasing restoration level until $\underline{A}$ is reached. See Figure 3.4 for a graphical exposition. This path provides a basic but satisfactory answer to the question of intragenerational equity within the subgroup of generations belonging to the $[0, \underline{T}_3]$ period. It is easy to verify that the transition period will be all the longer as p_0 (and thus $f(p_0)$) is high.

Let us finally notice that it is straightforward that the transition path ranks in the following order regarding their duration:

$$\underline{T}_2 > \underline{T}_3 > \underline{T}_1$$

Policy interpretation

This last example of gradual path could provide an intuitive guideline for sus-

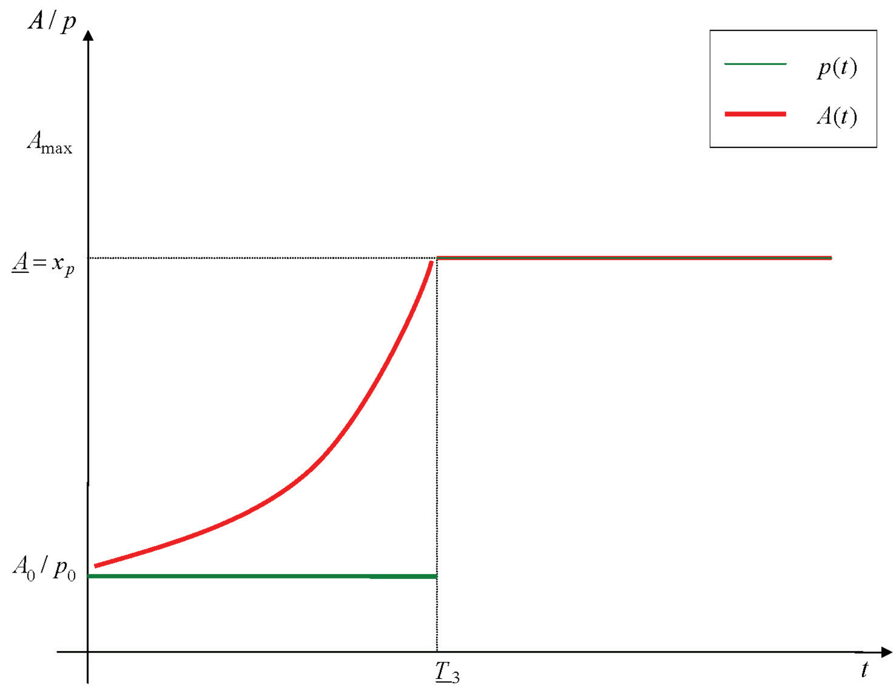

Figure 3.4: Green Golden Rule: Case 1. Path 3.

tainable pollution control regulation. In the (unlikely) event that a social consensus agrees on the necessity to implement the Green Golden Rule, path (3) could settle the equity issue that would undoubtedly arise among the generations concerned by the transition period. If we can accept the idea that all the future generations beyond date $\underline{T}_3$ (the steady state generations) will be better off than the "transition generations" and that this inequality is necessary to maximize the indefinite sustainable level of utility, then we could accept the notion of imposing undiscounted equity within each subgroup as a satisfying measure or social sustainability in parallel to the environmental sustainability guaranteed.

Case 2: $\underline{A} = A_{max}$

Let us note that in the real economic sphere, Case 2 is unfortunately more likely to occur than Case 1. Indeed, the limits of the fundamental neoclassic assumptions on the concavity of the utility function are well known and we can assume that there is a stricter bound[1] on the maximum level of assimilative capacity than on the private benefit optimum x_p so that it will be more likely to have $\min x_p, A_{max} = A_{max}$.

[1] The hypothesis that a firm restricts itself to a given level of pollution/production because it has reached its optimum is not backed up by much empirical evidence.

It is obvious that in this configuration, Proposition (3.2) does not hold anymore. Indeed, it is now shortsightedly "rational", microeconomically speaking, to overshoot the assimilative capacity in search of a higher immediate utility until the Turvey condition ($f' - D' = 0$) is met.

4) *Oscillating paths*

Practically, a "greedy" generation already enjoying the highest maintainable level of instantaneous utility $f(A_{max})$ might decide to increase its current utility by overshooting the assimilative capacity. This excess will translate into a reduction of the latter, that will have to be restored later on, probably by other generations. The asymptotic convergence towards the Green Golden Rule steady state could thus take the form, among many others, of a sinusoid path oscillating along the steady state line (see Figure 3.5). This particular path taken as an example illustrates the potential problems that may arise along the transition path. In this specific case the economy remains on the transition path indefinitely as it oscillates around the sustainable steady state, choosing either to pollute in excess of the assimilative capacity, thus degrading it and then polluting below the assimilative capacity to restore it.

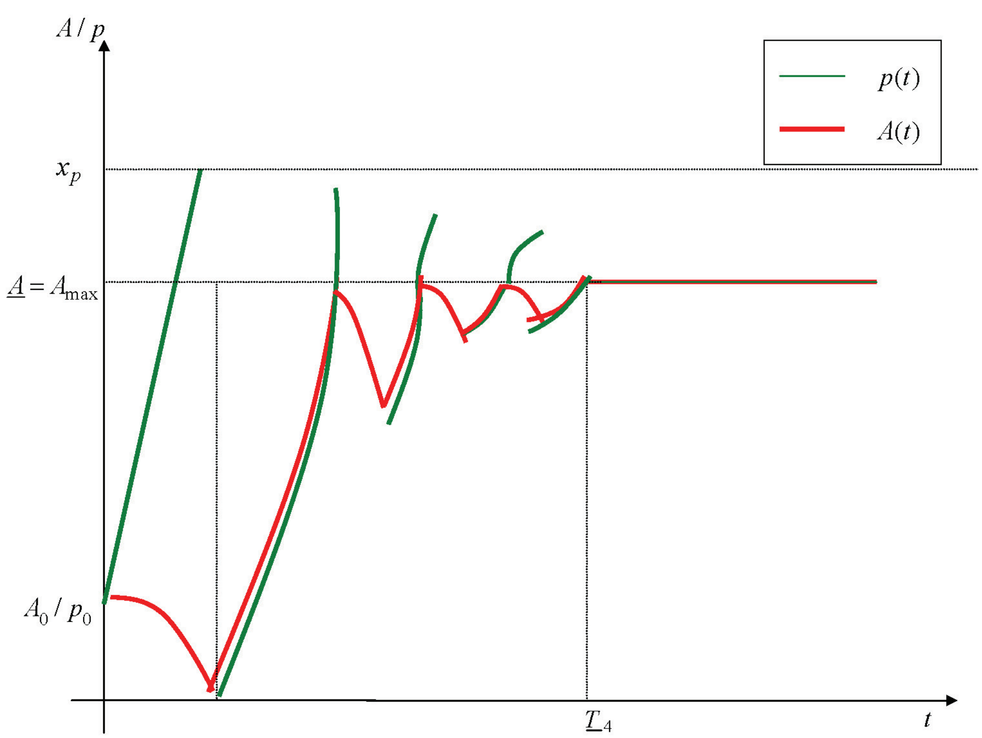

Figure 3.5: Green Golden Rule: Case 2. Path 4.

A brief conclusion on the Green Golden Rule

The exploration[1] of the Green Golden Rule as an alternative criteria to replace the present-biased discounted utilitarian framework has shown that this rule could in some occasions display a bias towards the future, and lacked a clear definition of the transition paths leading to the desirable steady state. This last point is particularly problematic in terms of intergenerational equity as this rule does not indicate how to share the burden of consumption-restriction between generations. That is why although this criterion leads eventually to a situation that looks intuitively as a sustainable management regime of pollution, it lacks the ability to yield operational landmarks on the economic trade-offs that are indispensable to design sound and efficient policies.

3.4 The Maximin criterion

3.4.1 Definition of the Maximin criterion

A significant part of the economic literature on intertemporal equity and sustainability has been built on the concept of equity developed in the theory of justice designed by Rawls (1971). Although Rawls himself acknowledged that the Maximin criterion he builds a case for can be applied only to intragenerational justice and not to intergenerational equity problems, economists like Solow have suggested an economic approach that is *"plus Rawlsien que le Rawls "* (Solow, 1974). The Maximin criterion has thus become a criterion for intertemporal optimization problems. According to this criterion, wealth (or utility) inequality between individuals (generations) is tolerable if and only if it improves the situation of the least wealthy individual. As he transposes the concept in the intergenerational context, Solow shows that the Maximin rule demands a constant level of utility: *"except possibly for trick cases, the max-min principle requires that consumption per head be constant through time. If consumption per head were higher for a later than for an earlier generation, then social welfare would be increased if the early generation were to save and invest less, or to consume*

[1] We shall revisit briefly the Green Golden Rule later (Chapter 4, Appendix B) as the shift of framework we operate in Chapter 4 will prove very useful to strengthen the robustness of our results compared to the "standard" Green Golden Rule literature.

capital, so as to increase its own consumption at the expense of the later generation. If consumption per head were higher for an earlier than for a later generation, then social welfare would be increased if the early generation were to consume less and, correspondingly, save and invest more, so as to permit higher consumption in the future".

Formally the Maximin criterion applied to intertemporal problems writes (Heal, 2000):

$$\max_{feasible\ paths}\{\min_{generations\ t}(Welfare_t)\}$$

where "Welfare$_t$" denotes a measure of the welfare enjoyed by society at time t. This measure can be the utility of current consumption $U(c_t)$ as in Solow (1974).

3.4.2 Application of the Maximin to our pollution control problem

The application of this criterion to our pollution control problem yields an unambiguous solution in Proposition (3.3):

Proposition 3.3. *Along the Maximin path, we must have* $\forall t$*:* $p(t) = A_0$ *and* $A(t) = A_0$*, in the no restoration as well as in the restoration case.*

Proof:

Case 1: No Restoration

Let us use as the baseline scenario the constant sustainable path such that for all t:

$$\{pt), A(t)\} = \{A_0, A_0\}$$

It is straightforward that the minimum level of welfare sustained by a generation along this path is equal to $f(A_0)$. Consequently we shall compare the Maximin level of welfare of all the other paths to $f(A_0)$. If at time t_1 society deviates from this path and decides to pollute beyond the assimilative capacity, thus depleting it, a higher level of welfare can be experienced for some time. But such a "deviant" path will inevitably lead to a sub-optimal situation compared to the baseline path with regards to the Maximin criterion, whether or not society chooses to start respecting the assimilative capacity at one point. If the assimilative capacity keeps on being depleted, there will eventually be a time t_2 such that $A_{t_2} = \tilde{A}$ has been reduced to the point that the social welfare $f(p) - D(p, A(t_2))$ will be indefinitely inferior to the baseline welfare $f(A_0)$, for any pollution level p, including the private maximum x_p.

$$\begin{aligned} & f(x_p) - D(x_p, \tilde{A}) < f(A_0) \\ & \forall p \in [0, x_p] \end{aligned}$$

This means that all generations coming after time t_2 will enjoy a welfare inferior to the welfare they could have enjoyed on the baseline sustainable path.

If society starts respecting the assimilative capacity at time t_3 before reaching $\tilde{A}$, then the maximum sustainable level of welfare available will be $f(A_{t_3})$, and since the assimilative capacity has been irreversibly depleted beforehand we have necessarily

$$\begin{aligned} & A_{t_3} < A_0 \\ & f(A_{t_3}) < f(A_0) \end{aligned}$$

This means that for all the generations coming after t_3, the maximum level of welfare that can be achieved will be less than the welfare guaranteed by the baseline sustainable path. We have thus shown that in the no-restoration case, any path deviating from the baseline path will imply at least one (but in fact many more) generation with a welfare level inferior to the one it would have had along the baseline path. The application of the Maximin criterion leads us to choosing a constant path, both economically and ecologically speaking, such that for all t:

$$\{p(t), A(t)\} = \{A_0, A_0\}$$

Case 2: Restoration

Let us keep the constant sustainable path (A_0, A_0) as the baseline. Two new options are available compared to the previous case: depleting and then restoring or directly restoring.

If at time t_1 society deviates from the baseline path and decides to pollute beyond the assimilative capacity, this will lead either to one of the two sub-optimal paths described in Case 1 or to a third restoration solution at time t_2. In that case, in order to restore the assimilative capacity a "sacrifice" in welfare will be necessary and some generations will enjoy a level $f(A_R)$ such that $A_R < A(t_2)$. Since $A(t_2) < A_0$, any of these paths combining depletion and then restoration is clearly sub-optimal compared to the baseline path in the light of the Maximin criterion.

If at time t_1' society deviates from the baseline path and decides to pollute strictly less than the assimilative capacity to restore it, this will lead at least one "sacrificing" (restoring) generation to experience a level of welfare inferior to the minimum welfare of the baseline case. In order to increase the assimilative capacity above A_0, a pollution level p_R such that $p_R < A_0$ is necessary, thus implying a level of welfare at time t_1' equal to $f(p_R)$. And since $f(p_R) < f(A_0)$, this restoring path is suboptimal whatever happens afterwards. Once again this path involving restoration is clearly sub-optimal compared to the baseline path in the light of the Maximin criterion. We can thus represent the Maximin path in Figure 3.6 and this path is valid for both the restoration and the no-restoration case.

Figure 3.6: Maximin path

3.4.3 Insights on the stock pollution interpretation

Although it is not as intuitive as in the Green Golden Rule Case, we can assert the type of solution that would arise from the application of the Maximin to our framework of stock pollution elaborated in Chapter 2. If the initial stock of pollution is below the degradation threshold ($Z_0 \leq \bar{Z}$), then it is clear that the economy will stabilize at this level, whether or not restoration is available. However if $Z_0 > \bar{Z}$ the pollution stock needs to be reduced below the degradation threshold, either through low emissions or through restoration. Indeed, although this reduction will entail a reduction of the welfare level of the present generations, the latter will still be higher than the welfare future generations would have enjoyed if society had let the assimilative capacity get completely depleted.

3.4.4 Policy interpretation and limits of the Maximin criterion

The last argument of the proof above reveals a well known characteristic of the Maximin criterion. We have shown that according to this criterion, it was never desirable to increase the initial level of assimilative capacity, even if this involved little sacrifice and even if it could lead to an indefinitely sustainable level of welfare superior to $f(A_0)$ (as the Green Golden Rule or the Overtaking criterion advocate). In more general terms, this criterion is an extremely conservative one. It prohibits any kind of investment above the strict maintenance level, whatever the return of this investment for all future generations may be[1]. This rejection *a priori* of any kind of trade-offs, a problematic feature for an economic criterion, has been underlined by Dasgupta and Mäler (1995). It is particularly true when we deal with investment in natural capital, that may be crucial to reach a sustainable situation as we have shown in Chapter 2.

Due to this impossibility to operate intertemporal Pareto-improving trade-offs, the Maximin criterion yields paths that are extremely dependant on the initial conditions (here the constant level of welfare is $f(A_0)$). This may lead to "poverty traps" if those initial conditions are not abundant. As Solow (1974) himself noted *"if the*

[1] To caricature, we could imagine an economy that has the possibility to invest in a magical asset with incredible returns on investment. The Maximin rule would not allow even an infinitesimal investment (sacrifice) that would later yield eternal happiness to the rest of humanity forever...

initial capital stock is very small, no more will be accumulated and the standard of living will be low forever". The same happens here with the assimilative capacity asset, no investment in natural capital is carried on since trade offs are not allowed. Translated in policy terms, this criterion does not allow a wide range of action and runs the risk of being discarded for its conservatism, particularly not welcome in a poor initial conditions situation.

3.5 Overtaking criterion and sustainability

3.5.1 Definition of the Overtaking criterion

The Overtaking criterion, very clearly exposed by Heal (2000, Chapter 5) finds its origins in the work of von Weizäcker (1967). The criterion attempts to suppress the bias in favor of the present introduced by the discount rate while avoiding the formal pitfalls that arise from zero-discount rate problems.

A path c_1 is said to weakly overtake a path c_2 if there exists a time T^* such that for all $T > T^*$, we have

$$\int_0^T u(c_1(t))dt \geq \int_0^T u(c_2(t))dt$$

3.5.2 Overtaking criterion and sustainable path

The Overtaking criterion proves to be an interesting alternative to avoid the bias against future benefits due to the positive discount rate applied. Although we are aware this criterion is never summoned when it comes to policy making, we find it relevant in our discussion to see if it can increase the sustainability of optimal paths.

In doing so, we will define a standard sustainable path that a strong environmental sustainability approach such as Daly's (1990) would advocate. Indeed, regarding the problem we are dealing with, an obvious incarnation of Daly's third sustainable development operational principle[1] based on the respect of the environment's assimilative

[1]Daly (1990) describes three operational principles of sustainable development that consist respectively in extracting exhaustible resources at the rate at which backstop technologies can offer durable substitutes, not harvesting natural resources at a higher rate than their natural regeneration

capacity would be a pollution path respecting at every single period the assimilative capacity of the ecosystems at stake. From now on we shall call this path the strong sustainable path, identical to the baseline case used in the Maximin section before. Our analytical approach will focus on the comparison of the set of paths satisfying the Overtaking criterion with the baseline discounted utilitarian path characterized in the Chapter 1.

3.5.2.1 First case

We will start our analysis with the initial case not allowing restoration. We demonstrate in Appendix A the following proposition:

Proposition 3.4. *In the no-restoration configuration, the strong sustainable path dominates all other feasible pollution paths, including the discounted utilitarian path.*

This result can be interpreted through the prism of optimal resource exploitation. Indeed it is very similar to Heal's results with this criterion (Heal, 2000, 6.4) which determines the optimal depletion path of a non-renewable resource under the Overtaking criterion. This criterion advocates a total and indefinite conservation of the resource which is tantamount to the conclusion stated in Proposition (3.4) if assimilative capacity is considered as a natural exhaustible resource.

3.5.2.2 The case with restoration

We may now define as the maximum strong sustainable set the set of pollution paths that increase the assimilative capacity through restoration until it reaches $\underline{A} = \min x_p, A_{max}$ and remains at this level indefinitely with $p = \underline{A}$.

We prove in Appendix A the following proposition:

Proposition 3.5. *In the configuration allowing restoration, any path belonging to the maximum strong sustainable set dominates all other feasible pollution paths, including the discounted utilitarian path.*

rate and emitting pollution within the sink capacity of the environment.

This result can also be compared to Heal's results (2000, 7.4), that states that in the case of a renewable resource, the overtaking criterion determines a path leading to the Green Golden Rule equilibrium point. This equilibrium does not correspond to the maximum level of stock attainable (the carrying capacity) but to the point where the marginal productivity of the stock equals the ratio of the marginal utility provided by the stock over the marginal utility provided by consumption. In our problem the marginal productivity of the renewable stock does not depend only on the total stock but also on the "rest" granted to the assimilative capacity by the polluter. This major difference, reflecting the peculiar nature of assimilative capacity as an exhaustible resource, is accountable for the discrepancy between those two sets of results.

The Overtaking criterion path leads thus to the same results as the Green Golden Rule. Once again the transition phase is not determined unambiguously by this criterion as it is the asymptotic behavior of the path that matters. Moreover, the ranking of paths it recommends remains incomplete as in some settings it will judge equivalent a constant utility path and an oscillating path (oscillating around the constant path value), whereas a discounted criterion would have been able to rank them. As such, this criterion does not provide much economic added value[1] to implement sustainable pollution control policies.

3.6 Constrained optimization, cost-efficiency analysis and sustainable tax

Now that we have explored alternative criteria that do not display the same biases as the discounted utilitarianism framework, it is necessary to explore another use of this framework. Instead of changing the optimality criteria, another solution in favor of sustainable trajectories might be to amend the cost-benefit method itself while keeping the discounted utilitarian framework. Turning cost-benefit analysis (CBA) into a cost-efficiency analysis, might indeed be a positive step towards more sustainable economic trajectories.

[1]In addition, it has been shown by Lauwers (1992) that this criterion actually displays impatience despite having a zero discount rate.

Insisting on determining optimal paths and checking *a posteriori* if those paths are sustainable, as it is done in the larger part of the pollution control literature does not seem an appropriate position to open economics to the multidimensional challenges of sustainability. We favor a view similar to the one advocated by Bond and Farzin (2007) who claim that sustainability concerns should be translated as constraints on the net present value optimizing problem, even though theoretical economists are not fond of constrained optimization and cost-effectiveness analysis. It seems only natural for a social planner committed to sustainability to set up an environmental objective as its top priority instead of global optimality. Once this environmental target is determined, a constrained optimization will prove very helpful to minimize the costs of achieving this target. It seems indeed more reasonable to use the tools of economics to design directly sustainable paths by conducting constraint based optimization (Woodward, 2000) rather than to hope that the unconstrained optimal paths will be sustainable. As such, cost-efficiency analysis, which is more or less tantamount in this context with constrained optimization, can be used as a powerful tool to implement environmental regulation at a minimal cost. It is worth noting that Pearce, whose intuition (1976) is at the origin of the present work, developed his initial analysis in another contribution (1988) where he advocated a similar shift from cost-benefit analysis to cost-efficiency analysis. We will restrict here our analysis to the flow pollution framework but constrained optimization on stock pollution problems and especially on greenhouse gases accumulation problems can be found in the literature (Ha-Duong et al., 1997).

Before proceeding to a detailed analysis of the perspectives offered by constrained optimization, we must mention that other "amendments" to the standard CBA analysis can be found in the literature, among which the introduction of an exogenous catastrophic risk and the explicit valuation of environmental services. We provide an analysis of the most standard approach of these two variations and question their true impact in Appendix B.

3.6.1 Preliminary clarifications

3.6.1.1 Definitions of CBA

Before we dwell into the critical analysis of cost-benefit analysis, it can be helpful to remind exactly what is meant by this term. Indeed, it denotes sometimes very different frameworks of economic analysis. The main source of confusion comes from the two distinctive applications of cost-benefit analysis. The most "operational" one is the cost-benefit analysis applied to private or public projects that require an investment and yields benefits discounted over time. The other application concerns the general net present value framework in which optimization of social welfare functions (including environmental impacts on utility) is conducted. For instance, the common framework adopted by most economists to deal with climate change is reckoned to be cost-benefit analysis. A cost-benefit analysis of the first type is a typical preliminary requirement[1] before implementing a large-scale project with potential social or environmental impacts, for instance the building of a dam such as the "Three Gorges" dam in China (Morimoto and Risako, 2004). Both methods imply complex ethical considerations about the compensation mechanisms at stake (Kaldor-Hicks criterion) that are discussed at length in Mishan (1972). As the Kaldor-Hicks criterion requires an outcome where the "winners" in a project could potentially compensate the "losers", it has been widely criticized on the ground that the "losers" are never actually compensated by the "winners". Another bone of contention is the legitimacy of trade-offs between radically different values (financial benefits vs. human health, or worse, vs. human life). Conversely they both share the advantage of acknowledging very explicitly the various stakeholders involved in an economic (political) decision. Although he worked very extensively on the first type of cost-benefit analysis, Pearce refers to the second type in his 1976 article. It must be clear by now that our work, that finds its root in that article, focuses explicitly on the second interpretation, that is most commonly called *social optimization* in the literature. From now on we shall refer to this second notion, tantamount to intertemporal discounted optimization, when we discuss the pros and cons of cost-benefit analysis, although many of the arguments developed here are relevant to both methodologies.

[1] It has been made mandatory by law for most public investments decisions in the United States.

3.6.1.2 The widespread dominancy of CBA over CEA: preferences revelation vs exogenous constraints

Among the economists community there is a widespread feeling of mistrust towards the use of constrained optimization. This defiant posture is based on the belief that such a method restricts the role of economic science to an analytical tool and deprives it partially from its normative power. Many economists oppose such an approach that tends to ignore what the "true" economic preferences of agents (or of a representative agent) would have chosen freewillingly. In doing so, the social planner under-exploits the potential of the economic science and its policies may result in a wasteful allocation of resources. It is true that the choice of the minimal amount of asset that should be preserved along an economic path is necessarily a political choice, hopefully based on ecological expertise, but it is an exogenous constraint nonetheless and as such it is not well accepted by standard economics habits. But as we shall suggest in the conclusion of Chapter 4, some environmental problems are so concerning that they actually demand the implementation of an exogenous constraint, similar to the safe-minimum standards mentioned earlier. We shall not dwell deeper in this fundamental debate about the role of economics in policy-making but it must be acknowledged that in this work we take the stance that economics provide great tools to achieve environmental regulation but that in some urgent cases, such as climate change, the social planner must step up and endorse the responsibility of the environmental target it judges a sustainable one and not let the "economic preferences" determine what should be preserved or not[1]. It must also be noted that from a strictly technical point of view, economists are somehow reluctant to work in a dynamic optimization framework with an inequality constraint on a state-variable. Considering the mathematical tools used, an inequality constraint on the state variable in this kind of setting has indeed an ambiguous bias on the optimal path, as it is not tantamount to simply "stabilizing" the path just when the constraint bites.

[1]It is somehow revealing to notice that the subfield of economics that makes the larger use of cost-efficiency analysis is the domain of health economics, where some objectives, such as curing a serious illness, are less prone to trade-offs than it is the case in other applications.

3.6.1.3 Constrained optimization and cost-efficiency analysis

We have shown in Section 3.2 that an explicit constraint on the level of environmental asset (namely assimilative capacity) seemed more straightforward than attributing an existence value to the asset when sustainability is an official objective of the social planner. Constrained optimization can thus prove to be a helpful tool to achieve a minimal target at the least cost. In doing so, it is very similar to the method of "cost-efficiency" analysis that is often proposed as an alternative to cost-benefit analysis in the literature on the appraisal and ranking of economic projets (Little and Mirrlees, 1974). The basic definition of cost efficiency analysis consists in achieving a given goal at a minimized cost whereas in constrained optimization, it is only the "floor" value of the asset that is specified. Consequently the latter can result in a level of asset actually preserved higher that the constrained lower bound if the utility trade-offs demand it. Nevertheless, considering the "unreliability", in a sustainability perspective, of the unconstrained steady-state levels of preserved assimilative capacity (A_{ss}) highlighted in Chapter 1, it is clear that constrained maximization should be resorted to when the "spontaneous" equilibrium level of asset is feared to be too low. Constrained optimization will thus merely provide a "safe minimum" (see Chapter 4) to ensure that the equilibrium level does not fall below a sustainability threshold. As such, constrained optimization, used in this "safe minimum insurance" perspective, can reasonably by identified with the cost-efficiency framework.

3.6.2 Analytical implementation of constrained optimization

3.6.2.1 The constrained maximization problem

We may now proceed to the actual resolution of our model (we use our flow-pollution model as the baseline for the sake of simplicity) in a constrained optimization framework.

Assuming that the social-planner wants to ensure a guaranteed level of assimilative capacity $\hat{A}$ to be transmitted indefinitely across time, the maximization problem solved in Chapter 1 now writes, keeping the exact same properties for all functions in the framework <u>allowing restoration</u>:

$$\max_p W = \int_0^{+\infty} \left(f\left(p(t)\right) - D(p(t), A(t))\right) e^{-\rho . t} dt \tag{3.2}$$

$$\tag{3.3}$$

$$1 > \rho > 0 \tag{3.4}$$

subject to:

$$\dot{A}(t) = -h(p(t), A(t)) \tag{3.5}$$

$$A(0) = A_0$$

$$A(t) \geq \hat{A} \;\forall\; t \tag{3.6}$$

In order for this problem to have a solution, we need to introduce another assumption on the initial level of assimilative capacity A_0:

$$A_0 \geq \hat{A} \tag{3.7}$$

If condition (3.7) is not respected then constraint (3.6) is immediately violated.

The sustainability constraint (3.6) must be integrated in the following current-value Lagrangian:

$$\mathcal{L} = f(p(t)) - D(p(t), A(t)) - \lambda(t)h(p(t), A(t)) + \omega(A(t) - \hat{A})$$

with λ again the co-state variable associated with the assimilative capacity and ω the Lagrangian multiplier associated with constraint (3.6) such that

$$\omega(t) \geq 0\,;\, \omega(t)(A(t) - \hat{A}) = 0$$

The resolution of this modified problem is not fully developed here as we shall focus solely on the interesting new optimality conditions that arise and on the difference between the final results and the conclusions from the former model.

3.6.2.2 First Order Conditions under constraint

$$f_p(p) = D_p(p, A) + \lambda h_p(p, A) \tag{3.8}$$

$$\dot{\lambda} = \lambda(\rho + h_A(p, A)) + D_A(p, A) - \omega \tag{3.9}$$

$$\omega(t)(A(t) - \hat{A}) = 0 \, , \ \ w \geq 0 \, , \ \ A - \hat{A} \geq 0 \tag{3.10}$$

In addition there is the transversality condition:

$$\lim_{t \to +\infty} e^{-\rho t} \lambda(t) A(t) = 0$$

Compared to the baseline problem we get an additional slackness condition (3.10) and the constraint on the state variable shows up in condition (3.9) in the Lagrangian multiplier ω. Consequently, the presence of this constraint will affect the shadow price of assimilative capacity $\lambda(t)$ along the optimal path that will differ, if the constraint is biting, from the shadow price in the unconstrained model.

3.6.2.3 Characterization of the optimal path

Facing a constrained problem like this, we need to distinguish two cases depending on whether or not the constraint bites along the optimal path.

Case I: Constraint never reached

If the functional forms are such that the constraint is never reached, then the problem amounts exactly to the problem studied in Chapter 1 with the additional condition (3.7) in the initial stock. However we have now the guarantee that the steady state level of assimilative capacity A'_{ss} will be higher that our lower bound $\hat{A}$. In that case the optimal path, starting at A_0, will be either a restoration path (respectively if ($\hat{A} \leq A_0 \leq A'_{ss}$) or a depletion path (if $\hat{A} \leq A'_{ss} \leq A_0$). Considering the very reasons why we believe resorting to constrained optimization, namely the fear of an insufficient level of preserved environmental asset, this case is the least relevant one.

Case II: Biting Constraint

Of higher interest is the case where the constraint starts biting at a date T_b. Two phases must be distinguished.

II. 1.: Before saturation

Before the saturation of the constraint we have $A - \hat{A} > 0$ and $\omega = 0$. It is easy to verify that during this phase the maximization problem is tantamount to the baseline case and for A_0 sufficiently high the optimal pollution path will deplete A until the minimum level $\hat{A}$ is reached. This is translated on the phase diagram with an optimal path following the same path as the unconstrained optimal path.

