Essais on Macroeconomics

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Essays on Macroeconomics

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I spent the last three years at Insee's department of economic analysis. Insee *de facto* hosted and financed the present research, but I also benefited from the proximity with the macroeconomic laboratory of the Crest. Before that, I spent three years in Insee's national accounts department and a year in the monetary policy research division of Banque de France. I believe this shows in the present dissertation, especially as some chapters exemplify the blurred frontier between pure academia and institutional research.

As any other candidate, I have been helped, advised, supported, guided, motivated, read and corrected, congratulated or contradicted and proved wrong, sometimes unwittingly, by a great number of friends, colleagues and co-authors. I hope they will recognize themselves in this short inventory and be assured of my gratitude.

The four chapters of this dissertation are available as working papers or published. Chapter 3 on the Euler equation is available in a light version as an Insee working paper. Chapter 4 on the Taylor rule in a monetary policy model with sticky wages has been revised and resubmitted to the B.E. Journal of Macroeconomics. Chapter 5 on the computation of Households Satellite Accounts has been published in the Review on Income and Wealth. Chapter 6 on structural reforms will soon be available as an Insee working paper together with two companion papers presenting our monetary union model and fiscal reforms analysis.

Other researches pursued over the last three years have not been included in this dissertation. In these papers I mainly deal with methodological questions related to macroeconomic measures (temporal disaggregation, projection of input-output tables or chain-linking). I believe macro-economists should not look down at such issues as good data is a prerequisite for any proper research. Without reliable data, economics would be confined to the construction of normative theorems without any ability to test their underlying hypothesis. I have also worked on the effect of technology and trade on employment and a model of matching between supply and multidimensional preferences. All these papers are available on my personal web page.

Contents

List of Tables

List of Figures

Chapitre

1 Résumé

Cette thèse aborde en quatre chapitres plusieurs thèmes distincts mais complémentaires de macroéconomie et plus spécifiquement d'équilibre général. L'accent est porté sur le comportement des ménages mais des questions de politique économique ou encore de méthodes de modélisation sont également traitées. Ces chapitres illustrent par ailleurs l'importance du lien entre théorie et données pour la science économique ainsi que la frontière entre recherche académique et institutionnelle.

Le chapitre 3 est consacré à l'adéquation entre les données et le modèle standard d'arbitrage entre consommation et épargne.

Le chapitre 4 généralise le principe de Taylor au cas d'un modèle de politique monétaire à salaires et prix rigides.

Le chapitre 5 valorise le travail domestique (tâches ménagères, soins à la personne...) des ménages en France en 1998 et 2010 et traite des problèmes méthodologiques associés à cette question.

Le chapitre 6 est consacré à l'évaluation des réformes structurelles dans les modèles d'équilibre général.

Mots clés : *consommation, équation d'Euler, politique monétaire, principe de Taylor, travail domestique, compte satellite des ménages, réformes structurelles, modèles DSGE*

1.1 Consommation, taux d'intérêt et anticipations : vers une réconciliation

Le chapitre 3 est consacré à l'adéquation entre les données et le modèle standard d'arbitrage entre consommation et épargne.

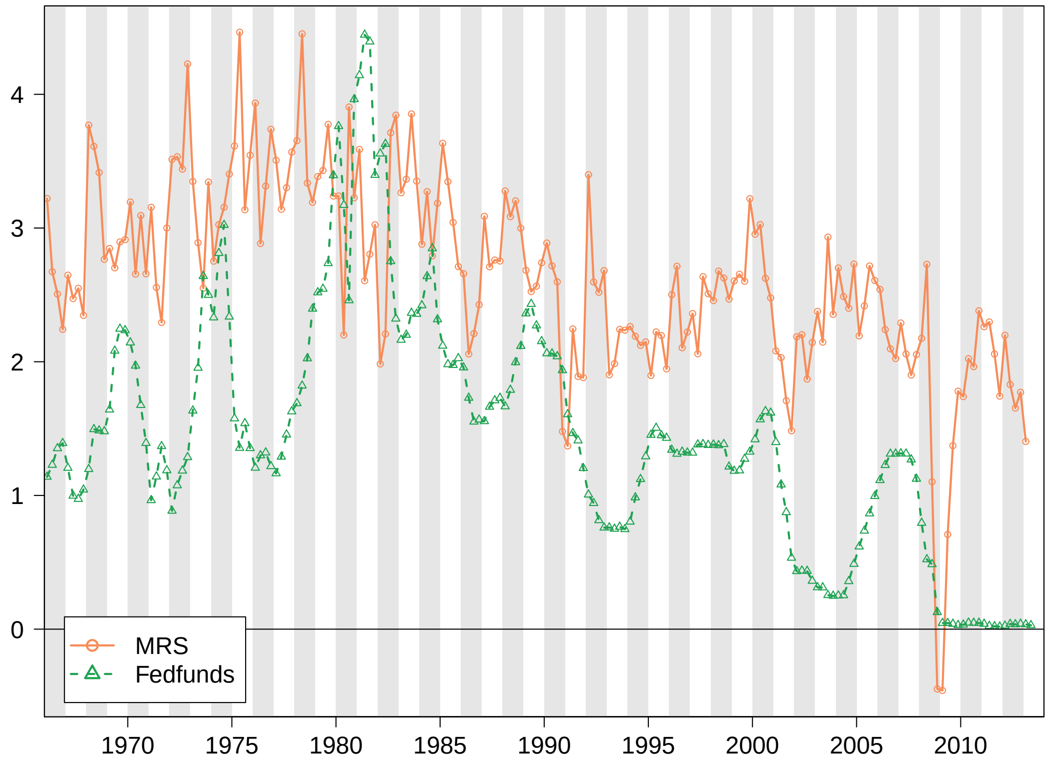
L'équation d'Euler décrivant l'arbitrage consommation-épargne d'un ménage représentatif constitue la pierre angulaire d'un grand nombre de modèles en macroéconomie, notamment de politique monétaire. La description de cet arbitrage au niveau agrégé comme individuel s'est construite sur la base de la théorie du revenu permanent et du cycle de vie. Bien que ces modèles aient connu d'importantes améliorations (formation d'habitude, épargne de précaution...), certains auteurs ont montré les limites de cette représentation, tant sur la base d'arguments théoriques (Carroll, 2000b), qu'empiriques (Canzoneri et al., 2007).

Carroll (2000b) dans son *requiem pour l'agent représentatif* interprète le rejet de l'équation d'Euler comme une incompatibilité entre des données agrégées et un modèle de comportement microéconomique, du fait d'une trop grande dispersion des utilités marginales des ménages. L'équation d'Euler serait fausse, non pas du fait du modèle, mais du fait de l'hypothèse trop forte de l'existence d'un ménage représentatif.

Pour autant, cette équation est un élément fondamental de nombre de modèles, notamment de politique monétaire et les théories du revenu permanent et du cycle de vie sont incontournables.

Attanasio (1999) référence les premiers articles consacrés à cette équation, notamment depuis (Hall, 1978). Ceux-ci font souvent l'hypothèse d'un taux d'intérêt constant et de l'équivalence certaine pour caractériser le lien entre le modèle et les données. La fonction d'utilité a été modifiée pour améliorer l'adéquation du modèle avec les données, en lien avec certains fait stylisés. Mankiw (1982) ou Bernanke (1985) étudient le cas de la consommation de biens durables. Abel (1990) et Gali (1994) s'intéressent à l'*equity premium puzzle* et introduisent des habitudes. Campbell and Mankiw (1990) introduisent une fraction de la population contrainte financièrement et consommant à chaque période son revenu tandis que le reste de la population arbitre entre consommation et épargne. Flavin (1981) s'intéresse à l'*excess sensitivity puzzle*, c'est-à-dire la réaction de la consommation à des variations passées du revenu. Campbell and Deaton (1989) à l'inverse, s'intéressent à l'*excess smoothness puzzle*, c'est-à-dire la trop faible réaction de la consommation à des variations du revenu permanent.

Méthodologiquement, la littérature sur l'équation d'Euler est divisée entre deux techniques : l'estimation par maximum de vraisemblance ou par méthode des moments généralisés, introduite par (Hansen and Singleton, 1982; Hansen, 1982). Dans ce chapitre, j'utilise la première méthode car je ne souhaite pas faire l'économie de l'estimation des anticipations qui entrent en jeu dans cette équation. Cette méthode permet en particulier



Graphique 1.1 – Taux d'intérêt et taux marginal de substitution (*MRS*)

d'étudier des anticipations qui ne soient pas parfaitement optimales et d'analyser la tension entre le modèle et l'hypothèse d'anticipations rationnelles. La méthode des moments généralisée ne permet pas cette entorse à la rationalité car, par construction, l'estimateur est construit sur une condition d'orthogonalité qui suppose identiques les anticipations du ménage et de l'économètre, conditionnellement à un jeu de données.

Ce travail s'inscrit dans le prolongement de (Canzoneri et al., 2007). Ces derniers ont montré pour les US que les données tendent à décrire les ménages comme réagissant à un taux d'intérêt non corrélé voire négativement corrélé au taux fixé par le banquier central. Ils qualifient ce résultat de *challenge* pour les modèles de politique monétaire dans la mesure où les données semblent incompatibles (sous hypothèse d'anticipations rationnelles) avec cette équation pourtant clé. Le Graphique 1.1 résume cette incompatibilité : le taux marginal de substitution intertemporelle de la consommation (MRS), contrairement aux prédictions du modèle, n'est pas du tout égal au taux d'intérêt de la politique monétaire (Fedfunds).

Ce rejet de l'équation d'Euler agrégée est un peu hâtif pour deux raisons.

Tout d'abord, les ménages ne réagissent pas forcément au taux d'intérêt de la politique monétaire, mais à des taux sur les marchés d'épargne et d'emprunt auxquels ils ont accès (prêts immobiliers, emprunts à la consommation, assurance vie, comptes de dépôt...). Dès lors, le différentiel entre ces taux et celui de la politique monétaire pourrait expliquer la corrélation faible ou négative mise en évidence.

