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# Market Mechanisms and Valuation of Environmental Public Goods

Arnaud Dragicevic

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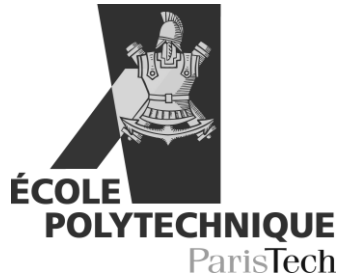
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## THÈSE

Pour l'obtention du grade de  
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Présentée et soutenue publiquement par

**Arnaud Z. Dragicevic**

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### **MARKET MECHANISMS AND VALUATION OF ENVIRONMENTAL PUBLIC GOODS**

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### **MÉCANISMES DE MARCHÉ ET ÉVALUATION DES BIENS PUBLICS ENVIRONNEMENTAUX**

Directeur de thèse : Bernard Sinclair-Desgagné

Membres du jury

Pr. Bureau, D., École Polytechnique et Conseil économique pour le développement durable (président)

Pr. Shogren, J., Université du Wyoming et Académie royale des sciences de Suède (rapporteur)

Pr. Sinclair-Desgagné, B., École Polytechnique, HEC Montréal et CIRANO (directeur)

Pr. Willinger, M., Université de Montpellier 1, Institut universitaire de France et LAMETA (rapporteur)

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# **Chapitre 0**

## **Introduction Générale**



"La nature n'est ni morale ni immorale, elle est radieusement, glorieusement, amoral." Théodore Monod

## 0.1. Le préambule

L'évaluation économique des biens et services publics environnementaux répond à un double objectif : en premier lieu, produire un ordre de grandeur, en des termes monétaires, des services rendus par l'environnement afin qu'ils soient incorporés dans les décisions publiques à leur juste valeur ; en second lieu, apporter des éléments qui permettent de bâtir des politiques de l'environnement tout en prenant en compte les préférences des agents économiques.

La thèse publique en évaluation économique développée dans ce manuscrit se compose de quatre essais. Le premier interroge la nature des préférences des agents économiques pour les biens publics sur un marché hypothétique. Le deuxième examine le bien-fondé des mécanismes d'enchères pour révéler les *préférences environnementales*. Le troisième considère la question de la sincérité des valeurs révélées en enchères répétées. Enfin, le quatrième appréhende ce qui motive les agents à financer un bien public, le financement et la valeur qu'ils attribuent au bien public étant des corrélatives, en dépit de l'intérêt rationnel à se comporter en passager clandestin.

La démarche scientifique transdisciplinaire qui consiste à mettre des concepts d'horizons divers en relation les uns avec les autres – démarche que nous nous sommes efforcés d'entreprendre tout au long de cette recherche – apporte des propositions à des questions soulevées en économie de l'environnement et plus généralement celle des biens publics. Les essais n'édifient pas de lois de la nature (quitte à y divertir d'éventuels détracteurs des sciences économiques) et ouvrent autant de débats qu'ils n'en closent. Toutefois, nous espérons qu'ils donnent une plus grande compréhension du comportement individuel vis-à-vis d'un bien public

en contexte d'échange marchand, et portent à la connaissance ce qui constitue la valeur économique de l'environnement<sup>1</sup>.

## 0.2. L'approche économique

L'environnement et les ressources naturelles fournissent aux agents des services essentiels tous les jours. Les pouvoirs publics ont nécessité de les évaluer pour budgéter les politiques environnementales. Si l'environnement a une valeur, il n'a pas de prix. Dès lors, comment justifier les montants d'investissements inhérents à sa gestion ainsi que les dépenses pour la mise à disposition des biens publics ? L'analyse économique permet de comparer les coûts et les bénéfices d'actions envers l'environnement, ce qui en fait un outil de décision robuste pour évaluer les politiques et mieux légiférer. Appliquée à l'environnement, l'approche économique se divise en régulation et évaluation. Elle observe et modélise les préférences des agents (eux-mêmes présumés conscients de leurs préférences) par rapport à leur cadre de vie, le milieu naturel dans le cas présent.

La régulation représente l'ensemble des règles qui ont pour but de maintenir l'équilibre du marché. L'absence de marché des biens publics implique l'intervention de l'État. La régulation devient alors la mise en place de règles de conduite qui permettent de maximiser le bien-être social. Les politiques s'appuient généralement sur la régulation, à travers la taxation des pollueurs imaginée par Pigou en 1929 ainsi que les compensations monétaires fixées par le droit commun. Cependant, l'absence de marché induit l'absence de prix, lequel est un vecteur d'information sur la valeur du bien. Il en résulte distorsions de valeur, coûts de transactions et asymétries d'information très coûteuses en efficacité. En effet, peu de politiques environnementales se basent sur le critère d'efficacité, notamment parce que les décideurs publics ont d'autres objectifs que l'efficacité économique, tels que l'équité ou bien la *soutenabilité* des systèmes de ressources (Freeman

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<sup>1</sup> S'agissant d'une thèse publique, malgré des efforts de vulgarisation, les quatre chapitres qui composent cette thèse comportent quelques passages techniques difficiles. Nous sollicitons l'indulgence du lecteur intéressé mais non-initié.

2003). Pourtant, l'analyse économique se propose justement d'éclairer le décideur sur les critères *a minima* lesquels permettent le développement durable ; et développement durable ne signifie pas développement souhaitable (Sinclair-Desgagné 2007a) qui relève d'autres grilles de lecture sociétales.

L'évaluation économique consiste à montrer que l'environnement a une valeur d'usage. Préserver cet usage intact revient à s'exprimer sur des projets qui impactent, positivement et négativement, le niveau de qualité environnementale, puis à arbitrer entre coûts et bénéfices. Il s'agit de l'analyse coût-bénéfice, basée sur la prise en compte des *équivalents monétaires* que les individus considèrent pertinents pour refléter leurs préférences (Gatzweiler et Volkmann 2007). Par exemple, cela signifie que les individus sont capables d'associer des valeurs monétaires à des niveaux de préservation d'un milieu naturel. Cette capacité d'association est la pierre angulaire de l'analyse économique sur les questions environnementales. Sans celle-ci, il apparaît impossible d'appliquer des principes économiques développés en théorie du bien-être. L'environnement naturel a donc une valeur économique, mais il n'y a toujours pas de consensus sur la nature de cette valeur ou sur les meilleurs outils pour la mesurer.

D'un côté, les économistes néoclassiques lient la valeur d'un bien ou service à l'utilité, ou la satisfaction des préférences, qu'il procure. Selon ce mode de pensée qu'on peut définir comme anthropocentrique, l'environnement a une valeur instrumentale, laquelle dépend des préférences des agents qui le considèrent comme un moyen et non comme une fin en soi (même un parc naturel est un moyen qui rend possibles la contemplation de la vie sauvage et la randonnée en milieu naturel). En effet, le socle de l'analyse coût-bénéfice repose sur la logique instrumentale. La somme qu'un individu est disposé à dépenser pour satisfaire ses préférences reflète la valeur qu'il accorde au bien. Il est donc possible de révéler la valeur du bien à travers sa demande (Bateman *et al.* 2002). Les économistes calculent ensuite le taux auquel un agent est prêt à substituer ce bien pour un autre (en l'occurrence, cet *autre* bien est le numéraire dans lequel sont mesurés les prix). Ce taux est capté par les indices de consentement-à-payer maximal (le CAP) et de consentement-à-recevoir minimal (le CAR). Les valeurs

économiques sont d'ordinaire révélées dans le cadre d'une institution fondée sur l'échange. Le principe est que tous les agents possèdent la même quantité d'informations sur le bien à valoriser, soit l'absence d'asymétrie d'information.

De l'autre côté, les environmentalistes accordent au milieu naturel une valeur de non-usage, c'est-à-dire une valeur intrinsèque ou *per se*. Or la valeur de non-usage est indépendante des prix du marché, si bien qu'elle ne peut pas être approximée autrement que par l'évaluation hors-marché. Sachant que les différentes natures de la valeur sont imbriquées dans ce qui serait la vraie valeur d'un bien ou service, O'Neill (1993) considère simplificateur d'utiliser un outil d'évaluation basé sur la *commensurabilité* et la représentation monétaire. Diamond et Hausman (1994) affirment même que les agents n'ont pas de préférences dites environnementales. Toutefois, les individus sont d'expérience disposés à payer ou recevoir une valeur monétaire pour un bien ou service environnemental, prouvant ainsi qu'ils sont prêts à substituer des biens entre eux, et donc à rendre comparables des biens privés avec des biens publics. Si la conversion monétaire était irrecevable, son refus serait observable quel que soit le contexte, ce qui n'est pas le cas. C'est pourquoi ont été introduits les marchés hypothétiques tels que l'*évaluation contingente* initiée par Ciriacy-Wantrup (1947).

L'autre problème concerne la nature publique des biens et services environnementaux. En effet, ils sont des biens publics, donc par définition non exclusifs, c'est-à-dire qu'aucun agent ne peut être exclu de leur consommation, et non rivaux, à savoir que l'usage d'un agent n'entrave pas celle d'un autre agent. Comme les biens publics ne s'échangent pas sur un marché, il en résulte absence du taux de substitution et du prix d'échange. Néanmoins, grâce aux marchés hypothétiques, l'agrégation des valeurs privées permet de construire une courbe de demande pour le bien public. Il est donc raisonnablement possible de baser les politiques environnementales sur les évaluations privées issues des enquêtes montées à cet effet.

Il est argué que les problèmes liés à l'environnement sont dus à l'absence de définition adéquate des droits de propriété. Le prétexte juridique a souvent

déplacé le débat des biens publics en dehors du sentier économique, par le fait que le CAP place l'agent en position d'acquéreur tandis que le CAR place l'agent en situation de propriétaire ; alors que le bien public correspond au statut intermédiaire de copropriété. De fait ou par défaut, les normes juridiques sont devenues l'instrument utilisé par les pouvoirs publics. Pourtant, les autorités régulatrices pourraient rétablir la logique du marché en régulant via des prix. L'idée que l'évaluation économique ne peut pas résoudre les questions de biens publics en raison de la logique marchande – qui serait inapte à traduire leur valeur sociale –, et que seul l'aménagement juridique des droits individuels à l'usage des biens publics en est capable, est sans véritable fondement. D'abord, si les prix sont incomplets, comme le montrent les externalités négatives souvent citées en exemple pour justifier l'échec des marchés, la juridiction l'est autant. Créer des marchés hypothétiques pour l'environnement, c'est ni plus ni moins prendre en compte ces externalités, et la surveillance des parties prenantes peut se substituer à l'autorité publique. Ensuite, la mise en place d'un arsenal juridique est onéreuse, et il appartient aux autorités régulatrices de minimiser les coûts d'administration, parce que d'autres politiques publiques peuvent être initiées et rétribuées par la réalisation de ces économies.

L'ère est à la rationalisation des dépenses publiques qui ont trop longtemps manqué dans les finances publiques, entraînant des gaspillages dont les coûts sont supportés par la société civile. Ainsi, Montgomery (1972) a démontré que le coût d'implémentation d'une politique environnementale par les instruments de marché tels que les droits d'émission était minimisé à l'équilibre. Également, d'après Sinclair-Desgagné (2007*b*), "il incombe à l'État de veiller au bon fonctionnement du mécanisme des prix [e]n réduisant le nombre de biens collectifs par l'instauration de conditions propices à la naissance et au fonctionnement de marchés efficaces." Rappelons qu'en situation de copropriété, de nombreuses décisions sont prises à la majorité, évitant le piège de l'unanimité qui ne peut exister en analyse économique compte tenu de l'hétérogénéité des préférences. Enfin, la démarche qui consiste à aller directement interroger les citoyens sur les questions environnementales n'est-elle pas la plus démocratique qui soit ?

### 0.3. Les méthodes d'élicitation

La méthode des préférences révélées déduit la valeur de l'environnement à partir des décisions prises par les agents économiques. Son ambition est d'observer le comportement effectif de l'agent, sensé traduire ses préférences et la valeur qu'il accorde à l'environnement. Cette méthode utilise les données du marché existantes pour extraire la valeur implicite d'un bien. De la sorte, Hotelling (1949) a proposé la méthode indirecte des coûts de transport pour évaluer la demande pour les loisirs dans les milieux naturels. Cependant, les préférences révélées ne fonctionnent que si on dispose de données du marché.

Il est souvent difficile d'obtenir des données du marché relatives aux questions environnementales, aussi une part importante des études repose-t-elle sur les préférences déclarées, à l'égal de l'évaluation contingente. L'évaluation contingente prend la forme d'une enquête d'opinion dans laquelle on demande aux individus de déclarer combien ils sont disposés à payer pour éviter une dégradation de l'environnement ou bien combien ils sont disposés à recevoir en compensation pour laisser faire cette dégradation. Les valeurs – assimilées aux prix du marché hypothétique – sont ensuite agrégées pour calculer la valeur monétaire globale. Le but de l'évaluation contingente a d'abord été de mesurer la disposition à payer pour assurer la disponibilité d'un service environnemental. Mais, la dégradation accrue de l'environnement a fait basculer cette littérature vers des études portant sur des dommages subis par le milieu naturel (voir Carson *et al.* 1992).

Même si la méthode permet de prendre en compte la valeur de non-usage (Walsh *et al.* 1984) défendue par les environmentalistes, sa limite réside dans le fait qu'elle est source de nombreux biais : risque de questions mal formulées qui orienteraient les réponses ; mauvaise perception du bien à évaluer ; réponse stratégique plutôt que sincère ; apparition de biais cognitifs incompatibles avec la rationalité. En effet, les individus valorisent un scénario hypothétique. L'absence des incitations du marché, qui prennent la forme des contraintes budgétaires et de mise en disponibilité des substituts, produit donc des données contestables. Par

exemple, les agents peuvent promettre des sommes destinées à la protection de l'environnement largement supérieures à celles qu'ils sont réellement prêts à payer (Diamond et Hausmann 1994, Hanemann 1994, Neill *et al.* 1994). Rien n'incite donc l'individu à donner sa vraie valeur lors d'une déclaration. Les préférences déclarées ont ainsi été accueillies avec pyrrhonisme, voire hostilité.

Lorsque les agents considèrent leurs déclarations inconséquentialistes, toutes les réponses se valent. Même en vertu de la sincérité des agents (ce qui demeure hypothétique), ceux-ci n'ont pas incitation à engager d'efforts cognitifs importants lorsqu'ils doivent formuler une déclaration, ce qui rend les valeurs déclarées potentiellement bruyantes ou biaisées. Dans le cas où les agents considèrent leurs déclarations conséquentialistes, ils sont incités à donner des réponses fictives, comme minimiser leurs CAP s'ils s'aperçoivent que le projet porte sur la création d'une nouvelle taxe, afin d'influencer les décideurs publics qui peuvent être dans la projection d'une réélection et donc dans l'opportunisme.

#### **0.4. Les enchères expérimentales**

Puisque les économistes doivent en tout état de cause éliciter des valeurs pour mener à bien des analyses coût-bénéfice et estimer les effets d'une politique publique sur le bien-être des agents (Boardman *et al.* 2005) pourquoi ne pas utiliser les mécanismes d'enchères ? En effet, les économistes s'intéressent aux enchères expérimentales depuis un certain temps déjà : Bohm (1972), Brookshire et Coursey (1987), Hoffman *et al.* (1993), Shogren *et al.* (1994), Shogren *et al.* (2001), Rozan *et al.* (2004), Lusk *et al.* (2007). La seule méthode capable à ce jour de combiner les avantages des préférences révélées avec la possibilité de construire un marché simulé est le mécanisme de ventes aux enchères. Simuler un marché en laboratoire, c'est créer un marché qui n'existe pas, pour quelques heures et avec quelques individus recrutés à cette fin. Cette création temporaire n'a pas d'autre finalité que d'observer le comportement des agents sur le marché, seul capable de révéler les CAP (Robin *et al.* 2007).

La valeur ajoutée des enchères expérimentales réside dans le fait qu'elles peuvent s'appliquer à n'importe quel type de bien non-marchand, ou évaluer les programmes sociaux enclins aux divergences d'intérêts (Heckman 2001). Bien que les mécanismes de marché de type ventes aux enchères aient initialement été conçus pour éliciter la valeur des loteries et tester la validité de l'utilité espérée (Becker *et al.* 1964), ils ont depuis été largement repris pour des biens réels, notamment la protection de l'environnement (Cummings *et al.* 1986).

Les enchères expérimentales mettent les individus en situation d'échange actif. Quand bien même ils prendraient en compte les données du marché et réviseraient leurs préférences en fonction de celles-ci, la compatibilité avec les incitations des mécanismes d'enchères induit un coût désincitatif à dévier des préférences sincères ; rappelons que toutes les conséquences monétaires issues des décisions sont réelles. Par ailleurs, les chercheurs peuvent y observer la manière dont les agents réagissent aux signaux publics tels que les prix de compensation. Ils ont à disposition des données directes – par opposition aux données indirectes à l'exemple des coûts de transport – afin de révéler la valeur économique d'un bien. Les problématiques résolues par des expériences d'évaluation sont nombreuses (Willinger 2001) mais nous nous contenterons de citer la différence entre le CAP et le CAR (Knetsch et Sinden 1984, Brookshire et Coursey 1987, Shogren *et al.* 1994, Shogren *et al.* 2001, Horowitz et McConnell 2003) ou encore l'effet de dotation (Samuelson et Zeckhauser 1988, Kahneman *et al.* 1990, Horowitz *et al.* 2005, Bischoff 2008).

Néanmoins, la validité externe des données de laboratoire est souvent remise en question. On accuse les expériences de simplisme ; on leur reproche l'effet de contexte éloigné de la réalité, c'est-à-dire un manque de reproduction fidèle des comportements des individus, comme dans une épicerie par exemple. Pour autant, le décideur public doit s'accommoder de l'absence du marché de référence. Il est inutile d'essayer de répliquer le marché réel en laboratoire, car la simplicité permet d'isoler de nombreux paramètres noyés dans la complexité du monde réel, ce qui améliore le contrôle de l'étude (Friedman et Sunder 1994). En effet, le marché simulé en laboratoire permet de contrôler les variables



décisionnelles qui pèsent sur le CAP et d'étudier l'impact d'une variation à la marge de l'une de ces variables décisionnelles, toutes choses étant égales par ailleurs (Robin *et al.* 2007). L'expérimentation en laboratoire doit donc être jugée sur la qualité de la compréhension des préférences qu'elle produit, non sur la qualité du facsimilé.

## **0.5. Le résumé de la thèse**

Après ce bref chapitre introductif, nous aborderons dans un premier chapitre la question de l'équivalence entre le CAP et le CAR. La disparité entre les deux indices a de profondes conséquences sur les prises de décision environnementales. Brown et Gregory (1999) mentionnent la formation des politiques de développement durable et l'allocation des droits. Tout autant, on peut se demander comment baser les décisions publiques si les valeurs sont qualifiées d'inconsistantes par rapport au choix rationnel ? Si la disparité était au départ associée aux carences de la méthode de mise en œuvre des enquêtes, les racines du problème s'avèrent être sensiblement plus profondes. Eu égard à l'évaluation des biens publics, nous pensons que la disparité est due à la substituabilité imparfaite entre les biens privés et publiques, ainsi qu'en raison de perceptions différenciées des agents économiques entre gains et pertes. C'est à cette problématique que le premier chapitre se consacre.

Ainsi, le Chapitre 1 traite de la disparité entre les indices CAP et CAR dans l'évaluation hors-marché. Dans la littérature, l'effet de substitution et l'effet de dotation sont tenus responsables de l'existence des disparités. Nous montrons que la substituabilité imparfaite dans la fonction d'utilité indirecte peut provoquer la disparité soit entre le CAP et le CAR – en raison du coût d'opportunité –, soit entre les gains et les pertes, où il s'agit d'évaluer une perte sèche. La mesure en termes relatifs accentue la substituabilité imparfaite, mais l'effet de substitution est borné dans le modèle d'aversion aux pertes.

Ce premier chapitre prépare le terrain pour le Chapitre 2, où nous évaluons un vrai bien public dans un contexte d'enchères expérimentales. Les offres d'achat

et de vente reflètent le CAP et le CAR, d'où leur importance. L'effet de dotation et le choix du meilleur mécanisme d'enchères y sont examinés. Les études en enchères expérimentales jusqu'ici menées ont porté sur des biens privés non marchands ; elles sont supposées divulguer ce qui se passerait en présence de biens publics, car il est *a priori* difficile d'envisager une expérience où le bien public est échangé (Robin *et al.* 2007). Nous y parvenons. Nous n'employons pas de valeurs induites mais laissons libre cours aux valeurs autoproduites par les sujets d'étude recrutés pour l'occasion. L'étude nous permet de vérifier si, sur des marchés simulés, bien privé non marchand et bien public sont évalués de manière identique.

Ainsi, nous évaluons l'impact de trois mécanismes d'enchère – le mécanisme Becker-DeGroot-Marschak (BDM), l'enchère au deuxième prix, et l'enchère aléatoire au *n*ième prix – dans l'évaluation des CAP et CAR privés d'un bien public pur. Nos résultats montrent que l'effet de dotation peut être éliminé en répétant le mécanisme BDM. Néanmoins, à l'échelle logarithmique, l'enchère aléatoire au *n*ième prix donne la vitesse de convergence vers l'égalité des indices de bien-être la plus élevée. Plus généralement, nous observons que les sujets d'étude évaluent les biens publics en se référant à l'avantage privé et subjectif qui résulte du financement du bien public.

Par la suite, le Chapitre 3 discute de la sincérité des préférences en enchères expérimentales répétées et traite des propriétés incitatives des mécanismes BDM et l'enchère aléatoire au *n*ième prix. Une propriété des mécanismes d'enchères est la compatibilité avec les incitations, dans laquelle un offreur a une stratégie faiblement dominante de soumettre une offre égale à sa valeur. Il a été prouvé que les deux mécanismes sont compatibles avec les incitations. En évaluation, on répète des sessions d'enchères pour donner aux offreurs l'opportunité d'apprendre le mécanisme de marché : leur donner du temps pour révéler leurs préférences. Or, ce procédé les contre-incite à adapter leurs préférences en fonction des prix publiquement signalés, si bien qu'il crée un risque de licitation stratégique (par opposition aux offres sincères). Si les offreurs s'engagent dans des stratégies déviantes pour faire face à l'incertitude sur la

valeur du bien public, les mécanismes d'enchères perdent leur propriété de compatibilité avec les incitations et révèlent de fausses préférences.

Lorsque les prix dépendent des offres soumises, c'est-à-dire en présence de mécanismes de marché répétés avec prix de compensation endogènes, l'hypothèse de l'indépendance des valeurs privées – sous-jacente à la compatibilité avec les incitations – est remise en question ; même si ce type de mécanismes fournit une participation active et un apprentissage du marché. Dans sa vision orthodoxe, le comportement marchand d'adaptation met en péril la compatibilité avec les incitations. Nous introduisons un modèle qui montre que les enchérisseurs licitent suivant l'heuristique d'ancrage et d'ajustement, dépendante d'une fonction de pondération séquentielle, laquelle prend en compte les contraintes de compatibilité avec les incitations sans rejeter les prix signalés issus des autres offres. En déviant de leur ancrage dans le sens du signal public, les enchérisseurs opèrent dans un équilibre corrélé.

En dernier lieu, Vatn (2005) estime que les préférences environnementales dépendent des normes sociales intériorisées : elles sont socialement contingentes. Comme le prouve l'expérience du Chapitre 2, les contributions privées aux biens publics sont issues d'une démarche d'évaluation. Elles sont conduites aussi bien par des incitations asociales que sociales. Si l'offre privée du bien public est stimulée à la fois par une rationalité qui dicte de ne pas contribuer au bien public et de profiter de l'effort fourni par la collectivité, et par l'appétit pour la reconnaissance sociale qui incite à se faire publiquement connaître en tant que généreux donateur, laquelle des deux motivations domine ?

Le Chapitre 4 fait ainsi la comparaison entre déculpabilisation et compétition pour le statut social dans la provision privée des biens publics. Lorsque les agents sont intrinsèquement impulsés, c'est-à-dire qu'ils contribuent essentiellement aux biens publics dans le but de soulager leur culpabilité d'avoir indirectement participé à leur dégradation, ils tendent à se comporter en passagers clandestins. En revanche, lorsque les agents sont extrinsèquement impulsés et se mettent en compétition pour atteindre du statut social qu'ils visent par le financement des biens publics à titre privé, leurs contributions deviennent des

compléments stratégiques. Dans ce cas, le niveau agrégé des biens publics croît avec la réduction des écarts de revenus entre les agents. Injecter de la compétition pour le statut social dans des fonctions d'utilité augmente les contributions aux biens publics, et donc leur niveau global, faisant de la concurrence une incitation féconde pour résoudre le problème du passager clandestin.

## **0.6. Les recommandations de politique publique**

Quatre recommandations découlent de ce travail de recherche, à savoir que nous suggérons de : (1) *conduire des expériences de marchés simulés et répéter des sessions de marché pour évaluer les préférences environnementales ; évaluer à la fois les deux indices de bien-être ; (2) privilégier les mécanismes d'enchères tels que BDM et l'enchère aléatoire au nième prix, pour la raison qu'ils sont capables de réduire, voire supprimer, l'écart initial entre les indices en sessions répétées ; si l'écart persiste, considérer les valeurs comme une fourchette révélée par l'ensemble des individus ; (3) tolérer l'influence des prix de compensation signalés sur la licitation, celle-ci révélant la rationalité limitée des individus plutôt que leur imposture ; (4) inciter à la provision privée des biens publics, et encourager la compétition pour le statut social par la mise en valeur de ce type d'actions, tout en s'assurant de transferts de revenu des individus à haut revenu vers des individus à bas revenu afin que la compétition existe.*

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## Imperfect Substitutability in Standard and Reference-Dependence Models

### Abstract

This chapter focuses on the disparity between willingness-to-pay and willingness-to-accept indices in nonmarket valuation. In the literature, the substitution effect and the endowment effect are presumed to cause the disparities. We show that imperfect substitutability in the indirect utility function can lead to disparity either between WTA and WTP – due to the opportunity loss – or between gains and losses, which reflects a net loss. Context-dependent valuation accentuates the imperfect substitutability, but the substitution effect is bounded inside the behavioral model of loss aversion.

Keywords: contingent valuation, WTP-WTA disparity, substitution effect, loss aversion

JEL Classification: D61, D81, Q51



"A thing does not have value because it costs, as people suppose; instead it costs because it has a value." Étienne Bonnot de Condillac

## 1.1. Introduction

The debate on how policy-makers compare the benefits derived from one public plan against another has been led by the cost-benefit analysis. In this debate, the quest for revealing nonmarket values induced the direct contingent valuation method based on the Hicksian  $C$  and  $E$ , *i.e.* the individual's maximum willingness-to-pay (or best offer) to guarantee the change and the minimum willingness-to-accept (or reservation price) to sacrifice the change.

Empirical and experimental studies have given evidence of large disparity between WTP and WTA, which makes impractical the use of values estimates derived from the contingent valuation. Experimental laboratory markets confirmed persistency in disparities (Knetsch and Sinden 1984; Brookshire and Coursey 1987). To justify the disparity, theorists invoked the substitution effect or the context-dependent endowment effect, and oriented the effects in rivalry. The substitution effect results from the agent's imperfect trade-off between private goods and public goods. The loss aversion output, that is, the endowment effect, makes agents value losses higher than equivalent gains. Morrison (1997) asserts that the endowment effect and the substitution effect play a combined role in the disparity.

To be loss averse, an agent has to consider herself an owner of the public good. In general, dealing with substitution rather than endowment allows to study the consumers' behavior without the constraint of the initial allocation of property rights. As Sinclair-Desgagné (2005) emphasizes, the property rights remain difficult to establish, guarantee, or to legitimate in public policies, whereas in a market, the price of a good or service signals the value of the resources; agents adjust their preferences and make necessary substitutions. We consider a gain in the environmental level as a non-essential right. In reverse, a compensation for a loss of the environmental level is an essential right that agents express by means of high valuation statements. This

distinction explains the difference between the standard disparity and the gain and loss disparity in terms of the property rights.

This chapter brings out three elements. First, through convex preferences or quasi-concave utility functions, where agents prefer mixed over extreme consumption bundles of private and public goods, we show that, akin to the standard disparity between WTP and WTA, the gain and loss disparity is prone to imperfect substitutability. Second, the nature of the disparities is different, simply because agents do not tolerate a loss in the same way they bear a foregone gain. Inside the neoclassical paradigm, the substitution effect works as an opportunity loss<sup>2</sup>: the lower the substitutability, the higher the opportunity loss. But the utility of an agent does not change along the indifference curve. At worst, an agent faces the status quo. On the contrary, when an agent is asked to value the loss of the public good and to weigh this loss against an equivalent gain, the opportunity loss becomes a net loss. The net loss is a critical change, for agents attach a high value to the goods or services they cannot regain. They use the status quo as a reference point to switch to a steeper indifference curve. This would be the endowment effect, or what Kahneman and Tversky refer to as loss aversion in their behavioral model. Finally, we emphasize that the substitution effect – proved to be infinite in the Hicksian context – is bounded inside the behavioral model of loss aversion.

We recall the basic account of the neoclassical model in Section 1.2, we provide clarifications of the substitution effect in Section 1.3 and the endowment effect in Section 1.4. We scrutinize loss aversion through imperfect substitutability in Section 1.5, and we study boundedness within imperfect substitutability in Section 1.6. Concluding comments are given in Section 1.7.