II. 2.: After saturation

Using a method similar to that of Cesar and de Zeeuw (1995, p.39) we can characterize the constrained optimal path: A straightforward steady state analysis, similar to the one lead in Chapter 1, shows that it exists a unique steady-state A'_{ss} that is likely to be below $\hat{A}$ if the social planner has ambitious sustainability goals. According to Feichtinger and Hartl (1985) in this configuration it is not optimal for A to bend back for $t \geq T_b$. In order for the economy to reach the steady state under the constraint $A \geq \hat{A}$, the optimal path must bend away from the unconstrained trajectory. We have thus at time T_b:

$$\dot{A}(T) = \dot{\lambda}(T) = 0$$

and the constrained steady state values are reached at time T. We have thus:

$$\omega = 0 \ \ and \ \ \dot{\lambda} = (\rho +)\lambda + D' \ \ \forall t \leq T$$
$$\omega > 0 \ \ and \ \ \dot{\lambda} = 0 \ \ \forall t > T$$

which means that $\dot{\lambda}$ will jump at time T by the amount ω where ω takes the following value:

$$\omega = \lambda(\rho + h_A(p, \hat{A})) + D_A(p, \hat{A})$$

Once the minimal level of assimilative capacity is reached we have $\omega > 0$ and $A - \hat{A} = 0$ along the optimal path. The economy must then stabilize at this level even if $\hat{A}$ is not the economic optimum. The resulting constrained optimal path appears clearly on Figure 3.7 and is quite easy to interpret intuitively. This path is tantamount to the unconstrained path until it reaches the lower bound $\hat{A}$.

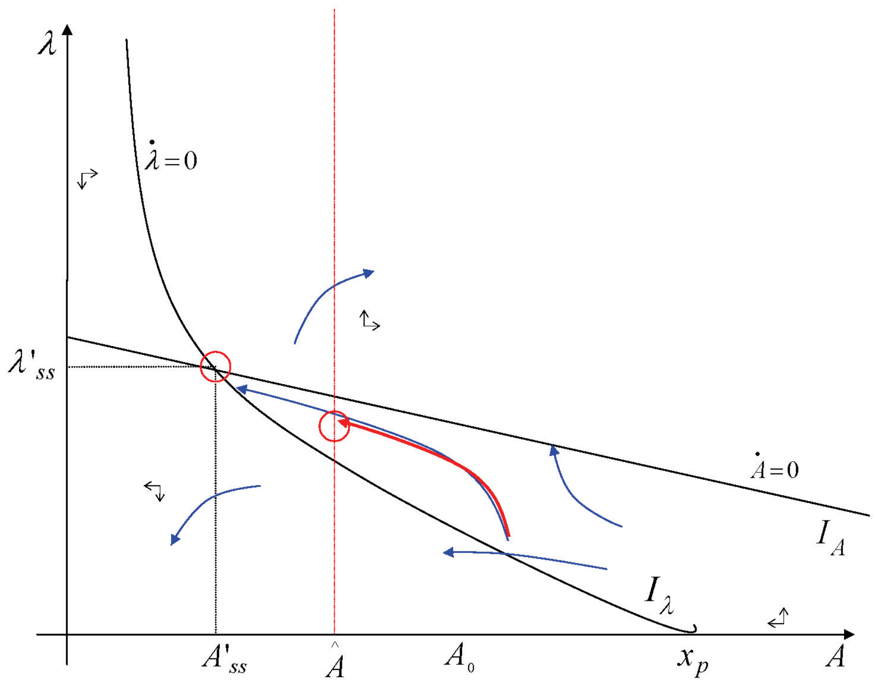

Figure 3.7: Optimal constrained pollution path with $\hat{A} > A'_{ss}$

3.6.2.4 A remark on the application of constrained optimization to stock pollution control

Ensuring that a minimum level of assimilative capacity is preserved all along the optimal path is a lot harder when it comes to stock pollution. Indeed, if the initial stock of pollution Z_0 is already above the degradation threshold and if the degradation function h is very "intensive", the assimilative capacity might be depleted too fast, in spite of reduced emissions or restoration efforts, to remain above a minimum threshold $\bar{\alpha}$ even if $\alpha_0 > \bar{\alpha}$. The constraint might thus be violated in the first periods of the model. If no restoration is available, then this violation is irreversible and it will never be possible to achieve an optimal solution respecting this constraint. If restoration is available then it is possible to restore back the assimilative capacity above the constraint threshold.

3.6.2.5 Sustainability tax and sustainable user cost

Quite intuitively, the constrained optimal path leads, through the depletion of assimilative capacity, to the lower bound $\hat{A}$. It is not so much the easily predictable final state of the economy, consisting in a sustainable level of production/pollution $p_c^* = \hat{A}$ associated with a sustained stock of assimilative capacity $\hat{A}$, that is of interest here. The added value of this constrained maximization is to characterize the shadow price of assimilative capacity $\lambda(t)$ that evolves differently from the baseline unconstrained case as condition (3.9) translates. This shadow price contributes to putting the economy on a maximized welfare path taking into account the sustainability constraint. As such, it represents a valuable theoretical landmark for the design of a sustainability tax. Such a tax would differ from the optimal tax studied in Chapter 1 as it would need to be stricter at one point to ensure that the sustainability constraint is never violated. Through this dynamic sustainability tax τ_S, the polluter is forced to internalize not only the externality dealt with in Chapter 1 but also the respect of the constraint (3.9).

Sustainable user cost

We will not develop in more details the process of internalization through the dynamic tax as it is strictly identical to the demonstration made in Chapter 1. However it is interesting to recall the results of Pearce (1988) relative to such a "sustainability" tax. Although he uses a somehow myopic optimization process as in his 1976 article, his approach regarding the implementation of a sustainable environmental regulation is in the same spirit as our sustainability tax. Pearce refers to the *sustainable user cost* as the additional cost levied on private profit that leads to a set of sustainable prices compatible with the indefinite preservation of a non negative assimilative capacity across time. According to the author, the inclusion of such a cost enables the social planner to establish a set of *optimal* prices within *sustainable* bounds. However Pearce's sustainable user cost, tantamount to a tax, is more radical than our dynamic tax τ_S as it aims at preventing any degradation of assimilative capacity from the start. Such a policy corresponds to a "strong sustainability" approach and could be implemented in our model if we set the lower bound constraint $\hat{A}$ to be equal to A_0.

Alternative economic instruments

Since Weitzman's famous article (1974) there is an on-going debate in the environmental economics literature on the respective advantages of taxes and tradable permits. A vast body of contributions compares both the economic and the environmental efficiency of these two sets of instruments under various conditions: different market structures (David and Sinclair-Desgagné, 2005), asymmetrical information (McKitrick, 1999), etc. The impact of these tools on innovation (Milliman and Prince, 1989) or on market distortions (Goulder, 1995) has also been widely discussed. In this section we have focused on the dynamic tax as the sole instrument to implement a sustainable pollution path, mainly because such a tax stems directly from the shadow price of assimilative capacity that our model is able to provide. However it would be only fair to consider an alternative implementation process of the desirable levels of pollution. The adequacy of tradable permits with the efficient implementation of sustainability goals ranging from biodiversity conservation to climate change mitigation has been underlined by Godard (2005) and prompts us to explore further this lead. We must obviously let aside the model of a sole representative farmer/polluter as tradable permits make no sense if there is only a single polluter[1] and we must consider a pollution problem with multiple agents using or overusing a common assimilative capacity. Since such a configuration is very similar to the framework of private exploitation of a common resource such as fisheries, we shall analyze it fully in the next chapter where we apply viability control to our pollution problem.

3.7 Sequential decision strategy

In terms of policy-oriented conclusions, we have seen that a cost-efficiency analysis based on a theoretical constrained optimization framework seems to be the most sensible approach of economic control of pollution. It is nevertheless worth mentioning a last kind of approach, from an applied point of view, that fits quite well the requirements of sustainability as we envision it, e.g., as the preservation of capacities, especially once uncertainty is taken into consideration. Sequential approaches are indeed the most intuitive way to address a problem with heavy irreducibilities and significant uncertainty, like climate change for instance.

[1]In this case it would be tantamount to a command and control norm.

3.7.1 Sequential approach of climate change

Given the major uncertainties surround the timing and impact of climate change as well as our potential to mitigate or to cope with it, some authors (Hammitt et al., 1992; Hua-Duong et al., 1997; Godard, 2007) advocate a sequential decision strategy. As explained by Godard (2007), such a strategy consists in *"identifying a kernel of short and medium term decisions"* that must be *"completed and revised at different time horizons"*. Abandoning the idea of a definitive infinite horizon optimization carried on now once and for all, this approach aims at reaching a goal (e.g., a ceiling of CO_2 concentration for example) at a minimized cost while integrating new information along the way[1]. In a way, sequential decision analysis can be defined as a cost-effectiveness analysis that is carried on in a non-deterministic framework and that encompasses future improvements in the available information. One of the salient features of the sequential approaches of climate change is the precautious acknowledgment of two kinds of irreversibility. On the one side, environmental irreversibility is extremely threatening, although the thresholds are not well identified yet we know that exceeding a certain level of concentration on the atmosphere might trigger a climate upheaval that will not be stopped however drastic emission reductions might be after that point. This environmental irreversibility might also characterize the impairing of some ecosystem services and the ocean and biosphere's assimilative capacity as we have seen in Chapter 2. On the other hand, there is a danger of irreversible investment in capital and technologies that could result in "lock-ins" (Hourcade et al, 2003) preventing us from adopting the most efficient technology when it becomes available.

3.7.2 Application to local flow pollution problems

This sequential approach seems particularly in accordance with the principles of sustainability as it seeks to preserve the capacity (seen here as kernel of viable decisions) of future generations to deal with a major issue, instead of transmitting these generations irreversible economic and ecological burdens. As such it seems to us as a

[1]This approach aiming at the preservation of a maximum of options (both in terms of environmental processes at work and in terms of investment in physical or human capital) can be somehow related to the viability approach (Martinet et al., 2007) we shall discuss in the next chapter as both strategies seek to keep a "viable" kernel of decisions.

very valuable framework to discuss the most suitable policies against climate change. Nevertheless it can also prove useful in a more local, less crucial problem such as the contamination of water courses by agricultural nitrates. In the empirical application of our flow pollution model, we have underlined the uncertainty surrounding some of the ecological processes at work, especially regarding the lixiviation of nutrients (widely discussed in agricultural economics literature) and the degradation function h. A "once and for all" optimization such as the one we conducted in Chapter 1 could therefore prove to be very damaging if the degradation process is not known with sufficient accuracy. As a result, the pollution level believed to be optimal and to lead to a sustainable equilibrium could in fact be much more damaging than expected and trigger rapidly the entire depletion of the assimilative capacity. We have already stressed that empirically it is difficult to assess the actual level of assimilative capacity, let alone the amount of degradation caused by a pollution excess. Consequently, a sequential decision approach would make a lot of sense in this context: a regular monitoring of water contamination would provide useful information on the actual lixiviated quantities and on the actual denitrification potential of the riparian buffer strip. An analytical development of this sequential approach would be beyond the scope of this work but it seems important to mention this approach as a valid and robust method to ensure sustainable development, if sustainable development is feasible.

3.8 Conclusion: optimality, sustainability and economic trade-offs

3.8.1 Optimality vs. Sustainability: a dead-end?

How can economics contribute to sustainable policies? This should be, *in fine*, the most important question environmental economists try to answer when they address the issue of sustainability. So far, the spontaneous reflex of conventional economists[1] has been to explore the compatibility between their standard optimal solutions (optimal pollution, optimal investment) and their own definition of sustainability.

[1] They have also engaged in highly complex and challenging axiomatical and ethical controversies on the mere content of the concept of sustainability (Asheim, 1994; Chichilnisky, 1996, among many others).

This approach, epitomized by Heal's discussion on "Optimality and Sustainability" (2001), and its limits have been thoroughly analyzed in the first sections of this chapter (Sections 2 to 5) on cost-benefit analysis and sustainability. After having stressed the intrinsical obstacles to sustainability contained in the discounted utilitarian framework, we have presented and applied to our basic model alternative optimality criteria that yield "optimal" strategies that seem more in line with an intuitive definition of sustainability and, most importantly, that are not subject to the "dictatorship of the present" that is conveyed by positive discount rates and that is hardly compatible with sustainable policies. We have shown that the Green Golden Rule[1] and the Overtaking criterion determine optimal solutions that demand the preservation, or even the restoration, of assimilative capacity while the Maximin criterion implied in some cases a status quo that prevented an improvement of environmental conditions. The first two criteria can thus fit quite well the perspective of sustainability as the general "preservation of capacities" and in particular, they allow for maintenance or investment in natural capital, reinstating a symmetry between the way man-made and natural capital are dealt with in economic models.

However, it is legitimate to argue that this "instinctive" economic approach to sustainability, namely scrutinizing optimal solutions for their sustainable features, might not be the most appropriate one policywise[2]. These efforts have given birth to very elegant papers and to some reassuring, although often quite trivial conclusions[3]. That is why we have explored alternative approaches such as cost-efficiency analysis or sequential strategy approach. Applying the former to our initial model, we have found intuitive results that could be integrated more easily in a sustainable environmental policy and that consider sustainability as an explicit goal, or at least as a binding constraint, instead of looking for the *coincidental sustainability* of "optimal" policies under very narrow and often meaningless technical conditions.

[1] As we have shown, the Green Golden Rule might imply a bias in favor of future generations when it comes to the transition path associated with the optimal final state. This bias could be attenuated through a careful consideration of the trade-offs at stake between the costs of restoration (or the costs of foregone benefits) and the ecological benefit for the assimilative capacity.

[2] All the more than actual economic or environmental policies rarely implement exactly the theoretically optimal strategies.

[3] A caricatural example would be a proposition claiming that '*'optimal growth is also sustainable if the discount rate is low enough and the marginal productivity of abatement capital is higher than the polluting emission factor of a unit of production"*.

3.8.2 A shift of focus to make sustainability an explicit goal?

In the light of this review of the standard arguments of system analysis applied to the economy-environment interactions, it appears that the conceptual framework of external effects, as efficient as it may be in many cases, might not reflect clearly enough the challenge of preserving (environmental) capacities. One of the main concern animating this work so far has been to draw attention to the finite biophysical dimension of assimilative capacity. In order to do so, we have represented it as a state variable in its own right in the standard models of flow and stock pollution. Studying the dynamics of this environmental stock we have noted various times that it was of a very similar nature to a natural resource, that can either be renewable (naturally or artificially) of non renewable. And although the analogy between assimilative capacity and a renewable resource is mentioned briefly in some contributions (Rotillon, 2005; Schubert and Zagamé, 1998), to our knowledge it had never been modelized as an explicit stock of natural resource before. That is why in the next chapter we will resort to the conceptual tools of natural resource management (maximum sustainable yield, viability theory, etc.) to complete our economic analysis of pollution problems involving limited assimilative capacity. In particular, our discussion of the viability approach will provide us with a good opportunity to develop the stances adopted in Section 6 on the relevancy of "exogenous constraints" introduced into economic optimization.

Appendix Chapter 3

Appendix A: Overtaking criterion proofs

From now on we will refer to F as the set of all feasible pollution paths. Those paths must respect the following constraints:

$$0 \leq p(t) \leq x_p$$
$$0 \leq A(t) \leq A_0$$
$$\dot{A}(t) = -h[p(t) - A(t)]$$

Let us call SP the strong sustainability path, SP is defined by the following properties and its feasibility is straightforward:

$$\text{SP: } \{p_s(t), A_s(t)\} \text{ s.t.} p_s(t) = A_0 \text{ and } A_s(t) = A_0 \forall \ t \geq 0$$

Since the assimilative capacity is respected all along the path, its level $A(t)$ remains constant and equal to A_0 while the private benefit is also constant and equal to $f(A_0)$ and no environmental damage ever occurs, ie $D(p(t) - A(t)) = 0 \ \forall \ t$.

Comparison between the utilitarian path *UP* and the sustainable path *SP*

We shall start by comparing, in the light of the Overtaking criterion, the optimal utilitarian path UP described in the previous section with the strong sustainable path. We will then show that the path SP is superior to any other feasible path and is thus the optimal path according to our new criterion.

According to the qualitative characterization of the optimal utilitarian path conducted before, we know that p^* is decreasing along this path (see Chapter 1). We can write:

$$\exists \ \bar{T} \ , \bar{T} \text{ finite and positive, such that:}$$
$$\forall \ t \geq \bar{T}, \ p^*(t) < A_0$$

To compare UP and SP we study the asymptotic behaviour, when T tends towards infinity, of the quantity $z(T)$, defined as follow:

$$\begin{aligned} z(T) &= \int_0^T \Phi(p_s(t), A_s(t))dt - \int_0^T \Phi(p^*(t), A^*(t))dt \\ &= \int_0^T [f(p_s(t)) - D(p_s(t) - A_s(t))]dt - \int_0^T [f(p^*(t)) - D(p^*(t) - A^*(t))]dt \end{aligned}$$

hence

$$\begin{aligned} z(T) &= \int_0^T [f(A_0) - D(0)]dt - \int_0^T f(p^*(t)) - D(p^*(t) - A^*(t))dt \\ &= \int_0^T [f(A_0) - f(p^*(t)) + D(p^*(t) - A^*(t))]dt \\ &= \int_0^{\bar{T}} [f(A_0) - f(p^*(t)) + D(p^*(t) - A^*(t))]dt + \int_{\bar{T}}^T [f(A_0) - f(p^*(t)) + D(p^*(t) - A^*(t))]dt \end{aligned} \tag{3.11}$$

On the set $[0,\ \bar{T}]$ we define $K(\bar{T})$ such that:

$$K(\bar{T}) = \int_0^{\bar{T}} [f(A_0) - f(p^*(t)) + D(p^*(t) - A^*(t))]dt$$

given the definition sets of p^* and A^*, and since $\bar{T}$ is finite, we have $K(\bar{T})$ bounded:

$$\exists\ K\ \in\ \Re, \text{such that}\ \ K(\bar{T}) < K \tag{3.12}$$

and on the set $[\bar{T},\ T]$ we have:

$$p^* < A_0$$

we can thus write

$$\begin{aligned} &\forall\ t \geq \bar{T} \\ &f(A_0) - f(p^*(t)) + D(p^*(t) - A^*(t)) > f(A_0) - f(A_0) + D(0) = 0 \end{aligned}$$

which yields

$$\lim_{T \to +\infty} \int_{\bar{T}}^T [f(A_0) - f(p^*(t)) + D(p^*(t) - A^*(t))]dt = +\infty \tag{3.13}$$

From equations (3.11), (3.12) and (3.13), we can conclude:

$$\lim_{T\to+\infty} z(T) > 0$$

According to the Overtaking criterion the strong sustainable path respecting systematically the assimilative capacity SP is superior to the utilitarian optimal path UP.

General optimality of the strong sustainable path *SP*

We will compare the SP path to two complementary subsets of the feasible paths. The first subset S_1 includes the paths that preserve indefinitely a positive level of assimilative capacity while the second subset S_2 consists of the paths that lead to the total depletion of the assimilative capacity.

$$S_1 \bigcap S_2 = \emptyset$$
$$S_1 \bigcup S_2 = F$$

The path SP obviously belongs to subset S_1 while the path UP belongs to subset S_2.

Comparison with the subset S_1

First we show that the path SP is superior to all other paths belonging to S_1. On these paths, there is necessarily a time when the pollution level $p_1(t)$ remains below or equal to the assimilative capacity $A_1(t)$. From that time on, theses paths remain on a steady state, with a level of assimilative capacity $A_1(t)$ striclty below A_0 or else the path would have to be identical to SP.

$$\forall \text{ paths } P_1, \{p_1(t), A_1(t)\} \in \{S_1 SP\}\ ,\ \exists\ T_1 \text{ positive such that :}$$
$$\forall\ t \geq T_1, p_1(t) \leq A_1(t) < A_0$$

Using the same method as in the previous subsection, we define:

$$\begin{aligned} z(T) &= \int_0^T \Phi(p_s(t), A_s(t))dt - \int_0^T \Phi(p_1(t), A_1(t))dt \\ &= \int_0^T [f(p_s(t)) - D(p_s(t) - A_s(t))]dt - \int_0^T [f(p_1(t)) - D(p_1(t) - A_1(t))]dt \end{aligned}$$

It is then straightforward that according to the constraints on the feasible paths, $K(T_1)$ is bounded with:

$$K(T_1) = \int_0^{T_1} [f(A_0) - f(p_1(t)) + D(p_1(t) - A_1(t))]dt$$

and similarly, given that $\forall\ t \geq T_1, p_1(t) < A_0$:

$$\lim_{T \to +\infty} \int_{T_1}^{T} [f(A_0) - f(p_1(t)) + D(p_1(t) - A_1(t))]dt = +\infty$$

Hence

$$\lim_{T \to +\infty} z(T) > 0$$

The path SP is superior to all other paths included in subset S_1 according to the Overtaking criterion.

Comparison with the subset S_2

Along the paths belonging to S_2, there is a time T_2 when the assimilative capacity is reduced to zero.

$$\forall \text{ paths } P_2, \{p_2(t), A_2(t)\} \in \{S_2 UP\}, \exists\ T_2 \text{ positive such that :}$$
$$\forall\ t \geq T_2,\ A_2(t) = 0$$

In a line of reasoning similar to the previous demonstrations, we break up $z(T)$ in two parts.

It is straightforward that $K(T_2)$ is bounded:

$$K(T_2) = \int_0^{T_2} [f(A_0) - f(p_2(t)) + D(p_2(t) - A_2(t))]dt$$

$K(T_2)$ could be negative, but this does not change the results.

And in addition:

$$\begin{aligned}\forall\ t \geq T_2\ \ f(A_0) - f(p_2(t)) + D(p_2(t) - A_2(t)) &= f(A_0) - f(p_2(t)) + D(p_2(t)) \\ &\geq f(A_0) - f(p_2(t)) + D(0) = f(A_0) - f(p_2(t))\end{aligned}$$

For all paths $P_2 1$ belonging to S_2 such that $p_2(t) < A_0 \;\forall\; t \geq T_2$ we have:

$$f(A_0) - f(p_2(t)) + D(p_2(t) - A_2(t)) \geq f(A_0) - f(p_2(t)) > f(A_0) - f(A_0)$$
$$f(A_0) - f(p_2(t)) + D(p_2(t) - A_2(t)) > 0$$

so that, for all paths $P_2 1$:

$$\lim_{T \to +\infty} \int_{T_2}^{T} [f(A_0) - f(p_2(t)) + D(p_2(t))]dt = +\infty \qquad (3.14)$$

We consider now the case of the rest of the paths belonging to P_2. At some time along those paths, from time T_2, we may have $p_2(t) \geq A_0$.

Let us call $\zeta(p)$ the following function defined on the set $[A_0, x_p]$:

$$\zeta(p) = f(A_0) - f(p) + D(p)$$

Since we know that:

$$\zeta(A_0) = f(A_0) - f(A_0) + D(A_0) > 0$$

In order to prove that the result (3.14) also holds on $[A_0, x_p]$ we just need to show that ζ is non decreasing on this set.

Given the properties of f and D, $\zeta(p)$ is C^2 and we have:

$$\zeta'(p) = D'(p) - f'(p) \qquad (3.15)$$

We have defined earlier $\bar{p}(A)$ such that:

$$D'(\bar{p}(A) - A) - f'(\bar{p}(A)) = 0$$

We can write (3.15) with $\bar{p}(0)$:

$$\zeta'(\bar{p}(0)) = D'(\bar{p}(0) - 0) - f'(\bar{p}(0)) = 0$$

If we can show that $\bar{p}(A) \leq A_0$ then we will have proven that ζ is non decreasing and positive on $[A_0, x_p]$.

Let us call $\hat{A}$ the value of A such that $\bar{p}(\hat{A}) = A_0$. This means that:

$$D'(A_0 - \hat{A}) - f'(A_0) = 0$$

By definition we have:

$$A_0 \geq \hat{A} \geq 0$$

Since we have demonstrated that $\bar{p}(A)$ is increasing in A we have:

$$\bar{p}(A_0) \geq \bar{p}(\hat{A}) \geq \bar{p}(0)$$

hence:

$$A_0 \geq \bar{p}(0)$$

From all this we conclude that equation (3.14) holds for all paths belonging to S_2.

In combination with the fact that $K(T_2)$ is bounded, we can reach a conclusion on the subset S_2:

$$\lim_{T \to +\infty} z(T) > 0$$

We can thus extend this result to the entire subset F and reach the following final conclusion: the strong sustainable path dominates SP all other feasible paths according to the Overtaking criterion.

Appendix B: *Ad hoc* amendments of cost-benefit analysis

In response to the critics attacking the standard framework of dynamic cost-benefit analysis, and in particular the essential incapacity of such a framework to guarantee the conservation of a minimum stock of environmental asset given that extinction can be optimal (Clark, 1973) with a given set of economic parameters, some amendments have been applied to cost-benefit analysis. In pollution problems, two significant amendments can be identified: the introduction of a potential catastrophe and the explicit recognition of the contribution of environmental assets to the economic well-being via their sole existence.

Uncertain exogenous catastrophical thresholds

The cost-benefit analysis approach of pollution problems is often reproached with its linear representation of environmental damage (Hediger, 2009). The non-convexity of ecosystem dynamics (Dasgupta and Mäler, 2003) is rarely accounted for in mainstream optimal pollution control and the solutions thus obtained are not as relevant as they ought to be. In addition to the sophisticated specification of assimilative capacity functions reviewed earlier, the concept of catastrophical thresholds is necessary in some cases (climate change or nuclear waste accumulation for instance) to give the right weight[1] to the potential environmental and/or human catastrophe at stake. We have noted earlier that our first model applies to local problems and that the extinction of the assimilative capacity does not imply a major catastrophe for society, only a definitive local loss of a natural capital delivering a valuable service. Including such a risk in our model is not as relevant as it might be for global environmental threats such as climate change. The Stern Review (Stern, 2006), discussed thoroughly in the next section, has thus included a chance of catastrophical outcome in the climate change scenarios its economic analysis is built upon.

Among the models of pollution control or growth with pollution reviewed earlier, some pay special attention to this risk (Cropper, 1976; Chevé and Congar, 2000). In their model once the pollution stock $Z(t)$ exceeds an unknown threshold level Z^*, determined by soft or hard uncertainty, a catastrophical event occurs and this

[1] The use of a positive discount rate, discussed in the next section, contributes to minimizing the economic importance of a catastrophical event in the long term.

catastrophical event means in economic terms zero consumption level indefinitely after the date of the catastrophe.

For instance in Cropper (1976), Z^* is a random variable with a probability density function $f(Z^*)$ distributed over $[0, \infty[$. If $Z \geq Z^*$ then the catastrophe happens and the level of utility is zero forever. If $Z < Z^*$ then the level of utility is described by a standard utility[1] function $U(c(t))$. Consequently the expected utility at time t writes:

$$\int_{Z(t)}^{+\infty} U(c(t)) f(Z^*) dZ^* = \Gamma(Z(t)) U(c)$$
$$s.t.\ \Gamma(Z(t)) = \int_{Z(t)}^{+\infty} f(Z^*) dZ^*$$

$\Gamma(Z(t))$ is thus the probability that the catastrophe has not happened given that the current pollution stock is $Z(t)$.

And the maximization problem for the social planner is:

$$\max J[C] = \int_0^{+\infty} U(c(t)) \Gamma(Z) e^{-\rho.t} dt$$
$$\text{s.t. } \dot{Z} = \Phi(C) - \alpha Z$$
$$\text{and } \Gamma(Z) = \int_Z^{+\infty} f(Z^*) dZ^*$$

The results of this "catastrophical" model are not unambiguous because it is subject to multiple equilibrium solutions. However the qualitative conclusions indicate that under such uncertainty (*"a small probability of large loss"* as Cropper puts it) society tends to accumulate pollution faster or use up the resource stock faster[2].

The notion of uncertainty dealt with here is clearly crucial when it comes to sensitive biological dynamics. As such, it will be addressed in the next chapter in the resource depletion framework but the uncertainty will shift from the pollution threshold to the initial assimilative capacity available, since the monitoring of this assimilative capacity is not always easy, or even possible[3]. The link between pollution

[1]The pollution stock does not enter the utility function as an argument, it exclusively determines the occurrence of the catastrophe.

[2]A simplified interpretation is that uncertainty "adds" a discount factor ($\Gamma(Z)$) to the standard maximization problem that is already discounted.

[3]As noted by Woodwell (1970), the exact level of assimilative capacity is generally discovered once it has been exceeded.

thresholds and resource depletion will be all the more relevant as it has been shown (Cropper, 1976) that *"if the size of the resource stock is uncertain, then society's optimal depletion problem is analogous to the catastrophe problem"*.

Explicit valuation of environmental services

Besides the sink function it provides and that can be valued in comparison to artificial substitutes, there is objectively no reason to value the mere existence of the assimilative capacity offered by microbial organisms in water-flows. However, if the assimilative capacity "stock" corresponds to the area of riparian ecosystems (Chapter 1) or carbon sequestrating forests (Chapter 2), it extinction is naturally tantamount to the loss of biodiversity as well as valuable environmental amenities. We mentioned this point while elaborating our *a minima* sustainability criterion and it offers a "leverage" to increase the weight of environmental assets in cost-benefit analysis. Analogous cases can be found in renewable resource management. A forest that provides a flow of commercialized timber sold on the market can also be valued *per se* because it offers significant amenities for tourists or it shelters endangered animal species. In this perspective, Heal (2000, p.36) suggests to add an argument to the social utility function so as to make this utility dependant also on the renewable resource stock level. The resource management program in the standard discounted utilitarian framework writes now

$$\max_{y} W(t) = \int_{0}^{+\infty} u(c(t), S(t))e^{-\rho.t}dt$$

where $c(t)$ is the current consumption of a part of the resource stock S(t) that follows its own motion equation. Following a similar approach, some fishery models add an extra valuation of the fish stock in its own right, independently of its commercial use, when the fish or shellfish involved have a recognized existence value as it is the case with blue whales for instance (Krutilla, 1967). The fishery optimization program thus becomes

$$\max_{y} W(t) = \int_{0}^{+\infty} p(t)y(t) - cy(t) + V(S(t))e^{-\rho.t}dt$$

where a harvest $y(t)$, carried on at a unit cost s, is sold at a market unit price p and $V(S(t))$ denotes the existence value of a stock of fish $S(t)$ that follows its own motion equation. Intuitively enough, this explicit valuation of the existence of the stock leads to a higher optimal stock preserved. Some authors such as Heal (2000) claim that this valuation improves the chances of the resulting optimal path to be sustainable. However it is straightforward in the simplified analytical models that in the end, the amount of environmental stock preserved depends essentially on the elasticity of substitution between the utility of an additional unit consumed and of an additional unit left *in situ*. For instance, in the basic model developed by Heal (2000) (including a separable utility function and rejecting the Inada conditions) a positive stock of the resource is preserved for ever if there exists S^* such that

$$u_c(0)\rho = u_S(S^*)$$

From a broader perspective, it appears clearly that the degree of "sustainability" of the consumption path will depend *a priori* on the economic parameters (the discount rate ρ and the *ex ante* properties of the utility function). As such, it seems to us very problematic to rely only on this explicit recognition to ensure a sustainable economic trajectory. The bottom line of such an approach is that one can decide beforehand, through the specification of the existence value function or the utility function, how much of the stock of natural asset will be ultimately preserved. Specifying the function is thus tantamount to choosing a steady state level, and since existence value functions and environmental arguments of utility functions are not very robust and are quite complex to calibrate, a large part of the final "results" depends on the somehow arbitrary construction of these functions. The compatibility or overlapping between optimality and sustainability that may arise is then not so much a coincidence or a "free solution" as an *ex ante* decision. It seems to us that it would be more straight forward to admit, or at least to suggest, that we as a society want to keep a minimum level of a given asset, and to conduct the corresponding constrained optimization. This would translate in our specific case with an explicit constraint on the state variable $A(t)$, such that for all t $A(t) \geq \hat{A}$. We shall explore this solution in Section 8. Consequently, we shall deny to these amendments the capacity to integrate fully the requirements of sustainable development and that is why we explore alternative ways to assess the sustainability of cost-benefit analysis-driven paths.