Par ailleurs, dans l'idée d'approcher un modèle sous anticipations rationnelles, Canzoneri et al. mesurent les anticipations des ménages à l'aide d'un modèle VARX estimé *ex ante*, i.e. les ménagent utilisent de manière optimale l'information publiquement disponible pour prévoir leur arbitrage consommation-épargne. Cette hypothèse biaise le test de l'équation d'Euler en faveur de son rejet. Notamment, un petit écart à la prévision optimale pourrait améliorer sensiblement le lien entre le modèle et les données. On pourrait parler alors de *quasi rationalité*.

Pour introduire des taux d'intérêts spécifiques aux ménages dans cette équation, je considère un modèle à *participation limitée*. La population se divise entre ménages arbitrant sur l'un ou l'autre des produits d'épargne ou d'emprunt, mais à la différence de la littérature sur l'*asset pricing puzzle* (Hansen, 1982; Hansen and Singleton, 1982; Campbell and Cochrane, 1999; Ferson and Constantinides, 1991; Hall, 1988), chaque ménage n'arbitre pas sur chaque marché financier. Cette approche montre pour les États-Unis que les données sont plus compatibles avec un modèle où les ménages réagissent pour environ 20% au taux sur les emprunts d'état et 80% sur le taux des crédits à la consommation. Le $R^2_{ajusté}$ de l'équation d'Euler passe de 0 à 30% en considérant ces taux spécifiques aux ménages.

Ce résultat, de ménages réagissant à des taux d'intérêt qui leurs sont spécifiques, peut paraitre trivial tant il est vrai qu'ils ne peuvent emprunter ou déposer directement de l'argent à la banque centrale. Cependant, il indique que l'écart entre les taux commerciaux et celui du banquier central ne peut être considéré comme faisant partie du résidu du modèle. Les modèles de politique monétaire doivent donc intégrer les frictions financières auxquelles les ménages font face et, ce faisant, affaiblir leurs mécanismes de transmission de la politique monétaire. En terme de mise en œuvre de la politique monétaire, ce résultat tend également à promouvoir des mesures non conventionnelles de politique monétaire, directement ciblée sur les conditions de marché des ménages.

La comparaison avec la France est également intéressante : tandis que les ménages américains réagissent principalement au taux d'intérêt sur les emprunts à la consommation ; en France, ils réagissent au taux d'intérêt sur les *livrets*. Ces résultats reflètent des différences profondes entre ces deux économies puisqu'aux États-Unis le taux d'épargne est plus faible et l'usage des cartes de crédit généralisé, tandis que les français disposent à plus de 90% d'un *livret* d'épargne (*Livret A, jeune, LDD, populaire*).

Les anticipations optimales introduisent cependant une assez forte volatilité dans le taux marginal de substitution intertemporelle de la consommation, comparativement à la volatilité des taux d'intérêts.

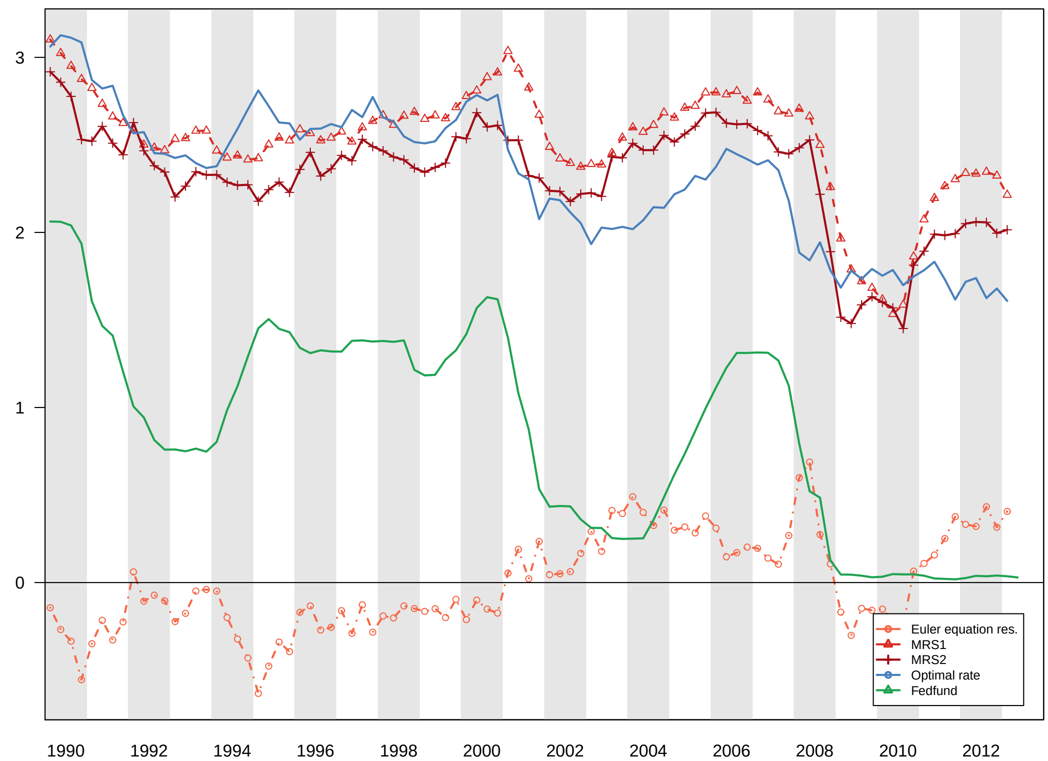
Pour permettre un écart à la rationalité des anticipations, j'estime le VARX des anticipations conjointement à l'équation d'Euler et non plus *ex ante*. Cet approche permet d'analyser les anticipations qui pourraient réconcilier l'équation d'Euler et les données. Le modèle joint tend à indiquer une rupture dans la formation des anticipations ou du comportement des ménages américains à la fin des années 80, c'est-à-dire à la fin de la grande modération. Les anticipations ainsi obtenues sont notamment plus lisses que les anticipations optimales ; un résultat qui peut être interprété dans le cadre de l'hypothèse des consommateurs inattentifs (Reis, 2006). Récolter et analyser l'information nécessaire à la formation d'anticipations est coûteux et il peut être optimal de ne pas réviser ces dernières à chaque période. En particulier, pour l'inflation qui est stable et modérée depuis les années 90, il peut s'avérer aussi efficace de former des prévisions naïves (Diron and Mojon, 2008), ou dans des termes plus proches de (Cochrane, 1989) la *quasi rationalité* serait suffisamment proche de la rationalité. Par ailleurs, mesurer les anticipations sur la base des enquêtes de conjoncture auprès des ménages permet également d'améliorer l'adéquation du modèle avec les données.

Sur la base des analyses précédentes, je teste un modèle où les ménages réagissent à une combinaison de taux d'intérêt sur les prêts à la consommation et sur les bons du trésor. Je suppose également que les ménages sont inattentifs (Reis, 2006). Je teste de surcroit les hypothèses de formation d'habitude (Abel, 1990; Gali, 1994), de ménages contraints (Campbell and Mankiw, 1990), d'épargne de précaution (Carroll et al., 1992; Kimball, 1990) et de préférences à la Uzawa ou d'esprits animaux. Seul le modèle d'épargne de précaution, où l'épargne est reliée au risque de chômage, est retenu par les données. La réponse des ménages sur leur perception des taux d'intérêt apporte également une information marginale pour l'adéquation du modèle aux données. Le modèle alors retenu amène à une conclusion plus positive (G 1.2). Le taux marginal de substitution (avec ou sans inattention vis-à-vis de la consommation, *MRS1 ou 2* respectivement) est très similaire au taux d'intérêt spécifique aux ménages (*optimal rate*) mais reste différent du taux d'intérêt du banquier central.

1.2 Le principe de Taylor est vérifié lorsque les salaires sont rigides

Le chapitre 4, co-écrit avec A. Blasselle, généralise le principe de Taylor au cas d'un modèle de politique monétaire à salaires et prix rigides.

Taylor (1993) propose de guider la fixation des taux directeurs à l'aide d'une règle de politique monétaire suivant laquelle le banquier central réagit à l'inflation et à l'écart de production. Dans le modèle néo keynésien simple, cette règle de Taylor est déterminante pour l'élimination des équilibres à tâches solaires : le principe de Taylor associé à cette



Graphique 1.2 – Taux d'intérêt et taux marginal de substitution (MRS)

règle stipule que le banquier central doit sur-réagir à l'inflation pour assurer l'existence et l'unicité de la solution sous l'hypothèse d'anticipations rationnelles.

Le modèle considéré dans le chapitre 4 intègre au modèle néo keynésien simple des salaires rigides (Erceg et al., 2000; Galí, 2008)

$$\pi_t^p = \beta E(\pi_{t+1}^p|t) + \kappa_p y_t + \lambda_p \omega_t \tag{1.1}$$
$$\pi_t^w = \beta E(\pi_{t+1}^w|t) + \kappa_w y_t - \lambda_w \omega_t \tag{1.2}$$
$$\omega_{t-1} = \omega_t - \pi_t^w + \pi_t^p + \Delta\omega_t^n \tag{1.3}$$
$$y_t = E(y_{t+1}|t) - \frac{1}{\sigma}(i_t - E(\pi_{t+1}^p|t) - r_t^n) \tag{1.4}$$
$$i_t = \Phi_p \pi_t^p + \Phi_w \pi_t^w + \Phi_y y_t + v_t \tag{1.5}$$

Les équations (1.1) et (1.2) sont les courbes de Phillips néo keynésiennes associées respectivement à l'évolution des prix et des salaires. L'équation (1.3) modélise de manière exogène l'écart entre les salaires et leur équivalent si ceux-ci étaient flexibles. L'équation (1.4) est l'équation d'Euler reliant l'évolution de la demande au taux d'intérêt réel. L'équation (1.5) est la règle de Taylor représentant la réaction du banquier central à l'inflation et la production.