## **1.2. The standard model**

According to Hicksian theory, an agent has preferences over nonnegative quantities of goods and her preference ordering is transitive, continuous,

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<sup>2</sup> By analogy to finance, consider the foregone opportunity to improve the level of the public as an opportunity cost of an interest in a bank account.

nondecreasing and convex<sup>3</sup>. Assume an agent has convex preferences for market private goods  $x$  and some public good such as the environmental quality  $q$ . She can vary the quantity of consumption of the  $x$ 's, whereas the quantity of  $q$  is taken to be fixed to her. Her preferences are quasi-concave in utility<sup>4</sup> of the  $x$ 's and represented by a continuous and nondecreasing utility function  $u = u(x, q)$  which is twice differentiable<sup>5</sup>. The agent faces a budget constraint based on her income  $y$  and the prices of the private goods  $p$ . She maximizes her utility subject to a budget constraint:

$$\max_x u(x, q) \quad \text{subject to} \quad \sum p_i x_i \leq y \quad [1]$$

According to [1], the program yields the Marshallian ordinary direct demand functions  $x_i$ . Substituting them as functions of  $(p, y)$  gives the indirect utility functions which represent agent's preference ordering.  $v(\cdot)$  is continuous, decreasing, and quasi-convex:

$$x_i = h^i(p, q, y) \quad \text{for } i = 1, \dots, n \quad [1a]$$

$$u(h(p, q, y), q) \equiv v(p, q, y) \quad [1b]$$

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<sup>3</sup> The completeness of preferences implies that utility is complete. When preferences satisfy completeness and transitivity, preferences are considered to be rational. In addition, when they satisfy continuity, the utility function is continuous. At last, when preferences are monotonic, utility is nondecreasing.

<sup>4</sup> A quasi-concave utility function means that preferences are convex, that is: for all  $x$  and  $q$ , and any  $\pi$ ,  $0 \leq \pi \leq 1$ ,  $u(\pi x + (1 - \pi)q) \geq \min\{u(x), u(q)\}$ . It ensures the preference ordering.

<sup>5</sup> This assumption eliminates kinks in the indifference curves.

The agent's consumption of  $(x, q)$  can also be obtained through a program that minimizes her expenditures  $(\sum_i p_i x_i)$  based on her utility level constraint  $u(x, q)$ :

$$\min_x \sum p_i x_i \quad \text{subject to} \quad u = u(x, q) \quad [2]$$

Its resolution gives the expenditure or cost function<sup>6</sup> or the minimum amount of income necessary to achieve an attainable utility level at least as high as  $u$ , given the price vector  $p$ :

$$e(p, q, u) \equiv \sum p_i g^i(p, q, u) \quad \text{for } i = 1, \dots, n \quad [2a]$$

The expenditure function is jointly continuous in  $(p, q, u)$ , strictly increasing in  $u$ , positively linear homogenous, and concave in  $(p, q)$ . Its derivative with respect to  $y$  gives the cost-minimizing demand function or the Hicksian compensated demand function that delivers optimal quantities at various prices. Moreover, the income is compensated in such a way as to leave utility unchanged:

$$x_i = g^i(p, q, u) \quad \text{for } i = 1, \dots, n \quad [2b]$$

So far, preferences are just as well represented by both the indirect utility function and the expenditure function:

$$u \equiv v[p, q, e(p, q, u)] \quad [3]$$

$$y \equiv e[p, q, v(p, q, y)] \quad [4]$$

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<sup>6</sup> The expenditure function is twice differentiable due to the assumption that the utility function is differentiable.

The standard Hicksian welfare measures deal with changes in prices (from  $p^0$  to  $p^1$ ) while  $q$  and  $y$  are left unchanged. These changes have an impact on the indirect utility functions. The compensating variation  $C$  is the maximum amount of income that could be taken from an agent who gains from a particular change while leaving her no worse-off than before the change. The equivalent variation  $E$  is the minimum amount that an agent who gains from a particular change would be willing to accept to forego the change after it has taken place.

**Definition 1.1.:**  $C$  and  $E$  are implicitly and explicitly defined by:

$$v(p^0, q, y) = v(p^1, q, y - C) \text{ and } C = y - e(p, q, u^0)$$

$$v(p^1, q, y) = v(p^0, q, y + E) \text{ and } E = e(p, q, u^1) - y$$

Now assume changes occur in the levels of the environmental quantity<sup>7</sup>. If  $q$  changes from  $q^0$  to  $q^1$ , the agent's utility changes from  $u^0$  to  $u^1$ :

$$u^0 \equiv v(p, q^0, y)$$

$$u^1 \equiv v(p, q^1, y)$$

These changes will also have an impact on the expenditure functions. The welfare measure is the change in expenditure necessary to hold the utility constant, at the two quantity sets. We can write  $C$  and  $E$  as the difference between the minimal expenditure before the change and minimal expenditure after the change given utility levels  $u^0$  and  $u^1$ :

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<sup>7</sup> This model does not consider irreversible environmental damages. Therefore, the individual can always increase the level of the environmental quality and then recover some utility level.

$$C \equiv C(q^0, q^1, p, y) = e(p, q^0, u^0) - e(p, q^1, u^0) = -\int_{q^0}^{q^1} \frac{\partial e(p, q, u^0)}{\partial q} dq \quad [3a]$$

$$E \equiv E(q^0, q^1, p, y) = e(p, q^0, u^1) - e(p, q^1, u^1) = -\int_{q^0}^{q^1} \frac{\partial e(p, q, u^1)}{\partial q} dq \quad [3b]$$

The superscripts 0 and 1 indicate the situation before and after the change.  $C$  equals the maximum amount of money an agent could give up in situation 1 without being worse-off than in situation 0.  $E$  equals the minimum amount of money an agent would require in situation 0 to attain the utility in situation 1.  $C$  and  $E$  depend on the starting and ending values of  $q$ , and the value of  $(p, y)$  at which the change takes places.

In terms of the indirect utility function,  $C$  and  $E$  are plugged as follows:

$$u^0 \equiv v(p, q^0, y) = v(p, q^1, y - C) \quad [4a]$$

$$u^1 \equiv v(p, q^1, y) = v(p, q^0, y + E) \quad [4b]$$

Be it with indirect utility function or expenditure function, the concepts of WTP and WTA can be derived from the Hicksian paradigm. Depending on the direction of the change,  $C$  and  $E$  may be positive or negative. When the change improves utility or  $\Delta u \geq 0$ ,  $C$  is the agent's maximum willingness-to-pay to guarantee the improvement and  $E$  is the agent's minimum willingness-to-accept to forego the improvement. When the change deteriorates utility or  $\Delta u \leq 0$ ,  $-C$  is the agent's minimum willingness-to-accept to tolerate the deterioration and  $-E$  is the agent's maximum willingness-to-pay to avoid it. This property is obtained by reversing the initial and final levels (see Table 1.1.) Indeed, Ebert (1984) proves that the welfare measures possess the property of circularity. Therefore,  $C$  and  $E$  are symmetric or  $C(q^0, q^1) = -E(q^1, q^0)$ . From [4a] and [4b], we get:

$$C(q^0, q^1, p, y) = -E(q^1, q^0, p, y)$$

$$E(q^0, q^1, p, y) = -C(q^1, q^0, p, y).$$

**Table 1.1. Four welfare indices**

<b>decline</b> ( $u^1 - u^0 \leq 0$ )	<b>growth</b> ( $u^1 - u^0 \geq 0$ )
$-C = \text{WTA}$	$C = \text{WTP}$
$-E = \text{WTP}$	$E = \text{WTA}$

As pioneered by Mäler (1974) and taken over by Hanemann (1991), suppose now an agent can pay for the environmental quality as if it were marketed. She thus pays for  $q$  in this hypothetical market at some implicit price  $\pi$ . The standard price flexibility of income can be interpreted as the income elasticity of demand for the environmental quality. We then fix the following programs:

$$\max_x u(x, q) \quad \text{subject to} \quad \sum p_i x_i + \pi q \leq y \quad [5]$$

$$\min_{x, q} \sum p_i x_i + \pi q \quad \text{subject to} \quad u = u(x, q) \quad [6]$$

From which we obtain the following indirect utility function and expenditure function:

$$v(p, q, y) \equiv \hat{v}[p, \hat{\pi}(p, q, u), u] - \hat{\pi}(p, q, u) \cdot q \quad [5a]$$

$$\hat{e}(p, \pi, u) \equiv \sum p_i \hat{g}^i(p, \pi, u) + \pi \hat{g}^q(p, \pi, u) \quad [6a]$$

The derivative of the Marshallian demand function with respect to  $(p, q, y)$  gives the indirect utility function. Inverted, it gives  $\pi$ , *i.e.* the inverse demand

function to obtain  $q$  supplied at  $\hat{\pi}(\cdot)$ . In this case, the agent's income must be supplemented so she can both afford  $q$  and the  $x$ 's:

$$x_i = \hat{h}^q(p, q, y + \pi q) \quad [5b]$$

$$\pi = \hat{\pi}(p, q, y) \quad [5c]$$

The derivative of the expenditure function with respect to  $(p, q, u)$  gives the Hicksian compensated demand function  $x_i$ . Inverted, it gives the inverse compensated demand price  $\hat{\pi}(\cdot)$ , that is, the price that would induce her to purchase  $q$  units if her income were increased:

$$x_i = \hat{g}^q(p, \pi, u) \quad [6b]$$

$$\pi = \hat{\pi}(p, q, u) \quad [6c]$$

The two inverse demand functions are:

$$\hat{\pi}(p, q, y) \equiv \hat{\pi}[p, q, v(p, q, y)] \quad [5'c]$$

$$\hat{\pi}(p, q, u) \equiv \hat{\pi}[p, q, e(p, q, u)] \quad [6'c]$$

From [5'c] and [6'c] it follows that:

$$\pi^0 \equiv \hat{\pi}(p, q^0, u^0) \equiv \hat{\pi}(p, q^0, y) \quad [7a]$$

$$\pi^1 \equiv \hat{\pi}(p, q^1, u^1) \equiv \hat{\pi}(p, q^1, y) \quad [7b]$$

[5] differs with [1] on  $-\hat{\pi}(p, q, u) \cdot q$ . The expenditure function and the compensated demand function are equal, thus the inverse compensated demand function for  $q$  becomes:



$$e_q(p, q, u) = \frac{\partial e}{\partial q} = -\hat{\pi}[p, q, e(p, q, u)] \quad [8]$$

The inverse demand  $\hat{\pi}(\cdot)$  measures shadow or virtual prices, or marginal valuation, or marginal WTP or WTA to pay for a unit of  $q$  by the agent, *i.e.* the marginal rate of substitution between the  $x$ 's and  $q$ . As the inverse of indirect utility functions yields the expenditures functions, the inverse of direct utility functions gives indirect expenditure functions. Combining [5'c] and [6'c] with [4a] and [4b] and using Shepard's Lemma yields the following:

$$C \equiv C(q^0, q^1, p, y) = -\int_{q^0}^{q^1} \frac{\partial e(p, q, u^0)}{\partial q} dq = \int_{q^0}^{q^1} \hat{\pi}(p, q, u^0) dq \quad [9a]$$

$$E \equiv E(q^0, q^1, p, y) = -\int_{q^0}^{q^1} \frac{\partial e(p, q, u^1)}{\partial q} dq = \int_{q^0}^{q^1} \hat{\pi}(p, q, u^1) dq \quad [9b]$$

Thus WTP and WTA can be expressed by way of the integral of inverse compensated demand curves for a change in quantities from  $q^0$  to  $q^1$ . The distinction between WTP and WTA is the level of utility the compensation is designed to reach:  $u^0$  and  $u^1$  respectively.

Welfare measures can also be defined by a distance function (Ebert 1984). The distance function is a utility function normalized by monetary income, *i.e.* a monotonic transformation of the direct utility function for fixed quantities:

$$d = d(x, q, u) \quad [10a]$$

$d(\cdot)$  is continuous, decreasing in  $u$ , increasing and positively linear homogenous, and concave in  $x$ . The Shephard's input distance function has been introduced to consumer theory and defined in terms of the utility function (Deaton

1979). The derivatives of  $d(\cdot)$  with respect to  $(x, q)$  give a set of inverse compensated demand functions:

$$x_i = a^i(x, q, d(x, q, u)) \text{ for } i=1, \dots, n \quad [10b]$$

$a^i$  is the normalized price of  $q$  with respect to income. The distance function can be interpreted as an indirect expenditure function. Indeed, duality results show that the expenditure (cost) function is a distance function derived from the indirect utility function (Blackorby *et al.* 1978). Apart from the monotonicity and definition over different arguments, the expenditure function and the distance function share the same properties. If we consider the distance function for quantity changes, it is a dual to the expenditure function for fixed quantities and can be used to examine the welfare effects of quantity changes. To recover (non-normalized) monetary measures, the welfare measures must be multiplied by income. Thus,  $C$  and  $E$  are defined by:

$$C \equiv y(d(x, q^0, u^0) - d(x, q^1, u^0)) \quad [11a]$$

$$E \equiv y(d(x, q^0, u^1) - d(x, q^1, u^1)) \quad [11b]$$

Using Shepard's Lemma, the latter reduces to:

$$C \equiv \int_{q^0}^{q^1} a(x, q, yd(x, q, u^0)) dq \quad [12a]$$

$$E \equiv \int_{q^0}^{q^1} a(x, q, yd(x, q, u^1)) dq \quad [12b]$$

$C$  and  $E$  are measured by the area under the compensated inverse demand curves from  $q^0$  to  $q^1$  with the old and new utility levels, respectively.

Let us now compare those areas in order to see whether positive and negative changes induce the same consumer behavior.

### 1.3. The substitution effect

When goods are available in a market at no cost, there is a regular intermediate monetary exchange of commodities, which involves a linear indifference curve for the  $x$ 's and  $q$ . If there is disparity, it depends on the constant price flexibility of income, *i.e.* the elasticity of the marginal valuation of  $q$  with respect to the  $x$ 's (or  $y$  that buys the  $x$ 's):

$$C_y = \frac{\partial C}{\partial y} = v_y(p, q^0, y) \int_{q^0}^{q^1} \hat{\pi}_u(p, q, u^0) dq \quad [13a]$$

$$E_y = \frac{\partial E}{\partial y} = v_y(p, q^1, y) \int_{q^0}^{q^1} \hat{\pi}_u(p, q, u^1) dq \quad [13b]$$

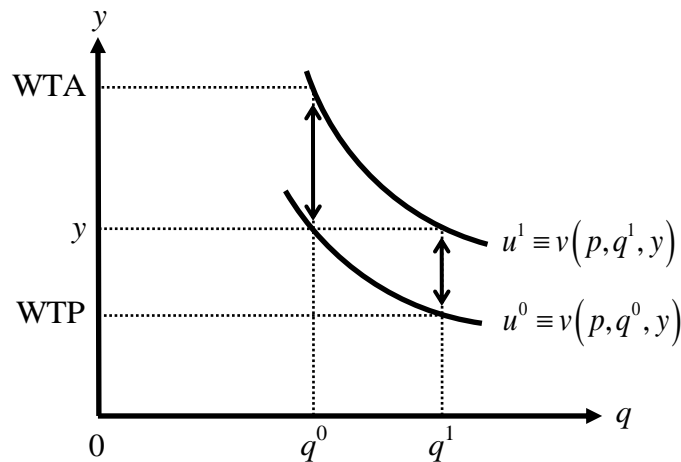
If  $C_y = E_y = 0$  and if  $e_{qu} = (\partial^2 e / \partial q \partial u) = 0$ ,  $E = C$ . Indeed, the second cross-partial derivative  $e_{qu}$  reflects the substitution effect. A null substitution effect involves linear indifference curves and null opportunity loss. Due to perfect substitutability, agents are indifferent to the variations of the public good, because they can always adjust the level of the  $x$ 's to maintain their utility constant. One interpretation could be that they feel unconcerned by the changes in the level of the public good. Another interpretation could be that they are unconditionally ready to substitute the public good with some private good. The usual proposition results from the above.

**Proposition 1.1.:** When the welfare change is induced by  $q$ , due to imperfect substitutability or low elasticity of substitution between  $q$  and the  $x$ 's, there is values disparity. It can be infinite in the limit.

Proof: In the appendix.

The substitution effect reflects the convex curvature of the indifference curve between  $q$  and the  $x$ 's and the convexity of the expenditure function in  $q$ . Ebert (1993) claims that quasi-concavity of the indirect utility function  $v(\cdot)$ , jointly with the normality of the public good, is necessary and sufficient to obtain WTA superior to WTP. If the combinations of  $(x, q)$  lead to the same level of utility, it is in the interest of an agent to have a convex mixture of goods, for it never decreases utility.

Fig. 1.1. illustrates a diminishing marginal rate of substitution between the  $x$ 's and  $q$  with quasi-concave utility functions. As  $q$  rises from  $q^0$  to  $q^1$ , utility increases from  $u^0$  to  $u^1$ . Displacements of the indifference curves reflect unitary income elasticity. As can be seen,  $WTA > WTP$ . The trade-off between environmental quality and the private good turns out to be less and less attractive: the marginal utility from environmental quality upgrading is diminishing. *Vice versa*, it means that the marginal loss of environmental quality is increasing. In any case, the lower the elasticity of substitution between  $q$  and the  $x$ 's – or to be more accurate, between  $q$  and the  $x$ 's that  $y$  buys – the broader the disparity.



**Fig. 1.1. A change in  $q$  and imperfect substitution with the  $x$ 's**

Regarding the distance function, in presence of a normal good, the inverse compensated demand curve  $a(x, q, u^0)$  lies below the inverse compensated demand curve  $a(x, q, u^1)$  for the reason that scale effects depend on the elasticity of substitution between  $q$  and the  $x$ 's. In the presence of two goods, Park (1997) finds that the Hicks elasticity of substitution equals the Allen-Uzawa elasticity of substitution. The difference between WTP and WTA thus arises whenever substitutability is imperfect.

What happens to the consumer's behavior if we now distinguish foregone gains from real losses?

#### 1.4. Imperfect substitutability and the endowment effect

Hanemann (1991) points out in his footnote 25 that Kahneman and Tversky's (1979) loss aversion, observed from some reference point, differs from the standard disparity. Indeed, in the Hicksian framework, preferences over consumption bundles are independent of initial endowments. In reference to the gain and loss perspective, Thaler (1980) proposed the term endowment effect. When an agent is endowed with a good, her reference point changes, she shifts her position on the map, and the shape of her indifference curve is altered.

If we adapt the standard framework to the loss aversion idea, a gain or a loss in  $q$  can be written  $q_+^1 \equiv q^0 + \Delta$  and  $q_-^1 \equiv q^0 - \Delta$ , with  $\Delta \geq 0$ . Assume agent's utility is affected by variations of the environmental quality level  $q$ . In this case, her utility  $u^0 \equiv v(p, q^0, y)$ , which now involves a single indifference curve, changes either to  $u^+$  in a case of a gain or to  $u^-$  in case of a loss:

$$u^+ \equiv v(p, q^0 + \Delta, y) \quad [14a]$$

$$u^- \equiv v(p, q^0 - \Delta, y) \quad [14b]$$

Bateman *et al.* (1997) define two additional measures, identified with some reference point. Regarding the first measure, the question is what additional amount of private consumption is as preferable as an increase in the environmental quality. This is the equivalent gain, equal to WTA. Regarding the second measure, the question is what loss of private consumption would be just as preferable as a decrease in the environmental quality. This is the equivalent loss, equal to WTP.

When the agent is endowed, fixing a gain and a loss in [3a] and [3b] or [4a] and [4b] gives the following relationships:  $C^+$  or compensating gain is the maximum amount she would pay to secure the gain;  $E^+$  or equivalent gain is the minimum amount she would accept to sacrifice the gain;  $-E^-$  or equivalent loss is the maximum amount she would give up to avoid the loss;  $-C^-$  or compensating loss is the minimum amount she would accept to tolerate the loss. The summary is recapitulated in Table 1.2.

**Table 1.2. Welfare indices and context-dependence**

loss ( $q_-^1 \equiv q^0 - \Delta$ )	gain ( $q_+^1 \equiv q^0 + \Delta$ )
$-C^- = WTA^- (\leq 0)$	$C^+ = WTP^+ (\geq 0)$
$-E^- = WTP^- (\leq 0)$	$E^+ = WTA^+ (\geq 0)$

Unlike the standard disparity *alias* ( $WTA^+ - WTP^+$ ) or ( $WTA^- - WTP^-$ ), where changes go in the same direction, a gain and loss disparity is computed differently, simply because we observe changes that depart in opposite directions from some reference point. Here, we subtract WTA to tolerate the loss and WTP to guarantee the gain or ( $WTA^- - WTP^+$ ). From [3a] and [3b], it follows:

$$-C^- - C^+ = \left[ e(p, q_-^1, u^0) - e(p, q^0, u^0) \right] - \left[ e(p, q^0, u^0) - e(p, q_+^1, u^0) \right] \quad [15a]$$

Since the utility function  $u(x, q)$  is quasi-concave in  $(x, q)$ , when  $q$  increases the expenditure function  $e(p, q, u^0)$  decreases, *i.e.* is convex in  $q$ , since less income is necessary to attain the fixed utility level. The second reaction is that the indirect utility function  $v(p, q, y)$  is quasi-concave in  $(q, y)$ , which means that the cross-partial derivative only implies the substitution effect  $e_{qq} = (\partial^2 e / \partial q^2)$ . As a matter of fact, the income effect – the spacing of the indifference curves – does not count, for gain and loss perspective involves a single indifference curve  $u^0$  observed from some positive and negative change, thus:

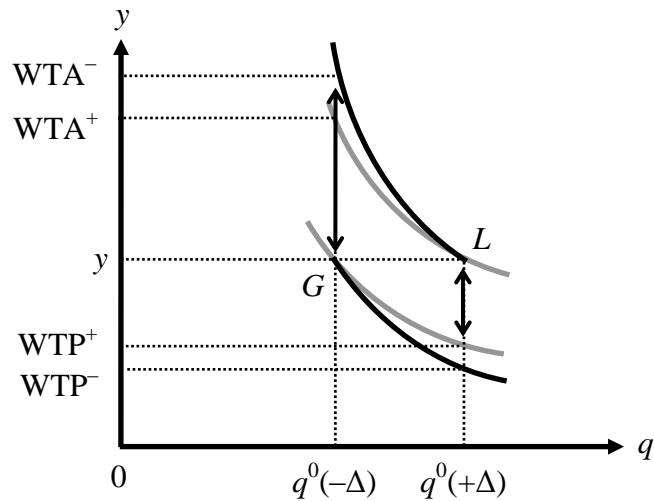
$$\text{WTA}^- - \text{WTP}^+ \geq 0 \quad [15b]$$

**Proposition 1.2.:** Imperfect substitutability between  $q$  and the  $x$ 's in the indirect utility function causes disparity either between WTP and WTA or between gain and loss, independently from the reference.

Proof: In the appendix.

Fig. 1.2. illustrates the four measures, observed from some reference coordinates  $(q^0, y)$ . Grey curves depict the same pre-endowed utility  $u^0$  observed from two reference points. Through the incursion of context-dependence, the utility changes from  $u^0$  to either  $u^+$  (a gain in utility) or  $u^-$  (a loss in utility). The reference point for  $\text{WTA}^+$  and  $\text{WTP}^+$  is  $G$ . Viewed from  $G$ , the distance from  $q^0$  to  $q_+^1 \equiv q^0 + \Delta$  is a gain in level of the environmental quality. For  $\text{WTA}^-$  and  $\text{WTP}^-$  the reference point is  $L$ . Viewed from  $L$ , the distance from  $q^0$  to  $q_-^1 \equiv q^0 - \Delta$  is perceived as a loss in level of the environmental quality. The endowment effect induces the pivoting of the indifference curve from the reference point, which illustrates the discontinuity in the slope from  $u^0$  to  $u^+$  or  $u^-$ . The steeper the indifference curve, the less the substitutability between  $q$  and the  $x$ 's that  $y$  buys.

The difference between  $WTA^+$  and  $WTA^-$ , which is essential to distinguish the standard disparity from the gain and loss disparity, lies in the way the loss is perceived. In the first case, the agent is asked to state her value to give up a gain from an increase in the environmental quality. This is an opportunity loss. This cannot be a right. In the second case, the agent is asked to state her value to suffer a loss from a decrease in environmental quality.



**Fig. 1.2. Reference-dependent preferences**

This is a net loss and it differs from the former. The difference is due to imperfect substitutability, for agents take more account of the goods they can not regain. When agents are asked to value their losses in monetary terms, the behavioral effect of loss aversion arises and they shift their indifference curves. They know they have the right to be compensated for the loss and claim this right in form of high  $WTA^-$  statements.

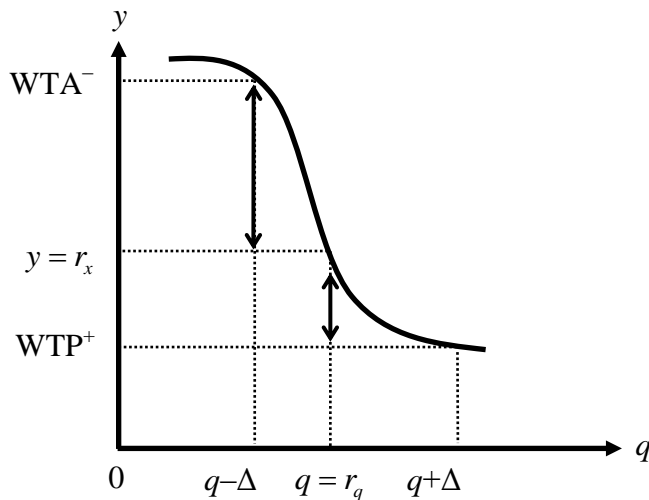
Transitivity implies that whenever  $WTA^- > WTA^+$  and  $WTA^+ > WTP^+$ ,  $WTA^- > WTP^+$ .



### 1.5. Imperfect substitutability and loss aversion

In their 1991 article, Tversky and Kahneman propose the behavioral reference-dependent theory as an alternative to the Hicksian theory of preferences. Outcomes are now valued using a value (utility) function where agents have preferences over goods relative to some reference level  $(r_x, r_q)$  seen as the status quo. According to them, (i) all is perceived as a gain or a loss; (ii) losses are weighted more heavily than gains or agents are loss averse; and (iii) the marginal value of gains or losses exhibits diminishing sensitivity. They assume that preferences are transitive, continuous and nondecreasing (but not convex).

If  $(r_x, r_q)$  stands for the reference points for consuming  $(x, q)$ , the utility function changes to  $u = u(x, q, r_x, r_q)$ ; the demand functions take the form of  $x_i = h^i(p, q, y, r_x, r_q)$  and  $x_i = h^i(p, q, u, r_x, r_q)$ ; the indirect utility is now  $v(p, q, y, r_x, r_q)$  just as is the expenditure function  $e(p, q, y, r_x, r_q)$ . These new functions are discontinuous at the reference point (Putler 1992). Fig. 1.3. shows a typical loss aversion curve observed within the context of welfare measurement.



**Fig. 1.3. Loss aversion in welfare measures**

The additive formulation of the constant loss aversion model used by Tversky and Kahneman gives the following indirect utility function:

$$u \equiv v(q, y) = R \left[ \lambda_i^q (q - r_q) + \lambda_i^y (y - r_y) \right] \quad [16]$$

with  $R' \geq 0$ . Parameters  $\lambda_i^q$  and  $\lambda_i^y$ , for  $i=1,2$  and  $\lambda_i \geq 1$ , are defined as coefficients of loss aversion for dimensions  $r_q$  and  $r_y$ . They magnify the disutility of losing some environmental quality. When the agent perceives a change as a gain, this coefficient amounts to  $\lambda_1 = 1$ , which means that the agent has neoclassical utility. When she perceives a loss, this coefficient amounts to  $\lambda_2 > 1$ . We can see that  $\lambda_2^q > \lambda_1^q (=1)$  and  $\lambda_2^y > \lambda_1^y (=1)$ .

**Definition 1.2.:** The change from the reference level  $r_q$  to either a gain  $r_q^+$  or a loss  $r_q^-$ , while  $r_y \equiv y$ , gives the following:

$$\begin{cases} \lambda_i^q = \lambda_1^q \text{ if } q - r_q \geq 0 \text{ and } \lambda_i^y = \lambda_1^y \text{ if } y - r_y \geq 0 \\ \lambda_i^q = \lambda_2^q \text{ if } q - r_q < 0 \text{ and } \lambda_i^y = \lambda_2^y \text{ if } y - r_y < 0 \end{cases}$$

In terms of coefficients of loss aversion, the welfare measures matrix becomes what is shown in Table 1.3.

**Table 1.3. Welfare indices in a gain and loss perspective**

<b>loss</b> ( $r_q^- \equiv r_q - \Delta$ )	<b>gain</b> ( $r_q^+ \equiv r_q + \Delta$ )
$\lambda_2^q \Delta / \lambda_1^y = \text{WTA}^-$	$\lambda_1^q \Delta / \lambda_2^y = \text{WTP}^+$
$\lambda_2^q \Delta / \lambda_2^y = \text{WTP}^-$	$\lambda_1^q \Delta / \lambda_1^y = \text{WTA}^+$

Since  $\lambda_1^q = \lambda_1^y = 1$ ,  $\frac{\Delta}{\lambda_2^y} < \lambda_2^q \Delta$  or  $\text{WTP}^+ < \text{WTA}^-$ . If we invert the function

$\mathbf{R}(\cdot)$ , i.e.  $y \equiv e(q, u)$  is the inverse of  $u \equiv v(q, y)$  with  $y = (\mathbf{R}/\lambda_i^y) + r_y$ , and differentiate it with respect to  $u$ , the following disparity arises:

$$e_u(q, u) \equiv \begin{cases} \mu'(u) & \text{if } \mu(u) \geq \lambda_i^q (q - r_q) \\ \frac{\mu'(u)}{\lambda_2^y} & \text{if } \mu(u) < \lambda_i^q (q - r_q) \end{cases} \quad [17]$$

where  $\mu(\cdot) = \mathbf{R}^{-1}(\cdot)$  and  $\mu' \geq 0$ .