Chapter 4

Pollution as a resource depletion problem: preserving environmental capacities through viable control

4.1 Introduction

As a result of the dramatic development and specialization it has experienced in the last decade, the field of theoretical environmental economics has grown more and more compartmentalized. If this evolution has allowed for a more accurate modelling and a better understanding of complex issues, it has diminished the incentives to encompass the relationships between the economic and the ecological sphere in a broad perspective. At a time when policy-making seems to favor the paradigm of sustainable development as a founding platform for action, theoretical environmental economics should try to keep in sight the salient features that a wide range of environmental problems have in common. Indeed the body of theoretical literature developed in one area may prove useful to address other problems that share some of these common features but that have been dealt with in a very different setting so far. In this paper we argue that such a shift of perspective could be particularly beneficial to the field of pollution economics where the framework of analysis of renewable resource[1] literature applies relevantly once we start to acknowledge the natural assimilative capacity at

[1] It must be reminded that a so-called "renewable resource" can nonetheless be completely depleted under some management regimes.

stake. Indeed, apart from a few exceptions that we shall discuss later, the finite nature of the environment's assimilative capacity is often ignored in pollution control problems whereas the depletion of this asset should be a serious concern. We suggest in this work to adapt the framework of exhaustible resource depletion to pollution problems via focusing on the assimilative capacity of ecosystems as a autonomous state variable. In particular we propose an original application of the viability framework to our simple pollution problem. Using a phenomenological model in discrete time, we explore the set of viable decisions that can be computed for a particular social, economic and ecological configuration.

This chapter suggests that in order to integrate more explicitly the requirements of sustainable development, a shift from the standard framework of pollution economics to the optimal resource management paradigm can be very helpful. This transposition is justified by our previous conclusions on the potential unsustainability of standard optimal pollution paths. We have shown indeed that if the initial environmental conditions are too low and for high discount rates, these paths lead to the optimal extinction of the resource. The criteria explored in the previous chapter (Green Golden Rule, Maximin, Overtaking Criterion) have failed to provide a robust alternative to the discounted framework but they have contributed to confirming the "intuitive" solution to a sustainable pollution path. Following this intuition and the various arguments that favor an association of assimilative capacity with a renewable resource threatened by mismanagement, we take this analogy one step further in order to point out the salient ecological features of pollution problems that are often overlooked.

In the light of our analysis, the framework of economic management of natural resources seems more suited than a basic external effects framework focusing on environmental damage to integrate the constraints that matter most to the operationalization of the main requirements of sustainability and in particular the preservation of capacities. Hence our proposal to bridge the gap between economics of pollution and renewable resource economics. The considerable developments of the latter can be of great interest in application to the management of assimilative capacity as a stock of renewable resource.

We shall start by recalling the features of assimilative capacity that have been highlighted in the previous chapters as typical of a renewable resource and we will discuss the minor adjustments necessary to transpose the pollution control problem from its initial framework to the management of a renewable resource framework. In Section 3 we build a very basic model to make apparent the new resource management problem formulated and we briefly review the immediate results that follow. Thanks to this new framework, we can carry out an original viability approach in Section 4 that addresses in part the limits of the previous approaches in terms of sustainability. Section 5 draws policy oriented conclusion from our formal analysis and concludes.

4.2 Definition of the assimilative capacity as a renewable resource

As noted by Brewer (1968), *"there are other natural resource problems that can usefully be explored within the context of stock resource. In particular, certain problems of pollution and environmental quality appear analogous to classical stock resource problems [...]"*. In this section we shall check that our definition of assimilative capacity as a specific type of renewable resource holds under rigorous scrutiny. This analogy has already been stressed by Rotillon (2005) or Kany and Ragot (1998, p.154) but to our knowledge it has never been formally expressed in a stylized model.

4.2.1 A renewable resource yielding a flow of environmental service and regenerating at a peculiar rate

4.2.1.1 Defining assimilative capacity as a natural resource

It has been demonstrated in details previously, for different concrete pollution problems, that the assimilative capacity yields an ecosystem service highly valuable for human society. Consequently, it can undoubtedly be classified as an environmental asset forming part of natural capital. The category of renewable resources is used in the environmental economics literature mainly in reference to animal or vegetal populations (fisheries and forestry are the standard fields of application). It is rather unusual to analyze in this framework an inanimate resource yielding an environmental function such as the assimilative capacity but it has been done. For instance, soil fertility (Mc Donnell, 1983; Hediger, 1999) or ecosystem resilience (Mäler et al., 2003) have been legitimately stylized as stocks of renewable resource on their own right (see General Introduction).

Regarding assimilative capacity, a few contributions on natural resource management note that in pollution problems the ecosystem's assimilative capacity fulfills the features of a renewable resource[1] (Rotillon, 2005; Schubert and Zagamé, 1998). In a recent discussion on sustainable resource management, Heal (2001) considers that some inanimate resources are actual renewable resources since "*soil fertility is renewed by microbial action if the soil is not used, and the air and bodies of water have the capacity to cleanse themselves as long as pollution is below a threshold level*". Since the assimilative capacity of ecosystems yields a service to society and is renewable under certain conditions described in the previous chapters, it seems only natural to characterize it as a renewable resource.

Finally, in some settings, the assimilative capacity of a given ecosystem is closely linked to a population of living organisms. The definition of assimilative capacity suggested by Pearce (1976) refers to the action of degrading agents, belonging to the bacterial or microbial sphere, especially in the pollution assimilation in lakes and watercourses. Moreover, the global assimilative capacity of the biosphere vis à vis CO_2 accumulation involves for a great part animal (plankton) or vegetal (forests) agents.

4.2.1.2 ***Harvesting*** **assimilative capacity**

If we accept the definition of assimilative capacity as a renewable resource, we must now introduce it into a general framework of human exploitation. Considered as an economic resource, it can thus be "harvested" by human activity. However the meaning of a "harvest" of assimilative capacity is not as intuitive as it can be for forests or fisheries. Given the dynamics of assimilative capacity described several times in this work, *harvesting assimilative capacity* consists in using the sink function by emitting polluting discharges into the ecosystems. The amount of assimilative capacity harvested is thus tantamount to the amount of polluting emissions, assuming the latter

[1]Rotillon (2005) thus states that *"pollution problems and natural systems regeneration can be assimilated to renewable resource exploitation problems. Rejecting gas in the atmosphere, acid rain on forests or nitrates in underground water is a form of exploitation of a natural asset using its assimilative capacity."*

are not in excess of the current assimilative capacity. In this modified framework, the actual "depletion" of the resource and its finite nature are made more explicit than in the strictly environmental externality framework. Consequently, the threat of overexploitation might be better coped with through improved monitoring and conservation policies. As Daly and Cobb (1989) warned, the "stock" of assimilative capacity, as a natural resource, has been even more overestimated than the supposed "inextinguishable" fish stocks.

If we compare it to the harvest of fish, the assimilative capacity harvested is directly valued on the market by a constant price[1]. However it can also be a productive factor among other factors (L(abor), R(esource), K(capital)) contributing to an industrial output Y, such that

$$Y = F(K, L, A, R) \tag{4.1}$$

In relation (4.1) A represents the *"industry's demand for a productive factor, namely for the waste assimilation services of the environment"* (Pethig, 1994).

4.2.2 The regeneration rate of assimilative capacity

One of the defining features of a renewable resource is its natural regeneration rate. The abundant fishery literature relies heavily on a logistic reproduction function such that

$$f(S(t)) = r(1 - \frac{S(t)}{K})$$

where $S(t)$ is the stock of fish at time t, r the constant rate of reproduction and K the constant carrying capacity[2] reflecting the fish population's habitat constraint.

It is clear that this natural regeneration rate is independent on the level of human harvest sustained by the resource. This harvest $h(t)$[3] is introduced afterwards to

[1]The equivalent of the market price of fish could be the market price of the agricultural output produced with fertilizers using up the assimilative capacity of the riparian buffer zone.

[2]As we noted in the General Introduction, this constant carrying capacity hypothesis can be legitimately questioned and a formal development of a dynamic carrying capacity model could offer a promising framework in which to address environmental degradation cycles highlighted by Hardin (1977), for instance in the case of the Sahel region's desertification process.

[3]The control variable denoting the harvest $h(t)$ must not be confused with the degradation function $h(p(t), A(t))$ used before in this work.

determine the net evolution of the stock.

$$\dot{S}(t) = r(1 - \frac{S(t)}{K}) - h(t)$$

The case of assimilative capacity is slightly more complex and displays analogous features with two types of standard problems: water resource replenishment and soil erosion. Both these models use a "recharge rate" such that the net dynamics of the "water stock" or the "soil stock" $S(t)$ under harvest $h(t)$[1] writes (McConnell, 1983)

$$\dot{S}(t) = R(t) - h(t) \tag{4.2}$$

$R(t)$ is an exogenous "recharge rate" that refills the stock periodically. For instance in soil conservation models (McConnell, 1983), it is assumed that the soil replenishes itself at a constant given annual rate[2]. This recharge can be either constant or can be a random variable. Assimilative capacity exploitation fits very well the definition of soil as a renewable resource established by Ciriacy-Wantrup (1968). According to this author, soil is a *"renewable resource with a threshold level below which resource depletion becomes irreversible"*[3]. As such, soil, and similarly assimilative capacity, can be exploited in a renewable sustainable way or can be irreversibly depleted. However the difference between these two resources is that we usually assume, according to ecological observations, that the assimilative capacity does not replenish itself by an exogenous amount $R(t)$, but that it "refills" whatever amount $h(t)$ has been harvested as long as $h(t) \leq A(t)$ but this replenishment is bounded above by $A(t)$ if $h(t) > A(t)$. This particular feature is analogous to one specific kind of water resource models (Fonseca and Flichman, 2002) where the future availability of the resource decreases if the extraction rate from an aquifer exceeds its rate if replenishment[4].

[1] In that case, $h(t)$ corresponds to the soil loss induced by agricultural practices or by the amount of water extracted from the aquifer.

[2] 2-5 tons/acre/year depending on soil type, weather, etc.

[3] In order to reflect this definition in equation (4.2), we would need to make the replenishment $R(t)$ decrease if many excessive harvests are conducted instead of keeping it constant as most models do.

[4] We have not identified in the soil or water management literature a modeling attempt of a replenishment rate that decreases if the previous harvest was in excess of this rate.

4.2.2.1 Case without restoration

Let us first consider the case with no restoration of assimilative capacity. When it is not needed at all, that is to say in a pollution-free world, the assimilative capacity remains constant. When the emissions of pollutants released in the ecosystem are below the assimilative capacity level, we have explained before that the assimilative capacity remains constant also. However when these emissions are in excess of the assimilative capacity then the latter decreases. We have characterized this reduction to be equivalent to the excess of p with respects to A. In resource management terms, this means that the regeneration of assimilative capacity is exactly equal to the amount of assimilative capacity *harvested* in the first case and is equal to its initial amount in the second case. The assimilative capacity always replenishes itself completely unless it is reduced by an excess of pollution.

Using the degradation function h introduced in Chapter 1 we know that

$$\dot{A}(t) = -h(p(t), A(t)) \tag{4.3}$$

Adopting a simplified functional form such as the one mentioned in the footnote of Section (1.2.4) and borrowed from Pearce (1976), e.g., $h(p, A) = h(p - A)$, we can write

$$\begin{aligned} p(t) > A(t) &\Rightarrow \dot{A}(t) = -h(p(t) - A(t)) < 0 \\ p(t) \leq A(t) &\Rightarrow \dot{A}(t) = -h(0) = 0 \end{aligned}$$

This can be rewritten in a single equation

$$\dot{A}(t) = -h(p(t) - \min\{A(t), p(t)\}) \tag{4.4}$$

To translate this motion equation in renewable resource terms, we chose a linear form of h such that $h(p - A) = \alpha(p - A)$. Hence

$$\dot{A}(t) = -\alpha(p(t) - \min\{A(t), p(t)\}) \tag{4.5}$$

This peculiar regeneration equation[1] appears to be quite similar to the ones used in common renewable resource programs if we choose $\alpha = 1$. In that case, the net replenishment of the renewable resource under harvest p writes

$$\dot{A}(t) = (\min\{A(t), p(t)\} - p(t)) \tag{4.6}$$

with $\min\{A(t), p(t)\}$ the particular form of natural generation, equivalent to the exogenous $R(t)$ in soil analysis or to the logistic growth $r(1 - \frac{S(t)}{K})$ in fisheries studies. Relation (4.5) translates in our natural resource framework with harvesting as

$$\dot{A}(t) = -\alpha(h(t) - \min\{A(t), h(t)\}) \tag{4.7}$$

and for the special case $\alpha = 1$ this yields:

$$\dot{A}(t) = (\min\{A(t), h(t)\} - h(t)) \tag{4.8}$$

As far as we know, such a peculiar regeneration rate expressed in (4.7) has never been modelized in the literature. The most original feature of our model is that for any harvest lower than the threshold, "nothing happens" as we explained it in "pollution terms" in Chapter 1, or, better said, the stock replenishes itself exactly back to its previous level.

4.2.2.2 Case with restoration

If we allow for an artificial restoration of the assimilative capacity, we need to modify slightly the dynamics equation (4.7). When restoration is available, it becomes possible to restore the past depletions due to pollution excesses in the following periods. In order to keep our model as simple as possible, and since we will adopt in the next section a damage function based on the flow of emissions instead of the stock, we will use the restoration mechanism designed in Chapter 1. According to the properties described in Chapter 1, we assume thus that if the emissions are strictly inferior to the

[1] It is easy to check that we can easily reestablish our standard results:

$$p(t) \leq A(t) \Rightarrow \dot{A}(t) = p(t) - p(t) = 0$$
$$p(t) > A(t) \Rightarrow \dot{A}(t) = A(t) - p(t) < 0$$

assimilative capacity, the latter increase, and this increase is given by the symmetrical degradation function $h(p, A)$. Hence

$$p(t) < A(t) \Rightarrow \dot{A}(t) = h(p(t), A(t)) > 0 \tag{4.9}$$

And since we have adopted in this section the most simple form for $h(p, A)$ with $\alpha = 1$, (4.9) writes

$$p(t) < A(t) \Rightarrow \dot{A}(t) = -(p(t) - A(t)) = A(t) - p(t) > 0 \tag{4.10}$$

Combining (4.10) with the other properties of the case without restoration that remain valid we have:

$$\begin{aligned} p(t) < A(t) &\Rightarrow \dot{A}(t) = A(t) - p(t) > 0 \\ p(t) = A(t) &\Rightarrow \dot{A}(t) = p(t) - p(t) = 0 \\ p(t) > A(t) &\Rightarrow \dot{A}(t) = A(t) - p(t) < 0 \end{aligned}$$

We can thus rewrite equation (4.5) without the min function and translate it in renewable resource terms

$$\begin{aligned} \dot{A}(t) &= -\alpha(h(t) - A(t)) \qquad (4.11) \\ \dot{A}(t) &= A(t) - h(t) \end{aligned}$$

Moreover, it must be noted, as in Chapter 1, that even through artificial restoration the assimilative capacity remains bounded above by A_{max} and we assume once again for the sake of clarity that $A_{max} > A_0$. The stock dynamics in (4.11) are much simpler in the restoration setting than in the case of no restoration.

4.3 The economic model

In this section we show a way to model the pollution problem involving assimilative capacity as a simplified resource management problem, in order to extend the results of the literature on renewable resources to environmental regulation problems.

4.3.1 Internalization of negative externalities and resource management

The contribution of soil to farm production is analogous to the contribution of assimilative capacity to polluting industrial or agricultural production. Indeed, from a social planner's perspective less assimilative capacity means less "damage-less" production, and consequently less overall social utility. The major difference is that it is generally assumed that farmers are aware of the detrimental impact of their chosen agricultural practices on the soil (Barbier, 1990) and that their future benefits are thus directly affected by their present decisions[1]. On the contrary, the private agents using up the assimilative capacity by their polluting activities are not aware, or at least not economically concerned with the depletion of this resource since the environmental damages that occur as the assimilative capacity is more and more exceeded are borne by society as a whole. Hence the need to internalize this negative externality in order to analyze this as a true resource management problem, just like the fishery literature advocates the internalization of the stock externalities by the private fishermen. A social planner regulating polluting activities is thus tantamount to a private farmer deciding its level of crop culture with knowledge of the soil dynamics[2].

4.3.2 Benefits and costs of harvest

We have exposed the renewable resource features of assimilative capacity in general and defined the "harvest" sustained by this resource. We now need to establish the benefits and costs associated with the harvest of assimilative capacity. We shall simplify here the approach that we conducted in the previous chapters in order to sharpen the focus on resource depletion.

[1] One of the biggest concern regarding the development of sub-saharian African countries is the lack of consideration for soil depletion and its impacts on future generations (see for example Soule and Sheperd, 2000).

[2] The natural "replenishment" of soil $R(t)$ depends on various factors (Crosson and Stout, 1983): rainfall, slope of land exploited, area with row crops, conservation techniques (terracing for instance). However the afferent literature generally assumes that the economic agent, usually the individual farmer, is able to decide exactly what quantity of soil it wants to deplete.

4.3.2.1 The basic fishery model

Let us recall the most basic framework[1] that is used in fishery economics (Brown, 2000).

$$\max_h \int_0^\infty \varsigma h(t) - ch(t)dt$$
$$\text{s.t. } \dot{S}(t) = f(S(t)) - h(t) \ \textit{and} \ h \leq h_{max}$$

The private benefits are directly linear in h and depend on the constant market price of fish ς[2] and on the constant unit cost of harvest c. The amount harvested is bounded above by a h_{max} that represents the maximum catch capacities of the harvesting industries, supposed constant.

This simplified approach can be refined by introducing a stock externality such that the unit cost of harvest increases if the stock of fish decreases, reflecting the increasing difficulty of catching preys in a less abundant population. This yields the following modified problem

$$\max_h \int_0^\infty \varsigma h(t) - c(S(t))h(t)dt \tag{4.12}$$
$$\text{s.t. } \dot{S}(t) = f(S(t)) - h(t) \ \textit{and} \ h \leq h_{max}$$

ith $c(S)$ a decreasing convex function.

This stock externality[3] is very important in fishery management because if it is acknowledged (or internalized) by fishermen, it leads to a higher steady state stock of resource since they become aware of their impact on the overall productivity of the asset.

[1] In slightly more complicated models, the control variable is often the catch "effort" e instead of the actual catch h.

[2] ς, the market price of the produced good must not be confused with $p(t)$ the control variable denoting pollution used in the previous models.

[3] The concept of stock externality in natural resource management, namely the fact that the individual exploiters of the resource ignore the effects of their actions on the stock of resource and on the future productivity of this stock, is clearly exposed by Farzin (1996).

4.3.2.2 Adapting the basic model to the pollution problem

In our case, the stock externality reflects the fact that a depletion of the stock of assimilative capacity reduces the "damage-less pollution threshold". In our approach of assimilative capacity as a renewable resource, this stock externality on the unit cost is a practical substitute for the environmental damage function used in pollution economics. If we reason in terms of overall benefits and costs at society scale, harvesting assimilative capacity has a cost when an excessive harvest induces environmental damage. This unit "cost" of extraction (and consequently the total cost) is nil as long as the assimilative capacity is respected. For the sake of clarity we work here with flow environmental damages, as our analogy with renewable resource would become formally untractable if we were to introduce the stock damages as we do in Chapter 2. This simplification should, once again, not affect the general scope of our approach in terms of resource management. Regarding the benefit function, we can also drop the utility function used in previous chapters for a more basic profit function linear in the market price for the industrial or the agricultural production. Finally, the upper bound for harvest h_{max} is equivalent to the private optimum x_p discussed in Chapters 1 and 2.

Case with no restoration

Hence the following very simple framework for assimilative capacity management in the irreversible case (using the dynamics of the stock variable $A(t)$ defined in equation (4.5):

$$\max_h \int_0^\infty \varsigma h(t) - C(A(t), h(t))h(t)dt$$
$$\text{s.t. } \dot{A}(t) = -\alpha(h(t) - \min\{A(t), h(t)\}) \; and \; h \leq h_{max}$$

with $C(A, h)$ such that $C_A < 0$, $C_h > 0$ and $C(A, h) = 0$ if $A \geq h$. The lower the assimilative capacity, the higher the cost for society for the same given level of harvest.

In order to stick to the simple resource management framework, we can replace the two variables $C(A, h)$ function by the one variable $c(A)$ function similar to the one used in fishery models in equation (4.12). This simplification deprives our model from the relative dimension of the damage function[1] based on the "excess harvest" *vis*

[1]The crucial feature of our model, e.g., the degradation of the assimilative capacity itself in case

à vis the assimilative capacity but it can be argued that the choice of a highly convex c function could partially compensate. With such a function, once the assimilative capacity is depleted from its initial level, even if only by a small amount, damages start to occur. We get thus

$$\max_h \int_0^\infty \varsigma h(t) - c(A(t))h(t)dt \tag{4.13}$$
$$\text{s.t. } \dot{A}(t) = -\alpha(h(t) - \min\{A(t), h(t)\}) \ \ and \ \ h \leq h_{max}$$

with $c(A)$ convex and decreasing. We could have also included the cost of inputs (the price of fertilizers for example) in the model but it is tantamount to substituting the selling price ς by a net selling price ($\varsigma' = \varsigma - \beta$) where β is the unit price of the input paid by the farmer. It is straightforward that such a substitution would not change the fundamental results at all.

Case with restoration

The only thing that differs in the case with restoration is the dynamics of the stock variable $A(t)$ defined in equation (4.11). Consequently, equation (4.13) becomes:

$$\max_h \int_0^\infty \varsigma h(t) - c(A(t))h(t)dt \tag{4.14}$$
$$\text{s.t. } \dot{A}(t) = \alpha(A(t) - h(t)) \ \ and \ \ h \leq h_{max}$$

Despite some simplifications that do not affect the general spirit of the resource management model features, we have shown how the depletion of assimilative capacity implies by polluting activities can be assimilated to the basic case of resource management program.

4.3.3 Exploring the resource management toolbox

The driving concern of our research, the implementation of sustainable environmental regulation designed through economic analysis, has led us (Chapter 3) to question the sustainability of the pollution paths in the mainstream literature that fails to acknowledge the scale of the pollution stock as well as the finite availability of natural sink functions. The shift of framework operated in this chapter enables us to look

of excess pollution, is maintained in the motion equation.

at the sustainability issue from a new perspective. We can now apply the standard results of the renewable resource management literature to basic pollution problems. Before building on the alternative approach of viable control, we review briefly the most standard tools of renewable resource management applied to our specific case.

4.3.3.1 Standard present value maximization

Whether it concerns fisheries (Clark and Munro, 1975; Brown, 2000), soil (Barrett, 1991; McDonnell, 1983; Barbier, 1990) or renewable resources in general (Heal, 2000; Cropper et al., 1979), the optimal management rule determined in a welfarian optimization process with a positive discount rate is well known. If the initial level of resource S_0 is lower than the equilibrium level S^*, the optimal strategy is to refrain from harvesting the resource so as to let it replenish until it reaches S^*. If $S_0 > S^*$, the optimal path recommends to deplete it down to S^*. Of course the level of S^* is determined by the discount rate and the utility/profit function. It is thus important to keep in mind that for a given set of values of the discount rate or a given set of functional forms, the optimal stock level S^* can be zero, implying that the extinction of the resource is economically optimal. The renewable resource is considered indeed as an economic asset and its rate of return must be equal to the returns on other assets in the economy. Its optimal extraction is thus governed, as assimilative capacity in our previous models, by a Hotelling like rule based on the discount rate (Brown, 2000). Differences in the optimal path leading to this optimal steady state resource level S^* are induced by the linearity of the Hamiltonian in the optimal control program. If the Hamiltonian is linear in the control variable as it is often the case in fishery models, the path leading to S^* is of the "bang-bang" type (Seierstad and Sydsaeter, 1987). In this case the optimal policy is to either harvest at maximum ($h = h_{max}$) or to refrain from harvesting anything ($h = 0$). This solution is typical of basic fishery economics. If the Hamiltonian is not linear in the control variable, then the optimal harvest level increases if $S_0 > S^*$ and decreases in case of an opposite initial situation. The results we obtained in our first two chapters focusing on assimilative capacity as an autonomous state variable are in total accordance with this general solution. As noted by Doyen and Pereau (2006), *"optimal control modeling for the sustainable management of renewable resource can be criticized because it may imply dictator of future or present [Heal, 2000] and favor extinction of stock as shown* *by [Clark, 1990]".* In particular, cases of optimal extinction will inevitably arise in presence of high discount rates and low initial conditions.

4.3.3.2 Maximum Sustainable Yield and reproductive surplus

The identification of assimilative capacity to a renewable resource allows us to characterize important parameters often summoned in this domain. In particular, we can attempt to define the Maximum Sustainable Yield and the reproductive surplus of our particular resource. The original specification of these concepts are developed formally in Appendix A as we wish to focus the core of this chapter on our original use of the viability framework. These specifications contribute very helpfully to an explicit consideration of the finite nature of assimilative capacity. As such, presenting a pollution problem in terms of reproductive surplus could favor the design of sustainable regulation policies, especially in presence of low initial conditions.

4.3.3.3 Robustness of the model: the Green Golden Rule revisited

Our original specification of the pollution problem in terms of renewable resource also allows us to revisit the Green Golden Rule criteria already tackled in Chapter 3. We are able to draw very convincing links between our results and the standard results of the GGR that have been originally obtained in such a renewable resource framework. Our results are presented in Appendix B and confirm the robustness of our proposition to address assimilative capacity as a renewable resource. But although we explore more deeply the properties of the optimal paths according to the Green Golden Rule, the main conclusions remain identical to what we have shown in Chapter 3. Consequently, our change of framework would not yield different results from our initial approach if we stick to the present value maximization criterion or to the Green Golden Rule. These criteria display the same shortcoming as they did in the original pollution "presentation". That is why we shall dedicate this chapter to an original criterion that is successfully expanding in the renewable literature: the viability framework.

4.4 An attempt at the Viability approach

In this section we introduce briefly the basic viability approach and we apply it to our assimilative capacity management problem. It gives us the opportunity to extend our economic model to other kind of considerations such as the size of the polluting/producing sector and to integrate social constraints in our sustainability requirements. What's more, it can palliate some of the shortcomings of optimal control applied to resource management, especially as far as optimal extinction threats are concerned. In addition to the controversial role of the discount factor, discussed in the previous chapter, the optimal control approach is restricted to a unique trajectory while the Green Golden Rule fails to *characterize explicitly* any of the paths that could lead to the desired final situation. Béné et al. (2001) recall indeed that *"the optimal solution path is generally unique, which does not allow for possible alternate strategies"* whereas viability approach determines a set of viable paths. Given the perspective we have adopted to address sustainability issues, *ie* ensuring the preservation of capacities, the Viability approach fits quite nicely our purpose.

4.4.1 The adaptation of useful conceptual tools to our model

In the last decade, very promising contributions, in the wake of the framework exposed by Aubin (1991), have articulated formally the conceptual foundations of Viability theory and applied them to natural resource management problems. When it comes to designing sustainable policies the new viability approaches, applied mainly to fishery management so far, provide a very interesting way to integrate various requirements of sustainable development (ecological, economic and social criteria) within the same framework. According to Doyen and Pereau (2006), the viability methods matches the definition of sustainable development as a *"development that meets the needs of the present without compromising the ability of future generations to meet their own needs"*. The viability approach rests on the concept of *viability kernel* (Béné et al., 2001), defined as *"the set of bioeconomic states that make it possible to satisfy the constraints throughout time, given the dynamics"* (Martinet et al., 2007). This set, consisting for example of two state variables $S(.)$ and $X(.)$, is such that *"there is at least one feasible path [...], associated with admissible decisions [...], that satisfies all the constraints along time"*. The direct implication of this definition is that whenever the economy starts from (or moves to) a state outside this viability kernel, it will never be possible again to meet the various constraints indefinitely: *"at least one of the constraints will not be respected after some finite time T, whatever decisions apply"* (Martinet et al., 2007).

So far in this work we have tried to emphasized that the current decisions on emission levels had a significant intertemporal impact on the conditions in which the future emission decisions will be taken. The salient feature of our model, especially in the irreversible configuration, is that an excess of pollution today reduces the assimilative capacity available tomorrow and thus the maximum damage-less pollution potential. We have shown that the only way to preserve indefinitely a positive level of assimilative capacity is to set the emission level equal to this maximum damage-less level. Therefore, an excess of pollution *vis à vis* the assimilative capacity is tantamount to a reduction of the viability kernel. As assimilative capacity is depleted, the sustainable level of exploitation of this resource gets lower and lower. What's more, if we add additional constraints on the resource management, for example a minimum production level for the polluters/farmers (as the authors quoted above do for the fishermen), the viability kernel will be smaller than the initial one focused on environmental sustainability only.

Another interesting feature of the viability approach that we will not develop here is the definition of "recovery paths" (Martinet et al., 2007). A recovery path is a trajectory that leads the economy back into the viability kernel if this economy has left the kernel after a sequence of non-viable decisions or if the initial conditions were outside the kernel *ex ante*. By definition it implies that at least one of the viability constraints will be violated along this path before it can reach back the kernel. Viability analysis offers interesting tools to characterize the properties of the various paths that can lead drive the economy back to the kernel. The "speed of recovery" or the intensity of the constraint violation along each set of paths can be used as valuable additional decision criterion. If the recovery has been set as a policy objective. For a fishery, this would translate as choosing between reducing drastically (below the minimum individual profit threshold) the income (via the harvest) of fishermen during the necessary period to let the fish population regenerate quickly, or to make marginal reductions to induce a slow regeneration process while the economic and social constraint are not violated considerably. If we look into our assimilative capacity problem with restoration, the question of a multi-criteria sustainable recovery path for the assimilative capacity, namely a path that does not forbid any economic activity[1], is a crucial one. The viability approach is well equipped to take into account not only the environmental/economic feasibility of a recovery path but also its political and social acceptability.

The viability framework appears thus as a very promising framework to analyze our resource management problem.

4.4.2 A basic viability model

Developing an exhaustive viable approach to our problem would be beyond the scope of this paper, all the more than numerical simulations play a significant role in this approach (see the one carried on by Martinet et al., 2007). However we shall propose here the foundations of viable approach to our "pollution turned resource" problem. To our knowledge, the viability approach has not yet been formally applied to pollution problem and our model at the frontier between resource and pollution economics could be a useful step in the diffusion of the viability approach to other kind of environmental problems. We will focus here on the case with restoration to avoid unnecessary technical difficulties. This analysis constitutes a first step in the application of the viability framework to pollution control and the method of resolution is directly inspired from the approach of Martinet et al. (2007). Our results, especially on the definition of the viability kernel, are indeed merely qualitative characterizations and should be completed in the future by numerical simulations based on relevant empirical parameters.