Dans le cadre de ce modèle, Erceg et al. (2000); Galí (2008) étudient la politique monétaire optimale en terme de bien être, en partant du postulat que le principe de Taylor se généralise. Celui-ci est en effet vérifié numériquement dans le cas des salaires rigides. Cependant, une démonstration analytique de ce principe n'avait pas encore été donnée. Dans le cas limite de ce même modèle en temps discret, Flaschel et al. (2008) démontrent que la condition d'existence et d'unicité de la solution est une généralisation du principe de Taylor. Dans ce chapitre nous généralisons leur résultat au cas plus usuel du modèle en temps discret.

Résultat Nous montrons dans ce chapitre que si la règle de Taylor suivie par le banquier central vérifie la contrainte (1.6) alors il existe une seule et unique solution au modèle ; en d'autres termes le banquier central élimine les équilibres à tâches solaires.

$$\Phi_p + \Phi_w + \Phi_y \frac{(1-\beta)(\lambda_p + \lambda_w)}{\kappa_w \lambda_p + \kappa_p \lambda_w} > 1 \tag{1.6}$$

La contrainte de validité de la règle de Taylor (1.6) dépend symétriquement de l'inflation des prix et des salaires : si le banquier central ne réagit pas à la production, la contrainte sur les paramètre de politique monétaire devient $\Phi_p + \Phi_w > 1$ ce qui coïncide avec l'analyse numérique de (Galí, 2008). Toujours conformément aux résultats numérique de (Galí, 2008), lorsque le banquier central réagit à l'écart de production ($\Phi_y > 0$), cette réaction relâche la contrainte ci-dessus. Ce relâchement est proportionnel à $\frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w}$. Ce coefficient est une fonction symétrique des pentes des courbes de Phillips des prix et des salaires : des agents plus impatients (β plus petit) ou des courbes de Phillips plus plates (λ ou κ plus faibles), facilitent la tâche du banquier central dans l'élimination des équilibres à tâches solaires.

Dans un modèle où seuls les prix sont rigides, la frontière d'indétermination de la solution est l'équation (1.7) (Woodford, 2001). La frontière (1.6) en est une généralisation directe.

$$\Phi_p + \frac{1-\beta}{\kappa_p}\Phi_y > 1 \tag{1.7}$$

Flaschel et al. (2008) montrent également que le principe de Taylor est valide dans le cas du modèle en temps continu. L'existence et l'unicité de la solution sont vérifiées sous la condition suivante :

$$\Phi_p + \Phi_w + \Phi_y \frac{\rho(\lambda_p + \lambda_w)}{\kappa_w \lambda_p + \kappa_p \lambda_w} > 1 \tag{1.8}$$

où ρ est liée au taux d'escompte par la relation suivante $1/\beta(t) = 1 + \rho t$. Dans (1.8) ρ est la limite $(1 - \beta(t))/t$ lorsque le pas de temps t tend vers zero, tandis que celui-ci vaut 1 dans (1.6). Nous montrons donc que le résultat de Flaschel et al. peut être passé à la limite, conformément à leur intuition.

Une hausse permanente de l'inflation sur les prix ($\tilde{\pi}$) induit une hausse identique de l'inflation salariale (équation (1.3)). Les courbes de Phillips (équations (1.1) et (1.2)) induisent une hausse proportionnelle de l'écart de production $\tilde{y} = \frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w}\tilde{\pi}$. Le taux d'intérêt augmente de $\tilde{i} = \left[\Phi_p + \Phi_w + \Phi_y \frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w}\right]\tilde{\pi}$ du fait de la règle de Taylor (1.5). Par conséquent, à l'instar du modèle néo keynésien standard sans rigidités salariales (Woodford, 2011, chapitre 4), la frontière d'indétermination de la solution définit un principe de Taylor dans le cas de chocs permanents : le *banquier central doit réagir plus que proportionnellement aux chocs inflationnistes permanents.*

Enfin, en présence de rigidités nominales sur les salaires et les prix, les deux inflations jouent un rôle similaire dans la définition d'une politique monétaire optimale (Galí, 2008; Erceg et al., 2000). Nous montrons qu'elles jouent également un rôle symétrique dans l'élimination des équilibres à tâches solaires. Cette conclusion est également *en contradiction avec la pratique de la majorité des banques centrales, qui semblent accorder peu d'importance à l'inflation salariale en tant que cible* (Galí, 2008).

Méthode Suivant le résultat de (Blanchard and Kahn, 1980), la démonstration du principe de Taylor généralisé est basée sur l'étude des valeurs propres du système dynamique donné par le modèle. Celui-ci peut être simplifié en un système à quatre inconnues. Il s'agit alors d'étudier la position des racines d'un polynôme de degré quatre par rapport au disque unité : il y a existence et unicité de la solution si et seulement si trois des valeurs propres du système ont un module supérieur à un.

Dans un premier temps nous montrons que dans le cas $\Phi_y = 0$, si $\Phi_p + \Phi_w = 1$ le modèle est à la limite de l'indétermination. Nous montrons alors qu'une déviation par rapport à ce cas assure l'unicité de la solution si est seulement si elle est positive. Dans le cas $\Phi_y \neq 0$ la limite de l'existence et l'unicité de la solution est $\Phi_p + \Phi_w = 1 - \theta$ avec $\theta = \Phi_y \frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w}$. De la même façon, l'étude des valeurs propres montre qu'une déviation par rapport à ce cas assure l'unicité de la solution si est seulement si elle est positive.

1.3 Un compte satellite des ménages français. Problèmes méthodologiques pour l'évaluation de la production domestique

Le chapitre 5, co-écrit avec D. Roy, estime un équivalent monétaire du travail domestique (tâches ménagères, soins à la personne...) des ménages en France en 1998 et 2010 et traite des problèmes méthodologiques associés à cette question.

La comptabilité nationale considère un dîner au restaurant comme une dépense de consommation d'un service. Ce n'est en revanche pas le cas d'un dîner préparé à domicile, parce qu'il n'y a pas d'échange monétaire. Ce n'est pas non plus le cas pour de nombreuses activités domestiques comme le ménage ou les soins aux enfants qui participent pourtant du bien-être matériel des ménages. L'estimation d'un équivalent monétaire pour ces activités domestiques apporte alors un éclairage différent sur la consommation et le revenu disponible des ménages.

En 2010, les Français ont consacré 41 milliards d'heures à des activités comme la cuisine, le ménage et les soins apportés aux enfants ou à des personnes dépendantes contre seulement 38 milliards d'heures pour le travail rémunéré.[1] Cela représente chaque semaine 15h en moyenne par personne, soit 765 heures par an (Tableau 1.1).

Nous estimons la valeur de ces tâches domestiques en France en 1998 et 2010 avec la méthode *input* basée sur l'enquête emploi du temps. Nous décrivons ces activités productives actuellement non couvertes par le système européen des comptes (SEC) dans un compte satellite des ménages suivant les recommandations d'Eurostat (2003). Conformément aux précédentes études dans plusieurs pays développés (Tableau 1.2), nous trouvons que la redéfinition de la frontière de production pour y inclure les tâches domestiques (ménages, cuisine, soins à la personne...) a un effet notable sur les principaux indicateurs macroéconomiques (l'équivalent de +33 % du PIB, de +50 % du revenu disponible brut, de +58 % de la consommation et de -4 points de taux d'épargne) et leur croissance (-5 points de croissance du PIB, -10 points de croissance du pouvoir d'achat).

Nous montrons également que les activités domestiques, ayant lieu hors cadre marchand sont nettement plus importantes en termes monétaires que leurs équivalents marchands. La production de repas *faits maison* est 8 fois supérieure à ceux consommés au restaurant : 459 contre 59 milliards d'euros. La différence est encore plus marquée pour l'entretien du logement : si le recours à un prestataire pour l'entretien du logement et du jardin ne représente que 6 milliards d'euros, le *fait maison* dans ce domaine est supérieur à 253 milliards d'euros. Pour les soins à la personne, la production domestique (92 milliards d'euros) dépasse la consommation effective en action sociale (67 milliards d'euros, y compris dépenses des administrations publiques).

[1] La population *active à domicile* inclus les retraités, les chômeurs, les parents au foyer, les adolescents...

Périmètre	restreint (I)	intermédiaire (II)	extensif (III)
Activités inclues	cuisine, vaisselle, entretien du logement, ménage, soins à la personne, lessive, gestion du ménage, conduite pour un tiers	(I) et courses, réparations du logement, jardinage, jeux avec les enfants	(II) et conduite pour soi, promenade d'un chien
Quotidien	2 h 07	3 h 04	3 h 53
Hebdomadaire	14 h 50	21 h 30	27 h 14
Part de bénévolat	3.7 %	3.8 %	5 %
Part des femmes	72 %	64 %	60 %

Champs : personnes de 11 ans et plus, France (excl. Guyane et Mayotte).
Source : Insee, Enquête emploi du temps 2010.

Tableau 1.1 – Temps de travail domestique moyen par personne en 2010 pour trois définitions

Ce travail soulève plusieurs problèmes méthodologiques qui ne font actuellement pas l'objet d'un consensus international et dont nous quantifions l'impact. Le choix le plus important pour l'harmonisation des évaluations est la définition de la production domestique. Une définition extensive du travail domestique induit un doublement du temps de travail correspondant (Tableau 1.1). Nous proposons l'usage d'une définition relativement restreinte pour laquelle chaque activité productive considérée fait l'objet d'un consensus et pour lesquelles la valorisation est moins sujette à une surestimation de la productivité. Une autre question d'importance est l'utilisation d'un salaire net ou super brut : la différence de valorisation du temps accordé aux tâches domestiques est de l'ordre de 40 % entre ces deux conventions.