The indirect utility function is quasi-concave because of the monotone transformation  $\mathbf{R}(\cdot)$  in [16]. Moreover,  $\lambda_i^q (q - r_q)$  is a concave function of  $q$ , which illustrates the gain and loss disparity with decreasing sensitivity to losses. Since  $\lambda_2^y > 1$ , when  $q$  increases  $e_u$  decreases, which implies the negativity of the derivative  $e_{qu}$  from changes in  $q$ . As a result, we get back to the standard disparity between WTP and WTA.

Recall that the curvature of the indifference curves shows diminishing marginal utility between the consumption bundles, and thus the standard WTA-WTP disparity. Furthermore, it generates disparity between gain and loss because of the imperfect substitution in the indirect utility function between  $y$  and some function of the environmental quality  $q$ . Through the discontinuity in the slope at the reference point, loss aversion theory implies convex indifference curves. On that subject, Hanemann (1999) argues that the assumption of quasi-concave utility function suffices to observe convexity. Quasi-concavity with inversely proportional disparity to the substitution effect can explain the disparity between gain and loss. The endowment effect within loss aversion can be explained through less than perfect substitutability.

The behavioral theory of loss aversion also works with distance effects. In terms of the distance function, it is a matter of distance between coordinates of some level of  $y$  or  $q$  and the agent's reference point. In this case, the function becomes  $d(x, q, u, r_x, r_q)$ . Adding coefficients of loss aversion into the distance function yields now a weighted distance function of the form:

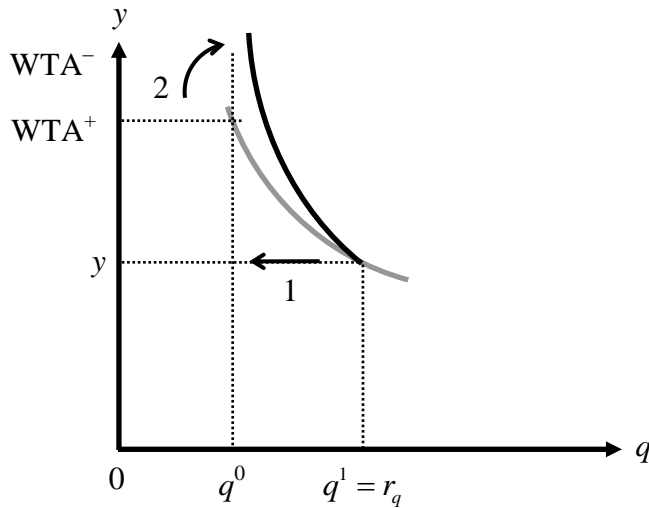
$$d \equiv d(x, q, u) = R \left[ \lambda_i^q (q - r_q)^p + \lambda_i^y (y - r_y)^p \right] \quad [18]$$

where  $p \geq 1$  denotes the metric. When  $p = 1$ , the distance is measured as the sum of weighted absolute differences. We then fall on [16]. The distance function recovers from the expenditure function. Therefore, imperfect substitutability can once again explain the gain and loss disparity.

### **1.6. Imperfect substitutability and boundedness**

Randall and Stoll (1980) demonstrate that the disparity between WTP and WTA is bounded by the ratio between the price flexibility of income and endowment. Cook and Graham (1977) assert that the compensation demanded for irreplaceable commodities, which we can assume to be imperfectly substitutable, depends on the initial level of wealth or endowment. As the probability of loss  $p \rightarrow 1$ , WTA, dependent on the income that buys the  $x$ 's, tends to infinity as the indifference curve is asymptotic to the vertical line at  $p = 1$ . This is what Amiran and Hagen (2003) also suggest in a slightly different manner: in presence of asymptotically bounded utility functions, there exists an initial level of wealth sufficiently high to produce an infinite  $WTA^-$ . Nevertheless, the substitution effect still plays a capital role, for it induces frictional trade-off between public and private goods. In terms of elasticity, the authors show that the income elasticity of the inverse compensated demand is bounded above and below by positive values independent of the amounts of public goods.

In case of reference-dependent preferences, we believe that imperfect substitutability accentuates the pivoting of the indifference curve, which in turn can produce infinite compensation demanded. We replace the nonsatiation assumption by the assumption that for each level of income  $y$ , the status quo  $q^1$  is strictly preferred to the net loss of the public good  $q^0$  or  $u(q^0, y) < u(q^1, y)$  with  $q^0 < q^1$ . A double outcome arises. The first outcome lies in the convex curvature of the indifference curve. In point of fact, imperfect substitutability induces a steeper slope for higher opportunity losses (see Fig. 1.4.: grey segment and arrow 1). The second outcome results from the enlargement of the substitution effect due to aversion of net losses, yielding clockwise rotation and, accordingly, a steeper slope of the initial indifference curve (see Fig. 1.4.: black segment and arrow 2).



**Fig. 1.4. Unboundedness of the compensation demanded**

Beyond some level of loss of the public good in view of their reference point, *i.e.*  $q^1 - \Delta \geq q^0$ , standard agents ask for an infinite monetary compensation. Formally, this yields a level of monetary compensation  $s$  strictly inferior to the disutility of the loss:

$$u(q^1 - \Delta, y) > z(q^0) + s > u(q^0, y) \quad [19]$$

**Proposition 1.3.:** In case of reference-dependent preferences and imperfect substitutability between  $q$  and the  $x$ 's that  $y$  buys, large net losses of the public good can be infinitely uncompensated.

Proof: In the appendix.

Hanemann (1999) points out that the wealth effect in Cook and Graham is not the income effect typically considered in consumer demand theory. While being true, let us recall that the income effect does not count within context-dependence. We therefore explain the infinite limit of  $WTA^-$  by the pivoting of the indifference curve from the reference (endowed) level of the public good. Again, this is a net loss perception magnifying the substitution effect. Contrary to Cook and Graham who find infinite  $WTA^-$  as the probability of losses moves towards one, our indifference curve is asymptotic to the vertical line at  $q^0$ , which shows infinite  $WTA^-$  when losses are severe and approach  $q^0$ . Unlike the previous models – which unquestionably consider substitutability as the mainspring for infinite monetary compensation – our design neither depends on the initial level of wealth or the initial endowment in market goods nor on the boundedness of the utility function. It rather depends on the severity of loss of the public goods combined with their unfeasibility to be perfectly substitutable.

In the behavioral loss aversion model, when an agent stands at  $(r_q, r_x)$ , that is, at the kink point,  $q$  and  $y$  are perfect substitutes, for she is equidistant to both references points and indifferent between the level of environmental quality and her income. Except these coordinates, any other point along the curve exhibits imperfect substitutability. As can be noticed in terms of distance minimization, above the kink point she substitutes the loss of the environmental quality with monetary compensation. Below is the opposite. Because of loss aversion, as  $(q - \Delta) - r_q < 0$  we have  $(q - \Delta - r_q) \partial u / \partial r_q \leq 0$ . When  $q - \Delta$  goes farther from  $r_q$ , additional decreases in

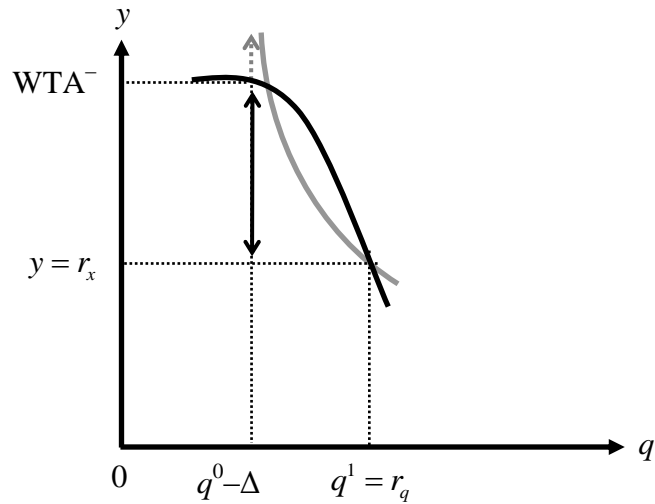
$q$  lead to smaller changes in utility, which yields  $\partial^2 u / \partial q^2 \geq 0$ . In other terms, diminishing sensitivity implies a lower substitution effect in both gains and losses. Conversely to the standard model, the marginal disutility from environmental quality downgrading is decreasing as the agent moves from the reference point, implying a bounded value for compensation.

**Proposition 1.4.:** Inside the behavioral theory of loss aversion, given constant diminishing sensitivity towards losses, the substitution effect is bounded.

Proof: In the appendix.

Hence, there is a difference between the Hicksian standard paradigm and loss aversion in the representation of context-dependence. If we superpose the indifference curves illustrating the willingness-to-accept to tolerate a loss, their respective curvatures reveal two types of behavior on the subject of losses. The grey segment represents the standard theory context-dependence. The black segment stands for the behavioral model of loss aversion (see Fig. 1.5.).

Inside the standard model, agents show increasing marginal disutility as  $q^1 - \Delta$  tends towards  $q^0$ . Inside the behavioral model, agents exhibit high loss aversion with small changes as regards their reference point, but they turn out to be less and less sensitive as  $q^1 - \Delta \geq q^0$ . Diminishing sensitivity of the marginal value of losses clarifies this phenomenon. The farther something moves from a reference point, the less additional changes should matter, which in our case surprisingly means increasing substitutability. This counter effect appears because agents are myopic, which makes them feel unconcerned by changes out of their visual field. As a consequence, they end up asking for a bounded amount of compensation, no matter the additional degradation of the environment.



**Fig. 1.5. Comparison between reference-dependent indifference curves**

The significance of it is non-negligible. In case of irreversible damages to the environment or high losses of public goods with regards to their initial level – commodities that we know to be imperfectly substitutable –, standard agents which turn out to be far-sighted will ask for an infinite monetary compensation, whereas loss aversion agents will ask for a bounded amount of compensation, and neither can adapt their reference points. While economists have long considered loss aversion to degenerate agents’ rational preferences, we see that past some level of changes in  $q$ , it limits their proclivity towards abnormal valuation.

### 1.7. Concluding remarks

Applied to market valuation of the public goods, this chapter dealt with imperfect substitutability in both standard welfare and reference-dependence theories. Imperfect substitutability in the indirect utility function can provoke disparity either between WTA and WTP or between gain and loss. Further, the same quasi-concave utility functions can explain the endowment effect.

What is the point of finding that imperfect substitutability plays a role in both the WTA and WTP disparity and the gain and loss disparity? According to the above,



it basically means that agents' unwillingness to substitute an environmental good or service increases with its defective substitutability. When agents substitute a public good for a private good, an opportunity loss appears and induces the standard disparity. In case the scenario to price is a loss instead of a foregone gain, loss aversion transforms the opportunity loss in a net loss, which enlarges the initial disparity, for people heavily value things they cannot regain. Experimental findings from Boyce *et al.* (1992) and Chapman (1998) support this conclusion. At last, the substitution effect observed from some reference point has a bound inside the behavioral model of loss aversion. Whether agents should have infinite values for severe or irreversible losses might be the topic that decides which model better values environmental preferences.

Yet, these common findings must be toned down. Valuing environmental goods or services calls for an understanding of the public and private benefits derived from the public good. This is partially ensured, as environmental commodities are unfamiliar to agents and their benefits for utility obscure in most cases. The risk of having naïve valuations is existent. Only an interactive market-like setting permits to surmount these limits, and hypothetical markets remain devoid of market interactions. Experimental markets are thus essential in the contingent valuation. In experimentation, the early disparity between welfare measures is redundant, supporting either of the two effects. But their confrontation occults the market efficiency which rules the economic valuation. Indeed, markets bound anomalies by means of *ad hoc* incentives, for they aid agents to correct their untruthful or naïve valuations. The next step consists in identifying, by probing into auction mechanisms, why some of them reduce the disparity better than others. As well, studying agents' context-dependent behavior faced with irreversible environmental damages and ambiguity – when they can adapt their reference points – is a matter of future research.

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## 1.9. Appendix

### Proof of Proposition 1.1.

The demonstration is as follows. After Randall and Stoll (1980), WTP and WTA for changes in public goods should not differ with small income effects. They bound  $(E - C)$  via the income elasticity of demand (or income elasticity of willingness-to-pay) of the public good. For example, when the price of a certain good one changes, the disparity amounts:

$$E - C = \int_{p_1^0}^{p_1^1} [e_{p_1}(p, q, u^1) - e_{p_1}(p, q, u^0)] dp_1$$

or

$$E - C = e_{p_1 u}(p, q, u) = g_u^1(p, q, u) = g_y^1(p, q, e(p, q, u)) \cdot e_u(p, q, u)$$

The income effect associated with good one – the second cross-partial derivative  $e_{p_1 u} = \partial^2 e / \partial p_1 \partial u$  – establishes the size of the disparity, the limit being the individual's income. The bounding method carries over to welfare measures of the quantity changes. The analogous result for a change in  $q$  gives the cross-partial derivative  $e_{qu} = \partial^2 e / \partial q \partial u$ , *i.e.* the substitutability of the nonmarket good by means of market goods:

$$E - C = \int_{q^0}^{q^1} [e_q(p, q, u^0) - e_q(p, q, u^1)] dq = -e_{qu}(p, q, u)$$

For a change in the public good's level, Hanemann (1991) demonstrates that the second cross-partial derivative  $e_{qu}$  reflects the substitution effect. Indeed, from [8] and the differentiation of the compensating demand function for  $q$ , we hold the derivative involved in changes in  $q$  that impact on  $u$ :

$$e_{qu}(p, q, u) = \frac{\partial^2 e}{\partial q \partial u} = \frac{\partial \pi(p, q, u)}{\partial u} = \frac{\hat{g}_y^q(p, \pi, \hat{e}(p, \pi, u)) \cdot e(p, \pi, u)}{\hat{g}_\pi^q(p, \pi, \hat{e}(p, \pi, u)) + \hat{g}_y^q(p, \pi, u) \cdot q}$$

By the Hicks decomposition, the precedent becomes:

$$e_{qu}(p, q, u) = \frac{\hat{g}_u^q(p, \pi, u)}{\hat{g}_\pi^q(p, \pi, u)}$$

This difference between WTP and WTA depends on the price flexibility of income and thus the ratio of the income elasticity of the ordinary demand function for  $q$  to the elasticity of substitution between  $q$  and the  $x$ 's<sup>8</sup>. The numerator represents the income effect of  $q$  in the hypothetical market, established from the derivative of the demand function with respect to income. The denominator is the own-price derivative of the compensated demand function for  $q$  and gives the aggregate Allen-Uzawa elasticity of substitution between  $q$  and the private goods weighted by the budget share of the same private goods.

Changes in prices and changes in  $q$  both vary with income and depend on a cross-partial derivative of the expenditure function. And when  $e_{qu} < 0$ , WTA is superior to WTP, for  $u(x, q)$  being quasi-concave in  $(x, q)$ . The disparity is more influenced by the substitution effect because of the Engel adding-up restriction, which requires that the sum of the income elasticities of demand for the  $x$ 's and  $q$ , weighted by their budget shares, equals unity: limiting the magnitude of the income effect.  $\square$

### Proof of Proposition 1.2.

We focus on the disparity between gain and loss.

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<sup>8</sup> That is Hanneman's (1991) equation 17.

A necessary and sufficient condition for the disparity between gain and loss to occur, *i.e.*  $WTA^- > WTP^+$ , is that  $v(p, q, y)$  is quasi-concave in  $(p, y)$  or that  $e(p, q, u^0)$  is a convex function of  $q$ . In this case, the second partial derivative  $e_{qq}$  must be strictly positive. Let us look at the expenditure function.

The disparity arises because of the convexity of the initial indifference curve  $u^0$ . It follows from [3a] and [3b] that:

$$-C^- - C^+ = [e(p, q_-, u^0) - e(p, q^0, u^0)] - [e(p, q^0, u^0) - e(p, q_+, u^0)]$$

Which gives:

$$\begin{aligned} 0 &\leq e(p, q_-, u^0) + e(p, q_+, u^0) - 2e(p, q^0, u^0) \\ 2e(p, q^0, u^0) &\leq e(p, q^0 - \Delta, u^0) + e(p, q^0 + \Delta, u^0) \\ e(p, q^0, u^0) &\leq \frac{1}{2} (e(p, q^0 - \Delta, u^0) + e(p, q^0 + \Delta, u^0)) \end{aligned}$$

In parallel,  $e(p, q^0, u^0)$  can be rewritten as:

$$e\left(p, \frac{1}{2}((q^0 - \Delta) + (q^0 + \Delta)), u^0\right)$$

Substituting the precedent into the general inequality gives:

$$\begin{aligned} e\left(p, \frac{1}{2}((q^0 - \Delta) + (q^0 + \Delta)), u^0\right) &\leq \frac{1}{2} (e(p, q^0 - \Delta, u^0) + e(p, q^0 + \Delta, u^0)) \\ e\left(p, \frac{1}{2}(q^0 - \Delta) + \left(1 - \frac{1}{2}\right)(q^0 + \Delta), u^0\right) &\leq \frac{1}{2} e(p, q^0 - \Delta, u^0) + \left(1 - \frac{1}{2}\right) e(p, q^0 + \Delta, u^0) \end{aligned}$$

From which directly follows the convexity of the expenditure function of  $q$ . The expenditure function being convex, we have  $e_{qq} > 0$ .

There is a disparity between gain and loss. □

### **Proof of Proposition 1.3.**

For  $q^0 \leq q^1 - \Delta$  with  $\Delta > 0$ , we have  $u_0 = u(q, y)$  and  $u^-(q, y) = u(q - \Delta, y)$ . Let us set  $z(q) \equiv \sup[y \mid u^-(q, y) = u(q, y)]$  for a level of monetary compensation such that the utility remains constant. For each  $q \geq 0$  we have  $u(q, y) \leq z(q)$ . The supremum  $z(q)$  is increasing in  $q$ . This says that for each level of income  $y$  and for  $q^0 < q^1$  we have  $u(q^0, y) < u(q^1, y)$  because the status quo is always preferred to the net loss of the public good.

Let us set  $z(q^1) - z(q^1 - \Delta) = s$  with  $s > 0$  being the compensation equal to WTA. With  $z(q^1)$  being the supremum for  $u(q^1, y)$  and  $z(q^1 - \Delta)$  being the supremum for  $u(q^1 - \Delta, y)$  is there  $y$  that gives  $u(q^1 - \Delta, y) > z(q^1 - \Delta)$  or  $u(q^1 - \Delta, y) > z(q^1) - s$ ?

We know that  $q^0 \leq q^1 - \Delta$  so for any  $q$  and  $y$  we have  $z(q^0) \leq z(q^1 - \Delta)$ . By definition  $z(q^1 - \Delta) = z(q^1) - s$  and  $z(q^0) + s \leq z(q^1)$ . Moreover, we also know that  $z(q^0) \geq u(q^0, y)$  because  $u(q, y) \leq z(q)$  so  $u(q^0, y) < z(q^0) + s$ .

As  $z(q^0) + s \leq z(q^1)$  we have  $z(q^0) + s \leq u(q^1, y)$  and  $z(q^0) + s < u(q^1 - \Delta, y)$ .

From the above we see that  $u(q^0, y) < z(q^0) + s < u(q^1 - \Delta, y)$  □

### Proof of Proposition 1.4.

Let us now prove that WTA is bounded within the behavioral model of loss aversion.

One way comes directly from the construction of the model: according to diminishing sensitivity, smaller changes in  $\Delta$  should be accompanied by smaller increases in  $y$ , the utility being constant, thus  $\partial^2 u / \partial q^2 \geq 0$ .

Another way is to look for a bound on the losses' side of the value function.  $\forall (q - \Delta) \in (0, r_q]$  and some value function  $v \in C(\mathbf{R}_+^*)$  on  $(0, r_q]$  which is concave and nonincreasing, one has:  $v(q - \Delta) \leq v'(r_q)(q - \Delta - r_q) + v(r_q)$ . The right-hand expression of the weak inequality is the tangent of  $v$  at  $r_q$ . When  $q - \Delta = 0$  it gives  $v(q - \Delta) \leq v'(r_q)(-r_q) + v(r_q)$  which is independent of  $q - \Delta$ . Hence, the losses' side of the value function is bounded. □





# Private Valuation of a Public Good in Three Auction Mechanisms

### Abstract

We evaluate the impact of three auction mechanisms – the Becker–DeGroot–Marschak (BDM) mechanism, the second-price auction, and the random  $n$ th-price auction – in the measurement of private willingness-to-pay and willingness-to-accept for a pure public good. Our results show that the endowment effect can be eliminated with repetitions of the BDM mechanism. Yet, on a logarithmic scale, the random  $n$ th-price auction yields the highest speed of convergence to welfare indices' equality. Overall, we observe that subjects value public goods in reference to their private subjective benefit derived from the public good funding.

Keywords: contingent valuation, WTP-WTA gap, auctions, public good private provision

JEL classification: C91, D44, Q53

"I have never known much good done by those who affected to trade for the public good." Adam Smith

## 2.1. Introduction

The experimental private provision of public goods based on the contingent valuation method is often used to value public goods such as health, safety or environment. Estimating preferences for public goods is however laborious, for individuals reveal behavioral biases during their valuation process.

In accordance with the Coase theorem (Coase 1960), neoclassical theory postulates that with null income effect and close substitutes, the willingness-to-pay (WTP), which is the price at which an individual is ready to buy a commodity, and the willingness-to-accept (WTA), which is the price at which an individual is ready to sell the same commodity, should be equal (Randall and Stoll 1980, Hanemann 1991). If the good is available in an active market at the market price, an individual's WTP and WTA should be similar. And if people face similar transaction costs, WTP and WTA should be similar among people as well. Yet, experimental research that stemmed from contingent valuation studies has found large disparities between the WTP and WTA. The endowment effect, or loss aversion, as a behavioral feature is often invoked to explain the disparity. It occurs when people offer to sell a commonly available good in their possession at a substantially higher rate than they will pay for the identical good not in their possession. The other effect, promoted to explain the disparity, is imperfect substitutability.

Two remedies help remove the initial disparity. The first corresponds to market settings. Market institutions serve as social tools that induce and reinforce individual rationality (Smith 1991). Gode and Sunder (1993) assert that an auction market exerts a powerful constraining force on individual behavior. Cherry *et al.* (2003) suggest that a dynamic market environment with repeated exposure to discipline is necessary to achieve rationality. When they act rationally, individuals refine their statements of value. List (2003a) provides evidence consistent with the notion that experience in bidding with an incentive-compatible auction can remove

the WTA/WTP gap. The second corresponds to market repetition. The motive for repeating auctions that are incentive-compatible is that individuals require experience to understand that sincere bidding is the dominant strategy (Coppinger *et al.* 1980) and to realize their true valuation of unfamiliar products (Shogren *et al.* 2000). Plott (1996) advances a discovered preference hypothesis argument, positing that responses reflect a type of internal search process in which subjects use practice rounds to discover their preferences. The experience they gain is reflected in their bidding behavior. Hence, the imperfect substitutability effect disappears when the value of the unfamiliar good is perfectly revealed.

Market-based mechanisms such as auctions are widely studied as a means of buying and selling resources. Auctions took part in the environmental valuation to answer two questions: (1) which effect counts the most in the WTP and WTA disparity? and (2) which of the auction mechanisms best removes this disparity?

At first, Kahneman *et al.* (1990) report experimental evidence of the endowment effect. They perform an experiment on WTP and WTA by way of hypothetical telephone inquiry, trading environmental improvements and preparedness for disasters. They find that randomly assigned owners of an item require more money to separate from their possession than random buyers are willing to pay to acquire it. To elicit individuals' estimates, they use a Becker-DeGroot-Marschak mechanism (BDM) – described later on – with random exogenous price feedback. According to their results, preferences are dependent on endowments, even in market settings.

Shogren *et al.* (1994) assert that Kahneman *et al.*'s experiment creates artificial scarcity. They find no evidence of the endowment effect on trading candy bars, for the values converge over time. But, in the experiment with contaminated food – a good with imperfect substitutes that can be considered as nonmarketed – they show that the discrepancy remains significant after iteration.

Later on, Shogren *et al.* (2001) test three auction mechanisms to trade candy bars and mugs and suggest that the auction mechanism can itself account for the conflicting observations in experiments. In their experiments, they show that the common early disparity between WTP and WTA in auctions is not to be called into

question. However, the gap ebbs away under the Vickrey's second price auction (SPA) and random  $n$ th-price auction (NPA) – see Section 2 for further details – while it lasts under the BDM mechanism, implying that the endowment effect can be eliminated with repetitions of some market mechanisms.

Horowitz (2006a) states that the BDM framework could be used to assess public WTP for public projects, with the distribution of costs equal to the project costs; and other valuation mechanism should be used if the behavioral evidence shows that mechanisms are equivalent. Lusk and Rousu (2006) suggest that NPA is preferable to BDM if the researcher is looking for true valuation above all. Lusk *et al.* (2007) conclude in their study of payoff functions that BDM and NPA "provide relatively strong incentives for truthful bidding for all individuals regardless of the magnitude of their true WTP".

Seeing that findings suggest that the auction mechanism *per se* accounts for the conflicting observations across market settings, Plott and Zeiler's (2005) conclusion that the results differ from unsound experimental procedures is incomplete.

This chapter builds on Shogren *et al.*'s (2001) results. Which auction mechanism is the best and fastest at reducing the gap? Which mechanism should be preferred over another? While Shogren *et al.* (1994) support Hanneman's results, assuming that the low substitution elasticity for the nonmarket good explains the WTA/WTP gap, they do not advocate the institution capable of properly valuing nonmarket goods. Likewise, Shogren *et al.* (2001) use only private goods to compare the influence of auction mechanisms. Only List (2003b) gives credit to the use of the random  $n$ th-price auction in valuing nonmarket private goods, but he does not state whether his results carry over to public goods.

We aim at studying private valuation of public goods without direct substitutes, so we put realistic public goods such as the carbon offset, which can be attained *via* tree planting, into auctioning. Public goods have two defining characteristics: non-excludability and non-rivalry. Offsetting carbon emission helps prevent the effects of climate change; it is considered as a public good because, once provided, everyone can enjoy the benefits without adversely affecting anyone else's

ability to do the same<sup>9</sup>. Rather than compulsory carbon trade, we institute voluntary trade to approach truthful valuation on both the bidder's (buyer's) and the offerer's (seller's) sides. On account of the common bias of *nescience*<sup>10</sup> in valuing unfamiliar or public goods, we remind the subjects that they are part of the milieu, which makes them indirectly and partly accountable for the current level of greenhouse gases, as they solicit industries to produce goods they are willing to consume at some environmental cost; in our case, it is the paper and energy used by students to achieve their education<sup>11</sup>.

By means of repetitive auction mechanisms, the initial disparity between WTP and WTA can be removed. Nevertheless, we obtain different results from preceding studies, in a sense that only the BDM mechanism is able to remove the gap in later bidding rounds. SPA and NPA, which are also incentive-compatible, do not succeed in removing the disparity between bids and offers. Still, when we submit our experimental results to the exponential regression, we notice that in spite of a large early gap, NPA yields the highest speed of convergence to welfare indices' equality, suggesting that it contains strong incentives for rational behavior. In addition, we observe that subjects are strongly motivated by the subjective private benefit from funding the public good (either due to warm-glow<sup>12</sup> or to a concern for being formally identified as a contributor of the public good).

The remainder of this chapter proceeds as follows. Section 2.2 describes the experimental design. Section 2.3 presents results and the analysis of data through standard and novel statistical tools. Section 2.4 provides discussion on how our results relate with existing work and present a new line of reasoning. We give some concluding remarks in Section 2.5.

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<sup>9</sup> We insured the public good characteristic by providing to every subject, after couple of weeks, an email feedback on the aggregate offset achievement.

<sup>10</sup> It reflects the absence of knowledge or the consideration that things are unknowable.

<sup>11</sup> The money released from trading (buying and non-selling) was sent to a non-governmental organization that launched a plantation of 1,404 Mangrove trees in Sumatra, Indonesia.

<sup>12</sup> Utility derived from the warm-glow (see Andreoni 1990) arises when the act itself of giving generates utility. It contrasts with the usual case where the individual only cares about the total amount of the carbon offset.

## 2.2. The experimental design

We want to evaluate the impact of three incentive-compatible auction mechanisms in the measurement of WTP and WTA for a public good without substitutes. Our experiments were conducted during three sessions at the École Polytechnique. Different subjects took part in each of the three sessions (three types of auction mechanism) for a total of 102 participants, divided in three groups of subjects, which in turn were arbitrarily divided into two subgroups of buyers and sellers. Each subject received an identification number she filled in on each bid or offer, enabling her to be tracked whilst preserving her anonymity. The initial endowment distributed to the buyers was put forward to fund tree planting. On the WTP market-side, each buyer received EUR 15 and was asked to state her bid for a certificate of one ton of carbon offset ( $\leq$  EUR 15). If she won the bid, trees were planted in her name (this was acknowledged by a certificate). On the WTA market-side, each seller was given a certificate of one ton of carbon offset she could keep, in which case trees were planted in her name, or sell. If she decided to sell the certificate on the offer she stated ( $\leq$  EUR 15), no trees were planted. Subjects ignored that the cost of offsetting one ton of carbon in a five-year period was EUR 15, which enabled to plant 36 trees<sup>13</sup>.

The parameters – recapitulated in the table below – of the experiments are the following: (i) 31 to 37 subjects participated per experiment; (ii) subjects were recruited among the voluntary students from the École Polytechnique<sup>14</sup>; (iii) the good put in auctioning was a certificate of one ton of carbon offset; (iv) none information on price was provided; (v) subjects received an initial balance of EUR 15 or a certificate of one ton of carbon offset as an endowment; (vi) ten trials per experiment were unfolded, one of which was randomly selected as the binding trial; and (vii) BDM, SPA and NPA auction mechanisms were tested.

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<sup>13</sup> In accordance with the system of reference applied by the non-governmental organization.

<sup>14</sup> Multi-cultural elite undergraduate students in science and engineering, salaried by the French Government. Their curriculum includes economics courses.