4.4.2.1 Modified model and sustainability constraints

Similarly to what we did in the previous sections, we solve the problem from the social planner's point of view, considering thus the social damage incurred by pollution excesses as the *harvesting cost* for society as a whole. This is equivalent to assuming that the polluters themselves suffer from the environmental damage they inflict, which can be the case if their use of nitrogen fertilizers pollutes the water they need

[1] We have seen that the most rapid approach path used in the previous sections and in Chapter 3 imposed zero production during the transition phase.

for other purposes. We now consider the case of various polluters instead of working with a single representative polluter. This enables us to tackle the "social" dimension of the economic problem. We shall base empirically this analysis on the nitrate contamination problem and refer to the polluters as farmers, selling their agricultural output for a constant market price p. The total number of farmers in activity at time t is denoted $N(t)$. In a stock context, the individual polluters could be replaced by industries and the nitrates pollution by CO_2 emissions. Let us call $h(t)$ the harvest per farm[1] at time t. The total harvest $H(t)$ writes:

$$H(t) = N(t)h(t) \tag{4.15}$$

We can adapt the dynamics of the stock $A(t)$ to a discrete setting using (4.11) and we get

$$A(t+1) = A(t) + \alpha(A(t) - H(t)) \tag{4.16}$$

We need to make explicit the dynamics of the other state variable $N(t)$. For this purpose we use the dynamics suggested by Martinet et al. (2007) for the fishing sector. It seems reasonable to assume that the evolution of the farming sector follows a more or less similar pattern of evolution[2].

$$N(t+1) = N(t) + \xi(t) \tag{4.17}$$

$$-\gamma_1 \leq \xi(t) \leq \gamma_2 \tag{4.18}$$

The change in size of the sector is limited both ways. It is bounded above by γ_2 for technological reasons and because of the inertia of capital. What's more, intrinsical social constraints limit the exit rate of exploitations from the sector. Social inertia and imperfect labor markets make it impossible for more than γ_1 exploitations to leave the activity at each period. This inertia in the adjustment of the sector's invested capital prevents a "bang bang" solution that has little empirical feasibility.

In addition to these features from Martinet et al. (2007), we need to introduce an upper limit N_{max} on the sector size since, contrary to what happens in fisheries, there

[1] We suppose here that the farmers are homogenous in their pollution impact, hence in their assimilative capacity harvest. What's more, they all "harvest" the same quantities at each period, which is not satisfactory in terms of empirical observations but it is a necessary simplification to conduct a viability analysis.

[2] Empirical data on the pork sector backs up this assumption as we shall discuss later.

can be only a finite number of exploitations using up the assimilative capacity (of a riparian zone for example).

$$N(t) \leq N_{max} \ \forall \ t$$

Let us also recall that the harvest per unit h is bounded above by the maximum harvest capacity h_{max} such that

$$h(t) \leq h_{max} \ \forall \ t$$

We will use the profit function used in the previous sections and presented in equation (4.14) and substitute the individual harvest $h(t)$ by the aggregated harvest $H(t)$. The net aggregated profit for society $\Pi(t)$ thus writes:

$$\Pi(t) = \varsigma H(t) - c(A(t))H(t)dt \tag{4.19}$$
$$\text{s.t. } A(t+1) = A(t) + \alpha(A(t) - H(t)) \ \ and \ \ h(t) \leq h_{max}$$

Let us recall that $c(A)$ is a decreasing convex function. Combining (4.19) with (4.15) and adding (4.17) we get

$$\begin{aligned} \Pi(t) &= \varsigma h(t)N(t) - c(A(t))h(t)N(t) \qquad (4.20)\\ &\text{s.t. } A(t+1) = A(t) + \alpha(A(t) - H(t)) \\ &N(t+1) = N(t) + \xi(t) \\ &N(t) \leq \bar{N} \ \ and \ \ h(t) \leq h_{max} \end{aligned}$$

Multi-level sustainability constraints

One of the major assets of viability analysis is the multi-criteria definition of sustainability it allows. Sustainable development can be encompassed as a set of ecological, economic and social constraints. We can thus impose our standard ecological constraint used so far, namely the indefinite conservation of a minimum level of assimilative capacity:

$$A(t) \geq A_{min} > 0 \ \forall \ t \tag{4.21}$$

On the social level, we can impose a constraint aiming at keeping employment at a given level, which translates in terms of a minimum sector size:

$$N(t) \geq N_{min} > 0 \ \forall \ t \tag{4.22}$$

Finally economic efficiency can be obtained by setting a constraint on the average net profit per unit of exploitation (assuming that farmers suffering also from environmental damage). This net profit per unit is equal to the ratio of the aggregated profit $\Pi(t)$ over the number of exploitation $N(t)$. For the sector to be viable in an economic sense (it must be noted that the environmental cost is present in this net profit) we need

$$\begin{aligned} &\pi(t) \geq \pi_{min} > 0 \ \forall \ t \\ &\text{with } \pi(t) = \frac{\varsigma H(t) - c(A(t))H(t)}{N(t)} = \varsigma h(t) - c(A(t))h(t) \end{aligned} \tag{4.23}$$

Let us call K this set of constraints. The viability constraints (4.21), (4.22) and (4.23) are respected if and only if $(A_t, N_t, \xi_t, \pi_t) \in K$.

4.4.2.2 Viability analysis

Viability analysis tries to answer the question of the compatibility between the set of constraints defined above [(4.21), (4.22) and (4.23)] and the dynamics of the state variables [(4.17) and (4.16)]. The viability kernel consists thus in the set of bioeconomic states from which there exist intertemporal decisions that respect our sustainability constraints. In order to get a better characterization of this viability kernel, we follow an approach directly inspired from Martinet et al. (2007).

Minimum harvest

We start by deriving the ecological conditions under which the minimum net profit constraint (4.23) is respected. Economic viability requires that

$$\varsigma h(t) - c(A(t))h(t) \geq \pi_{min} \tag{4.24}$$

Let us call $\underline{h}(A)$ the minimum harvest that must be collected in order to satisfy (4.24) for a given A. It is straightforward that if the net profit per unit of harvest $(\varsigma - c(A))$ is negative, the harvest must be zero in order to avoid negative net profit, e.g., net losses. Let us call $A^{\#}$ the level of A for which $(\varsigma - c(A)) = 0$. In order to avoid complexities that do not add insightful information to the model we will assume that $A^{\#} < A_{min}$ so that we do not have to worry about $A^{\#}$ in the constrained set K.

Let us define $\hat{A}$ such that

$$c(\hat{A}) = 0$$

We will also assume that $A_{max} < \hat{A}$ so that we do not have to worry about $\hat{A}$ in the feasible set. We have thus

$$\underline{h}(A) = \frac{\pi_{min}}{p}$$
$$\forall\ A > \hat{A}$$

and

$$\varsigma - c(A) > 0\ \forall A > A^{\#} \tag{4.25}$$
$$\varsigma - c(A) \leq 0\ \forall A \leq A^{\#}$$
$$c(A^{\#}) = \varsigma \tag{4.26}$$
$$\varsigma - c(A) = \varsigma\ \forall A > \hat{A}$$

If $(p - c(A))$ is positive then according to (4.24) we have $\underline{h}(A) = \frac{\pi_{min}}{p-c(A)}$. Hence

$$\underline{h}(A) = \max(0, \frac{\pi_{min}}{\varsigma - c(A)})\ \forall\ A \leq \hat{A} \tag{4.27}$$

This minimal harvest necessary to satisfy the microeconomic constraint decreases when the stock of the resource A increases (since the stock externality diminishes). It is bounded below by $\underline{h}_{inf} = \frac{\pi_{min}}{p}$ as for a high enough stock of resource $\hat{A}$ the stock externality (the environmental damage in our pollution problem) disappears and it is bounded above by h_{max} as we will see.

Relation (4.27) can be rewritten as

$$\underline{h}(A) = \frac{\pi_{min}}{\varsigma - c(A)}\ \forall A > A^{\#}$$
$$\underline{h}(A) = 0\ \forall A < A^{\#}$$

This bang-bang like solution means that if the assimilative capacity level is too low, there will be no “harvest” of assimilative capacity, e.g., no pollution, and a maximum harvest (pollution) otherwise.

We can represent graphically this minimum harvest level in function of the resource stock focusing on the case where $A_{min} > A^{\#}$ (Figure 4.1)[1]. In that case we can ignore what happens for $A < A^{\#}$ and we get:

$$\underline{h}(A) = \frac{\pi_{min}}{p - c(A)} \ \forall \ A \in [A_{min}, A_{max}] \tag{4.28}$$

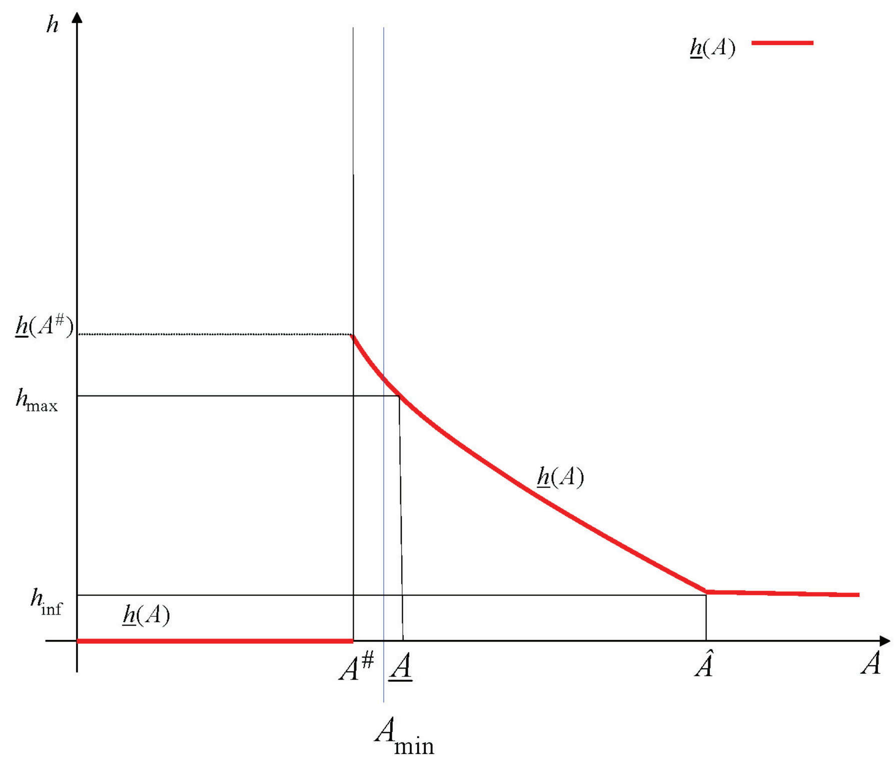

Figure 4.1: Minimal individual harvest level with $A_{min} > A^{\#}$

Minimum resource level

Since by definition we have $h(t) \leq h_{max}$ for all t, we can derive the following induced constraint on the minimal resource stock necessary to respect the economic

[1]To avoid a continuity problem in $A^{\#}$ we restrict our analysis to the configuration where the ecological sustainability constraint A_{min} is high enough so that we are in the Figure 4.1 setting with a smooth $\underline{h}$-curve on the feasible interval.

constraint (4.23):

$$\underline{A} = c^{-1}(\varsigma - \frac{\pi_{min}}{h_{max}}) \tag{4.29}$$

Considering the restrictions assumed earlier we know that this unique level $\underline{A}$ is such that

$$A^{\#} \leq \underline{A} \leq \hat{A}$$

$\underline{A}$ denotes the minimum level of resource stock for which it is possible, combining with the maximum level of harvest h_{max}, to get the minimum level of individual profit π_{min}. The illustration of this minimum resource level in Figure 4.1 is of particular interest as it shows that in order to satisfy indefinitely an economic constraint, it is necessary to preserve a minimum stock of resource. This issue is totally evacuated by the standard present value optimization.

Given the definition of h_{max} we can write

$$\underline{h}(\underline{A}) = h_{max}$$

Graphically, this is reflected by the fact that the image of $\underline{A}$ on the $\underline{h}$-curve is the maximum feasible harvest h_{max}. This means that for any resource level A lower than $\underline{A}$, it is not possible, given that the harvest is bounded above by h_{max}, to satisfy the microeconomic constraint (4.23).

Given the properties of $c(.)$, $c^{-1}(.)$ is also decreasing. Consequently, since $\frac{\pi_{min}}{h_{max}} > 0$, we have:

$$c^{-1}(\varsigma - \frac{\pi_{min}}{h_{max}}) > c^{-1}(p)$$

which, according to (4.29) and (4.26), is equivalent to

$$\underline{A} > A^{\#}$$

We still need to compare $\underline{A}$ and A_{min}. If $\underline{A} > A_{min}$ then the ecological constraint will be automatically respected if the economic constraint is respected. Conversely, if $\underline{A} < A_{min}$, respecting the ecological constraint will ensure that the economic constraint is respected also. In Figure 4.1 we choose arbitrarily to have $\underline{A} > A_{min}$.

The vicious circle of unsustainability

In this subsection we investigate the conditions under which an environmental vicious cycle such as those we have highlighted in Chapter 1 and 2 is likely to occur in this framework and how environmental unsustainability can feed from social viability requirements. Since it is the total harvest H that is of interest when it comes to the dynamics of the resource, it is useful to compare this aggregated level to the assimilative capacity level. The aggregated harvest level induced by the satisfaction of constraint (4.23), given that the sector size is constant and equal to $\tilde{N}$, can be easily computed for any A as

$$H(A,\tilde{N}) = \tilde{N}\underline{h}(A)$$

We draw on Figure 4.2 the aggregated harvest curve $H(A,\tilde{N})$ that corresponds simply to the points of the $\underline{h}$-curve multiplied by $\tilde{N}$.

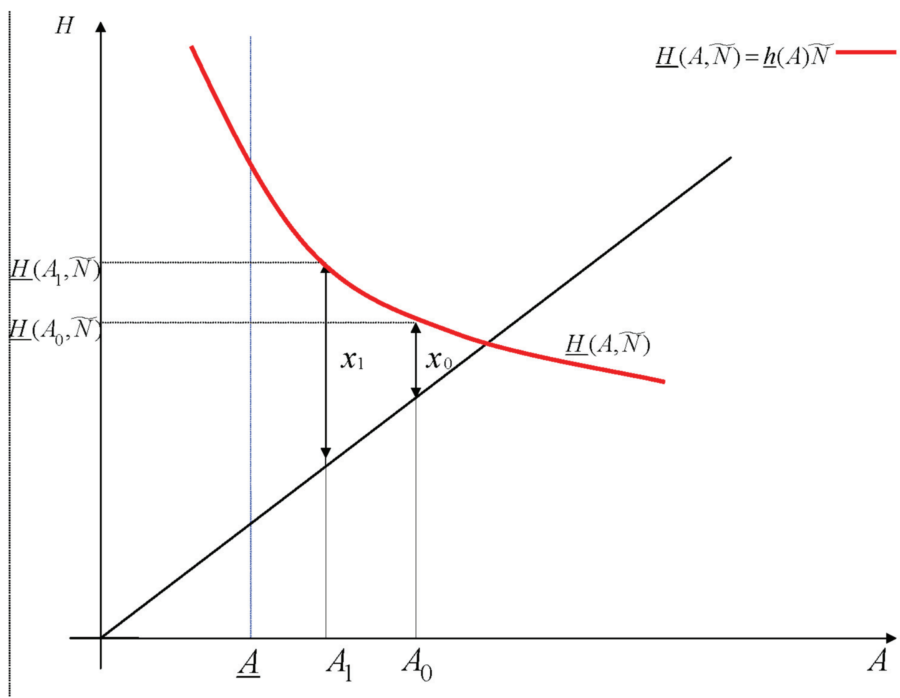

Figure 4.2: Minimal aggregated harvest and unsustainability vicious circle

Let us suppose that the sector size $\tilde{N}$ (the total number of agricultural exploitations in the nitrate contamination case) is so large that we have, as shown on Figure 4.2, $H(0) = (A_0, \tilde{N}) > {A_0}^1$. Figure 4.2 allows us to see that there is an excess of har-

[1]The $H(0)$ point is above the bisectrix for $A = A_0$.

vest (pollution) compared to the resource stock. Taking into account the dynamics of A in (4.16) this figure sheds some acute light on the degradation process through constant economic activity that is at the core of our work. Indeed, we know that at the new period we are going to have $A_1 < A_0$.

This depletion has a twofold negative effect. On the one hand it means that the minimum individual harvest level $\underline{h}(A_1)$ will have to be higher than before ($\underline{h}(A_1) > \underline{h}(A_0)$). On the other hand it means that if the sector size is kept constant at its level $\tilde{N}$, the aggregated harvest $H(A_1, \tilde{N})$ will be once again in excess towards the assimilative capacity A_1 but also that this excess (noted x_1 on the graph) will be larger than the previous excess x_0, which entails an even larger depletion than the one happened before.

It is easy to check that such a mechanism, very similar to the one highlighted initially by Pearce (1976) in his myopic model, will go on until the resource stock is reduced below $\underline{A}$ which will mean that it is no longer possible to guarantee a minimum profit per individual exploitation and by then the ecological constraint (4.21) will have been violated as well if we assume $\underline{A} < A_{min}$. Thus if the current stock of the resource is not sufficiently high or if the sector size is too large, then in order to insure a minimum level of individual profit, *overharvesting* will occur, degrading the assimilative capacity available at the next period, which will trigger even more *overharvesting* at the next period to keep the profit above the minimum threshold π_{min}.

As such this graphical representation of the first step of the viability approach highlights the potential vicious circle that may occur and jeopardize both ecological and economic sustainability.

4.4.2.3 Viable steady states

The next step of the viability analysis consists in identifying the stationary states that satisfy all the sustainability constraints. We shall represent the viable steady states in the A/N plane.

First of all the feasible set is defined by the $OPQR$ box, ensuring that the resource and the sector do not exceed their respective material limits A_{max} and N_{max}. Then this feasible set is reduced to the ecological-social constraint set $O'P'QR'$ as the ecological and social constraints (4.22) and (4.23) are materialized respectively on this plane by the horizonal dotted line $N = N_{min}$ and the vertical dotted line $A = A_{min}$.

We suppose for our graphic illustration that $\underline{A} < A_{min}$ so that it is the ecological constraint that is biting first as the resource stock is depleted. Figure 4.3 illustrates this partially-constrained set. Now in order to identify steady states satisfying all

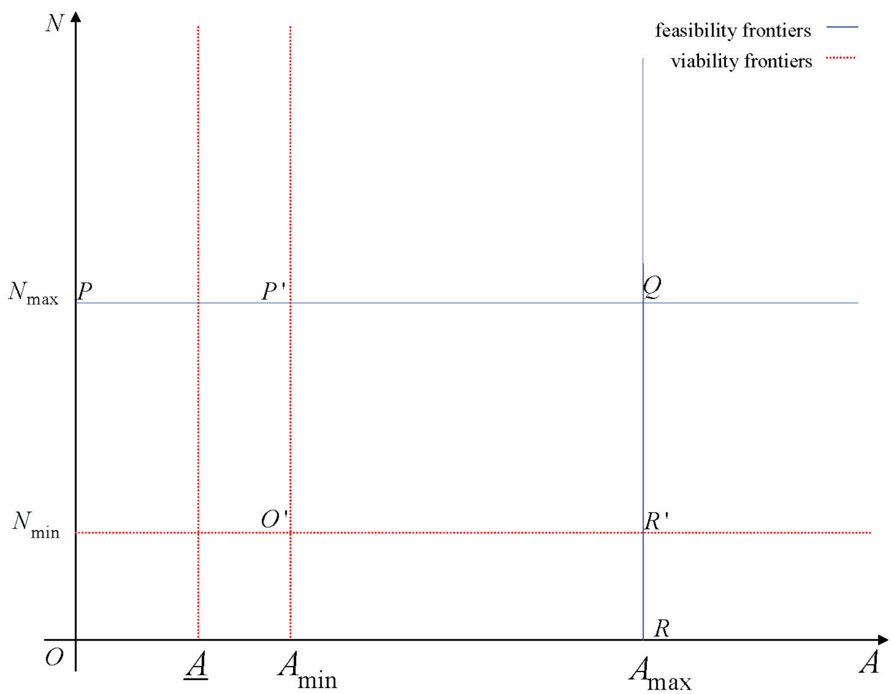

Figure 4.3: Feasible and viable constrained set

constraints, we must define the set of combinations of N and A that are within the $O'P'QR'$ box and that can be associated with harvest levels h and sector variations ξ such that the stocks are stationary and the microeconomic constraint is respected. In order to do so we study the system including both the dynamics of the state variables and the induced constraints determined in the previous subsection.

At the steady state we must have

$$A(t+1) = A(t)$$

$$N(t+1) = N(t)$$

which implies

$$H(t) = A(t) \tag{4.30}$$
$$\xi(t) = 0$$

Condition (4.30) implies

$$h(t) = \frac{A(t)}{N(t)} \tag{4.31}$$

This relation states quite intuitively that at the steady state, the level of harvest per unit exploitation $h(t)$ must be set equal to the share of assimilative capacity available per unit of exploitation $\frac{A(t)}{N(t)}$. We can now set admissible pairs (N_{ss}, h_{ss}) with respect to the resource stock $A(t)$. In order to define the set of viable steady states, we study the extreme cases corresponding to the minimum harvest $\underline{h}(A)$ and the maximum harvest h_{max}. We represent these frontiers on Figure 4.4. The idea underlying this approach is to study the feasible combinations of resource and sector size that satisfy the whole set of constraints K.

Materialization of the $\underline{h}$-frontier

The slope of the $\underline{h}$-frontier in the A/N plane is obtained through relation (4.31) using the expression of $\underline{h}(A)$ in (4.28). Let us denote $N_{f1}(A)$ the value of N for a given A such that $(A, N_{f1}(A))$ is on the $\underline{h}$-frontier. $N_{f1}(A)$ must respect both the stationary constraint 4.31 and the minimal microeconomic constraint (4.28). We must thus have:

$$N_{f1} = \frac{A}{\underline{h}(A)} \tag{4.32}$$
$$= \frac{A}{\frac{\pi_{min}}{\varsigma - c(A)}} \tag{4.33}$$
$$= \frac{A(\varsigma - c(A))}{\pi_{min}} \tag{4.34}$$

Let us call $\phi(A)$ the function such that

$$N_{f1}(A) = \phi(A) = \frac{A(\varsigma - c(A))}{\pi_{min}}$$

We show in Appendix C that ϕ is increasing under reasonable assumptions and we restrict our analysis to a concave form of ϕ. We can thus draw the following $\underline{h} - frontier$

on the A/N plane using some specific landmarks such as

$$N_{f1}(A^{\#}) = 0$$
$$N_{f1}(A) = \frac{Ap}{\pi_{min}} \;\; \forall \;\; A \geq \hat{A}$$
$$N_{f1}(\underline{A}) = \frac{\underline{A}}{h_{max}} \tag{4.35}$$

The economic interpretation of this frontier confirms important intuitions already hinted at earlier in this work. A point on the $\underline{h}$-frontier corresponds to a combination of sector size and resource level such that if the individual exploitation rate is at the minimum level necessary to satisfy constraint (4.23) then the resource level will be stationary. Given the dynamics of A, this implies that to any point above this frontier corresponds a A/N combination with a sector size (resource level) so high(respectively low) that even if the individual rate of harvest is at its minimum level necessary to respect the microeconomic constraint, the resource stock will decrease since the aggregated level of harvest will be strictly higher than the assimilative capacity. The latter will thus be lower at the next period and even more prone to depletion. However this does not mean that it is not possible to adopt a viable intertemporal strategy from this zone as we will discuss in the subsection on the viability kernel. Below this frontier, the A/N combinations are such that it is possible either to guarantee the respect of the economic constraint while increasing the resource level ($h = \underline{h}(A)$ and $H = N\underline{h} < A$) or to ensure the stationarity of the resource level while enjoying a higher level of harvest ($h > \underline{h}$ and $H = Nh = A$).

Materialization of the h_{max}-frontier

Let us denote $N_{f2}(A)$ the value of N for a given A such that $(A, N_{f2}(A))$ is on the h_{max}-frontier. More concretely, $N_{f2}(A)$ is the maximum sector size that allows stationary ecological dynamics and a maximum individual harvest level. The slope of the h_{max}-frontier in the A/N plane is easily obtained through relation (4.31) and yields a linear relation between N_{f2} and A:

$$N_{f2}(A) = \frac{A}{h_{max}}$$

We can thus draw the h_{max}-frontier on the A/N plane. We know from (4.35) that two frontiers will intersect for $A = \underline{A}$. We also know that in the constrained-feasible zone

$O'P'QR'$ we must have $N_{f2} < N_{f1}$. Indeed, for a given A, the level of N satisfying the stationary condition for $h = h_{max}$ is necessarily strictly lower than for $h = \underline{h}$ (except for $A = \underline{A}$ when they are equal).

The economic interpretation of this frontier is of particular interest. A point on the h_{max}-frontier corresponds to a combination of sector size and resource level such that if the individual exploitation rate is at its maximum level h_{max} then the resource level will be stationary[1]. Given the dynamics of A, this implies that any point above this frontier can either combine a lower level of harvest with a stationary resource level:

$$h < h_{max}$$
$$H = Nh = A$$

or keep the harvest level at its maximum and trigger the depletion of the stock:

$$h = h_{max}$$
$$H = Nh_{max} > A$$

Symmetrically, to any point below this frontier corresponds a A/N combination with a sufficiently high (respectively low) resource level (sector size) such that even if the individual rate of harvest is at its maximum, the aggregated level of harvest will be strictly lower than the assimilative capacity that will thus be restored. This increase in the resource stock is materialized by the blue arrow pointing to the right in Figure 4.4.

$$H(t) < A(t) \Rightarrow A(t+1) > A(t)$$

Consequently, the area II between this frontier and the constraint and feasibility frontiers (box $O'P'QR'$) corresponds to feasible non-stationary states, ecologically and socially viable where the resource level will increase whatever the intensity of harvest. It is also an economically viable area since it is below the $\underline{h}$-frontier characterized earlier which means that within this area it is always possible to find a level of harvest that satisfies the microeconomic constraint for a given A and N. This area can thus

[1] This frontier does not tell us directly if the microeconomic constraint can be respected but we can verify graphically that since $N_{f2} < N_{f1}$ on the feasible set, it is the case for all points on or below the frontier.

be determined as a part of the viability kernel since from that point there always exists a strategy that enables to stay within the constraints and to reach a viable steady state. For example, increasing progressively the size of the sector will move the state upwards within the viable steady states zone. Such an increase can be done through choosing $\xi(t) > 0$.

Characterization of the viable stationary states

Computing the results of this subsection, we can characterize analytically the viable stationary states set. First of all, we know that they must belong to the $O'P'QR'$ zone in order to respect the feasibility and the socio-ecological constraints. Secondly, combining the economic interpretation of the zones delimited by the two microeconomic-constraint-frontiers, we can conclude that to any point between these two curves can be associated a harvesting strategy[1] that turns them into steady-states while respecting constraint (4.23). Considering our analysis of the areas outside those frontiers, there cannot be any other feasible and viable steady states. The set of admissible steady states respecting the three constraints of sustainability is thus the striped zone (I).

4.4.2.4 Defining the viability kernel

Adapting the definition of Béné et al (2001) to our problem, we can state that the viability kernel corresponds to the set of all initial conditions $[(A_0, N_0)]$ such that there exists at least one trajectory starting from (A_0, N_0) that stays in the set of constraints K and leads to a steady state. As noted by Béné et al. (2001) *"the viability kernel differs from the niches in that, for the kernel, regulations through changes in effort can take place, thus allowing the viability to be enlarged"*. In our configuration, this means that from a state in the viability kernel a sustainable strategy can be adopted that implies variations in the intensity of the harvest, namely in the pollution levels. Such variations can be allocated in time such that the assimilative capacity stock can regenerate during a phase before being exploited at its maximum level.

Since we do not resort to numerical analysis we will characterize our viability

[1] It is important to keep in mind that other non-stationary strategies can be chosen from these points, leading to either the depletion or the restoration of the resource. This is a distinctive feature of the viability approach in comparison to the unique path corresponding to a given point in an optimal control analysis.

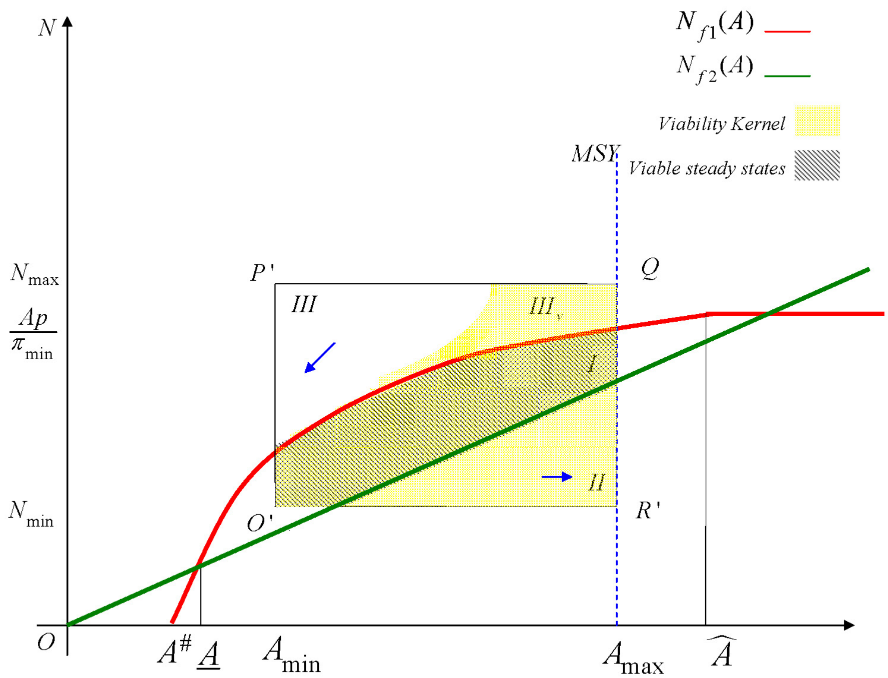

Figure 4.4: Viable stationary states and viability kernel

kernel through the elimination of the non-viability kernel zones. First of all it is obvious that for any point located outside the $O'P'QR'$ zone at least one feasibility or sustainability constraint is immediately violated. The viability kernel must thus be included in the $O'P'QR'$ zone.

Secondly, it is straightforward that any point belonging to the viable stationary state set also belongs to the viability kernel. It is indeed always possible to find at least one strategy (including a stationary one) that leads from one of these points to the indefinite respect of the sustainability and feasibility constraints. Area I is thus included in the viability kernel.