D'autres questions pourtant largement débattues ont un impact plus marginal : la différence entre le travail domestique valorisé sur la base d'un salaire spécialisé (cuisinier, chauffeur, maçon...) ou généraliste (homme à tout faire) n'est que de 1 %. Dans le cas de la France, l'utilisation du SMIC comme salaire de référence pour les tâches domestiques nous semble à proscrire car il interdit toute comparaison internationale.

Ces questions méthodologiques ne sont pas anodines du point de vue de l'interprétation économique. Un compte satellite nécessite de redéfinir la consommation finale dont nous reclassons 23 % en consommation intermédiaire et 2 % en investissement pour la définition du travail domestique retenue. La réalisation des travaux domestiques est en effet liée à l'achat de biens (liquide vaisselle, légumes, viandes...) et de services (électricité, eau...). Certains biens durables des ménages sont également utilisés comme capital productif (électroménager, voiture).

Au-delà de la définition du champ du travail domestique, la question de la valorisation de ce temps a un effet direct sur l'évaluation du revenu disponible, de la consommation et de l'épargne associés. Nous montrons que le taux d'épargne des ménages (16 % suivant le SEC), une fois corrigé pour tenir compte de la production domestique, peut être égal à 11 % ou 13 % selon la méthode utilisée, voire -2 % si on ne prête pas attention à la cohérence du cadre comptable.

			Révisions			Revenu et taux d'épargne		
Pays	Source	Année	PIB	Cons.	FBCF	Revenu	SEC$^{\triangle}$	non-SEC$^{\triangle}$
USA	a	1946	+50%	+63%	+50%	+59%	8.3%	10.8%
USA	b	1965	+39%	+49%	+50%	+49%	8.6%	11.5%
USA	a	1997	+36%	+34%	+54%	+38%	1.8%	8.5%
USA	b	2004	+27%	+26%	+48%	+32%	1.8%	4.2%
Finland	c	2001	+40%	+59%	+60%	+81%	-1.2%	0.2%
Finland	d	2006	+39%	+55%	+47%	+77%	-1.8%	-0.3%
Pays Basque	e	1993	+49%	+74%*	-	-	-	-
Pays Basque	e	1998	+39%	+64%*	-	-	-	-
Pays Basque	e	2003	+33%	+56%*	-	-	-	-
UK	f	2000	+63%*	+95%*	+98%*	+93%*	4.2%	-6.9%*
Finlande	g	2001	+36%	-	-	-	-	-
Allemagne	g	2001	+43%	-	-	-	-	-

* : calcul des auteurs, - : non disponible
$^{\triangle}$: non-SEC correspond à l'extension du SEC dans le cadre du compte satellite.
a : (Landefeld and McCulla, 2000), b :(Landefeld et al., 2009), c :(Varjonen and Aalto, 2006), d :(Hamunen et al., 2012), e :(Eustat, 2004), f :(Holloway et al., 2002), g : (Ruger and Varjonen, 2008)
FBCF : formation brute de capital fixe des ménages, Cons. : consommation finale des ménages, dont consommation individualisable sauf pour le pays Basque

Tableau 1.2 – Comptes satellite des ménages pour 5 pays : impact sur les principales variables macroéconomiques

1.4 Réformes structurelles dans les modèles DSGE

Le chapitre 6, co-écrit avec B. Campagne, est consacré à l'évaluation des réformes structurelles dans les modèles d'équilibre général.

L'évaluation des réformes structurelles est devenue un exercice standard à l'aide des modèles dynamiques et stochastiques d'équilibre général (DSGE). Le FMI, la Commission Européenne, l'OCDE, la BCE entre autres institutions ont mené leurs propres analyses sur la base de leurs modèles. D'Auria et al. (2009); Roeger et al. (2008); Varga et al. (2014) réalisent ces exercices pour les pays membres de l'Union Européenne à l'aide du modèle Quest III de la Commission, Annicchiarico et al. (2013) se concentrent sur le cas italien. Des études similaires ont été menées pour le FMI (Bayoumi et al., 2004; Everaert and Schule, 2006, 2008) et pour l'OCDE (Cacciatore et al., 2012). Ces analyses évaluent des gains de PIB potentiel de 5 à 10% suite à une hausse de la concurrence sur le marché des biens et du travail.[2] Dès lors, ces institutions recommandent la mise en œuvre de telles réformes pour raviver la croissance, notamment en Europe. Les mécanismes de ces gros modèles sont cependant complexes à démêler. Par ailleurs, malgré le consensus sur l'effet qualitatif des réformes, la quantification des résultats varie du simple au double entre les modèles.

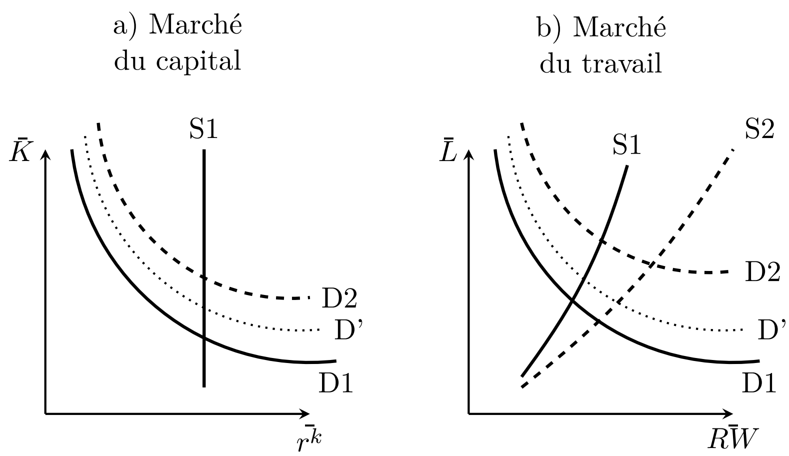
Ces modèles sont généralement construits sur la base du modèle de Smets et Wouters (Christiano et al., 2005; Smets and Wouters, 2002, 2003). Au sein de ce modèle, les entreprises et les ménages maximisent leur objectif intertemporel (profit ou utilité) en interagissant sur les marchés des biens, du travail et du capital. Des rigidités nominales sur les prix et les salaires (Erceg et al., 2000) donnent au modèle une dimension néo keynésienne. Enfin, la possibilité d'épargner via un actif sans risque induit un arbitrage intertemporel entre consommation et épargne. Il s'agit là d'un modèle de cycles économiques et sa pertinence pour l'analyse des réformes structurelles n'est pas évidente.

Dans ce cadre, nous analysons l'effet des réformes structurelles sur le marché des biens et des services, à long terme et dans la transition. À l'instar des études précédentes, nous montrons qu'une hausse de la concurrence sur le marché des biens ou du travail induit une hausse de la production à long terme mais un coût transitoire, résultat déjà démontré par Jonsson (2007) et Matheron and Maury (2004) notamment. Malgré les différences de choix de modélisation, les mécanismes associés à ces réformes restent comparables à ceux de (Blanchard and Giavazzi, 2003).

À la suite d'une libéralisation du marché des biens (Graphique 1.3), la perte de pouvoir monopolistique des entreprises se traduit par une baisse de leurs profits et une part plus forte de la valeur ajoutée distribuée aux facteurs de production (capital et travail). Afin de contrebalancer la hausse associée du coût marginal (baisse de la marge intensive), les entreprises augmentent leur production (hausse de la marge extensive, translation de la demande de facteurs de D1 vers D' puis D2 par effet de second tour). Toutefois, cette hausse de la production, et donc du revenu des ménages, se traduit simultanément par un effet de richesse qui vient réduire l'offre de travail des ménages (de S1 vers S2), et atténuer les gains de production.

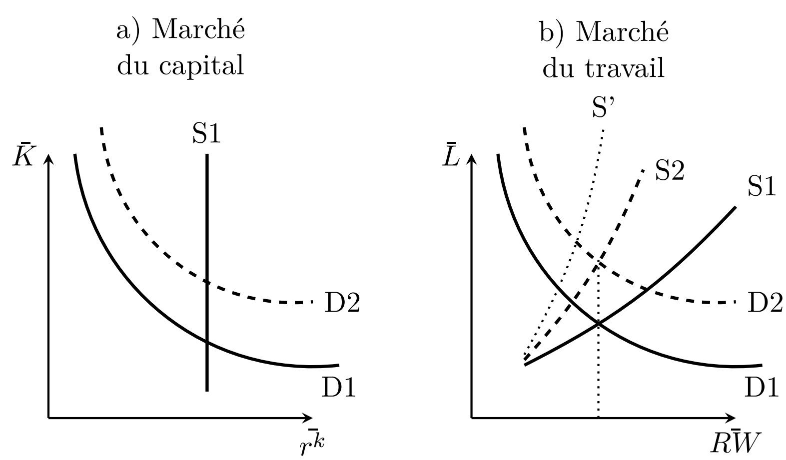
À la suite d'une libéralisation du marché du travail (Graphique 1.4), la perte de pouvoir de marché des travailleurs se traduit par une hausse de leur offre de travail (de S1 vers S'). Cette hausse de l'offre, et donc du niveau d'équilibre, implique alors une hausse de la productivité marginale du capital et du travail et donc de la demande en facteurs de production (D1 vers D2). Au total, les niveaux d'équilibre plus élevés des facteurs de production (travail et capital) induisent une hausse de l'activité et du revenu des ménages. Ceux-ci ajustent leur arbitrage consommation-loisir en réduisant partiellement leur offre

[2] Une hausse de la concurrence est modélisée par une hausse de la substituabilité entre biens ou travailleurs engendrant une baisse de l'ordre de 15 points des taux de marges sur ces marchés.



Graphique 1.3 – Effet d'un baisse du pouvoir de marché des entreprises sur le marché du capital et du travail

de travail (de S' vers S2).