Comments on the experimental protocol: our goal is to question auction mechanisms' influence on the gap between WTP and WTA, and not to divulge the gap itself, for we consider it as an established fact, so we decide to put an upper-bound on the sellers' choices in order to monitor which of the three market settings best replies to the early disparity. The bounds and endowments definitely create an anchoring effect, but there is no reason that it affects differently the three incentive-compatible mechanisms. Then, we publicly suggested to the subjects that revealing truthful preferences is a neutral strategy which will not penalize them. At last, we pooled all performed rounds in the measurement of the gap.

<b>Market environment</b>	<b>BDM</b>	<b>SPA</b>	<b>NPA</b>
Auctioned goods	CO <sub>2</sub> offset certificate	CO <sub>2</sub> offset certificate	CO <sub>2</sub> offset certificate
Initially endowment	EUR 15	EUR 15	EUR 15
Sellers' bound	EUR 15	EUR 15	EUR 15
Number of trials	10	10	10
Retail price information	None provided	None provided	None provided
Optimal responses explained	Suggested	Suggested	Suggested
Practice round performed	Pooled	Pooled	Pooled
Subject participation	Voluntary	Voluntary	Voluntary
Number of subjects	37	34	31

*The Becker–DeGroot–Marschak mechanism (BDM)*

Becker, DeGroot, and Marschak (1964) introduce a mechanism under which buyers (respectively sellers) simultaneously state the highest (respectively lowest) amount they are willing to pay (respectively accept) for the good. In our experiment, each buyer and seller was asked to give, for each of the ten trials, independently and privately, her WTP or WTA by marking an "x" on a recording sheet that listed price intervals, such as in the following illustration. The price intervals ranged from EUR 1–15, in increments of EUR 0.5. After collecting recording sheets from buyers and sellers, the monitor randomly selected one price from the list. If a buyer was willing to pay at least the random price for the certificate of one ton of carbon offset, she bought the item at that price. Otherwise, she did not buy the item. If a seller was



willing to accept a price lower than or equal to the random price for the certificate of one ton of carbon offset, she sold the item at that price. Otherwise, she did not sell the item.

	I will buy (sell)	I will not buy (sell)
If the price is EUR 0.0	--	--
If the price is EUR 0.5	--	--
If the price is EUR 1.0	--	--
If the price is EUR 1.5	--	--
...		
If the price is EUR 14.0	--	--
If the price is EUR 14.5	--	--
If the price is EUR 15.0	--	--

The random price, all bids and offers, and the number of buyers and sellers willing to buy and sell at the random price were made public after each trial. At the end of the experiment, one of the trials was randomly selected as the binding trial for the take-home pay.

*The second-price auction mechanism (SPA)*

Under the Vickrey (1961) second-price auction, bidders and offerers operated simultaneously. Buyers were asked to record, for each of the ten trials, privately and independently, the maximum they were willing to pay for the certificate of one ton of carbon offset. In this case, buyers wrote a numerical value on the recording sheet. The monitor collected values and, after each trial, made all bids public, as well as the identification number of the highest bidder and the market-clearing price (second highest bid). The monitor gave each seller a certificate of one ton of carbon offset. For each trial, sellers wrote their minimum WTA to sell the certificate. After each trial, the monitor publicly diffused all offers, the identification number of the lowest offerer and the market-clearing price (second lowest offer). Like with BDM, after the tenth trial, the monitor randomly selected one of the trials as the binding trial for the take-home pay for both buyers and sellers.

### *The random $n$ th-price auction mechanism (NPA)*

The random  $n$ th-price auction is conducted as follows (bidders and offerers operate simultaneously): (i) for each trial, each bidder submits a bid (resp. an offer) on a recording sheet; (ii) all bids are ranked from lowest to highest, all offers are ranked from highest to lowest; (iii) the monitor selects a random number  $n \in (2, N]$  with  $N$  the number of bidders; (iv) the  $n-1$  buyers who made the highest bids buy the certificate of one ton of carbon offset at the  $n$ th-price and the  $n-1$  sellers who made the lowest offers sell the certificate of one ton of carbon offset at the  $n$ th-price. The value of  $n$ , all bids and offers, the buying and selling price, and the number of buyers and sellers willing to buy and sell at the random price, are made public after each trial. Once again, after the tenth trial, the monitor randomly selects one of the trials as the binding trial for the take-home pay for both buyers and sellers.

The BDM, SPA and NPA mechanisms are incentive-compatible. It is not in a buyer's interest to understate her WTP; if the random buying price falls between the stated WTP and the true WTP, the buyer foregoes a beneficial trade. It is also not in a buyer's interest to overstate true WTP; if the random buying price is greater than the true value but less than the stated value, the buyer is required to buy the good at a price greater than her true WTP. The reasoning is identical for the seller.

A complementary remark on NPA can be made. Contrary to SPA, subjects have a nonnegative probability of winning the auction, which engages off-margin bidders and offerers who usually consider that they will be excluded from the market. As well, the endogenously determined market-clearing price prevents bidders and offerers from using the random market-clearing price as an indicator.

### **2.3. The results**

Table 2.1. presents the summary statistics of the experimental results under BDM, SPA and NPA. In all experiments, bidding behavior in the initial trial does

**Table 2.1. Summary statistics of the BDM, SPA and NPA mechanisms**

Auction	Value measure		Trial										
			1	2	3	4	5	6	7	8	9	10	
<b>BDM</b>	WTP N=19	Mean	6.18	7.11	7.82	8.11	8.29	8.66	8.39	8.71	8.82	8.61	
		Median	5.00	5.50	6.50	6.50	7.00	7.00	7.00	7.50	7.50	7.50	
		Variance	12.51	15.52	15.39	15.43	15.09	15.86	15.27	14.62	14.37	17.74	
	WTA N=18	Mean	10.53	9.47	9.56	8.42	8.92	8.69	9.53	9.19	8.67	8.06	
		Median	10.00	10.00	10.00	8.75	9.50	9.75	10.00	10.00	9.75	8.25	
		Variance	6.07	12.34	18.03	18.60	20.95	21.53	19.75	16.86	17.79	20.97	
	Ratio of mean WTA/WTP			1.70	1.33	1.22	1.04	1.08	1.00	1.13	1.06	0.98	0.94
	<i>t</i> -test of means <sup>a</sup>			-3.85	-1.46	-0.83	0.27	0.06	0.46	-0.39	0.09	0.58	0.91
	<b>SPA</b>	WTP N=17	Mean	3.47	3.91	4.69	5.43	5.68	5.71	6.01	6.50	5.46	6.59
Median			3.00	4.10	5.00	5.60	5.80	6.05	7.00	7.00	7.00	7.00	
Variance			9.64	6.68	5.52	5.42	6.15	7.71	8.86	14.50	12.56	10.04	
WTA N=17		Mean	10.66	8.74	8.47	9.07	8.59	9.82	9.40	8.32	9.52	9.23	
		Median	10.00	9.00	8.00	9.00	7.00	10.00	8.00	8.00	8.00	8.00	
		Variance	16.60	19.56	14.03	22.27	20.72	29.45	29.44	32.86	26.44	30.86	
Ratio of mean WTA/WTP			3.07	2.23	1.81	1.67	1.51	1.72	1.57	1.28	1.75	1.40	
<i>t</i> -test of means <sup>a</sup>			-5.28	-3.41	-3.06	-2.35	-1.78	-2.30	-1.78	-0.59	-2.21	-1.20	
<b>NPA</b>		WTP N=15	Mean	3.97	3.98	4.77	4.93	4.77	5.19	6.18	6.12	6.85	6.72
	Median		2.50	4.00	5.00	5.12	5.14	5.01	7.00	6.50	7.00	7.26	
	Variance		12.67	6.92	4.83	4.30	5.40	6.33	5.81	6.54	7.77	10.03	
	WTA N=16	Mean	10.75	10.52	10.29	10.22	9.86	9.05	9.17	9.14	9.23	9.37	
		Median	10.50	10.00	9.74	9.65	8.77	8.50	8.49	8.35	8.09	8.50	
		Variance	10.19	6.99	6.32	9.46	10.31	13.75	16.67	13.30	14.08	20.64	
	Ratio of mean WTA/WTP			2.71	2.64	2.16	2.07	2.07	1.74	1.48	1.49	1.35	1.39
	<i>t</i> -test of means <sup>a</sup>			-5.06	-6.45	-6.21	-5.17	-4.60	-2.87	-1.90	-2.10	-1.40	-1.33

$H_0$ : Mean WTP – Mean WTA = 0;  $H_1$ : Mean WTP – Mean WTA < 0

<sup>a</sup> *t*-test: reject  $H_0$  at the 5% level

not contradict the endowment effect: the mean offer  $\overline{WTA}$ <sup>15</sup> is significantly greater than the mean bid  $\overline{WTP}$ <sup>16</sup>. Still, with experience gained through repetitive auctioning under the BDM mechanism,  $\overline{WTA}$  offers decrease and  $\overline{WTP}$  bids increase over time<sup>17</sup>. The  $\overline{WTA}/\overline{WTP}$  ratios thus decline throughout the ten trials, falling from 1.70 in trial 1 to 0.94 in trial 10 (see Fig 2.1.), which corresponds to  $\overline{WTP}$  increase of 39% and  $\overline{WTA}$  decrease of 23%. Concerning variances, we notice that the dispersion around the mean increases for both  $\overline{WTP}$  (42%) and  $\overline{WTA}$  (245%) from trial 1 to trial 10. In trials 4–10, a *t*-test shows that we cannot reject the null hypothesis that  $\overline{WTP}$  and  $\overline{WTA}$  come from the same distribution at the  $p < 0.05$  level. Under BDM, value measures are not statistically different, signifying that the disparity fades away.

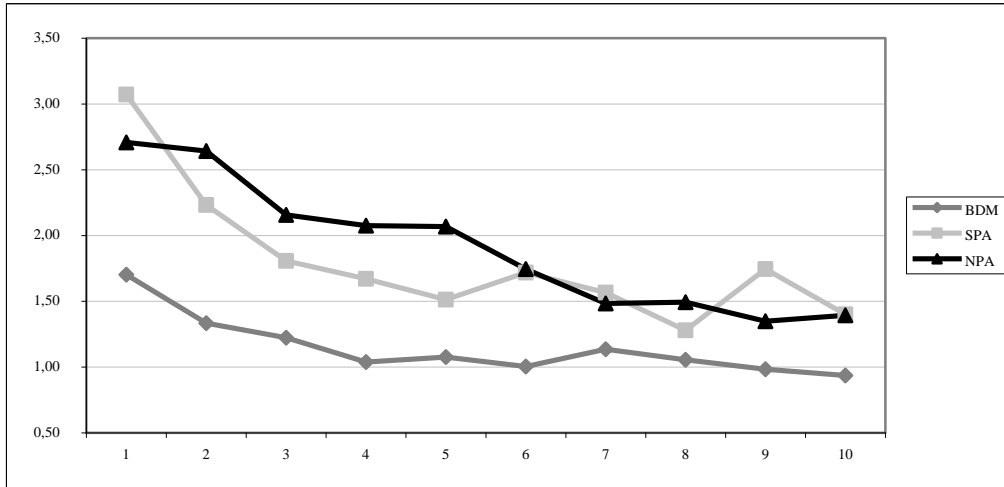
Under NPA and SPA, the mean selling price exceeds the mean buying price for all ten trials. This also holds for the median bids. We observe similar starting and ending values of the welfare indices. The  $\overline{WTA}/\overline{WTP}$  ratios remain above one, ranging from 1.35 to 2.71 under NPA, and from 1.28 to 3.07 under SPA (see Fig. 2.1.). Bids respectively increase by 69% and 90%; offers decrease by 13% in both experiments. The dispersion around  $\overline{WTP}$  follows a different path under NPA and SPA. The dispersion around  $\overline{WTA}$  amplifies under both auction mechanisms from trial 1 to trial 10 (NPA: 103%; SPA: 86%). On the contrary, the dispersion around  $\overline{WTP}$  remains quasi-stationary under SPA (4%) but decreases under NPA (–21%) throughout the trials, which suggests a degree of homogenization between the bids. In all trials, we reject the null hypothesis that  $\overline{WTP}$  and  $\overline{WTA}$  are equal at the 5% level of a *t*-test. However, we point out that ratios decrease over time approaching the value of one in latter trials.

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<sup>15</sup> The over-bar signifies mean value.

<sup>16</sup> This is also confirmed by the analysis of the medians.

<sup>17</sup> Though they never reach the outside market price, *i.e.* the upper bound of EUR 15, such as in Bohm *et al.* (1997).



**Fig. 2.1.**  $\overline{WTA}/\overline{WTP}$  disparity from trial 1 to trial 10

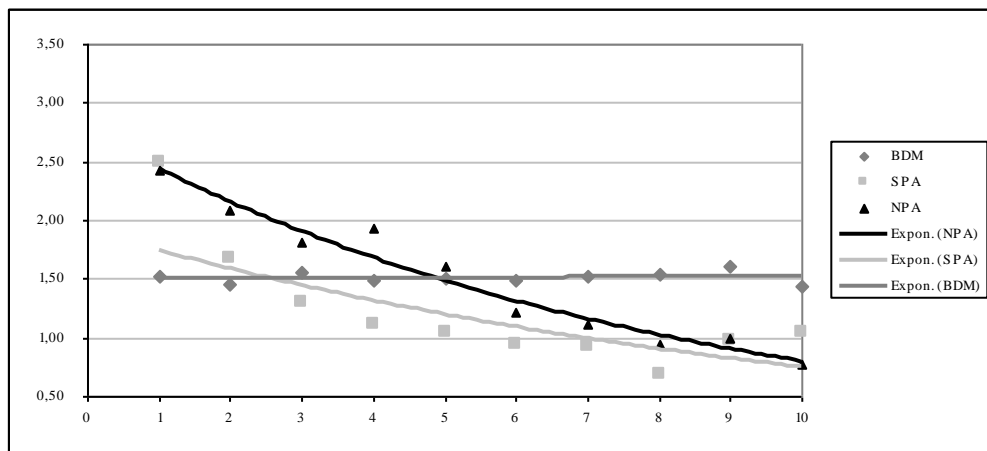
Let us now take a further insight in our results and those of the mug experiments from Shogren *et al.* (2001). At first sight, we obtain contradictory results. In our experiment, the gap disappears under BDM, whereas in theirs, BDM is the only mechanism unable to remove the early gap.

Our findings show that repetitions under the BDM mechanism can remove the endowment effect, as long as it steers people's behavior. Likewise, they suggest that the auction mechanism *per se* can account for the conflicting observations, as we clearly observe different paths of equalization of  $\overline{WTP}$  and  $\overline{WTA}$ . We introduce an innovative tool to study the path of gap removal: the exponential regression on the  $\overline{WTA}/\overline{WTP}$  ratios.

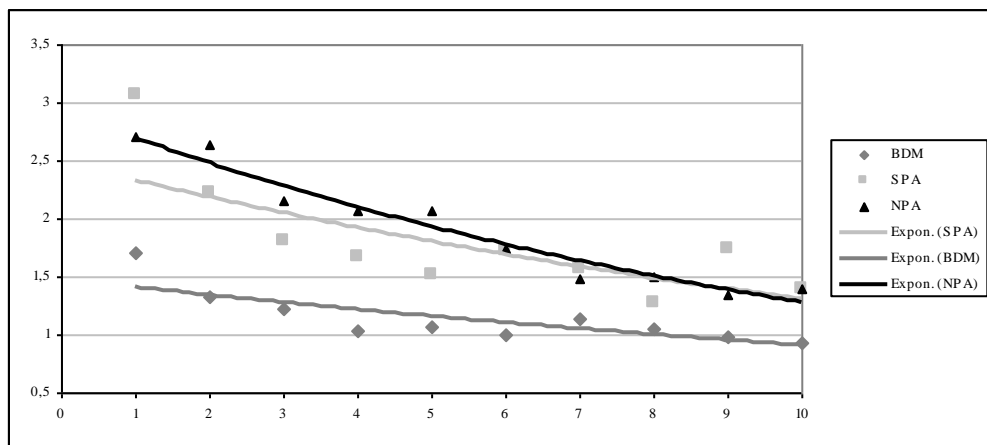
An exponential regression is of a form  $y = be^{ax}$  with  $x$  the variable along the  $x$ -axis,  $y$  the regressed values of  $\overline{WTA}/\overline{WTP}$ ,  $a$  the amplitude of the decrease (or speed of convergence to equality) and  $b$  the  $y$ -intercept of regression. The function is based on the function linear regression, with the  $y$ -axis logarithmically scaled.  $R$ -square gives information on the exponential relationship between ratios.

We apply this method to Shogren *et al.*'s (2001) mug experiments (see Fig. 2.2.) and to our experiments (see Fig. 2.3.). The exponential regression is

used for two reasons: first, it allows observing phenomena with rapid variations, such as in our experiments; second, it allows observing the decrease path to equality, that is, the way ratios tend to one. We try to unearth the mechanism able to remove the gap as quickly as possible, whatever the initial ratio. We can thus consider the highest coefficient of decrease as the highest speed of convergence to welfare indices' equality (see Table 2.2.).



**Fig. 2.2. Exponential regression of  $\overline{WTA} / \overline{WTP}$  disparity from Shogren *et al.*'s (2001) mug experiments**



**Fig. 2.3. Exponential regression of  $\overline{WTA} / \overline{WTP}$  disparity**

Shogren *et al.*'s (2001) data from BDM provides no exponential relationship between sequential ratios, but ours does. Although the  $y$ -intercept of regression starts with the same value (both 1.5), the gap disappears in our experiment (illustrated by the speed of convergence  $-0.04$ ) but stays stationary in the mug experiment (null speed of convergence).

We find in both data that NPA provides the best exponential relationship between ratios (respectively  $R^2 = 0.95$  and  $R^2 = 0.96$ ) and the highest speed of convergence to equality (respectively  $-0.08$  and  $-0.12$ ) in time. Under SPA, the exponential relationship between ratios (respectively  $R^2 = 0.61$  and  $R^2 = 0.63$ ) and the speed of convergence to equality (respectively  $-0.06$  and  $-0.09$ ) are significant but lower. Sudden leaps of increase of the  $\overline{WTA}/\overline{WTP}$  ratio under SPA – due to off-margin bidders – explain the differences in  $R^2$  with regard to NPA. It is worthwhile noticing that SPA comes out as the "worst" active market mechanism even though it is frequently used in experimental environments to reveal agents' preferences. Under BDM, our experiment and Shogren *et al.*'s (2001) experiment both obtain the lowest results in terms of exponential relationship<sup>18</sup> and speed of convergence to equality. Therefore, the orderings of convergence in our experiments and those of Shogren *et al.*'s (2001) are alike.

**Table 2.2. Exponential regression statistics**

Auction	Regression statistics	Our experiments	Shogren <i>et al.</i> 's mug experiments
BDM	Speed of convergence ( $a$ )	$-0.04$	$-0.00$
	$y$ -intercept of regression ( $b$ )	1.5	1.5
	$R$ -square	0.69	0.00
SPA	Speed of convergence ( $a$ )	$-0.06$	$-0.09$
	$y$ -intercept of regression ( $b$ )	2.5	1.9
	$R$ -square	0.61	0.63
NPA	Speed of convergence ( $a$ )	$-0.08$	$-0.12$
	$y$ -intercept of regression ( $b$ )	2.9	2.8
	$R$ -square	0.95	0.96

<sup>18</sup> The low exponential factor with the BDM is partially explained by the initial smaller difference between WTP and WTA.

If the initial gap is due to the choice of the market mechanism, then the choice of BDM is appropriate, for it produces the smallest initial gap. But, if we are to urge the auction mechanism able to rapidly deflate an excessive initial  $\overline{WTA}/\overline{WTP}$  gap in a market-clearing price setting, we suggest the use of NPA which involves most of the bidders in the auctioning. Indeed, as for the model of exponential regression, the BDM mechanism would not have equalized the welfare indices if the starting ratio were more of SPA or NPA's magnitude. This appears all the more sound, provided the BDM mechanism is a passive market-like setting with only minor adjustments in bidding behavior<sup>19</sup>. Indeed, NPA applies competitive pressure to the participating bidders. A bidder cannot avoid acting strategically since her best bid depends on the competing bids. By bidding more aggressively, the bidder improves her chances of winning the auction. As far as SPA is concerned, the unevenness in the decrease of the gap jeopardizes its robustness.

#### 2.4. Discussion

Our results support the standard thesis that market mechanisms can remove or at least sturdily reduce the initial disparity between WTP and WTA. However, some points need to be clarified.

Let us first focus on the specificity of the good in sale. Under NPA and SPA, the number of traded tons of carbon offset in a period is independent of the bids and offers submitted by the subjects. In any case, in SPA, one ton of carbon offset is bought and sold; in NPA,  $n-1$  tons of carbon offset are bought and sold. As a result, free-riding is likely to occur, since a subject's bid cannot affect the total public good provision while it affects her payment (buying a certificate is costly). On the contrary, under BDM, subjects' choices affect the total provision of public good. Indeed, if a seller chooses a minimum selling price higher than the randomly selected price, she will keep her certificate and one more ton of carbon

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<sup>19</sup> See footnote 7 in Shogren *et al.* (2001).



will be offset. The same reasoning applies for buyers. Put differently, subjects know they can influence the amount of carbon offset under BDM as their probability of winning the right to buy one certificate is independent of other bidders: the higher the private bid, the higher the chances that a ton of carbon is offset.

This difference between BDM on the one side and NPA and SPA on the other side allows identifying two distinct motivations for funding the public good. First, there is "the public good motivation": a subject wants to buy or keep a certificate because it allows offsetting one ton of carbon for the community. Second, there is "the private good motivation": a subject wants to buy or keep a certificate because she wants to own a certificate and be associated to the offsetting even though this does not change the number of tons of carbon offset (she either wants to derive a warm-glow from altruism or wants to gain social status through the public good funding). Individuals often provide more public goods than traditional economic theory predicts. Public goods are then considered as impure public goods, which are products or services that combine both public and private benefits.

In BDM, both motivations for funding the public good are present, whereas in NPA and SPA, only the private good motivation is present. Let us consider  $\bar{s}$  – the mean value of all bids (WTP) and offers (WTA) – as the mean value of the public good. After computation, we observe that  $\bar{s}$  over the ten rounds is strictly higher with BDM (8.57) than with SPA (7.26) or NPA (7.63). Locally, at the last period, the values are respectively 8.34, 7.91 and 8.09. These results indicate that the private good motivation is extreme compared to the public good motivation, *i.e.* subjects are mainly paying for enjoying warm-glow or being identified as contributors of the carbon offsetting. If we take  $\bar{s}$  of BDM as a benchmark value of the public good, the surplus of the BDM value compared to SPA and NPA values corresponds to the value of the public good motivation which lies in the interval  $[0.94, 1.31]$ . Since the public benefit for an individual is negligible, individuals mostly derive some private benefit from the public good.

These results are thus consistent with microeconomic analysis, where the private benefit governs the decisions of economic agents.

Contrary to the observations where repeat-play public good games produce declining contribution over time (see Andreoni (1988) and Caldas *et al.* (2003)),  $\bar{s}$  is increasing in our experiments. As a matter of fact, if we regress  $\bar{s}$  with the number of periods, we obtain a small but strictly positive correlation coefficient (BDM: 0.18; SPA: 0.13; NPA: 0.15). In standard public good games, the fall is motivated by free-riding and discouragement of high type players to pursue alone the provision of public good. We propose two explanations for the rise we observed. First, the funded public good does not only concern the subjects but the population outside the experiment. Therefore, the free-riding attitude of some participants does not alter subjects' motivations since they do not specifically contribute for these free-riders (while they do in public good games). Second, as already mentioned, the private good motivation outperforms the public good motivation, which also explains the absence of the usual decline in subjects' bids<sup>20</sup>.

For all these reasons, we decide to focus on the private value dimension of the public good in the following discussion.

Contrary to NPA or SPA, the initial gap under the BDM mechanism is closer to one in both Shogren *et al.*'s (2001) and our experiments. As  $\overline{WTA}$  is similar under the three auction mechanisms in the first trial, this observation comes from a high starting  $\overline{WTP}$  under BDM, *i.e.* shorter distance to cover from bids to offers. Given that BDM and NPA both share the properties of incentive-compatibility and the possibility for every bidder to offset a ton of carbon, the explanation could come from the unambiguous distribution of prices and payoffs under the BDM mechanism, whereas under NPA there is ambiguity in view of the unknown bids of the opponents (see Sarin and Weber (1993)).

Another explanation could come from the theory of disappointment aversion. In a recent article, Horowitz (2006b) relates that under BDM an

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<sup>20</sup> One could argue that bids increased because of the house money effect. However, Clark (2002) finds no evidence of it in a public good experiment.

individual may report a higher value than the true one, simply because she is more disappointed from not receiving the good than from receiving it at a relatively high price, which induces her to report a higher bid to increase the chance of winning the auction. This could explain why the bids under BDM started higher earlier than the bids under NPA and SPA; subjects knew from the very beginning that they were bidding against a market-clearing price issued from a known ceiling market-clearing price.

Let us also mention a proposition from Milgrom and Weber (1982) that could be spoken for our results. The authors state that common uncertainty about the value of a good creates affiliated private values, especially in case of unfamiliar goods. This is because early trials send information from which high-bidders induce low-bidders to revise their preferences and increase their bids, the logic being that there are some common but unknown characteristics of the item released with bids. Our experimental protocol does not permit to validate or invalidate this hypothesis, but we can specify that all subjects received the same amount of information on the nature of the unfamiliar good before the auction took place<sup>21</sup>. Although the mimesis phenomenon could explain rising low bids under SPA and NPA just after the start-off, our BDM experiment shows higher early bids; therefore, the logic of common uncertainty could only relate to the latter bidding rounds. Moreover, the dispersion around the mean from trial 1 to trial 10 increases in all experiments, partly refuting the argument of affiliated private values. The only case of dispersion fall that could challenge independent values' validity deals with  $\overline{WTP}$  in the NPA mechanism.

## 2.5. Concluding remarks

We examined three mechanisms that could rectify the initial gap between WTP and WTA in the trading of a public good. From simple observations of the disparity ratios, we observe different results from Shogren *et al.* (2001) and can

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<sup>21</sup> The market price effect, implied by affiliated private values, disappears when bidders receive nonprice information about the good before the experience is conducted (List and Shogren 1999).

conclude either that their findings – which suggest the validity of SPA and NPA in valuing private goods – are local, or that the public goods are subject to a different bidding behavior.

We think that under a quasi-market setting such as the BDM mechanism subjects understood the fact that they could influence the level of the public good and behaved accordingly. In active markets with endogenous market-clearing prices such as NPA, no subject could influence the level of the public good which acted as a disincentive to augment the level of public good. Our results show a disparity dropped with repetition under the three mechanisms, suggesting that the economic theory of rationality within markets operates. And yet, the theory implies a perfect equality between WTP and WTA, which seems not to be guaranteed when funding a public good. Research must deal with this.

Value measures approached equality principally for the reason that bids considerably increased throughout trials. Since offers moderately decreased in time, signifying a modest remedy to loss aversion, we could think of markets as systems which lift the subjects' regret not to acquire the good. Two-sided market value would then be somewhere between the behavioral exaggerations of loss aversion and disappointment aversion. These unforeseen questions necessitate further research.

In addition, more experimental research on private and public values of a public good should be conducted. For example, we could identify more accurately the private good and public good motivations by explicitly insisting on the fact that bids cannot affect the size of the provision of public goods in NPA and SPA. As well, we could conduct experiments where subjects would be purposely deprived from any proof of having financed the public good; that way, we could distinguish between the desire to finance the public good and the desire to be identified by others as a generous contributor to the public good.

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## 2.7. Appendix

You are about to participate in an experiment about decision making. You are not allowed to speak to your neighbors during the experiment.

All human activities release greenhouse gases, including CO<sub>2</sub>, that provoke the global warming. This warming endangers the planet, its inhabitants, its ecosystems and biodiversity. One way to fight against global warming is to plant trees. The key elements are the following: the forested surfaces are a carbon trap; young forests store much more carbon than old forests, for trees absorb CO<sub>2</sub> as they grow; forests preserve plant and animal biodiversity.

An NGO has launched a project of carbon offsetting by funding the reforestation projects. The purpose is to offset carbon emissions by buying off your own emissions. The compensation is acknowledged by a certificate of one ton of carbon offset.

During your education at the École Polytechnique ParisTech, you have received and printed, and will certainly do it over in the future, number of documents required for your schoolwork; it is also the case with your consumption of energy (such as light, heating, power supply for computers, etc.) Because you are contributing to the emissions through your consumption of paper and energy *via* your indirect demand for their manufacturing and distribution, we want to value your willingness to buy off your CO<sub>2</sub> emissions.

To this end, we will use a mechanism of purchasing and selling certificates of one ton of CO<sub>2</sub> offset, such as the ones we currently hold in our hands.

In couple of weeks, we will get in touch with you by email to inform you about the number of offset tons of CO<sub>2</sub> according to your decisions.

We will now conduct an experiment. As you came into the class, some of you were designated as sellers while others were designated as buyers. Indeed, each of you randomly drew a number which decided between buyer and seller. Please keep this number until the end of the experiment: it will serve us to track you on the information cards. In the end of the experiment, during the imbursement, please give us back your numbers.

Only one trial will be binding. We will repeat the experiment ten times. After the tenth trial, the youngest person in the room will randomly draw a number between 1 and 10, which will designate the binding trial.

Please feel free to interrupt us and ask any question you might have in mind.

Without further delay, we are going to read you the instructions concerning the conduct of the experiment. Let us start with those of you who are buyers.



## BDM MECHANISM

### Buyers

You own EUR 15. You can now participate in an auction in order to buy a certificate of one ton of CO<sub>2</sub> offset. If that is your wish, please submit a bid. The bid you submit can range between EUR 0 and EUR 15. If you decide to buy the certificate, trees which are planted on your behalf (acknowledged by your name on the certificate) will compensate one ton of CO<sub>2</sub>.

