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Intégration des énergies renouvelables au réseau de distribution d'électricité

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FOREWORD

This thesis has been financed by the Centre Scientifique et Technique du Batiment (CSTB), a French public research and assessment organization tasked with developing and disseminating new knowledge to improve the quality and efficiency of buildings. The impetus behind this foray into the inner workings of the power system is the expected development of "zero-energy" or "positive-energy" buildings that not only strive to limit their energy consumption but also supply energy on-site, usually via renewable-based converters. If it reaches a significant level, the integration of such buildings into the power system will have impacts on its planning and operation. In this light, the objective of this work, from the standpoint of the CSTB, is to provide a methodology to assess those impacts in a systemic manner, which means taking into account the power system as a whole with a particular focus on the interactions between its various subsystems. While the power systems in various countries display conceptual similarities, their practical implementation, especially in terms of the technical characteristics of the generators and the regulatory environment, are the results of historical processes and are thus specific to each countries. As a consequence, the methodologies and tools developed in this contribution could be adapted to various power systems, whereas the case studies are heavily influenced by the characteristics of the French power system.

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ACRONYMS

AC Alternating Current

DC Direct Current

EPIC Etablissement Public à caractère Industriel et Commercial, or Commercial and Industrial Public Company

EDF Electricité De France

CEA Commissariat à l'Energie Atomique

IEA International Energy Agency

TSO Transmission System Operator

DSO Distribution System Operator

DG Distributed Generation

DN Distribution Network

APEI Advanced Power Electronic Interface

PV Photovoltaic

EROI Energy Return On Investment

IPCC International Panel on Climate Change

LCOE Levelized Cost Of Electricity

FIT Feed-In Tariffs

ADN Active Distribution Network

EPS Electric Power System

SMES Super Magnetic Energy Storage

DER Distributed Energy Resources

CPS Centralized Power System

OPF Optimal Power Flow

CG Centralized Generation

AS Active Substations

MV Medium Voltage

LV Low Voltage

MINLP Mixed-Integer Non-Linear Problem

LTMP Long-Term Master Problem

QCQP Quadratically Constrained Quadratic Problem

SOC Second-Order Cone

OLTC On-Load Tap Changer

1 INTRODUCTION

This introduction will be organized in the following manner. First we will describe the various phases of the evolution of the French power system, from the decentralized system of the early days to the large-scale deregulated current system and passing through the state-owned monopoly. We will focus particularly on the interplay between technological improvements and political orientations as the determining factor explaining power system organization and functioning. Subsequently we introduce a potentially disruptive set of technologies that is the focus of this thesis: distributed generation, with a specific attention brought to the complementary instances of weather-dependent renewable generators and storage. Due to intrinsic qualities that we set forth, renewable energies have enjoyed a broad political support that has turned into various subsidy mechanisms that we also present. In turn, the development that this has spurned will have possibly adverse impacts on the rest of the power system that we depict in detail. Distributed storage units, their characteristics and the applications they could have in the context of large distributed renewable generation penetration are specified next. Due to this expected expansion of distributed generation, the assumptions underpinning the current planning and operation of the power system, and especially the distribution system, are called into question. To cope with this, various strategies relying on differing assumptions concerning the level of control and observability of the future distribution network have been put forward in the literature. After describing them summarily, we identify a potential area of improvement that leads us to set out the list of objectives that we have assigned ourselves as well as the organization of the rest of this contribution.

1.1 EVOLUTION OF THE POWER SYSTEM

1.1.1 *Early days, 1880-1910*

The first commercial applications of electricity were developed in competition with other energy vectors, such as pneumatic or hydraulic transmission to power motors or gas for street lights. Each type of load (street lights, factory motors, streetcars, etc) had differing voltages and had to be supplied with its specific generators and lines. Moreover these generators had to be located nearby for low-voltage loads. It was already known that higher voltages would allow transmission on longer distances, in turn permitting to take advantage of economies of scale and remote hydropower sources. However, there did not exist any practical mean to transform the voltage and the only solution to transport electricity on long distances was thus to supply many loads connected in series, which was done, for example, with light systems at the Universal Exposition in 1878 (11 kilometers) or with a line between the Moutiers hydropower plant and the Lyon street light infrastructure in 1906 (125 kV, 230 kilometers). Both alternatives (local supply or loads connected in series) had limitations that did not allow electricity to supplant existing energy vectors easily, a fact illustrated by, for example, the fierce competition between electricity and gas lightning in the UK during the 1880s, see [11]. As an telling anecdote, we can refer to the Opera Avenue in Paris, whose lighting system was electrified in 1878 before going back to gas in 1882.

1.1.2 *Regional AC transmission, 1910-1946*

In this context, the invention of the transformer, that can be dated back to 1884-1885, along with the invention of an efficient AC motor by Tesla in 1888, proved to be a pivotal moment in the history of electricity. The advantages were twofold : first, low-voltage loads could be connected to remote generators and, second, loads with differing voltages and thus differing uses could be supplied with the same network. One of the first example of a commercial implementation is the AC line linking the Niagara Falls hydropower plant and the city of Buffalo, NY in 1895. This did not end the War of Currents between AC and Direct Current (DC) immediately (for example the Moustiers-Lyon line mentioned above was DC) but AC gained an advance that proved decisive. From an economic standpoint, this resulted in significant reduction in the costs of electricity due to effects of scale in generation, higher load factors and reduced capital investments in networks. Consequently, the first decades of the 20th century saw a continuous expansion of electricity use, with, for example, the number of electrified municipalities in France going from 7000 in 1919 to 36500 in 1938, with a particularly high growth between 1927 and 1933 (18210 to 33567 electrified municipalities). In parallel, the yearly electricity produced went from 1.8 TWh in 1914 to 21 TWh in 1938. Figure 1 depicts the state of the French power system in 1930, with the several high-voltage line allowing the connection of load centers with remote power sources on a regional scale. At the end of the 1930s, there existed around 200 companies concerned with production, 100 with transmission and 1150 with distribution [12]. However, the sector was fragmented only in appearance, as most of the companies in production and transport belonged to large holdings (Union d'Electricité, Energie Industrielle, groups Péchiney or Empain) that thus acted as regional quasi-monopoly. The consequence is a series of

Figure 1: French network and main generators in 1930, source : [1]

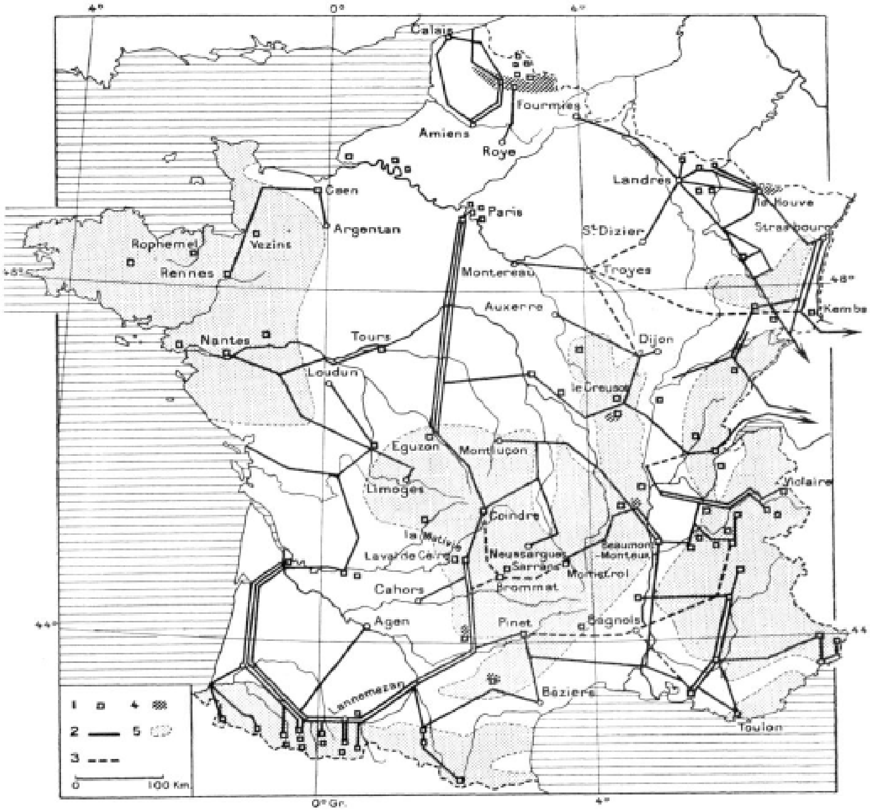


FIG. 1. — LE RÉSEAU ÉLECTRIQUE FRANÇAIS.

D'après la Carte de France, Centrales, réseaux de distribution, lignes de transport, dressée par la SOCIÉTÉ FINANCIÈRE ÉLECTRIQUE. — 1, Usines. — 2, Lignes à 55 000 volts et au-dessus. — 3, Lignes projetées ou en construction. — 4, Bassins houillers. — 5, Montagnes susceptibles d'aménagement hydro-électrique. — Échelle, 1 : 8 300 000.

anti-competitive behavior regarding investments in generators (large and capital-intensive investments, especially in hydroelectricity, that could lower electricity costs are avoided) and transmission (inter regional connections that could permit a better matchup of generators and loads are discarded), see [13] for details, that did not allow the development of the power system to its full potential and set the stage for the next phase : nationalization.

1.1.3 *National monopoly : 1946-1996*

The idea that the power sector had to be regulated was not a new one : as early as 1906, June 15th, a law established that the distribution of electricity was a public service with the equipment being the property of municipalities that could concede its exploitation to the private sector. On July 19th, 1922, another law was enacted to allow the State to either invest himself or force private companies to invest in transmission lines if it deemed it necessary "to ensure a more complete utilization and better sharing out of electrical energy". However, this law was never put to use and the anti-competitive behaviors in the power sector continued unabated, with high prices and a high geographical inequality in the access to power. The situation deteriorated rapidly at the beginning of the 1930s and led, in July 1937, to the signings of resolutions by a parliamentary commission on electricity, the federation of electrified municipalities and the association of mayors urging the government to enforce "a significant decrease in high-voltage electricity prices" and "the creation of a public entity tasked with sharing out high voltage energy between production plants and distribution networks". The first response of the government was the the enactment of a decree (May 2^{nd}, 1938) allocating 350 million Francs to the creation of a publicly-owned transmission network in eastern France. Fearing a possible nationalization of electricity transmission, the private sector joined the government in talks that ended up with an agreement formalized by a decree on the 17^{th} of June. The major measure was the creation of the "Groupement Financier d'Electricité", an entity tasked with collecting funds from electricity companies and emitting bonds to finance a 3 billion Franc development program with a particular focus on hydropower and interregional connections. Other measures concerning prices and working conditions were also introduced and the threat of nationalization was fended off for the time.

This did not last long as the end of World War II ushered in a new era. The fundamental issues stemming from the control of the power sector by a few "electricity trusts" were not solved by the pre-war attempts at regulation. This time around, the difference was that the political landscape had changed, with the nationalization not only defended by the communist party and unions but also supported by the Général de Gaulle since 1944 and listed as an objective in the Conseil national de la Résistance political program. Thus, on April 18^{th}, 1946, following the initiative of Marcel Paul, the then minister of industrial production, a law was passed to create Electricité de France, a vertically and horizontally integrated state-owned company ("établissement public à caractère industriel et commercial" or Etablissement Public Ã caractÃšre Industriel et Commercial, or Commercial and Industrial Public Company (EPIC) to be accurate) tasked with ensuring the production, transport and distribution as well as imports and exports of electricity. While all the transmission system was incorporated into EDF, some parts of the distribution system already under municipal control and some generators -usually small or used for self-consumption by industries- remained independent. The creation of EDF had multiple objectives [14]:

- Operating the power system on a national scale in an integrated manner and with a public service goal regarding price setting, equality of access and quality of supply

- Planning and carrying out the rebuilding and then expansion of the power system
- Financing, usually through debts, these investments
- Balancing revenues and expenditures through rational organization and price settings

Of course, these objectives could not be attained all at once, and we can distinguish three phases in the history of EDF.

1.1.3.1 *1946-1950*

Administrative and organizational integration of nationalized companies, investment in production and transport and management of shortages are the main priorities of the newly integrated utility. In 1950, the situation is back to normal concerning the supply-demand equilibrium, but France still lags behind in term of overall access to electricity (10 million households connected in 1948, 140 kWh of annual mean consumption compared to 390 kWh in the UK or 590 kWh in Switzerland). The theoretical foundations for electricity tariff, based on marginalist theory calculation are established.

1.1.3.2 *1950-1970*

Large investments in production (hydro and thermal), transport (380-400 kV) and distribution system allow France to fill the gap with other industrial countries. At first thermal and hydro are developed at virtually the same rate, but in the middle of the 60s, hydro capacity increase stalls as the geographical opportunities for low-cost hydro decrease. In the middle of the 70s, 20 million households are connected (for a total population of 52 millions) which means that only highly isolated sites are still not supplied by the centralized network. Starting in 1951 for industries and 1956 for households, an electricity price based on long-term marginal costs is applied.

1.1.3.3 *After 1970*

The development of the power system is not focused on electrification anymore, but rather on increasing the quality and reliability of supply, as well as increasing the total output to allow new uses (in particular thermal uses) of electricity to develop. This coincides with the beginning of the industrial phase of the civil nuclear program. After a long opposition between EDF and the Commissariat Ã l'Energie Atomique (CEA), the light-water technology (boiling and pressurized were considered, but only pressurized water reactors were eventually built) was chosen and the first 6 reactors were commissioned at the end of 1969. While a further expansion was already planned, the oil shock of 1973 precipitated the commissioning of new nuclear plants,

Figure 2: Electricity production by source, source : EDF

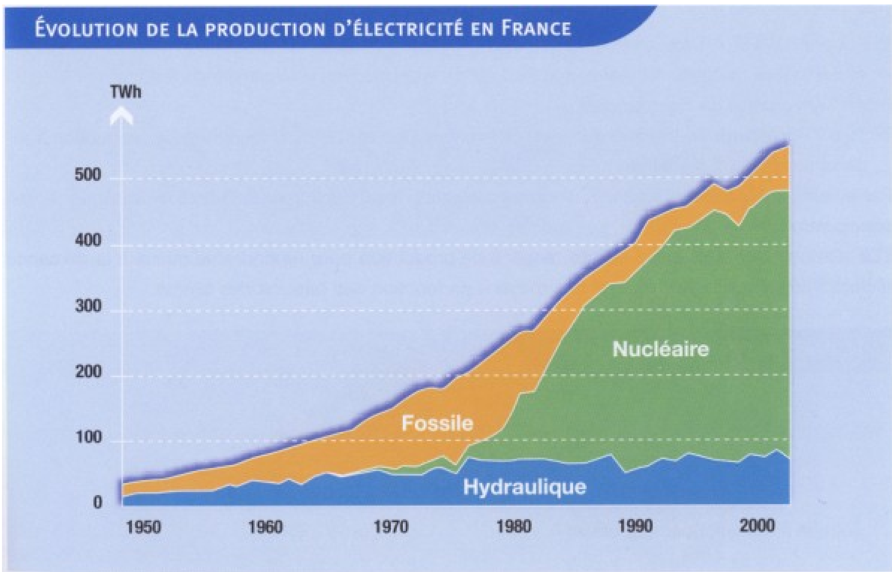
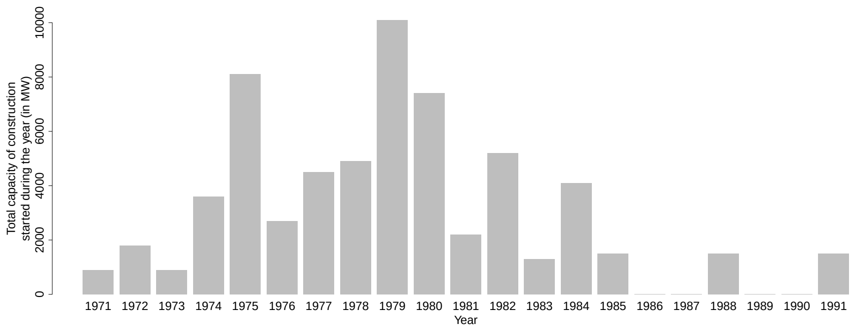


Figure 3: Evolution of the start of constructions of nuclear reactors in France from 1971 to 1991, source : IEA



with energetic independence a major concern at that time. The start of construction of new reactors continued at a rapid pace during the 1970s, as can be seen in Figure 3. In parallel, the density of the 400 kV is greatly increased to accommodate the new generation means, see Figure 4. The beginning of the 80s marks a shift in the perception of nuclear with the incidents in Three Mile Island in 1979 and Saint-Laurent in 1980, but it is the Chernobyl accident of 1986, in conjunction with a decrease in expected load growth, that puts a stop to the French nuclear program. Commissioned plants are still built but no new plant is planned until 2004.

1.1.4 *Deregulation: 1996-Now*

Starting with Chile in 1981, a trend of liberalization, restructuring and privatization of the power sector can be identified, a part of a much

(a) 1960 (b) 1978 (c) 1990

Figure 4: Evolution of the high-voltage (400 kV) network in France, source : RTE

broader movement toward a market-based organization of the economy. This trend has taken various forms depending on the country considered but common denominators can be outlined:

- "Unbundling" of previously vertically integrated companies into separate (at least from an accounting viewpoint) entities tasked with the production, transmission, distribution and retailing
- Preservation of tightly-regulated natural monopolies in transmission and distribution
- Creation of an open wholesale energy and ancillary service market
- Creation of a regulatory agency tasked with overseeing regulated activities (transmission and distribution) and ensuring an efficient market operation

It may be an understatement to say that France did not take the lead in the liberalization of its electricity market. However, after directives were enacted by the European Union, it started on this path and we can highlight the following milestones:

- December 19^{th}, 1996: directive 96/92 organizing the interior electricity market by the "accounting unbundling" of the vertically integrated utilities, the creation of open markets for production and retailing, the establishment of rules for transmission and distribution network access and for exchanges between countries.
- February 10^{th}, 2000: the French law transcribing directive 96/92 is enacted and the "accounting unbundling" as well as the opening of the retail market for large customers are carried out.
- June 26^{th}, 2003: directive 2003/54 imposes, among other things, the "legal unbundling" and a time frame for the opening of the whole retail market.

- July 1st, 2004: the retail market for all professional customers and municipalities is opened
- August 9th, 2004: EDF is turned from an EPIC to a "Société Anonyme". Part of its capital is introduced on the stock exchange on November 21st, 2005. To this date, the French state still owns around 84% of the capital.
- September 1st, 2005: RTE, the French Transmission System Operator (TSO), becomes a subsidiary of EDF
- July 1st, 2007: the retail market for all customers is opened
- January 1st, 2008: ErDF, the French Distribution System Operator (DSO) who operates 95% of the distribution network, becomes a subsidiary of EDF
- July 13th, 2009: directive 2009/72 was meant to enforce the "ownership unbundling", but France obtained the possibility to let the network operators be owned by utilities in exchange for strict rules guaranteeing their financial and technical autonomy. RTE obtained its agreement from the French regulatory agency to operate within this framework in January 2012.

As a consequence of these laws and decrees, the French power system can be considered nominally liberalized and restructured. Despite this, a meaningfully competitive environment did not ensue. The fundamental issue behind this state of affairs is the fact that, having acted as a natural monopoly forced to sell to its retail clients at its long-term marginal cost, EDF can not be dislodged solely by opening up its market. Indeed, its competitors cannot have access to electricity at a price sufficiently low to enter the retail market either through the wholesale market because the price establishes itself at the short-term marginal cost of the incumbent, which is higher than its long-term marginal cost, or through its own investment in generation because it would need to invest at once in a generation fleet on the scale of the incumbent's one to take advantage of both the effect of scale due to large generators and that obtained through integrating the operation of generators with complementary technical and economic characteristics (e.g. base, mid-base and peak plants). To remedy this fact and comply with requests from the European Commission to effectively open up its electricity market, a law was passed to establish a "new organization of the electricity market" (Nouvelle Organisation du Marché de l'Electricité or NOME law) on December 7th, 2010. It consists in creating a mechanism (Accès régulé à l'électricité nucléaire historique or ARENH) that forces EDF to sell up to a 100 TWh a year of the electricity it produces to its competitors at a price "that reflects its economic operation", in complete opposition with the principles of pure and perfect competition. Of course, such a price is open to interpretation depending on the methodology employed to calculate it. Unsurprisingly, EDF advocated a method taking into account the cost of the generation fleet over its whole life that lead to a 49,5 euros/MWh result, while its competitors favored a net accounting value approach (i.e considering the current level of amortization) that resulted in a 39 euros/MWh. The very visible hand of the government set it in a decree at 40 euros/MWh before changing it to 42 euros/MWh to "take into account the consequences of the Fukushima accident". Such a paradoxical approach led Marcel Boiteux, CEO of EDF from 1967 to 1987, to declare that "we opened up the market to competition to decrease the prices, but now we have to increase the prices to allow competition".

In spite of all the efforts to dislodge EDF from its dominant position and 14 years after the first deregulation law, we are still in the presence of an incumbent actor that owns around 85% of the generation capacity, produces about 90% of the yearly energy and supplies 93% of the residential customers and 92% of the other classes of customers [15], at a price that is, for most of them, still regulated by the state. Moreover it is subject to public service requirement such as the obligation to buy renewable at a rate fixed the State or to propose specific tariffs to underprivileged households. This set of observation is the reason why, in this contribution, we have chosen to consider the distributed generators and storage units that could be deployed not as market participants operating in a pure and perfect competitive environment, but rather as parts of a larger generation fleet operating with an aim to minimize the total costs while respecting constraints on reliability and quality, in accordance with the public services requirements Electricité De France (EDF) is subjected to.

1.2 DISTRIBUTED GENERATION

In the preceding section, we have explored the evolution of the power system from a decentralized infrastructure to one in which most generators are large-scale and connected to the transmission system. We will now consider a potential reversal of this trend that could be brought about by the advent of new technologies for distributed generation.

1.2.1 *Definition*

As a general definition of distributed generation, we follow [16] by considering that Distributed Generation (DG) 'is an electric power source connected directly to the distribution network (Distribution Network (DN)) or on the customer site of the meter'. In this thesis, we will focus on a subset of the distributed generators comprised of the renewable-based, weather-dependent generators and the storage units.

1.2.2 *Distributed renewable generators*

1.2.2.1 *Characteristics*

We will concentrate on distributed renewable generators that convert wind or solar energy into electricity before injecting it into the DN. We make the hypothesis that they are connected to the DN through an Advanced Power Electronic Interface (Advanced Power Electronic Interface (APEI), see [17]) that can control active power injection up to a limit dependent on the weather condition and reactive power within the limits imposed by the APEI's apparent power. It should be noted that, due to the power source they tap, such generators have only fixed cost. As a consequence, any decrease in their output below the limit allowed by the weather conditions represents an economic loss for their operators.

1.2.2.2 *Drivers explaining their development*

The photovoltaic effect was discovered in 1839 and put to practical use by the space industry during the 60s, while the first electricity-generating wind turbine can be dated back to 1887. However, it was not until rather recently that their large-scale integration into the power system was considered. We can classify the reasons behind this change into three categories:

- Environmental aspects: starting in the 70s, it became apparent that an economic development based on the exploitation of non-renewable resources (in particular for energy generation) could not go on indefinitely in a finite world. The seminal work of [18] has been influential in propagating this viewpoint while the 2004 update of their report [19] confirmed the validity of the models used. As long as they attain an Energy Return On Investment (Energy Return On Investment (EROI), [20] and [21]) sufficient to allow the "continued economic activity and social function" of the society they supply, renewable energies could be a solution to this issue [22]. Available data on EROI presents large variability but [23] indicates a range of 3-10:1 for solar and an average of around 18:1 for wind energy, which is compatible with the preliminary results in [22]. In parallel, the growing concerns over the potential consequences of climate change, epitomized by the creation of the International Panel on Climate Change (IPCC) in 1988 and the release of their first report in 1990 [24], represent another purpose of renewable energy expansion. Indeed, Figure 5 taken from [25] shows the dominant role of the power sector in greenhouse gases emissions, while

Figure 5: Evolution of direct global greenhouse gas emissions, by sector, 1970-2004

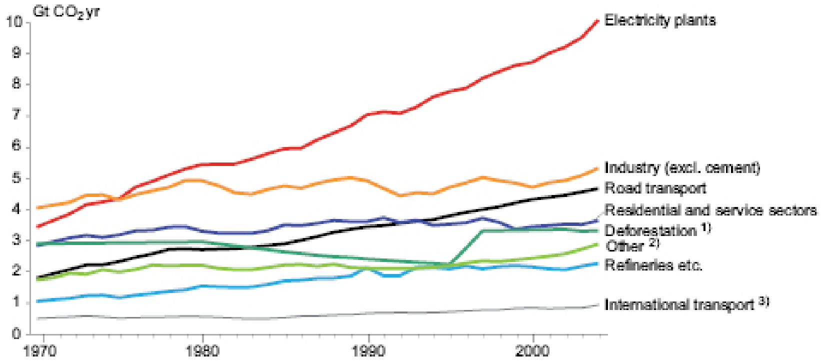


Figure 6: Life-cycle greenhouse gas emissions for selected power sources

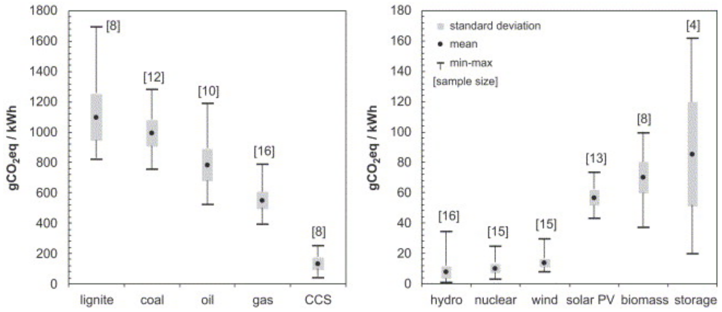


Figure 6 extracted form [26] illustrates the desirable properties of renewable energies in this respect.

- Social preferences: the deployment of distributed generators, owned by individual households or at a community level [27] corresponds to a new, more horizontal organization of the economy that is emerging [28]. In addition, visions of energy autonomy -100% reliance on renewable energies- like [29] or [30] create powerful narratives facilitating their social acceptance and increasing citizens' active involvement.