Thirdly we have shown that for any point below the h_{max}-frontier, it will always be possible to respect the microeconomic constraint while letting the resource stock increase. This stock will increase all the more rapidly (right pointing blue arrow on the graph) as the aggregated harvest level is low, if for example the sector size remains constant and if the individual harvest is inferior to h_{max}. As a consequence any point within the $O'P'QR'$ box and below h_{max}-frontier belongs to the viability kernel. Area II is thus included in the viability kernel.

Finally we must analyze what kind of paths the A/N combinations located in zone

III (within the $O'P'QR'$ zone and above the $\underline{h} - frontier$). As shown before, it is not possible for any state in this zone to ensure the respect of the economic constraint *and* to be at a stationary state at the same time. However this does not mean that it is not possible, from at least a subzone of zone III, to adopt an intertemporal strategy that respects from the start and indefinitely all the constraints, e.g., a viable intertemporal path. Let us call III_v such a subset of zone III, knowing that defined as such, zone III_v belongs to the viability kernel. We will obviously not be able to determine exactly the limits of the area III_v but we can develop some intuitions to get a rough qualitative idea of this subpart of the viability kernel. We have explained that from any point in zone III, respecting the microeconomic constraint implies depleting the assimilative capacity (blue left-pointing arrow on the graph). If the sector size does not change, the assimilative capacity will thus quickly reach and go beyond the ecological threshold A_{min} which prevents the path from being viable. It is therefore necessary (but not sufficient yet), for a path starting in zone III to be viable that this path implies a decrease in the sector size. The main challenge is to ensure that this sector size reduction can be implemented quickly enough so as to reach the steady state zone I before the ecological constraint is violated. Such a path will be "south-west" oriented (red diagonal arrow on the graph) and its "success" depends on the rigidity of the variations allowed in the sector size.

As we have stressed at the beginning, social and physical limits impose a lower bound on the decrease in N. At each period, the sector can only diminish by a maximum amount γ_1. In order for a path starting in zone III to be viable all along, we thus need γ_1 to be high enough relatively to the depletion speed of A, depending on the parameter α as well as on the current levels of N and A and on the economic constraint π_{min} so that the sector size can be reduced before the ecological constraint is violated. We cannot characterize more precisely these conditions without a numerical simulation but we can try to draw roughly the III_v viable subzone with simple economic intuitions.

Intuitively, the sector size will have more time to decrease before the resource level collapses if the initial stock of resource is high and the sector size is low. The "most likely to be viable" area in zone III must then be the bottom right-hand corner. As we move to the left from this area, following the $\underline{h}$-frontier, the sector size gets smaller, so that it is still easy to switch to the steady state zone in little time, but the resource stock also gets lower, which increases the risks of violating the ecological constraint if γ_1 is too small, e.g., if the sector can decrease only very little at a time. As we move upwards, the sector size gets larger and larger, which makes it more complicated to reach a steady state before violating the ecological constraint. It is of course all the more complicated as we are on the left of the III zone, as the initial resource stock is lower. Three cases must be distinguished to characterize the subset III_v.

Case 1: $III_v = \emptyset$

If the exploiting sector is too rigid, e.g., if γ_1 is too small, relatively to the depletion rate of the resource ($\alpha(H - A)$) caused by the individual harvest level necessary to respect the economic constraint, then it will not be possible, from any point in zone III, even the ones closest to the $\underline{h}$-frontier, to adopt an indefinitely viable strategy. Zone III_v is thus the empty set in this case.

Case 2: $III_v = III$

If the exploiting sector is extremely flexible, e.g., if γ_1 is extremely large, relatively to the depletion rate of the resource ($\alpha(H - A)$) caused by the individual harvest level necessary to respect the economic constraint, then it will be possible, from any point in zone III, even the ones furthest from the $\underline{h}$-frontier, to adopt an indefinitely viable strategy. Zone III_v is thus the whole zone III in this case.

Case 3: $III_v \subset III$

If the exploiting sector is flexible enough, e.g., if γ_1 is large enough, relatively to the depletion rate of the resource ($\alpha(H - A)$) caused by the individual harvest level necessary to respect the economic constraint, then it will be possible, from several points in zone III to adopt an indefinitely viable strategy while this will be impossible from other points in zone III (especially the ones located in the top left hand corner). Considering the economic intuition developed above, we can draw roughly the form of such a zone III_v on Figure 4.4.

We can thus characterize the viability kernel as the yellow shaded area consisting in the subsets I, II and III_v. Figure 4.4 illustrates the viability kernel for the case $III_v \subset III$. Of course the viability and feasibility constraints might be such in some cases that it is not possible, from any initial situation, to launch an intertemporal strategy that will respect the viability constraints all along. This means that the viability kernel can be sometimes equal to the empty subset.

Maximum Sustainable Yield and viability

Of particular interest is the comparison between the viability kernel and the Maximum Sustainable Yield resource level A_{MSY} defined in Appendix A. We have shown with relation (4.36) that

$$A_{MSY} = A_{max}$$

The Maximum Sustainable Yield is represented on Figure 4.4 by the vertical feasibility constraint $A = A_{max}$. It is thus included in the viability kernel.

4.4.2.5 Sensitivity analysis of the Viability kernel

Let us discuss briefly the sensitivity of the viability kernel to the variations of the exogenous parameters and constraints of the model. We will focus on three relevant parameters: p, γ_1 and α, and on the three sustainability thresholds π_{min}, N_{min} and A_{min}.

Market price and minimum profit variations

If the market price for the resource harvested, p, increases it will mechanically (see equation (4.34)) increase the slope of the $\underline{h}$-frontier. Symmetrically, equation (4.34) also tells us that a reduction in the minimum profit π_{min} has the same effect on the slope of the $\underline{h}$-frontier, only with a higher magnitude. Any of these variations will thus imply a larger viable steady state zone I. If zone III_v is not confounded with zone III it will increase the size of the viability kernel as can be observed on Figure 4.5. Otherwise it has no impact on the viability kernel. From the opposite point of view, a reduction on the market price or an increase in the minimum profit would have the opposite effect on the viable steady state zone and on the viability kernel. The economic interpretation of this sensitivity is quite straightforward: if the minimum harvest level necessary to ensure a minimum profit decreases (respectively increases) either because the same amount of resource sells for a higher (lower) market price ς or because the minimum profit defined as economically viable π_{min} is lower (higher), then the subset of A/N combinations from which an indefinitely viable path can be adopted is larger (smaller). This means that a change in the economic conditions of the harvesting activity can make it either easier (harder) to achieve global viability.

Concretely, a decrease in π_{min} is very unlikely to happen as the global economic conditions both nationally and internationally tend to favor higher minimum viable profits. An increase in this minimum viable profit, which is more likely, puts more pressure on the environment *ceteris paribus*. Considering the possible increase in the market price, if the latter is not due to a stronger demand, but to increased production costs or to (environmental) taxation, this will not produce an increase in the actual price earned by the producer[1] and thus the viability kernel will remain the same.

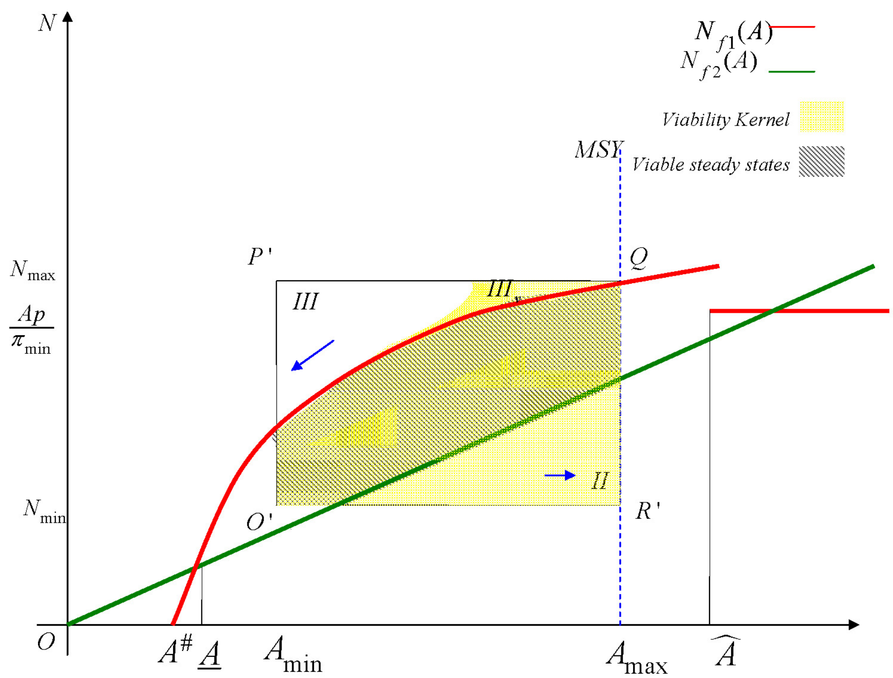

Figure 4.5: Increased viability kernel for a higher market price

Variations of the degradation factor α ***and of the maximal downsizing factor*** γ_1

The degradation factor α determines the speed at which the assimilative capacity is depleted (or restored) when the aggregated harvest is in excess of the assimilative capacity. The restoration effect is relevant only in zone II and there is no constraint-violation threat for the paths starting in that zone[2]. Symmetrically, the parameter γ_1 determines the maximum downsizing speed of the sector. These parameters are of great importance when it comes to zone III as it is their relative value that will

[1]If the product is already taxed, an additional price increase by the producer might lead to a decrease in the demand for the good.

[2]That is also why the parameter γ_2 is not a problematic one.

determine if there exists a viable path from a given point in that zone. An increase (respectively a decrease) in α, just like a decrease (respectively an increase) in γ_1 will thus entail a reduction (increase) of the subzone III_v and thus of the viability kernel in general. This interesting feature shows an unusual but crucial link between the social/insitutional rigidity of a polluting sector and the viability of the economic trajectories available.

Variations of the ecological and social thresholds

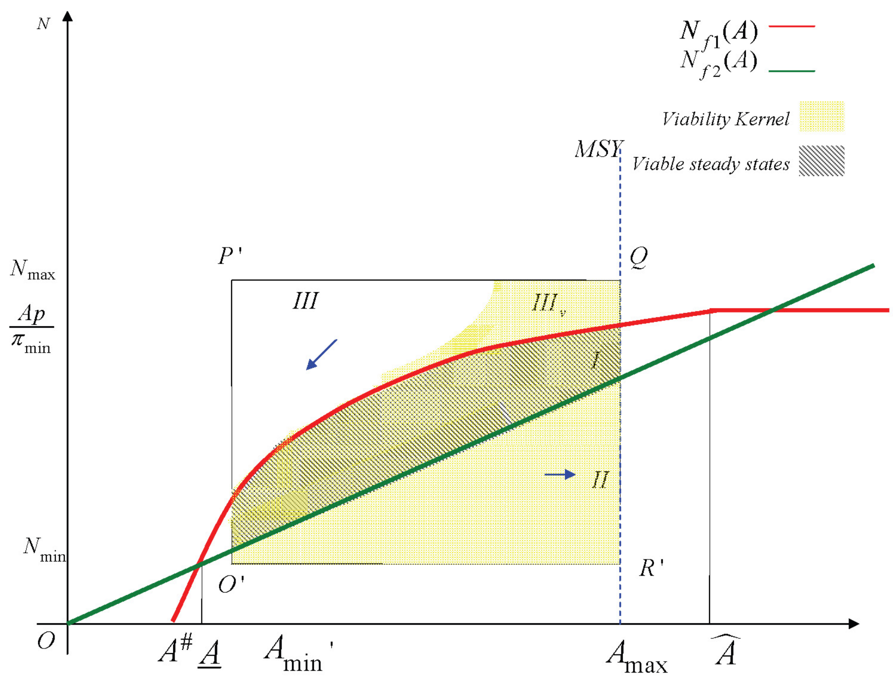

Figure 4.6: Increased viability kernel for lower viability constraints

The ecological and social thresholds A_{min} and N_{min} delimit the local zone in which the viability kernel will be included. As such, it is clear that a lowering of the social demands regarding the size of the sector or a decrease on the requirements of natural capital conservation will allow more flexibility for the viable economic strategies. As shown in Figure 4.6, reduced viability constraints A_{min} and N_{min} allow for a larger $O'P'QR'$ zone which implies a larger zone II and a larger zone III. What's more, a larger zone II directly enlarges the viability kernel and so does a larger zone III if the $\frac{\gamma_1}{\alpha}$ ratio permits it. Intuitively enough, the looser the viability constraints, the larger the viability kernel.

4.5 Conclusion: viable control, assimilative capacity management and sustainable policies

4.5.1 Viable control: a promising alternative

The results we obtain in this promising framework are of crucial value in the exploration process of sustainable regulation of pollution problems initiated in our research.

A wide range of viable strategies

The first concrete advantage of the viability framework developed in Section 4 is the large panorama of strategies that it offers compared to the restricted unique solutions of optimal control. In terms of concrete policy application, the range of options branded as viable by viable control is much more realistic than the unique optimal path. As such, our attempt at applying this viable approach to a pollution problem, even if it still needs refinements that will be done in further work, usefully contributes to broadening the perspective of the decision maker. In doing so, it proves capable of highlighting one or more sustainable strategies towards an environmental or a social goal without sacrificing other dimensions such as microeconomic constraints.

Intergenerational equity

A second advantage is that the viable paths defined are not subject to the influence of the discount rate or of an arbitrary weight given to the present and the future (as it is the case with the Chichilnisky criterion for instance). In this sense, the sustainability of these paths is not subject to an *a priori* decision on intertemporal equity. Such a "freedom" is highly valuable considering the heated debates on discounting that are especially vocal when it comes to climate change (see Chapter 3). Subsequently, the pitfall of intergenerational equity can be avoided and our vision of sustainability as the "preservation of capacities" fits adequately into this framework. This setting allows us indeed to focus on the preservation of actual capacities that may be irreversibly degraded without having to "negotiate" intergenerational trade-offs.

Integration of various capacities

This attempt to apply viable control to our problem has proved quite fruitful in terms of highlighting interesting relations between the size of a producing/polluting sector such as agricultural exploitation and the viable use of a resource such assimilative capacity. It has brought to light the crucial role played by the flexibility/rigity[1] of the activity sector, if the latter adapts slowly to the ecological conditions around, it can lead to environmental collapse or at least threaten environmental viability. As such the viability framework is a privileged setting in which to integrate the various capacities (economic, environmental) that are at stake in the implementation of sustainable policies. In vulgar terms, this approach is able to encompass all three "pillars" of sustainable development: environmental, economic and social dimensions. The various sustainability constraints can indeed be made explicit (definition *ex ante* of A_{min}, N_{min} and π_{min}) inasmuch as the number of variable can be computed mathematically. Moreover, the different levers available to guarantee a multi-dimensional viable situation, as well as their interconnections, appear very clearly in this framework. Among the options available to ensure a viable activity, the decision maker can regulate the total size of the sector by imposing a legal limit on the number of exploitations. In our nitrate-contamination case, this could be easily done through the restriction of the authorized cultivated areas for nitrate-intensive cultures (corn, pork breeding). As the sensibility analysis shown, an action on the individual profit level could also enlarge the viability kernel and limit the risks of crisis. This could be achieved for example with a subvention system resulting in higher market prices for the harvested resource, e.g., for the polluting good, while controlling the maximum harvest h_{max} to avoid beefing caught off guard by excess aggregated harvest. The viability grid (the $A - N$ plane in our approach) of analysis offered by the viability framework seems of promising interest when it comes to ensuring multidimensional sustainability without loosing sight of the interdependency between the variables as a compartmentalized multi-criteria approach might do.

[1]This feature can be illustrated with the famous "pork cycle" in French agriculture that denotes the large variations of the pork exploiting sector following equally large market price variations (Porin and Mainsant, 2000). Since pork-exploitation is a well-known nitrate producing industry, we can subsume that in that particular case, and under cautious regulation, the sector could adapt rapidly enough in order to avoid massive depletion of riparian buffer assimilative capacity.

Recovery paths

Of additional interest would be the study of the recovery path from a state outside the viability kernel. This "crisis" situation might be due either to initial conditions that were spontaneously outside the viability kernel or to an external shock on one of the state variables (an environmental or an economic "catastrophe" drastically reducing A or N for example). In order to reach or to come back within the viability kernel, different strategies are available, but they all imply, by definition, a violation of one or several sustainability constraints during the recovery phase (Martinet et al., 2007). The feasible recovery paths are determined by the dynamics (see the blue arrows on Figure 4.4) on each state variable and are heavily dependent on the parameters γ_1, γ_2 and α. When it comes to choosing a recovery path, the interesting issue arising concerns the criteria that are used to classify the potential recovery paths and more specifically the concept of *time of crisis* developed by Doyen and Saint-Pierre (1997). The recovery approach of Martinet et al. (2007) relies on minimizing the recovery period during which one or more constraints are violated. Conducting a thorough analysis of the recovery paths is unfortunately way beyond the scope of this paper, in part for technical reasons acknowledged by the authors quoted, but scrutinizing carefully the recovery paths in a viable framework is of great interest to shed some light on the irreversibility issues at stake and on the interdependency between economic, social and ecological viability. We shall insist on the role of these recovery paths when low initial conditions are at stake in the Conclusion of Part II.

Appendix Chapter 4

Appendix A: Maximum Sustainable Yield and Reproductive Surplus

Within the debate on sustainability animating the literature on renewable resource, specific concepts claiming to ensure sustainable resource management have appeared. The most sensible approach advocated by ecologists is based on the concept of Maximum Sustainable Yield (*MSY*). The Maximum Sustainable Yield can be defined as the largest harvest that can be taken from a species stock over an indefinite period. In the case of a standard Gordon-Shaefer fishery model, it is equal to h_{msy}, as shown in Figure 4.7:

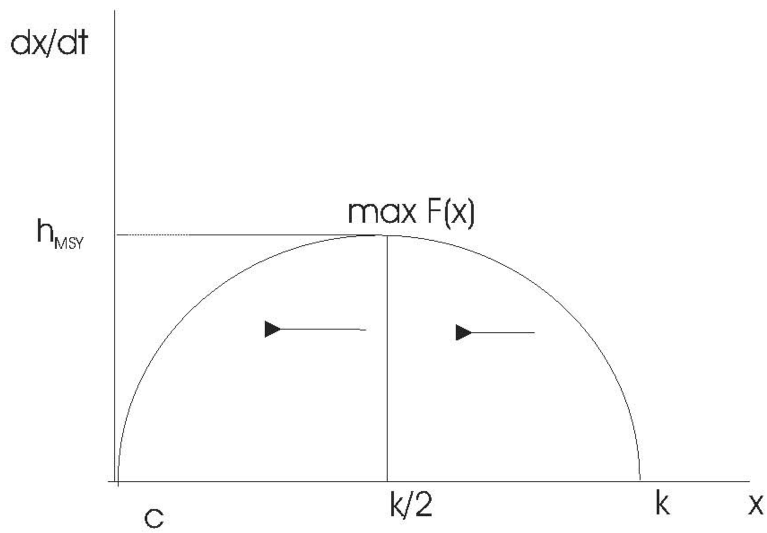

Figure 4.7: Maximum Sustainable Yield with a logistic function (Xepapadeas, 2000)

The companion concept of *MSY* is the *reproductive surplus* (Hilborn et al., 1995). According to these authors the *"reproductive surplus [...] is determined by the balance between births, deaths, and somatic growth"*. Consequently, *"the reproductive surplus [...] can be harvested on a sustained basis"*. This concept is declined along the different categories of renewable resources. In the case of forestry management it translates into annual allowable cuts and the annual reproductive surplus for wild life conservation.

Considering the analogy successfully drawn previously between pollution control and resource management, the next logical step in our analysis of sustainable pollution paths is to explore the application of the Maximum Sustainable Yield approach to our modified model of assimilative capacity management. The features of our peculiar resource depletion problem are displayed in the previous section. Equations (4.7) and (4.11) translate the dynamics of the reproductive surplus of the stock of assimilative capacity. Although we have carefully pointed out that the "natural" regeneration rate of assimilative capacity depended on the harvest sustained by the resource, we can easily see that this "replenishment" (any amount harvested less than the assimilative capacity is replenished at the next period) reaches a maximum for the corner solution $h(t) = A(t)$ for any given $A(t)$. This reproductive surplus is thus all the higher when the assimilative capacity level is high. In order to maximize the reproductive surplus, we must thus maximize the assimilative capacity level. In our case, the relation between the reproductive surplus and the stock of resource can be expressed graphically as an increasing linear function as shown in Figure 4.8.

Considering the upper-limit on A in the restoration setting, this means that the Maximum Sustainable Yield is obtained for $A = A_{max}$:

$$A_{MSY} = A_{max} \tag{4.36}$$

The maximization of the reproductive surplus in our resource management framework yields a solution similar to the alternative criteria explored in the previous chapter (Maximin, Green Golden Rule and Overtaking in the irreversible case, only Green Golden Rule and Overtaking in the reversible case): reaching and conserving the highest possible level of assimilative capacity and harvesting (respectively polluting) the maximized reproductive surplus (the maximum damage-less amount). Consequently we must also distinguish this time two configurations, one allowing restoration of the assimilative capacity and the other assuming that any decrease in the level of assimilative capacity is irreversible. The corresponding sustainable strategies are identical to the one designed in Chapter 3 and we will simply recall them here and translate them into renewable resource terms.

Proposition 4.1. *In the irreversible case, the reproductive surplus is maximized through the following strategy: the harvest at each period must be equal to the maximum reproductive surplus corresponding to* A_0*. For all* t *we must have* $h(t) = A_0$*.*

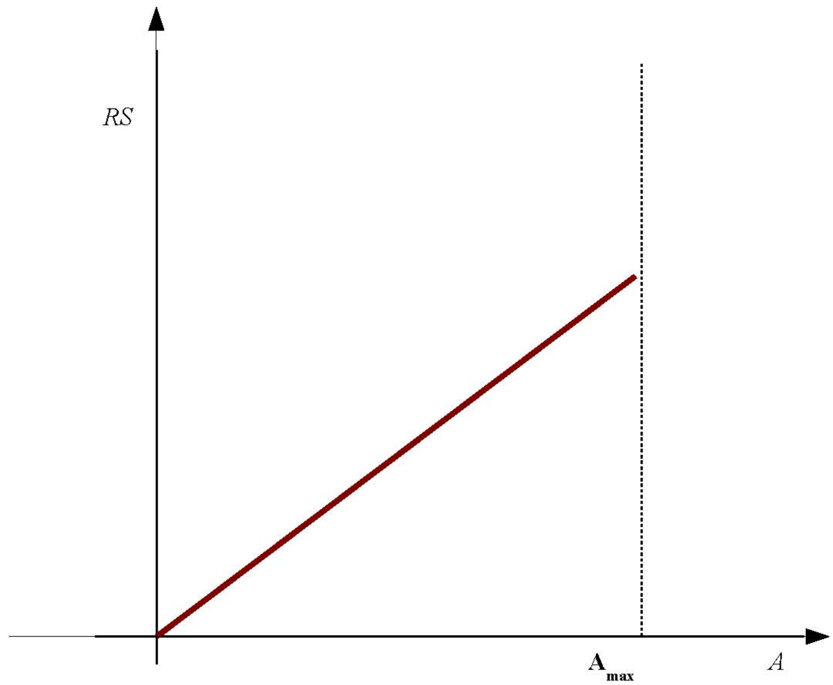

Figure 4.8: Reproductive surplus of assimilative capacity as a renewable resource.

This harvesting strategy guarantees an indefinite sustainable profit equal to ςA_0 *at each period and an overall present value profit equal to:*
$\int_0^\infty e^{-\rho t}(\varsigma A_0 - c(A_0, A_0)A_0)dt = \int_0^\infty e^{-\rho t}(\varsigma A_0)dt = \frac{\varsigma A_0}{\rho}$

Regarding the reversible case, it must be noted that the ambiguity on the transition phase leading to the asymptotic optimal steady state remains in this setting as well. We will not explore again many different transition paths here as we have done in Chapter 3 Section 5. We shall thus stick to the first path studied in 5.2.2, the most rapid approach path.

Proposition 4.2. *In the reversible case, the reproductive surplus is maximized through the following strategy: during a first phase the increase in* A *is maximized, which means, according to (4.10), that* $h = 0$*, e.g., that artificial restoration is conducted, at a cost equivalent to the foregone profits that should have been yielded by a damage-less harvest* $h = A$*.*
Once A_{max} *has been reached, at time* $\bar{T}$*, the Maximum Sustainable Yield* $h = A_{max}$ *is harvested indefinitely. The maximum sustainable profit is equal to* ςA_{max} *and the corresponding overall present value profit is equal to:*

$\int_0^{\bar{T}} e^{-\rho t}(\varsigma 0 - c(A(t),0)A(t))dt + \int_{\bar{T}}^{\infty} e^{-\rho t}(\varsigma A_{max} - c(A_{max}, A_{max})A_{max})dt)$.
This can be easily reduce through an integration by parts to $\varsigma A_{max}(1-\rho\bar{T})$.

In both cases, the profit yielded by the Maximum Sustainable Yield strategy is heavily dependant on the discount rate. The higher ρ, the lower the total present value profits.

As far as the restoration case is concerned, it is clear that in policy terms the most rapid approach path chosen here is difficult to implement because it implies refraining from harvesting (polluting) hence refraining from producing, especially if the transition period is long. Although we must keep in mind that this absence of profit is analyzed from the social planner point of view[1] and does not necessarily forbid any economic activity, the profit function $\varsigma A_{max}(1-\rho\bar{T})$ clearly shows that a long transition period entails lower profits (since the profits are nil during the transition period). Given our concern to keep the results as general as possible thanks to general functional forms, we cannot analyze more deeply in continuous time the transition phase of the Maximum Sustainable Yield strategy. However, if we switch back to discrete time we can gather some interesting information on the influence of certain parameters on this strategy. Adapting our dynamics to discrete time we can translate equation (4.10) into:

$$A_{T+1} = A_T + \alpha(A_T - h(T)) \tag{4.37}$$

Since $h(T) = 0$ for all T during the first phase of a Maximum Sustainable Yield strategy we have

$$A_{T+1} = A_T + \alpha A_T \;\; \forall T < \bar{T}$$

and

$$A_T = (1+\alpha)^T A_0 \tag{4.38}$$

[1]We consider that the private profits of the harvesting activity are beneficial to the whole society and that our restoration mechanism is tantamount to an additional costly restoration effort $r(t)$ such that $\dot{A}(t) = -\alpha(p(t) - A(t)) + r(t)$ where r would be paid by the government or by society as a whole (See Barbier, 1990). The private economic agents are not necessarily targeted as the ones that must restrain from making profit. See the initial description of the restoration process in Chapter 1.

Thanks to (4.38) we can now determine the length of the transition period $\bar{T}$. By definition we know that

$$A_{\bar{T}} = A_{max}$$
$$A_{\bar{T}} = A_{max} = (1+\alpha)^{\bar{T}} A_0$$

which gives us

$$\bar{T} = \frac{ln(\frac{A_{max}}{A_0})}{ln(1+\alpha)} \qquad (4.39)$$

We can now examine the role played by the different parameters. $\bar{T}$ is increasing in A_{max} and decreasing in A_0. This is obviously very intuitive as the higher the maximum stock of resource, the longer it takes to reach it *ceteris paribus* and conversely the higher the initial stock, the shorter the transition period. The parameter α defines the degradation impact of pollution excesses and symmetrically the restoration factor when pollution is strictly inferior to the assimilative capacity. In (4.39) we can clearly see that the higher this factor, the lower the transition period. In terms of political acceptability (as mentioned in the previous footnote, this political acceptability concerns society as a whole, not just the harvesters), the Maximum Sustainable Yield strategy in the restoration case will thus be easier to implement if α is high enough.

Conclusion: the Maximum Sustainable Yield as a useful indicator in favor of resource conservation

Our analysis of the Maximum Sustainable Yield of the assimilative capacity leads to ambiguous conclusions. On the one hand it has highlighted the interest of restoring artificially (or letting restore naturally through "rest periods") the resource to its maximum level A_{max}, such that its reproductive surplus (if exploited fully) is maximized. This MSY indicator does not provide an exhaustive management method *per se*, but it can nevertheless play a helpful role in policy design. First it has the advantage of stressing once more the finite dimension of the assimilative capacity resource so as to escape the "cowboy" economy paradigm. Second, it is a concept that appeals to biologists and conservationist and that can bridge the gap between the former and the other experts involved in the decision making process, especially economists. The mere presence of the MSY in the discussions surrounding environmental regulation can contribute to focusing a little more on the sustainability dimension, and maximizing the reproductive surplus plays in favor of environmental capital conservation and basic intergenerational equity.

On the other hand, given the peculiar dynamics of assimilative capacity whose regeneration rate depends on the harvest it must sustain, advocating a level of harvest h that corresponds to the Maximum Sustainable Yield might be risky. The slightest monitoring error could indeed lead to an excessive harvest inducing a depletion of the resource and triggering a possible unsustainable cycle. Considering our previous commentaries on safe minimum standards, the most reasonable use of the Maximum Sustainable Yield indicator would be to justify roughly the maintenance of a high enough level of assimilative capacity and to enforce a harvest level lower than the expected available reproductive surplus, in order to leave a "safety margin" to the resource regeneration. According to Barrett (1991), keeping such a safety margin that would leave the resource stock within admissible bounds both in the short and long run is a consistent step towards sustainability.

Appendix B: The Green Golden Rule revisited

In the previous chapter, we explored the alternative criteria to the discounted utilitarian framework in the context of our pollution problem. The shift or framework operated in this chapter will allow us to dig deeper on the properties of one of these criteria: the Green Golden Rule. As it was originally designed to cope with resource management issues, our new focus on assimilative capacity as a special kind of renewable resource brings to light interesting results that are completely in accordance with the seminal literature on the Green Golden Rule. For this section, we let aside our simplified resource exploitation profit function defined in Section 3 and used in our viability approach and we retain the more complex $c(A, h)$ damage/cost function.

The fundamental results

As intuition indicates, the optimal resource depletion path according to the Green Golden Rule matches the results obtained with this criterion applied to the pollution control framework.

We can reproduce the demonstration in this new context that corresponds more precisely to Heal's (2000) exposition of the alternative criterion.

$$U(h, A) = ph - c(A, h)$$

The maximum sustainable utility level demands that

$$h(t) = R(A(t)) = A(t)$$

and, since $c(A, A) = 0$ for all A:

$$\max_A U(A, A) \Leftrightarrow \max_A pA$$

$$\max_A pA \Rightarrow \begin{cases} A = A_0 \ \ (no\ \ restoration) \\ \\ A = A_{max} \ \ (restoration) \end{cases}$$

Proposition 4.3. *If restoration is not available, then according to the Green Golden Rule,* $A* = A_0$ *and* $p^*(t) = A* = A_0 \ \forall t$.

Proposition 4.4. *If restoration is available, then according to the Green Golden Rule,* $A^* = A_{max}$ *and* $p^*(t) < A(t) \ \forall A(t) < A_{max}$ *and* $p^*(t) = A_{max}$ *if* $A(t) = A_{max}$

These results are in total accordance with the ones obtained in Chapter 3. The results show the same shortcomings regarding the approaching path in the restoration case. There are many paths that approach the Green Golden Rule but that yield very different utility levels along the way (see Chapter 3 Section 5).