**To submit a bid, please fill in the following table and mark an "x" for each price at which you are (and are not) willing to buy the certificate.**

Rules: your maximum bid is ranked among all bids. Bids are classified in ascending order. We randomly select one price from the price list, which becomes the displayed price. You buy a certificate if your bid is higher than or equal to the displayed price.

Example: We randomly draw EUR 6. Since your bid is higher than or equal to EUR 6, you buy the certificate and pay EUR 6.

	I will buy	I will not buy
If the price is EUR 0	x	
If the price is EUR 0.5	x	
If the price is EUR 1.0	x	
...	...	
If the price is EUR 8.5	x	
If the price is EUR 9	x	
If the price is EUR 9.5		x
...		...
If the price is EUR 14.0		x
If the price is EUR 14.5		x
If the price is EUR 15.0		x

*Nota bene:* the higher your bid, the higher your chances of buying the certificate. Since you ignore the displayed price *ex ante*, giving your own value of one ton of CO<sub>2</sub> offset enables you to buy the certificate if your value is higher than the displayed price, and prevents you from buying otherwise.

### Sellers

You own a certificate of one ton of CO<sub>2</sub> offset. You can now participate in an auction in order to sell your certificate. If that is your wish, please submit an offer. The offer you submit can range between EUR 0 and EUR 15. If you decide to sell the certificate with your name on, no ton of CO<sub>2</sub> will be offset.

**To submit an offer, please fill in the following table and mark an "x" for each price at which you are (and are not) willing to sell the certificate.**

Rules: your minimum offer is ranked among all offers. Offers are ranked in descending order. We randomly select one price from the price list, which becomes the displayed price. You sell a certificate if your offer is lower than or equal to the displayed price.

Example: We randomly draw EUR 10. Since your offer is lower than or equal to EUR 10, you sell the certificate and earn EUR 10.

	I will sell	I will not sell
If the price is EUR 15.0	x	
If the price is EUR 14.5	x	
If the price is EUR 14.0	x	
...	...	
If the price is EUR 5.0	x	
If the price is EUR 4.5	x	
If the price is EUR 4.0		x
...		...
If the price is EUR 1.0		x
If the price is EUR 0.5		x
If the price is EUR 0.0		x

*Nota bene:* the lower your offer, the higher your chances of selling the certificate. Since you ignore the displayed price *ex ante*, giving your own value of one ton of CO<sub>2</sub> offset enables you to sell the certificate if the displayed price is higher than your value, and prevents you from selling otherwise.

## **RANDOM NTH-PRICE AUCTION**

### **Buyers**

You own EUR 15. You can now participate in an auction in order to buy a certificate of one ton of CO<sub>2</sub> offset. If that is your wish, please submit a bid. The bid you submit can range between EUR 0 and EUR 15. If you decide to buy the certificate, trees which are planted on your behalf (acknowledged by your name on the certificate) will compensate one ton of CO<sub>2</sub>.

**To submit a bid, please specify on the information card the price at which you are willing to buy the certificate.**

Rules: your bid is ranked among all bids. Bids are classified in ascending order. We randomly select a number between 2 and  $n$  ( $n$  being the total number of bids). In other words, we randomly draw one of the bids and look at its rank. If your bid

is contained in  $n-1$  highest bids, you buy a certificate at the displayed price: the  $n$ th price.

Example: twenty bids are submitted. We randomly draw seven, that is, the seventh-highest bid in the increasing order. You buy a certificate at a displayed price (seventh-highest bid) if your bid is contained in the six highest bids.

*Nota bene:* the higher your bid, the higher your chances of buying the certificate. If your bid is randomly drawn, your bid becomes the displayed price imposed to the  $n-1$  highest bidders. Since you ignore the displayed price *ex ante*, giving your own value of one ton of CO<sub>2</sub> offset enables you to buy the certificate if your value is higher than the displayed price, and prevents you from buying otherwise.

## **Sellers**

You own a certificate of one ton of CO<sub>2</sub> offset. You can now participate in an auction in order to sell your certificate. If that is your wish, please submit an offer. The offer you submit can range between EUR 0 and EUR 15. If you decide to sell the certificate with your name on, no ton of CO<sub>2</sub> will be offset.

**To submit an offer, please specify on the information card the price at which you are willing to sell the certificate.**

Rules: your offer is ranked among all offers. Offers are ranked in descending order. We randomly select a number between 2 and  $n$  ( $n$  being the total number of offers). In other words, we randomly draw one of the offers and look at its rank. If your offer is contained in  $n-1$  lowest offers, you sell a certificate at the displayed price: the  $n$ th price.

Example: twenty offers are submitted. We randomly draw six, that is, the sixth-lowest offer in the decreasing order. You sell your certificate at a displayed price (sixth-lowest offer) if your offer is contained in the five lowest offers.

*Nota bene:* the lower your offer, the higher your chances of selling the certificate. If your offer is randomly drawn, your offer becomes the displayed price imposed to the  $n-1$  lowest offers. Since you ignore the displayed price *ex ante*, giving your own value of one ton of CO<sub>2</sub> offset enables you to sell the certificate if the price is higher than your value, and prevents you from selling otherwise.

## **SECOND-PRICE AUCTION**

### **Buyers**

You own EUR 15. You can now participate in an auction in order to buy a certificate of one ton of CO<sub>2</sub> offset. If that is your wish, please submit a bid. The bid you submit can range between EUR 0 and EUR 15. If you decide to buy the

certificate, trees which are planted on your behalf (acknowledged by your name on the certificate) will compensate one ton of CO<sub>2</sub>.

**To submit a bid, please specify on the information card the price at which you are willing to buy the certificate.**

Rules: your bid is ranked among all bids. Bids are classified in ascending order. If your bid is the highest, you buy a certificate at a displayed price: the second-highest bid.

Example: ten bids are submitted. The highest bid is EUR 13. The second highest bid is EUR 11. The bidder who proposed EUR 13 buys the certificate and pays EUR 11.

*Nota bene:* the higher your bid, the higher your chances of buying the certificate. Since you ignore the displayed price *ex ante*, giving your own value of one ton of CO<sub>2</sub> offset enables you to buy the certificate if your value is higher than the displayed price, and prevents you from buying otherwise.

## **Sellers**

You own a certificate of one ton of CO<sub>2</sub> offset. You can now participate in an auction in order to sell your certificate. If that is your wish, please submit an offer. The offer you submit can range between EUR 0 and EUR 15. If you decide to sell the certificate with your name on, no ton of CO<sub>2</sub> will be offset.

**To submit an offer, please specify on the information card the price at which you are willing to sell the certificate.**

Rules: your offer is ranked among all offers. Offers are ranked in descending order. If your offer is the lowest, you sell a certificate at a displayed price: the second-lowest offer.

Example: ten offers are submitted. The lowest offer is EUR 5. The second lowest offer is EUR 7. The seller who proposes EUR 5 sells her certificate and earns EUR 7.

*Nota bene:* the lower your offer, the higher your chances of selling the certificate. Since you ignore the displayed price *ex ante*, giving your own value of one ton of CO<sub>2</sub> offset enables you to sell the certificate if the displayed price is higher than your value, and prevents you from selling otherwise.



# Endogenous Market-Clearing Prices and Reference Point Adaptation

### Abstract

When prices depend on the submitted bids, *i.e.* with endogenous market-clearing prices in repeated-round auction mechanisms, the assumption of independent private values that underlines the property of incentive-compatibility is to be brought into question; even if these mechanisms provide active involvement and market learning. In its orthodox view, adaptive bidding behavior imperils incentive-compatibility. We introduce a model which shows that bidders bid according to the anchoring-and-adjustment heuristic, contingent on a sequential weighting function, which neither ignores the incentive-compatibility constraints nor rejects the posted prices issued from others' bids. By deviating from their anchor in the direction of the public signal, bidders operate in a correlated equilibrium.

Keywords: auctions, incentive-compatibility, rank-dependence, reference point, heuristic, bounded rationality, correlated equilibrium

JEL classification: C73, D44, D81, D83

"Verum esse ipsum factum<sup>22</sup>."  
Giovanni Battista Vico.

### 3.1. Introduction

To know how much an individual is willing to pay for some item or for the provision of public services, and to assess how individuals would behave in the real world, economists now learn from experiments of repeated-round auctions. In this way, experimental auctions have been used to examine economic issues such as the disparity between willingness-to-pay and willingness-to-accept (Kahneman *et al.* 1990, Shogren *et al.* 1994, Shogren *et al.* 2001a) or preference reversals (Cherry *et al.* 2003, Cox and Grether 1996).

In the presence of an active market, rational behavior ensues from repetition. In experimental repeated-round auctions, individuals repeatedly bid for the same good. One of the arguments supportive of repeating auctions is that practice allows bidders to learn about the auction format and form values in a market-like setting, which improves the accuracy of value estimates (Alfnes and Rickertsen 2003, Hayes *et al.* 1995, Lusk *et al.* 2001). Plott (1996) formulated the *discovered preference hypothesis* which says that preferences converge to the same underlying preferences – respectful of expected utility – regardless of the market mechanism. These underlying preferences are discovered after bidders repeatedly take decisions, receive feed-back on the outcomes of their decisions, and are given incentives to discover which actions best satisfy their preferences. Discovered preference hypothesis suggests an equality of mean bids across rounds. Since anomalies to standard theoretical requirements are the results of bidders' irrationality, only later market trials reveal the true preferences.

Experimentalists want individuals to reveal their preferences truthfully. Therefore they use incentive-compatibility constraints, where truthfully announcing private information is an optimal strategy for all individuals participating in the auction mechanism. Incentive-compatibility is dependent on

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<sup>22</sup> The true itself is made.

the restrictive assumption that individuals have independent private values. In strategic interactions under incomplete information, different types of bidders – such as high- or low-value types – select from a menu of strategies. In principle, incentive-compatibility forbids the possibility that a given type of bidder mimics the behavior of other types and adjusts her bids to theirs.

One of the critics against the incentive-compatibility is the argument of uncertainty (Horowitz 2006a). After an individual reports her bid, she faces uncertainty over her chances to win the auction and over the final cost she will incur. On the assumption that the absence of affiliation is verified, repeating auctions in experiments reduces the uncertainty faced by bidders, because repeated-rounds provide market feedback from which they learn their preferences and produce reliable value estimates.

Knetsch *et al.* (2001) find that bids are influenced by the choice of auction mechanism. They show that willingness-to-pay (WTP) bids submitted in the later rounds of a second-price auction are significantly higher than those submitted in the later rounds of a ninth-price auction. Shogren *et al.* (2001a) report that mean WTP bids increase in repeated second-price and random *n*th-price auctions, but not in a repeated BDM mechanism (the Becker-DeGroot-Marschak mechanism, described later on). Lusk and Rousu (2006) find that the BDM mechanism is less accurate than NPA (random *n*th-price auction, described later on) in generating bids consistent with true values and recommend the use of endogenous clearing-price mechanisms when estimating nonmarket goods and services. Indeed, under BDM, the price is determined separately from the bids, preventing interactions between bidders plus providing poor market learning. As such, bidders have no opportunity to perform in a competition that normally imposes discipline on their bidding behavior (Bohm *et al.* 1997). *Ergo* market anomalies and violations of economic theory are fostered (Lusk and Rousu 2006, Lusk and Shogren 2007). Still, only a default of interaction makes the independence of bids certain, as the probability of winning does not depend on others' preferences. Shogren and Hays (1997) assert that “the repeated signals sent by the endogenous market price



contaminate individual bids into unreliable and unreasonable beacons of true preferences”.

Under BDM, the distribution of clearing prices is often known in advance. When the price distribution is fixed in reference to the common endowment, the ambiguity of the potential price disappears. On the contrary, under NPA, the distribution of prices depends on what her opponents are ready to pay for the good. The  $n$ th highest bid will be linked to the highest value. A bidder thus bids as if she held the highest private value conditional on her subjective estimation of the distribution of her opponents’ private values; she assesses her opponents and their expected valuations for the good. As a result, a complementary issue on uncertainty appears: uncertainty over the bids of opponents. Of course, bidders should always bid sincerely because the randomness of the market-clearing price prevents them from fixing on a stable cost such as with BDM (Shogren *et al.* 2001*b*), but they are counter-incited to chase other bidders’ *true* valuations.

Several previous experimental studies advocate that affiliation between private values is factual. List and Shogren (1999) unearth affiliation between naïve bidders for new goods and influence of posted prices. Similarly, Bernard (2005) finds affiliation, loss of information about bidders’ initial values and recommends the use of single-round auctions. Indeed, if the object of the experiment is to elicit actual preferences and to test them for consistency, price information is a potential source of contamination (Cubitt *et al.* 2001). Cox and Grether (1996) discover that bids are positively correlated with previous market-clearing prices. Although it can simply prove interaction between the learning processes of different subjects, it can also be the result of imitation. Knetsch *et al.* (2001) and Cubitt *et al.* (2001) also report experimental results which imply that bids are influenced by observations of past prices and by expectations of future prices. They argue that the provision of price information in repeated auctions induces cross-subject contamination. This is all the more unsurprising, for posted prices are the norm, unlike bargaining (Hanemann 1994).

In this chapter, we relax the assumption of private values’ independence in the repeated-round auctions such as BDM and NPA, when the market-clearing

prices are made public at the end of each round. Instead of using game-theory learning models, we introduce a behavioral model that shows that bidders bid according to the anchoring-and-adjustment heuristic which neither ignores the rationality and incentive-compatibility constraints, nor rejects the posted prices issued from others' bids. Bidders simply weight information at their disposal and adjust their discovered value using reference points encoded in the sequential price weighting function. The general hypothesis is that selection among strategies is adaptive, in that a decision maker will choose strategies that are relatively efficient in terms of effort and accuracy as task and context demands are varied. For unfamiliar choices, individuals make up a decision rule at the moment they need to use it (Bettman 1988). Of particular interest is the finding that under time constraints, some heuristics are more accurate than a normative procedure such as expected value maximization (Payne *et al.* 1988). In fact, real people are cognitive misers: they tend to choose in the simplest way possible (Hanemann 1994). Put to the test, our model shows that bidders and offerers are sincere boundedly rational utility maximizers. Still, they act rationally even if they operate inside a correlated equilibrium. Instead of handling affiliation of values after market prices are revealed<sup>23</sup>, we prefer to speak in terms of reference point adaptation and posted prices' weighting mechanisms.

The chapter is organized as follows. Section 3.2 introduces the auction mechanisms. Section 3.3 deals with the interactions among bidders and the incentive compatibility constraints. Section 3.4 presents a method for adjusting reference points according to a sequential price weighting function. Section 3.5 examines the empirical validity of such a model. Section 3.6 concludes.

### **3.2. Auctions and incentive-compatibility**

The BDM mechanism (Becker *et al.* 1964, Shogren *et al.* 1994) and the random *n*th-price auction (Shogren *et al.* 2001) are two market based mechanisms

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<sup>23</sup> Another restatement proposed by Morrison (2000) is the *leading*, which is the following of the randomly chosen exchange price.

often used in experiments. Determination of the market clearing-price and the expected payoff, which ensues from the market price, is different in the two mechanisms. In theory, they perform the same. In practice, this assertion no longer holds true.

Under BDM, an individual reports a bid for a good; a price is then exogenously and randomly drawn from a price list. If the individual bids above the price, she receives the good and pays the drawn price. If the individual bids below the price, she does not receive the good and pays nothing. The mechanism is regarded as a quasi-market mechanism, its market price being exogenously determined.

Under NPA, the market price is endogenously determined. The mechanism works as follows (see Shogren *et al.* 2001, List 2003): each bidder submits a bid; all bids are rank-ordered; a number between 2 and  $n$  ( $n$  being the number of bidders) is randomly selected as a market-clearing price; a unit of the good is sold to each of the  $n-1$  highest bidders at the  $n$ th-price drawn from the bids. Because of the endogenous market price, NPA is considered to be a full-active market.

Following the induced value payoff theory, whatever the auction mechanism, an individual faces the following payoff rule:

$$\begin{cases} v_i - p & \text{if } p < b_i \\ 0 & \text{if } p \geq b_i \end{cases}$$

where  $v_i$  is bidder  $i$ 's value,  $b_i$  her bid, and  $p$  the market price. Whenever optimal bidding arises with  $b_i = v_i$ , an auction mechanism is said to be incentive-compatible. Put differently, an auction is truth-telling when the individual pays a price independent from what she bids. As Lusk and Shogren (2007) point out, the incentive to value truthfully can easily be proved.

When the individual  $i$  bids, she is ignorant of the price she will pay. So she draws an estimate of the price from the probability density function  $f_i(p)$  with support  $[p_\bullet, p^\bullet]$  and the cumulative distribution function  $F_i(p)$  where

$b_i \in [p_\bullet, p^\bullet]$  corresponds to the bid. The rational individual submits a bid that maximizes her expected payoff which corresponds to her expected utility  $u_i$ , which is twice continuously differentiable and increasing<sup>24</sup>:

$$\begin{aligned} E[u_i] &= \int_{p_\bullet}^{b_i} u_i(v_i - p) dF_i(p) + \int_{b_i}^{p^\bullet} u_i(0) dF_i(p) \\ &= \int_{p_\bullet}^{b_i} u_i(v_i - p) f_i(p) dp + \int_{b_i}^{p^\bullet} u_i(0) dp \end{aligned} \quad [1]$$

The first integral describes the expected payoff for random prices below her bid (where she expects a positive surplus). The second integral describes the expected payoff for random prices between her bid and the maximum possible bid (where she expects a loss). The maximum over  $b_i$  occurs when the derivative of  $E[u_i]$  with respect to  $b_i$  is null:

$$\frac{\partial E[u_i]}{\partial b_i} = u_i(v_i - b_i) f_i(b_i) = 0 \quad [2]$$

where  $u_i(0) = 0$ . When  $b_i = v_i$ , the probability distribution that the individual's bid equals the price is strictly positive or we assume positive support on  $[p_\bullet, p^\bullet]$ . The individual maximizes her expected utility when she bids her true value.

In BDM, the market-clearing price is drawn from a uniform distribution with the probability density function  $f(p)$  and a cumulative distribution function  $F(p)$ . Bidders have different values but face the same price which is modeled as the mean of the price distribution in the support of  $b_i$ . The probability of winning the auction given  $i$ 's bid is  $F(b_i)$ . Taking her bid as given, the price that  $i$  expects to pay conditional upon winning is:

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<sup>24</sup> Assumptions that satisfy the von Neumann-Morgenstern utility function.

$$f(p | p < b_i) = \int_{-\infty}^{b_i} p \frac{f(p)}{F(b_i)} dp \quad [3a]$$

Then her expected utility is her expected payoff:

$$E[\pi_i] = \left[ v_i - \int_{-\infty}^{b_i} p \frac{f(p)}{F(b_i)} dp \right] F(b_i) \quad [3b]$$

In NPA with  $n$  bidders, one of the bidders' values, from the uniform distribution<sup>25</sup> with PDF  $g(v)$  and CDF  $G(v)$ , is independently drawn at random and set as the market price. Conditional on  $v_i$  being the  $n$ th value, the chance that a bid from the opponents is drawn as the  $n$ -order statistic is  $(n-1)/n$ . The probability of winning given  $i$ 's bid is  $G(b_i)$ . Taking her bid as given, the price that  $i$  expects to pay conditional upon winning is:

$$g(p | v < b_i) = \frac{n-1}{n} \left[ \int_{-\infty}^{b_i} v \frac{g(v)}{G(b_i)} dv \right] \quad [4a]$$

Her expected utility is her expected payoff:

$$E[\pi_i] = \left[ v_i - \frac{n-1}{n} \int_{-\infty}^{b_i} v \frac{g(v)}{G(b_i)} dv \right] G(b_i) \quad [4b]$$

The BDM and NPA are proved to be incentive-compatible (Kahneman *et al.* 1990, Shogren *et al.* 2001b). Lusk *et al.* (2007) analyze the cost of misbehaving or deviating from truthful bidding in terms of foregone expected earnings, and show that suboptimal bidding has equivalent effects for BDM and NPA. For a uniform distribution of values, the incentive to bid their value is

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<sup>25</sup> This time, the distribution comes from others' bids, not from a price list.

identical for both high- and low-type individuals, engaging all bidders to value truthfully.

Let  $\pi_i^*$  be individual  $i$ 's optimal payoff, which is achieved when an individual submits a bid equal to her value. Consider a bidder with a valuation slightly above or under  $v_i$ . The deviation is profitable only if deviating is costless. The expected cost of deviating from  $b_i = v_i$  to  $b_i = v_i \pm \varepsilon_i$ , with  $\varepsilon_i > 0$ , is given by  $\hat{\pi}_i$ :

$$E[\hat{\pi}_i(v_i, b_i, \varepsilon_i)] = E[\pi_i^* | v_i, v_i] - E[\pi_i | v_i, v_i \pm \varepsilon_i] \quad [5]$$

Equation [5] represents the expected loss of an individual who does not bid her true value. It is a non-negative number that equals zero when  $\varepsilon_i = 0$ . For both the BDM and NPA, the derivative of the expected cost of deviating with respect to  $\varepsilon_i$  at the point where  $v_i = b_i$  yields:

$$\left. \frac{\partial E[\hat{\pi}_i]}{\partial \varepsilon_i} \right|_{v_i=b_i} = 0 \quad [6]$$

Equation [6] states that only bidding sincerely is costless. If a bidder deviates and bids above her value, she may increase her chance of winning the auction, but her payoff will be negative even if she wins the auction. If a bidder deviates and bids under her value, she loses the auction and has zero payoff, which means that she loses the chance of winning the auction with some positive payoff. It is useful to think of the magnitude of deviation at the disposal of the bidder, which is the difference between her value and the highest bid. This would be the amount by which she could reduce her bid and still take part to the trades, or increase her bid to augment her chances of winning without supporting negative payoffs, once the distribution of high bids is known.

In spite of the theoretical incentive-compatibility equivalence between elicitation mechanisms that employ endogenous and exogenous clearing prices, empirical evidence suggests that the two approaches generate divergent results. If the market price is based upon the preferences of other bidders, the risk of deviating from truthful bidding comes out. It is hard to distinguish between refining and copying, not only for experimentalists but for bidders too.

### 3.3. Interactive incentive-compatibility

Standard game models prescribe dominant strategies. Each individual has beliefs about the types of other individuals, how each individual values the good, and these beliefs are independent rational expectations, so the individuals' bidding strategies are constrained not to evolve. Indeed, incentive-compatibility requires that truth telling is best averaged over the types of other bidders in the auction.

Incentive-compatibility constraints guarantee that it is optimal for the bidder to make a bid (send a signal to announce her type) truthfully. Let us consider two bidders  $i = 1, 2$  with unit demands, which are *ex ante* identical. Their valuations  $v_1$  and  $v_2$  are independent, that is, each bidder's beliefs about the type of the other bidder are independent of the other bidder's belief distribution. Let  $b_1$  and  $b_2$  denote the outcomes of the bidders' strategies  $\sigma_1$  and  $\sigma_2$ . The auction mechanism specifies the probability  $f_i(b_1, b_2)$  that the good is carried by  $i$  at price  $p_i(b_1, b_2)$ . Let  $\sigma_1^*(\cdot)$  and  $\sigma_2^*(\cdot)$  denote Bayesian Nash equilibrium strategies in the auction mechanism. For bidder 1, the rationality constraint is that, for each  $v_1$  and for each  $b_1$  belonging to the support of  $\sigma_1^*(v_1)$ :

$$E_{v_2} E_{\sigma_2^*(v_2)} [v_1 f_1(b_1, b_2) - p_1(b_1, b_2)] \geq 0 \quad [7]$$

The rationality constraint ensures that the bidder is willing to participate in the auction only in the case of nonnegative payoffs, since withdrawing from the

auctioning gives her null expected payoff. The probability distribution can be understood in different ways. Provided that bidder 1 controls  $b_1$  but not  $b_2$ , we can think of a bidder as choosing a conditional probability distribution  $f_1(b_1|b_2)$ , where  $b_2$  has some exogenous probability distribution. Another interpretation is that  $f_1(b_1, b_2)$  is the result of a very complicated information mechanism by which the bidder learns and updates her beliefs about  $b_2$ . Finally, it can be understood as bidder  $i$ 's actions over time.

The incentive-compatibility constraint is such that, for each  $v_1$ , each  $b_1$  in the support of  $\sigma_1^*(v_1)$  and each deviation  $\hat{b}_1$ :

$$E_{v_2} E_{\sigma_2^*(v_2)} [v_1 f_1(b_1, b_2) - p_1(b_1, b_2)] \geq E_{v_2} E_{\sigma_2^*(v_2)} [v_1 f_1(\hat{b}_1, b_2) - p_1(\hat{b}_1, b_2)] \quad [8]$$

The left-hand side of the constraint is the expected payoff if she reports her true bid  $b_1$ , and the right-hand side of this constraint is the expected payoff if she deviates and reports  $\hat{b}_1$ . The idea here is that when bidder 1 bids  $\hat{b}_1$  instead of  $b_1$ , her payoff changes but the resulting probability distribution over  $b_2$  does not change, since she cannot control  $b_2$ , and hence she gets a different expected payoff. The incentive-compatibility constraint asserts that her expected payoff from honesty is not less than her expected payoff from deviating, *i.e.* by deviating she cannot gain more. The same applies to bidder 2. If the two bidders announce untruthful types  $(\hat{b}_1, \hat{b}_2)$ , the probability of winning the auction is:

$$\hat{f}_i(\hat{b}_1, \hat{b}_2) \equiv E_{\{\sigma_1^*(v_1), \sigma_2^*(v_2)\}} [f_i(b_1, b_2)] \quad [9]$$

The expected price is:



$$\hat{p}_i(\hat{b}_1, \hat{b}_2) \equiv E_{\{\sigma_1^*(v_1), \sigma_2^*(v_2)\}} [p_i(b_1, b_2)] \quad [10]$$

The incentive-compatibility constraint ensures that a Bayesian Nash equilibrium for both bidders is to announce the truth ( $\hat{v}_1 = v_1$  and  $\hat{v}_2 = v_2$ ). Regardless of how  $f_i(b_1, b_2)$  occurs, if it violates the incentive-compatibility constraint, the bidder is not maximizing her expected payoff.

Hausch (1986) asserts that an individual has an incentive to underbid in sequential auctions, *i.e.* to provide misleading information about her valuation of the good in the first round to deceive her opponents, in order to secure winning in the second round. Jeitschko (1998) demonstrates that bidders face a trade-off between increasing the probability of winning the early auction and increasing expected payoffs in the later auction. As a corollary, bidders place lower bids in the early auction, because they are aware of the learning effects.

However, there is a strong information requirement. Each bidder must know the distribution of types of all the other bidders as well as the ability to determine the Nash strategies of every other bidder in the auction. In practice, equilibrium computation is usually infeasible. Moreover, the distribution over the possible types of  $n$  individuals in repeated-round auctions is complex and makes the space of types go of hand. One could calculate the equilibrium, but in the absence of common knowledge of type space and prior beliefs, it is unlikely to expect it (Saran and Serrano 2007). As a consequence, it is pragmatic to stress that individuals observe how others value the good, and some kind of equilibrium emerges (Boutillier *et al.* 2000)<sup>26</sup>.

Theorists assume incentive-compatibility in the strict case of independent private values, which means that the individual's value is independently drawn from a commonly known distribution. In this case, the individual has only a prior on her signal. The setting of independent private values is reasonable for domains in which individuals' valuations are unrelated to each other, depending only on their signals. But when the bidder's valuation depends on both her signal and

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<sup>26</sup> Recent literature shifts the analysis to the *ex post* equilibrium so any type space fits.

others' signals, those signals are likely to be affiliated: a phenomenon known as *affiliated values* pioneered by Milgrom and Weber (1982)<sup>27</sup>. For example, if a signal from an individual is a high value, this will increase the probability that other individuals will have high signals as well. As a consequence, a higher value for one bidder makes higher values for other bidders more likely (Kagel 1995). Values are drawn from an affiliated distribution if the posted price – which serves to signal the relative value of the good – shifts bids' distribution. Corrigan and Rousu (2006) make a distinction between bid affiliation and value affiliation, and prefer the bid affiliation as a broader concept. According to them, positive correlation between bids may not be caused by positive correlation between values: experimentalists observing bids, bid affiliation is a more relevant concept.

When individuals actively interrelate, such as under NPA, they cannot circumvent estimating the probability distributions over maximal bids of other bidders and their chances of winning the auction given their true value. If the individual observes that others' bids are higher than her own, she learns she has little chance of winning the auction. In this case, the literature shows that individuals tend to submit higher bids afterwards (Fox *et al.* 1998, Cummings and Taylor 1999, List 2001). Likewise, Corrigan and Rousu (2006) experimentally find that posted prices have a statistically and economically significant impact on bids submitted in subsequent rounds. Furthermore, according to their study, the bidder's propensity to increase her bid is independent of her initial bid.

Individuals combine their own signal with the signals received from others, which creates affiliation between values or bids (Klemperer 1999). For that reason, their value is given by:

$$v_i = \alpha t_i + \beta \sum_{j \neq i} t_j \quad [11]$$

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<sup>27</sup> Let  $x = (x_1, \dots, x_n)$  be the vector of signals observed by the bidders. Let there be another vector of signals containing information important to value the good. Bidders' values for the good are affiliated if  $v_i = u_i(s, x)$ . Otherwise, that is  $v_i = x_i$ , bidders' values of the good are privately and independently distributed.

where  $t_i$  is bidder  $i$ 's signal,  $t_j$  is  $j$ 's signal,  $\alpha$  is the weight assigned to  $i$ 's signal and  $\beta$  the weight assigned to  $j$ 's signal, with  $\alpha \geq \beta$ . It is non-realistic to believe that individual  $i$  ignores others' signals. Her private value does not remain independent thus  $\beta \in (0,1]$ . Finally, the individual does not bid her true value, and her over- or underbidding depends on the magnitude of  $\beta$ . Since the individual does not know other bidders' signals, she forms expectations on them.