- Economic competitiveness: in a positive feedback loop, the public support for renewable energies, embodied in various subsidies, has created expanding commercial outlets for manufacturers, allowing them to invest in larger production facilities and research and development, which has then led to reduction in production costs and increased interest in their development. Figure 7 and 8 display the evolution of the levelized cost of electricity (Levelized Cost Of Electricity (LCOE)) for photovoltaic and wind respectively, while Figure 9 compares it to other power sources.

Figure 7: Historic and projected evolution of levelized cost of electricity for solar in Germany, 2005-2020, source: [2]

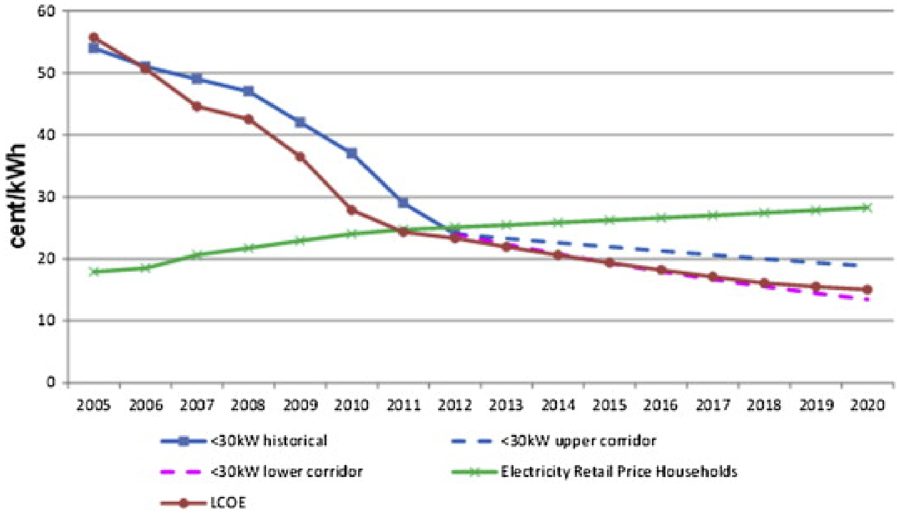


Figure 8: Evolution of levelized cost fo electricity for wind, 1980-2010, source: [3]

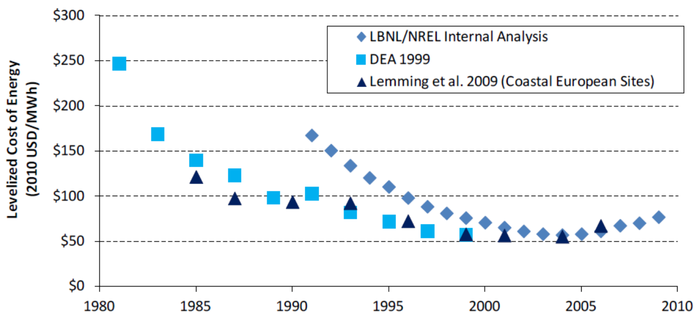
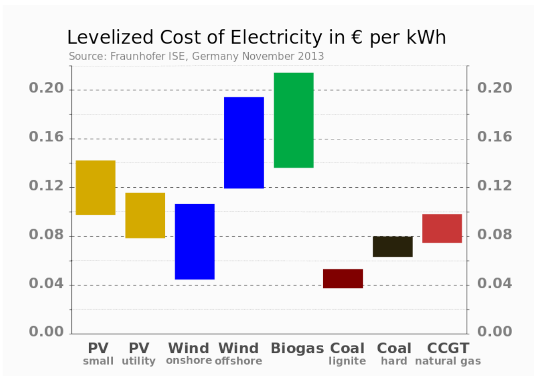


Figure 9: Comparison of levelized cost of electricity for various type of generators, source: Fraunhofer Institute



1.2.2.3 *Structures of incentives*

In some countries, the public support in favor of a larger integration of renewable energies to the power system has turned into a political will to provide a regulatory environment favoring it. Each country introduced a specific set of rules, but they can be grouped into three broad categories, with France having adopted a mix of the first three ones:

- feed-in-tariffs (Feed-In Tariffs (FIT)): all the electricity generated is injected into the grid and sold to the utility at a price fixed by law for the expected lifetime of the installation [31]. The utility is forced to buy the electricity and is compensated, usually through a specific tax scheme. The idea is to set the price at a value high enough to make the investments profitable, but not too high to contain the costs passed on to electricity end-users. This has not always been done very well, especially in the French case, where FITs for Photovoltaic (PV) were massively increased in 2006 to catch up the delay and stayed high until 2010 (see Figure 10), at a time when PV costs were significantly lower, creating an undue profitability and a predictable surge in connection requests (see Figure 11). Fearing ballooning costs for end-users, the government suspended the mechanism until it came up with a self-correcting scheme with changes in tariffs every three months, very successful at containing costs by effectively slashing the yearly installed capacity to 650 MW in 2013. For PV, it is usual to create several classes of FITs according to the size of the installation, so that not only utility-scale projects are carried out.

- premium: in a manner very similar to feed-in tariffs, producers are paid a rate fixed by law that is not supposed to cover all the costs but rather is supplementary to the revenue coming from the wholesale market.

- net-metering: it is in essence equivalent to the FIT scheme, with the only difference being that only the net energy injected in the network is paid at a premium rate [32]. However, if the rate for net-energy injection is the same as the FIT, as is the case in France, net-metering is a net loss for generator operators as the cost of retail electricity is usually lower than the FIT.

- tax credit: like the first two methods, it is a price-based policy. The main differences are the fact that costs are borne by the state instead of the utility and that they are due at the construction instead of spread-out over the lifetime of the plant.

- renewable portfolio standard: it consists in mandating utilities to incorporate a minimum percentage of renewable energy to

Figure 10: Evolution of FITs in France for various type of installations, 2002-2014 source: photovoltaique.info

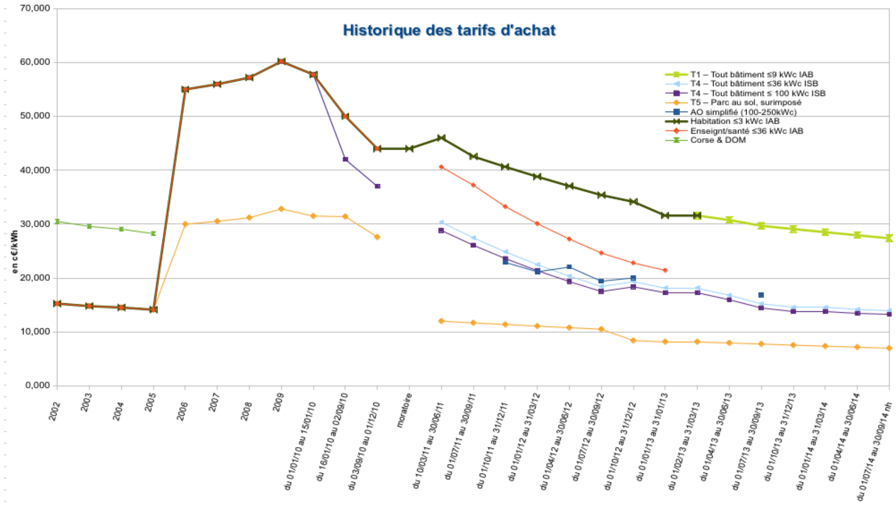
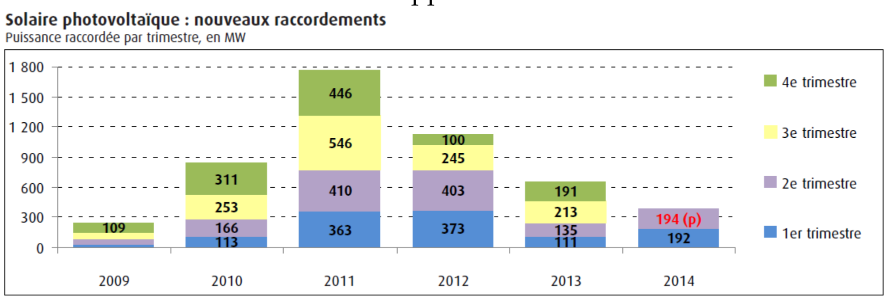


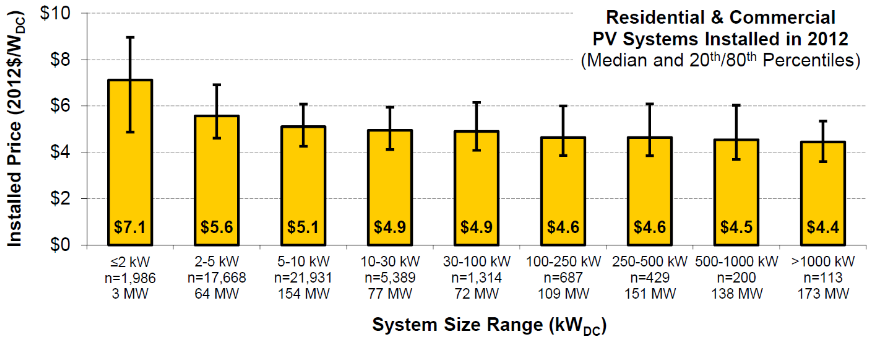
Figure 11: Evolution of installed PV capacity, 2009-2014, source: Commissariat Général au Développement Durable



their supply and is thus a quantity-based policy. It is generally supported by the issuance of renewable energy certificates to eligible producers that are then traded on a specific market, a feature that allows utilities to meet their requirements in the most cost-effective manner, according to proponents of this mechanism [33].

Each type of incentive has specific advantages and drawbacks, but, in general, price-based mechanisms are more effective to jump-start a market by removing part of the risks for investors (see [34] and [35]) at the cost of having less controllable outcomes in terms of installed capacity and the associated financial burden. On the contrary, renewable portfolio standards allow a better control of costs but can sometimes fail to attain their goal in terms of capacity addition, depending on details of their implementation [36].

Figure 12: Installation price of residential and commercial PV according to their size in the USA, 2012, source: [4]



1.2.2.4 *Motivations for a distributed deployment*

A particular decision to invest in a renewable generator is the result of a complex process involving considerations on technical and financial aspects of the installation, the financial status and the personal preferences of the potential investor and the local and national regulatory environment. Nonetheless, two aspects specific to renewable generators can be identified to explain their propensity to be deployed in a distributed fashion:

- The energy converters used are modular, thus allowing even small installations to enjoy, to a certain extent, the effects of scale in upstream manufacturing. Larger plants still have additional effects of scale, but they become tenuous above 100 kW and vanish around 1 MW and higher, see Figure 12 and Figure 13. The consequence is that investments in renewable are accessible to a wide array of potential investors.

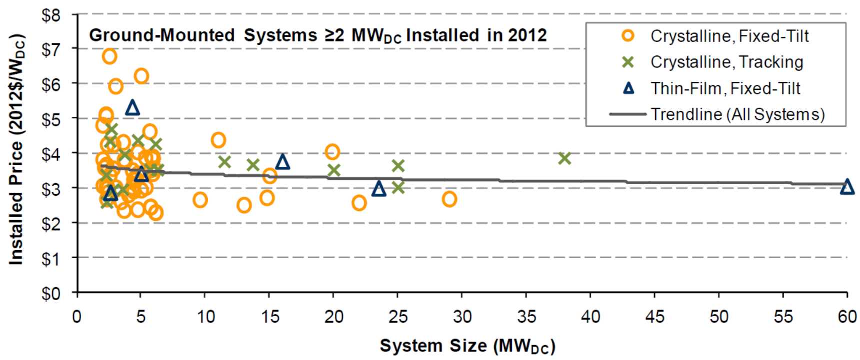
- The primary energy they use is inherently diffuse. This ensures that a large array of siting opportunities with suitable weather condition exist.

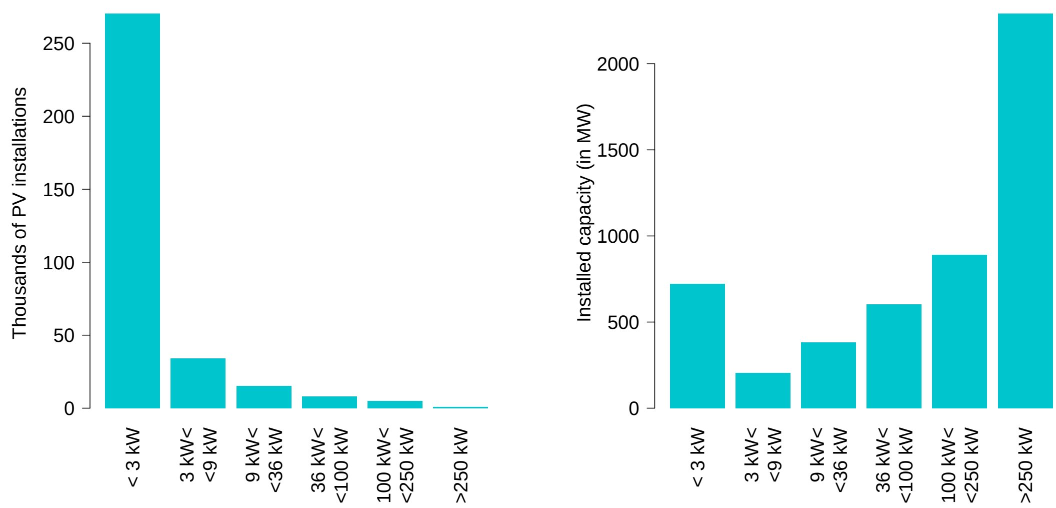
Combining these two characteristics with a size-dependent FIT warrants the availability of many potential sites of various sizes that could achieve a LCOE low enough to be profitable. This can explain why a significant proportion of the PV generators are connected to the distribution network, as Figure 14 illustrates (the category "> 250 kW" contains all the installations connected to the transmission system but also some generators connected to the distribution system), and why these are rather well shared out across the country, as Figure 28.

1.2.2.5 *Impacts on the Power System*

Since the interconnection of regional areas and the concentration of generators at the transmission level, the distribution network has

Figure 13: Installation price of utility-scale PV according to their size in the USA, 2012, source: [4]



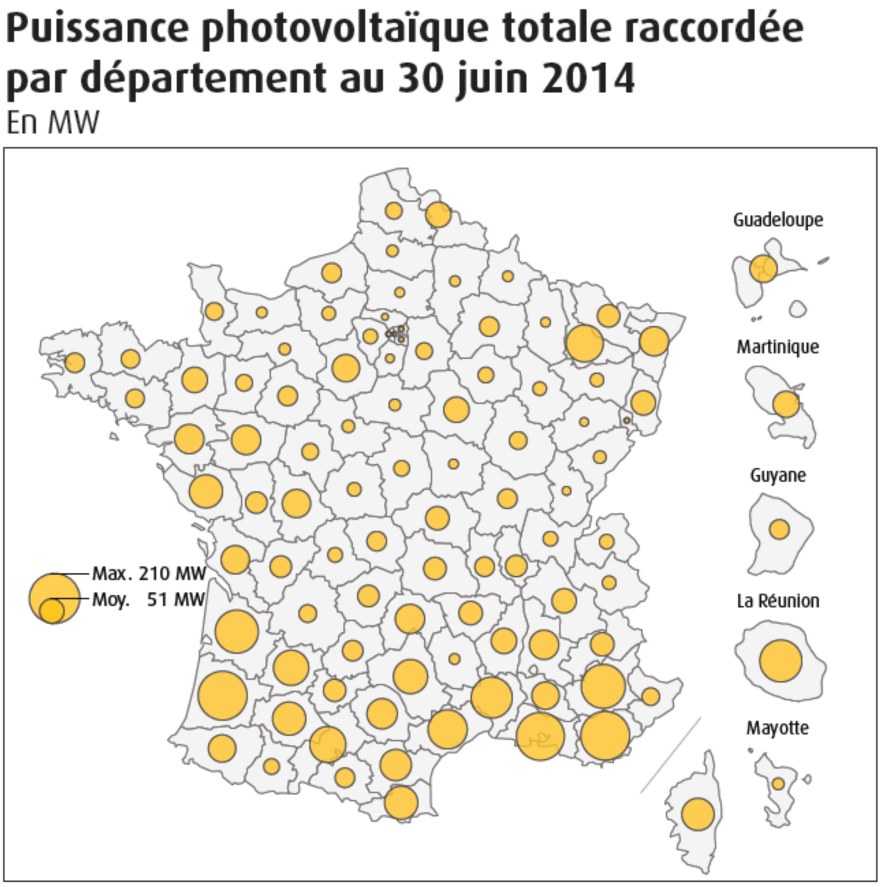


(a) In terms of number of installations

(b) In terms of installed capacity

Figure 14: Distribution of the size of PV installations in France, June 2014, source : Commissariat Général au Développement Durable

Figure 15: Installed capacity by region,June 2014, source : Commissariat Général au Développement Durable



been planned and operated in a passive manner and under the assumption of unidirectional power flows. If we exclude research experiments, the dominant mode of integration of distributed renewable generators is the fit-and-forget approach [37], that consists in reinforcing the network to alleviate any potential local constraint and then letting the generator inject active power in the grid at the maximal output allowed by the meteorological conditions and at a unity power factor. When the penetration of DG remains marginal at a local and national level, this method is justified as the existing margins in network and power plant capacity help restrain the costs associated to passive DG connection while also avoiding the large investment in software, communication, measurement and control technology needed for an active distribution network (Active Distribution Network (ADN)) [38] and [39]. However, when the level of penetration becomes significant relative to the other determinants of the power system, DG will have impacts on both the local distribution network and the centralized power system, denoted respectively local Electric Power System (EPS) and area EPS in [40], where a comprehensive review of these impacts is undertaken.

Impacts on the distribution network:

- Voltage profile: the assumption on unidirectional power flows ensures that the voltage will be decreasing between the substation and the loads, allowing the voltage downstream of the substation to be set at a high value, see Figure 16. However, when DG injection surpasses load, voltage will increase, as in Figure 17, compelling changes in the management of voltage profiles [41–43].

- Line flows: if DG produces reliably during peak load, it can help defer investment in new lines to alleviate line intensity constraints. Conversely, if its maximal output entails reverse power flows higher in magnitude than the maximal flows associated to the preexisting load, investments will need to be carried out [44, 45].

- Line losses: as line flows change, line losses will too, albeit in a quadratic manner. This will influence the operational expenses of the distribution system operators [46, 47].

- Short-circuit current: depending on the relative location of the DG, the protective device considered and the fault envisioned, DG will either increase or decrease the short-circuit current. The protection and coordination need to be adapted in consequence [48–50].

- Power quality: DG can cause the injection of harmonics and direct currents or provoke flickers [51–53]

Impacts on the centralized power system:

- Transmission system: in essence, the impacts on the transmission are the same than those on the distribution system. However, the fact that they result in costs or benefits and the amplitude of those will of course depend on the determinants of the transmission system (aggregated load profiles, thermal limits, impedances,...). As a consequence, it is conceivable that a given DG could bring benefits to the distribution system and costs to the transmission system, if for example, the aggregated load profile at the different levels have non-coincidental peaks.

- Power plant operation: DG will modify the net load profile that centralized power plant are expected to serve. As a consequence, power plants are likely to operate at different set-points and, if the variation in DG output are more dynamic than usual load variation, with more load-following duties [54]. In systems with very high penetration of DG and inflexible base load power plants (typically nuclear or coal), conflict can arise between the must-run constraints of these plants and the injection of DG. Figure 18 illustrates this phenomenon by displaying the output of the various type of plants as well as the prices in energy markets that become negative PV and output are high and must-run constraints prevent conventional generators from being turned-off.

- Reserve sizing: standby and spinning reserve are used to compensate short-term, unpredictable variations in load and power plant availability. The introduction of DG will change the amplitude and dynamic nature of the variations in the net load. In a manner comparable to load-following, albeit at a shorter time frame, the minimum level of standing and spinning reserve will need to be reevaluated [55–57].

- Spinning reserve provisioning: spinning reserve are usually supplied by conventional generators operating below their maximal output. As DG will displace conventional generators, the remaining will need to operate farther from their optimal set-point or new means of reserve provisioning will have to be introduced [58].

1.2.3 *Distributed storage units*

Storage is often presented as the natural companion to intermittent renewable generators as, intuitively, it could be used to store energy when production is high compared to the load and discharge it otherwise, allowing the intermittent generators to act as dispatchable

Figure 16: Voltage in a distribution without DG, source: [5]

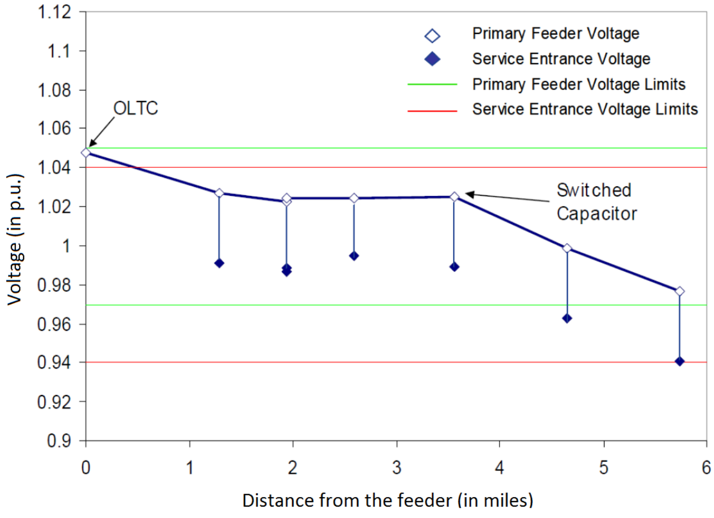


Figure 17: Voltage in a distribution without DG, source: [5]

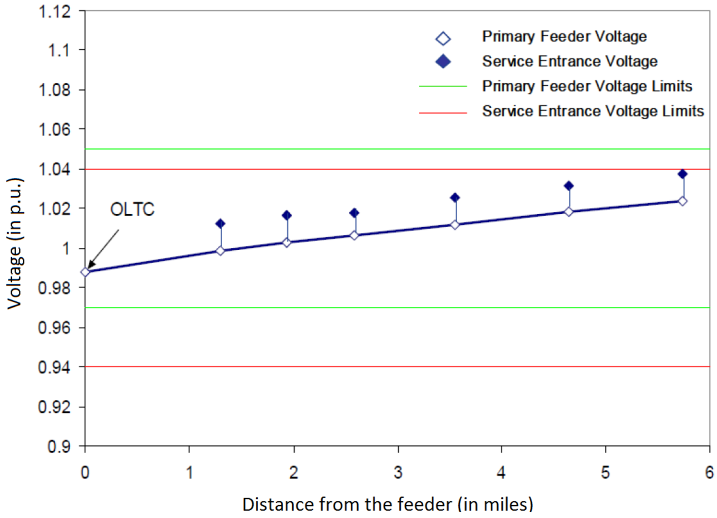
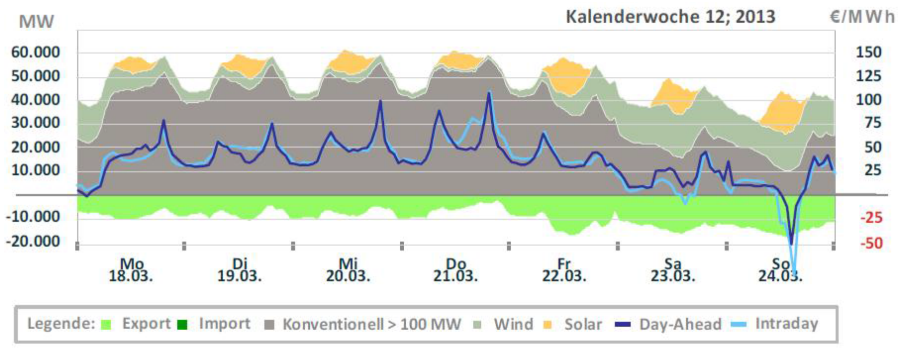


Figure 18: Output of PV, wind and conventional power plants with the coincident market prices, source: [6]



ones [59], thus eliminating their main negative feature. In this section, we will endeavor to nuance this viewpoint first by presenting the defining characteristics of storage units before detailing the roles they could play in the power system. To clarify, we are not opposing distributed storage units that could be controlled and coordinated to distributed renewable generators that could only be deployed in an uncoordinated manner, but rather separating distributed renewable generators, that could be deployed for reasons other than optimizing the power system operation (see Section1.2.2.2) and thus maybe in an uncoordinated fashion, from distributed storage units, whose only purpose is optimizing power system operation and whose deployment should thus be coordinated with power system operation.

1.2.3.1 *Characteristics*

Electricity is notoriously difficult to store and so most of the storage technologies, to the exception of supercondensator and superconducting magnetic energy storage (Super Magnetic Energy Storage (SMES)), consists in converting the electrical energy into another form of energy to store it before converting it back when needed. Table 1 describes the most common storage technologies and the form of energy they are converting electricity to. We include pumped-hydro and compressed-air energy storage despite the fact that, in their current commercial form, these systems require a connection to the transmission system due to their size because current developments may allow small-scale versions of these technologies to be viable in the future, see [60, 61] Detailed descriptions of each storage technologies are available in [8].

STORAGE TECHNOLOGY	ENERGY FORM
Supercondensator	Electrostatic field
SMES	Magnetic field
Solid state batteries	Chemical
Flow batteries	Chemical
Pumped Hydro	Potential
Flywheel	Kinetic
Compressed-air energy storage	Pneumatic

Table 1: Storage technologies and their associated energy form

To compare the various storage technologies between themselves, several parameters can be considered:

- Power and energy capacity: while it is technically conceivable to consider a lead battery storage system of several hundred

MW or a pumped-hydro storage system of a few kW, implementation and economic constraints impose minimal and maximal power and capacity for storage systems according to the technology considered, that are summarized in Figure 19. It should be noted that only existing large-scale pumped-hydro and compressed-air energy storage are displayed, and that the figures concerning these technologies could evolve significantly in the future.

- Energy density: In a context where storage is a potential candidate for connection at the distribution level where space and load-bearing capacity may be limited, the volume and weight of storage systems may be an influential factor. Figure 20 displays the energy density in weight and volume of the various technologies.