The main difference with our previous application of the Green Golden Rule is that the pollution path that complies with the Green Golden Rule under the assumption of artificial restoration available does not depend on x_p anymore. This is a straightforward consequence of the linearization of the benefit function we operated to fit the problem into the most common configuration of the renewable resource framework.

Robustness of the results compared to the standard conclusions

If we return temporarily to a concave utility function such that

$$U(A,h) = f(h) - c(A,h)h$$

we can shed light on the previous results through a geometrical resolution. Based on the argument of Heal (2000) we can determine graphically the optimum level of A under the Green Golden Rule criterion. Via some simplifications, we can assimilate our utility function $U(A,h)$ to a standard double-variable utility function and thus introduce indifference curves on Figure 4.8 in order to get Figure 4.9. In order to satisfy the sustainability constraint, we must look for values that lie on the curve $h = R(A) = A$, e.g., on the main bisectrix. The intersection between this subset and the highest possible indifference curve gives us the maximum sustainable utility level[1].

Consequently, at this point basic microeconomics tell us that the marginal rate of substitution between h and A equals the marginal rate of transformation of the $R(A)$ curve (equal to one in our specific case). We must thus have

$$\frac{U_A}{U_h} = -R'(A)$$

We can verify analytically that our results satisfy this condition with the following proof. Let us focus on the restoration case and we assume that $x_p \leq A_{max}$, the Green Golden Rule must give us, according to Chapter 3 Section 5, $A^* = x_p$. The dynamics of A in this configuration read

$$h \leq A \Rightarrow \dot{A} = A - h$$

[1] In standard renewable resource frameworks with a logistic $R(.)$ function, this maximum sustainable utility level is not necessarily identical to the Maximum Sustainable Yield level, but in our case given the linearity of $R(A)$, they are identical.

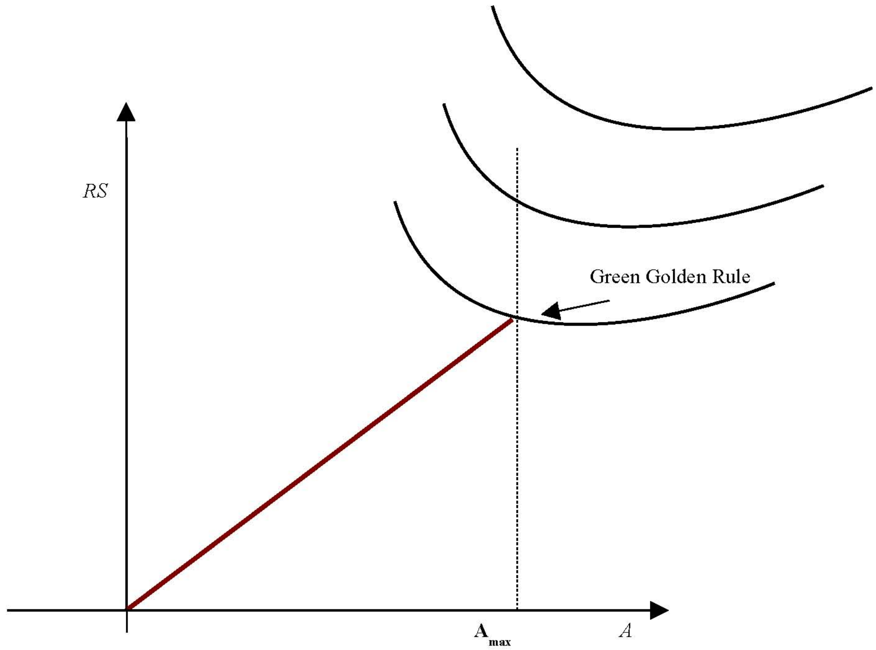

Figure 4.9: Graphical representation of the Green Golden Rule

Rewriting it with a regeneration function R(.), corresponding here to the identity, we get

$$h \leq A \Rightarrow \dot{A} = R(A) - h$$
$$R'(A) = 1$$

$$U_A = \frac{dU(A,h)}{dA} = -c_A(A,h).h$$
$$U_h = \frac{dU(A,h)}{dh} = f'(h) - (c_h(A,h).h + c(A,h))$$

Since $h = A = x_p$ we have $f'(h) = 0$ and $c(A,h) = 0$

$$f'(h) - (c_h(A,h).h + c(A,h)) = -c_h(A,h).h$$

Given that

$$c_A(A,h) = -c_h(A,h)$$

We get

$$\frac{U_A}{U_h} = \frac{-c_A(A,h).h}{-c_h(A,h).h}$$

$$\frac{U_A}{U_h} = \frac{c_h(A,h).h}{-c_h(A,h).h} = -1 = -R'(A)$$

$\frac{U_A}{U_h} = -R'(A)$ is the fundamental condition of the Green Golden Rule optimization. We are thus able to show that for the restoration case (that does not display discontinuities in the resource dynamics), the standard condition for the Green Golden Rule optimum (Heal, 2000, p.53) is satisfied.

Appendix C: Properties of function ϕ

We have

$$\phi(A) = \frac{A(p - c(A))}{\pi_{min}}$$
$$\frac{d\phi(A)}{dA} = \frac{p - c(A) - Ac'(A)}{\pi_{min}}$$

Given the properties of $c(.)$, we know that $c'(A) > 0$ and $c"(A) > 0$. We have thus $-Ac'(A) > 0$ and we know from condition (4.25) that for any $A > A^{\#}$ we have $p - c(A) > 0$.

$$\phi'(A) = \frac{d\phi(A)}{dA} > 0 \tag{4.40}$$

We can derivate once more with respect to A and get

$$\phi"(A) = \frac{d^2\phi(A)}{d^2A} > 0 = \frac{-2c'(A) - Ac"(A)}{\pi_{min}} \tag{4.41}$$

The sign of $-2c'(A) - Ac"(A)$ remains ambiguous but since it is beyond the scope of this work to conduct an exhaustive viability analysis, we will assume for the sake of clarity that $-2c'(A) - Ac"(A) \leq 0$.

Hence

$$\phi"(A) \leq 0 \tag{4.42}$$

From (4.40) and (4.42) we can deduce, in the specific case we restricted our analysis to, that ϕ is increasing and concave. It can easily be verified on Figure 4.4 that choosing a convex form of ϕ does not modify the general range of our results.

Conclusion of Part II: The need for exogenous constraints in sustainable economic analysis

Recovering sustainability against the curse of initial conditions

This second part of our work stemmed from the unsatisfying assessment of the pollution control discounted optimization carried out in Part I. All through Chapter 3 and 4, we have striven to suggest alternative methods to address the "unsustainability traps" that may arise in the discounted optimal paths. The first set of alternative criteria tested in Chapter 3 has revealed interesting ways to reestablish intergenerational equity and to reach pollution recommendations that are more in line with an intuitive idea of sustainable pollution control policies. Nevertheless we have pointed out in the conclusion of that chapter the shortcomings of these tools that account in part for the continuing predominance of discounted cost-benefit analysis in the economics discipline landscape. Exploring even further the analogy between assimilative capacity and a renewable resource, we have managed to build an adequate framework to apply viable control to our "pollution-turned-resource" problem in Chapter 4. This original approach proved particularly promising to found sustainable policies. In addition to its various assets described in the conclusion of Chapter 4, it displays a very attractive property regarding initial conditions. Indeed this framework frees the economy from the "curse of initial conditions" that we consider as the most serious obstacle between dynamic cost-benefit analysis and sustainability, along with the discount rate.

As shown in Chapter 4 (Section 4), we can escape through viable control the "tyranny" of a unique optimal path, and more importantly the doom outcome linked with low initial conditions. This significant breach of sustainability requirements stressed in Part I is nowhere to be found in a viable framework. If the initial conditions are so low that they either violate the viability constraints or that they place the economy outside the viability kernel, the conclusion in terms of policy is not to conduct an "optimal extinction" of the resource, but to consider the most efficient and most rapid recovery paths. It is thus of crucial interest to explore in deeper details the perspective of recovery paths in such a framework and to illustrate them with relevant numerical simulations.

Exogenous constraints vs autonomous normative production

It could be argued against viability analysis and constrained cost-benefit analysis, the two options we favor at the end of this exploration throughout environmental economics, that setting up exogenous thresholds can pose serious theoretical and political problems. Although it is obviously way beyond the scope of this paper to address the debate on the normative autonomy of economics, we believe that the analysis demands to take at clear stance on this issue at one point. It is indeed impossible to adopt a coherent position on the issue we have been tackling all along without making explicit one's acceptance or refusal of exogenous norms that can constrain economic analysis. As mentioned in Chapter 3, some economists tend to mistrust any kind of exogenous norm whether it is political or ecological. According to them the core of economic thinking rests on revealed preferences and the latter are the sole indicator for allocating resources in an economy. We expressed earlier our disagreement with this point of view and the exploration process lead in the first two parts of this dissertation have strongly corroborated our stance. Exogenous constraints are indispensable to found sustainable policies and the mere essence of an economic analysis of environmental problems consists in acknowledging the existence of such constraints.

The way these limits are determined must of course be scrutinized to ensure both democratic processes and expertise but environmental economics can no longer pretend, especially in presence of potential disasters such as climate change, to emancipate from any external constraint. Recommendations from economists are of course never translated literally, if at all, into policymaking. They are just one element of a larger decision making framework including lobbies of any kind, electoral pressure, hard science expertise, public opinion and ideological values. As such the choices they support are *a priori* subject to external political factors. At the end of the day economics are thus always subject to exogenous norms and should not express reluctance to integrate them *ex ante* as they will eventually bite *ex post*. And, more importantly, the so-called "hard science" constraints that can serve as exogenous constraints are endogenously determined by the economic system we live in. The recent shift[1] of the "official position" of the government of the United States regarding the threat of greenhouse gases is quite a vocal example of the complex imbrication between the economy and the environment that is analyzed by Godard (2009) in an *"entangled hierarchy"* framework.

This idea has been discussed regarding the general paradigm organizing economics as a discipline and it can be worthy to conclude on this generalization. The need to finally acknowledge exogenous biophysical or social constraints can be explained by the necessary reversal of hierarchical dependency between the biosphere global system and the economic subsystem called upon by Passet (1979) from a system theory point of view. It has also been formulated by Polanyi (1976) who recommends a shift from *formal economics*, disconnected from biophysical realities, to *substantial economics* acknowledging that the survival of humankind depends on the survival of the biosphere in general. This idea of *substantial economics* can be associated with the *existence theorem* evoked by Pearce and Turner (1990) that puts the survival of mankind as the fundamental *ex ante* constraint on any economic analysis. Conventional economics have developed their theoretical paradigm taking the viability of the economic system for granted. As noted by Barbier (1990): *"the emphasis on irreversible environmental degradation and the possibility of ecological collapse, [...] resurrects the notion of absolute natural resource scarcity, which seemed to have been so successfully buried by the classical and early neo-classical theorists"*. It is exactly this concept of "absolute

[1] van Kote, G., "Les Etats-Unis reconnaissent la dangerosité des gaz à effet de serre", *Le Monde*, April 18th 2009.

scarcity that we have been trying to shed light on with the explicit integration of assimilative capacity into our pollution models. Therefore our support of a viable or a constrained approach of pollution problems is not at all disturbed by the requirement of exogenous social and ecological constraints. We shall thus conclude this argument with a quote[1] from Malinvaud (1985) that recalls this need of economics to accept exogenous constraints:

"In so far as it is a positive, that is explanatory science, economics must analyze the behavior of agents who enjoy some freedom but are subject to the constraints imposed on them by nature and institutions."

Some conclusive thoughts on the operationalization of sustainable development

In this second part of our dissertation we tried to cast a fresh look on pollution control considered from an economic viewpoint. This attempt to shift from standard pollution economic tools to a renewable resource management framework was based on our driving concern to make more explicit the finite dimension of assimilative capacity. Despite some necessary simplifications, this change of perspective has allowed us to apply various resource management methods ranging from the maximum sustainable yield indicator to viable control, that have contributed to this explicit acknowledgement of the depletion threatening this crucial sink function. The implications of this shift of framework in concrete terms of policy consists also in making more explicit, through “harvesting” quotas, the exhaustible property of assimilative capacity and the intertemporal consequences that are at stake even in flow pollution problems. From the renewable resource angle, it is easier to make a case for sustainable policies that demand the preservation of (environmental) capacities. What’s more this new standpoint allows the use of concepts and terms that may find more echo among the non-economist experts involved in a decision-making process. Experts belonging to biological or ecological disciplines will be more at ease to discuss the conditions of exploitation and regeneration of a natural resource than the concept of optimal pollution.

[1] I wish to thank Olivier Godard for pointing out this quotation.

More generally, the conclusions of this chapter argue in favor of a definition of sustainability that guarantees the preservation of capacities and that acknowledges the finite dimension of many environmental assets. However environmental concerns need to be completed with explicit recognition of social and economic thresholds in a comprehensive definition of sustainability. That is why we judge the viability framework as a promising setting to design sustainable policies.

Part III

Natural capital revisited

Chapter 5

Endogenous depreciation of natural capital

5.1 Introduction

As a last consistent step of our analysis, we replace the sustainability issues at stake into the context of economic development and natural capital. The purpose of this conclusive chapter[1] is twofold. On the one hand we broaden the range of our economic analysis through the introduction of capital accumulation in a discounted optimization framework. In doing so, we palliate one of the limit of the optimal pollution control models developed in Part I and we can study the trade-offs at work between investment in *economic capacities* and maintenance of *environmental capacities*. On the other hand we build on the many insights gathered throughout the four previous chapters on the status and the dynamics of various environmental services to suggest a bottom-up approach that characterizes formally the endogenous depreciation of natural capital. The convergence of these two driving concerns result in the proposition of a simple economic model with both physical and natural capital. In its most simple phenomenological form, this model contributes to a more complete exposition of the environmental degradation cycles analyzed in Chapters 1 and 4 as we identify alternative natural capital management strategies under a survival constraint. Replaced in a standard discounted optimization framework, we shed some light on the interesting

[1]The stylized model developed in Section 4 has been inspired by an ongoing joint work with Robert Cairns and Vincent Martinet on land use and dynamic ecological productivity.

trade-offs between consumption, investment in physical capital and maintenance of natural capital. Our driving approach of sustainability as the preservation of capacities is partially challenged by the explicit introduction of economic capacities into this last model. When the model if contextualized in the case of a developing countries with low initial manufactured capital endowments and little technology, the option to overexploit natural capital in order to feed physical capital accumulation reveals less "unsustainable" that it seems. Under a binding survival constraint, preserving at all cost natural capital could indeed drive the economy into severe poverty traps. Our analysis will thus be mostly dedicated to the cases of economies with low initial physical capital.

5.1.1 Natural capital: adapting an ambiguous concept

Our constant concern to acknowledge explicitly the ecosystem services contributing to social welfare, initiated with the study of pollution assimilative capacity, leads us quite naturally to turn to the notion of natural capital to capture the economic role of the environment on an aggregated level. However, the conceptual leap consisting in putting together all environmental goods and services together and placing them alongside physical capital in a neoclassical production function is not as easy as writing down $F(K, N)$ instead of $F(K)$. On the contrary, the attempt to fit "Nature" into a category of capital raises, as could be expected, various theoretical and empirical difficulties. This task is all the more complex as the standard notion of capital itself is not as stabilized as macroeconomic textbooks let us believe. Any rigorous attempt to define, measure or analyze formally the so-called natural capital must cope with contradictory views on the original concept of capital. It is beyond the scope of this paper to review exhaustively the numerous debates that are symptomatical of the fuzzyness that surrounds the notion of capital. Let us just recall how this ambiguity is embodied in the seminal debate between the *fundist* and the *materialist* approach (Hicks, 1946). The former infers the aggregated "stock" of capital as a proxy for the sum of all the future valuable services yielded by the productive structures, in a process analogous to financial assets. The latter favors a physical count of concrete heterogenous assets despite obvious aggregation obstacles. These obstacle can be partly overcome with a monetary valuation based on the purchase cost of each asset[1].

[1]Theoretically, the fundist and the materialist value of capital should be equated in an economy at the optimum but this is clearly not the case in real economies.

5.1.2 The definition of natural capital: *Fundists* vs. *Materialists*

Beyond the (too?) general definition of natural capital as *"the range of functions the natural environment provides for humans and for itself"* (Ekins et al., 2003), the fundist-materialist dichotomy is clearly reflected in the competing definitions of natural capital. Interestingly enough, both definitions imply a specific organic definition of sustainability as we will see shortly. A "fundist" approach of natural capital focuses on the value of the flows of environmental goods of services provided periodically and valued through market prices (timber, fish, etc.) and non-market prices (environmental amenities valued through contingent valuation, waste assimilation valued through substitution costs, etc.). A "materialist" approach of natural capital consists in the recognition of all the heterogenous environmental assets available gathered in a physical stock. This "inventory" can hardly be aggregated since no purchase costs can be used to value these "free" assets. This conception of natural capital corresponds to a "patrimonial" vision that has been translated operationally into *satellites accounts* added to standard national accounting indicators (see for instance Lintott, 1996). We will not develop into more details the "materialist" approach as by definition it is a topic of ecological, biophysical and geological studies rather than a relevant subject of economic analysis. On the other hand, we will develop our own "intuitive" approach of the *fundist* definition of natural capital based on the insights on the economic status of environmental services we collected throughout this work.

5.1.3 Organic conceptions of sustainability

Nonetheless we shall compare briefly the conceptions of sustainability that stem from these two definitions of natural capital. From the materialistic view of natural capital arises quite intuitively a strong sustainability conception (see Chapter 1) that requires the preservation of all natural assets[1]. Conversely, a fundist approach leads naturally to a weak sustainability definition that allows for a degree of substitution between manufactured and physical capital. Indeed, the focus on the value of the flows of services produced contributes to merging more easily all the sources of services. A

[1]In some weaker definitions, strong sustainability requires the conservation of critical assets that are hard but not impossible to assess (see the CRITINC project in Ekins et al., 2003).

constant level of utility can thus be more easily guaranteed, no matter which capital provided the services. Although weak sustainability seems more intuitive from a purely economic point of view, the considerable uncertainty that characterizes natural capital dynamics is a serious argument in favor of strong sustainability. This uncertainty should inspire great caution at the time of substituting partially a fragile natural asset with manufactured capital. This reasonable caution is also reinforced by the irreversibility of the depletion of certain forms of natural capital (Dietz and Neumayer, 2007). Our personal concern for sustainability through the preservation of environmental capacities can be interpreted at this stage of our analysis as a middle way between these two definitions. We favor a valuation of ecosystems through the economic value of the services they provide while requiring nonetheless the preservation of environmental capacities, e.g., the maintenance of a positive "stock" of environmental assets.

5.1.4 Outline : a twofold approach of natural capital with a focus on development strategies

The contribution of this conclusive chapter is twofold. The two main objects of study belong to the same field but they are independent nonetheless. In a first preliminary part we carry out our personal exploration (Section 2) of natural capital from a fundist perspective, addressing in particular the operational estimation of natural capital variations. This conceptual contribution has no direct impact on the formalized model that it precedes. In a second part we generalize the original dynamics proposed earlier for the formal analysis of assimilative capacity into a neoclassical growth model with both manufactured and natural capital as production factors. It is important to note that in this second part of our analysis we shall not wake up the ghosts of the Cambridge-Cambridge Controversy (Stiglitz, 1974) on the nature of capital and we will simply adopt the common implicit postulate of this kind of models consisting in treating capital stocks as actual concrete physical stocks yielding physical flows, just as the Dasgupta-Solow-Stiglitz model do (Dasgupta and Heal, 1974; Solow, 1974; Stiglitz; 1974) when they introduce the flow of exhaustible resource entering the production function. Our approach will build (Section 3) on the capital utilization literature that developed a promising formalization of endogenous depreciation of capital. This simple model (Section 4) completes our previous studies that did not include capital accumulation (with the exception of our viability approach in Chapter 4). As mentioned in the General Introduction we will focus first on economies with low initial endowments of capital that are at the edge of environmental degradation cycles. Before turning to optimal trajectories we will compare (Section 5) two development strategies: one applying "immediate sustainability" on the environment while the other degrades the stock of natural capital through excessive utilization in order to reinvest the additional output in manufactured capital. We will show that the second strategy can actually drive the economy away from future environmental degradation cycles whereas the first one is bound to provoke "green poverty traps". In Section 6 we shall replace the problem in a dynamic optimization framework to shed some light on meaningful necessary conditions for optimal paths to be sustainable. Section 7 concludes this chapter with open questions on the adequacy of the discounted optimization framework to yield sustainable decision rules. This last step consistently completes our long-haul sustainability study driven by the concern for the preservation of capacities as we will extend the concept of capacities to productive capacities through the introduction of manufactured capital.

5.2 A contribution to the definition of natural capital and natural capital dynamics

In this section we explore the fundist definition of natural capital in the wake of our personal insights on the status of environmental services in economic analysis. This conceptual description aims at enriching the understanding of the issues at stake when natural capital is involved and of the available tools for empirical monitoring of natural capital. However we do not advocate the systematic use of this approach, or of the subsequent weak sustainability definition, for the design of sustainable policies. This dimension will be dealt with independently in Section 4.

5.2.1 The basic approach to natural capital

Before we advance any further in our study of natural capital, it is necessary to gather the illustrations of natural capital that are sprinkled around in this work into a systematic definition. So far we have identified as natural capital heterogenous assets and environmental functions such as fish and forest populations, pollution assimilative capacity, erosion protection, soil fertility, fresh water sources, recreational amenities, biodiversity conservation, etc. A common definition given to natural capital is the set of all environmental assets. It is frequently declined into three categories: environmental services, resource uptake and waste disposal (England, 1998). Sometimes the surface of actual land available is also considered as part of natural capital (Azqueta and Sotelsek, 2007), however it can be argued that without land no activity at all would be possible and as such land consists more in an existence condition rather than in actual capital (but the soil of the land is definitely a part of natural capital). In the rest of our analysis, we will use the term "environmental services" or "ecosystem services" to embrace all forms of production of services and goods that are either used as input material for production or that are valued by society, whether it is a quantity of fish that can be harvested, a water purification function or the existence of blue whales. Finally we are convinced that exhaustible resources should be banished from the category of natural capital to respect the consistency of the concept of *capital*. The rationale for this exclusion can be found in Appendix A. We shall thus restrict our scope to renewable natural capital (El Serafy, 1989).

5.2.2 Developing the fundist definition of natural capital

The choice of envisioning nature through the prism of capital, an economic concept usually applied to man-made machines, implies an approach of the environment determined by human needs. If they do not provide direct or indirect services (such as biodiversity conservation or even existence value), natural elements (such as landscapes) and living organisms[1] are not a part of natural capital. The difficulty to identify exactly which ecosystem or which unit of an ecosystem provides which service or resource is quite blatant. It seems very complex to categorize all the natural

[1] A beautiful landscape in a place for ever unreachable to humans and ugly deep sea creatures with no trophic relation to "useful" ecosystems can somehow be seen as a caricatural examples of non natural capital.

forms, living or not, on the planet that provide a flow or a service to society and even more to aggregate all of them into one stock as it is done with manufactured capital. This aggregation into a single stock is all the more problematic as the *"flow of services from ecosystems requires that they function as whole systems"* (England, 2000). Consequently it is the structure of the ecosystems that determine the value of natural capital and a continuous stock cannot represent adequately the threshold effects that are likely to take place if the structure of the ecosystem is affected[1].

However the concept of natural capital can be apprehended from a reverse angle. The functional definition of capital that applies to any economic or financial asset is "a stock that yields a flow of goods or services into the future". Although we do not know (and we might never know) how each ecological unit on the planet contributes to human wellbeing, we have begun to realize and to measure the extent of the ecosystem services we can benefit from[2] in a sustainable way. Instead of a strenuous attempt to identify and evaluate exhaustively all concrete natural elements contributing to society's wellbeing, natural capital can be defined as a theoretical quantity, an unobservable artefact, that is inferred from the level of environmental functions available at a rate of exploitation that does not alter them. If we build natural capital as a virtual matrix of the economic valuable goods and services we can receive from nature indefinitely, we can make it a much more accurate and operational concept. Let us deduce the level of natural capital from the current level of periodic ecosystem services rather than trying to evaluate *ex post* these services from an endless list of imbricated natural elements. What we wish to capture here is an unobservable global stock that is positively correlated to the total flows of valuable environmental services. For operational purposes, the absolute level of this stock does not need to be established, as it is its variations that are of interest. In this sense, our approach of natural capital shares common ground with the Gross Domestic Product (GDP) index, although we insist that in our case it would be vain to seek to determine an absolute value for the (virtual) stock of natural capital. As it is the case for GDP, estimating the absolute levels of the stock of natural capital does not matter much.

[1]The concept of resilience (Holling, 1973) could prove useful here to study the level of services provided by the ecosystem at its new equilibrium after an ecological flip. The analysis of the nutrient assimilation function of shallow lakes (Mäler et al., 2003) that is mentioned in Chapter 2 is a vocal illustration of these jumps in the level of natural capital.

[2]Environmental functions such as fresh water supply or soil fertility have been acknowledged and enjoyed for thousand of years but it is only recently that we have begun to appreciate the climate regulation service provided jointly by our atmosphere and biosphere.

The significant index that can contribute to orient sustainable policies and test the efficiency of economic tools is the variation of this stock of natural capital.

Let us illustrate our approach with an example. Halieutic resources constitute natural capital inasmuch as they provide each year a flow of services (namely a productive industrial/alimentary input: fish). The economic value of this form of capital should not be based on the total fish population at a given time valued at its current market price. It should rather be a computation of the flows of "halieutic services" that can be enjoyed in the future. Obviously the level of these future flows are affected by the current regime of management: excessive harvest will diminish the future available flows and, *ceteris paribus* the stock of virtual natural capital. We will address this issue below, making explicit how the present conception of natural capital contains *ex ante* the notion of a sustainable use of resources. It must be noted that we came to realize after having built up this personal approach of natural capital that it relates closely to the definition of sustainability given by Hueting and Reijnders (1998) that consists in a *"use of the vital functions (possible uses) of our biophysical surroundings in such a way that they remain indefinitely available"*.

5.2.2.1 The ambiguous impact of technical change

The present suggestion of this approach to capital natural displays nevertheless a few shortcomings. This definition contains in its premises the idea of a viable use of natural resources and as such it requires a significant amount of information on the actual ecological level of sustainable use of a variety of environmental assets. More importantly, technical progress fits ambiguously into this approach. Innovations that diminish the pressure on the environment for a given level of production increase the level of potential environmental services that can be provided indefinitely. "Green" innovations that increase the energy efficiency or reduce the waste generated by a productive process do increase the value of the flow of future services (not the physical amount of the flows themselves) as they allow to produce/consume more for the same level of environmental pressure. This kind of technical change stimulates the demand for sustainable environmental services and causes their value to rise. According to our definition, the stock of natural capital rises also.

Conversely, technical change improving the substitution technologies that palliate the need for environmental services have an opposite effect on the stock of natural capital. Indeed cheap access to artificial solutions to absorb pollution, sequestrate carbon or hinder soil erosion induces a fall of the value of the original environmental services for lack of demand. When they are out-competed, environmental services lose their (scarcity) value and this triggers a mechanic drop of the stock of natural capital. An extreme outcome of this process would be a world where cheap artificial substitutes have replaced all environmental functions, inducing a value of natural capital equal to zero even if many environmental "assets" are still present. This caricatural hypothesis is immediately contradicted by the fact that many environmental functions are not technologically substitutable, or that their substitution would be so expensive, even if the technology exist, that there is no economic inventive to develop it on a large scale original environmental services[1]. In particular biodiversity conservation[2], climate regulation[3], or recreational and aesthetical amenities can be hardly replaced by artificial counterparts. This ambiguous impact of the two strains of technical change has naturally consequences on the empirical estimation techniques if substitution costs methods are used to value environmental services. Future work on the compatibility between our approach of natural capital and technical change is clearly needed but would be beyond the modest scope of this work. The insights of Hueting and Reijnders (1998) on environmental functions seem to offer a useful framework to fit better technical change into the next steps of our analysis of natural capital in future work.

We will nevertheless choose to ignore technical change in our formal analysis, in part to avoid additional technical difficulties in a two state variable optimal control model, but also to keep our focus on a pessimistic scenario. Far from being convinced by the technological optimism often brandished as the ultimate instrument against climate change by a school of environmental economists (Nordhaus, 2006), we wish to keep the focus of our model on the most pressing development problems involving environmental degradation cycles for lack of manufactured, technical or human capital. Counting on environmentally oriented technical change when it comes to

[1]The economic advantage of the Catskills water management scheme over an artificial purification system mentioned in Chapter 1 epitomizes this economic feasibility issue.

[2]Although the recent launch of the Svalbard Global Seed Vault in Norway seems to pave the way for efficient artificial techniques of biodiversity conservation.

[3]As mentioned in the General Introduction, most of the very vocal geo-engineering projects have failed so far.

studying a poor country's development options does not seem judicious. In addition, the technical innovations developing countries have benefited from have more often than not accrued the pressure exerted on their natural capital and disturbed ecological equilibria (cf. the example of Sahel desertification in the General Introduction).

5.2.3 From environmental degradation to natural capital depreciation

5.2.3.1 A rigorous use of "depreciation"

This exploration of the fundist conception of natural capital can prove useful to encompass the biggest concern regarding natural capital: its depreciation. The term depreciation must be understood exactly in the same sense as when it is applied to manufactured capital: a decrease in the capacity to yield flows of services with a positive economic value. The depreciation of natural capital must thus be derived from an observed decrease in the ecosystem services yielded. According to our definition, this depreciation does not necessarily follow the physical degradation of natural capital. The occurrence and the extent of natural capital depreciation must be inferred from the actual diminution of an environmental function, whether or not it is due to human intervention. For instance let us imagine that a large part or the population of bees disappears suddenly in an agricultural zone where pollination plays a crucial role in economic activity. If this disappearance does not affect the usual course of pollination[1], then the level of natural capital remains the same. If there is a reduction in the pollination service, then the virtual natural capital level has suffered a depreciation. In this example this depreciation can be easily computed using future (discounted) flows of substitution costs[2]. It is thus entirely legitimate to speak of natural capital depreciation rather than of degradation. However it is obvious that a physical degradation will often entail economic depreciation and that this economic depreciation will often be due to a physical degradation, but as our example showed it is not systematic.

[1]For the sake of the example we assume that the bee has no other implication in human activities, in particular through trophic chains.

[2]In the United States there exists a market for artificial pollination provided by truckloads of bees delivered to farms.

Our discussion here is made on theoretical grounds and does not favors the systematic use of this specific conception of natural capital in policy-making. Indeed, as we discussed in the Introduction, this conception fails to consider the irreversibility and the uncertainty that surround natural capital dynamics.

5.2.3.2 Endogenous depreciation of natural capital

Even though it consists in the same phenomenon, the depreciation sustained by natural capital does not follow the same pattern as the standard time-dependent depreciation of manufactured capital which we will question afterwards.