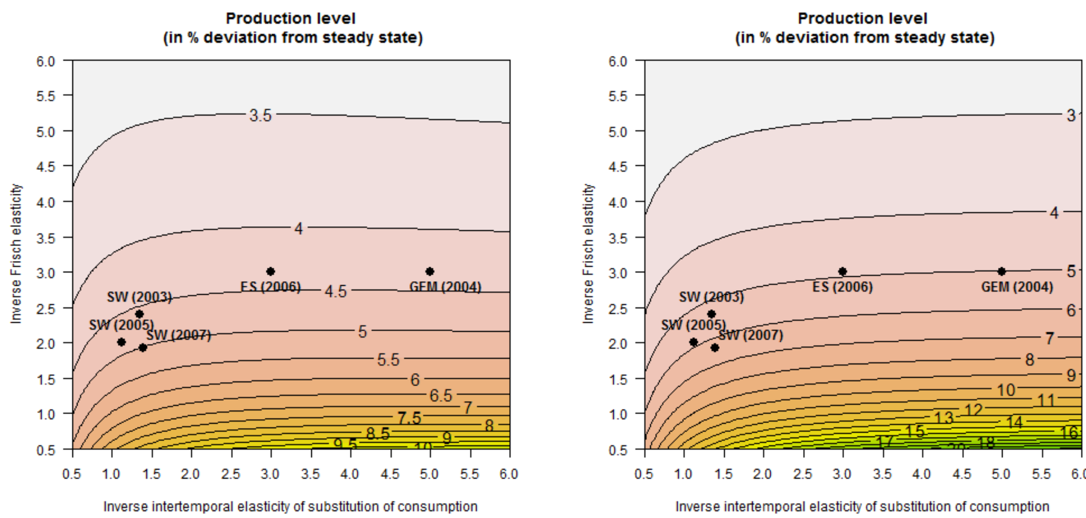


Graphique 1.4 – Effet d'un baisse du pouvoir de marché des travailleurs sur le marché du capital et du travail

On constate dès lors que l'arbitrage consommation-loisir est un élément clé de l'effet de ces deux types de réformes. Nous montrons que des variations pourtant usuelles de la calibration de l'utilité des ménages et en particulier de l'élasticité de Frisch induisent des modifications importantes de l'effet à long terme des réformes (écart de plusieurs points de pourcentage, Graphique 1.5).

L'introduction de formation d'habitudes dans l'utilité des ménages et le choix entre des habitudes *additives* ou *multiplicatives* induit également une grande variabilité des effets à long terme. Ces deux choix de modélisation sont pourtant indiscernables dans un modèle de cycles (Carroll, 2000a). Par ailleurs, bien que bénéfiques à la production, ces réformes peuvent être néfastes en terme de bien être pour des calibrations usuelles du modèle.

Enfin, l'introduction de ménages contraints financièrement (dits non-Ricardiens), usuelle dans les modèles institutionnels, permet aussi une analyse stylisée des inégalités entre ménages. Les réformes sur le marché des biens, en pénalisant les profits des entreprises et donc les dividendes versés aux ménages actionnaires (les ménages Ricardiens uniquement), conduiraient ainsi à une réduction des inégalités. Au contraire, les réformes sur le marché du travail seraient neutres, celles-ci impactant les deux types de ménages de manière identique.



(a) Réforme sur le marché des biens (b) Réforme sur le marché du travail

σ_c correspond à l'inverse de l'élasticité intertemporelle de substitution et σ_l à l'inverse de l'élasticité de Frisch. Les réformes structurelles simulées sont celles de (Everaert and Schule, 2006), c'est à dire un alignement du niveau de concurrence sur les 3 meilleurs *élèves* européens (Royaume-Unis, Suède et Danemark).
SW (YYYY) désigne la calibration du modèle par Smets et Wouters (YYYY) pour la zone euro. Notre calibration de base suit (Smets and Wouters, 2005). *GEM (2004)* correspond à (Bayoumi et al., 2004) et *ES (2006)* à (Everaert and Schule, 2006).

Graphique 1.5 – Effet de réformes structurelles en points de pourcentage en fonction de la calibration de la fonction d'utilité

Au final, nous tirons deux conclusions principales de nos résultats : tout d'abord, les recommandations de politiques économiques sur la base de ces modèles doivent faire un usage plus large des analyses de sensibilité des résultats ; plus fondamentalement deux aspects de ces modèles doivent être approfondis : les mécanismes sur le marché du travail à long terme et la différence entre les élasticités (Frisch et substitution) à long terme et au cours du cycle.

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Chapter

2 Introduction

This dissertation does not investigate in depth a specific question. In four chapters, at first sight disconnected, I touch many aspects of macroeconomics. This approach exemplifies the variety of themes, tools and sub-fields of this science but also the context in which I pursued these researches. Each chapter is an attempt at contributing to the economic literature and knowledge on households' aggregate behaviour, monetary policy, measurement of economic activity, economic modelling...

Main results

In Chapter 3, I provide directions to reconcile the standard macroeconomic model of households' consumption-savings behaviour with the data.

In Chapter 4, coauthored with A. Blasselle, I extend a normative result of monetary policy and describe the stabilizing effect of the Taylor rule (defining the central bankers interest rate decision) in a model with staggered wages as well as prices.

In Chapter 5, coauthored with D. Roy, I estimate the economic value of domestic work (house chores, personal care...) taking place outside the market and revise key macroeconomic indicators accordingly.

In Chapter 6, coauthored with B. Campagne, I consider the evaluation of structural reforms performed with standard macroeconomic models and show how sensitive their quantification is to the modelling of households.

Chapter 3 deals with the Euler equation relating consumption, interest rates and expectations to describe households' consumption-savings behaviour. Despite some strong cases built against it, the Euler equation relating consumption, interest rates and expectations remains a cornerstone of monetary policy models.
Under rational expectations, a baseline test of this equation based on the monetary policy rate has a very low fit to the data ($R^2_{adj} \sim 0$). Compared to this baseline, tests using interest rates specific to households' savings and borrowing markedly improve the model's fit ($R^2_{adj} \sim 30\%$). In addition, rational expectations in their most simple form feed too much volatility into the Euler equation. A I estimate a model where households are inattentive, where households react to a combination of the personal loans and the Treasury bill rate but also to their own perception of interest rate as reported in the Michigan survey, and where precautionary savings are linked to the risk of unemployment. This model replicates cyclical co-fluctuation and generates similar volatilities of the interest rate to which households react and their marginal rate of intertemporal substitution since 1990.

In Chapter 4, we consider the textbook neo-Keynesian model with staggered prices and wages in discrete time. In line with results established in the case of staggered prices only, we prove analytically that the Taylor principle holds in this case. When both contracts exhibit sluggish adjustment to market conditions, the policy maker faces a trade-off between stabilizing three welfare relevant variables: output, price inflation and wage inflation. We consider a monetary policy rule designed accordingly: the Central Banker can react to both inflations and the output gap. In addition to generalizing the Taylor principle we show that the frontier of determinacy embeds the frontier derived with staggered prices only, generalizes the frontier of determinacy in the limit case of continuous time and is symmetric in price and wage inflations.

In Chapter 5, we estimate in a household satellite account (HHSA) the value of French domestic production in 2010 and 1998, using the input method and following Eurostat's recommendations. In line with previous studies, we find that extending the system of national accounts (SNA) frontier of production to domestic activities (house chores, cooking, care...) has a sizeable effect on key macroeconomic indicators (+33% GDP, -5 p.p. GDP growth, +50% disposable income, +58% consumption, and -10 p.p. of purchasing power growth). We conduct a sensitivity analysis to various methodological issues which have not yet been settled by an international benchmark. Quantitatively, the two most important issues are the boundary of household production -we favour a relatively narrow definition- and the use of a gross or a net wage -we prefer gross wage-. However, estimates are much less sensitive to otherwise greatly debated issues such as which substitute wage to use.

In Chapter 6, we use a standard DSGE model of the euro area to shed a new light on a popular exercise in these times of crisis: structural reforms evaluation. We provide with a detailed analysis of the underlying mechanisms and proceed to a sensitivity analysis. The simple redefinition of households' utility can lead to additional gains or losses of a few percentage points in output following goods or labour markets deregulations. In addition, welfare analyses show that policy recommendations for structural reforms are less clear-cut than those solely based on output gains. Introducing *non Ricardian* agents allows stylized yet informative inequality analyses showing that goods market reforms reduce inequalities while labour market reforms are neutral. All in all, our results advocate for the extensive use of sensitivity analyses for policy purposes. More fundamentally, they advocate for a rethink of the mechanisms at play in the long run (and in particular on the labour market) and of differences in elasticities (Frisch and intertemporal substitution of consumption) between the long run and over the business cycle.

Relation to the literature

Each chapter is rooted in a specific literature.

Macroeconomic models for consumption and savings A seminal paper for the literature on households consumption and savings as described by the Euler equation is Hall (1978) testing whether US consumption data follow a random walk as the *permanent income-life cycle theory* predicts. Since then, macroeconomic data have been confronted to this theory repeatedly leading to successive refutations, validations and improvements in households' consumption models. Early papers, surveyed by Attanasio (1999), have investigated the properties of consumption within this framework under the assumption of constant real interest rate and often certainty equivalence (Hall (1978) in particular). Many alternative utility functions have been used to reconcile consumption data with the permanent-income hypothesis and understand some major stylized facts. Mankiw (1982), and Bernanke (1985) study the specificities of durables consumption. Abel (1990), and Gali (1994) investigate the equity premium puzzle with habit formation. Campbell and Mankiw (1990) consider a population with only a fraction of households following the model, while others, financially constrained, consume their current income in each period. Flavin (1981) investigates the *excess sensitivity puzzle*, that is consumption reacting to lagged changes in income. Campbell and Deaton (1989) investigate the *excess smoothness* *puzzle*, that is consumption not responding one-to-one to shocks to permanent income. (Carroll et al., 1992; Kimball, 1990) investigate the effect of uncertainty on households behaviour and the fact that for a given permanent income an increasing volatility of revenues is a motive for precautionary savings. (Chetty, 2004; Chetty and Szeidl, 2007) expand (Grossman and Laroque, 1990) analysis of illiquid consumption in durables. They isolate committed expenditure in households' consumption. Such models deeply modify households behaviour, modifying households risk aversion and generating smoother aggregate consumption decisions. (Reis, 2006) considers a cost for households in forming expectations. In this model it can be optimal for households to be *inattentive* and only reset their choices periodically. The behaviour of households is deeply modified and this model can explain the *excess sensitivity* and *excess smoothness* puzzles.