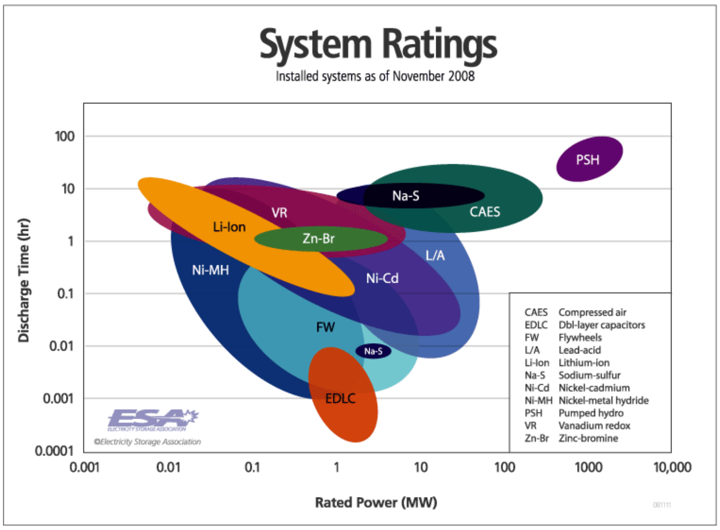
- Capital cost: the relevance of this factor is self-evident. When dealing with storage, it is usual to separate the cost into two components: one linked with power and one with capacity. Indeed, the various storage technologies can have very contrasting discharge duration and so there is no linear relationship between these two parameters. Moreover, we will see that some applications of storage will be focused on power while others will be limited by the energy available and so the cost of both aspects have to be differentiated, a feat achieved in Figure 21.

- Efficiency and lifetime: for some applications such as price arbitrage, efficiency is an essential parameters as it determines the profitability that can be expected. The lifetime (expressed in term of the number of cycles that can be achieved at a given depth of discharge) is a crucial factor whatever the application is as it dictates when a storage system will need to be replaced and has thus a great influence on the overall cost associated to the implementation of a storage system for a given application and a given operation duration. Figure 22 illustrates both aspects.

- Response time: it is the time taken by the storage device to reach its full output. This parameter will influence the suitability of a given storage technology for applications demanding a high reactivity, such as spinning reserve provisioning. Ranges of estimates for this parameter are available in Table 2.

- Technological maturity: electricity storage is a field in which technologies at very differing stages of development co-exist. Thus it is useful to have an idea of their maturity so as to, for example, estimate the potential for future cost reduction or the reliability of implementation feedback. Figure 23 gives an overview of this aspect.

Figure 19: Power and Energy Capacity, source: Energy Storage Association



STORAGE TECHNOLOGY	ENERGY FORM
Supercondensator	< 1/4 cycle
SMES	< 1/4 cycle
Solid state batteries	< 1/4 cycle
Flow batteries	< 1/4 cycle
Pumped Hydro	sec-min
Flywheel	< 1 cycle
Compressed-air energy storage	sec-min

Table 2: Estimates of response time for various storage technologies, source : [10]

1.2.3.2 *Applications*

In the section 1.2.2.5, we have summarily described the various type of impacts that the introduction of distributed renewable generators could have on the power system. With their ability to control injection and consumption and their potentially distributed implementation, storage systems have applications that can be conceived as symmetrical to the impacts of distributed renewable generators and that we will now specify, along with potential technical alternatives. For a thorough discussion of this subject in a broader context, the reader is advised to consult [8, 62, 63].

Applications in the distribution system:

- Voltage profile: in a straightforward manner, storage systems can help control voltage by consuming active power when the local production is too high and injecting it otherwise. More-

Figure 20: Weight and Volume Energy Density, source: Energy Storage Association

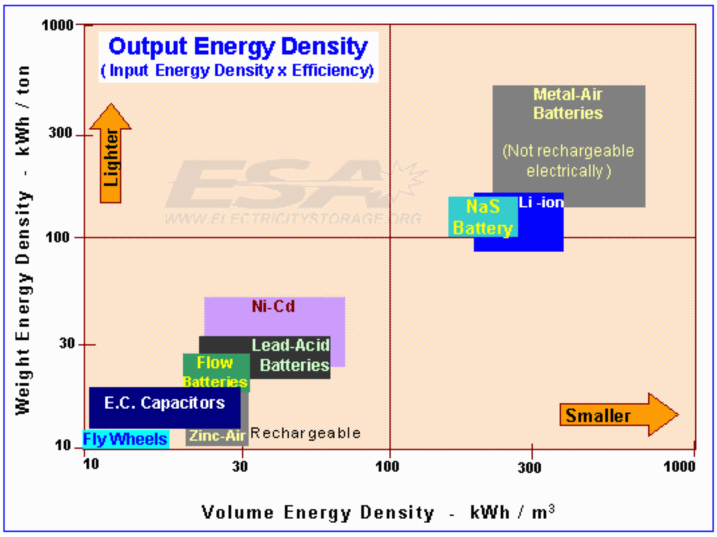


Figure 21: Capital Cost per Unit Power and Energy, source: Energy Storage Association

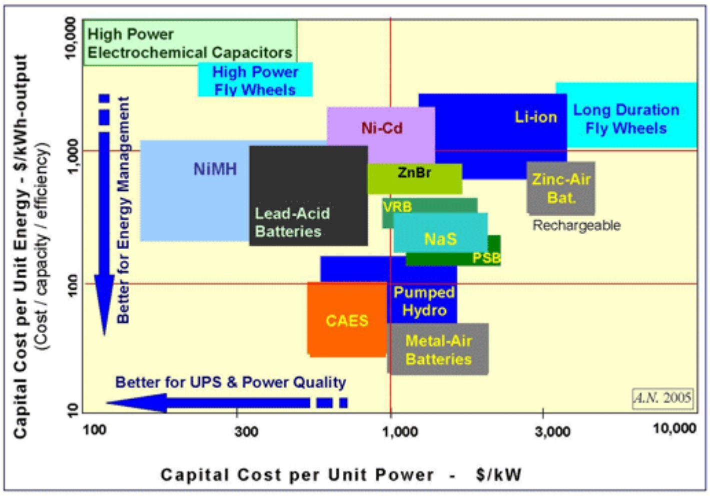


Figure 22: Efficiency and Lifetime, source: Energy Storage Association

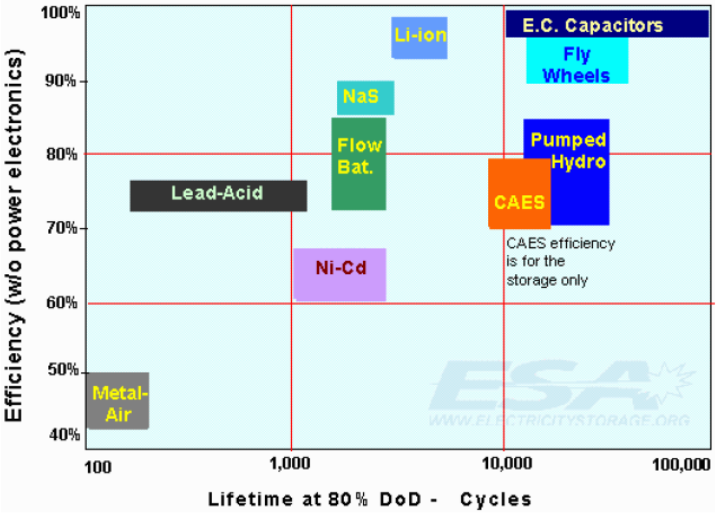
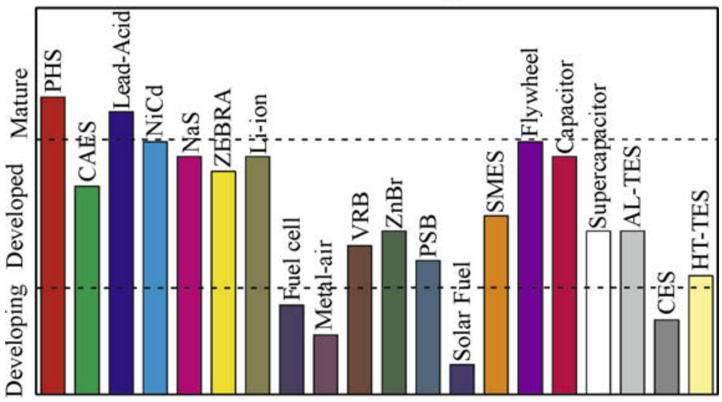


Figure 23: Technological Maturity of Storage Technologies, source: [7]



over, if storage systems are connected to the grid through APEIs and the R/X ratios of the lines are suitable, they can help control voltage by managing reactive power, at the expense of possibly higher losses [64]. For this application, storage has to be compared with other suitable alternatives or combination thereof: reactive power control through intermittent distributed generation APEIs, on-load tap changer management, adaptation of FACTS devices to the distribution network, demand response or dispatchable distributed generation such as fuel cells or reciprocating engines.

- Line flows: distributed storage system can smooth out line flows, thus allowing investment deferral on upstream distribution network equipment, a service that could also be achieved by demand response and other forms of dispatchable distributed generation.

- Line losses: as line flows change, line losses will too, albeit in a quadratic manner. This is more likely to be a by-product of another application (e.g. peak shaving for investment deferral), as operating a storage system solely for loss reduction purposes is not usually a sensible approach considering the level of losses in the DN and the efficiency of the storage systems (i.e. storage losses would be higher than prevented network losses).

Application in the centralized power system:

- Transmission system: in essence, the applications of distributed storage for the transmission system are the same than those for the distribution system. However, the fact that they result in costs or benefits and the amplitude of those will of course depend on the determinants of the transmission system (aggregated load profiles, thermal limits, impedances,...). As a consequence, it is conceivable that a given distributed storage could bring benefits to the distribution system and costs to the transmission system, if for example, the aggregated load profile at the different levels have non-coincidental peaks, see Figure 24.

- Power plant operation: In a system where intermittent generators co-exist with inflexible baseload generators, storage can be used to minimize the total running cost of the system by, for example, storing energy when intermittent production is high to prevent the stopping of generators with high start-up cost and discharging it at the peak load to avoid the use of generators with high running cost. In essence, this is the same application for which 5 GW of pumped-hydro are used today in France : storing the excess nuclear energy during the night to discharge it at the evening peak. The only difference being that, with the introduction of intermittent generation, the net load is

Figure 24: Non-coincidental peak for system and transformer load, source: [8]

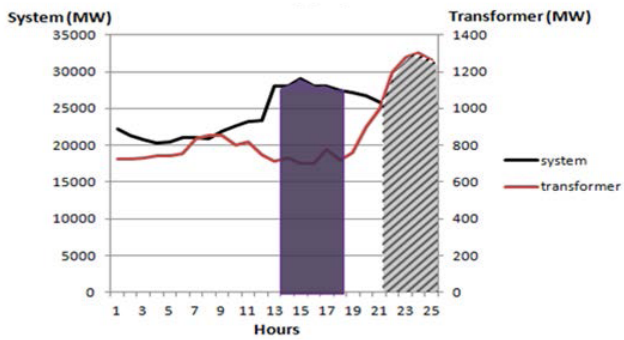
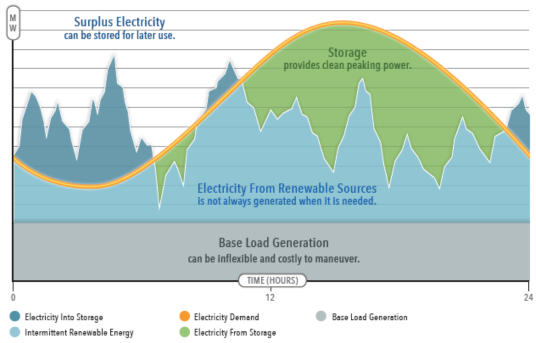


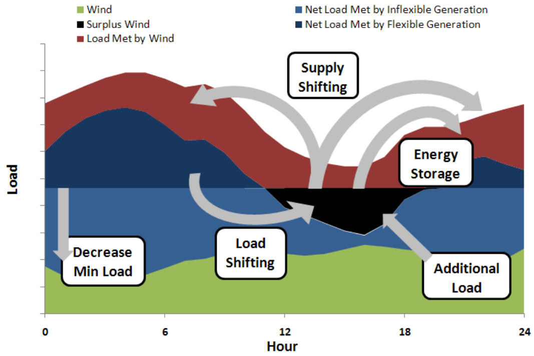
Figure 25: Use of storage in combination with intermittent and inflexible generators, source: NRStor



expected to be more variable, leading to more opportunities for cost-effective shifting of energy by storage systems. Figure 25 displays a stylized view of this application. It should be noted that, for this application, storage is in competition with other alternatives such as intermittent generation curtailing, demand response or investment in flexible generation, as exemplified in Figure 26.

- Load-following and reserve provisioning: Storage technologies with suitable response times could be used to replace conventional generators for the provisioning of these ancillary services [65, 66]. Moreover, storage systems are able to provide both downward and upward reserve, a feature that will become relevant in grids with high intermittent generator penetration [67, 68]. Such ancillary services could also be provided by demand response, intermittent generation curtailing or flexible generators.

Figure 26: Various options for matching load and demand in systems with intermittent and inflexible generation, source: [9]



1.3 EXISTING APPROACHES FOR IMPACT MITIGATION

1.3.1 *Local control*

In the preceding section, we have introduced the general concept of distributed generation before considering in more detail two specific subsets: distributed renewable generators and storage systems. In particular, we have reviewed the impacts that distributed renewable generators could have on the power system as well as the roles that storage systems could fulfill.

Many approaches have been put forward to try and mitigate the more immediate impacts of renewable distributed generation at the distribution level (i.e voltage rise) and increase the acceptable level of penetration by abandoning the fit-and-forget approach while relying on local measurement and control so as to avoid massive changes in distribution network architecture. Most of them can be classified into one of the three categories, based on the physical quantity controlled:

- Reactive power compensation: controlling reactive power injection of distributed generation inverters so as to maintain voltage inside the desired limit [69–72].

- On-load tap changer control: controlling the tap setting of the transformer to modify its output voltage and thus the voltages in the rest of the network [73–76].

- Generation curtailment: disconnecting or limiting the output of distributed generators as a last resort to mitigate voltage rise [77–79]

Compared to the fit-and-forget methodology, these techniques allow for a much higher level of penetration of distributed generation (as much as 72% in a specific case study according to [80]). However, this could still be improved by more centralized supervision and control capabilities, the subject of the next section, which could also unlock the potential ability to provide impact mitigation and benefits not only to the local distribution network but also to other areas of the power system.

1.3.2 *Active Distribution Network*

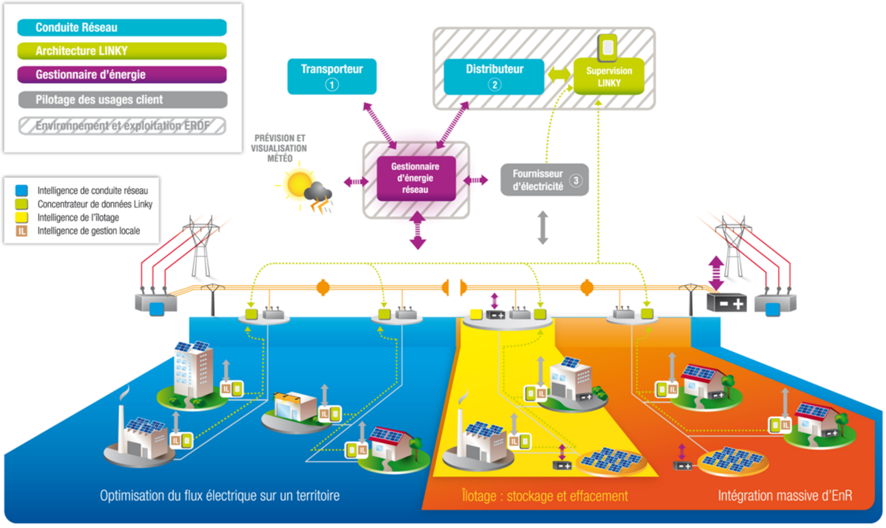
1.3.2.1 *Definition*

The earliest reference to the concept of Active Distribution Network (ADN) that we found in the literature is [81]. According to the author, it is a concept based "on a vision in which distribution networks have the physical structure and the control options that were traditionally only found in transmission systems". On a conceptual level, this is justified by the fact that, if a significant level of DG penetration is envisioned, the role of the distribution network will not anymore be to only serve loads but rather to interconnect them with generation, in a manner similar to the transmission system. Since then, more concrete and applicable definitions of ADN have been proposed, such as in [82–84] or [85]. For our purpose, we will adopt the latest version of the definition put forward by the CIGRE SC6 Working Group [86] and taken up in [87]: "Active distribution networks (ADNs) have systems in place to control a combination of distributed energy resources (Distributed Energy Resources (DER)s), defined as generators, loads and storage. Distribution system operators (DSOs) have the possibility of managing the electricity flows using a flexible network topology. DERs take some degree of responsibility for system support, which will depend on a suitable regulatory environment and connection agreement." In particular, this entails the existence of sensors, actuators and the associated information and communication infrastructure to allow the supervision and control of the system. An example of the architecture of an ADN is take from the Nice Grid demonstrator project and reproduced in Figure 27

1.3.2.2 *Existing Approaches for ADN management*

In parallel to the drive to define accurately what the technical features of an ADN could be, a flurry of work has been published to propose various management strategies able to take advantage of the control capabilities of ADNs to maximize the benefits they bring to the power system at the operational stage. We now propose a classification of these approaches according to which aspect of power system operation they put their emphasis on. We have chosen to use existing concepts to avoid the useless creation of terminology but the drawback is that, due to the lack of widely agreed upon definitions, there may exist conflicts between our definitions and some uses in the literature.

Figure 27: Possible Architecture of an Active Distribution Network, source: Nice Grid Project



- The Microgrid, where DG and distribution network are operated in coordination with an aim to satisfy local network constraints and objectives [88], [89], [90], with an emphasis on potential planned or unplanned islanded operation.

- The Commercial Virtual Power Plant, in which DG capabilities are aggregated in order to bring services to the centralized power system [91], [92], [93], [94], generally in the form of market participation [95]

- The Technical Virtual Power Plant [96], [97], [98] where DG and distribution network are grouped according to electrical and geographical determining factors (usually all elements downstream of a considered substation) and operated in coordination to satisfy local network constraints and objectives while considering the interaction with the Centralized Power System (CPS) through the possibility to buy and sell electricity considering market prices (with a price taker assumption) or time-of-use prices.

In the Commercial Virtual Power Plant concept, the local distribution network constraints and objective are ignored which implicitly means that the benefits and impacts DG could bring to the distribution systems cannot be modeled. Conversely, in the Microgrid approach, the emphasis on local network constraints and objectives prevents from evaluating DG interactions with the centralized power system. Therefore, the Technical Virtual Power Plant approach is the most aligned with our purpose as it takes into account distribution network constraints and objectives and models the influence of the centralized power system operation on DG operation. However, hidden in the price taker assumption or the utilization of time-of-use prices is the fact that the influence of DG active operation on CPS operation cannot be apprehended through such a framework. This is not an issue when only marginal DG penetration (at the scale of the whole power system considered) is envisioned as centralized power system operating points would not be meaningfully altered by DG operation. Nonetheless, large DG penetration at the national level will influence centralized power system operation, in particular with regards to the interaction between DG and inflexible centralized generators, and an assessment of the costs and benefits of DG has to take this into account, especially considering our particular focus on the French power system, in which the centralized power system is still operated in an integrated manner by a single actor.

Independently of the issue outlined above, the similarities between the current transmission network and the future ADN lead us to assume that the Optimal Power Flow (Optimal Power Flow (OPF)), a framework first introduced in 1962 by [99] and now widely used for the planning and operation of the transmission network, will prove useful as a tool to manage the integrated operation of DG and ADN. However, it has been remarked by several authors such as [100] and [101] that some characteristics inherent to the distribution system (in particular high R/X ratios and the radial nature of its topology) prevent us from applying traditional transmission system OPF algorithms, such as the Newton-Raphson method implemented in, for example, the Matpower package [102]. Moreover, simplifications commonly used in the planning of the transmission system, such as the linearization of the power flow constraints, are known to produce poor results with high R/X networks.Thus, and despite recent efforts that we will detail in the relevant chapter, we are still lacking a methodology to solve the OPF in the distribution network that is satisfactory in terms of convergence properties, low computational requirement and versatility, either to establish control strategies at the operational stage or simulate the operation at the planning stage.

1.3.2.3 *Integration at the planning stage*

In the sections dealing with the impacts and benefits of DG, we have observed that some of those have consequences not only at the operational stage but also at the planning stage (transmission or distribution network investment deferral or investment in new production capacity). As a consequence, a fully integrated assessment of DG cannot be achieved without considering the planning stage. In the transmission system today, a planning option is usually evaluated by simulating the operation of the system using a simplified version of the OPF implemented at the operational stage. Due to the lack of such a methodology for the distribution system, integration at the planning has only been envisioned under the assumption of passive operation [103–106] or with limited control [107].

1.4 OBJECTIVES

In this introduction, we have started by describing the evolution of the power system from its inception up until the current situation of a highly centralized power system. Then we discussed the development of distributed generation, with a particular focus on distributed renewable generators and distributed storage systems, the drivers behind their development and their impacts on various aspects of power system operation and planning. We continued by relating the existing propositions to mitigate these impacts and harness these benefits, from decentralized control warranting little investment and change in distribution network management to integrated operation relying on a yet-to-be developed supervision, communication and control infrastructure that would turn the current passive distributed network into an active one. This investigation has prompted us to identify several areas of the existing literature that we could improve upon, leading us to assign ourselves the following scientific objectives:

- Propose a mathematical framework defining the fully integrated operation of DGs, ADNs and the centralized power system as the solution to an optimization problem under constraints.
- Develop a decomposition and coordination methodology able to turn the large-scale, non-linear, mixed-integer optimization problem resulting from the objective above into a series of smaller and less complex master and sub-problems.
- As the sub-problems turn out to be single-stage OPFs in the distribution system, propose an algorithm able to solve them while guaranteeing convergence, low computational cost and versatility.
- Advance a technique able to identify beforehand which sub-problems will effectively influence the objective value, so that only those are solved in order to evaluate a planning option in the most computationally effective manner.

In addition to the objectives outlined above that have led to novel contributions, a separate work has been undertaken to obtain relevant datasets. Indeed, developing a methodology to assess the impacts of DG without applying it to realistic data, especially concerning network characteristics and load profiles, is moot. For load profiles, I took advantage of the work by my colleagues at the PERSEE research center, while network data was obtained by establishing a partnership with ErDF in order to carry out a first attempt at a taxonomy of the

French medium-voltage distribution network, based on the principles outlined in [108].

1.5 OUTLINE

The remainder of this contribution is organized around three articles submitted for publication, each occupying a specific chapter:

- Chapter 2 contains our attempt at fulfilling the first two objectives in the form of the article "Distributed Generation Operational Benefit Assessment by Coordinated Economic Dispatch and Distribution Optimal Power Flow".
- Chapter 3 is our answer to the third objective termed "Optimal Power Flow of a distribution system based on increasingly tight cutting planes added to a second order cone relaxation"
- Chapter 4 proposes a solution to the fourth objective in an article labeled "A criticality criterion to decrease the computational burden in multistage distribution system optimal power flow"

T	the number of time steps, page 45
N^{ADN}	the number of Active Distribution Network (ADN), page 45
$\mathbf{x}^{CPS}$	a vector containing the control variables of the Centralized Power System (CPS), page 45
$\mathbf{x}_t^{CPS}$	a vector containing the control variables of the Centralized Power System (CPS) at time step t, page 45
$\mathbf{u}^{CPS}$	a vector containing the state variables of the Centralized Power System (CPS), page 45
$\mathbf{u}_t^{CPS}$	a vector containing the state variables of the Centralized Power System (CPS) at time step t, page 45
$\mathbf{x}_j^{ADN}$	a vector containing the control variables of the j-th ADN, page 45
$\mathbf{x}_{t,j}^{ADN}$	a vector containing the control variables of the j-th ADN at time step t, page 45
$\mathbf{u}_j^{ADN}$	a vector containing the state variables of the j-th ADN, page 45
$\mathbf{u}_{t,j}^{ADN}$	a vector containing the state variables of the j-th ADN at time step t, page 45
$\mathbf{x}_j^{AS}$	a vector containing the control variables of the substation linking the j-th ADN to the CPS, page 45
$\mathbf{x}_{t,j}^{AS}$	a vector containing the control variables of the substation linking the j-th ADN to the CPS at time step t, page 45
$\mathbf{u}_j^{AS}$	a vector containing the state variables of the substation linking the j-th ADN to the CPS, page 45
$\mathbf{u}_{t,j}^{AS}$	a vector containing the state variables of the substation linking the j-th ADN to the CPS at time step t, page 45
$N^{Nodes,j}$	the number of nodes in ADN j, page 47
$N^{Lines,j}$	the number of lines in ADN j, page 47
$P_t^{AS,j}$	active power flowing through the substation linking the j-th ADN to the CPS at time step t, page 47

$Q_t^{AS,j}$	reactive power flowing through the substation linking the j-th ADN to the CPS at time step t, page 47
$P_{i,t}^{Load,j}$	active load at node i of the j-th ADN at time step t, page 47
$Q_{i,t}^{Load,j}$	reactive load at node i of the j-th ADN at time step t, page 47
$P_{i,t}^{DG,j}$	active injection of distributed generation at node i of the j-th ADN at time step t, page 47
$Q_{i,t}^{DG,j}$	reactive injection of distributed generation at node i of the j-th ADN at time step t, page 47
$r_k^{ADN,j}$	resistance of line k of the j-th ADN, page 47
$x_k^{ADN,j}$	reactance of line k of the j-th ADN, page 47
$z_k^{ADN,j}$	impedance of line k of the j-th ADN, page 47
$I_{k,t}^{ADN,j}$	Square of current flowing through line k of the j-th ADN to the CPS at time step t, page 47
$\mathbf{l}_j$	Linear function linking the AS state variables to the ADN control variables in ADN j, page 47
$\mathbf{x}_{t,j}^{ADN,MP}$	a vector containing the result of the master problem concerning the control variables of the j-th ADN at time step t, page 48
$\mathbf{x}_{t,j}^{ADN,SP}$	a vector containing the result of the sub-problem concerning the control variables of the j-th ADN at time step t, page 48
$\gamma_i^{ST,j}$	storage charging efficiency of storage unit located at node i of ADN j, page 52
$P_{i,t}^{ST,j,+}$	charge power of storage unit located at node i of ADN j at time step t, page 52
$P_{i,t}^{ST,j,-}$	discharge power of storage unit located at node i of ADN j at time step t, page 52
$P_{i,t}^{ST,j}$	algebraic injection of storage unit located at node i of ADN j at time step t, page 52
$P_{i,t}^{IP,MP,j}$	Master problem set-point for intermittent generator located at node i of ADN j at time step t, page 53
$P_{i,t}^{IP,j}$	active power injection of intermittent generator located at node i of ADN j at time step t, page 53

$P_{i,t}^{ST,MP,j}$ Master problem set-point for storage unit located at node i of ADN j at time step t, page 53