It is true that natural capital is affected by an intrinsic natural obsolescence as illustrated by the decline of photosynthesis productivity of primary forests for instance. However tt can be legitimately argued that this long term obsolescence is negligible[1] compared to the short term depreciation caused by anthropogenic overexploitation of environmental services and resources. Ecological evidence tends to indicate that *without human intervention*, environmental capital displays a *"substantial degree of constancy or even increase"* (England, 2000) rather than linearly declining. For instance in a "state of nature" new species outnumber, or at least compensate, the number of naturally extinct species (Raup, 1986). The depreciation of natural capital is thus an endogenous process that depends mainly on the intensity of use of the environmental functions available. We have shown this for the assimilative capacity function in the first two parts of this work. Emitting no pollution at all or pollution within the bounds of the current assimilative capacity (in terms of stock or flow) does not affect the latter. Conversely, a level of pollution beyond the assimilative capacity will trigger a reduction of this capacity. In terms of natural capital, this means that if the rate of utilization of natural capital respects a threshold, no depreciation will occur as the environmental function will be unaffected. If it exceeds this threshold, then there will be an economic loss in terms of ecosystem services and a subsequent depreciation of natural capital.

As we will show in Section 2, this mechanism of endogenous depreciation can be found in many forms of natural capital: renewable resource harvest, soil productivity,

[1]The only relevant natural obsolescence that seems to come into play in the time scale of human society is the carbon sequestration cycle of forests. The management of forests as carbon sinks requires indeed optimal planning policies (Schubert and Ragot, 2008).

fresh water reserves, etc. We can thus generalize it to the concept of natural capital as a whole. Endogenous depreciation of natural capital in an aggregated model means that the depreciation rate sustained by the natural capital stock will be an increasing function of the rate of utilization of this capital. Such a generalization inevitably incurs simplifications and we are aware that not all legitimate forms of natural capital will follow such a pattern of depreciation. Our original attempt to unify a wide range of natural capital depreciation phenomena provides nevertheless a simple analytical tool to highlight fundamental trade-offs in a balanced physical-natural capital growth model.

Our modeling proposal of natural capital, and in particular the use of the concept "endogenous depreciation" stems exclusively from the exploration we carried out in the previous chapters of this work that lead us to develop a thorough analysis of on specific environmental function, namely the assimilative capacity. We realized progressively that this idea had been formulated before by authors such as Costanza and Daly (1992)[1] but as far as we know it has never been formalized in a neoclassical growth model, with the exception of Rodriguesa et al. (2005).

5.2.3.3 Measuring natural capital depreciation

The decline of natural capital during the last century has been asserted by empirical observations (Goudie, 1993) based on a physical approach of natural capital (deforestation, erosion, ground water pollution, etc.). We have all the reasons to believe that this physical degradation has also been accompanied by economic depreciation of natural capital. In order to assess the environmental sustainability of an economic path, natural capital variations, net of exogenous shocks such as the discovery of a crucial environmental function such as climate regulation, can prove to be useful indicators. It is fundamental here to focus on the variations to evaluate the properties of a path and to abandon any attempt to measure absolute levels of capital. Empirical measures of these "amounts" of natural capital (Dixon and Hamilton, 1996) face many conceptual and practical obstacles. In particular, if mineral reserves and animal populations can be inventoried roughly without too many difficulties, the environmental services such as waste assimilation or erosion protection are much harder to monitor.

[1]Costanza and Daly assert that the *"excessive harvest of ecosystem good can reduce renewable natural capital's ability to produce services and to maintain itself"*.

The literature on Environmental Accounting (UN, 1993; Lange, 2004) and Genuine Savings (Hamilton, 1994) acknowledges the numerous pitfalls that surround natural capital estimations such as double counting or lack of valuation methods for non-use values. Attempts at a gross estimation of the planet's total natural capital such as Costanza et al. (1997) have raised tremendous controversies but have not provided any added value for the operationalization of sustainability[1]. As for today, achieving an actual aggregated measurement of the stock of natural capital remains highly unlikely (England, 1998). But it is the variations of natural capital across time that matter, not its absolute level. For instance, Pearce and Atkinson (1993) show[2] from natural and manufactured capital depreciation and investment data that among 18 countries monitored, only 8 displayed non-declining stocks of total capital. Applied empirical estimations are obviously needed to test how operational our approach of natural capital proves to be, they will be the subject of immediate extensions of this dissertation.

5.2.4 Restoration of natural capital?

Since our personal bottom-up approach of natural capital through environmental functions was initiated and largely built on the arguments developed for the special case of pollution assimilative capacity, we could extend the process to ponder over the possibility of natural capital restoration. Assimilative capacity restoration has indeed been widely discussed throughout this work as an additional tool that can considerably improve the sustained level of welfare under given economic conditions. It seems only natural to explore the generalization of our various local examples of environmental restoration to the aggregate level of natural capital: soil fertility, waste assimilation, carbon sequestration, etc. The natural and artificial methods to achieve the restoration of environmental functions have bloomed over the past decades in the filed of ecological engineering. A vast panorama of the local applications is described in Aronson et al. (1997). In terms of an estimation of natural capital as an artefact reflecting the current level of sustainable use of environmental functions, restoration implies an increase in this sustainable level and, as such, entails a rise in the natural

[1]The extremely large number they attribute to the total ecosystem value of the planet can be simply seen as a lower bound approximation of the infinite value of the survival of the human species

[2]They ignore population growth and technological change in their analysis.

capital value. But since some forms of natural capital do not exhibit this ability to be restored and given that our model will not allow for critical levels of natural capital, it seems more reasonable to adopt a conservative view of natural capital evolution for now. We shall thus keep this restoration option as a promising lead for future works[1] and restrict in this initial analysis the dynamics of natural capital to the irreversible degradation or constance of capital.

5.3 Natural capital utilization and depreciation

Our search for a way to formalize on an aggregated level the dynamic feature common to a broad range of environmental assets amounted to translate the fact that the degradation of natural capital depended on the intensity at which it was exploited. Comparing this rather straightforward proposition to the standard representation of physical capital depreciation, we came to realize two important elements regarding the status of capital in economic theory.

First of all, as noted above it would make no sense to apply to natural capital a linear constant depreciation rate such as the one systematically applied to physical capital in every growth model. There is indeed no reason to believe that ecological equilibria would inevitably decline to a lower functioning level just under the action of time when no anthropogenic perturbation is present. On the contrary, it can be assumed that an ecosystem safe from any human intervention or exogenous shock (earthquake, hurricane, pest, etc.) would deliver a constant level of environmental functions. The constant depreciation rate δ must be refined to account for these dynamics.

This immediate assessment raises some questions on the consistency of the constant depreciation rate when it comes to physical capital itself. It would be indeed quite naive to accept this depreciation rate as a realist representation of the way physical capital depreciates. This constant rate does capture the part of depreciation that is due to *obsolescence* but implicitly assumes that the rate of utilization of capital is the same at each period. In doing so, it ignores away the variations of utilization that might characterize the use of a given capital, at the local or at the aggregated level.

[1]A description of the formalization of restoration mechanisms at stake is given in Appendix B.

Based on this suspicion, we have explored the relevant literature and in particular a sub-branch of capital theory that addresses precisely this point. We find it worth of interest to disclose the main features of the capital utilization literature that open promising leads to improve the modeling of natural capital.

We must recall that as far as our stylized model is concerned, we will retain the general implicit assumption in this kind of models that consider both forms of capital as concrete aggregated stocks of "productive structures". In the rest of our analysis $K(t)$ and $N(t)$ will denote respectively the stock of manufactured and natural capital at time t.

5.3.1 Capital utilization theory

The development of a specific body of economic theory on capital utilization[1] since the seminal work of Keynes (1936) on "user cost" reflects an effort to go beyond the implicit assumption running all throughout the economic literature on factor demand theory and growth models that identifies the current stock of capital and the capital service drawn from this stock at the period. Standard economic models simplify away the range of utilization of capital by firms and the possibility to vary the intensity of the capital services as inputs in the production process. In doing so, they ignore the firms' capacity to modulate the rate of use of its physical facilities through different working shifts or overtime work. This strong assumption leads in turn to a second implicit *ex ante* decision on the process of depreciation of capital. Indeed, it seems only natural to relate the rate of physical depreciation of capital to the intensity of actual capital utilization, beyond time-dependent obsolescence. If this capital utilization, commonly defined as the ratio of capital services to the stock of capital (Chatterjee, 2005), is exogenously set once and for all to one, e.g., the stock of capital is always utilized at its maximum capacity, the endogenous depreciation of capital can no longer be reflected in the model. Hence the systematic use of an exogenous fixed depreciation rate of capital δ, such that the total depreciation sustained by the stock of capital at time t is equal to $\delta K(t)$. This rate does not account for the capital utilization choices of the firms. This can explain a part of the discrepancy between the theoretical predictions and empirical observations on maintenance investment (Licandro et al.,

[1] See the survey by Winston (1974) and the review in the recent work of Chatterjee (2005).

2001) as well as on the rate of convergence of growth (Chatterjee, 2005; Dalgaard and Hansen, 2005).

The literature on capital utilization attempts to reinstate the rate of capital utilization as a significant decision variable for the firm and subsequently to endogenize the rate of capital depreciation in dynamic models. These contributions have been completed and illustrated by empirical analysis focusing on the management of working shifts by industrial firms. Their results tend to confirm the role played by capital utilization and its consequences in terms of maintenance investment in the intertemporal programs of managers.

The introduction of capital utilization rate has proved particularly useful in the real business cycle literature (Greenwood et al., 1988; Burnside and Eichenbaum, 1996). It has also contributed to explain the absence of convergence observed empirically among countries with similar endowment of capital due to different rates of utilization (Aznar-Marquez and Ruiz-Tamarit, 2001) and the smaller growth rates of developing countries due to suboptimal workshifts allocation of their capital (Kim and Winston, 1974; Winston, 1971). It also plays an important role in the distortion range of benefit tax net of depreciation (Zhu, 1995) and in the measure of comparative advantages in international trade (Betancourt et al., 1985) as these advantages depend partially on the international differences in the willingness of workers to engage in shift-work.

5.3.2 Extending capital utilization to natural capital: maximum sustainable utilization rate and endogenous depreciation

The crucial concern driving capital utilization theory is the possibility of a variation in the flow of capital services drawn from a stock of capital, and more generally the introduction of a "degree of liberty" between this flow and the stock it stems from. This intensity variable determines in turn the physical depreciation of the stock of capital such that at time t the depreciation rate $\delta(u(t))$ is an increasing function of the rate of utilization $u(t)$ (Calvo, 1975). This specification reflects the faster depreciation of the capital stock due to longer work-hours of capital (Zhu, 1995).

This range of decision regarding the intensity of capital utilization is of great interest when it comes to analyzing natural capital. As noted previously, the actual flows of natural capital services stemming from the overall stock of natural capital depends very much on the rate of exploitation/utilization of this stock. In this context, identifying these flows with the stock of natural capital itself would thus result even more misleading than it is for manufactured capital.

Subsequently, the extinction of renewable resource stocks due to unsustainable harvest rates, the feedbacks of pollution accumulation on the assimilative capacity of the environment[1] and the degradation of soil productivity due to intensive agriculture (Barbier, 1990) illustrate, each in their domain, the impact of the rate of utilization on the evolution of the natural capital stock. This impact is all the more decisive on the dynamics of the capital stock as they are very little opportunities to "invest" in natural capital and thus to offset this degradation as can be done with manufactured capital. These "stock externalities" have been of course largely addressed in their specific branch of the literature but to our knowledge they have never been articulated in a general model of natural capital utilization. Capital utilization theory, in our opinion, offers a very fitting framework to unify all these mechanisms as expressions of *endogenous depreciation of natural capital*. The motivation of the present contribution is to draw on this body of literature in order to sketch a theoretical model of multiple factor production that accounts for these predominant features of natural capital, allowing for a wide latitude in the utilization of this capital and, consequently, for an endogenous rate of depreciation.

5.3.3 Maximum Sustainable Utilization Rate

The concept of natural capital provides an interesting field of extension for capital utilization theory inasmuch as it requires the expansion of the endogenous depreciation function in two directions. This endogenous depreciation function needs indeed to be built around a "maximum sustainable utilization" threshold. It must be reminded that given the real processes at work reflected in this rate of physical capital utilization, it is commonly accepted in the standard manufactured capital utilization theory that the

[1] At the local scale such feedbacks can be found in the eutrophication processes of shallow lakes (Mäler et al., 2003). At a global scale, they can be observed in the degradation of the carbon sinks of the biosphere due to the contribution of greenhouse gases emissions to oceans' acidification.

depreciation rate $\delta(u(t))$ is strictly positive for any strictly positive rate of utilization[1]. There is thus no range of utilization that would not entail a depreciation of the stock of capital while allowing capital services to be used in the production process. On the contrary, whatever forms it takes, natural capital is systematically characterized by a maximum sustainable level of exploitation, that is to say a rate of exploitation of the capital services that leaves the stock of natural capital untouched from one period to another[2]. From now on we shall note $v(t)$ the rate of utilization of natural capital at time t.

In the case of renewable resources harvesting[3] this "maximum sustainable utilization rate" (MSUR), noted $\tilde{v}$ is tantamount to the maximum sustainable yield. We will assume throughout this paper that given the actual ecological conditions we are currently facing, only the "left part" of the logistic function (see Figure 4.7) commonly used to represent renewable resources natural regeneration will be considered. We believe indeed that nowadays it is a little too optimistic to show concern for a decrease in renewable resources productivity due to very high stocks of resources (except maybe for cases such as wild boars but for such a population our resource exploitation problem would fall under the category of pest regulation).

As far as pollution control is concerned, this MSUR corresponds to the amount of pollutant the environment is able to absorb and process naturally without being impaired and diminished at the next period. Regarding soil productivity, the MSUR can be thought of as the intensity of agricultural practices (use of fertilizers, pesticides, fallow periods, etc.) that conserves the same soil productivity from one harvest to another. Finally the application of MSUR to environmental amenities would require that the utilization of these amenities, in terms of congestion and landscape degradation for instance, leaves the same capital of "amenities" to be enjoyed the next period.

In terms of endogenous depreciation, this translates into the fact that the depreciation of natural capital entailed by a rate of utilization equal to the MSUR is nil. More importantly, any rate of utilization higher that the MSUR causes a strictly pos-

[1] In order to account for the "natural" decay of manufactured capital in most models (Calvo, 1975; Rumbos and Auernheimer, 1997) $\delta(u) > 0$ for all $u \geq 0$.

[2] We have excluded exhaustible resources from natural capital, see Appendix A.

[3] We leave aside non-renewable resources as they can hardly be identified to capital and correspond more accurately to "inventory" in the nature-firm analogy, see Appendix A.

itive depreciation. This MSUR is assumed constant overtime such that the absolute level of environment services available indefinitely decreases as the stock of natural capital decreases.

Once again we shall assume deterministic patterns of natural capital depreciation although we are very aware that uncertainty characterizes many of the ecological processes at work in this degradation of environmental functions. Moreover our formal analysis will be carried out under the assumption that no natural capital restoration is possible. However it must be noted for future work that the framework we develop here is fitted to encompass such a restoration through "negative depreciation". We explore this option in Appendix B following the method used in Chapter 1.

5.4 A model of endogenous depreciation of natural capital

5.4.1 Endogenous depreciation of physical capital in neoclassical growth models

Let us recall the main improvement added by the capital utilization models in neoclassical growth models as found in Calvo (1975) or Rumbos and Auernheimer (1997).

The utilization rate, denoted u, expresses the intensity of use of the actual stock of capital K and is commonly defined as the ratio between capital flow services κ and this capital stock K at time t. Putty-clay models generally admit that the rate of utilization can vary between periods but is constant during one unit of time.

$$u(t) = \frac{\kappa(t)}{K(t)}$$

Given the physical meaning of u, we have $0 \leq u \leq 1$, which means that the exploitation of manufactured capital can never be in "overshooting" with regards to the available stock of capital. We shall modify this domain of definition in our application to natural capital.

In an actual production process it is the flow of capital services κ, and not the stock $K(t)$ that enters the production function along side of other "flow" factors such as L (labor) or R (resource input). Therefore the production function should write

$$Y(t) = F(\kappa(t), L(t), R(t), ...)$$

Using the capital utilization variable $u(t)$ this reads

$$Y(t) = F(u(t)K(t), L(t), R(t), ...)$$

With this notation, we see that the standard models that do not allow for capital utilization implicitly set the rate of utilization to a constant equal to one such that $\kappa(t) = K(t)$ at all time.

Formalization of endogenous depreciation

As a consequence of the introduction of the utilization rate, the depreciation rate of capital is no longer constant but it is an increasing convex function of u. The physical degradation of capital, ignoring obsolescence, is indeed directly related to the "wear and tear" impact of the intensity of utilization. Hence the following depreciation function $\delta(.)$:

$$\delta(u) > 0 \;\; \forall \;\; u > 0$$
$$\delta'(u) > 0 \; ; \;\; \delta"(u) > 0$$

Let us note that in this configuration $\delta(u)$ is always strictly positive, which reflects the fact that physical equipment wears out even if they are not used.

The law of motion of capital in a standard capital utilization growth model thus writes:

$$\dot{K}(t) = F(u(t)K(t), L(t), R(t), ...) - c(t) - \delta(u(t))K(t) \tag{5.1}$$

5.4.2 A growth model with natural capital utilization

We feel that this concept of capital utilization is particularly fitted to account for the use and management of various forms of natural capital. It can be usefully expanded to cover the negative feedbacks of the exploitation rate of environmental assets. We shall denote $v(t)$ the rate of natural capital utilization at time t.

Imitating equation (5.1), the law of motion of natural capital $N(t)$ writes:

$$\dot{N}(t) = -\delta(v)N(t)$$

In our extension, the domain of definition of v needs not be restricted to $[0, 1]$. Indeed the phenomenon of "overexploitation" of a renewable resource (or overshooting pollution beyond the environment's natural assimilative capacity) can be interpreted in terms of capital utilization as an "overutilization" of this capital, which would be tantamount, in the realm of physical capital, to an exploitation of a machine more than seven days a week. We establish thus the following definition set, where $\bar{v}$ is the maximum intensity of overexploitation, such that:

$$v \in [0, \bar{v}]$$

Let us now define $\tilde{v}$ as the maximum sustainable rate of utilization defined previously. Accordingly we have

$$0 < \tilde{v} < \bar{v} \tag{5.2}$$
$$\delta(\tilde{v}) = 0$$
$$\delta(v) > 0 \quad \forall \; v > \tilde{v}$$
$$\delta(v) = 0 \quad \forall \; v < \tilde{v} \tag{5.3}$$

Equation (5.14) reflects the fact that the depreciation rate is nil even for utilization rates strictly lower than the MSUR. We show afterwards how this property can be modified to account for restoration of natural capital. When utilized at its maximum sustainable utilization rate (MSUR) $\tilde{v}$, strictly positive, the stock of natural capital does not depreciate. As noted above, such a "neutral" rate cannot be found in physical capital endogenous depreciation function. The distinctive features of function $\delta(.)$ are illustrated in Figure 5.1. Considering the specification above, we shall write the production function F, assuming that only natural capital utilization is subject to variation to avoid unnecessary complexities, thus setting the utilization rate of physical capital to the fixed value of one and the physical capital depreciation rate to

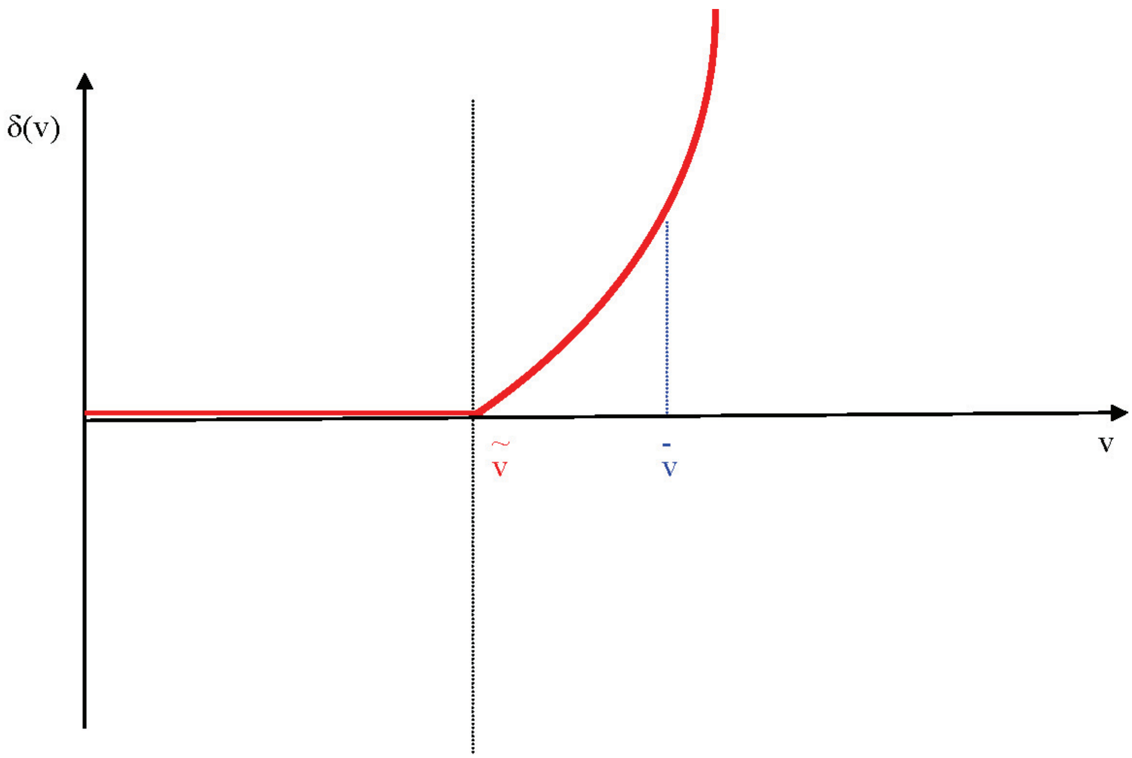

Figure 5.1: Endogenous depreciation function

the constant value b.

$$Y(t) = F(K(t), v(t)N(t))$$
$$\dot{K} = F(K(t), v(t)N(t)) - c(t) - bK(t)$$

where $c(t)$ is the level of consumption at time t and $F(.,.)$ the production function. This production function, homogenous of degree one, respects the standard properties of neoclassical growth models and in particular $F_1 > 0$, $F_2 > 0$, $F_{11} \leq 0$, $F_{22} \leq 0$, $F_{12} > 0$, $F_{21} > 0$ where F_i denotes the derivative of F with respect to the i^th argument of the function. Concretely this standard function is such that the marginal productivity of manufactured capital decreases when the stock of manufactured capital increases and this productivity increases when intensive natural capital (vN) increases. Symmetrically the marginal productivity of natural capital decreases when the stock of intensive natural capital (vN) increases and it increases when manufactured capital increases. Finally both factors are essential to production and $F(K, 0) = F(0, eN) = 0 \forall\ (K, N)$. A simple functional form for this production function would be a Cob-Douglas function such that $F(K, eN) = (K)^{\alpha}(eN)^{\beta}$ where the positive parameters α and β are both inferior to 1.

We suppose that the utility of the representative agent only[1] depends on current consumption such that $U(t) = U(c(t))$. This utility function respects the most standard assumptions of the literature. Population is supposed constant here but we are aware that this is a significant limitation since we intend to address sustainability issues, and more specifically, questions of economic development in less industrialized countries.

5.5 Development strategies under a survivability constraint

The main concern of this second part of the chapter is to observe the characteristics of optimal and sustainable strategies in a framework with both manufactured capital accumulation and natural capital depreciation. We do not aim at elaborating an exhaustive technical analysis but simply to use our phenomenological model designed in the previous section to shed some light on the sustainability issues that have been driving this work. In particular, the threat of environmental degradation cycles should orient our research on the options of development available to the less industrialized countries. As hinted by the "rubberbands model" (Giraud and Loyer, 2005) shown in the General Introduction, the lack of manufactured capital sometimes leads to the overexploitation and the depletion of natural capital. In this section our analysis will focus on situations with low endowments of manufactured capital. Addressing explicitly this kind of problems also demands a pragmatic appraisal of economic trajectories, especially as far as minimum consumption levels are concerned. It seems indeed crucial to introduce a minimum threshold of consumption c_{min} that guarantees the survival of the population. Slight variations of consumptions levels have marginal impacts on well-being in industrialized countries but they can have disastrous human consequences on poor regions. In that case, the marginal utility consumption should be infinite in the vicinity of c_{min}. This does not modify dynamic optimization process as the utility function can be easily redesigned to shift the infinite marginal utility

[1] A natural extension of this model is to introduce natural capital also as an argument of the utility function such that $U(t) = U(c(t), N(t))$.

from 0 (standard Inada condition) to c_{min}. But the absolute necessity to obtain this consumption can significantly reduce the range of development strategies and place these economies on environmental degradation paths.

Let us leave side discounted optimization for now and explore a purely phenomenological model. We consider an economy with a low initial stock of capital K_0 and a given stock of natural capital N_0. The purpose of this subsection is to explore the development strategies available with a survivability constraint on the consumption level outside the framework of discounted optimization. For the sake of clarity we will translate our model into a discrete setting. The productive factors are subject to the following dynamics:

$$K_{t+1} - K_t = F(K_t, v_t N_t) - c_t - bK_t$$
$$N_{t+1} - N_t = -\delta(v_t)N_t$$

It must be noted that in this setting without restoration there is no rational motive for v to be inferior to $\tilde{v}$, we shall thus have

$$\tilde{v} \leq v_t \leq \bar{v} \ \forall \ t$$

Let us assume that at time 0, due to the low capital stock and insufficient technology, we have

$$F(K_0, \tilde{v}N_0) \leq c_{min}$$

Given the current production infrastructures of the economy, the output that can be obtained without depreciating natural capital (we shall refer to this choice of $\tilde{v}$ as non-degrading whereas any $v > \tilde{v}$ will be "degrading") is inferior or just equal to c_{min}. We shall now compare two development strategies under this output limitation and the survivability constraint: a non-degrading strategy and a depreciating one. Intuitively the former should be "sustainable" and the latter "unsustainable".

"Green poverty traps"

If the economy applies a strong "sustainability" criterion that requires natural capital to be non-declining, it will choose $v_t = \tilde{v}$ at all times. Concretely this means that the community will systematically respect the biophysical limits of its environment. If manufactured capital does not depreciate, e.g., if $b = 0$ then both stocks of capital and the level of output will remain constant over time. If this output meets the minimum needs of the population, *ie* $F(K_0, \tilde{v}N_0) = c_{min}$), then the preservation of natural capital and the survival of the population are ensured but the latter will enjoy a very low level of consumption (by definition its minimum). However if manufactured capital does depreciate ($b > 0$), which is the most realistic assumption, except for really basic tools, there will be no output in excess to allocate to capital maintenance since

$$F(K_0, \tilde{v}N_0) - c_0 = 0$$

At the next period manufactured capital will be reduced and the level of potential output without natural capital degradation will decrease to

$$F(K_1, \tilde{v}N_1) = F(K_0 - bK_0, \tilde{v}N_0) < F(K_0, \tilde{v}N_0) = c_{min}$$

This diminution will make it impossible for the community to obtain its minimum level of consumption c_{min} and the survivability condition will be violated. Even if we relax the survivability condition and allow consumption to fall below c_{min} for brief periods of time, it is clear that the depreciation of capital will go on and the economy will be pulled into a "green poverty trap" where environmental capacities are maintained constant but production and consumption fall quickly below the survivable minimum.

"Hartwick strategies"

The other strategy consists in overexploiting natural capital, in order to stimulate manufactured capital accumulation and future output. Choosing $v_1 > \tilde{v}$ increases the output in such a way that it is strictly superior to c_{min}. It is quite straightforward that if the excess output is consumed ($c_0 = F(K_0, v_0N_0) > c_{min}$) then the economy will stumble into a "dirty poverty trap" as both natural capital and manufactured capital will decline under the respective effects of endogenous ($\delta(v_0)$) and lack of maintenance (b). This trap will be "faster" than the above mentioned "green trap" where only manufactured capital declined. However if the excess output gained from exerting extra pressure on the environment (through excessive pollution or overfishing for example) is immediately used to maintain and invest in manufactured capital, it can lead to brighter future. This strategy follows the well known Hartwick rule (1977).

First of all if the decision to overuse natural capital is taken, the rate of utilization $v_0 > \tilde{v}$ must be high enough to respect the following condition

$$N_0 F_2(K_0, v_0 N_0) > bK_0 \tag{5.4}$$

This condition paradoxically requires a high enough rate of overexploitation. If this condition is not respected then the extra output would be entirely allocated to the maintenance of manufactured capital and no net investment would be achieved. This would lead to an absurd situation at the following period with a lower stock of manufactured capital and a strictly lower stock of natural capital. The level of overexploitation would therefore need to be increased in order to satisfy the survivability constraint, causing a sharper diminution of natural capital while manufactured capital is barely maintained. Very quickly this would bring the economy at time T to the doom point where even the maximum rate of overexploitation $\bar{v}$ is not sufficient to guarantee survivable consumption:

$$F(K_T, \bar{v} N_T) < c_{min} \tag{5.5}$$

If condition (5.4) is satisfied, then the additional output drawn from a high enough exploitation rate can be reinvested while the survivability condition is respected. Let us consider a pure "Hartwick" strategy that requires the reinvestment of all additional output into manufactured capital. In our setting, the irreversible degradation of natural capital that fuels the additional output can be interpreted as the consumption of capital to fit Hartwick's seminal rule of weak sustainability. This reinvestment will have a multiplier effect. First if the marginal productivity of manufactured capital is high enough compared to the marginal productivity of natural capital (it should be the case since we work in the neighborhood of low initial stocks of capital), the potential output will increase. Mechanically a higher potential output will correspond to an even higher depreciating output. The second effect corresponds to the increase in the productivity of natural capital that is triggered by a higher stock of physical capital (see the properties of F above). This will partly compensate the degradation of natural capital incurred by the initial overuse. As a result it will be profitable with this strategy to overuse natural capital for the period of time necessary to remain in a set where (5.5) does not occur. Progressively the marginal productivity of manufactured capital increases while natural capital productivity decreases (after a certain point where the multiplier effect of manufactured capital vanishes, depending on the shape of the production function). When these two marginal productivities are equated, it is not worthy to keep on overexploiting natural capital and the capital utilization rate must be set at $v = \tilde{v}$. There, if it is not too late, the stock of natural capital will be sustained at a positive level N_p and the economy will be following an accumulation path away from green and dirty poverty traps. Of great interest is this improvised Hartwick strategy regarding our general discussion of sustainability and economic analysis. Although it implies an initial period of overexploitation of natural capital, we have shown that this strategy is unambiguously better suited to avoid environmental degradation and poverty traps. Trying to preserve immediately the environmental functions at stake can have a very detrimental effect on the long term sustainability of the economy, both in terms of output growth and preservation of actual environmental assets preserved.