Learning preferences by repeating bidding is part of the methodological consensus. However, learning may also provoke unintended effects that challenge *stricto sensu* the constraints of incentive-compatibility. The reasoning is quite intuitive. An individual is given an initial endowment she uses as a reference to submit her bid. She reveals her value upon her preferences and this initial amount. Provided that a randomly selected round is chosen as the binding round in experimental repeated-round auctions, the individual bids in reference to the same endowment at the beginning of each round. In theory, this cannot compromise the property of demand-revealing. Nevertheless, she is told all the bids and the market price before submitting her bid in the next round, and revealing their distribution provokes an adaptive bidding behavior. Indeed, the individual extracts information on value perception from price formation in the auction, and price posting makes her update her values iteratively without fear of deviation.

It is hard to believe that the process by which an individual maximizes her expected utility is one of assigning an independent value to the good after market information has been revealed. Assuming independent distributions implies that the individual is assumed to reason as if the bids for subsequent rounds were issued from independent beliefs. In other words, such a basic bidder is insensitive to strategic implications of varying  $\varepsilon_i$  in [5] and to the information content of  $t$  in [11]. Indeed, even if signals are mostly irrelevant to the payoffs, it is hard to exclude the possibility that they may find themselves into the equilibrium, which suggests existence of a correlated equilibrium (Aumann 1974). Moreover, Bayesian rational players play a correlated equilibrium as long the Harsanyi

common prior assumption is verified (Aumann 1987)<sup>28</sup>. We think that the individual builds a bidding policy by which her bid is conditioned on the outcome of earlier rounds. Henceforth, the uncertainty is over opponents' bids. With an endogenous market-clearing price, the individual forms beliefs on the unknown distribution of the highest bid according to others' preferences. Her uncertainty over the parameters of this distribution is reflected by her prior distribution over the probability space of bid distributions. But, the use of equilibrium to describe the uncertainty relies on the existence of a type space as common knowledge, which is an important limitation.

### **3.4. The behavioral model**

Consider dynamic settings where bidders interact repeatedly. We call a rule of behavior an adaptive heuristic. Invariably making the same choice is a sort of heuristic but not an adaptive one, since it is not responsive to a situation. At each stage, a bidder plays a strategy which is optimal against the distribution of the past actions of other bidders. Adaptive heuristics are boundedly rational strategies<sup>29</sup>. However, in the long run, such simple strategies yield highly sophisticated and rational behavior (Hart 2005).

Now consider an individual who is aware of the strategic implications inbuilt in the auction, such as the effects of varying expectations on the adjacency of potential opponents' values to hers. We believe that instead of using a single bidding policy at every round, individuals use the distribution of bids they've observed at earlier rounds to update their bidding policy and their estimate of the true distribution of high bids. Their bidding strategy in the next round is based on the updated distributions and all individuals play a Nash equilibrium in a Bayesian

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<sup>28</sup> Common prior only requires the bidders' mutual beliefs on the fundamentals of the interaction be elicited, like expected payoffs entailed by the possible actions.

<sup>29</sup> Learning dynamics are levels of full rationality, whereas evolutionary dynamics are completely irrational actions. Adaptive heuristics are in-between.

manner<sup>30</sup>. If the individual updates her bidding policy based on past observations, her true bids at early rounds are not reflective of the bids she submits at latter rounds, which means she is learning based on observations drawn from a nonstationary distribution. It has been shown that myopic learning models such as *fictitious play* – which is an adaptive heuristic – converge to a stationary distribution despite the initial nonstationarity (Fudenberg and Levine 1998). In a fictitious play, the individual is enabled to learn if she can realistically win the auction given her true value. She learns by observing the history of past bids – prior to the beginning of the next round – and forms a belief about her opponents’ bids in the next period. She believes that her opponents are using a stationary strategy which is the empirical distribution of past bids, and thus updates her beliefs, her best reply and bid, computing a new bidding policy based on earlier outcomes. Although truth-telling is theoretically proved to be optimal, computing optimal bids as best replies defies the assumption of true valuation<sup>31</sup>.

Instead of using these learning models, let us introduce a descriptive behavioral model based upon reference point adaptation. We introduce a parallel model to rank-dependent expected utility, because we consider agents to derive utility from changes in wealth relative to their reference point. If an agent perceives her payoff to be higher than the reference point, she perceives a gain; and perceives a loss, otherwise. We exploit the idea of linear and non-linear probability weighting and propose a sequential information weighting because we assume that strategic bidders convert objective linear weighting into subjective nonlinear decision weights.

In this case, let us assume that bidders adjust their starting values. Anchoring-and-adjustment is a heuristic that influences the way individuals intuitively assess probabilities. According to this process, individuals start with a

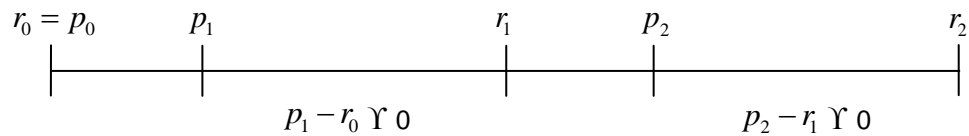
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<sup>30</sup> In the long run, irrational behavior can lead to Bayesian rationality (Aumann 1987).

<sup>31</sup> Shlomit *et al.* (1998) analyze a repeated first-price auction in which the types of the players are determined before the first round and do not vary in time. When each player uses a fictitious play learning scheme, the equilibrium vector of bids is the same as in a one-shot auction with the types of players being common knowledge. However, their players are too basic for they do not attempt to learn their opponents’ types or to hide their own types.

reference point (the anchor) and make adjustments to it to reach their estimate<sup>32</sup> (Tversky and Kahneman 1974). In the case of repeated one-shot auctions, their true value is a reference point that bidders discovered in time, *i.e.* people use practice rounds to refine their values with regard to their vague or naïve start. Deviation from true value is then an adjustment from the self-generated anchor in order to win the auction in the late rounds. When bidders long to increase either their payoffs or their probability of winning the auction, given that a rational agent is programmed to maximize her payoff, deviating can be considered rational.

The reference point is formed after observing the last posted price. The bidder thus makes her bid in  $i + 1$  according to  $r_i$ . Depending on whether  $p_i > r_{i-1}$  or  $p_i < r_{i-1}$ , she scales her bid up and down, respectively. The adaptation of the reference point corresponds to the following phase diagram:



Arkes *et al.* (2008) term the adaptation of the reference point the rule where bidders shift their reference point in the direction of a realized outcome. If the reference point is  $r_0$  and the price is  $p_1$ , the difference between  $p_1$  and  $r_0$  should be equal to the difference between  $p_2$  and  $r_1$ , or  $p_1 - r_0 = p_2 - r_1$ . This is standard rationality. It is due to the linear shape of the utility function where bidders are indifferent to rank-dependence. We term this the uniform or linear adaptation of the reference point. If the utility function  $v$  is linear, the reference point is a weighted average of posted prices. With  $p_0 = r_0$  as the anchor in a fictitious period  $i = 0$ , the next bid is formulated along with:

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<sup>32</sup> Einhorn and Hogarth (1985) have also considered the anchoring-and-adjustment process to describe how people make judgements under ambiguity; their adjustment is made according to some probability  $p$  which could come from any distribution.

$$\bar{r}_i = \frac{1}{i} \sum_{i=1}^n w_i p_i \quad [12]$$

The expected gain of deviating or adapting the reference point from  $b_1 = v_1$  to  $\hat{b}_1 = \hat{v}_1$ , conditional on  $b_2$  is given by  $\hat{\pi}_1$ :

$$E\left[\hat{\pi}_1\left(b_1, \hat{b}_1, v_2\right)\right] = E\left[\pi_1\left|\hat{b}_1, b_2\right.\right] - E\left[\pi_1\left|b_1, b_2\right.\right] \quad [13]$$

There are several competing notions of rationality, and one among them is the correlated equilibrium, which has the advantage of being reasonable, simple and is guaranteed always to exist. The rationality constraint says that a bidder has no reason to bid in case of null payoff. Since losing in auctioning means absence of payoff, increasing the probability of winning the auction and consequently the chance of earning some positive payoff by deviating is rational. In parallel, a rational bidder seeks to maximize her payoff which is the difference between her value and the cost of the item. If by deviating, a bidder increases her expected payoff with some extra gain, she is acting rationally.

In terms of interactions between two players, the deviation of player 1 is such that, for all  $b_1$  and  $\hat{b}_1$  in  $\sigma_1(v_1)$  and all  $b_2, \hat{b}_2$  in  $\sigma_2(v_2)$ :

$$E_{v_2} E_{\sigma_2(v_2)} \left[ v_1 f_{1,2}(\hat{b}_1, \hat{b}_2) - p_1(\hat{b}_1, \hat{b}_2) \right] \geq E_{v_2} E_{\sigma_2(v_2)} \left[ v_1 f_{1,2}(b_1, \hat{b}_2) - p_1(b_1, \hat{b}_2) \right] \quad [14]$$

If joint distribution  $f_{1,2}$  with  $\sum_{v_1} \sum_{v_2} f_{1,2}(b_1, \hat{b}_2) = 1$  is a correlated strategy, equilibrium is achieved when no player ignores the public signal, which is to make an expected gain from deviating with some positive probability, given that others follow this rule as well. This implies that deviating is worthwhile only if a public signal such as a posted price recommends doing so and all submit to it because the suggested strategy is the best in expectation. The right hand-side expression is when player 1 is the only one not to follow the recommendation

issued from the public signal and chooses some bid  $b_1$  instead of  $\hat{b}_1$ , provided the endogeneity of the market-clearing price.

**Proposition 3.1.:** When bidders follow the public recommendation leading them to rationally deviate from their anchor, there exists a correlated equilibrium.

Proof: In the appendix.

The incentive-compatibility constraint ensures that truthful bidding maximizes utility. Let us now consider this point. Following the work on rank dependent expected utility (Bleichrodt and Pinto 2000) and reference point adaptation (Arkes *et al.* 2008, Baucells *et al.* 2008), we introduce a model of sequential decision analysis. First, let us recall the existing decision theoretic background.

According to cumulative prospect theory (Tversky and Kahneman 1992), people weight outcomes when they choose between lotteries. Let  $(\theta_1, p_1; \theta_2, p_2; \dots; \theta_{n-1}, p_{n-1}; \theta_n, p_n)$  be a lottery that yields outcome  $p_i$  with probability  $\theta_i$ . A lottery can be defined as a set of  $n$  outcomes  $(p_1, p_2, \dots, p_{n-1}, p_n)$  with respective probability  $(\theta_1, \theta_2, \dots, \theta_{n-1}, \theta_n)$ . The rank-dependent expected utility of this lottery is a junction between the value or utility function  $v(\cdot)$  and the weighting function  $w$ :

$$v(\theta_1, p_1; \theta_2, p_2; \dots; \theta_{n-1}, p_{n-1}; \theta_n, p_n) = \sum_{i=1}^n v(p_i) w_i \quad [15]$$

where

$$w_i = w\left(\sum_1^i \theta_i\right) - w\left(\sum_1^{i-1} \theta_{i-1}\right) \quad [16]$$



in particular  $w_1 = w(\theta_1)$ . The weighting function  $w$  is increasing with  $w(0) = 0$  and  $w(1) = 1$ . It is a function of the cumulative distribution at  $\theta_i$  and  $\theta_{i-1}$ . If  $w$  is an identity transformation and corresponds to a positive linear transformation of  $v$ , the rank-dependent expected utility theory is equivalent to the expected utility theory. In this case, bidders are considered rational: they have linear or uniform preferences for money, separately from the rank position. Tversky and Kahneman (1992) rather take  $w(\theta)$  as nonlinear, that is, a monotonic s-shaped function, which implies deviations from linearity and irrationality because of insensitivity to or misperceiving of mean probabilities. That takes the form as follows:

$$w(\theta) = \frac{\theta^\gamma}{\left(\theta^\gamma + (1 - \theta^\gamma)\right)^{\frac{1}{\gamma}}} \quad [17]$$

for  $0 < \gamma < 1$ . This one-parameter specification gives more weight to the worst and best events so  $w(\theta_i) > \theta_i$  for  $i$  close to 1 or  $n$ .

By analogy, we assume that bidders weight all the sequential information at their disposal to build their bidding strategy, in particular their anchor and the posted market-clearing prices. Bidders start with an outcome  $p_0 = r_0$  which is their original reference point and which corresponds to their subjective and asocial valuation of a good. Put differently, their first reference point is their value after the practice rounds: a true value issued from discovered preference hypothesis. In repeated-round auctions, bidders are told the market-clearing price – which can be endogenous to the bids – before submitting their next bid, so all posted prices correspond to subsequent outcomes of the outcome set.

Instead of ordering outcomes from worst ( $i = 1$ ) to best ( $i = n$ ) as in cumulative prospect theory, we assume that bidders sort the outcomes backwards, from the latest to the anchor, according to  $p_i \rightarrow p_{n-i+1}$ , with  $i$  being the rank of the round. Posted prices arrive following the sequence of rounds. Therefore, the price vector is sequentially sorted. By analogy to the probability weighting function

(Einhorn and Hogarth 1985, Tversky and Wakker 1995), we assume a sequential price weighting function, such that bidders give a weight of  $1/n$  to each price, with  $n$  the length of the price sequence. We define the sequential rank-dependent function.

**Definition 3.1.:**  $W(A) \geq W(B)$  whenever  $A \supset B$ . If  $W$  is additive, *i.e.*  $W(A \cup B) = W(A) + W(B)$  for all disjoint outcomes  $A$  and  $B$ , then it is a weighting measure. A sequential price weighting measure is a strictly increasing function  $w: [0,1] \rightarrow [0,1]$  with  $w(0) = 0$  and  $w(1) = 1$ . A weighting measure  $W$  on  $P$ , with  $P$  the outcome space, is a function whose components are included in  $[0,1]$  such that  $W(\emptyset) = 0$  and  $W(P) = 1$ .

Bidders rank prices following the mirror reflection. Henceforth, the sequential sorting is:  $(\bar{p}_1, \bar{p}_2, \dots, \bar{p}_{n-1}, \bar{p}_n) \rightarrow (\bar{p}_n, \bar{p}_{n-1}, \dots, \bar{p}_2, \bar{p}_1)$ . Ranks are then accumulated such that:

$$(\bar{\theta}_n, \bar{\theta}_{n-1}, \dots, \bar{\theta}_2, \bar{\theta}_1) := \left( \frac{1}{n}, \frac{2}{n}, \dots, 1 - \frac{1}{n}, 1 \right) \quad [18]$$

where  $1/n$  corresponds to the weight of the latest posted price, and the last increment corresponds to the weight of the anchor. A sequential weighting function is introduced to transform the ranks into cumulative decision weights:

$$\left( w(\bar{\theta}_n), w(\bar{\theta}_{n-1}), \dots, w(\bar{\theta}_2), w(\bar{\theta}_1) \right) := \left( w\left(\frac{1}{n}\right), w\left(\frac{2}{n}\right), \dots, w\left(1 - \frac{1}{n}\right), w(1) \right) \quad [19]$$

Following [16], the weighting factor is an increment between two rounds<sup>33</sup>:

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<sup>33</sup> Uniform or linear weighting results as a special case, and we have  $w(i/n) - w(i-1/n) = w(1/n)$  for all  $i$ .

$$w_i = w(\bar{\theta}_i) - w(\bar{\theta}_{i-1}) := w\left(\frac{i}{n}\right) - w\left(\frac{i-1}{n}\right), \quad i = 1, \dots, n \quad [20]$$

The cumulative prospect theory suggests an s-shaped weighting function that overweighs extreme outcomes which occur with small probabilities and underweighs average outcomes which occur with high probabilities. In lieu of this point, we assume that bidders overweight the beginning and the end of time series<sup>34</sup>. Indeed, one reference point in the context of stock investment is the starting point which enjoys a privileged role (Spranca *et al.* 1991). As well, investors partially update their reference point after a stimulus is presented to a price between the purchase price and the current price, but they do it incompletely (Chen and Rao 2002)<sup>35</sup>.

The sequential weighting function presented in Fig. 3.1. is s-shaped: it is steep near 0 and 1 and mild in-between. Thus, a low interval  $[0, 1/n]$  and a high interval  $[1 - (1/n), 1]$  have more impact than a middle interval  $[1/n, 1 - (1/n)]$ .

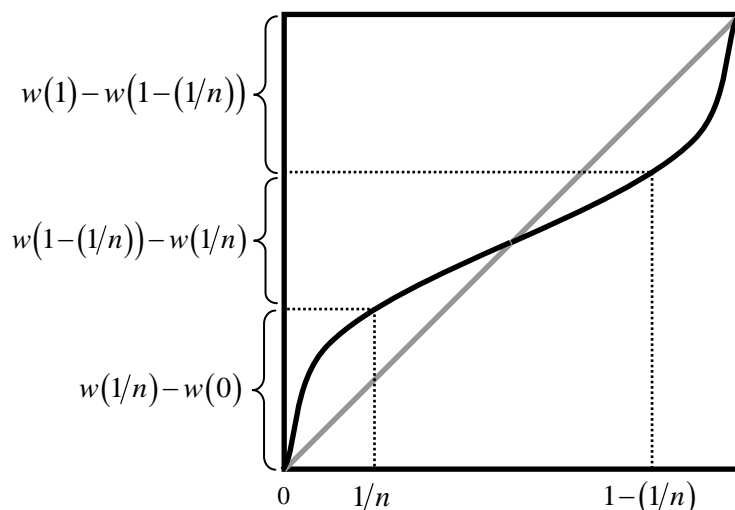
To compute her next bid, the bidder takes into account a reference point, and adjusts her estimates upon the weighted sequential price vector. If the posted price is higher than her latest reference point, she revises her value and her bid upwards to increase her chance of winning the auction, given that she learns that she earns a null payoff with her previous bid: where she does not maximize any utility. This could simply mean that she has a higher reservation price for a good than the bid she posted in the first round. If the posted price is lower than her latest reference point, she will revise her value and her bid downwards in order to

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<sup>34</sup> We are drawn to the s-shaped decision-weighting function partly because of convenience to represent some non-linear weighting.

<sup>35</sup> Another reference point used by individuals is the historical peak (Gneezy 2005) and expectations about future outcomes (Koszegi and Rabin 2006).

augment her payoff, as she learns that she can deviate and still take part to the trades: where she maximizes her expected gain and accordingly her utility<sup>36</sup>.



**Fig. 3.1. The sequential price weighting function**

As we can see, introducing the sequential price weighting function modulates the linear or uniform adaptation of the reference point. In point of fact,  $w$  is s-shaped, so the latest posted price and the anchor will most impact the valuation of the reference point. Their respective weights amount  $w(1/n)$  and  $1 - w(1 - (1/n))$ . The rest is distributed among the in-between, that is,  $w(1 - (1/n)) - w(1/n)$ .

This model lies between evolutionary dynamics and adaptive heuristics. In the evolutionary literature, *inertia* means that the bidder will invariably repeat a bid in  $i + 1$  she used in  $i$ . If her bid is sincere, it implies that she is always bidding truthfully. In our case, she will adjust her bid in the direction of the last posted price, and an adaptive rule based on the posted prices has an important component of heuristics. Since we are dealing with posted prices issued from others' bids,

<sup>36</sup> Aumann has argued that rationality should be examined in the context of rules rather than acts, *i.e.* rules of behavior that are better to other rules.

linear or uniform weighting supposed to reveal rationality (Van de Kuilen 2009) no longer holds. Bidders with well-defined preferences exploit the market mechanism to discover their true preferences. If their preferences satisfy standard theoretical requirements, the discovered preference hypothesis implies that irrationality is the results of individuals' errors, and these can be reduced by market experience. However, only later market trials can reveal their true preferences. According to this rationale, when the bidder has discovered her value, moving from it becomes irrational. In fact, because the bidder has discovered her preferences, adjusting her bid upon posted prices cannot be considered rational, for truth-telling is rational and affiliating private values on public signaling is not. Although we accept the model of discovered preferences, because we consider it to reveal the anchor, we believe that bidders can partially adapt their reference point according to posted prices and still be sincere.

We thus model the concept of inertia as high weighting of the anchor, which stands for truthful bidding and high regard to freshly discovered preferences. Adjustment means adaptive rule based on adaptation of the reference point in the direction of the posted price. It helps a bidder to maximize her expected payoff, which is after all the only purpose that matters to rationality. From the above, the two components simply suggest that sincere bidders are *boundedly rational*. Once a bidder has discovered her preferences, she is considered insincere only if she scales her references point upon the posted prices issued from others' bids with uniform sequential weighting, *i.e.* null inertia, where her anchor – a result of discovered preferences hypothesis – would be drowned by the sequence of posted prices. The following proposition comes into existence.

**Proposition 3.2.:** A bidder is truth-telling inasmuch as she behaves as a (boundedly rational) utility maximizer<sup>37</sup>, *i.e.* so long as she bids pursuant to the sequential s-shaped weighting function.

Proof: In the appendix.

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<sup>37</sup> This can be connected to the equation [11] where  $\alpha \geq \beta$ .

The correlation between bids comes from the commonly observed history of play and each bidder's actions are determined by the history. Uniform weighting means that at round  $i$  each bidder knows the history of the repeated one-shot auction; that is, each bidder uniformly considers all prices that were posted in all previous rounds. We consider bidders to be sincere if they have limited memory and confine their reference point adaptation to their anchor and the latest posted price. S-shaped weighting mechanism reflects such a bidding strategy.

Our model predicts that different-type bidders will pursue a similar rule as they get into interactions *via* endogenous market-clearing prices, no matter what their anchors are. Of course, preferences are no longer invariable in time due to the local weighting function, but this guarantees the high weight given to freshly discovered preferences. Besides, bidders still seek to maximize their expected payoff. Although bidders would orthodoxically be regarded as irrational, this model shows that sincere bidders are just boundedly rational.

### **3.5. The empirical study**

Let us now test the empirical relevance of the sequential weighting function. We reprocess the home-grown data from the BDM and NPA experimental auctions on the carbon offset (regarded as an unfamiliar good) realized by Dragicevic and Ettinger (2009). We analyze the five – out of ten – last rounds because we consider bidders and offerers to have discovered their preferences after a sufficient number of practice rounds. If bidders or offerers are to deceive and compute their bids or offers insincerely, they reasonably do it from this point of time.

Under BDM, the market-clearing price is exogenously and randomly chosen from a price list, so the value of the good is worth any market-clearing price. If every posted price is uniformly weighted, subjects are naïve. Under NPA, the market-clearing price is endogenously and randomly chosen, so the value of

the good is worth anybody's value participating in the auction. If every posted price is uniformly weighted, subjects are insincere because they are copying others' values.

As shown in equations 21 and 22, we correspondingly compute the following theoretical bids:

$$b_i = \frac{1}{i} \gamma_m \left( b_1 + \sum_{i=1}^n p_i \right) \quad [21]$$

$$b_i = \left( 1 - \gamma_h \frac{n-1}{n} \right) b_1 + \gamma_m \frac{n-2}{n} \left( \sum_{i=1}^{n-1} p_i \right) + \gamma_h \frac{1}{n} p_n \quad [22]$$

We estimate bids and offers of the subsequent round according to the uniform and s-shaped reference point adaptations previously explained. We use one-parameter specification factors  $\gamma_m = 0.61$  for moderate weighing and  $\gamma_h = 0.69$  for high weighting from Tversky and Kahneman (1992)<sup>38</sup>.

**Table 3.1. Unitary sequential weight coefficients**

Round estimate	Uniform weighting			S-shaped weighting		
	Anchor	In-between	Last Price	Anchor	In-between	Last Price
10th (5–8) <sup>39</sup>	0.102	0.102	0.102	0.425	0.102	0.115
Normalization	0.167	0.167	0.167	0.449	0.107	0.121
9th (5–7)	0.122	0.122	0.122	0.448	0.122	0.138
Normalization	0.200	0.200	0.200	0.471	0.128	0.145
8th (5–6)	0.153	0.153	0.153	0.483	0.153	0.173
Normalization	0.250	0.250	0.250	0.503	0.159	0.180
7th (5–5)	0.203	0.203	0.203	0.540	0.203	0.230
Normalization	0.333	0.333	0.333	0.555	0.209	0.236

<sup>38</sup> We rather use linear  $\gamma(1/n)$  instead of power  $(1/n)^\gamma$  factoring, because the anchor gets underweighted otherwise.

<sup>39</sup> (. . .): in-between rounds.

**Table 3.2. Summary statistics of the uniform and s-shaped theoretical estimates**

Auction mechanism	<i>n</i> th round	WTP bids				WTA offers			
		7	8	9	10	7	8	9	10
<b>BDM</b>	First bid or offer (5th round)	8.29	8.29	8.29	8.29	8.92	8.92	8.92	8.92
	Last posted price ( <i>n</i> – 1)	1.50	5.00	6.50	13.50	1.50	5.00	6.50	13.50
	Average real bid or offer	8.39	8.71	8.82	8.61	9.53	9.19	8.67	8.03
	Average bond between two rounds		0.32	0.11	–0.21		–0.35	–0.56	–1.05
	Uniform bid or offer average estimate	7.92	7.20	7.06	8.15	8.13	7.35	7.18	8.25
	Average bond between two rounds		–0.72	–0.14	1.09		0.78	–0.17	1.07
	<i>t</i> -test* of bonds between two rounds		7.24	1.34	–3.87		1.19	–0.49	–5.02
	Average SSE <sup>40</sup> (uniform residual)	7.10	10.39	12.19	12.77	10.87	12.81	12.96	13.83
	S-shaped bid or offer average estimate	7.88	7.53	7.47	8.24	8.23	7.85	7.77	8.53
	Average bond between two rounds		–0.35	–0.06	0.77		–0.38	–0.08	0.76
	<i>t</i> -test* of bonds between two rounds		5.04	0.98	–2.98		0.08	–0.69	–4.11
	Average SSE (s-shaped residual)	3.85	5.49	6.62	7.33	6.56	7.17	5.90	7.04
<b>NPA</b>	First bid or offer	4.77	4.77	4.77	4.77	9.86	9.86	9.86	9.86
	Last posted price ( <i>n</i> – 1)	1.50	8.51	7.84	7.03	10.00	5.00	5.88	7.96
	Average real bid or offer	6.18	6.12	6.85	6.72	9.17	9.14	9.23	9.37
	Average bond between two rounds		–0.06	0.73	–0.12		–0.03	0.09	0.14
	Uniform bid or offer average estimate	4.14	5.33	5.83	6.04	9.11	8.09	7.65	7.71
	Average bond between two rounds		1.19	0.50	0.21		–1.02	–0.44	0.07
	<i>t</i> -test* of bonds between two rounds		–2.41	0.67	–0.60		3.19	2.06	0.09
	Average SSE (uniform residual)	7.78	5.06	6.64	7.92	10.80	9.90	12.36	19.62
	S-shaped bid or offer average estimate	4.37	5.21	5.50	5.60	9.39	8.64	8.37	8.42
	Average bond between two rounds		0.84	0.29	0.10		–0.76	–0.26	0.05
	<i>t</i> -test* of bonds between two rounds		–1.77	0.50	–0.40		2.66	1.42	0.10
	Average SSE (s-shaped residual)	6.41	4.22	5.84	6.14	8.66	6.56	7.27	14.81

\*  $H_0$ : The difference between experimental and theoretical average bonds is zero at 5% significance.

<sup>40</sup> SSE: the sum of the squares of the residuals.



We normalize the sequential weights to one (Table 3.1.) in order to compute the reference point from which the bid or offer is figured out and to compare it to the real bid or offer (Table 3.2.). We study both the (insincere) uniform weighting and the (sincere) s-shaped weighting.

Table 3.2. presents the summary statistics of the uniform and s-shaped theoretical estimates and their comparison to the experimental results of trials 7–10. The WTP market-side is analyzed as follows. If the real bid is greater than or equal to the theoretical bid, the bidder overbids regarding her reference point. If the real bid is lower than the theoretical bid, the bidder underbids regarding her reference point. When the bidder overbids, she values the good more than what her reference point suggests. She increases her chances of winning the auction but decreases her expected payoff regarding her true value. If the uniform residual is higher than the s-shaped residual, the bidder is considered insincere. The WTA market-side is analyzed as follows. If the real offer is greater than the theoretical offer, the offerer overoffers regarding her reference point. Otherwise, she underoffers. When the offerer underoffers, she values the good less than what her reference point suggests. She increases her chances of winning the auction but decreases her expected payoff regarding her true value. If the uniform residual is higher than the s-shaped residual, the offerer is considered insincere.

Our first investigation reveals that within BDM, only 26% of offerers and 22% of bidders stick to their discovered value. Within NPA these figures even collapse to 13% for both offerers and bidders, making the auctioning tactical until the last round. Let us now see whether agents' strategies are based upon market-clearing prices by looking at the average bonds in bids and offers between two rounds. Given that we believe that agents incorporate public signals into their bidding and offering strategies, we analyze the impact of posted prices on their bids and offers, *i.e.* their freshly discovered preferences. We thus look at Student's  $t$  distribution between experimental and theoretical data and regard whether they fit. With NPA and under both adaptation weightings, the theoretical bonds in bids and offers are not significantly different from the real bonds in bids and offers. The  $t$ -test fails to reject the null hypothesis that the theoretical bonds in offers and the real bonds in offers

come from the same distribution at the  $p < 0.05$  level. With BDM, under both adaptation weightings, the theoretical bonds in offers are not significantly different from the real bonds in offers. We do not reject the null hypothesis that the theoretical and experimental data are equal at the 5% level of a  $t$ -test. On the contrary, the theoretical bonds in bids are significantly different from the real bonds in under both adaptation weightings. Here we reject the null hypothesis at the 5% level of a  $t$ -test. The estimated distribution does not fit very well with the real BDM bidding distribution.