$I_{i,t}^{j}$ Square of current flowing through the line ending at node i of ADN j at time step t, page 53

$P_{i,t}^{j}$ active power flowing through the line ending at node i of ADN j at time step t, page 53

$Q_{i,t}^{j}$ reactive power flowing through the line ending at node i of ADN j at time step t, page 53

$c(i)$ set of downstream node(s) of node i

$f(i)$ unique upstream node of node i

$Q_{i,t}^{IP,j}$ reactive power injection of intermittent generator located at node i of ADN j at time step t, page 53

$Q_{i,t}^{ST,j}$ reactive power injection of storage unit located at node i of ADN j at time step t, page 53

$V_{i,t}^{j}$ voltage magnitude at node i of ADN j at time step t, page 53

$I_{i}^{max,j}$ maximal intensity flowing in line ending at node i of ADN j, squared, page 53

V_{j}^{min} maximal voltage magnitude in ADN j, page 53

V_{j}^{max} minimal voltage magnitude in ADN j, page 53

$V_{j}^{min,OLTC}$ maximal voltage magnitude at the root node of ADN j, page 53

$V_{j}^{max,OLTC}$ minimal voltage magnitude at the root node of ADN j, page 53

$S_{i}^{max,IP,j}$ maximal apparent power injection of intermittent generator located at node i of ADN j, page 54

$S_{i}^{max,ST,j}$ maximal apparent power injection of storage unit located at node i of ADN j, page 54

$P_{i,t}^{IP,Max,j}$ maximal active power injection of intermittent generator located at node i of ADN j at time step t, page 54

$u_{t}^{Nuc,l}$ Number of on-line nuclear plant of type l at time step t, page 55

$\mathbf{x}$ the vector of control and state variables, page 69

$\Delta(\mathbf{x})$ the convex objective function representing the interests of the DGS operators, page 69

P_j^{load}	the active power load at node j, page 70
Q_j^{load}	the reactive power load j, page 70
$j \in [1, J]$	the index of a node, where J is the node count and 1 is the root node, page 70
$f(j)$	the father node of node j, exists and is unique for all nodes except the root node, page 70
$c(j)$	the set of children nodes of node j, page 70
$P_{i,j}$	the active power flowing from node i to node j, page 70
$Q_{i,j}$	the reactive power flowing from node i to node j, page 70
$I_{i,j}$	the square of the current flowing from node i to node j, page 70
$r_{i,j}$	the resistance between node i and node j, page 70
$x_{i,j}$	the reactance between node i and node j, page 70
$z_{i,j}$	the impedance magnitude between node i and node j, page 70
V_j	the voltage magnitude at node j, page 70
P_j^{st}	the active power injected by the storage inverter at node j, page 70
Q_j^{st}	the reactive power injected by the storage inverter at node j, page 70
P_j^{pv}	the active power injected by the PV inverter at node j, page 70
Q_j^{pv}	the reactive power injected by the PV inverter at node j, page 70
$I_{i,j}^{max}$	the maximal intensity that may flow from node i to node j
$P_j^{st,sp}$	the set point for the active power injected by the storage inverter at node j, page 70
$P_j^{pv,sp}$	the set point for the active power injected by the PV inverter at node j, page 70
$S_j^{max,st}$	the maximal apparent power of the storage inverter at node j, page 70
$S_j^{max,pv}$	the maximal apparent power of the PV inverter at node j, page 70

$\mathcal{A}$	set of permissible tap-settings, page 89
$V_{i,t}^{j,\alpha}$	approximation of voltage at node i of ADN j at time step t for tap-setting α_0, page 89
$\Delta V_{i,t}^{j,\alpha}$	estimation of the voltage limit violation during time step t at bus i for the tap setting α in ADN j, page 90
$\Delta V_{i,t}^{j,Q}$	reactive voltage change potential during time step t at bus i in ADN j, page 90
$\Delta V_{i,t}^{j,\alpha,Q}$	local compensated voltage violation during time step t at bus i in ADN j, page 90
$C_V^{t,j}$	voltage-criticality criterion at time step t in ADN j, page 91
$I_{i,t}^{j}$	current constraint violation during time step t, in the line ending in i, in ADN j, page 92
$I_{i,t}^{j}$	downstream reactive power capability during time step t, for the line ending in i, in ADN j, page 92
$I_{i,t}^{j}$	compensated current violation during time step t, for the line ending in i, in ADN j, page 92
$C_I^{t,j}$	current criticality criterion at time step t for ADN j, page 92

2 DISTRIBUTED GENERATION ASSESSMENT

2.1 INTRODUCTION

2.1.1 *Current DG deployment*

Many definitions of distributed generation (DG) have been proposed or investigated in, for example, [109], [110] or [16]. While various aspects of distributed generation such as its rating, purpose, ownership or technology may be used to define it, we follow [16] by considering that DG "is an electric power source connected directly to the distribution network (DN) or on the customer site of the meter." Several drivers can be identified to explain the current and projected increase in DG penetration, such as its improving competitiveness [111], the depletion of conventional fossil fuels [112], the need to decrease greenhouse gases emissions [113] or the diminishing social acceptance of new transmission lines [114]. In the countries where these drivers have been translated into a political will, regulations favoring the deployment of primarily renewable DG have been implemented, such as feed-in tariffs [31], net-metering [32] or portfolio standards [115]. These policies have the objective to increase the wholesale output of the targeted electric power sources and are often associated with a "fit-and-forget" approach to grid connection [37]. In general, it consists in reinforcing the local network so as to alleviate any expected constraint violation, which allows the distribution system operator to keep a passive operation mode. When the penetration of DG remains marginal at a local and national level, this method is justified as the existing margins in network and power plant capacity help restrain the costs associated to passive DG connection while also avoiding the large investment in software, communication, measurement and control technology needed for an active distribution network (ADN) [38] and [39].

2.1.2 *Potential impact of large DG penetration*

If the level of DG penetration becomes large enough, the deployment of DG will affect the operation, and subsequently the planning, of the whole power system through impacts on, for example, centralized generation (Centralized Generation (CG)) scheduling [116] [117] [92], security and reliability [118], DN investments [119], DN losses [120] or the choice of protection scheme [121]. These impacts may be positive or negative depending on a complex interplay between the characteristics of the legacy power systems, those of the DG considered and the way their deployment is integrated in the planning and operation of the power system. In turn, this may lead to benefits or costs incurred by the various stakeholders of the power system (end-users, transmission and distribution system operators, power plant operators, etc) at differing time horizons (from real-time operation to long term planning), depending on the compatibility of geographical and temporal production profile of the DG(s) considered with the other determinants of the local and global power system. In this light, a strategy aimed only at increasing bulk DG output can only lead coincidentally to optimal outcomes in terms, for example, of minimizing levelized cost of electricity under security and reliability constraints. As an example, in Germany, admittedly the most aggressive country in renewable DG deployment, the lack of coordination between DG, transmission and central power plant planning has already spawned undesirable outcomes for some actors such as low and sometimes negative market prices affecting power plant profitability, rapidly rising retail prices impacting final consumers and transmission congestions hampering the further deployment of DG. Consequently, a more advanced coordination between DG deployment and power system planning and operation is needed to take full advantage of the benefits DG could bring. If large level of DG penetration are contemplated, not only should the passive distribution network be turned into an active one, the decoupling between the planning and operation of the distribution system and those of the rest of the power system should be questioned, as its validity is entirely dependent on the assumption of unidirectional power flows.

2.1.3 *Existing approaches*

Several trends can be identified in the literature advocating for a more meaningful integration of DG deployment and power system planning and operation. For our purpose, we may classify them in three categories :

- The Microgrid, where DG and DN are operated in coordination with an aim to satisfy local network constraints and objectives [88], [89], [90], with an emphasis on potential planned or unplanned islanded operation.
- The Commercial Virtual Power Plant, in which DG capabilities are aggregated in order to bring services to the centralized power system (CPS) [91], [92], [93], [94], generally in the form of market participation [95]
- The Technical Virtual Power Plant [96], [97], where DG and DN are grouped according to electrical and geographical determining factors (usually all elements downstream of a considered

substation) and operated in coordination to satisfy local network constraints and objectives while considering the interaction with the CPS through the possibility to buy and sell electricity considering market prices (with a price taker assumption) or time-of-use prices.

- The coordinated planning of DG and DN, either with passive DG operation [103], [104], [105], [106] or some level of control [107]

The first three aim at enabling the full capabilities of DG to maximize the operating benefits (or minimize the operating costs) they bring to the DN or the CPS, while the last one focuses on integration at the planning stage. While integration at the planning stage is a worthy goal, DG full benefits can not be achieved if a passive or almost passive operation mode is postulated. Thus, this present contribution will focus on integration at the operational stage as an initial step toward fully integrated DG deployment, in the vein of the approaches outlined in category 1, 2 and 3. In the Commercial Virtual Power Plant concept, the local distribution network constraints and objective are ignored which implicitly means that the benefits DG could bring to the distribution systems cannot be modeled. Conversely, in the Microgrid approach, the emphasis on local network constraints and objectives prevents from evaluating DG interactions with the CPS. Therefore, the Technical Virtual Power Plant approach is the most aligned with our purpose as it takes into account DN constraints and objectives and models the influence of the CPS operation on DG operation. However, hidden in the price taker assumption or the utilization of time-of-use prices is the fact that the influence of DG active operation on CPS operation cannot be apprehended through such a framework. This is not an issue when only marginal DG penetration (at the scale of the whole power system considered) is envisioned as CPS operating points would not be meaningfully altered by DG operation. Nonetheless, large DG penetration at the national level will influence CPS operation and an assessment of the operational benefits of DG has to take this into account. Filling this gap is the precise purpose of our contribution that will take the form of a coordinated economic dispatch and distribution system optimal power flow.

2.1.4 *Outline*

The remainder of the thesis will be organized as follow. First we present a theoretical model of the integrated operation of centralized and distributed generation as a large scale non-linear mixed-integer optimization problem. We then propose a decomposition of this problem based on the observation that only a subset of the variables defining the operation of DG and DN are linked to the operation of the centralized power system. We obtain a master problem defining the operation of the centralized power system incorporating DG capabilities and a set of sub-problems governing the operation of each active distribution network considered. Afterwards, we introduce a case study and various assumptions upon which the rest of our developments will rely. This leads to a further decomposition of the master problem into a long-term unit commitment and a short-term scheduling. We subsequently describe in more detail the optimization problems we will solve followed by a presentation of the results of the case study accompanied by a discussion of some of their features. We conclude by reflecting on the benefits brought about by our approach, its limitations and the subsequent improvements that could be devised.

2.2 OPTIMIZATION MODEL AND DECOMPOSITION

2.2.1 *Theoretical model*

As an ideal theoretical model, the operation of the power system including DG and ADN can be formally described as the solution to an optimization problem with an objective function reflecting political orientations and compromises (e.g. minimizing the cost of electricity generation and/or minimizing its environmental impacts) under quality requirement constraints (e.g. voltage level), security constraints (e.g. reserve provisioning) and technical constraints (e.g power plant output variation or network flow equations). Additionally, the time horizon envisioned has to surpass the duration of the temporal couplings introduced by, for example, plant maintenance scheduling, hydro availability and load patterns. Moreover, individual time steps sufficiently small to capture relevant variations (on the order of an hour) have to be considered.

For our purpose, we separate the power system considered into three conceptual entities :

- the Centralized Power System (CPS) which contains the transmission system, the generators and loads connected to it and the parts of the distribution system that remain in a passive operation mode.
- the Active Distribution Networks (ADN) that are constituted by the portions of the distribution system equipped with a control and observation infrastructure as well as the DGs and load connected to it.
- the Active Substations (Active Substations (AS)) linking the ADNs to the CPS

For each entity, control and state variables can be defined that may include, for example, generator active and reactive outputs, line active and reactive transits, voltage magnitudes and angles or tap-changer settings. It is self-evident that no control variables are defined for the parts of the distribution system operating in passive mode but it should be noted that no state variable are considered either, as it is assumed that decisions taken at the planning stage ensure that operational constraints are always respected. Considering this, the theoretical model we introduced above can be expressed as the minimization of an additively separable function of the control variables in the ADN and CPS (see (1)), under a set of constraints concerning control and state variables. This set of constraints can be divided into three components :

- (2) contains all the constraints bearing on CPS variables and thus includes AS state variables
- (3) models the temporal couplings in DG operation, such as the limits on storage capacity
- (4) can be divided in N^{ADN} by T sets of constraints bearing on the ADN and AS control and state variables, in each ADN and for each time step

This can be formally represented in the following manner :

$$\min \sum_{t=1}^{T} \left(f_t \left(\mathbf{x}_t^{CPS} \right) + \sum_{j=1}^{N^{ADN}} f_{t,j} \left(\mathbf{x}_{t,j}^{ADN} \right) \right) \tag{1}$$

$$\mathbf{g} \left(\mathbf{x}^{CPS}, \mathbf{u}^{CPS}, \mathbf{u}_1^{AS}, ..., \mathbf{u}_{N^{ADN}}^{AS} \right) \geqslant 0 \tag{2}$$

$$\mathbf{h} \left(\mathbf{x}_1^{ADN}, ..., \mathbf{x}_{N^{ADN}}^{ADN} \right) \geqslant 0 \tag{3}$$

$$\mathbf{h}_{j,t} \left(\mathbf{x}_{j,t}^{AS}, \mathbf{u}_{j,t}^{AS}, \mathbf{x}_{j,t}^{ADN}, \mathbf{u}_{j,t}^{ADN} \right) \geqslant 0 \quad \forall j \in [\![1, N^{ADN}]\!] \quad \forall t \in [\![1, T]\!] \tag{4}$$

$\mathbf{g}$ and $\mathbf{h}_{j,t}$ are non-linear non-convex vector-valued functions due to, at least, the presence of network constraints. If nothing else, $\mathbf{x}_t^{CPS}$ will contain variables with integer constraints to model start-up costs or minimum up-time. Considering the number of time steps and the number of nodes in a distribution network (e.g 30 million customers in France), even a limited conversion to an ADN would entail the existence of billions of variables. Thus, we are faced with a large scale, non-linear, non-convex, mixed-integer optimization problem. As it cannot be solved directly, decomposition techniques are contemplated next.

2.2.2 *Principles of decomposition and coordination*

Decomposition and coordination techniques have been pioneered by the work of Dantzig and Wolfe [122] concerning linear programming problem and Benders [123] dealing with mixed-integer programming, with the work of, for example, [124], [125] or [126] extending their field of applicability. It consists in separating a problem into a master problem and a series of sub-problems that are then solved in an iterative manner in order to obtain the solution to the original problem. These techniques are particularly suited to problems that present an underlying block structure but cannot be completely separated due to the presence of complicating variables (i.e variables present in several blocks) and/or complicating constraints (i.e constraints depending on variables in several blocks). For a thorough discussion of this subject from the point of view of engineering applications, we advise the reader to peruse [127]. In general, we can identify three types of purposes, or mix thereof, driving the application of a decomposition technique to a problem presenting a suitable structure :

- Change in nature : the optimization problems obtained present particular features allowing them to be solved by specific algorithms that could not be applied to the original problem
- Change in scale : the sheer change in scale brought about by the decomposition may allow previously untractable problems to be solved or the attainment of the optimal solution to be sped up
- Political or organizational concerns : in situations where heterogeneous actors are involved, the operational structure in which the problem considered is to be solved may impose a distributed implementation, with a decomposition in line with the distribution of management responsibilities (control variables) and data accesses (state variables and problem parameters) among the various actors.

2.2.3 *Application to CPS and DG coordinated operation*

2.2.3.1 *Decomposition techniques in power system operation*

Decomposition techniques have long been applied to CPS optimization, be it for operational (see [128],[129], [130] or [131]) or planning purposes (see [132], [133] or [134]). In particular, the most relevant approaches for our purpose are those in the vein of [135], where the generation schedule for the day-ahead market is obtained by iteratively solving a master problem of unit-commitment along with sub-problems verifying that network constraints are satisfied.

2.2.3.2 *Assumptions on AS variables*

In the remainder, we consider that the AS control variables are the tap-changer settings of the transformers and the state variables are the active and reactive power flowing through the transformers. In addition, we have the following relationships, $\forall j \in [\![1, N^{ADN}]\!]$, $\forall t \in [\![1, T]\!]$:

$$P_t^{AS,j} = \sum_{i=1}^{N^{Nodes,j}} \left(P_{i,t}^{Load,j} - P_{i,t}^{DG,j}\right) + \sum_{k=1}^{N^{Lines,j}} r_k^{ADN,j} I_{k,t}^{ADN,j} \quad (5)$$

$$Q_t^{AS,j} = \sum_{i=1}^{N^{Nodes,j}} \left(Q_{i,t}^{Load,j} - Q_{i,t}^{DG,j}\right) + \sum_{k=1}^{N^{Lines,j}} x_k^{ADN,j} I_{k,t}^{ADN,j} \quad (6)$$

Thus we can consider as a first approximation neglecting ADN network losses that AS state variables are a linear function of ADN control variables and (2) becomes:

$$\mathbf{g}\left(\mathbf{x}^{CPS}, \mathbf{u}^{CPS}, \mathbf{l}_1(\mathbf{x}_1^{ADN}), ..., \mathbf{l}_{N^{ADN}}(\mathbf{x}_{N^{ADN}}^{ADN})\right) \geqslant 0 \quad (7)$$

2.2.3.3 *Structure of the problem in relation to decomposition*

The optimization problem considered has a structure constituted by N^{ADN} by T block of single-step ADN network constraints in equation (4) along with a block of constraints related to the operation of the CPS including DG capabilities in equation (7) through their influence on AS state variables and considering the temporal couplings they are subjected to in equation (3). Consequently, we can consider the ADN control variables as the complicating variables. Therefore this problem has the desired form to be decomposed into a master problem dealing with the operation of the CPS including DG capabilities and temporal couplings but ignoring local ADN constraints, and a series of sub-problems modeling ADN single-step operations.

2.2.3.4 *Motivations for decomposition*

CHANGE IN SCALE Due to the temporal and electro-geographical decoupling, the resulting master and sub problems will feature significantly less variables and constraints than the original problem. In addition, following restrictive assumptions on DG characteristics detailed in the next section, a further reduction in the size of the master problem will be achieved by way of an aggregation of some ADN control variables.

CHANGE IN NATURE Separating the operation of the CPS from that of the ADN allows, on one hand, to take part of the very large existing literature on CPS operation. On the other hand, we will subsequently show that, under some hypothesis regarding the ADN, the sub problems can be reformulated in a manner that render them amenable to the algorithm outlined in [136]. Consequently, we obtain a master and sub problems for which solution algorithms are readily available, whereas the original problem was intractable.

POLITICAL OR ORGANIZATIONAL CONCERNS The regulatory environment in which the deployment of DG should be envisaged is still unclear as of yet. Nonetheless, it seems highly unlikely that the secular trend in electricity industry vertical disintegration will end anytime soon. Consequently, a decomposition separating the operation of the ADN from that of the CPS is compatible with the expected structure of the electricity market.

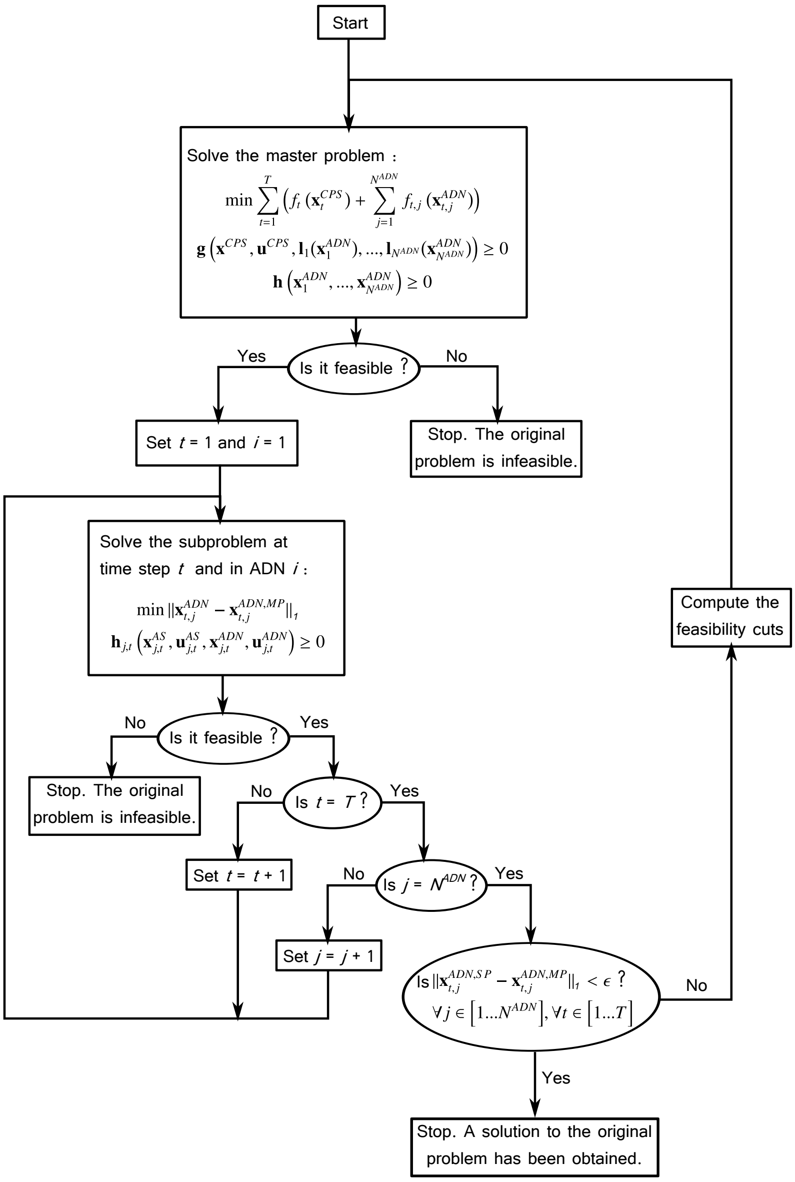
2.2.3.5 *Algorithmic implementation*

The principle of the algorithm we propose is based on Collaborative Optimization, a bi-level optimization procedure. It consists in first solving a master problem that will set target values for system-level variables (here the $\mathbf{x}^{CPS}$,$\mathbf{u}^{CPS}$ and $\mathbf{x}_1^{ADN}$,...,$\mathbf{x}_{N^{ADN}}^{ADN}$), followed by a series of sub-problems (one in each ADN considered and for each time step). The objective of the sub-problems is a compromise between minimizing the distance with the corresponding system-level target values ($\mathbf{x}_{t,j}^{ADN,MP}$) and local ADN objectives (i.e active loss minimization) while satisfying local constraints (e.g. distribution network equations, voltage limits or line intensity limits). Afterwards, if the distances between the system-level targets values and the results of the sub-problems ($\mathbf{x}_{t,j}^{ADN,SP}$) falls below a given termination criterion ϵ, a solution to the original problem has been obtained and the procedure is stopped. If not, cuts are computed and added to the master problem and the process is repeated. The nature of the cuts will be detailed in 2.3.3.7, after further assumptions on the case study are laid out. Figure 28 presents a schematic view of the algorithm we propose.

2.3 CASE STUDY

While the principles of the decomposition algorithm we introduced in the preceding section are general and could be adapted to the peculiarities of a variety of power systems where ADN deployment is envisioned, its practical implementation and, in particular, the mathematical formulation of the optimization problems considered are predicated upon further assumptions concerning the case study. The aim of this section is thus to make these hypothesis and their consequences on the formulation of the optimization problems explicit.

Figure 28: A1gorithm for Coordinated CPS and ADN operation



2.3.1 *Context*

All numerical data concerning the current French power system have been taken from the TSO's website [137]. Currently, the French power system serves an annual load of roughly 480 TWh, with an energy mix dominated by nuclear (74,8%), hydro (11,8%) and thermal power (8,8%), where renewable energies other than hydro have a marginal impact, following 8 years of fluctuating governmental support. In this context, a wide-ranging energy transition bill has been proposed with, regarding the power system, two main objectives: reducing the share of nuclear energy to 50% by 2025 while increasing the contribution from renewable energies to 40% by 2030. As the practical implementation of this energy transition bill is still unclear, in particular concerning installed capacity of the various power sources, debatable hypothesis have had to be made, so as to obtain a working scenario. In this light, Table 3 presents the energy produced and installed capacity in 2012 for the various sources, as well as the hypothesis retained for the projected capacity in 2025, while Figure 29 depicts the weekly mean power produced by each of the main sources.

TYPE	ENERGY IN 2012, IN TWH	CAPACITY IN 2012, IN GW	CAPACITY IN 2025, IN GW
Nuclear	404.7	63.1	42
Hydro	63.8	25.4	30
Thermal	47.6	27.8	15
Wind	15.1	7.4	40.8
PV	3.8	3.5	44.8
Weekly Storage	N.A	+4.9/-4.2	+4.1/-4.1
Interconnections	-44	+13.7/-15.7	+22/-18

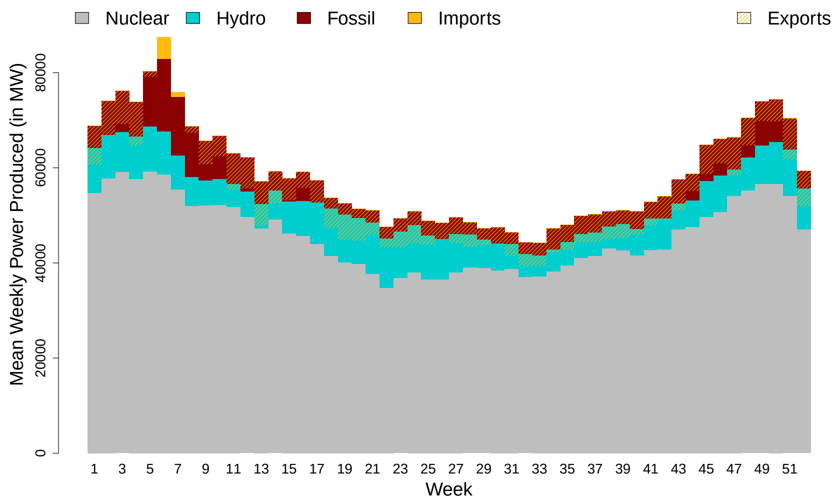
Table 3: Electricity production and capacity by source

2.3.2 *ADN and DG operation*

2.3.2.1 *Assumptions on ADN*

In this case study, we will model only the medium-voltage network, assuming a three-phase balanced operation, a radial structure and the presence of an on-load tap-changers at the transformer substation, for which a continuous approximation is used. The data needed (impedance, line intensity limits and Medium Voltage (MV)/Low Voltage (LV) transformers maximal loading) are taken from real networks located in three distinct geographical zones (urban, semi-urban and rural) for a total of 728 nodes, 192 of which have MV/LV transformers connected to them. The hourly active and reactive loads at each MV/LV transformer are obtained by summing simulated individual residential and tertiary sector loads until the maximal annual apparent aggregated loads reaches the corresponding MV/LV transformers

Figure 29: Weekly Mean Power Produced by Source (2012)



maximal loading. No MV customers are considered due to confidentiality considerations in data collection. The per unit values of network characteristics are obtained with a base of 20 kV for voltages and 1 MVA for powers. As we do not want to consider only a marginal penetration of ADN that would not modify the operation of the CPS substantially but we do not have access to extensive network data, we apply a scaling factor to the physical quantities corresponding to the ADN when they are considered in the CPS operation. A summary of the characteristics of the networks considered is presented below.