From a methodological point of view, an important contribution to the literature on consumption and savings is the GMM approach pioneered by (Hansen and Singleton, 1982; Hansen, 1982). However, because I test expectations departing from optimality, I use a maximum likelihood approach as in (Hall, 1988).

Relaxing the constant interest rate hypothesis Canzoneri et al. (2007) test the first order condition of households maximization program under different utility specifications against US data. Their main result is that when the Euler equation is assumed to hold and the interest rate is treated as the unknown variable, the counter-factual interest rate to which the representative household seems to respond is quite different from the monetary policy rate. In most cases it is negatively correlated with it, a result they pose as a *challenge for monetary policy models*, which are largely built on the Euler equation. Carroll (2000) in his *requiem for the representative consumer* interprets the rejection of the Euler equation as an incompatibility between macro data and a microfounded model, due to the differences in the marginal behaviour of individuals. The Euler equation on consumption could be wrong, not because of the form of the utility function but because the representative agent assumption is heroic.

In Chapter 3 of this dissertation, I build on (Canzoneri et al., 2007) and show how relaxing the strong assumptions made on interest rates and expectations in this literature may reconcile theory and the data.

Monetary policy rule In (Taylor, 1993), John Taylor advocates the use of monetary policy rules where the Central Banker reacts to both price inflation and output as a benchmark to be used judgementally. His design of a Wicksellian rule has been extensively studied since then in the context of neo-Keynesian models.

Authors have first considered models with staggered prices only and have investigated both questions of welfare optimization and uniqueness of the solution (also refered to as solution determinacy or ruling out sun spot fluctuations). These questions are independent of one another: optimal rules do not necessarily avoid sun-spot fluctuations (Clarida et al., 1999) but in both respects, it has been shown that the Taylor rule has appealing properties. The properties of the Taylor rule go beyond these two results as it is also key, for instance, in a model with adaptive learning (Bullard and Mitra, 2002).

In the simplest neo-Keynesian model with staggered prices only, the Taylor rule can be proved optimal in terms of welfare under some assumptions (Rotemberg and Woodford, 1999). It is also key in ruling out sun-spot fluctuations (Woodford, 1987, 2001) provided that the Taylor rule verifies the following principle: the Taylor principle states that the Central Banker should overreact to inflation to ensure the uniqueness of the solution under rational expectations.

Extensions to models with both staggered prices and wages have emphasized the welfare optimization problem while assuming, backed on numerical simulations, that the Taylor principle still holds in these cases (Erceg et al., 2000; Galí, 2008). Flaschel et al. (2008) prove that the Taylor principle holds in the same model but on the limit case of continuous time. Although the authors expect their demonstration in continuous time to be informative of the discrete time case, they acknowledge in their concluding remarks that their intuition is *not unchallenged*.

In Chapter 4 we prove that the Taylor principle holds in the textbook model for monetary policy with both staggered prices and wages in discrete time. A result which complements (Erceg et al., 2000; Galí, 2008) welfare analysis and directly relates to solution determinacy result in continuous time (Flaschel et al., 2008) and with staggered prices only (Woodford, 2001).

Beyond GDP A sizeable fraction of economic activity is not recorded by usual macroeconomic data. This is in particular the case for domestic work due to the definition of the frontier of production in the national accounts. This question of the frontier of production is part of a larger debate on the purpose of national accounts between a measure of economic activity and welfare (Vanoli, 2002), revived recently by the Sen-Stiglitz-Fitoussi report on the measurement of economic progress (Stiglitz et al., 2009).

In the literature on the valuation of domestic work, one may find references dating back to the 19th century (Charlotte Perkins Gilman (Women and Economics 1898)), or to the 1930s and 1940s such as Margaret Reid (Economics of Household Production, 1934), Wassily Leontief (The Structure of the American Economy, 1941), cited in (Ironmonger, 2000) or S. Kuznets, L. Epstein and W. I. King, H. Kirk, W. C. Mitchell cited by Chadeau and Fouquet (1981), Alfred Marshall (Principles of economics: An introductory volume, 1920), Arthur Pigou (The Economics of Welfare, 1932) in (Abraham and Mackie, 2006) or Lindahl *et al* (1937), Wesley C Mitchell *et al* (1921), Kuznets (1941) in (Vanoli, 2002).

Hawrylyshyn (1976) reviews some of the early quantitative studies on domestic work, from the second half of the 20th century. They mostly deal with the US but pioneer quantifications were performed in Nordic countries as well (Denmark, Norway, Sweden). Over the last 30 years, many authors have investigated this issue, mainly through Time Use

Surveys and the valuation of hours worked for domestic production. We found references to 27 national or regional economies[1] where at least hours of domestic work have been converted to monetary equivalents. Chadeau (1992) reviews such work in 7 countries; Goldschmidt-Clermont and Pagnossin-Aligisakis (1995) and Goldschmidt-Clermont and Pagnossin-Aligiasakis (1999) do so in 14 countries.

Over the last 15 years or so, the focus has shifted from the valuation of productive time to the construction of household satellite accounts (HHSA) as suggested by the SNA (IMF et al., 1993). In addition to the long debated questions already raised by hours worked and their valuation, the articulation of HHSA with other national accounts yields specific issues expounded in (Eurostat, 2003), (Varjonen and Aalto, 2006) for the European input approach, in (Abraham and Mackie, 2006), (Nordhaus, 2006) for the US input approach and (Holloway et al., 2002) for the UK's output approach. Using this HHSA framework, Landefeld and McCulla (2000), Landefeld et al. (2009), Varjonen and Aalto (2006), Hamunen et al. (2012), Eustat (2004), Holloway et al. (2002) and Ruger and Varjonen (2008) have estimated domestic production for the US, Finland, the Basque Country, the UK and Germany, respectively and revised key macroeconomic indicators accordingly.

In Chapter 5 of this dissertation, we perform a similar exercise for France in 1998 and 2010 and evaluate the quantitative impact of several methodological choices for which a consensus has yet to emerge.

Structural reforms The model used in Chapter 6 is based on (Christiano et al., 2005) and (Smets and Wouters, 2003, 2005, 2007). Within this model, firms and consumers maximize their objective (utility or profit) by interacting on the goods, labour and capital markets with both prices and wages rigidities introducing neo-Keynesian features in the model *à la* (Erceg et al., 2000). The model also integrates risk free assets to ensure an intertemporal trade-off and real rigidities on the capital market. This model is the core of many large scale institutional models. Despite different economic mechanisms, this model provides results in line with stylized facts obtained in deregulation-oriented models such as (Blanchard and Giavazzi, 2003).

The effects of structural reforms have been documented by D'Auria et al. (2009); Roeger et al. (2008); Varga et al. (2014) for EU member states and for Italy by Annicchiarico et al. (2013) both in the R&D version of the Quest III model. The IMF, the OECD or central banks have also conducted their own evaluations (Bayoumi et al., 2004; Everaert and Schule, 2006, 2008; Cacciatore et al., 2012; Forni et al., 2010). Their simulations concur to output gains of 5 to 10% following an average 15 points decrease in mark-ups. In a monetary union, (Everaert and Schule, 2006; Gomes et al., 2013) estimate positive spillovers from these reforms of approximately 1% output. In addition, these

[1] Australia, Austria, Basque Country, Bulgaria, Canada, Finland, France, Germany, Hungary, Italy, Israel, Japan, Luxembourg, Madrid, Mexico, Nepal, Netherlands, New-Zealand, Norway, Portugal, Russia, Slovenia, South-Africa, Sweden, Switzerland, United-Kingdom, United-States

simulations also point to welfare gains. However, these long term gains are partially offset by transitory losses (Jonsson, 2007; Matheron and Maury, 2004).

We conduct a sensitivity analysis with respect to the modelling of households. As we consider long term growth, we follow (King et al., 2002) recommendations and consider a non separable utility function in consumption and leisure, more precisely we borrow from (Trabandt and Uhlig, 2011) a specification with constant Frisch elasticity. We also consider the introduction of *non Ricardian* households *à la* (Campbell and Mankiw, 1990) as these are usually included in large scale institutional models.

Connections between chapters

Despite the eclecticism of these four chapters, many connections are possible between them.

Chapter 3 and 4, on the Euler equation and the Taylor rule respectively, both address a monetary policy question. The first one from an empirical point of view, stressing the need for coherence between theory and the data. The second one from a purely theoretical point of view, builds a normative result on the conduct of monetary policy.

Chapter 4 and 6 both rely on a specific class of models, namely DSGE models which have been developed in the last 20 years in reaction to the Lucas critique. Chapter 3 also relates to this literature as it investigates the ability to describe households consumption-savings behaviour in such models through the Euler equation.

Chapter 3, 5 and 6, contrary to chapter 4 on the Taylor rule, are closely linked to the data, they respectively confront the data with theory, built data to describe an otherwise unrecorded activity or use theory to quantify the effect of a reform. These three chapters also share a common focus on households behaviour, whether it is to better describe it (Chapter 3 and 5) or to point out its major relevance in quantifying the impact of structural reforms (Chapter 6).