Let us now compare the residuals. In the first place, we examine the average WTA estimates. Under BDM, although the average real offers are overoffered, we notice that the average s-shaped SSE (6.67; 24% of residuals less than 1) is lower than the average uniform SSE (12.62; 4% of residuals less than 1), showing that the offerers are sincere and weight their anchors heavily. Under NPA, even if the average real offers are overoffered, the average s-shaped SSE (9.33; 33% of residuals less than 1) is lower than the average uniform SSE (13.17; 30% of residuals less than 1), which suggests truthful offering. Yet, the values of the two weighting mechanisms are close, which is unsurprising since the market-clearing prices are issued from the offers. We see that the difference between refining and imitating is thin but real.

Secondly, we observe the average WTP bids. Under BDM, we can see that the average s-shaped SSE (5.82; 41% of residuals less than 1) is lower than the average uniform SSE (10.61; 30% of residuals less than 1), showing that the bidders weight their anchors enough to remain sincere, even though the average real bids are overbid. Under NPA, in spite of the fact that the average real bids are overbid, the average s-shaped SSE (5.65; 23% of residuals less than 1) is lower than the average uniform SSE (6.85; 18% of residuals less than 1), which also suggests truthful bidding. Again, the thin difference between the two weighting mechanisms shows that bidders take into account the posted prices not to behave insincerely but to increase their expected payoff.

We then regress on the first bid and the list of posted prices, which allows us to obtain respective  $\gamma$ -factors from equations 21 and 22 and compare them to those of Tversky and Kahneman. The least squares regression results are presented in Table

3.3. All estimates are significant, *i.e.* all  $p$ -values amount less than 0.001, and all  $R$ -squares are higher than 0.9. Despite the fact that they are comparable, we find that each one of the market-sides has its own  $\gamma$ . We do not identify  $\gamma_m$  in s-shaped weighting because the factor oscillates around zero and is not significant; therefore, bidders and offerers simply weight the anchor and the last posted-price, which proves their sincerity as well as the relevance of our descriptive model. As one notices, the regression factors we used to compute theoretical estimates are higher than those usually elicited in the gain and loss perception. However, they are in accordance with experimental data surpassing our predictions.

**Table 3.3.  $\gamma$ -factors statistics**

$\gamma$ estimate <sup>41</sup>	Uniform weighting		S-shaped weighting	
	BDM	NPA	BDM	NPA
Bidders	1.18 (0.02)	1.24 (0.03)	1.24 (0.03)	1.16 (0.06)
Offerers	1.19 (0.02)	1.17 (0.03)	1.15 (0.03)	1.21 (0.07)

Let us now discuss about the implications of the differences between the uniform and s-shaped estimates and the real bids or offers. Because market-clearing prices are exogenously determined under BDM, even though the risk of a uniform reference point adaptation exists, it does not compromise the incentive-compatibility. In experiments, bidders and offerers are foreseen as sincere. At worst, they are naïve, for it is irrational to run after luck. On the WTA market-side, the average s-shaped SSE is lower than the average uniform SSE, indicating that offerers are sincere: they refine their values in time. We observe the predominance of the s-shaped weighting on the WTP market-side as well. Finally, we denote that the average SSE is higher on the offerers' side than on the bidders' side. This is due to loss aversion of three out of eighteen offerers, who systematically proposed a ceiling WTA. When we ignore them, the average SSE is similar between both market-sides.

<sup>41</sup> Standard errors in parentheses.

Because market-clearing prices are endogenously determined under NPA, not only does the risk of a uniform reference point adaptation exist, but it compromises the incentive-compatibility of the market mechanism. Bidders and offerers are foreseen as potentially insincere. On the WTA market-side, the average s-shaped SSE is lower than the average uniform SSE, suggesting that the offerers are sincere. Furthermore, the average SSE is higher on the offerers' side than on the bidders' side. This can be explained by loss aversion of two out of sixteen offerers, who proposed a ceiling WTA in each round, and by one offerer who had no stable strategy. When we ignore them, the average SSE remains above the SSE of the buyers' market-side, but is similar to the average SSE of the BDM sellers' market-side. We also observe lower average s-shaped SSE on the WTP market-side, which illustrates sincere bidding. The NPA average buyers' SSE is the lowest, all auction mechanisms and market-sides taken into account.

At last, we notice that average WTP estimates, under both auction mechanisms, are beneath the average real bids: the real bids and offers are always higher than what the model suggests. Given that the bidders and offerers were confirmed to be sincere, we believe this is due to the reference point adaptation overstated by the combination of regret and competitive pressure. Indeed, theory of disappointment aversion (Horowitz 2006b) says that a bidder reports a higher value than the true one, simply because she is more disappointed from not receiving the good than from receiving it overpriced<sup>42</sup>. Let us recall that the WTP and WTA value measures approach equality more by virtue of the steady increase of the buyers' bids than the weak decrease of the sellers' offers. When we proceed to the computation of the NPA s-shaped estimates on the WTP market-side, but consider the posted prices issued from the offers instead of the bids, not only do we obtain an average SSE of 4.17 but also average WTP estimates almost equal to the real bids<sup>43</sup>. This unexpected result can stand for a high influence of the sellers' clearing prices on the bidders'

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<sup>42</sup> An alternative formulation of joy-of-winning was tested by Goeree *et al.* (2002) but they find that it does not add anything to the explanation of overbidding.

<sup>43</sup> This cannot hold with BDM, because the market clearing price is the same for both market-sides and because it is exogenously and randomly drawn from a known list.

reference point adaptation. It can also mean that the WTA posted prices – unlike the WTP posted prices – incorporate a behavioral effect of loss aversion combined with competitive pressure, which works as a catalyst, from the very beginning. The magnitude of disappointment aversion and loss aversion would then be similar, and a way to verify it is to call to mind the likeness of  $\gamma$ -factors between bidders and offerers. The excess in these values could be the quantitative measure of the competition’s pressure.

To end this section, let us verify if it is worthwhile deviating such as suggested by rational deviation. We compute the percentage of expected bidders and offerers who win some positive payoff by sticking to their anchor, and the percentage of expected bidders and offerers who win some positive payoff by deviating from their anchor according to the last market-clearing price. We then do the same computation on real payoffs obtained by not deviating and deviating from the anchor. The results presented in Table 3.4. show that deviating pays, since both expected and real deviating gainers outnumber.

**Table 3.4. Comparison between extra expected and real winners from deviation**

per cent	BDM WTP	BDM WTA	NPA WTP	NPA WTA
Extra expected deviant gainers	2.63	4.17	10.00	10.94
Extra real deviant gainers	0.00	2.78	3.33	3.13

Second, we measure up the average expected payoffs with and without deviation with real payoffs with and without deviation. The results are presented in Table 3.5. We observe that deviating is in general gainful, for only BDM offerers are penalized for having moved from their anchor (they get a negative payoff on average) which is unsurprising in view of the fact that the exogenous market-clearing price makes it inevitably a naïve strategy. Within NPA, adjusting the discovered value upon posted prices paid in both expected and real scenarios.

**Table 3.5. Comparison between extra expected and real gains from deviation**

<b>on average</b>	<b>BDM WTP</b>	<b>BDM WTA</b>	<b>NPA WTP</b>	<b>NPA WTA</b>
Extra expected gain from deviation	0.13	-0.36	0.72	0.09
Extra real gain from deviation	0.13	-0.22	0.26	0.08

### **3.6. Concluding remarks**

The validity of incentives for truthful value revelation is questioned whenever someone's probability of winning depends on the moves of others, such as with endogenous market-clearing price auctions. Still, should this imply that results obtained from experiments in the random  $n$ th-price auction have no meaning because of the risk of uniform reference point adaptation? It amounts to saying that experimentalists have to choose between the absence of market learning under BDM and the risk of dependence of private values that exists under NPA.

In repeated-round experimental auctions, the private-value-independence assumption behind the incentive-compatibility may be unrealistic and malapropos. When bids get correlated, the observed bid for a good after a round impacts the estimated price of the good at the next round. Individuals then revise their beliefs to reflect this information. With endogenous market-clearing prices, we believe that bidders start their valuation with a naïve anchor – their first reference point – and then adjust their value using market reference points encoded in the sequential price weighting function. They sort prices from present to past and weight these prices using an s-shape function.

Although quite simple and sometimes looked upon with a critical eye, our behavioral model underlines the validity of incentive-compatibility of both the BDM and NPA auction mechanisms. Contrary to conventional models, it shows that accounting for posted prices without rejecting the incentive-compatibility enables to differentiate sincere from insincere bidding or offering. Until some proper method

enables to distinguish learning from affiliating, we believe incentive-compatibility need not be excluded in presence of reference point adaptation, as long as one verifies the heavy weighting of the anchor. We thus suggest a different form of rationality within incentive-compatibility constraints where the correlated equilibrium plays a key role.

Nevertheless, the nature of the market-clearing price plays a significant role. When it is endogenous, *i.e.* issued from the bids of offers instead of drawn from a uniform price list, subjects tend to fix it and refine their reference point according to it, even if it is randomly chosen. In detail, our results suggest that bidders tend to overstate their bids as if posted prices were of the WTA level, because these incorporate lifting behavioral effects. Accordingly, market discipline and competition seem not only to reveal preferences and to moderate early loss aversion, but also to unveil belated disappointment aversion and competitive pressure which can arise when buyers interactively value an unfamiliar nonmarket good. This avenue of research requires more attention.

Instead of condemning behaviors that tie in and considering the NPA mechanism as a lesser evil, we believe that the good approach is to investigate conditions under which incentive-compatibility constraints can be remade. In this case, the notion of truth, which is undeniably contingent on human perception, convention, and social experience, should be reformulated. Our model is one attempt.

### 3.7. References

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### 3.8. Appendix

#### Proof of Proposition 3.1.

Let us give evidence through a numerical example that deviation in repeated-round auctions, in terms of adaptation of the reference point, is preferable. Suppose an initial reference point value  $r$ . The last posted price amounts  $r+1$ . The bidder faces an equiprobable trend of the value, *i.e.* bullish  $r+1$  or bearish  $r-1$ . She can update her reference point according to the last posted price. Updating a reference point equals to the average differential such as:

$$v \equiv \frac{(r-r+1)+0.5(r-1-r)+0.5(r+1-r)}{3}$$

Let us now compare cases with and without adaptation of the reference point. In spite of the last posted price, the reference point is not updated and remains at  $r$ . In this case, the expected value is:

$$\dot{v} \equiv (r-r+1)+0.5(r-1-r)+0.5(r+1-r) = \left(1-\frac{1}{2}+1\right) \times \frac{1}{3} = 0.5$$

The reference point is updated to  $r+1$  due to the last posted price. In this case, the expected value is:

$$\ddot{v} \equiv (r+1-r+1)+0.5(r-1-r+1)+0.5(r+1-r+1) = (2+0+1) \times \frac{1}{3} = 1$$

One can directly see that  $\dot{v} < \ddot{v}$ , which implies a larger expected payoff by reference point updating, or rationally deviating. Therefore, adaptation of the reference point is preferred to the current state of affairs.

Consider the following payoff matrix with a mixed strategy. Now suppose a third trusted party posts the market-clearing price which reveals some public signal. Each player has an incentive to rationally deviate instead of sticking to her anchor with some positive probability, if the public signal instructs to do so, by learning she has no chance of winning any positive payoff. While each player wants to deviate and to increase her expected payoff, she hopes that the other player does not act alike, as her payoff can depend on the bid of the other player: illustrating the endogeneity of the market price. Therefore, we have:

- Each player discovers alone her preferences and thus her expected payoff  $\pi$ .
- A player deviates to increase her expected payoff to  $\pi + 1$  if she finds out that her expected payoff is close to zero, but she hopes that the other player does not move from her own anchor; conversely, she expects a payoff of  $\pi - 2$  if she stays while the other player deviates.
- The market price comes from the bids. When a player follows the market price or  $v \rightarrow p$ , she risks a null payoff because  $v - p = \pi$ , explaining the absence of payoff in the last cell where all bids converged.

	stay	deviate
stay	$\pi, \pi$	$\pi - 2, \pi + 1$
deviate	$\pi + 1, \pi - 2$	$0, 0$

The third party only tells each player what she is supposed to do. There is a correlated equilibrium if no player refuses to follow the instruction. So if the row player receives the signal ‘deviate’ given she has no chance to win some positive payoff, she has no incentive not to follow, because she can make a positive payoff by deviating from her anchor, which is better in expectation. The row player assigns a positive conditional probability of 0.5 to each of the two pairs of signals (stay, deviate) and (deviate, deviate). If the column player follows the same rule, the (uncorrelated) expected payoff of the mixed strategy equilibrium by:

- staying is:  $\frac{1}{2}(\pi) + \frac{1}{2}(\pi - 2) = \frac{1}{2}(2\pi - 2) = \pi - 1$
- deviating is :  $\frac{1}{2}(\pi + 1) + \frac{1}{2}(0) = \frac{1}{2}(\pi + 1) = 0.5\pi + 0.5$

The expected payoff must be positive, that is, the row player's expected gain of staying must verify  $\pi > 1$ , whereas her expected gain of deviating must verify  $\pi > -1$ . Therefore, she is better-off by deviating, meaning that she is better-off by seeking the higher expected payoff. The game being symmetrical, the column player has no incentive not to follow her instruction either. We know that a player will never refuse to follow the recommendation resulting from the public signal in case of higher expected payoff.

If we now look at the correlated equilibrium, as there is necessarily one, it yields probabilities of  $\frac{1}{3}$  to each combination that yields some positive outcome. In this case, the expected payoff verifies  $\pi$  greater than  $\frac{1}{3}$ , which in return weakly dominates the strategy of staying (sticking to the anchor). Provided that expected payoffs are increased, each player takes the public price into account, making the decisions correlated and bids follow the same trend, which induces 'affiliation of values' in view of the standard rationality. □

### **Proof of Proposition 3.2.**

Then, let us show that a bidder who updates her reference point is sincere if she assigns a high weight on the anchor or she is subject to high inertia. Suppose a low-type bidder value and posted market-clearing prices issued from high-type bidders, such that market prices are greater than the anchor. For the purpose, let us once again take a numerical example. Assume a weight of 0.69 for the anchor and the last posted price and a weight of 0.61 for the in-between, after the losses and

gains factors in Tversky and Kahenman (1992). Assume there are five rounds at stake.

With an unnormalized uniform weighting, we obtain the following cumulative weighting:

$$\underbrace{0.61\left(\frac{1}{6}\right)}_{\text{last price}} + \underbrace{0.61\left(\frac{1}{6}\right) + 0.61\left(\frac{1}{6}\right) + 0.61\left(\frac{1}{6}\right) + 0.61\left(\frac{1}{6}\right)}_{\text{in-between posted prices}} + \underbrace{0.61\left(\frac{1}{6}\right)}_{\text{anchor}}$$

$$= 6 \times 0.102 = 0.610$$

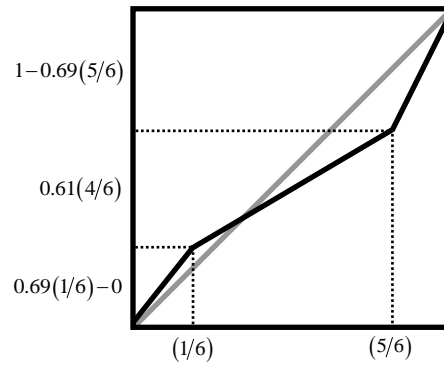
As one can see, each round receives an equal weight, which means that the anchor is drowned in time by the sequence of posted prices.

With an unnormalized s-shaped weighting, we obtain the following cumulative weighting:

$$\underbrace{\left(0.69\left(\frac{1}{6}\right) - 0\right)}_{\text{last price}} + \underbrace{\left(0.61\left(1 - \frac{1}{6}\right) - 0.61\left(\frac{1}{6}\right)\right)}_{\text{in-between posted prices}} + \underbrace{\left(1 - 0.69\left(1 - \frac{1}{6}\right)\right)}_{\text{anchor}}$$

$$= 0.115 + (4 \times 0.102) + 0.425 = 0.947$$

This graphical representation corresponds to the example of cumulated ranks on the  $x$ -axis and accumulated weight on the  $y$ -axis:



As we can see, the anchor, *i.e.* the right-hand side of the graph, receives the highest weight. If the asocial valuation gets a high weight on the topic of the value refinement in time, the bidder's valuation is not fully captured by the sequential market-clearing prices. Regarding our low-type bidder, the risk of deviating from her anchor, while she updates her reference point, is lower with the s-shaped weighting than with the uniform weighting. □





## Chapter 4

# Competitive Private Supply of Public Goods

### Abstract

This chapter compares guilt alleviation and competition for social status in the private provision of a public good. When agents are intrinsically impelled, that is, they mostly provide the public good in order to alleviate their guilt, they tend to free-ride. In contrast, when agents are extrinsically impelled and compete for social status, their provisions become strategic complements. In the latter case, the aggregate level of the public good increases as the disparity between agents' incomes shrinks. Injecting competition for social status into utility functions increases provisions to a public good, and hence its aggregate level. Market competition thus creates incentives to overcome the free-riding issue.

Keywords: public good private supply, guilt relieving, social status, competition, income transfer

JEL Classification: A13, C7, H41

"Guilt is the price we pay willingly for doing what we are going to do anyway." Isabelle Holland

#### 4.1. Introduction

The voluntary offset market enables agents to pay for their negative externalities issued from carbon emissions by investing in projects that reduce emissions or sequester carbon, such as tree planting or renewable energy. The reduction of carbon emissions is a public good because, once provided, agents can enjoy the benefits devoid of rivalry, without excluding anyone from its consumption.

Some people believe that the voluntary offset market is inefficient. One of the arguments put forward is that offsetting validates polluting behavior. Likewise, offsetting is said to operate like charities: voluntary supplies never provide enough public good because of the free-rider incentive. And when private arrangements finance a public good, free-riding on other people's provisions is rational.

However, free-riding is limited to some extent because agents who purchase offsets may also derive private benefits. Olson (1965) advances the hypothesis that free-riding can be overcome through social incentives. According to him, agents do not privately supply a public good for its direct material benefit, but to achieve social objectives like prestige or respect; this would explain why individuals do less free-riding than what the economic theory suggests. Following this rationale, Hawkes *et al.* (1993) show that in ancient times hunters and gatherers tended to share their resources because the cost of exclusion from the group – where every agent prefers a supplier to a consumer as a neighbor – was too high to risk, thus making resources a public good.

This impure approach of pro-social behavior has been modeled by Andreoni (1990) who justifies private provisions in terms of warm-glow or joy-of-giving. Our approach differs from Andreoni's and rejoins Olson's, for we consider social status gained by agents who privately supply a public good from its relative perspective. As a matter of fact, supplying to the public good can generate benefits of guilt relief – which we find more convincing than warm-glow – and/or social status. In the first

case, agents want to feel better about themselves, because they want to recover self-esteem after producing a public bad. If an agent feels guilty, because she believes she bears responsibility for carbon excesses, then guilt alleviation through carbon offsetting is a private benefit derived from the supply of the public good. Despite the private benefit, the motivation for it is internal. It is thus an intrinsic incentive. Since guilt arousal is positively related to donation intention (Hibbert *et al.* 2007), guilt alleviation has positive impacts on environmental awareness. Then, agents compete to be formally acknowledged as being the most concerned about the public good. This prosocial behavior can be due to social pressure and norms and corresponds to an extrinsic incentive. An agent who offsets receives a proof acknowledging her provision to the public good. She thus sends a signal to make other agents aware of her polluting abatement. Following this rationale, producers will also promote their offsets as part of their corporate social responsibility policy (Kotchen 2009).

People have a preference for showing altruism in situations that facilitate broadcast opportunities, and the provision of a public good is certainly one such situation (Smith and Bliege Bird 2000). *De facto*, what type of incentives should be introduced to increase private provisions? Are competitive settings such as auctions a good solution to the inefficient provision of a public good? Do agents become more generous by guilt or by craving for social status?

If high status brings with it high earnings, then status seeking behavior can be explained as a part of economic behavior (Ball and Eckel 1998). According to *competitive altruism*, despite the dearness of being publicly generous, agents can promote their generosity as potential exchange partners, reaping the benefits later on (Roberts 1998). Agents also refuse transactions that are in their best economic interest when they feel they are an insult to their dignity (Bénabou and Tirole 2006). Experimental literature has confirmed the role of individual status as an incentive affecting market outcomes (Ball *et al.* 2001) and donors (Duffy and Kornienko 2005). Because of the rivalry and excludability in social hierarchy, agents have to compete before attaining some desired social status: if an agent desires to be the first or among the first in some venture, she might have to make the most efforts to reach her goal. Making the most efforts means that she has knowledge of her challengers and of the

efforts she has to invest. In this case, how does competition influence an agent's voluntary supply of a public good? Competitive mechanisms, such as contests, have shown to increase the voluntary provision of a public good (Kolmar and Wagener 2008).

This chapter investigates how competition influences private provisions of the public good when agents are stirred by an intrinsic impulse, meaning that they mainly maximize utility from guilt relief, as opposed to when they are stirred by an extrinsic impulse, suggesting that they mainly maximize utility from social status. Our public good game unveils several results: first, we find that when status seeking dominates guilt relief, private provisions become strategic complements: an attribute which increases the aggregate level of the public good. Then, we prove sufficient conditions for existence and uniqueness of a Nash equilibrium. At last, when agents behave according to their best-response functions, we find that the aggregate level of the public good depends on the disparity between agents' incomes, which – depending on the nature of the provisions – induces a particular income transfer policy.

We give a basic account of the social status function and present the public good game in Section 4.2. We provide a model of logarithmic best-response functions and describe explicit properties of a Nash equilibrium in Section 4.3. Concluding comments are given in Section 4.4.

## 4.2. The public good game

Let us first introduce the social status function. Consider  $n$  agents who produce the public good by devoting some of their endowment  $w$  into the public good  $g$ . Following Frank (1985), let us suppose that each agent cares about her social status with respect to the other  $n - 1$  agents.

**Definition 4.1.:** The social status function is a continuously differentiable function  $s_i \equiv s(g_i, g_{-i})$  where  $g_i$  is the provision of agent  $i$ ,  $g_{-i}$  is the provision of other agents. The level of the provision to the public good determines social status. If  $f(g)$  is the density function for  $g$  values which determines the social status of the

agents and  $g_0$  is the smallest provision to the public good  $g$  among the  $n$  agents, then an agent with  $g_0 < g_n$  will have a social status function such that:

$$s(g_n) = \int_{g_0}^{g_n} f(g) dg \quad [1]$$

where  $f(g)$  increases as  $g$  moves towards the maximum value of its domain.

Let us consider two agents  $i$  and  $j$ , with  $j \neq i$ . Let  $w_i$  be agent  $i$ 's endowment, let  $x_i$  denote her consumption of the private good, let  $G$  be the aggregate level of public good and let  $g_i$  account for her provision to the public good. The aggregate level of public good is the sum of the two agents' provisions  $G = g_i + g_j$ . Agent  $i$ 's social status is determined by her relative contribution  $s_i = g_i - g_j$ <sup>44</sup>. Agents have preferences represented by the following utility function:

$$u_i \equiv u_i(x_i, G, s_i) \quad [2]$$

Considering agent  $j$ 's provision  $g_j$  as exogenous, agent  $i$  maximizes her utility by solving the following program:

$$\max_{x_i, g_i} u_i(x_i, G, s_i) \text{ subject to } x_i + g_i = w_i \text{ and } g_i \geq 0 \quad [2']$$

Let us now determine the Nash equilibrium of the public good game. Each agent's best-response function fully specifies her equilibrium strategy. This strategy involves choosing a level of private supply to the public good, the private supply of the other agent being exogenous. We first analyze the best response functions of each

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<sup>44</sup> According to Auriol and Renault (2008), social status is a scarce resource: increasing an agent's status requires that another agent's status decreases.

agent. We thus study the two motives for contributing to the public good: to relieve guilt and to acquire social status.

Assume the marginal utility from the provision to the public good to be:

$$H_i(x_i, G, s_i) = \frac{\partial u_i}{\partial G} + \frac{\partial u_i}{\partial s_i} - \frac{\partial u_i}{\partial x_i} \quad [3]$$

The first term denotes the marginal utility from the public good. The second term represents the marginal efficacy of social status. The last term is the marginal fall in the consumption of private goods. We then make three assumptions on  $H$ .

$$\text{A1: } \frac{\partial H_i}{\partial x_i} = \frac{\partial^2 u_i}{\partial x_i \partial G} + \frac{\partial^2 u_i}{\partial x_i \partial s_i} - \frac{\partial^2 u_i}{\partial x_i^2} > 0 \quad [4]$$

A1 says that an increase of income increases the marginal utility of the supply of the public good. The assumption is referred to as the normality assumption because it is satisfied if we assume that both private and public goods are normal with respect to income. It simply says that agent  $i$ 's demand for the public good increases with income and her demand for private goods does not decrease with income.

$$\text{A2: } \frac{\partial H_i}{\partial G_i} = \frac{\partial^2 u_i}{\partial G^2} + \frac{\partial^2 u_i}{\partial G \partial s_i} - \frac{\partial^2 u_i}{\partial G \partial x_i} < 0 \quad [5]$$

A2 states that the marginal utility of the public good decreases with  $G$ . As a matter of fact, if the level of the public good increases independently of agent  $i$ 's supply, there is no incentive to contribute to the public good. This is a formal foundation for the free-riding issue. Considering negative externalities, it simply means that any agent can compensate for the damage caused, and all agents can profit from its reparation.

$$\text{A3: } \frac{\partial H_i}{\partial s_i} = \frac{\partial^2 u_i}{\partial s_i \partial G} + \frac{\partial^2 u_i}{\partial s_i^2} - \frac{\partial^2 u_i}{\partial s_i \partial x_i} < 0 \quad [6]$$

A3 implies that an increase in social status creates negative incentives: the agent tends to reduce her supply to the public good, because she no longer has to compete for social status.

According to the previous assumptions and following the work on warm-glow by Andreoni (1990), we now consider that individuals obtain guilt relief and social status from their private supply of the public good. Following the first order condition, agent  $i$ 's best response, that is,  $r_i(w_i, g_i)$ , is to have  $g_i$  such as:

$$r_i \equiv H(w_i - g_i, g_i + g_j, g_i - g_j) = 0 \quad [7]$$

A Nash equilibrium of the public good game is a couple of strategies  $g_i^*, g_j^*$  such that each strategy is the best response to the other agent's strategy:

$$g_i^* = r_i(w_i, g_j^*) \text{ with } j \neq i \quad [8]$$

Let us now look at the second order condition to see whether contributing to the public good does in fact maximize an agent's function. The second order condition is satisfied for:

$$\frac{dH_i}{dg_i} = \frac{\partial H_i}{\partial G_i} + \frac{\partial H_i}{\partial s_i} - \frac{\partial H_i}{\partial x_i} < 0 \quad [9]$$

The sign of the differential implies a diminishing marginal utility of the public good as the agent supplies the public good. Negativity depends on three terms. The first term measures the outcome of any provision to the public good on the marginal utility of the public good. This is our indicator of free-riding. The second term values the outcome of a shift in the social status on the marginal utility of the public good. It



allows us to study the interactions between the aggregate level of the public good and social status in the utility function. The third term assesses the impact of a decrease in private goods' consumption on the marginal utility of the public good.

Let us now consider the effect of agent  $j$ 's supply on the marginal utility of agent  $i$ 's supply:

$$\frac{dH_i}{dg_j} = \frac{\partial H_i}{\partial G_i} - \frac{\partial H_i}{\partial s_i} \quad [10]$$

This effect is ambiguous, for the first term is negative while the second one is positive. The first term denotes a typical free-riding issue: an increase of agent  $j$ 's provision reduces agent  $i$ 's incentive to contribute; except that the second term denotes status seeking, thus an opposite effect, as social status decreases with agent  $j$ 's supply. Indeed, agent  $i$  suffers from the reduction in the level of public good due to carbon emissions, thus any private provision that increases the public good also increases agent  $i$ 's utility. Provided that any supply removes her feelings of guilt, she can free-ride on others' provisions and allocate all her endowment to the private goods instead. This is a counter-incentive to supply the public good. In parallel, agent  $i$  suffers from status loss in social hierarchy every time others supply the public good. Therefore  $g_j$  is also an incentive to contribute in order to maintain the level of social status.

The sign of the best-response function slope of agent  $i$  is:

$$\frac{\partial r_i}{\partial g_j} = \frac{dH_i/dg_j}{-dH_i/dg_i} \quad [11]$$

The sign depends on which effect prevails: guilt relieving or status seeking. According to the terms of Bulow *et al.* (1985), if free-riding dominates social hierarchy or  $\partial r_i/\partial g_j > 0$ , we are in the presence of strategic substitutes, and strategic complements *vice versa*. Despite the fact that in standard public good games (even in the presence of an impure public good) the only effect at stake is free-riding and

public good provisions are always strategic substitutes: injecting competition for social status converts the provisions into strategic complements in some cases.

A Nash equilibrium is a set of provisions that satisfies the aggregation of supplies. Let us prove the existence and uniqueness of a Nash equilibrium. For a Nash equilibrium between agents to exist, one must verify:

$$\frac{dH_i/dg_j}{-dH_i/dg_i}, \frac{dH_j/dg_i}{-dH_j/dg_j} \in (-1,1) \quad [12]$$

The slopes of the best-response functions are bounds within the interval  $(-1,1)$ . The binding conditions are sufficient for the existence of a unique Nash equilibrium.

**Proposition 4.1.:** If [12] is satisfied, there exists a unique Nash equilibrium.

Proof: In the appendix.

Let us now see what happens when the policy of income transfer is instituted. Consider the ratio which confronts the two motives involved in the public good's supply. The expression returns to an *intrinsic impulse coefficient* such as:

$$\gamma_i = \frac{\partial H_i / \partial x_i}{(\partial H_i / \partial x_i) - (2\partial H_i / \partial s_i)} \quad [13]$$

The numerator measures the marginal utility of the public good and stands for the intrinsic (contrite) impulse of guilt relief to supply the public good. It depends on agent  $i$ 's income and thus on her opportunity loss when she doesn't purchase the private goods. Here, agent  $i$  is indifferent between consuming her own supply or benefiting from agent  $j$ 's supply of the public good. In Andreoni's terminology, this phenomenon means pure altruism or selflessness of agent  $i$ . Here, we consider the numerator as a measure of free-riding on others' provisions.