TYPE	URBAN	SEMI-URBAN	RURAL
Number of nodes	136	256	336
Number of MV/LV transformers	31	75	86
Mean R/X	1.543	1.882	2.326
Mean line intensity (in Amps)	410	249	209
Yearly active load (in GWh)	9.4	11.0	13.8
Scaling factor	200	200	200

Table 4: Characteristics of considered ADN

2.3.2.2 *Assumptions on DG*

In the remainder, we consider two types of DG :

- Intermittent generators are connected to the MV network through Advanced Power Electronic Interface (APEI, see [17]) that can control active power injection up to a limit varying at each time step, with a curtailing cost. Reactive power can be absorbed or injected provided that the apparent power remains below the inverter's limit. We consider only one type of such generators : photovoltaic (PV). The apparent power installed, yearly energy and number of nodes equipped are available in Table 5. To test our algorithm in extreme conditions, we have chosen very large levels of penetration as the total energy produced is four times the total load in the networks.

- Storage units are also connected to the MV network through APEIs and can thus absorb or inject active and reactive power as long as maximal apparent power constraints are respected. Active power injections and absorptions are also subject to time-coupling constraints on the total amount of energy stored, considering round-trip efficiency. Furthermore, we assume that all distributed storage units can be grouped into two classes of devices, inside which discharge duration (maximal energy stored divided by maximal discharge power) and efficiency are homogeneous.

$$0 \leqslant \sum_{t=1}^{k} \left(\gamma_i^{ST,j} P_{i,t}^{ST,j,+} - P_{i,t}^{ST,j,-} \right) \leqslant C_i^{Max,j} \tag{8a}$$

$$P_{i,t}^{ST,j} = P_{i,t}^{ST,j,-} - P_{i,t}^{ST,j,+} \tag{8b}$$

TYPE	URBAN	SEMI-URBAN	RURAL
Number of nodes equipped	22	50	45
Total PV apparent power (in MVA)	28.7	15.3	40.6
Yearly available PV output (in GWh)	43.6	23.2	61.5
Total Storage apparent power (in MVA)	5.8	8.8	5.7

Table 5: Installed storage and PV characteristics, by network

CLASS	I	II
Discharge duration (in hours)	3	7
Round-trip efficiency (in %)	95	80

Table 6: Distributed storage characteristics

2.3.2.3 *Mathematical formulation*

The objective is to minimize a compromise between the distance to the set-points obtained from the master problem and the losses in the

ADN. Due to the radial nature of the network, the number of lines is the number of nodes minus one, the root node is assigned the first label and the objective function can be written in the following way, $\forall j \in [\![1, N^{ADN}]\!], \forall t \in [\![1, T]\!]$:

$$\min \sum_{i=1}^{N^{Nodes,j}} \left|P_{i,t}^{IP,MP,j} - P_{i,t}^{IP,j}\right| + \left|P_{i,t}^{ST,MP,j} - P_{i,t}^{ST,j}\right| + \sum_{i=1}^{N^{Nodes,j}} r_i^j I_{i,t}^j \quad (9)$$

Due to the assumption on radial and balanced operation of the ADN, we can use the branch flow model as power flow equations [138], $\forall i \in [\![2, N^{Nodes,j}]\!]$:

$$P_{i,t}^j - r_i^j I_{i,t}^j - \sum_{k \in c(i)} P_{k,t}^j - P_{i,t}^{Load,j} + P_{i,t}^{ST,j} + P_{i,t}^{IP,j} = 0 \quad (10a)$$

$$Q_{i,t}^j - x_i^j I_{i,t}^j - \sum_{k \in c(i)} Q_{k,t}^j - Q_{i,t}^{Load,j} + Q_{i,t}^{ST,j} + Q_{i,t}^{IP,j} = 0 \quad (10b)$$

$$U_{f(i),t}^j - U_{i,t}^j - 2\left(r_i^j P_{i,t}^j + x_i^j Q_{i,t}^j\right) + {z_i^j}^2 I_{i,t}^j = 0 \quad (10c)$$

with :

$$I_{i,t}^j = \frac{{P_{i,t}^j}^2 + {Q_{i,t}^j}^2}{U_{f(i),t}^j} \quad (11a)$$

$$U_{i,t}^j = {V_{i,t}^j}^2 \quad (11b)$$

We take into account the following constraints on the network :

$$I_{i,t}^j \leqslant I_i^{max,j} \quad (12a)$$

$$V_j^{min} \leqslant V_{i,t}^j \leqslant V_j^{max} \quad (12b)$$

$$V_j^{min,OLTC} \leqslant V_{1,t}^j \leqslant V_j^{max,OLTC} \quad (12c)$$

Storage and intermittent generation power injections are constrained by their inverter apparent power. In addition, we add a cut to the active power injection of storage to avoid it being lower than the master problem set point when it is negative and higher otherwise. The same is done for intermittent power generators, of course only for positive values. This is done to avoid oscillations between the solutions of the master and sub-problems.

$$
\begin{aligned}
& {P_{i,t}^{ST,j}}^2 + {Q_{i,t}^{ST,j}}^2 \leqslant {S_i^{max,ST,j}}^2 && \text{(13a)}\\
& {P_{i,t}^{IP,j}}^2 + {Q_{i,t}^{IP,j}}^2 \leqslant {S_i^{max,IP,j}}^2 && \text{(13b)}\\
& P_{i,t}^{IP,j} \in [0, P_{i,t}^{IP,MP,j}] && \text{(13c)}\\
& P_{i,t}^{ST,j} \in \begin{cases} [P_{i,t}^{ST,MP,j}, S_i^{max,ST,j}] & \text{if } P_{i,t}^{ST,MP,j} < 0 \\ [-S_i^{max,ST,j}, P_{i,t}^{ST,MP,j}] & \text{if } P_{i,t}^{ST,MP,j} > 0 \end{cases} && \text{(13d)}
\end{aligned}
$$

Such a formulation of the sub-problems, where the non-convexity is concentrated in Equation 11a and the objective is a compromise between loss minimization and a convex function of the injections in the network, is amenable to the algorithm described in [136]. To clarify further, the absolute value used in the definition of the objective function, when combined with the restriction on the injection region in Equation 13, reduces itself to a linear, and thus convex, function of the injections. Moreover, the non-convexity in Equation 11b is of no consequence, as the entire optimization problem can be solved using $U_{i,t}^j$ as variables, while the voltage magnitude can be derived *a posteriori*. While the convergence to the optimal solution is ultimately guaranteed through the cutting plane method proposed, we choose instead a compromise between optimality and computational requirements by setting the termination criterion defined in [136] to 0.005.

2.3.3 *CPS operation*

In this contribution, our goal regarding the CPS operation is not to obtain a detailed model of the operation of each power plant but rather to capture the overall behavior of the CPS while keeping computational requirements as low as possible. To this end, we make the following set of assumptions.

2.3.3.1 *Transmission network*

Considering the focus of our contribution on a methodology allowing the coordination between CPS and ADN operation, modeling the transmission network would bring little additional illustrative value at the cost of a large increase in computational and data requirements. We have thus decided against for this thesis while bearing in mind that this will be a necessary improvement for a more realistic assessment of DG. Consequently, only an equation in which all power sources and loads are aggregated ensures the adequacy of the supply to the demand in the CPS at each time step.

2.3.3.2 *Time horizon*

The optimization problem associated with yearly operation of a large CPS at a one-hour resolution is a multi-million Mixed-Integer Non-Linear Problem (MINLP) that cannot be solved directly. Conversely, separating it in weekly independent problems would preclude from modeling relevant phenomena, such as the interplay between seasonal patterns of load and water availability and the scheduling of refueling and maintenance operations for nuclear power plants. As a compromise, we choose to separate the master problem into two successive optimization problems.

- A long-term (one year), low-resolution (one-week time steps), mixed-integer problem (Long-Term Master Problem (LTMP)) tasked with setting the on/off status of nuclear power plants, the amount of hydro energy produced during a week as well as the variation in the state-of-charge of long-term storage
- A short-term (one week), high-resolution (one-hour time steps), continuous problem that takes as constraints the outputs of LTMP and obtains the operating set-points of CPS control variables (nuclear, hydro and thermal power output, interconnection usage, centralized PV, wind and short-term storage outputs) and the target values for ADN control variables (active injection of distributed PV, wind and short-term storage)

2.3.3.3 *Nuclear*

LONG TERM The fleet of nuclear power plants is separated into two groups according to their maximal output. To each group is associated an integer-valued variable that represents the number of plants committed for a given week. This variable is constrained to be below the total number of plants at each time step, while its sum over the whole time horizon has an upper bound reflecting the need for maintenance and refueling. It is then multiplied by the intermediate power corresponding to each group to obtain the total output considered in the supply-demand equilibrium. The total cost is considered to be a linear function of the power output. Furthermore, the variations in the number of power plants committed are smoothed out by constraining the difference between two steps as well as the difference between two successive 4-week moving averages, as described in Equation 14.

$$\forall k \in [\![1, T^{LT} - 7]\!], \forall l \in \{1,2\}, -2 \leqslant \sum_{t=k}^{k+3} u_t^{Nuc,l} - \sum_{t=k+4}^{k+7} u_t^{Nuc,l} \leqslant 2 \tag{14}$$

SHORT TERM In the short-term problem, the total output for each group is constrained between the associated minimal and maximal power multiplied by the number of units committed obtained in the long-term problem. The output variation between two hours is similarly constrained. For nuclear power plants, O&M costs are usually higher than fuel costs [139], meaning that, once a plant is committed, not operating at its maximal output represent a loss of value. To reflect this, a negative marginal cost or curtailing cost is applied to the nuclear power output in the short-term problem.

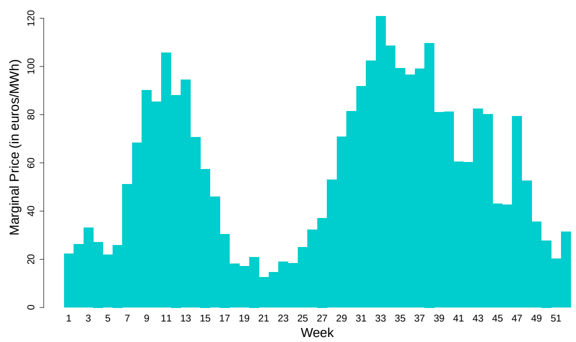
GROUP	I	II
Number of units	20	16
Availability factor	95 %	95 %
Long-term running cost (in euros/MWh)	25	25
Short-term curtailing cost (in euros/MWh)	10	10
Maximal power (in MW)	1000	1350
Minimal power (in MW)	850	1100
Intermediate power (in MW)	925	1225
Maximal hourly variation (in MW)	10	25

Table 7: Nuclear fleet characteristics

2.3.3.4 *Hydro*

LONG TERM Scheduling the use of hydro-power resources is a complex task involving interactions between meteorological conditions influencing water inflows, reservoir capacities, links between cascading reservoirs and other uses of water. As this is not our present focus, we have decided to hide this complexity by using virtual marginal prices that vary each week as a proxy for the scarcity of the hydro-power resource. This methodology is used by operators (see [140]), but we are lacking the underlying data that could allow us to mimic this approach (technical characteristics of thermal units, value of lost load). As a consequence, the prices we use are derived using historical data on hydro-power production obtained from the French TSO. We compute them by considering that the prices vary linearly with historical hydro output, with the minimum price (corresponding to the maximal hydro output) set at the value of the lowest marginal cost (i.e. nuclear) and the maximum price set at the value of the highest marginal cost. The result is plotted in Figure 30. Moreover we constrain the weekly output to lie between the minimal and maximal value recorded for the corresponding week of 2012, scaled to account for the increase in hydro capacity considered in our scenario. This is done to account for the presence of run-of-river hydro (varying mini-

Figure 30: Weekly hydro virtual marginal prices



mal output) and the fact that not all reservoir capacities are available throughout the year (varying maximal output).

SHORT TERM In the short term problem, we constrain the total output for a week to be the same of that obtained in the long term problem while the hourly output is constrained in the same manner than the weekly output in the long-term problem.

2.3.3.5 *Fossil*

Fossil fuel do not play a major role in the french electricity mix and are not expected to in the foreseeable future. Consequently, we choose to ignore the non-linear operational constraints associated with such generators in favor of a more simplistic model. We split these generators into two groups : one with high marginal cost but no constraints on output variation and one with lower marginal cost and constraints on output variations between two hours.

GROUP	I	II
Maximal power (in MW)	8000	7000
Marginal cost (in euros/MWh)	60	150
Maximal hourly variation (in MW)	1000	None

Table 8: Fossil plant characteristics

2.3.3.6 *Interconnections*

We adopt a simplified interconnection model by aggregating all the interconnections and considering a unique market price, that of the German market in 2012. The weekly availability of interconnections are taken from 2012 historical data and scaled to reflect the expected increase in interconnection capacities envisioned in our scenario. In the long-term problem, hourly market prices are averaged over a week.

2.3.3.7 *DG capabilities*

Due to the fact that we consider only distributed storage units that have a short discharge duration with regards to the time horizon of the short-term problem, we neglect their influence on the long-term problem. This is even more so with intermittent generators that do not have time-coupling constraints.

To decrease the computational burden, we aggregate the variable corresponding to DG active power injection into six variables: one for wind generators, one for photovoltaic generators and two for each of the type of distributed storage (charge and discharge have to be separated to take into account imperfect efficiency). Intermittent generators can allow their output to vary between zero and the maximal value permitted by the meteorological conditions, with a curtailing cost set to 25 euros/MWh. Variables corresponding to aggregated storage units are subject to the same time-coupling constraints that individual units would (see (8)).

After the sub-problems have been solved and in case the distances to the target values is not zero, simple linear cuts are added to the master problem that remove the interval between the target value and the result of the sub-problem from the feasibility domain. Along with Equations 13, this ensures that the same part of the feasibility domain cannot be visited twice, allowing for an easier convergence at the expense of possibly sub-optimal solutions.

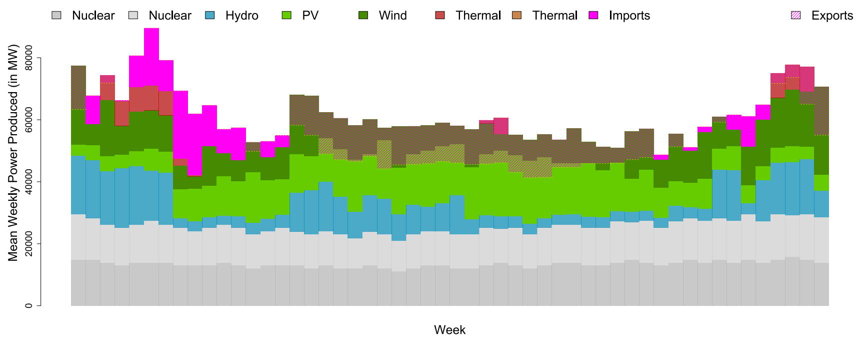
2.3.3.8 *Nature of the master problems*

Following the various assumptions stated above, we obtain a long-term master problem that is a mixed-integer linear problem and short-term master problems that are linear. Consequently, they can be solved efficiently using off-the-shelf solvers, in our case MOSEK [141].

2.4 RESULTS

With this case study, we have obtained results relative to active and reactive power transit, voltages, active and reactive power injections for 728 nodes over a time-horizon of 8736 hours. Consequently, the aim of this section is not to present an exhaustive account of these results but rather to highlight some noteworthy aspects, while the detailed results will be made available in separate data files.

Figure 31: Long-term master problem



2.4.1 *Master Problem results*

The result of the long-term master problem can be depicted in its entirety as it concerns only 52 time steps, see Figure 31. We can observe that during the winter, the load being high due to a significant use of electrical heating in France, imports, fossil generation and hydro contribute substantially to the supply-demand equilibrium. Conversely, lower load and high availability of photovoltaic and hydro contribute to large exports of electricity during spring and summer.

For the short-term master problem at the first iteration, we present the results concerning three differing weeks :

- Week 8 is a winter week with a high load, high PV production and small hydro resources. Its supply-demand equilibrium thus relies heavily on imports, fossil and storage when PV does not produce (Figure 32).

- Week 18 is spring week with a moderate load, high renewable energy contribution and thus large exports. Both imports and exports are carried out, which hints that storage may be used for price arbitrage (Figure 33). Nuclear output is decreased during the week-end, as load is low and export are saturated, as well as during the week when photovoltaic output is high.

- Week 52 is a winter week with high renewable production and exports with storage contributing considerably to the supply-demand equilibrium (Figure 34). Nuclear output is modulated in case where renewable production is high and exports saturated

To illustrate the effect of the cuts added to the short-term master problem after the sub-problems have been solved, we also present on separate graphs the storage output and PV output resulting from the short-term master problem at iteration 1 and 2, along with limit on their output stemming from the sub-problems. We choose week 13

Figure 32: Short-term master problem, Week 8

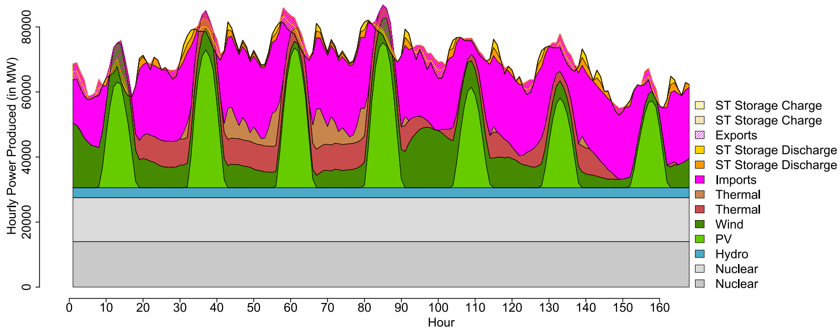


Figure 33: Short-term master problem, Week 18

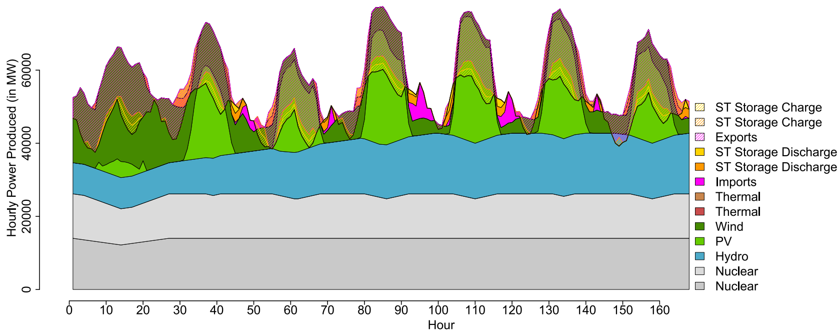


Figure 34: Short-term master problem, Week 52

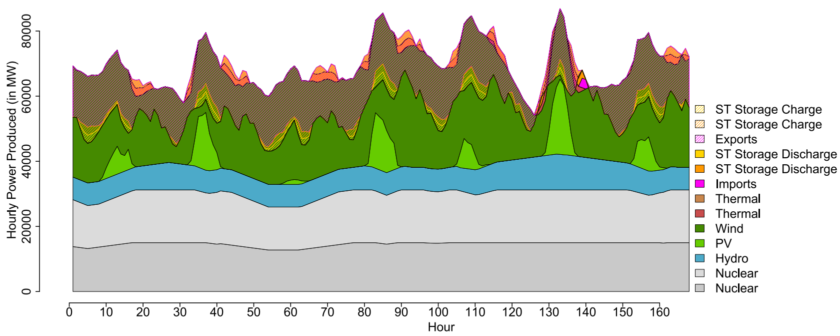


Figure 35: Storage output of type 1 at iteration 1 and 2

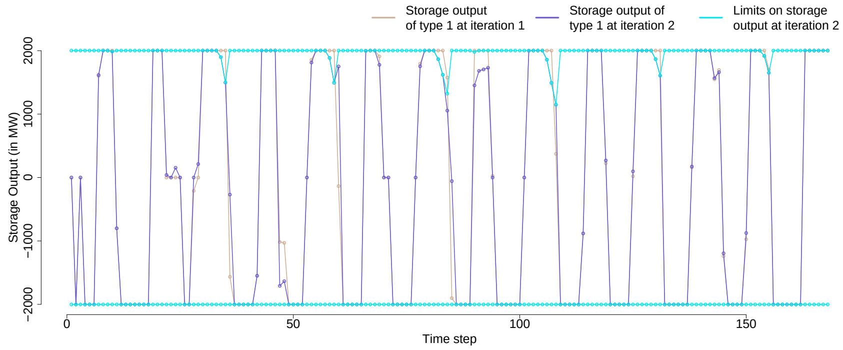
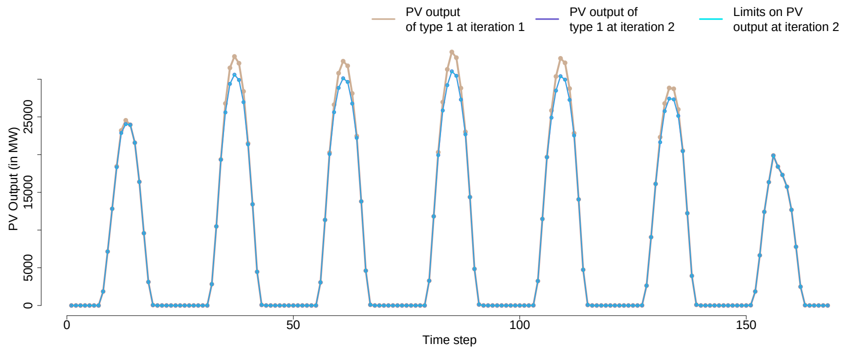


Figure 36: PV output at iteration 1 and 2



for storage and week 11 for PV as it exhibits the biggest difference in output between the two iterations.

For PV, the observation is straightforward : the output is decreased whenever the limit at iteration 2 is lower than the result at iteration 1, and unchanged otherwise. For storage, the output is also decreased whenever the limit at iteration 2 is lower than the result at iteration 1, but the output also changes at other time steps in order to satisfy the time-coupling constraint ensuring the limit on storage energy capacity.

2.4.2 *Sub-problem results*

We present seven different types of results concerning the sub-problems :

- The total yearly deviation from the set-points, for each network and at each iteration (Figure 37).
- The maximum of the total deviation from the set-points in each time step, for each network and at each iteration (Figure 38).
- The number of time steps for which the deviation was higher than 0.1 p.u., for each network and at each iteration (Figure 39).

Figure 37: Total yearly deviation

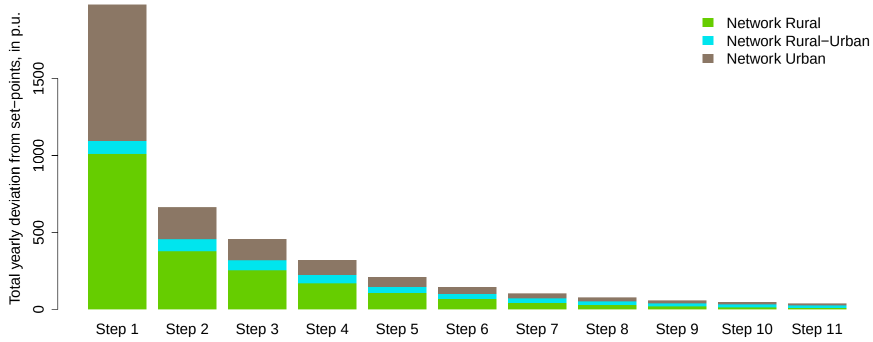
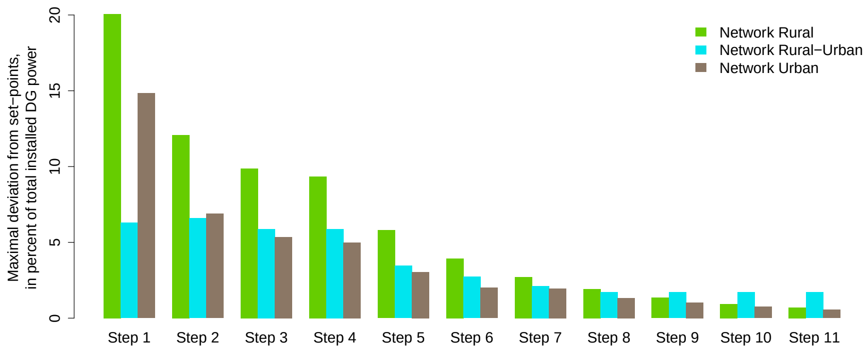


Figure 38: Maximal deviation



- The total photovoltaic energy curtailed as a percentage of the yearly energy produced, for each network and at each iteration (Figure 40).
- The total storage curtailed as a percentage of the yearly energy stored, for each network and at each iteration (Figure 41).
- An example of power flows resulting from the sub-problem calculation at the time step with the highest set-point deviation for the rural network in Figure 42). Each node in the network is positioned on the X-axis according to its electrical distance from the root node (i.e. the sum of impedances) and on the Y-axis according to the voltage magnitude. The lines between the nodes have a width proportional to the intensity flowing through them and a color reflecting the ratio of this intensity to its maximal value with 0% in deep blue, 99% in red and 100% in black.