Also, if chapter 4 on the Taylor rule does not treat the most current concern on the conduct of monetary policy.[2] Other chapters address economic questions which have recently gone beyond academic circles. In chapter 3, the use of rational expectations to describe economic behaviour and in particular the way household form their expectations is questioned. This feature of macroeconomic models made the headlines of economic newspapers after the burst of the latest crisis as a reason for the economists' failure to anticipate it (the arguments for this claim where however not convincing). In this chapter, the confrontation with the data suggest simpler expectation mechanisms could better describe households. Chapter 5 addresses another mediatic question: measuring

[2]Since the subprime crisis, research on monetary policy has been more focused non conventional measures and the zero lower bound than on the relevance of sticky wages for its conduct in normal times.

economic activity beyond GDP. This topic, put forth by Stiglitz et al., has been greatly debated outside academia. Chapter 5 follows one of the report's recommendations by incorporating domestic production into standard measures of economic activity. Chapter 6 investigates the evaluation of structural reforms called for by institutions such as the IMF, the European Commission, the ECB or the OECD as solutions to the economic crisis in Europe. This chapter borrows these institutional tools and analyses the mechanisms at play in such reforms. This allows for a robustness test of their recommendations (a test they do not pass).

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Chapter

3

Consumption, interest rates and expectations: a reconciliation

Despite some strong cases built against it, the Euler equation relating consumption, interest rates and expectations remains a cornerstone of monetary policy models.
Under rational expectations, a baseline test of this equation based on the monetary policy rate has a very low fit to the data ($R^2_{adj} \sim 0$). Compared to this baseline, tests using interest rates specific to households' savings and borrowing markedly improve the model's fit ($R^2_{adj} \sim 30\%$). A model of limited participation to financial markets, where households are inattentive, and where precautionary savings are linked to the risk of unemployment performs even better. It replicates cyclical co-fluctuation and generates similar levels and volatilities of the interest rate to which households react and their marginal rate of intertemporal substitution since 1990.

JEL-code : *E21, D91*

Keywords: *Euler equation, consumption, rational expectations, inattentiveness*

Introduction

The Euler equation on consumption is a cornerstone of monetary policy models. In the permanent income hypothesis, when households are assumed to be rational, their consumption scheme should verify the following equation:

$$1 = \beta E_t \left(\frac{U_c(t+1)}{U_c(t)} \frac{1+i_t}{\Pi_{t+1}} \right) \tag{3.1}$$

where $U_c(t)$ is the marginal utility of consumption at time t, i is the nominal interest rate at which households can lend and borrow and Π the inflation rate of consumption prices. E_t is the expectation operator describing how households form their expectations over future variables at time t.

Relaxing the constant interest rate hypothesis Canzoneri et al. (2007) test this first order condition of the representative household maximization program under different utility specifications against US data. Their main result is that when the Euler equation is assumed to hold and the interest rate is treated as the unknown variable, the counter-factual interest rate to which the representative household seems to respond is quite different from the monetary policy rate. In most cases it is negatively correlated with it, a result they pose as a *challenge for monetary policy models*, which are largely built on the Euler equation.[1]

This rejection of the Euler equation is too restrictive in two respects.

First, if households were reacting to another interest rate than those of the money market (mortgage, deposit, private loans...), the spread between this rate and the monetary policy instrument could account for the negative or weak correlation they observe. Such a result would have strong implications for monetary policy analysis and the way it is modelled (in the new neoclassical synthesis for instance). It would rehabilitate the Euler equation but dampen the transmission mechanism between the policy maker and households.

Second, Canzoneri et al. measure households' expectations through a VARX model estimated *ex ante*, i.e. households use optimally all the information publicly available to adjust their consumption-savings trade-off. This assumption can be relaxed in many ways to try not to reject the Euler equation. I inquire one possibility. Contrary to estimating the expectation VARX *ex ante*, which puts an infinite weight on the minimisation of expectation errors relatively to the Euler equation residual, I estimate the Euler equation and expectations altogether, putting the same weight on both types of residuals.

[1] In (Poissonnier, 2015b) I replicate their results. I show that they also hold on French data. While they tested their robustness to the definition of the utility function considered, I show that the result is also robust to the use of several consumption bundles.

I build on Canzoneri et al.'s analysis and test household specific interest rates. I find that households' behaviour is better described by interest rates other than the monetary policy one (specifically a combination of the interest rate on personal loans and the Treasury bill); indicating that households encounter sizeable frictions on the credit and loans market which shall not be relegated to a residual.

However, there remain sizeable discrepancies between the Euler equation and the data. In particular, rational expectations as approximated by a VARX model feed too much volatility into the Euler equation and between 1972 and 2013 there has been some structural changes either in households expectation formation process or their behaviour.

Building on these *intuitions*, I test a model of limited participation to financial markets (i.e. where households react to a combination of the personal loans and the Treasury bill rate but also to their own perception of interest rate as reported in the Michigan survey), where households are inattentive (Reis, 2006), in particular as regards inflation, and where precautionary savings are linked to the risk of unemployment. This set of modelling assumptions yield a close link between the *MRS* and the interest rate to which households are subject ($R^2_{adj} = 35\%$), replicates cyclical co-fluctuation and generates similar levels and volatilities of the two rates; a result in contrast with Canzoneri et al.'s rejection.

Since Hall (1978), macroeconomic data have been confronted to the permanent income-life cycle theory repeatedly leading to successive refutations, validations and improvements in households' consumption models. Early papers, surveyed by Attanasio (1999), have investigated the properties of consumption within this framework under the assumption of constant real interest rate and often certainty equivalence. Many alternative utility functions have been used to reconcile consumption data with the permanent-income hypothesis and understand some major stylized facts. Mankiw (1982), and Bernanke (1985) study the specificities of durables consumption. Abel (1990), and Gali (1994) investigate the equity premium puzzle by modifying the utility function. Campbell and Mankiw (1990) consider a population with only a fraction of households following the model, while others, financially constrained, consume their current income in each period. Flavin (1981) investigates the *excess sensitivity puzzle*, that is consumption reacting to lagged changes in income. Campbell and Deaton (1989) investigate the *excess smoothness puzzle*, that is consumption not responding one-to-one to shocks to permanent income.

The literature on the Euler equation is split between the maximum likelihood estimations and the GMM approach pioneered by (Hansen and Singleton, 1982; Hansen, 1982). Because in this paper I test expectations departing from optimality, I use maximum likelihood estimation. Indeed, the GMM approach relies on the assumption that households expectations are identical to the econometricians, so that conditional on the proper set of instruments an orthogonality condition is built.

Part of the literature based on GMM estimations investigate the asset pricing puzzle. If households arbitrate between consumption and savings on several markets, the Euler equation should be verified for the return on these assets at the same time. Although I consider several households specific interest rate, I work on a weaker assumption of limited participation. While any combination of returns should verify the Euler equation under the CAPM hypothesis, I investigate whether a particular combination of returns does, that is whether the population can be split between fractions arbitrating on one or another market.

Carroll (2000) in his *requiem for the representative consumer model* interprets the rejection of the Euler equation as an incompatibility between macro data and a micro-founded model due to the differences in the marginal behaviour of individuals. The Euler equation on consumption could be wrong, not because of the form of the utility function but because the representative agent assumption is heroic. Using macroeconomic data to capture the value of structural parameters is not the purpose of this paper: Attanasio and Weber (1993) show that estimates of the intertemporal elasticity of substitution on aggregate data are systematically lower than estimates on average cohort data. Attanasio and Weber (1993, 1995) show that tests on aggregate data tend to reject micro-founded models specification too often. However, microeconomics comes against its own difficulties (Carroll, 2001; Ludvigson and Paxson, 2001) while macroeconomic models remain widespread and have crucial implications for the policy maker, both monetary and fiscal. Thus, the *challenge* posed by the aggregate Euler equation on consumption is worth investigating.

The remainder of this paper is organized as follows: in section 3.1 I expose the framework of the present analysis; in section 3.2 I test the Euler equation against interest rates specific to households borrowing and saving; in section 3.3 I focus on expectations; in section 3.4 I combine results from the previous sections an propose a model which convincingly reconciles the Euler equation and rational expectations.

3.1 A framework for testing the Euler equation under limited participation and near rationality

The Euler equation relates households' expectation for consumption growth and inflation to the interest rate at which they can borrow or save. I first consider the standard CES utility function. Under a log-normality hypothesis of forecasting errors, or a first order approximation on these error, equation (3.1) can be linearised (see Appendix 3.B).

$$i_t = \delta + \sigma E_t \Delta c_{t+1} + E_t \pi_{t+1} \tag{3.2}$$

with σ the relative risk aversion parameter or $1/\sigma$ the intertemporal elasticity of substitution parameter, $E_t \Delta c_{t+1}$ consumption growth expectation (Δlog) and $E_t \pi_{t+1}$ inflation expectations.[2]

Such an equation is commonly introduced in monetary policy models but is not assumed to holds exactly. A residual to this equation is often interpreted as a preference shock, a demand shock from households part introduced through a time varying discount factor. There are however more reasons for the Euler equation not to hold on aggregate. First, even if each individual behaves in accordance to this equation, on aggregate it should not be exactly verified because households wealth vary and so does their marginal utility of consumption (Carroll, 2000). Second, under its linear form, the constant captures the risk associated to the forecast error. It is often assumed to be constant but changes in the volatility of the economic environment can imply through this channel a precautionary savings behaviour (**?**). This volatility can account for a significant share of fluctuations (Bloom, 2009). Third, equation (3.2) is based on a separable utility function, but more general utility functions imply non separability in with labour (Mankiw et al., 1985) or in time through durable consumption (Mankiw, 1982; Bernanke, 1985; Ferson and Constantinides, 1991; Grossman and Laroque, 1990) or through habit formation (Abel, 1990; Gali, 1994; Constantinides, 1990; Campbell and Cochrane, 1999; Ravn et al., 2006).