This interesting result underlines a curious tradeoff that consists in the conversion of natural capital into manufactured capital through overexploitation of the latter. Such a strategy, impossible to observe in our previous models with no capital accumulation, can actually prevent the occurrence of environmental degradation cycle, although it seems at first like an unsustainable policy.

Finally it must be noted that if natural capital restoration was allowed (see Appendix B), it would undoubtedly amplify the appeal of this Hartwick strategy. Indeed it is clear that if an economy has the possibility to restore natural capital, it is more prone to degrade it considerably in a first phase before starting restoring it back once it has reached a sufficient level of industrial development.

This partial analysis deserves obviously more rigorous developments and the most fitted framework to carry out a thorough study of these strategies could be viable control. As we have experienced in Chapter 4, it is the most adequate context to integrate various constraints and it would respond quite well to the imbrication of consumption survival levels and the two capital stocks dynamics. This extension is a major part of our post-doctoral research projects.

5.6 Optimal preservation of natural capital

Let us know replace the economic and ecological dynamics in a discounted optimization framework so that we question the optimality of sustainable paths. This problem proves much less tractable than the standard two factor problem involving manufactured capital and labor as the population growth rate determining labor is often considered an exogenous parameter. Here it is not possible to work with an intensive form of manufactured capital as both productive factors are endogenously determined by the optimal controls. The modest objective of this section is simply to try and underline necessary conditions for an optimal path to be sustainable and to characterize roughly the other optimal paths, restricting our analysis to non cyclical paths.

5.6.0.1 The social maximization problem

Considering the functions described in the previous section, the social planner's objective is to choose consumption levels and natural capital utilization levels that maximize intertemporal welfare given initial stocks of capital. Its maximization program writes

$$\max_{c,e} \int_0^\infty U(c(t))e^{-\rho t}dt$$

subject to

$$\dot{K}(t) = F(K(t), v(t)N(t)) - c(t) - bK(t) \tag{5.6}$$

$$\dot{N}(t) = -\delta(v)N(t) \tag{5.7}$$

$$N(t_0) = N_0 \ , \ \ K(t_0) = K_0$$

Note that we use a standard constant rate of depreciation b for the depreciation of manufactured capital and ρ is the constant social discount rate.

The current value Hamiltonian for this problem writes

$$\mathcal{H}(t) = U(c(t)) + \lambda(t)(F(K(t), v(t)N(t)) - c(t) - bK(t)) + \mu(t)(-\delta(v)N(t))$$

where λ and μ are the respective shadow prices of physical an natural capital. It can be noted right away that given the contribution of capital to the production function,

$\mu > 0$ and $\lambda > 0$ as long as $F_1 > b$.

The first order conditions for this problem are[1]

$$U'(c) = \lambda \tag{5.8}$$

$$\lambda F_2(K, vN)N = \mu\delta'(v)N \tag{5.9}$$

$$\dot{\lambda} = (\rho + b - F_1(K, vN))\lambda \tag{5.10}$$

$$\dot{\mu} = (\rho + \delta(v))\mu - e\lambda F_2(K, vN) \tag{5.11}$$

Using (5.9), condition (5.11) can be rewritten as

$$\dot{\mu} = (\rho + \delta(v) - v\delta'(v))\mu \tag{5.12}$$

Condition (5.8) is the standard Ramsey rule on the trade off between consumption and investment (valued through the shadow price of capital λ).

Condition (5.9) offers an original look on the efficient management of natural capital. An interesting interpretation at the margin of this condition can be given.

Let us work in the neighborhood of the MSUR $\tilde{v}$ such that increasing utilization will trigger capital depreciation.

Increasing marginally the rate of utilization of natural capital yields a higher output given by $NF_2(K, vN)$. This additional output is entirely redirected to consumption (there is no variation in the other terms of (5.1)) and can thus be valued by the marginal utility $U'(c)$, or, equivalently on the optimal path, by the shadow price of capital λ. The additional welfare provided by this increase of utilization is thus $\lambda F_2(K, vN)N$. However this increased utilization causes an acceleration in the degradation of natural capital measured by $\delta'(v)N$ and valued straightforwardly by the shadow price of natural capital μ. It is thus Pareto-improving to increase the utilization rate as long as the additional welfare is greater than the subsequent welfare loss. Hence the optimality condition (5.9).

The dynamics of λ correspond to the standard dynamics of neoclassical growth models.

The dynamics of μ given by (5.12) show that for low enough values of v, and in

[1]When no ambiguity can arise we shall drop the time index of the variables.

particular lower than the MSUR $\tilde{v}$, $\dot{\mu}$ will be negative. This reflects the fact that a decrease in the stock of natural capital following overutilization will increase the marginal productivity of natural capital and will thus increase the shadow value μ of one unit of this capital.

The following transversality conditions must also hold

$$\lim_{t\to\infty} \lambda(t)K(t) = 0$$
$$\lim_{t\to\infty} \mu(t)N(t) = 0 \tag{5.13}$$

5.6.1 Partial characterization of the optimal paths

Due to the complexity problem with two endogenous state variables, we will simply sketch general patterns of optimal paths and observe the treatment of natural capital along them. In particular the absence of restoration from our model constrains the optimal path to a bounded level of natural capital. The structure of the problem might give rise to cyclical paths but we will restrict our analysis to monotonous paths for now.

5.6.1.1 Preliminary result

To distinguish the various configurations that may arise, we need to define the function χ such that

$$\chi(v) = \rho + \delta(v) - v\delta'(v)$$

Given the convexity of $\delta(.)$ simple calculations show that χ is strictly decreasing.

Assuming that it exists, let us define $\hat{v} > 0$ such that

$$\chi(\hat{v}) = 0$$

Let us note that given the properties of χ the higher ρ, the higher $\hat{v}$ and the higher $\delta'(.)$ the lower $\hat{v}$.

As χ is decreasing we have

$$v < \hat{v} \Rightarrow \rho + \delta(\hat{v}) - \hat{v}\delta'(\hat{v}) > 0$$
$$v > \hat{v} \Rightarrow \rho + \delta(\hat{v}) - \hat{v}\delta'(\hat{v}) < 0$$

Using (5.12) and the positivity of μ this yields

$$v < \hat{v} \Rightarrow \dot{\mu} > 0$$
$$v > \hat{v} \Rightarrow \dot{\mu} < 0$$

First let us note that $\dot{\mu} = 0 \Leftrightarrow v = \hat{v}$. Since the dynamics of N in equation (5.7) tell us that $\dot{N} = 0 \Rightarrow v = \tilde{v}$, we can immediately conclude that the optimal path will lead to a steady state only if a very restrictive condition is respected. A steady state where $\dot{\mu} = \dot{N} = 0$ can only be reached if the *ex ante* condition $\tilde{v} = \hat{v}$. This condition depends on the exogenous value of $\tilde{v}$ and on the form of δ. As such this special case ($\hat{v} = \tilde{v}$) will be ignored and we will not focus on a steady state analysis (a balanced growth path could be a working alternative).

This absence of steady state rules out a stationary "sustainable" solution but it does not rule out an "even more" sustainable path where natural capital would be preserved at a constant level while physical capital would increase, thus increasing via marginal productivity the shadow price of natural capital. We can deduce that given the configuration of the problem the remaining non sustainable path will be of two kinds depending on whether capital utilization increases or decreases (without ever reaching $\tilde{v}$) along the way. Let us start by defining a necessary condition for the sustainable path before turning briefly to the non sustainable ones.

5.6.1.2 Necessary conditions for optimal sustainable paths

Let us call a sustainable path any monotonous optimal path that preserves indefinitely a positive level of natural capital, even if an initial phase of degradation has occurred. This preservation of environmental capacities must be distinguished from the asymptotical depletion that occurs in all other cases. Along such a path, the capital utilization rate necessarily either starts at or decreases until it reaches $\tilde{v}$. Let us assume that $v(T) = \tilde{v}$, with $T \geq t_0$. Given the feasible control set it will remain at

$\tilde{v}$ indefinitely while the level of natural capital will remain at $N(T)$. To respect the transversally condition (5.13), a path with

$$\lim_{t\to\infty} N(t) = N(T)$$

requires that μt tends towards 0. Along a monotonous sustainable path, μ must thus be decreasing once v has reached the non-degrading level $\tilde{v}$. Considering the preliminary result above, this means that we must have $\tilde{v} > \hat{v}$.

Condition 2. *A first necessary ex ante condition for a sustainable path to be optimal is that* $\tilde{v} > \hat{v}$.

For a given $\tilde{v}$, determined by ecological processes, this condition is more likely to be met for low $\hat{v}$, that is to say for low discount rates and high levels of $\delta'(.)$ that can be interpreted as the intensity of the threshold in the environmental degradation process. This necessary condition fits quite well the usual intuitive role of the discount rate and of the intensity of marginal degradation. We can conclude in particular that in this more general model with capital accumulation, a sustainable path will never be optimal for high discount rates.

Condition 3. *Along an optimal sustainable path there necessarily exists a finite time* T_s *such that* $\dot{K}(t) \leq 0 \ \forall \ t \leq T$.

The proof of this condition is given in Appendix C. This condition establishes that the economy will never settle at the non-degrading exploitation rate $\tilde{v}$ if it is in a manufactured capital accumulation mode. A simplified somehow candid interpretation of this condition considering only monotonous paths is that for economies with a low initial stock of manufactured capital, and thus a need to increase it initially, it will never be optimal to adopt a non-degrading utilization rate. We can thus conclude that optimal programs for economies with low endowments of manufactured capital will involve the depletion of natural capital, whatever the discount rate applied.

5.6.1.3 Hartwick strategies

Here we shall not achieve an exhaustive characterization of all possible optimal paths but simply shed some light on the trade offs at stake along a non sustainable path, e.g., a path along which natural capital is depleted asymptotically. According to our previous results, these paths are more likely to occur for high discount rates and low initial endowment of capital. We will focus on the optimality of the “Hartwick strategies” described above. Such strategies favor natural capital depletion at high initial rates that decrease in time. These high utilization rates generate an excess of output that is reinvested in manufactured capital in a proportion that leaves enough consumption to respect the first order condition (5.8).

According to (5.10), investment in manufactured capital is carried out until the marginal productivity of manufactured capital is lower than the adjusted rate of discount $(\rho+b)$. It must be noted that this marginal productivity decreases as the stock of manufactured capital increases but also as a result of natural capital depreciation. This accumulation of capital stops either when the marginal productivity of capital is lower than $(\rho + b)$ or when an additional loss of natural capital, valued at its increasing shadow price μ, would not compensate the additional output gained. At this point it is difficult to draw more insights from the model to characterize the optimal paths without entering technicalities that are beyond the scope of the conclusive propositions of this chapter.

However, the distinctive feature of optimal Hartwick strategies, compared to the wider set of Hartwick strategies envisioned in the previous section, is that they inevitably lead to the depletion of natural capital. In our anterior discrete-time approach, we did not associate the total degradation of natural capital with Hartwick strategies as the most intuitive conception was that they could at some point adopt a non-degrading rate of utilization. Only unsustainable Hartwick strategies are optimal whereas we pointed out that the original property of the Hartwick strategies set was to include policies that looked unsustainable initially but that were actually the source of much more sustainable situations.

5.7 Concluding remarks on optimality, sustainability and the preservation of capacities

5.7.1 A contribution to the fundist definition of natural capital

From a conceptual standpoint, this chapter attempted to formalize in a conceptual definition some insights about the fundist definition of natural capital. Inferring an arbitrary unobservable stock level of natural capital that yields the observable flows of ecosystem services of any kind facilitates the apprehension of its variations across time that should serve as landmark to design and validate sustainable development policies. Additional adjustments of this definition are still needed, especially regarding the inclusion of technical change and the relations with sustainability indicators based on capital theory such as Genuine Savings (Hamilton, 1994).

5.7.2 Environmental degradation cycles and Hartwick survival strategies

From a policy oriented standpoint we have drawn interesting insights on natural capital management in development policies. Placed in a developing country context, our stylized system dynamics, outside an optimization framework, prove helpful in the determination of the causing factors and favorable conditions of environmental degradation cycles and poverty traps. Our rather counter-intuitive strategy comparison obtained in Section 5.5 inspires a twofold conclusion. On the one hand, Hartwick strategies seem to correspond quite well to the historical development pattern of countries. They overuse their immediate environment at first and rely on this extra output to accumulate manufactured capital so as to place their economy on a growth path with less and less environmental pressure[1]. If this additional output is not reinvested then environmental degradation cycles take place and doom the chances of development faster than the non-degrading strategy. With the exception of the problem of anthropogenic climate change, that suffers no ecological bargaining, our formalization of Hartwick strategies could advocate the right of developing countries to start their

[1]If the arguments of the Environmental Kuznets Curve are to be trusted (Cole et al., 1997).

economic development on unsustainable tracks before they are industrialized enough to lower their pressure on natural capital[1].

The current enthusiasm of western countries for restoration techniques and geo-engineering evoked earlier in this chapter can also be interpreted as the second phase of development that follows the overexploitation of natural capital. The second insight concerns this blurry notion of unsustainable economic track. Here we point out the ambiguity that surrounds the dynamic definitions of sustainability. In our model it appears that (environmentally) unsustainable control decisions eventually place the economy on a path to a sustainable situation while rigid "sustainable" trajectories systematically lead to green or dirty poverty traps. This remark opens a field of discussion on the "subject" of sustainability: does it characterize a control decision, a path, an equilibrium situation?

5.7.3 A last word on optimality and sustainability

Finally from a methodological standpoint, we concluded our exploration of the compatibility between discounted optimization and sustainable policies in a broader framework involving capital stocks. Our candid analysis is limited inasmuch as it does not account for cyclical behaviors but we have nevertheless been able to establish an exogenous and an endogenous necessary conditions for optimal paths to be sustainable. In particular we have robustly assessed the classic impact of discount rate on the occurrence of optimal sustainable path. But more importantly we have shown that even for low enough discount rates, it will never be optimal for economies with low initial endowments of manufactured capital, intrinsically bound to increase it to avoid "green poverty traps", to adopt a sustainable path at some point in time. This result is all the more interesting as it completes symmetrically the conclusions of the pollution control analysis carried out in Part I. Indeed we have highlighted repeatedly the crucial role played by initial *environmental* capacities in the sustainability of optimal paths in Chapters 1 and 2. The present result corroborates our mild mistrust in discounted cost-benefit analysis when initial *economic* capacities are too low.

We are well aware of the specific limitations and restricted scopes of the various

[1]This result stems uniquely from a production perspective, it does not imply any direct consideration of the differential of environmental *preferences* that distinguishes rich and poor countries.

models we have summoned to study sustainable pollution regulation policies, and beyond that, a sustainable management of ecosystem services. Consequently we do not claim in any way to have definitely resolved the optimality vs. sustainability debate. However the conclusions we have brought forward in various settings throughout this work inspire us reservation towards the adequacy of discounted cost-benefit analysis when it comes to founding sustainable policies. We shall not explore any further the issue of the discount rate as it is a very documented debate. We believe nevertheless that an important contribution of our work has been to bring to light the crucial role played by initial conditions on the sustainability of optimal trajectories. For economies with low initial environmental or economic capacities, sustainability, even in its *a minima* form, will never be optimal.

A pragmatic interpretation of this property would be to recommend not using discounted cost-benefit when the respective initial capacities of the economy are “too” low. Such an approach does make sense on a local scale but it can be argued that applying cost-benefit analysis only when safe minimum ecological or capitalistic thresholds are respected amounts *in fine* to a cost efficiency analysis with exogenous conservation objectives.

A pessimistic interpretation of this dependency to initial conditions would be to recall that intuitively if a decision framework claims to be an adequate analytical tool for sustainable policies, it must obviously deliver sustainable policies for any kind of viable initial conditions. By necessity, sustainable paths should adapt to the economy’s characteristics, not the contrary. Excluding *a priori* a set of situations from the sustainability kernel seems in contradiction with the definition of sustainability, however blurry this definition may be.

We are thus comforted in our belief that economic instruments are indispensable tools to achieve sustainability, and achieve it at the least cost of course, but that their normative claims should remain bounded by exogenous political and biophysical constraints.

Appendix Chapter 5

Appendix A: Exhaustible resources: active capital or inventories?

Considering our concern to assess the status of natural capital into economic models, and especially its comparison with manufactured capital, it is crucial to characterize rigorously what does form part of natural capital in its true economic sense. The functional definition of capital, an asset that yields a periodic flow of services, proves quite useful to sort out true capital from other categories of natural resources. Exhaustible resources (mineral and fossil fuel reserves) provide only "one-shot" goods[1] whereas typical renewable assets offer periodic flows of services and goods (ecosystem functions, vegetal and animal harvest). This property led El Serafy (1989) and Costanza and Daly (1992) to distinguish non renewable capital from renewable natural capital. Based on this distinction it is clear for us that non-renewable resources such as oil, coal or minerals do not constitute true economic capital. The flows extracted from these stocks are a source of additional income that must not be falsely interpreted as Hicksian income, e.g., as *"the maximum amount of produced output that can be consumed at some point in time while maintaining constant wealth"* (Hicks, 1946). According to El Serafy (1989), renewable natural capital is "*analogous to machines and is subject to entropic depreciation*" while non-renewable capital is *"analogous to inventories and is subject to liquidation"*. As such these flows can produce revenue just like goods coming from a firm's inventory produce revenue but their source should not be considered as capital.

Only under one condition can these resources somehow turn into actual capital: if the extracting economy follows the Hartwick rule (1977). If it reinvests into physical or human capital its entire rents drawn from the sales of exhaustible resources, then these resources can qualify, *a posteriori*, as natural capital. This reference to the Hartwick rule does not presume of any of the conclusions of Hartwick in terms of optimality and sustainability. All we do here is observe the very peculiar nature of exhaustible resources, similar to a continuous positive exogenous income shock like

[1]Most of these stocks of exhaustible resource do not yield any services until this good is extracted, as would an hypothetical "sterile" forest that could not be regrown after harvests but that would still yield periodical services until it is entirely cut down.

money falling from the sky. These resources just evaporate if they are not reinvested, and that is the case in many developing countries, especially resource-rich ones[1]. In addition, before or after their transformation they do not yield any services beyond their productive input and do not display multifunctionality like the rest of natural assets do. As a result, we personally favor the exclusion of exhaustible resources from the concept of natural capital[2].

Appendix B: Negative depreciation as restoration

Let us come back briefly on the possibility to restore natural capital evoked previously. If we were to consider this option from an aggregated point of view, we could indeed extend the range of the endogenous depreciation function $\delta(v)$ to negative values. This negative depreciation, causing mechanically an increase in natural capital, would be triggered by a rate of capital utilization lower than a given threshold. This threshold must obviously be inferior to the MSUR but for the sake of simplicity it can also be identified with it as it was the case with our assimilative capacity restoration. Applying the concept of capital utilization to natural capital thus brings forward the idea of capital "overutilization" and capital "underutilization", relatively to the MSUR. Consequently we could characterize the depreciation function δ in the following alternative manner:

$$\begin{aligned} &\delta(\tilde{v}) = 0 \\ &\delta(v) > 0 \;\; \forall \;\; v > \tilde{v} \\ &\delta(v) < 0 \;\; \forall \;\; v < \tilde{v} \end{aligned} \tag{5.14}$$

[1]A very interesting counter-example is Botswana: the government in this country is constitutionally bound to reinvest all the rent gained from mineral extractions, diamonds in particular, since its independency in the 1960's (Lange, 2004).

[2]But whether they are considered part of natural capital or not, it is crucial for the sake of economic policy and indicators to exclude the income they generate from the gross domestic product and other measurements proxies of development.

which translates immediately into the following dynamics of N

$$v(t) = (\tilde{v}) \Rightarrow \dot{N} = 0$$
$$v(t) > (\tilde{v}) \Rightarrow \dot{N} < 0$$
$$v(t) < (\tilde{v}) \Rightarrow \dot{N} > 0$$

Graphically this new property implies the extension of the depreciation function shown in Figure 5.2. Following the interpretation of restoration costs we developed in Chap-

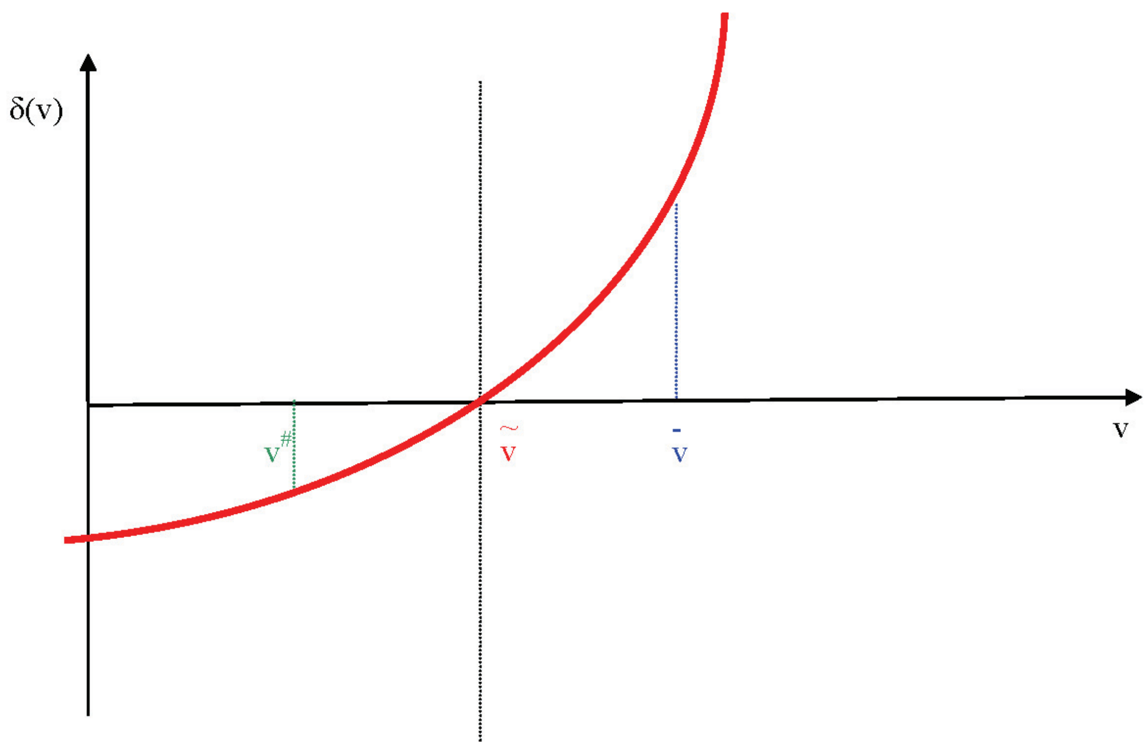

Figure 5.2: Endogenous depreciation function with restoration

ter 1, we can establish a twofold ecological and economic rationale behind this mechanism. We can consider that a utilization rate $v^{\#}$ lower than the MSUR lets the ecosystems "rest", and that the level of future environmental function they provide subsequently rises, similarly to a "fallow" sequence for agricultural soil or to a recovery period for a fish population. This conception of "natural regeneration" would imply that the aggregated stock of natural capital is globally "underutilized", but it does not mean that each and every one of the planet's ecosystem services are exploited in a sustainable way. Local "underutilization" might offset local "overutilization".

The second way to look at restoration through capital underutilization involves thinking in terms of foregone production and artificial restoration. Just as we could value assimilative capacity restoration cost through foregone benefits in Chapter 1, we can reinterpret the meaning of $v < \tilde{v}$ as an *ex post* image of an economic decision involving $v = \tilde{v}$.

Let us imagine that they are ecological restoration techniques (see Aronson et al., 2007, for various examples) available at a given cost. Society can find it optimal to exploit natural capital at the MSUR and to allocate part of the output to restoration. Instead of adding complexity to the model by introducing the restoration level as an additional control variable, we can write this restoration level as the difference of depreciation rates between a virtual level of capital utilization $v^{\#}$ and the MSUR, such that the restoration impact is $(\delta(\tilde{v} - \delta(v^{\#}))N = -\delta(v)N > 0$.

The cost of this restoration can then be given in terms of foregone production by the output differential caused by $v^{\#} < \tilde{v}$. This output differential $[F(K, \tilde{v}N) - F(K, v^{\#}N)]$ must be valued at the shadow price of production/consumption λ. In the real economy we have thus a capital utilization rate equal to $\tilde{v}$, an output equal to $F(K, \tilde{v}N)$, a net restoration impact of $\delta(v^{\#})N$ and a restoration cost equal to $[F(K, \tilde{v}N) - F(K, v^{\#}N)]$ in output units[1]. In our model this situation boils down to setting the utilization rate at $v^{\#}$. If the depreciation function is calibrated correctly, both interpretations, in terms of natural regeneration or costly artificial restoration amount to the same control variable choice $v = v^{\#}$.

The first order conditions (5.9), (5.10) and (5.11) can be easily interpreted in a symmetric manner to our discussion above in order to shed some light on the efficiency conditions of natural capital maintenance. In order to restore natural capital, a decrease in v is necessary. This decrease relatively to the MSUR means giving up a part of the output that could have been obtained without causing a decline in environmental productivity. This foregone output, voluntarily given up, is equal to $NF_2(K, vN)$ and can be valued once again using λ. This maintenance effort, similar to an investment in natural capital, thus costs $\lambda NF_2(K, vN)$ and yields an actual restoration equal to $\delta'(v)N$. This resulting restoration must be valued at the shadow price of natural capital μ. Consequently, it is efficient to restore natural capital until the marginal cost of restoration $\lambda NF_2(K, vN)$ is equated with the marginal benefit

[1]Along the optimal path the value of output units and consumption units is equalized to λ.

of restoration $\mu\delta'(v)N$. Note that our model does not allow for simultaneous maintenance and depreciation at the aggregated level unlike some manufactured capital utilization models such as Licandro et al. (2001) do.

We do not explore any further this potential extension because various forms of environmental services do not exhibit such a reaction to reduced exploitation rate. For instance the value of services yielded by semi-wild landscapes or natural recreational amenities would decrease without anthropogenic maintenance. The productivity of domesticated cattle would also be reduced with less human intervention. However for a range of adequate environmental functions this restoration provides a very promising framework to analyze the status of natural capital in the production process.

Appendix C: Necessary condition for optimal sustainable paths

The proof of condition (3) is based on the behavior of the system when v approaches $\tilde{v}$ from above. On this approach path the first order condition (5.9) must be respected:

$$\lambda F_2(K, vN) = \mu\delta'(v) \tag{5.15}$$

We have shown that we have necessarily $\dot{\mu} < 0$ on an optimal sustainable path. Moreover, as $\tilde{v}$ must be approached from above using the convexity of $\delta(.)$, $\delta'\dot{(v)} < 0$. The right hand side of (5.15) decreases on the approach path. On the left hand side v is decreasing by definition and since $v > \tilde{v}$ we have $\dot{N} < 0$. Given the properties of $F(.,.)$, this implies *ceteris paribus* an increase in $F_2(K, vN)$. In order to compensate the decrease of the right hand side, it is thus necessary that either λ or K decrease also sufficiently. Let us show now that on the approach path λ increases at some point, so that it is necessary in any case to have K decrease on the approach path.

We have from (5.10)

$$\dot{\lambda} = (\rho + b - F_1(K, vN))\lambda$$

$\dot{\lambda} < 0$ implies that $F_1(K, vN) > \rho + b$. Along the approach path vN decreases, which will eventually lead $F_1(K, vN)$ to be lower than $\rho + b$ and λ will increase.

We have thus proven that on the approach path leading the optimal path to the non-degrading control $\tilde{v}$ and a preserved stock of natural capital we must necessarily have $\dot{K} < 0$.

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Résumé : La présente thèse est partie d'une interrogation initiale sur la compatibilité avec les exigences du développement durable des modèles économiques d'optimisation de la pollution. Elle est centrée sur les modèles dynamiques de régulation de la pollution et de gestion du capital naturel. A l'aune d'un critère *a minima* de soutenabilité demandant une préservation suffisante de la capacité d'assimilation de la pollution par l'environnement, nous analysons dans une première partie les trajectoires de pollution optimale dans un cadre de maximisation intertemporelle de l'utilité actualisée. Les modèles théoriques standard dynamiques de contrôle de la pollution de flux (Ch.1) et de stock (Ch.2) sans accumulation de capital sont modifiés pour permettre une valorisation explicite de cette capacité d'assimilation. Ils sont illustrés respectivement par le rôle des zones ripariennes dans l'absorption des nitrates lixiviés et par la capacité d'assimilation du CO_2 par la biosphère. Cet examen montre que la compatibilité entre les exigences de durabilité n'est pas assurée dans tous les cas, ce qui nous conduit à rechercher des formes de modélisation différentes, davantage en phase avec les objectifs du développement durable. Dans une deuxième partie nous rapprochons les bases d'une approche de la régulation de la pollution de celles de la gestion d'une ressource naturelle renouvelable. Cette perspective nous conduit à explorer les alternatives au critère de la maximisation de l'utilité actualisée représentées par des critères comme la Règle d'Or Verte, le Maximin (Ch.3) ou la Viabilité (Ch.4). A partir des intuitions réunies au cours des deux premières parties, nous proposons dans la dernière partie (Ch.5) un modèle simplifié de croissance avec capital physique et capital naturel. Ce modèle, caractérisé par la dépréciation endogène du capital naturel, nous permet d'élargir aux capacités économiques notre réflexion sur la soutenabilité en termes de préservation des capacités environnementales.

Mots-Clés : Pollution optimale, Capacité d'assimilation, Changement climatique, Capital naturel, Développement durable, Fonctions environnementales, Viabilité.

Abstract : This dissertation originates in an initial questioning of the compatibility between the requirements of sustainable development and the economic models of optimal pollution control. It addresses dynamic models of pollution control and natural capital management. With respect to an *a minima* sustainability criterion based on the preservation of the pollution assimilative capacity of the environment, we analyze, in the first part, the optimal pollution paths in an intertemporal discounted utility maximization framework. The standard dynamic theoretical models of flow (Ch.1) and stock pollution control (Ch.2) without capital accumulation are modified to allow for the explicit valuation of this assimilative capacity. They are illustrated respectively by the role of riparian zones in the absorption of lixiviated nitrates and the CO_2 assimilative capacity of the biosphere. This review shows that the compatibility with the requirements of sustainability is not ensured in all cases, which leads us to investigate different modeling frameworks, more inline with the objectives of sustainable development. In a second part we found an approach of pollution control similar to the management of a renewable natural resource. This perspective allows us to explore the alternative options to the discounted maximized utility criterion proposed by the Green Golden Rule or the Maximin (Ch.3) or Viability (Ch.4). Based on the insights collected in the first two parts, we propose in the last part (Ch.5) a simplified model of growth with physical capital and natural capital. This model, characterized by endogenous depreciation of natural capital, allows us to extend our analysis in terms of environmental capacities preservation to the preservation of economic capacities.

Keywords : Optimal pollution control, Assimilative capacity, Climate change, Natural capital, Sustainability, Ecosystem services, Viability.