First, we can observe that the total deviation decreases monotonously to a negligible value, ensuring that we have obtained a feasible solution. As far as the maximal deviation is concerned, the general trend

Figure 39: Number of time steps with significant deviation

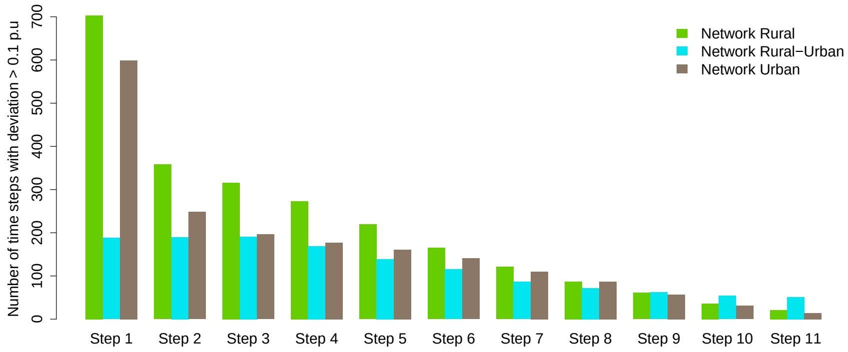


Figure 40: Photovoltaic energy curtailed

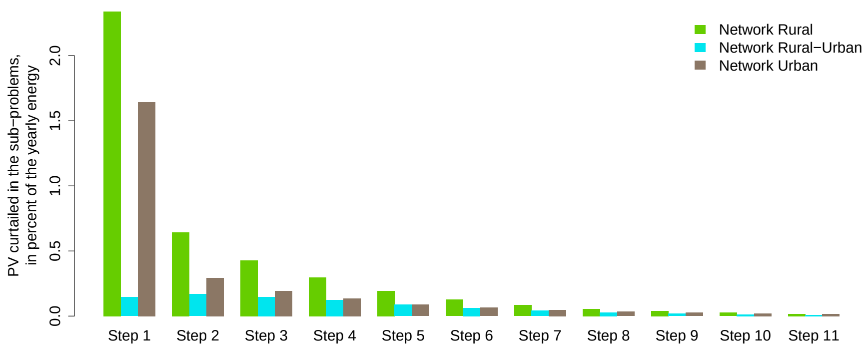


Figure 41: Storage curtailed

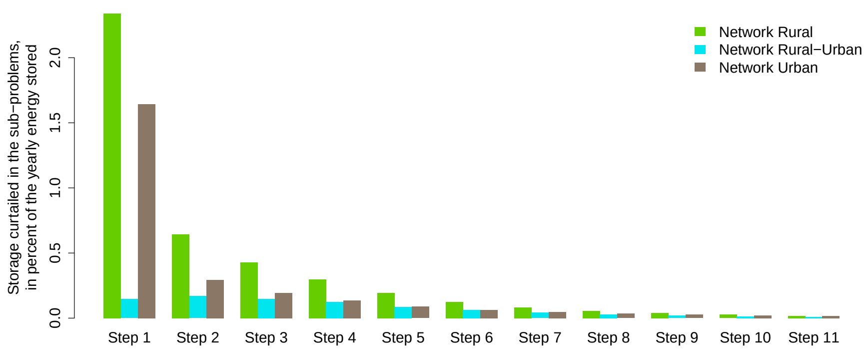
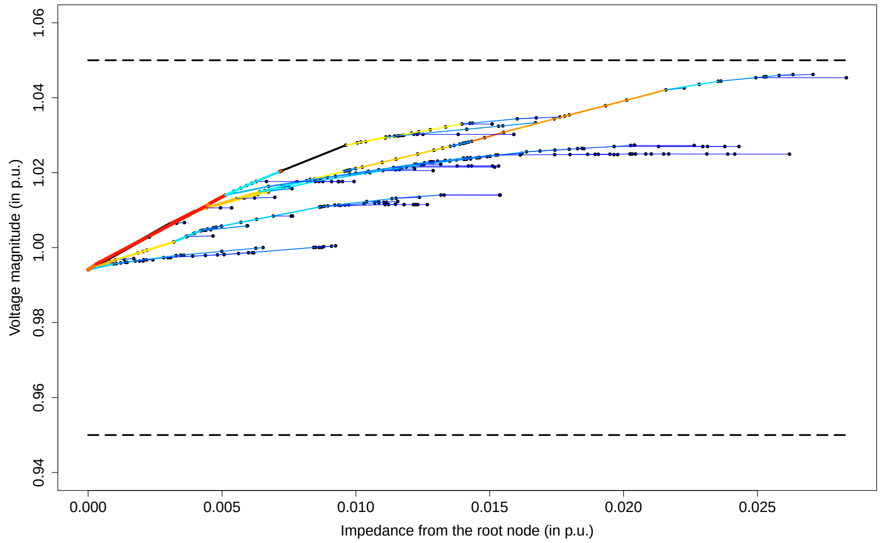


Figure 42: Sub problem power flows for the rural network at time step 5303



in the decrease may be interrupted by sudden increases. This is due to the presence of storage, as a cut associated to a previous sub-problem solution in a given time step may force the storage to change its output in another time step, thus creating the possibility of a deviation in a time step where previously there was none. We should also note that, despite the very high level of penetration, the photovoltaic energy curtailed remains rather low compared to the total energy produced.

As far as power flows inside ADN are concerned, we can remark that, in the time step presented here, the limiting factor was not the voltage constraints but rather the line intensity constraints. Moreover we can see that the voltage downstream of the substation is set at a high value in order, all things else equal, to minimize losses in the network. Nonetheless, maximal allowed voltages are not attained in the ADNs as a consequence of the use of a termination criterion of 0.005.

2.5 CONCLUSION AND PERSPECTIVES

We have presented a model of the operation of distributed generation integrated in a centralized power system as the solution to a large-scale mixed-integer nonlinear optimization problem. Facing the intractability of such a problem, we have proposed a decomposition and coordination procedure that turns this problem into a series of economic dispatches in the centralized power system and optimal power flow in the distribution system. After introducing further hypothesis related to the case study we consider, we showed that both of these optimization problems can be solved with existing techniques. We then gave further details on the case study we aim at solving before presenting some of the results to illustrate our methodology. These results show that our algorithm is able to obtain a satisfactory solution in a fairly realistic case study.

However several limitations of our approach can be identified. First and foremost is the issue of the infeasibility of the original problem. While if the one of sub-problems turns out to be infeasible, we can conclude that the original problem is infeasible, this is not true for the short-term master problem after the first iteration, due to the cuts added to reflect the results of the sub-problems. This is not a major issue when the set-points resulting from the master problem solution enter in conflict with the constraints of the ADN only a few hundred hours during the year, as a feasible solution is likely to be found by the optimizer, usually taking advantage of the flexibility during the time-step unaffected by the cuts. However, when the number of time step with cuts relative to the total number of time step becomes significant, it may lead to an unjustifiably infeasible problem. It remains to be seen if such a situation can arise in a practical setting or, from an alternative viewpoint, if sensible planning decisions can lead to an operation in such conditions. Other limitations that can be pointed out concern mainly the hypothesis and the model we use: no transmission network model, no low-voltage distribution network, three-phase balanced operation of MV network, assumption that loads and generation by renewable are perfectly known *a priori* and a crude model of centralized power plants.

Going forward, several aspects need to be addressed:

- Integrating the proposed simulation of DG operation in planning studies to evaluate the impacts of various planning decision such DG siting and sizing or line reinforcements

- Increasing the breadth of the case studies, in particular regarding the size and number of the ADN considered, to assess in more detail the domain of applicability of our approach

- Improving the quality of our model while keeping computational requirements reasonably low. A first step in this direction could be to take into account the transmission network using a DC approximation or to develop a methodology to model the influence of uncertainties in renewable production and load.

- Developing an adaptive stopping criterion considering the differing level of precision at which the various problems are solved

- Trying to extend our approach to the low-voltage distribution system, possibly by introducing another level of decomposition and coordination

3 OPTIMAL POWER FLOW IN A DISTRIBUTION SYSTEM

3.1 INTRODUCTION

With the aim to increase the sustainability of the electric power system, the share of renewable energies in the production mix is scheduled to increase in the future. For example, the European Union has set goals for its member states in order to attain a 20% share of renewable energy in its final energy consumption by 2020, and some countries have taken even more ambitious stances. This target will be partially met by integrating significant amounts of dispersed renewable energy generators (mainly photovoltaic (PV) and wind power) to the distribution grid. These developments will have a considerable impact on the design and operation of the electric system, both at the national and local level and so new tools will be needed to assist in the planning and operation of at least the distribution network. Indeed, as the current passive distribution network turns into an Active Distribution Network (ADN) (see [142] for a definition) with the introduction of partially and totally controllable generation and storage means, planning studies based solely on power flows for extreme load conditions will not be adapted anymore.

Considering the similarities between the current transmission network and the future ADN, it is a safe bet to assume that the Optimal Power Flow, a framework first introduced in 1962 by [99] and now widely used for the planning and operation of the transmission network, will prove useful for this purpose. However, it has been remarked by several authors such as [100] and [101] that some characteristics inherent to the distribution system (in particular high R/X ratios and the radial nature of its topology) prevent us from applying traditional transmission system OPF algorithms, such as the Newton-Raphson method implemented in, for example, the Matpower package [102]. Moreover, simplifications commonly used in the planning of the transmission system, such as the linearization of the power flow constraints, are known to produce poor results with high R/X networks.

In this context, the general objective pursued in this paper is to provide a methodology solving the single-stage OPF problem in a medium-voltage balanced radial distribution system to simulate DGS, with the purpose to integrate it in planning studies. This last conditions entails the need for a methodology that privileges speed of convergence and accuracy over precision, as it is expected to be run a large number of times to compare various planning options over long periods and with hour-long time steps, with significant uncertainties in input data. Additionally, its implementation should be versatile in terms of objective function and constraints to accommodate the uncertainties surrounding the future regulatory environment of the distribution system.

Of course, OPF in the distribution system have been envisioned for purpose other than the one described here, such as real time control [143] or optimal DG placement [144]. Nonetheless they all share an underlying common structure of Quadratically Constrained Quadratic Problem (Quadratically Constrained Quadratic Problem (QCQP)) that we will describe below.

3.1.1 *OPF as a QCQP*

Traditionally, OPF based on bus injection model have been associated with meshed transmission systems and branch flow models have been used for radial distribution systems, see [145] and [146] for an up-to-date view on the subject. It has been recently proven that both can be cast as QCQP that are non-convex due to the presence of quadratic equality constraints, in the form of a rank constraint in the bus injection model [147] and power flow constraints in the branch flow model [148]. It is thus a NP-hard problem and cannot be solved in polynomial time while guaranteeing global optimality in general. A wide range of techniques have been employed including conjugate gradient, successive quadratic programming, branch-and-bound, Lagrange relaxation, interior point methods, simulated annealing, genetic algorithm and particle swarm optimization that represent various compromises between optimality and convergence speed and between versatility and tailoring for a specific problem (see [149] for a review of deterministic algorithms and [150] for non-deterministic ones). Considering our emphasis on convergence speed and versatility, and the parallel development of high performance Second-Order Cone (SOC) solvers and studies of quadratic convex relaxations of the OPF problem, we have chosen to focus our interest on them.

3.1.2 *Convex relaxations*

In the same manner that the equivalent QCQP of an OPF depends on which model is used, the corresponding convex relaxations will also depend on the model chosen.

1. Convex relaxation based on bus injection model are obtained by relaxing the rank constraint, as explained in [147]. We then obtain a semi-definite problem with a size proportional to the square of the size of the initial problem

2. Convex relaxation based on branch flow model. These relaxations are done in two steps. The first is a relaxation of bus angle constraints that is always exact for radial network [148] and the second relaxes quadratic equalities to convex inequalities to yield a SOC problem of a size equivalent to that of the original problem.

In the case of a radial network, [145] has shown that these relaxations are equivalent. Consequently, we choose the SOC option as the size of the resulting problem is smaller. This is supported by the findings in [145], where the authors have compared computational time to solve both relaxations on radial networks. While these were equivalent for small networks (9 to 39 nodes), the SOC relaxation was two orders of magnitude faster for the 300 bus test case.

3.1.3 *Outline*

For the remainder of this paper, we will focus on the SOC relaxation applied to a branch flow model of a balanced radial distribution system. We will start by stating a mathematical representation of the problem and present some of the conditions under which this relaxation may be exact, the focal point of most of the existing literature. We will then illustrate in which situations these conditions cannot be met on a simplified case study and introduce the cutting plane concept we will use in these instances, before proving its theoretical convergence to global optimality. We continue by introducing a termination criterion so that the algorithm is able to reach a satisfactory solution in a few iterations of a SOC solver, before analyzing its behavior on several case studies. We will conclude by reflecting on the applicability of the presented algorithm for our purpose and on the ways its performance and representativity of the problem can be enhanced.

3.2 THE OPF MODEL AND ITS SOC RELAXATION

3.2.1 *Mathematical Model*

Drawing on the work of [138], we may formulate the OPF problem we wish to study with the following set of equations. Bus voltage angles are eliminated in accordance with [138].

The objective is :

$$\min \ \alpha \, \Delta(\mathbf{x}) + \beta \sum_{j \in [1,J]} r_{f(j),j} I_j \tag{15}$$

with $\alpha \geqslant 0$ and $\beta \geqslant 0$. It is thus a linear combination between a function representing the interests of the DGS operators and a function representing those of the distribution system operator (DSO) (e.g minimizing losses).

The branch flow model equations :

$$P_{f(j),j} - r_{f(j),j} I_{f(j),j} - \sum_{k \in c(j)} P_{j,k} - P_j^{load} + P_j^{st} + P_j^{pv} = 0 \tag{16a}$$

$$Q_{f(j),j} - x_{f(j),j} I_{f(j),j} - \sum_{k \in c(j)} Q_{j,k} - Q_j^{load} + Q_j^{st} + Q_j^{pv} = 0 \tag{16b}$$

$$U_{f(j)} - U_j - 2\left(r_{f(j),j} P_{f(j),j} + x_{f(j),j} Q_{f(j),j}\right) + {z_{f(j),j}}^2 I_{f(j),j} = 0 \tag{16c}$$

with :

$$I_{f(j),j} = \frac{{P_{f(j),j}}^2 + {Q_{f(j),j}}^2}{U_{f(j)}} \tag{17a}$$

$$U_j = {V_j}^2 \tag{17b}$$

We take into account the following constraints on the network :

$$I_{f(j),j} \leqslant I_{f(j),j}^{max} \tag{18a}$$

$$V_j^{min} \leqslant V_j \leqslant V_j^{max} \tag{18b}$$

$$V_1 \in V_{OLTC} \tag{18c}$$

Storage and PV power injections are constrained by their inverter apparent power :

$$P_j^{st\,2} + Q_j^{st\,2} \leqslant S_j^{max,pv\,2} \tag{19a}$$

$$P_j^{pv\,2} + Q_j^{pv\,2} \leqslant S_j^{max,st\,2} \tag{19b}$$

$$P_j^{pv} \in [0, P_j^{pv,sp}] \tag{19c}$$

$$P_j^{st} \in \begin{cases} [P_j^{st,sp}, S_j^{max,st}] & \text{if } P_j^{st,sp} < 0 \\ [-S_j^{max,st}, P_j^{st,sp}] & \text{if } P_j^{st,sp} > 0 \end{cases} \tag{19d}$$

In this representation, the non-convexity is concentrated in (17a) in the form of quadratic equalities. As an aside, the non-convexity in Equation 17b is of no consequence, as the entire optimization problem can be solved using $U_{i,t}^j$ as variables, while the voltage magnitude can be derived *a posteriori*. In the same manner as [138], we relax these to convex quadratic inequalities (20) to obtain the SOC relaxation.

$$I_{f(j),j} U_{f(j)} \geqslant {P_{f(j),j}}^2 + {Q_{f(j),j}}^2 \tag{20}$$

In the remainder of the paper, we will simplify the notation by replacing the index $f(j), j$ by j as it is unequivocal in radial networks.

LINE ID	$f(j)$	j	$r_{f(j),j}$ (IN P.U.)	$x_{f(j),j}$ (IN P.U.)
1	1	2	0.07882	0.04016
2	2	3	0.01072	0.03338
3	3	4	0.01829	0.06719
4	4	5	0.05393	0.04028
5	5	6	0.06239	0.07796
6	3	7	0.04172	0.00345
7	7	8	0.07030	0.04562
8	8	9	0.02258	0.00282
9	8	10	0.08972	0.05255

Table 9: Line parameters of the test network

3.2.2 *Mathematical Conditions ensuring the exactness of the relaxation*

Recently, the study of convex relaxations to the -distribution system-OPF has received considerable attention. Practically all this attention has been directed at establishing various sets of conditions on the objective function and the feasible injection region under which it is exact and we can cite [138], [151], [152], [153], [154] and [155] as prime examples of such a focus. In one of the two most encompassing approach for our purpose, [155] proves, for tree networks, that the Pareto-front of the feasible injection region is the same as that of its SOC relaxation. Consequently, if the objective function is convex and monotonically increasing in each active power injection and the initial OPF feasible, the relaxation will be exact. In parallel and using a different mathematical approach, it is proven in [154] that, if the objective function is strictly increasing in the active power injected at the substation and there can be no binding upper bound on voltages (and the mild condition C1 holds), the relaxation will be exact.

3.2.3 *Examples of inexact relaxations in a simplified case study*

In this section, we will test various instances of the objective function in simple case studies based on a 10 bus network and try and provide a practical understanding of the condition ensuring the exactness of the relaxation and explain why that may be antagonistic with the operation of the future ADN, which will justify the focus of this work on inexact relaxations. The topology and line characteristics of the 10 bus network are available in Table 9. One time step is modeled, with the active and reactive loads in 10. We study four different configurations in terms of distributed generation or storage that are described in Table 11.

NODE ID	P_j^{load} (IN P.U.)	Q_j^{load} (IN P.U.)
1	0	0
2	0.0168	3.10^{-4}
3	0.0197	4.10^{-4}
4	0.0186	5.10^{-4}
5	0.0162	5.10^{-4}
6	0.0150	4.10^{-4}
7	0.0277	4.10^{-4}
8	0.0223	3.10^{-4}
9	0.0297	5.10^{-4}
10	0.0262	3.10^{-4}

Table 10: Node parameters of the test network

PARAMETERS	CONFIG. 1	CONFIG. 2	CONFIG. 3	CONFIG. 4
Type	PV	PV	Storage	Storage
Node	3	3	4	4
Apparent Power (in p.u)	0.2	2	0.3	1
Set-point	0.2	2	-0.3	-1

Table 11: Configurations of the test network that will be studied

For our present purpose, we consider the root node to have a fixed voltage magnitude of 1.03 p.u. and the lower and upper bounds on bus voltage magnitude to be 0.95 and 1.05. We also make the hypothesis

$$\Delta(\mathbf{x}) = \sum_{j \in [1,J]} |P_j^{pv,sp} - P_j^{pv}| + |P_j^{st,sp} - P_j^{st}| \tag{21}$$

meaning the DGS operator have a preferred set-point from which they want to minimize the absolute difference. Here, we would like to emphasize the fact that the absolute values in the objective function formulation, combined with the constraints on injections in Equation 19, reduce to linear, and thus convex, functions of the injections. The three objective functions we study are then fully defined by the values taken by α and β :

1. $\alpha = 0$ and $\beta = 1$
2. $\alpha = 1$ and $\beta = 0$
3. $\alpha = 1$ and $\beta = 1$

The first objective function has the desirable property to be increasing over all the feasible injection region (the losses in the network are in fact the sum of all injections in the network, including the injection at the substation). Consequently, it can be used to determine the feasibility of an OPF problem, as it fulfills the requirement from [155]. However, in the presence of controllable active power injection in the form of storage devices or PV system that allow curtailment and that are large enough to create reverse power flows, minimizing losses can be contradictory with the objective of the storage or PV operator. Indeed, PV operators may be willing to allow the curtailment necessary to respect the network constraints if, for example, they are compensated by lower connection costs. But curtailing beyond that level may be counterproductive, as the losses prevented will end up being only a fraction of the PV energy curtailed. The same reasoning can be applied to storage operators, considering the benefits they bring to the electric system as a whole over one cycle of operation.

This is the justification for the second objective function, for which only the point of view of the distributed generator operators is taken, with the aim to minimize the distance with their optimal set-points. The last objective function is a compromise between loss and set-point absolute distance minimization and so a compromise between the interests of the DSO and those of the distributed generator operators.

3.2.4 *Results*

For the remainder, we define the gap at the solution of the SOC in the following manner :

$$rGap(\mathbf{x}) = \sum_{j \in [1:N]} r_{f(j),j} \left(I_j U_{f(j)} - \left({P_j}^2 + {Q_j}^2 \right) \right) \tag{22}$$

We remark first that the line maximal intensity have been chosen not to be binding in any of the case studies, as focusing on the voltage constraints is the main issue for our present purpose. More precisely, as line intensity constraints are imposed on I_j, they will always be respected by the actual power flows resulting from the relaxation, its exactness notwithstanding.

To analyze the effect of the various objective functions, we will display five different types of result :

1. the gap, for each objective function and configuration in Table 12
2. the power injected by either the storage or PV unit, at the optimal point of the relaxation, in Table 13
3. the network active power losses, at the optimal point of the relaxation, in Table 14
4. the voltages at each node at the optimal point of the relaxation (full line), called V_j^{SOC} hereafter, in Figure 43
5. the voltages at each node, obtained from a power flow with the active and reactive power injections fixed at their optimal value of the relaxation (dashed line), denominated V_j^{PF} in the remainder, in Figure 43

The SOC relaxation optimal solutions are calculated using the MOSEK optimization software [141] through its R [156] interface, with the relevant tolerances fixed at 10^{-5}. The fourth result could be obtained by solving a modified relaxation, minimizing the losses with the active and reactive power injections fixed in the same way and the first and second constraints in 13 removed. However it is faster to use a backward-forward sweep method based on Equations 16 and 17, provided it is, as the SOC solver, implemented in a compiled language.

3.2.5 *Interpretation*

We start by observing that, as expected for feasible OPFs, the relaxation is exact for the loss minimization objective in every case, and the losses at the optimal point are in fact substantially smaller than the their initial value. We also note that the voltage constraints are

OBJECTIVE	CONFIG. 1	CONFIG. 2	CONFIG. 3	CONFIG. 4
Loss minimization	0	0	0	0
Set-point absolute distance	0.01	0.12	0	0
Compromise	0	0.10	0	0

Table 12: Maximum gap (in p.u.) for various objective functions and configurations

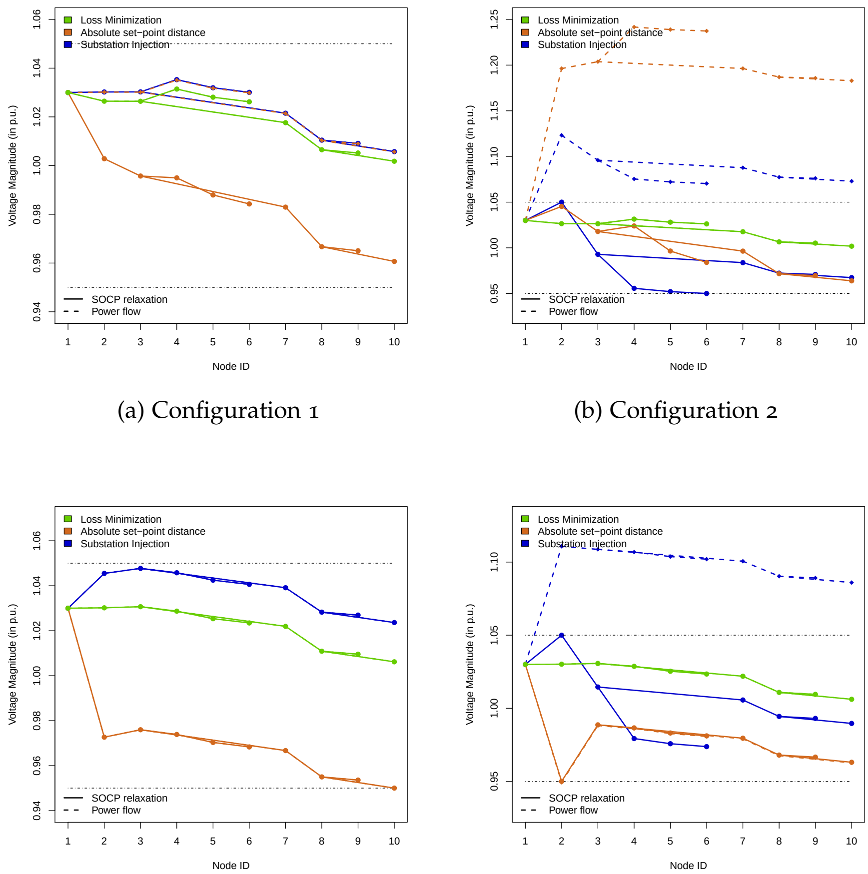
OBJECTIVE	CONFIG. 1	CONFIG. 2	CONFIG. 3	CONFIG. 4
Initial Value	0.20	2.00	-0.30	-1.00
Loss minimization	0.17	0.17	0.19	0.19
Set-point absolute distance	0.20	2.00	-0.23	-0.54
Compromise	0.20	1.94	-0.23	-0.49

Table 13: Optimal active power injection for various objective functions and configurations

OBJECTIVE	CONFIG. 1	CONFIG. 2	CONFIG. 3	CONFIG. 4
Initial Value	0.0028	0.4633	0.0527	0.5406
Loss minimization	0.0026	0.0026	0.0021	0.0021
Set-point absolute distance	NA	NA	0.0427	0.2315
Compromise	0.0028	NA	0.0427	0.1624

Table 14: Network Losses for various objective functions and configurations

Figure 43: Voltages obtained by SOCP relaxation and power flow for various objective functions



(a) Configuration 1

(b) Configuration 2

(c) Configuration 3

(d) Configuration 4

far from being binding and that the active power injection have been modified heavily in some cases, thus hinting that PV and storage power has been curtailed more than is necessary to respect network constraints. Moreover, in configuration 2, PV has been curtailed by 1.83 p.u while the loss prevented (compared to their initial value) are only 0.46 p.u. The same situation is encountered for configuration 4, where storage has been curtailed by 1.19 p.u while the losses prevented are only 0.54 p.u.

Concerning the second objective function, we observe two distinct behaviors : when the active power injection set-point is positive (i.e for PV), the relaxation is inexact while it is exact when the injected active power set-point is negative (i.e charging of storage). This can be understood by noticing that, inside the feasible injection region, the objective function is increasing with the active power injection when the set-point is negative and decreasing otherwise. Consequently, the optimal solution will be in the pareto front of the feasible injection region and the relaxation exact only when all set-points for active power injection are negative.