The denominator represents the influence of social status on the marginal utility of the public good and stands for the extrinsic (social) impulse of status seeking to supply the public good. Just as with the numerator, it depends on agent  $i$ 's income, but it depends on social status above all, that is, marginal utility of the public good derived from her own provision (analogue to Andreoni's impure altruism). Given that status is acquired by relative provisions, the effect of social status counts twice. First, consuming more of the  $x$ 's decreases agent  $i$ 's provision to the public good and thus her social status; second, more of  $g_j$  implies lower social status for agent  $i$ , all else being equal. For those reasons, the intrinsic impulse coefficient is inversely proportional to status seeking.

**Proposition 4.2.:** An income transfer from agent  $j$  to agent  $i$ , such that  $dw_i = -dw_j > 0$  increases  $G$  if and only if  $\gamma_i > \gamma_j$ .

Proof: In the appendix.

Agents are unwilling to perfectly substitute their provisions to offset a transfer. If  $\gamma_i > \gamma_j$  then agent  $i$  can be considered to be less status seeking than agent  $j$ . Hence, the policy of income transfer will increase (decrease or not change) the aggregate level of the public good if and only if the income gainer is less status seeking than (more status seeking than or equally status seeking than) the income loser. This proposition is comparable to that of Andreoni, but our interpretation is different. In fact, since competition for social status encourages agents to supply the public good, only an increase in income will motivate the lower income agent to supply more<sup>45</sup>, for it enables her to compete for social status. Without transfer, her position discourages her to race for social status and she can only relieve her guilt. The direct consequence is free-riding on other agents' provisions. Another way of understanding the proposition is: since the higher income agent proves – with a

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<sup>45</sup> For example, OECD (2007) suggests monetary transfers in benefit of low income households when imposing environmental taxes.

higher level of supply which reflects higher income – to be more extrinsically impulsed, she does not have to contribute more to the public good. She is in no doubt to hold the social status *ex ante*.

Our model is a way-out to Andreoni’s impure altruism and warm-glow giving. What he calls pure altruism, we identify as guilt relief and free-riding, while his impure altruism corresponds to our willingness to compete for social status, which is observable *via* any non-anonymous donation. The model is thus an alternative and a more realistic way to explain prosocial behavior.

### 4.3. The explicit logarithmic model

#### 4.3.1. The program

Following the model by Kumru and Vesterlund (2008) and Munoz-Garcia (2008), agents have preferences represented by the following separable nonlinear utility function:

$$u_i(x_i, G, s_i) = \ln(x_i) + \ln(\alpha_i G + \beta_i s_i) \quad [14]$$

where  $G = g_i + g_j$  and  $s_i = g_i - g_j$ . Private goods are included in the first term, while provisions are included in the second term which is nondecreasing in  $g_i$ . The latter measures utility derived from guilt relief based on the aggregate level of the public good  $G$  and social hierarchy  $s_i$  which are separable.

We assume that individuals originate guilt relief from their private supply of the public good. Agent  $i$ ’s preferences when she provides the public good by  $g_i$  are defined by:

$$\alpha_i(g_i + g_j) \text{ for } j \neq i \quad [15]$$

The expression denotes the utility that agent  $i$  gets from supplying to the public good and the aggregate level of the public good scaled by a specific index  $\alpha_i \geq 0$ . The aggregation of provisions corresponds to the public good dimension of the utility function. We assume that some willingness to relieve guilt is stated by either agent<sup>46</sup>. For example, either agent could relieve guilt with a single symbolic coin when participating in charity auctions.

Agent  $i$  gets utility from social status when she provides the public good by  $g_i$ <sup>47</sup>. Her status is given by the distance between her provision and that of agent  $j$ 's such as:

$$\beta_i (g_i - g_j) \text{ for } j \neq i \quad [16]$$

Agent  $i$  enhances her status in the social hierarchy if her provision outdistances agent  $j$ 's; otherwise, her social status deteriorates. The status is scaled by a specific index  $\beta_i$ , with  $\beta_i \geq 0$ , which measures agent  $i$ 's willingness to acquire social status. When agents provide identical provisions, the term vanishes. In the equilibrium, agent  $i$  knows whether she acquires social status through her private supply of the public good ( $g_i > g_j$ ). The explicit maximization program is then:

$$\begin{aligned} \max_{x_i, g_i} u(x_i, (g_i, g_j), s) &= \ln(w_i - g_i) + \ln[\alpha_i (g_i + g_j) + \beta_i (g_i - g_j)] \\ \text{subject to } x_i + g_i &= w, \quad g_i \geq 0 \end{aligned} \quad [17]$$

The first term represents the utility derived from the consumption of private goods  $x_i$ . The second term corresponds to the utility that agent  $i$  obtains from her supply of the public good. Agent  $j$ 's provision is both a strategic substitute and a strategic complement of agent  $i$ 's utility. As a strategic substitute, two obvious

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<sup>46</sup> Social comparison theory suggests that individuals have a need to compare themselves to individuals whom they deem are similar to them (Goethals 1986).

<sup>47</sup> A status-based model of market competition has already been introduced by Podolny (1993).

interpretations come out. First, agent  $i$  suffers from the public good diminishment due to carbon emissions, thus any private provision that increases the public good also increases agent  $i$ 's utility indexed by  $\alpha_i$ . Second, since any provision removes her feelings of guilt, she can free-ride on others' provisions and allocate all her endowment to the consumption of the private goods. To consider agent  $j$ 's provision as a strategic complement is to consider that agent  $i$  suffers from status loss in social hierarchy every time agent  $j$  provides the public good. Therefore, agent  $j$ 's private provision decreases agent  $i$ 's utility.

### 4.3.2. Reaction functions

Now suppose both agents decide to submit their provisions to the public good. Given  $g_j$ , differentiating  $u(\cdot)$  with respect to  $g_i$  gives  $r_i$ , agent  $i$ 's best-response function:

$$r_i(w_i, g_j) = \frac{1}{2} w_i - \frac{A_i}{2} g_j \text{ if } g_j < \max \left\{ \frac{w_i}{A_i}, -\frac{w_i}{A_i} \right\} \quad [18]$$

$$\text{where } A_i = \frac{\alpha_i - \beta_i}{\beta_i + \alpha_i}.$$

Whether  $r_i$  is constrained depends on the level of  $g_j$ . For small values of  $g_j$ , agent  $i$  allocates a part of her income to the supply of public good. For sufficiently high values of  $g_j$ , agent  $i$  can supply either nothing or her full income. Whichever occurs depends on the sign of  $\beta_i - \alpha_i$ .

**Corollary 4.1.:** The difference between  $\beta_i$  and  $\alpha_i$  determines whether provisions are strategic substitutes or strategic complements.

Proof: In the appendix.

When  $\alpha_i > \beta_i$  or  $A_i > 0$ ,  $r_i$  is the best-response only when  $g_j \leq w_i/A_i$ , which is a nonnegative number. If agent  $j$  surpasses this threshold, agent  $i$  has fairly no incentive to make positive provisions. In point of fact, even a quasi-null level of  $g_i$  (nonnegative by definition) enables agent  $i$  to maximize her utility by allocating her income to more private goods while alleviating her guilt through agent  $j$ 's provisions. We could think of an individual who pays tribute to the collective high efforts in providing the public good while ending up self-pleased by giving a single coin.

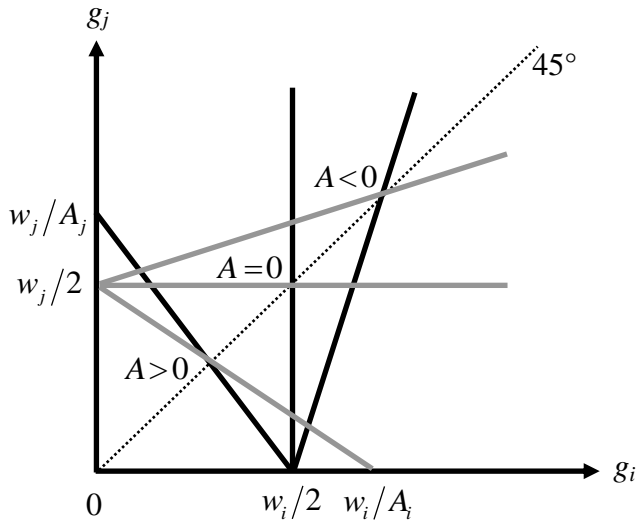
When  $\alpha_i = \beta_i$  or  $A_i = 0$ , agent  $i$  is equally concerned by guilt alleviation and social hierarchy. This time,  $r_i$  is equal to  $(1/2)w_i$  for  $g_j \in [0, \infty]$ . Her provision is always the half of her income, but she has no incentive to contribute more than that. This is the behavior of an autonomous agent who disregards the provisions of the opponent. We could think of an individual who invariably contributes to the public good in order to alleviate her guilt – because some moral obligation incites her to do so –, but who does not discredit the positive spillover on her social rank, even if she is not centered upon the social ranking matter. This agent is either blind or denies the possibility of acting as a free-rider.

At last, when  $\alpha_i < \beta_i$  or  $A_i < 0$ ,  $r_i$  holds if  $g_j \leq -w_i/A_i$ , otherwise  $r_i = w_i$  and agent  $i$  allocates her full income to the supply of the public good. Provisions are then strategic complements: every time agent  $j$  increases her supply, agent  $i$  has an incentive to increase her supply to stay in the race for the social status up to the point where her full income is spent.

According to the foregoing results, Fig. 4.1. illustrates the best-response functions which meet at the bisection line, observed from symmetric cases  $\alpha_i = \alpha_j = \alpha$  and  $\beta_i = \beta_j = \beta$ . Each best-response function – initiated from the reference point which is the opponent's null provision – is v-shaped, *i.e.* separated into two segments following opposite slopes.

The black straight lines depict agent  $i$ 's best-response functions. The grey straight lines depict agent  $j$ 's best-response functions. We have three cases: (i) when the intrinsic impulse dominates  $A > 0$ , their best-response functions decrease in their

opponents' provisions and the public good is weakly provided, for respective provisions are less than  $w_i/2, w_j/2$ ; (ii) when the extrinsic impulse dominates  $A < 0$  their best-response functions increase in their opponents' provisions and the public good is highly provided, equilibrium provisions' exceed  $w_i/2, w_j/2$ ; (iii) when impulses are identical  $A = 0$ , equilibrium provisions are  $w_i/2, w_j/2$ .



**Fig. 4.1. Agents  $i$ 's and  $j$ 's best-response functions**

Let us now differentiate  $r_i$  with respect to  $\alpha_i$  to observe the elasticity of the best-response function *vis-à-vis* guilt:

$$\frac{\partial r_i}{\partial \alpha_i} = -\frac{\beta_i g_j}{(\alpha_i + \beta_i)^2} < 0 \quad [19]$$

The derivative  $\partial r_i / \partial \alpha_i$  is strictly negative and agent  $i$ 's best-response function decreases in  $\alpha_i$ , for all  $g_j$ . Since the benefit from the consumption of the public good is negligible and contributing to the public good is an opportunity cost by cause of less consumption of  $x_i$ , agent  $i$  has no incentive to provide the public good so agent  $j$



could ease her guilt. In the same way, agent  $j$ 's provision is a substitute to her own, which encourages her to free-ride. As a result, as  $\alpha_i$  goes up, agent  $i$  decreases her provision. The same reasoning applies to agent  $j$ .

**Corollary 4.2.:** Agent  $i$ 's best-response function decreases in  $\alpha_i$ .

Let us now differentiate  $r_i$  with respect to  $\beta_i$  to observe the elasticity of the best-response function *vis-à-vis* social status:

$$\frac{\partial r_i}{\partial \beta_i} = \frac{\alpha_i g_j}{(\alpha_i + \beta_i)^2} > 0 \quad [20]$$

The derivative  $\partial r_i / \partial \beta_i$  is strictly positive and agent  $i$ 's best-response function increases in  $\beta_i$ , for all  $g_j$ . As  $\beta_i$  goes up, agent  $i$  is emulous and considers agent  $j$ 's provision a threat to her status in the social hierarchy, which makes her increase her provision. In consequence, the higher  $\beta_i$ , the higher the agent  $i$ 's provision. The same reasoning applies to agent  $j$ .

**Corollary 4.3.:** Agent  $i$ 's best-response function increases in  $\beta_i$ .

### 4.3.3. The equilibrium

At a Nash equilibrium  $(g_i^*, g_j^*)$  each agent's provision is her best response to the other's. We first consider an interior equilibrium where both agents' provisions are strictly positive but inferior to their incomes:  $0 < g_i^* < w_i$ ,  $0 < g_j^* < w_j$ . At such equilibrium, provisions amount:

$$\begin{cases} g_i^* = \frac{2w_i - A_j w_j}{4 - A_i A_j} \\ g_j^* = \frac{2w_j - A_i w_i}{4 - A_j A_i} \end{cases} \quad [21]$$

where  $A_i = \frac{\alpha_i - \beta_i}{\beta_i + \alpha_i}$  and  $A_j = \frac{\alpha_j - \beta_j}{\beta_j + \alpha_j}$ .

In this case, the aggregate level of the public good in equilibrium, that is,  $G^* = g_i^* + g_j^*$ , amounts:

$$G^* = \frac{1}{4 - A_i A_j} \left[ (2 - A_j) w_i + (2 - A_i) w_j \right] \quad [22]$$

As one can detect, when agents apply their best-response functions, the aggregate level of the public good depends on the relative distance between the social status and guilt relief indices.

**Corollary 4.4.:** When  $A_i$  increases, (i) the equilibrium provision of agent  $i$  decreases; (ii) the aggregate equilibrium quantity of public good decreases; (iii) and the equilibrium provision of agent  $j$  increases (decreases) if  $A_j > (<) 0$ .

Proof: In the appendix.

A policy of income transfer from agent  $j$  to agent  $i$  such that  $dw_i = -dw_j = 1$  impacts the aggregate quantity of public good:

$$dG = dg_i + dg_j = \left[ \frac{\partial g_i}{\partial w_i} - \frac{\partial g_i}{\partial w_j} \right] + \left[ \frac{\partial g_j}{\partial w_i} - \frac{\partial g_j}{\partial w_j} \right] \quad [23]$$

That is:

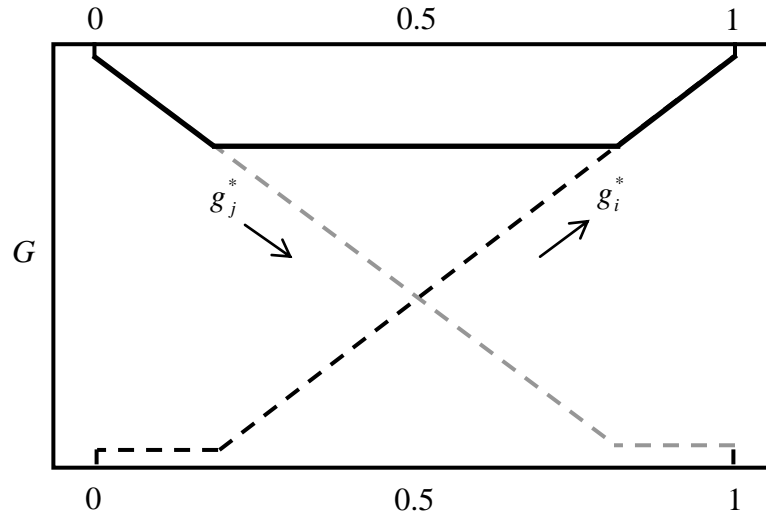
$$dG = \frac{1}{4 - A_i A_j} [A_i - A_j] \quad [23']$$

$$\text{where } A_i - A_j = 2 \frac{\alpha_i \beta_j - \alpha_j \beta_i}{(\alpha_i + \beta_i)(\alpha_j + \beta_j)}$$

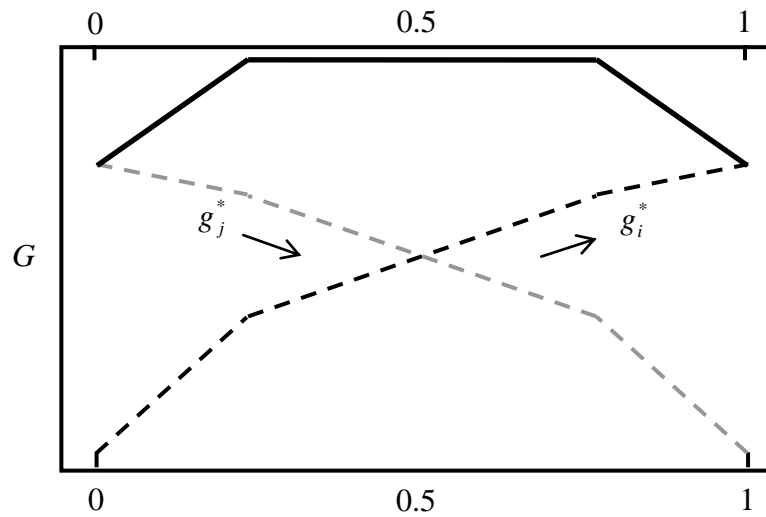
**Corollary 4.5.:** At interior equilibrium, an income transfer from agent  $j$  to agent  $i$  increases (decreases) the aggregate level of the public good if and only if  $\alpha_i \beta_j - \alpha_j \beta_i > (<) 0$ .

In equilibrium, when  $\beta_i = \beta_j = 0$  and each agent is indifferent between her supply and the other's supply, we have  $dG = 0$  which is the standard result of neutrality obtained by Warr (1983).

Let us now consider corner solutions with either null or full-income provisions. In corner equilibria, in case of strategic substitutes, one of the agents provides a null supply. In the case of strategic complements, one of the agents allocates her full income to the public good supply. If we analyze income transfers at the corner equilibria in the symmetric case, Figs. 4.2. and 4.3. depict provisions with respect to the income inequality. The  $x$ -axes denote agents' shares of total income:  $w_i / (w_i + w_j)$ ,  $w_j / (w_i + w_j)$ . The  $y$ -axis represents the aggregate level of the public good. The total income is fixed. The broken black curve represents the provision of agent  $i$  while the broken grey curve represents the provision of agent  $j$ . The broken grey curve decreases while the broken black curve increases as the transfer between agents  $j$  and  $i$  occurs. The equality arises at 0.5. The solid black curve illustrates the sum of provisions, *i.e.* the aggregate level of the public good.



**Fig. 4.2. Income transfer with strategic substitutes**

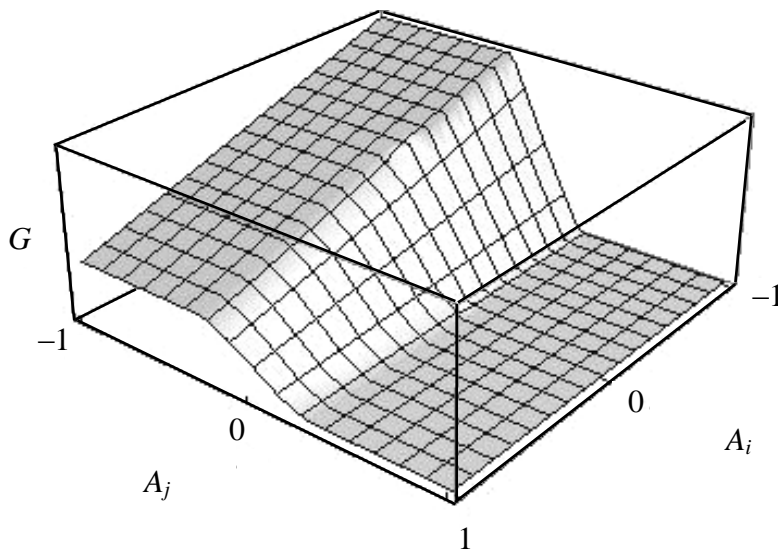


**Fig. 4.3. Income transfer with strategic complements**

In the case of strategic substitutes (the standard scenario in public good games), where guilt relief prevails, the aggregate level of the public good decreases as the incomes' disparity shrinks. Indeed, at a corner solution, the lower income agent invariably free-rides on the supply of the higher income agent. If the income is transferred from the lower income agent to the higher income agent, the latter should

allocate the extra income into the public good supply<sup>48</sup> and the aggregate quantity should increase. This is a similar result to Theorem 5 from Bergstrom *et al.* (1986) who show that equalizing income by transferring income from contributors to non-contributors will decrease the equilibrium supply of the public good, in the case of a pure public good ( $\beta_i = 0$  in our case).

In the case of strategic complements (the novel scenario in public good games), where status seeking prevails, the aggregate level of the public good decreases as the agents' income disparity grows. This time, the lower income agent allocates her full income to the supply of public good in order to gain social status, thus saturating her supply capacity, whereas the higher income agent contributes less than her full income. An income transfer from the higher income agent to the lower income agent should increase the quantity of public good, because the lower income agent should allocate the money transfer to the provision of the public good.



**Fig. 4.4. The aggregate level of provisions**

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<sup>48</sup> For example, this suggests that cutting taxes on the higher income agent and raising taxes on the lower income agent may increase private supply.

At last, Fig. 4.4. shows the aggregate provisions to the public good in view of  $A_i$  and  $A_j$ , in both interior and corner equilibria. The kinks in the slope correspond to corner equilibria. When  $A_i < 0$ , agent  $i$ 's provision is a strategic complement, and strategic substitute otherwise.  $G$  decreases with  $A_i$ .

#### 4.4. Concluding remarks

When agents privately provide a public good, agents profit from donations to alleviate their guilt. Because guilt relief entails opportunity costs, agents refuse to pay for others' guilt or simply profit from efforts of the others, and this leads to free-riding and weak provision of the public good: a phenomenon amply covered by the economic literature.

The private provision of the public good is stimulated by the private benefit of the public good such as obtaining some social status, which *ex post* seems intuitive. In the case where provisions become strategic complements, a policy of income transfers from the higher income agent to the lower income agent should increase the aggregate level of the public good. Its purpose would be to activate the competition for social status, which increases the public good's level and thus social welfare.

Our model can stand for the wanting theoretical background which explains why agents under-react to the income transfer, that is why lower income agents over-contribute and higher income agents under-contribute, in both experiments from Chan *et al.* (1996) and Maurice *et al.* (2009). Indeed, neither has considered contributions as strategic complements. Maurice *et al.* (2009) suggest the anchoring phenomenon explains their results, but we prefer the social status phenomenon as a way to explain pro-social behavior. Besides, Chan *et al.* (1996) themselves conclude that the explanation for experimental results might be in a model where agents react to their competitors. Their intuition meets up with ours.

Therefore, status (market) competition in the form of auctions can be an answer to free-riding<sup>49</sup>. Our results could explain the institution of charity auctions, honor rolls of donors and the construction of socially responsible finance indices. More generally, it could relate to why institutions make use of agents' willingness to demonstrate their generosity if not their apparent selflessness. To some extent, our model could be an illustration of the theory of crowding out of intrinsic motivations by extrinsic incentives. Further work consists in verifying the relevancy of these findings with field data.

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<sup>49</sup> This is an opposite result to Holländer (1990) who finds that opening a market for the collective good lowers its provision.

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## 4.6. Appendix

### Proof of Proposition 4.1.

First,  $(g_i^*, g_j^*)$  is a Nash equilibrium if and only if  $g_i^*$  is a fixed point of the function:

$$x \rightarrow r_i(r_j(x, w_j), w_i) \text{ and } r_j(g_i^*) = g_j^*.$$

Second, if  $\frac{\partial r_i}{\partial g_j} \in (-1, 1)$  and  $\frac{\partial r_j}{\partial g_i} \in (-1, 1)$ ,  $x \rightarrow r_i(r_j(x, w_j), w_i)$  has a unique fixed point and

$$\frac{\partial r_i}{\partial g_j} = \frac{\frac{dH_i}{dg_j}}{-\frac{dH_i}{dg_i}}, \quad \frac{\partial r_j}{\partial g_i} = \frac{\frac{dH_j}{dg_i}}{-\frac{dH_j}{dg_j}} \quad \square$$

### Proof of Proposition 4.2.

Consider the ratio which measures the relative incentives to contribute to the public good:

$$\gamma_i = \frac{\frac{dH_i}{dx_i}}{\left(\frac{dH_i}{dg_j}\right) - \left(\frac{dH_i}{dg_i}\right)} = \frac{\frac{\partial H_i}{\partial x_i}}{\left[\left(\frac{\partial H_i}{\partial G}\right) - \left(\frac{\partial H_i}{\partial s_i}\right)\right] - \left[\left(\frac{\partial H_i}{\partial G}\right) + \left(\frac{\partial H_i}{\partial s_i}\right) - \left(\frac{\partial H_i}{\partial x_i}\right)\right]}$$

First of all, it is worth writing  $\gamma_i$  according to the partial derivative of the reaction function  $r_i$ :

$$\gamma_i = \frac{\frac{\partial H_i}{\partial x_i}}{\left(\frac{\partial H_i}{\partial x_i}\right) - \left(\frac{2\partial H_i}{\partial s_i}\right)} = \frac{\frac{\partial H_i}{\partial x_i}}{\left(\frac{\partial H_i}{\partial g_j}\right) - \left(\frac{\partial H_i}{\partial g_i}\right)} = \frac{\frac{\partial r_i}{\partial w_i}}{1 + \left(\frac{\partial r_i}{\partial g_j}\right)}$$

The income transfer corresponds to  $dw_i = -dw_j > 0$ . At the unique equilibrium  $G^*(w_i, w_j)$ , agent  $i$ 's provision satisfies  $r_i(g_j^*, w_i) = g_i^*$  and differentiation of this relation gives

$$dg_i^* = \frac{\partial r_i}{\partial g_j} dg_j^* + \frac{\partial r_i}{\partial w_i} dw_i.$$

Since  $dg_j^* = dG^* - dg_i^*$ , we have  $dg_i^* = \frac{\partial r_i}{\partial g_j} (dG^* - dg_i^*) + \frac{\partial r_i}{\partial w_i} dw_i$ . That is:

$$dg_i^* = \frac{\frac{\partial r_i}{\partial g_j}}{1 + \frac{\partial r_i}{\partial g_j}} dG^* + \frac{\frac{\partial r_i}{\partial w_i}}{1 + \frac{\partial r_i}{\partial g_j}} dw_i = \frac{\frac{\partial r_i}{\partial g_j}}{1 + \frac{\partial r_i}{\partial g_j}} dG^* + \gamma_i dw_i,$$

A similar expression holds for  $dg_j^*$  and summing both expressions gives:

$$\left( \begin{array}{cc} \frac{\partial r_i}{\partial g_j} & \frac{\partial r_j}{\partial g_i} \\ 1 - \frac{\frac{\partial r_i}{\partial g_j}}{1 + \frac{\partial r_i}{\partial g_j}} - \frac{\frac{\partial r_j}{\partial g_i}}{1 + \frac{\partial r_j}{\partial g_i}} \end{array} \right) dG^* = \gamma_i dw_i + \gamma_j dw_j = (\gamma_i - \gamma_j) dw_i,$$

Because of [12], the first factor of the left hand side is positive thus:

$$dG^* > 0 \Leftrightarrow \gamma_i > \gamma_j$$

□

**Proof of Corollary 4.1.**

Given  $g_j$ , differentiating  $u(\cdot)$  with respect to  $g_i$  gives best-response  $g_i^*$ . At an interior solution the first order condition is satisfied:

$$-\frac{1}{w_i - r_i} + \frac{\alpha_i + \beta_i}{\alpha_i(r_i + g_j) + \beta_i(r_i - g_j)} = 0.$$

Therefore,  $(\alpha_i + \beta_i)(w_i - r_i) = \alpha_i(r_i + g_j) + \beta_i(r_i - g_j)$ , and

$$r_i(g_j, w_i) = \frac{1}{2}w_i - \frac{1}{2} \frac{(\beta_i - \alpha_i)}{(\beta_i + \alpha_i)} g_j = \frac{1}{2}w_i - \frac{1}{2}A_i g_j$$

This equation holds if the right hand side is between 0 and  $w_i$  which is the case if  $g_j \leq w_i/A_i$  when  $A_i > 0$  (i.e.  $\alpha_i > \beta_i$ ) and  $g_j \leq -w_i/A_i$  when  $A_i < 0$  (i.e.  $\alpha_i < \beta_i$ ).

When  $(\beta_i - \alpha_i) = 0$ ,  $r_i = (1/2)w_i$  for any  $g_j \geq 0$ . The same reasoning applies to agent  $j$ . □

**Proof of Corollary 4.4.**

At an interior equilibrium, the two following equations are satisfied:

$$\begin{cases} 2g_i^* + A_i g_j^* = w_i \\ A_j g_i^* + 2g_j^* = w_j \end{cases}$$

where  $A_i = \frac{\alpha_i - \beta_i}{\alpha_i + \beta_i}$ ,  $A_j = \frac{\alpha_j - \beta_j}{\beta_j + \alpha_j} \in [-1, 1]$ .

The aggregation of provisions amounts:

$$\begin{bmatrix} g_i \\ g_j \end{bmatrix} = \frac{1}{4 - A_i A_j} \begin{bmatrix} 2 & -A_i \\ -A_j & 2 \end{bmatrix} \begin{bmatrix} w_i \\ w_j \end{bmatrix} = \frac{1}{4 - A_i A_j} \begin{bmatrix} 2w_i - A_i w_j \\ 2w_j - A_j w_i \end{bmatrix}$$

And the total provision is:

$$G^* = \frac{1}{4 - A_i A_j} \left[ (2 - A_j) w_i + (2 - A_i) w_j \right]$$

When  $\beta_i = \beta_j = 0$  and agents are exclusively intrinsically impulsed

$$G^* = \frac{2w_i - w_i + 2w_j - w_j}{4 - 1} = \frac{1}{3} (w_i + w_j)$$

□