The use of the third objective function brings a positive result in terms of exactness of the relaxation for configuration 1. To explain this, we can remark that, when the set-points are positive, and the injection can not be higher than the set-point, minimizing the compromise function is in fact the same as minimizing the power injected at the substation because it is, algebraically, equal to the sum of active power injections and losses in the network :

$$P_{1,2} = \sum_{j\in[1,J]} r_j I_j + \sum_{j\in[2,J]} \left(P_j^{load} - P_j^{st} - P_j^{pv}\right) \tag{23a}$$

$$= \sum_{j\in[1,J]} r_j I_j + \sum_{j\in[2,J]} \left(|P_j^{pv,sp} - P_j^{pv}| + |P_j^{st,sp} - P_j^{st}|\right) + C \tag{23b}$$

Moreover, due to the low upper bound on injection in configuration 1, there can be no binding upper voltage constraints (this can be checked by running a power flow with all injections at their maximal value) and condition C1 from [154] holds, thus ensuring the exactness of the relaxation (see Theorem 1 from [154]). For configuration 2, the use of the compromise function only leads to a solution with a lower gap, without rendering it an exact relaxation because the upper bound on injection is high enough to entail binding upper voltage constraints. For configuration 3 the result is left unchanged, because, due to the absence of binding network constraints, the result was already a compromise. For configuration 4, the result is changed compared to the preceding objective function as the storage is curtailed by a further 0.05 p.u, allowing a decrease in losses of 0.07 p.u which is still an improvement in terms of overall energy efficiency.

We have shown that there exist situations where the interests of the DSO -minimizing losses and respecting network constraints- and the interests of the operator of the distributed generators and storages can be contradictory. In these situations, satisfying the interests of the DG operators (e.g minimizing the distance with the set points) may lead to inexact SOC relaxations. The use of the compromise objective function presented the obvious advantage of balancing the objective of the DSO with that of the DG and storage owners. Additionally, it allowed for an exact relaxation when set-points are positive and there can be no upper voltage binding constraint. Extracting a physically meaningful power flow respecting network constraints when power injection can lead to over-voltages will be the focus of the next section.

3.3 EXTRACTING A PHYSICALLY MEANINGFUL SOLUTION

Insofar as we tackle the case of inexact relaxations, our work is in the vein of the recent efforts by [157], [158], [159] and [160]. However, the comparison ends here as the four articles above address the more general problem of meshed networks, at the expense of the use of a more computationally demanding SDP relaxation. Moreover, our methodology relies on a cutting plane concept, while [157] and [158] propose a branch-and-bound approach, [159] develops a heuristic to extract rank-1 solutions from low-rank solutions and [160] uses an approximation of a rank-minimization algorithm.

In this section, we start by defining the methodology we implement and proving its convergence. We then illustrate it with detailed results on a simplified case study. By combining observations on these results with a qualitative assessment of the accuracy requirements of the expected OPF application, we subsequently propose a stopping criteria allowing the attainment of a satisfactory solution.

3.3.1 *Principle of the cutting planes*

The principle behind the use of cutting planes can be dated back to Gomory who first introduced them in 1958 to solve mixed integer linear programming [161]. The idea is to solve a relaxation of the original problem (for Gomory, ignoring the integer requirement, here, the SOC relaxation), then, if the relaxation is not exact, adding a linear inequality to the relaxation that will eliminate the first solution obtained from the feasible set while keeping the optimal solution of the non-relaxed problem in the feasible set. Then this method is applied iteratively and new cuts are added at each step, until a termination criterion is met. Of course, the complexity of such a methodology lies in finding a way to calculate the cutting planes that is computationally tractable and fulfills the requirements mentioned above (the cut is then deemed valid) and we will now describe the approach we put forward.

3.3.2 *Definition of the cuts*

To improve the readability of the following section, we define S as the set of feasible solutions to the original problem, $\tilde{S}_k$ as the set of feasible solutions of its SOC relaxation minus the points eliminated by the cut imposed at step k, starting at step 0 with no cut. Moreover :

$$rI(\mathbf{x}) = \sum_{j \in [1,J]} r_{f(j),j} I_j \tag{24a}$$

$$rL(\mathbf{x}) = \sum_{j \in [1,J]} r_{f(j),j} \frac{{P_j}^2 + {Q_j}^2}{U_{f(j)}} \tag{24b}$$

$$\dot{\mathbf{x}} = \operatorname*{argmin}_{x \in S} \; \alpha \, \Delta(\mathbf{x}) + \beta \, rI(\mathbf{x}) \tag{24c}$$

$$\tilde{\mathbf{x}}_k = \operatorname*{argmin}_{x \in \tilde{S}_k} \; \alpha \, \Delta(\mathbf{x}) + \beta \, rI(\mathbf{x}) \tag{24d}$$

To focus ourselves on interesting situations, we make four assumptions in the remainder :

1. S is not empty
2. the solution obtained at step k is not exact
3. $\alpha > 0$ (otherwise the relaxation would be exact at step 0)
4. $\beta > 0$

We then define the cut at step k in the following manner :

$$rI(\mathbf{x}) \leqslant rL(\tilde{\mathbf{x}}_{k-1}) \tag{25}$$

It is straightforward to see that the current optimal point is eliminated from the feasible set (the second assumption ensures that $rL(\tilde{\mathbf{x}}_{k-1})$ is strictly inferior to $rI(\tilde{\mathbf{x}}_{k-1})$) and thus the next subsection will consist in proving that this cut keeps $\dot{\mathbf{x}}$ in $\tilde{S}_k$.

3.3.3 *Proof of the validity of the cuts*

What we know from the existing literature is that minimizing the losses over $\tilde{S}_{k-1}$ will lead to a solution in S, provided that the cut at step $k-1$ is valid. Moreover, we have observed that the objective representing the interests of the DGS operators can be contradictory to the objective of loss minimization and thus lead to an inexact relaxation. Intuitively, what happens in such situations is that we have not reduced the losses "enough" to allow for a feasible solution to be obtained, which is the informal justification for adding cuts of the form (25). We will now formally prove that the cut defined in (25) is valid for all step k by induction. The base case for $k = 0$ is plain, as $S \subset \tilde{S}_0$. The fact that the validity of the cut at step $k-1$ entails the validity at step k is proven in three step :

$$rI(\dot{\mathbf{x}}) < rI(\tilde{\mathbf{x}}_{k-1}) \tag{26a}$$

$$\Delta(\dot{\mathbf{x}}) \geqslant \Delta(\tilde{\mathbf{x}}_{k-1}) \tag{26b}$$

$$rI(\dot{\mathbf{x}}) < rL(\tilde{\mathbf{x}}_{k-1}) \tag{26c}$$

STEP 1 We prove (26a) by contradiction. Let us assume that $rI(\dot{\mathbf{x}}) \geqslant rI(\tilde{\mathbf{x}}_{k-1})$. Due to theorem 7 in [155] and the validity of the cut at step $k-1$, we know the following :

$$\exists!\, \hat{\mathbf{x}} \in S : \hat{\mathbf{x}} = \underset{\substack{\mathbf{x}\in\tilde{S}_{k-1} \\ rI(\mathbf{x})\geqslant rI(\tilde{\mathbf{x}}_{k-1})}}{\operatorname{argmin}} \quad rI(\mathbf{x}) \tag{27}$$

We also have :

$$\tilde{\mathbf{x}}_{k-1} = \underset{\substack{\mathbf{x}\in\tilde{S}_{k-1} \\ rI(\mathbf{x})\geqslant rI(\tilde{\mathbf{x}}_{k-1})}}{\operatorname{argmin}} \quad rI(\mathbf{x}) \tag{28}$$

Due to the uniqueness in (27), we obtain $rGap(\hat{\mathbf{x}}) = rGap(\tilde{\mathbf{x}}_{k-1})$, which is in contradiction to the second assumption and the fact that $\hat{\mathbf{x}}$ belongs to S.

STEP 2 If the cut at step $k-1$ is valid, $\dot{\mathbf{x}}$ belongs to $\tilde{S}_{k-1}$. $\dot{\mathbf{x}}$ cannot have an objective value that is lower than that of $\tilde{\mathbf{x}}_{k-1}$, as this would contradict the definition of $\tilde{\mathbf{x}}_{k-1}$ and the assumption of the inexactness of the relaxation. Consequently, we have :

$$\alpha\left(\Delta(\dot{\mathbf{x}}) - \Delta(\tilde{\mathbf{x}}_{k-1})\right) \geqslant \beta\left(rI(\tilde{\mathbf{x}}_{k-1}) - rI(\dot{\mathbf{x}})\right) \tag{29}$$

Combining this with (26a), we obtain (26b)

STEP 3 $\tilde{\mathbf{x}}_{k-1}$ is a pareto efficient solution for the minimization of the vector valued objective function :

$$x \to (\Delta(\mathbf{x}), rL(\mathbf{x}), rGap(\mathbf{x})) \tag{30}$$

$$x \in \tilde{S}_{k-1} \tag{31}$$

because it is the solution of a minimization of a linear combination of the element of the above function. By definition of the pareto efficient solutions, we have :

$$rGap(\tilde{\mathbf{x}}_{k-1}) = \underset{\substack{\mathbf{x}\in\tilde{S}_{k-1} \\ rL(\mathbf{x})\leqslant rL(\tilde{\mathbf{x}}_{k-1}) \\ \Delta(\mathbf{x})\leqslant\Delta(\tilde{\mathbf{x}}_{k-1})}}{\operatorname{argmin}} \quad rGap(\mathbf{x}) \tag{32}$$

Due to the fact that $rGap(\mathbf{x})$ cannot be negative and $\dot{\mathbf{x}}$ belongs to $\tilde{S}_{k-1}$, we have :

$$rGap(\dot{\mathbf{x}}) = 0 = \underset{\substack{\mathbf{x}\in\tilde{S}_{k-1} \\ rL(\mathbf{x})\leqslant rL(\dot{\mathbf{x}}) \\ \Delta(\mathbf{x})\leqslant\Delta(\dot{\mathbf{x}})}}{\operatorname{argmin}} \quad rGap(\mathbf{x}) \tag{33}$$

From the second assumption, we know that $rGap(\dot{\mathbf{x}}) < rGap(\tilde{\mathbf{x}}_{k-1})$. We combine this with Equation (26b) to obtain :

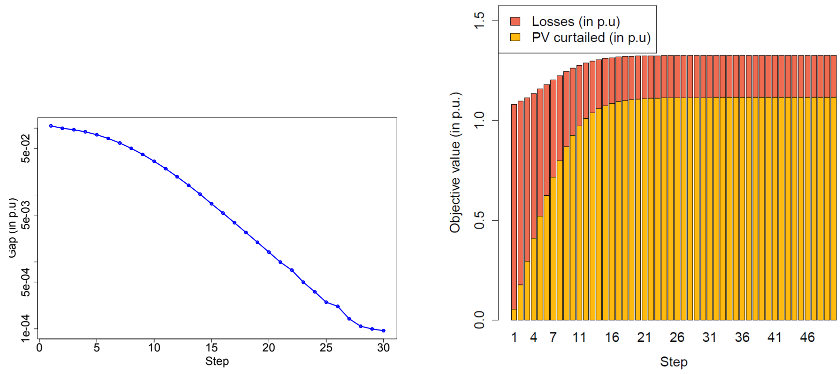
$$rL(\dot{\mathbf{x}}) < rL(\tilde{\mathbf{x}}_{k-1}) \tag{34}$$

As, by definition of $\dot{\mathbf{x}}$, $rGap(\dot{\mathbf{x}}) = 0$, we have demonstrated (26c) and the validity of the cuts at each step k.

3.3.4 *Illustration of the convergence on configuration 2*

We start by applying our methodology to the case study defined above for which the relaxation is inexact. The results provided in Figure 44 for the first 30 steps are $rGap(\tilde{x}_k)$, $rI(\tilde{x}_k)$ and $\Delta(\tilde{x}_k)$. $rGap(\tilde{x}_k)$ is plotted in Figure 45a with a logarithmic scale. $rI(\tilde{x}_k)$ and $\Delta(\tilde{x}_k)$ are represented as a stacked bar graph in Figure 45b, with the total height of the bars being the objective value at the considered step. First, we can observe that $rGap(\tilde{x}_k)$ decreases consistently during the process while the objective value of the relaxation converges asymptotically to the optimal value of the original problem. Moreover, the trade-off between losses and set-point deviations evolves as expected, with high losses and low set-point deviations for the first steps and conversely for the last ones.

Figure 44: Voltages obtained by SOCP relaxation and power flow for various objective functions



(a) Gap Reduction

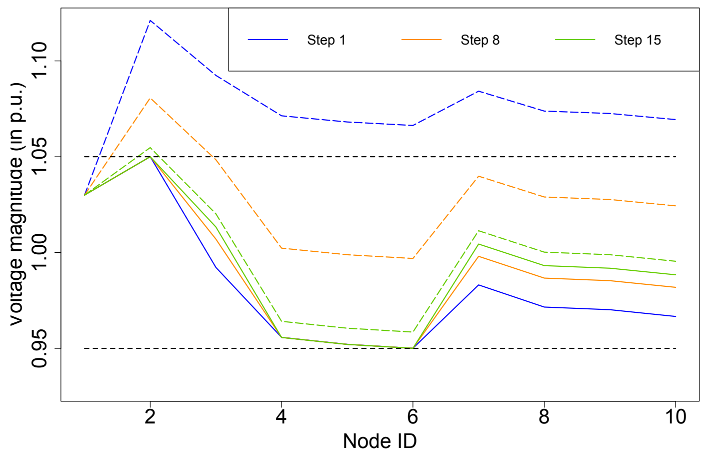
(b) Objective value evolution

In a manner comparable to 3.2.4, we further illustrate the convergence properties of our algorithm by drawing V_j^{SOC} and V_j^{PF} for steps 1, 8 and 15 in Figure 45. In coherence with the continuous decrease of $rGap(\tilde{x}_k)$, we witness that V_j^{SOC} and V_j^{PF} get closer to each other as the algorithm approaches the optimal solution. We must now emphasize the fact that, while the algorithm converges to the optimal solution of the original problem, each solution obtained will necessarily lead to over-voltages, as illustrated in Figure 45. Consequently, obtaining a solution that satisfies the network constraints in a finite number of steps will require a compromise between closeness to the optimal solution and computational cost that will be the subject of the next section.

3.3.5 *Stopping Criterion*

To overcome the fact that the injection set points obtained will lead to over-voltages, we propose to decrease the upper voltage bound by

Figure 45: Voltages obtained by SOCP relaxation and power flow for various time steps of the algorithm applied to configuration 2



ϵ_{Cons}	5.10^{-4}	1.10^{-3}	5.10^{-3}	1.10^{-2}	15.10^{-3}
Number of Steps	24	20	14	12	10
Objective Value	1.33	1.34	1.37	1.41	1.47

Table 15: Trade-off between computation time and optimality, with a stopping criterion allowing the achievement of a solution respecting network constraints

a given ϵ, and to stop the process when the original voltage bound is respected. With this procedure, network constraints are guaranteed to be respected in a finite number of steps, at the expense of an overvaluation of the objective function. The choice of ϵ will thus be a trade-off between execution time and optimality, as illustrated in Table 15, with high values of ϵ leading to a fast solution with a high objective value and conversely for small ϵ.

3.4 CASE STUDY

3.4.1 *Definition of the configurations studied*

First, we must make clear that we agree wholeheartedly with the claim in [162] that "practical systems operating at normal conditions" will exhibit exact relaxations for the associated OPF models and so we do not intend to study them. In other words, the case studies we present here have an overt bias towards dealing with very large active power injections at times of low loading in future hypothetical

CASE ID	1	2	3	4	5	6	7	8	9	10	11	12
Network size	10				69				189			
Total load (in p.u.)	0.2				13				13.8			
Buses Equipped	2		5		14		35		38		95	
Total PV (in p.u)	1.2	2.5	2.4	2.9	64	86	85	171	133	222	388	776

Table 16: Characteristics of the case studies

distribution systems with very high levels of distributed generation penetration. We are well aware that, in practical settings, network reinforcements would probably be implemented before such high levels of PV curtailing are attained, but the objective of such a focus in this section is not to draw conclusions on the necessity of PV curtailing, network reinforcement or storage development in general, but rather to assess the ability of our algorithm to attain satisfactory solutions in adverse conditions and when some characteristics of the input that we deem relevant are modified. These characteristics are : the number of nodes in the network, the number of nodes equipped with PV and the ratings of the PV inverters. The network data for the considered case studies are based on the 10-bus network described above, the 69-bus network in [163] and the 189-bus network in [164]. For each network, we study four configurations of PV injection (two number of nodes equipped and two combinations of PV inverter ratings). The buses equipped are selected in decreasing order of apparent load, again in the spirit of studying the worst conceivable situations. Moreover, the active power injection set-points are assumed to be at the maximal value allowed by the inverter rating, the voltage bounds are -/+5% around the nominal value and the voltage at the root node is fixed at +3% of the nominal value. Table 16 details these twelve configurations.

3.4.2 *Analysis of the results*

The results are presented in Table 19, Table 18 and Table 17, where are displayed, respectively, the objective value, $\max V_j^{PF}$ and the percentage of PV curtailed for the SOC relaxation without cuts and at the termination step for two settings of ϵ. Table 20 displays the mean duration of a single SOC optimization for each case, along with the number of steps necessary to fulfill the termination criterion.

We can first remark that, at constant network size, the number of node equipped has no influence on the duration of an individual step - variation on the order of the millisecond should not be overinterpreted as these are close to the measurement precision. We also observe that, across the various network sizes, the variation in individual step duration is fairly limited, as multiplying the network size

Case ID	1	2	3	4	5	6	7	8	9	10	11	12
SOCP relaxation	5.2%	34%	2.0%	5,6%	24%	22%	21%	26%	20%	25%	29%	39%
$\epsilon = 5.10^{-3}$	20%	65%	28%	32%	27%	32%	29%	45%	71%	35%	38%	53%
$\epsilon = 1.10^{-2}$	27%	71%	31%	34%	29%	36%	33%	51%	74%	37%	40%	57%

Table 17: PV curtailment for the SOCP relaxation without cuts, and for two values of the stopping criteria

Case ID	1	2	3	4	5	6	7	8	9	10	11	12
SOCP relaxation	1.088	1.192	1.210	1.233	1.071	1.143	1.106	1.370	1.070	1.110	1.211	1.453
$\epsilon = 5.10^{-3}$	1.050	1.050	1.050	1.050	1.050	1.050	1.050	1.050	1.048	1.049	1.050	1.050
$\epsilon = 1.10^{-2}$	1.049	1.049	1.050	1.050	1.050	1.050	1.050	1.050	1.047	1.049	1.050	1.050

Table 18: $\max V_j^{PF}$ for the SOCP relaxation without cuts, and for two values of the stopping criteria

Case ID	1	2	3	4	5	6	7	8	9	10	11	12
SOCP relaxation	0.54	1.71	1.48	1.95	29.4	44.0	39.0	98.5	45.7	99.3	213.7	533.6
$\epsilon = 5.10^{-3}$	0.60	1.85	1.54	2.00	30.4	45.2	40.3	101.4	49.6	104.4	220.7	547.1
$\epsilon = 1.10^{-2}$	0.65	1.90	1.58	2.03	30.9	45.7	40.8	101.5	53.7	109.3	225.8	550.7

Table 19: Objective value for the SOCP relaxation without cuts, and for two values of the stopping criteria

Case ID	1	2	3	4	5	6	7	8	9	10	11	12
Mean step duration (in ms)	9	10	11	9	26	27	26	26	52	53	57	54
$\epsilon = 5.10^{-3}$	9	13	108	118	35	61	49	85	3	10	18	51
$\epsilon = 1.10^{-2}$	7	9	25	74	16	17	18	37	3	8	4	19

Table 20: Mean duration of individual SOCP steps and number of SOCP steps before termination, for two values of the stopping criterion

by, respectively, 7 and 19 leads to an increase in duration by a factor of, respectively, around 2.5 and 5.5, which is very encouraging in terms of scalability of our approach. To be clear, this is not a feature of our algorithm *per se* but rather a consequence of being able to draw on the performance of existing SOC solvers. Concerning the ability of our algorithm to fulfill its purpose, we may start by observing that a satisfactory solution in terms of network constraints has been obtained in a finite number of steps for each of the case study and termination settings, as the results in Table 18 exhibit no remaining over-voltages. Moreover, the results in Table 17 and Table 19 are unsurprising, with more stringent termination criterion leading to better solution in terms of objective function or PV curtailing and increased level of penetration -all things else equal- leading to higher curtailing. However, when trying to draw conclusion on a potential link between the input data and the number of steps, the situation is more muddled. While, as could be expected, more stringent criteria, higher level of penetration or number of nodes equipped lead, for a given network, to higher number of steps needed, the variation in the amount of steps between the various network does not seem to be linked to the number of nodes in the network. This may be discomfiting at first but might be explained by the fact that there are many parameters in the network definition that can influence the efficiency of our algorithm that we do not isolate in these case study, such as the R/X ratio of the lines, the dispersion of the value of the loads and PV generator injections or the position of such generators relative to the root node. Consequently, additional investigations of this particular issue are in order.

3.5 CONCLUSION

After reviewing the existing literature that focuses on proving the existence of conditions ensuring the exactness of the SOC relaxation of the distribution system optimal power flow, we have illustrated how these could be incompatible with the efficient operation of future distribution systems with high penetration of distributed resources. We have then introduced an algorithm consisting in iteratively adding linear cuts to this SOC relaxation, before proving its theoretical convergence to the optimal solution. We continued by illustrating its practical implementation on various case studies while proposing a termination criterion allowing the achievement of a solution satisfying the network constraints in a finite number of steps. In the worst situation (biggest network, highest number of nodes equipped, highest penetration and best precision), a satisfactory solution was obtained in around 2.5 seconds. If we link this observation with our previous work in [165], where we present a methodology to define the optimal set-points from the point of view of the services rendered to the transmission system, we have obtained a practical way to estimate the yearly cost function of an ADN. Moreover if we implement the strategy described in [166], which consist in solving the distribution OPF only when absolutely needed, we can estimate that, in worst case scenarii, the calculation of the objective value for one set of planning options could be done in a few minutes, which is a promising result considering our initial objective.

4 CRITICALITY CRITERION

4.1 CONTEXT

As the interest for distributed generation and storage grows, so does the need for new tools to assist in the planning of the distribution network. Indeed, as the current passive network transforms into an Active Distribution Network (ADN) [38, 39] with the introduction of partially and totally controllable generation and storage means along with a supervision and control infrastructure, planning studies based solely on power flows for extreme load conditions will not be adapted anymore. Moreover, if controllable generation and storage systems are used not only to render services to the local ADN but also to the rest of the power system, power flows at maximum load and minimum generation and *vice versa* will also not be sufficient for the simple reason that they cannot be known *a priori* by examining the determinants of the local network. Considering the similarities between the current transmission network and the future ADN, it is a safe bet to assume that the Optimal Power Flow, a tool first introduced in 1962 by Carpentier [99] and now widely used for the planning and operation of the transmission network, will prove useful for this purpose. Consequently, the adaptation of the OPF concept and resolution algorithms to the distribution network has been the subject of numerous publications in the last decade such as [167–170]. Among the structural differences between transmission and distribution network, the radial topology and higher R/X ratio of line impedances are often identified as the main obstacles to the direct use of transmission network OPF algorithms to distribution systems as explained in [171]. Moreover, the DC approximation of the OPF that is often used in transmission planning studies is not accurate enough in the distribution system due to the high R/X ratio. Thus, in a planning study for which OPFs have to be solved for each hour of the year and multiple planning options evaluated, the computational burden can become dissuasive. And so, heuristics that allow to decrease the computational burden while controlling the loss of optimality can have practical applications, as demonstrated in [172].

4.2 OUTLINE

In the work presented here, we make use of the characteristics of the problem studied to propose a methodology allowing us to solve the computationally intensive distribution OPF only when it significantly influences the result of the evaluation of a given planning option. First, we will describe the problem studied and its characteristics and explain why the computation of a certain number of time steps can be avoided. Then, we detail the various calculations that allow us to estimate beforehand which time steps will be significant before illustrating the validity of our methodology on a wide array of test cases. Finally, we propose a variable termination criterion that allow the user to choose between various compromises in terms of computational gains and accuracy.

4.3 METHODOLOGY

4.3.1 *Problem studied*

4.3.1.1 *General structure*

In [173], we have formally defined the integrated operation of distributed generation (DG), active distribution networks and the centralized power system (CPS) as the solution to a large-scale, nonlinear, mixed-integer optimization problem. Facing the intractability of such a problem, we have proposed a decomposition and coordination procedure thanks to which it is replaced by a series of master problems, akin to economic dispatches of the CPS including aggregated DG capabilities but neglecting ADN constraints, and sub-problems, which are single-stage distribution OPF with the objective to minimize a compromise between the distance to the set-points resulting from the master problem and the losses in the ADN. The master problem is a classical problem in the literature for which we adopt a two-stage formulation combining a mixed-integer problem and a series of linear problems that can be readily solved by existing commercial solvers. The sub-problems are amenable to a solution by an algorithm we introduced in [136], an iterative cutting-plane procedure taking advantage of existing second-order cone programming software.

4.3.1.2 *Application to planning studies*

In the remainder, we will adopt the convention that a time step is deemed critical for a given ADN if it is infeasible or if the active power have to be modified from their master problem set-points in order to satisfy the ADN constraints. It is considered non-critical otherwise.

When the simulation is used for operational purposes, evidently the sub-problems have to be solved even for non-critical time steps to obtain the set-points for reactive powers and the setting of the tap changers. However, in planning studies, only two questions need to be answered by a simulation of the operation:

- Are the constraints satisfied at all time ?

- What is the running cost ?

On account of our CPS model relying on a global supply-demand equilibrium constraints and the influence of the variations in ADN losses due to active management neglected, active powers will be the only result from the feasible sub-problems that will influence the running cost. As a consequence, the sub-problems need to be solved solely for the critical time steps. We will now provide a methodology able to classify the time steps from the most likely to be critical to the least likely. In turn, this will allow us to solve the sub-problems in the order defined thusly until a non-critical time step is encountered, so as to avoid solving unnecessary distribution OPF.

4.3.2 *Criticality criteria*

In essence, determining beforehand which time steps are most likely to be critical is equivalent to predicting during which time steps either voltage or line current constraints might be violated if active power are not allowed to change. Adopting this viewpoint, we propose two criteria aiming at quantifying the likelihood of a time step to be either voltage-critical (i.e. critical with binding voltage constraints) or current-critical (i.e. critical with binding current constraints).

4.3.2.1 *Voltage criticality*

In the model we adopted, predicting if a voltage constraint will be violated unless active powers are modified is tantamount to determining if the On-load tap changer (On-Load Tap Changer (OLTC)) and the reactive powers are able to correct the voltage violations incurred from the behavior of the loads, distributed generation and storage systems resulting from the master problem solution. Consequently, it requires an estimation of the voltage levels dependent on OLTC settings and of the reactive power compensation capabilities.

VOLTAGE VIOLATION AND OLTC INFLUENCE In order to take into account the influence of the on-load tap changer, we define A as the set of permissible tap-settings and $V^{j,\alpha}_{i,t}$ the substation downstream voltage at time step t corresponding to the tap position α in ADN j. The first step of the criticality criterion calculation is to obtain a power flow for a chosen initial tap setting α_0 and then expand the result to all possible tap settings for each bus i by making the following approximation:

$$V^{j,\alpha}_{i,t} \approx V^{j,\alpha_0}_{i,t} + V^{j,\alpha}_{1,t} - V^{j,\alpha_0}_{1,t} \tag{35}$$

We then obtain an estimation of the voltage limit violation during time step t at bus i for the tap setting α in ADN j in the following way:

$$\Delta V_{i,t}^{j,\alpha} = \begin{cases} V_{i,t}^{j,\alpha} - V_j^{max} & \text{if } V_{i,t}^{j,\alpha} > V_j^{max} \\ 0 & \text{if } V_j^{max} \leqslant V_{i,t}^{j,\alpha} \leqslant V_j^{max} \\ V_j^{min} - V_{i,t,\alpha}^{j} & \text{if } V_{i,t}^{j,\alpha} < V_j^{min} \end{cases} \tag{36a}$$

This gives us an estimation of the absolute voltage violation to be compensated at each bus and for each permissible tap-setting.

REACTIVE POWER VOLTAGE VIOLATION COMPENSATION CAPABILITY We base our reasoning on the concept of Voltage Change Potential introduced in [174] and aimed at evaluating the local voltage variations caused by the connection of a given distributed energy resource. It is based on a linearization of the network equations, under the assumption that the actual voltage change is small compared to the pre-existing voltage, and is defined as follows:

$$\Delta V = \frac{X \times Q + R \times P}{V} \tag{37}$$

where X is the driving-point reactance, R the driving-point resistance, Q the reactive power injected at the considered bus, P the active power and V the pre-existing voltage. For our purpose, we define the Reactive Voltage Change Potential during time step t at bus i in ADN j as follows:

$$\Delta V_{i,t}^{j,Q} = \frac{X_i^j \times \left(Q_{i,t}^{ST,j} + Q_{i,t}^{IP,j}\right)}{V_{i,t}^{j}} \tag{38}$$

FORMAL DEFINITION OF THE CRITERION We now have at our disposal a measure of the voltage violations taking into account the role of the OLTC and an estimation of the reactive power local compensation capability. We can combine them in order to define a local compensated voltage violation in the following way:

$$\Delta V_{i,t}^{j,\alpha,Q} = \Delta V_{i,t}^{j,\alpha} - \Delta V_{i,t}^{j,Q} \tag{39}$$

As we need a single measurement by time step in order to establish our criticality criterion, we define it as the minimum of the L1-norm of the compensated voltage violation, while respecting constraints on reactive power availability and permissible tap-settings. We can then formally express the voltage-criticality criterion at time step t for ADN j as:

$$C_V^{t,j} = \min_{Q_{i,t}^{IP,j}, Q_{i,t}^{ST,j}, \alpha} \sum_{i \in N^{Nodes,j}} \left| \Delta V_{i,t}^{j,\alpha,Q} \right| \tag{40a}$$

$$0 \leqslant Q_{i,t}^{ST,j} \leqslant \sqrt{{S_i^{max,ST,j}}^2 - {P_{i,t}^{ST,j}}^2} \tag{40b}$$

$$0 \leqslant Q_{i,t}^{IP,j} \leqslant \sqrt{{S_i^{max,IP,j}}^2 - {P_{i,t}^{IP,j}}^2} \tag{40c}$$

$$\alpha \in A \tag{40d}$$

CALCULATION As α is a discrete variable, we can define, for each $\alpha \in A$:

$$C_{V,\alpha}^{t,j} = \min_{Q_{i,t}^{IP,j}, Q_{i,t}^{ST,j}} \sum_{i \in N^{Nodes,j}} \left| \Delta V_{i,t}^{j,\alpha,Q} \right| \tag{41}$$

while respecting Equations 40b and 40c. We can observe that, for a fixed α , the terms in the sum are independent from one another. Thus, we also have:

$$C_{V,\alpha}^{t,j} = \sum_{i \in N^{Nodes,j}} \min_{Q_{i,t}^{IP,j}, Q_{i,t}^{ST,j}} \left| \Delta V_{i,t}^{j,\alpha,Q} \right| \tag{42}$$

where:

$$\min_{Q_{i,t}^{IP,j}, Q_{i,t}^{ST,j}} \left| \Delta V_{i,t}^{j,\alpha,Q} \right| = \begin{cases} \Delta V_{i,t}^{j,\alpha,Q_{\max}} & \text{if } V_{i,t}^{j,\alpha,Q_{\max}} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{43a}$$

with :

$$\Delta V_{i,t}^{j,\alpha,Q_{\max}} = \Delta V_{i,t}^{j,\alpha} - \frac{X_i^j \times \left(\sqrt{{S_i^{max,ST,j}}^2 - {P_{i,t}^{ST,j}}^2} + \sqrt{{S_i^{max,IP,j}}^2 - {P_{i,t}^{IP,j}}^2} \right)}{V_{i,t}^j} \tag{44}$$

We can thus easily calculate $C_{V,\alpha}^{t,j}$ for each α and then obtain $C_V^{t,j}$ by choosing the minimum value of $C_{V,\alpha}^{t,j}$.

4.3.2.2 *Current criticality*

The calculation of the current criticality criterion is, in principle, similar to that of voltage criticality: identifying the potential constraint violation and estimating if available reactive power can compensate it. The main difference is that we ignore the influence of the OLTC, as its effect (i.e. small modification of losses due to voltage level variations) is not significant for our purpose. As a consequence, the criterion can be calculated along the following steps:

To avoid the introduction of unnecessary notations, we work with the square of current flowing through the line ending in i, $I^j_{i,t}$. Thus, the current constraint violation:

$$\Delta I^j_{i,t} = \begin{cases} I^j_{i,t} - I^{max,j}_i & \text{if } I^j_{i,t} > I^{max,j}_i \\ 0 & \text{otherwise} \end{cases} \tag{45a}$$

Downstream reactive power capability:

$$\Delta Q^j_{i,t} = \sum_{k \in N^{Down,i,j}} \sqrt{{S^{max,ST,j}_i}^2 - {P^{ST,j}_{i,t}}^2} + \sqrt{{S^{max,IP,j}_i}^2 - {P^{IP,j}_{i,t}}^2}) \tag{46a}$$

where $N^{Down,i,j}$ is the set of nodes downstream of i, including it. The compensated current violation is thus expressed as:

$$\Delta I^{j,Q}_{i,t} = \begin{cases} \Delta I^j_{i,t} - \frac{{\Delta Q^j_{i,t}}^2}{{V^j_{i,t}}^2} & \text{if } \Delta Q^j_{i,t} < \left| Q^j_{i,t} \right| \\ \Delta I^j_{i,t} - \frac{{Q^j_{i,t}}^2}{{V^j_{i,t}}^2} & \text{otherwise} \end{cases} \tag{47a}$$

Finally, the current criticality criterion at time step t for ADN j is simply the sum of individual compensated constraint violations:

$$C^{t,j}_I = \sum_{i \in N^{Nodes,j}} \Delta I^{j,Q}_{i,t} \tag{48a}$$

4.4 APPLICATION

4.4.1 *Case Study*

To exemplify our approach, we apply our methodology to an array of case studies based on three distribution network models. The first two are taken from our previous contribution [173], while the third one is based on the 69-node in [163], for which the resistances have been multiplied by a factor two. We replaced the Rural network from [173] because the three networks happened to exhibit only binding current constraints and were thus not useful in illustrating the voltage criticality criterion. For each of the three networks, we studied ten configurations in terms of photovoltaic and storage dimensioning. We have focused on extremely high levels of penetration with photovoltaic set-points representing up to 700% of the annual load,

Figure 46: Number of Critical Time Steps for the 30 Case Studies

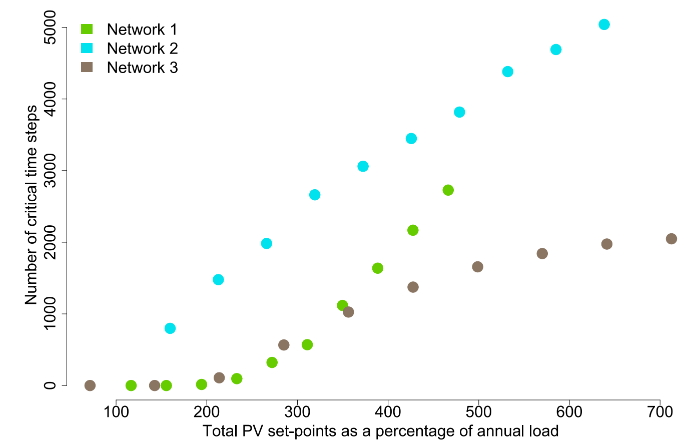
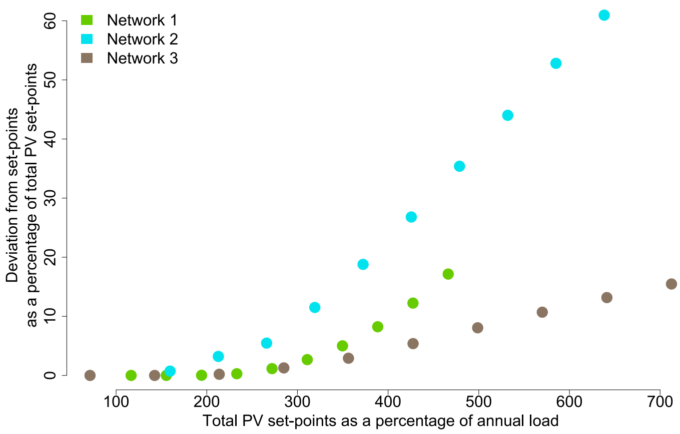
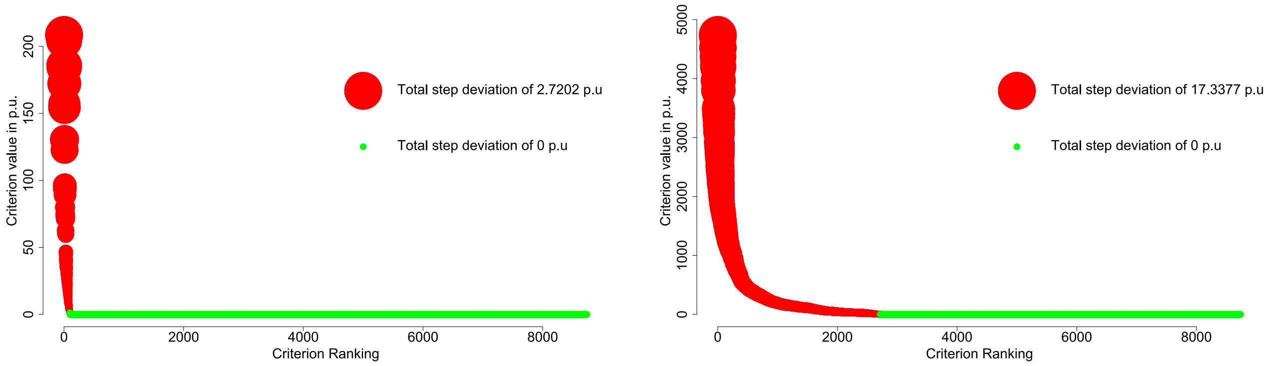


Figure 47: Total Deviation from Set-points for the 30 Case Studies



so as to try out our approach in the most unfavorable conditions and not because these could be representative of realistic situations. To illustrate the properties of the case studies in terms of criticality, Figure 46 displays the evolution of the number of critical time steps as a function of the level of penetration for the three networks, while Figure 47 displays the evolution of the total deviation from set-points. As we are working with finite precision, we modify slightly the definition of a critical time step to consider only those for which the total deviation is higher than 0.01 p.u.

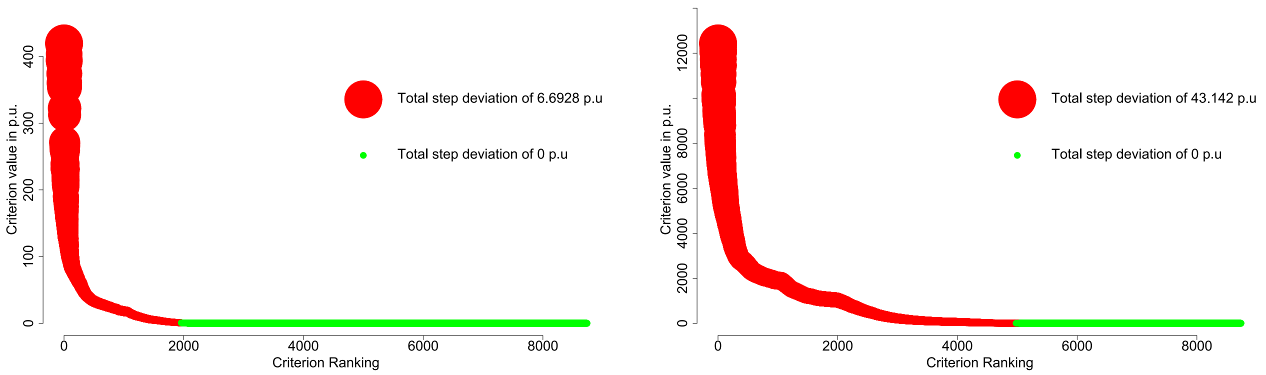
For each of the case study considered, we calculate both criticality criteria and decide which one to use according to the number of time step it is nonzero. As could be expected, the current-criticality criterion is used for the first two networks, while the voltage-criticality one is used for the last. Then we can rank the time step as a function of the chosen criterion, and Figure 48, 49 and 50 present the rankings obtained for six selected test cases, spanning the three networks. Crit-



(a) Case 4

(b) Case 10

Figure 48: Criticality Ranking for Two Cases of Network 1, with the Size of the Points Proportional to the Total Deviation



(a) Case 3

(b) Case 10

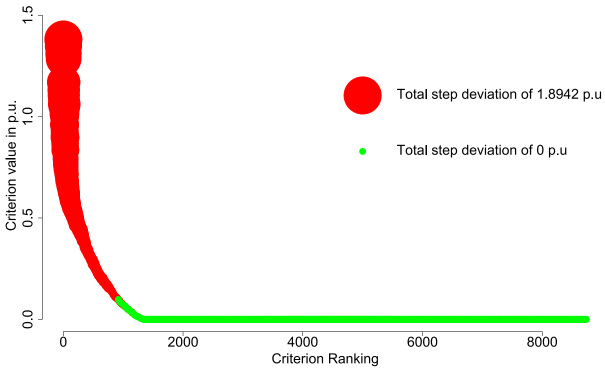
Figure 49: Criticality Ranking for Two Cases of Network 2, with the Size of the Points Proportional to the Total Deviation

ical and non-critical time steps are represented by, respectively, red and green dots, whose size is a linear function of the total deviation in the corresponding time step.

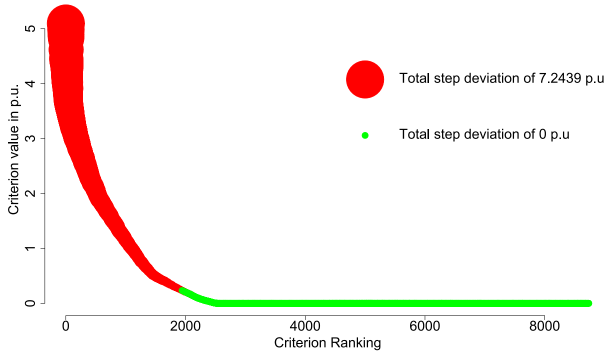
We can observe that the criticality criteria fulfill their role, as non-critical time steps are separated from critical ones in all the test cases considered.

4.4.2 *Compromise between accuracy and computational gains*

In the preceding section, we have chosen an arbitrary threshold between critical and non-critical time steps to illustrate the workings of the criteria. Now, we will consider this threshold to be variable and examine the trade-off between accuracy and computational gain



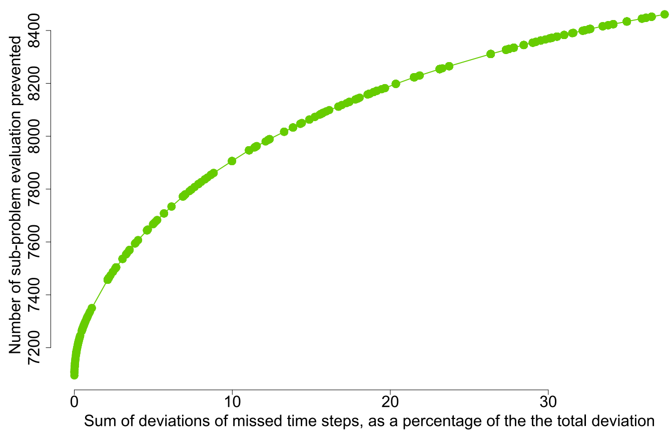
(a) Case 5



(b) Case 10

Figure 50: Criticality Ranking for Two Cases of Network 3, with the Size of the Points Proportional to the Total Deviation

Figure 51: Compromise Between Accuracy and Computational Gains for Case 10 of Network 1



that this entails. We select the three test cases corresponding to the maximal penetration of DG for each network. Then, for each value of the threshold considered, we compute the number of prevented sub-problem computation as well as the sum of deviations corresponding to these time steps. For each test case, the threshold is made to vary between 0.01% and 9% of the total installed DG power and the results are displayed in Figure 51, 52 and 53.

The calculation of the criticality criteria is roughly equivalent to solving the sub-problem for 30 time steps, and so the computational gains brought about by our approach are obvious, independently of the level of accuracy desired. Furthermore, for each of the test cases, the accuracy decreases with the increase in the number of prevented sub-problem computations, which is expected. Moreover, setting the threshold as a percentage of the installed DG power seems to allow

Figure 52: Compromise Between Accuracy and Computational Gains for Case 10 of Network 2

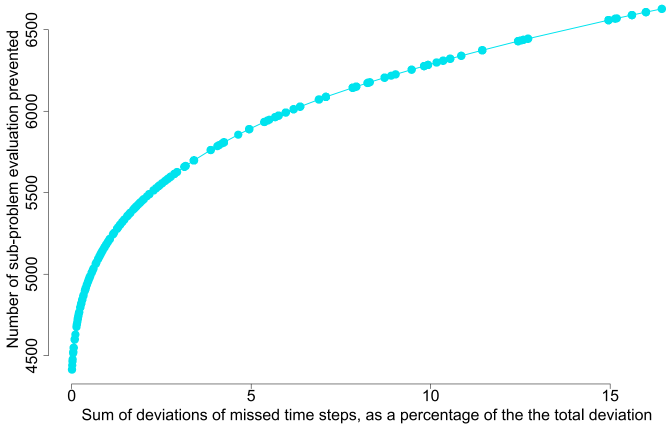
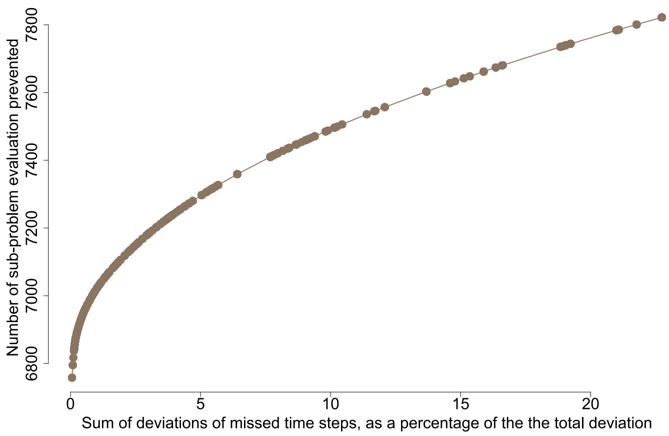


Figure 53: Compromise Between Accuracy and Computational Gains for Case 10 of Network 3



to span a similar range of trade-off independently of the test case, which hints that this a suitable way of setting the threshold.

4.5 CONCLUSION

We have presented a methodology aimed at significantly reducing the computational burden associated with the resolution of optimal power flows for distribution systems, in order to render it more practical for planning purposes. We have verified its validity for 30 case studies spanning a wide range of DG penetration and network characteristics. In addition, we have proposed a variable threshold that allows the user to choose between various compromises in terms of computational gains and accuracy. For the future, further testing of the methodology is envisioned, with a particular focus in trying to design cases in which both current and voltage constraints are binding either in the same time step or in different time step so as to explore the interplay between the two criteria. Moreover, as the model of CPS operation will be improved to take into account the influence of the sub-problem evaluation on network losses, this procedure will be improved to include an estimation of the variation in losses for unsolved time steps. Finally, it will be integrated, along with [173] and [136] to provide a fully functional framework to assess the impacts of DG on the power system operation for planning purposes.

5 CONCLUSION

5.1 ACHIEVEMENTS

In Chapter 1, we started by describing the evolution of the power system from the decentralized system of the early days to the current liberalized large-scale system. We then introduced and presented in detail the characteristics of a potentially disruptive technology: distributed generation. Subsequently we reviewed the existing approaches to integrate them in the power system, in particular from an operational standpoint. This prompted us to identify several areas that could be improved, setting the stage for our contributions.

In chapter 2, we have presented a model of the operation of distributed generation integrated in a centralized power system as the solution to a large-scale mixed-integer nonlinear optimization problem. Facing the intractability of such a problem, we have proposed a decomposition and coordination procedure that turns this problem into a series of economic dispatches in the centralized power system and optimal power flow in the distribution system. After introducing further hypothesis related to the case study we consider, we showed that both of these optimization problems can be solved with existing techniques. We then gave further details on the case study we aim at solving before presenting some of the results to illustrate our methodology. These results show that our algorithm is able to obtain a satisfactory solution in a fairly realistic case study.

In chapter 3, we have focused on finding a global solution to the single-stage distribution optimal power flow using a second-order cone programming approach. We started by reviewing the existing literature that focuses on proving the existence of conditions ensuring the exactness of the second-order cone relaxation of the distribution system optimal power flow, and continued by illustrating how these could be incompatible with the efficient operation of future distribution systems with high penetration of distributed resources. We have then introduced an algorithm consisting in iteratively adding linear cuts to this SOC relaxation, before proving its theoretical convergence to the optimal solution. We continued by illustrating its practical implementation on various case studies while proposing a termination criterion allowing the achievement of a solution satisfying the network constraints in a finite number of steps. In the worst situation (biggest network, highest number of nodes equipped, highest penetration and best precision), a satisfactory solution was obtained in around 2.5 seconds.

In chapter 4, we have presented a methodology aimed at significantly reducing the computational burden associated with the resolution of optimal power flows for distribution systems, in order to render it more practical for planning purposes. We have verified its validity for 30 case studies spanning a wide range of DG penetration and network characteristics. In addition, we have proposed a variable threshold that allows the user to choose between various compromises in terms of computational gains and accuracy.

5.2 PERSPECTIVES FUR FUTURE RESEARCH

While the objectives that we set out at the beginning of this contribution can be considered fulfilled, there are still many improvements that could be envisioned for the future. Without any pretense at exhaustiveness, here are the areas that could be investigated:

- Integrating the proposed simulation of DG operation, including single-stage OPF solution and the criticality criteria, in planning studies to evaluate the impacts of various planning decision such DG siting and sizing or line reinforcements
- Improving the quality of the DG operation model while keeping computational requirements reasonably low. A first step in this direction could be to take into account the transmission network using a DC approximation or to develop a methodology to model the influence of uncertainties in renewable production and load.
- Developing an adaptive stopping criterion considering the differing level of precision at which the various problems are solved
- Trying to extend our decomposition and coordination approach to the low-voltage distribution system, possibly by introducing another level of decomposition and coordination
- Proposing a tailored scaling method for resolution of the single-stage OPF so that accuracy can be improved
- Extend this methodology to three-phase unbalanced networks
- Extend the use of the criticality criteria to estimate losses in unresolved time steps
- Develop more comprehensive test cases, in particular concerning distribution network characteristics, to control the robustness of the approaches we put forward

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Intégration des énergies renouvelables dans le réseau de distribution d'électricité

RESUME : De nombreux pays ont mis en place des politiques plus ou moins ambitieuses de développement des énergies renouvelables électriques à court et moyen terme, tandis que, en parallèle, des scénarii de prospective à long terme envisagent un approvisionnement électrique majoritairement d'origine renouvelable. Certaines de ces énergies sont intermittentes et non contrôlables et, du fait des faibles économies d'échelle dont elles bénéficient, peuvent être développées de manière décentralisée, intégrées au bâtiment et raccordées au réseau de distribution, à l'opposé des moyens de production actuels.

Cependant, l'ensemble des procédures et méthodes de planification et d'opération actuel est basé sur l'hypothèse d'une production centralisée et d'un réseau de distribution passif. Dans ce contexte, cette thèse contribue au développement d'une méthode de simulation d'un réseau de distribution actif en proposant trois avancées: une méthode de décomposition-coordination permettant la simulation intégrée du système centralisé et du réseau de distribution, un algorithme de résolution du problème des flux optimaux dans le réseau de distribution et deux critères de criticité diminuant les besoins en capacité de calcul de la simulation si elle est employé dans un cadre de planification.

Mots clés : Energies renouvelables, stockage, optimisation

Integration of renewable energies in the electricity distribution system

ABSTRACT : Many countries have set up mechanism support in favor of renewable energy development at a short or long term, while, in parallel, long-term scenarios of predominantly renewable energy provisioning are studied. Some of these energies are intermittent and non-dispatchable and, due to the low economies of scale they exhibit, can be deployed in a decentralized manner, integrated in buildings and connected to the distribution system.

However, the methods and tools currently used to plan and operate the power system rely on hypothesis of centralized generation and passive distribution network. In this context, this thesis contributes to the development of a method aimed at simulated an active distribution network by proposing three novel approaches: a decomposition and coordination method allowing the integrated simulation of the centralized power system and the active distribution network, an algorithm to solve the single-stage optimal power flow in the distribution system as well as two criticality criteria allowing to decrease the computational burden of the simulation when it is used for planning purposes.

Keywords : Renewable energies, storage, optimization