The latter has received a lot of attention because it provides a solution to the equity premium/risk free rate puzzles (Abel, 1990; Gali, 1994; Constantinides, 1990; Abel, 1999; Campbell and Cochrane, 1999). In the context of monetary policy models, habit formation also implies a conveniently gradual response of private consumption to monetary policy shocks (Fuhrer, 2000). Habit formation can also explain why savings may be high in a growing economy (Carroll et al., 2000).
Assuming external habit formation, equation (3.2) becomes:

$$i_t = \delta + \sigma E_t \Delta c_{t+1} + \alpha \Delta c + E_t \pi_{t+1} \tag{3.3}$$

with $\alpha = -\gamma(1-\sigma)$ and $0 < \gamma < 1$ the habit formation parameter.

[2] Inflation is defined in accordance with consumption as the growth rate (Δlog) of the considered consumption bundle deflator.

The Euler equation assumes that households can arbitrate between consumption today and tomorrow. Another *popular* modelling assumption for households has been proposed by Campbell and Mankiw (1990): a fraction of the population is financially constrained and consumes its income within the period. Introducing this limited participation to financial market in an heterogeneous agent model modifies in particular the aggregate response to fiscal shocks (Mankiw, 2000; Galí et al., 2007). Being financially constrained, these hand-to-mouth consumers are considered as the poor, however Kaplan et al. (2014) show that such a behaviour can also describe rich households whose financial wealth is not liquid.
Assuming that hand-to-mouth households earn a constant share of total income μ, equation (3.2) becomes

$$i_t = \delta + \sigma \left(E_t \Delta c_{t+1} - \mu \frac{SR_t - SR_{t-1}}{(1 - SR_t)(1 - SR_{t-1} - \mu)} \right) + E_t \pi_{t+1} \tag{3.4}$$

with SR the aggregate savings ratio of households.

In this paper I consider a less restrictive form of *limited participation*. The population may be split between households engaged in different financial market (mortgage, savings, consumption loans...) so that on aggregate, the interest rate to which the representative consumer reacts is a combination of household specific interest rates. Although I consider returns on various financial instruments, this paper departs from the CCAPM literature (Campbell, 2003): all households do not arbitrate on all financial instrument returns, i.e. the Euler equation does not hold simultaneously for all interest rates but for aggregate consumption and an aggregate asset. Is this respect I test a more restrictive model than Hansen (1982); Hansen and Singleton (1982); Campbell and Cochrane (1999); Ferson and Constantinides (1991); Hall (1988) who consider simultaneously several rates of returns and assume households arbitrate between them.
Under the assumption that each households arbitrates between consumption and savings on one interest rate, equation (3.2) becomes

$$\bar{i}_t = (1 - \sum_{rates \neq fed} \gamma^{rate}) i_t^{fed} + \sum_{rates \neq fed} \gamma^{rate} i_t^{rate} = \delta + \sigma E_t \Delta c_{t+1} + E_t \pi_{t+1} \tag{3.5}$$

with $\bar{i}$ an interest rate combination, with weights assumed constant. As it is written, the interest rate combination imposes that the weights sum to one. In addition, one may impose, as these weights should reflect consumption shares, that these weights are constrained between 0 and 1.

From an econometric point of view, it is noteworthy that while the residual of the Euler equation due to the aggregation of individual consumption growth and more generally the demand shock is naturally correlated to consumption growth, the aggregation approximation for interest rates or missing components in this combination (taxation, management fees, differences in liquidity) imply a residual which is correlated to the interest rates. Either interpretation of the residual imply opposite estimations in terms of explained variable ($\bar{i}_t - E_t\pi_{t+1}$ or $E_t\Delta c_{t+1}$). The approach usually favoured regresses consumption growth on the Fedfunds and as such considers the residual as orthogonal to this interest rate. Nevertheless as this residual should in particular capture the spread between the Fedfunds and the interest rate to which households are subject, this orthogonality assumption is quite strong. For this reason, I favour the approach regressing an optimal interest rate combination on expected consumption.

More problematic for the estimation is the expectation operator. Indeed, expectations are not directly measured which calls for elaborate estimation strategies. Hansen (1982); Hansen and Singleton (1982) proposed a GMM estimator built on the orthogonality of the Euler equation with the information set of the representative household. Despite its statistical properties this estimator has a major drawback for the present exercise: with a GMM estimation, I could neither extract the expectations nor allow them to depart from optimality. For this reason, I use a maximum likelihood estimation and as in (Fuhrer, 2000; Canzoneri et al., 2007), I assume that households are forming their expectations of consumption growth and inflation through a VARX model:

$$Y_{t+1} = \Lambda(L)Y_t + \Gamma X_t + \epsilon_{t+1} \tag{3.6}$$

with $Y = [\Delta c, \pi]$[3] $\epsilon = [\epsilon^c, \epsilon^\pi]$ is the error of the VARX model and is by assumption identified to the expectation errors of consumption and inflation ($E_t Y_{t+1} = \Lambda(L)Y_t + \Gamma X_t$).

Plugging expectations in the Euler equation is a compatibility check of two key elements of monetary policy models, namely the Euler equation and rational expectations. In addition to the CES function considered here, Canzoneri et al. (2007) have tested the Euler equation implied by other forms of utility functions in a similar approach, but conclude that these functions also invalidate the Euler equation on aggregate data. In (Poissonnier, 2015b) I confirm their results with other specifications of the VAR, with other definitions of the consumption bundle and on French data. To further analyse the compatibility of the Euler equation (3.5) (possibly with habit formation and hand-to-mouth consumers) and the VARX model (3.6) I thus focus on the household specific interest rate combination $\bar{i}$ in the full model:

$$\bar{i}_t = \delta + \sigma E_t\Delta c_{t+1} + E_t\pi_{t+1} + \begin{cases} \alpha\Delta c_t \\ -\sigma\mu\frac{SR_t - SR_{t-1}}{(1-SR_t)(1-SR_{t-1}-\mu)} \end{cases} + \zeta_t \tag{3.7}$$

$$\text{with } \bar{i}_t = (1 - \sum_{r\neq fed}\gamma^r)i_t^{fed} + \sum_{r\neq fed}\gamma^r i_t^r$$

$$Y_{t+1} = \Lambda(L)Y_t + \Gamma X_t + \epsilon_{t+1} \tag{3.8}$$

$$\text{and by assumption} \quad E_t Y_{t+1} = \Lambda(L)Y_t + \Gamma X_t$$

[3]The VARX model includes two lags and exogenous regressors (X, see also Appendix 3.A.2).

Considering the three residuals (ζ_t, ϵ^c_{t+1}, ϵ^π_{t+1}), the system (3.7) (3.8) can be estimated by maximum likelihood. Let $[\zeta_t \epsilon^c_{t+1} \epsilon^\pi_{t+1}]$ follow a normal distribution with zero mean and Ω variance, conditional on the model, we have

$$\begin{bmatrix} \bar{i}_t - [\sigma\, 1](\Lambda(L)Y_t + \Gamma X_t) - \ldots - \delta \\ Y_{t+1} - \Lambda(L)Y_t - \Gamma X_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \Omega \right) \tag{3.9}$$

In section 3.2 I focus on the interest rate combination $\bar{i}$ and consider expectations (3.8) estimated *ex ante*. However, even if I assume the shocks to be independent, the fact that the Euler equation includes expectations derived from the other equations should prevent me from treating the shocks sequentially. Estimating expectations *ex ante* is then equivalent to putting an infinite weight on the minimisation of ϵ relative to the minimization of ζ and bias the test of the Euler equation towards its rejection. Indeed, small deviations from the estimation *ex ante* of the VARX model, within its confidence interval, may improve the fit to the Euler equation. To investigate this possibility of *near rationality*, I estimate the system (3.9) altogether in section 3.3. Near rationality is here the other side of the coin considered by Cochrane (1989): I do not consider near rationality as small deviations from the optimal arbitrage, i.e. the Euler equation, but small deviations from the optimal forecast of consumption and inflation required to make such an arbitrage.

3.2 The Euler equation with households-specific interest rates

In this section, I consider households-specific interest rates in testing the Euler equation. The interest rates I consider are depicted on Figure 3.1. The correlation of real monetary policy rate and the spreads on households-specific rates are reported in Table 3.1. It is noteworthy that households-specific spreads are negatively correlated to real monetary policy rate, which could *a priori* account for the results exposed by Canzoneri et al. (2007) and also motivate this analysis where this spread is not assumed orthogonal to the monetary policy stance.

US interest rates (Figure 3.1) I use the Effective Federal Funds Rate (Fedfunds), a mortgage rate *(30-Year conventional mortgage rate)*, a car loans rate *(Finance Rate on Consumer Instalment Loans at Commercial Banks, New Autos 48 Month Loan)*, a personal loans rate *(Finance Rate on Personal Loans at Commercial Banks, 24 Month Loan)*, a deposit rate *(3-Month Certificate of Deposit: Secondary Market Rate)* and the 3 month treasury bill *(3-Month Treasury Bill: Secondary Market Rate)*. I add to this list the null return on cash holdings.[4] Apart from the mortgage rate, maturities of these interest rates are short. The mortgage rate is included because wealth effects are empirically stronger

[4] This rate has been tested but discarded as the results are not interpretable in terms of Euler equation.

real FedFunds	1	-	-	-	-	-
spread Mortgage	-0.3	1	-	-	-	-
spread Car Loans	-0.43	0.91	1	-	-	-
spread Pers. Loans	-0.46	0.84	0.88	1	-	-
spread Cert. Deposit	-0.2	0.38	0.35	0.38	1	-
spread Treasury Bill	-0.46	0.73	0.76	0.84	0.52	1

Table 3.1 – Correlation of the interest rates (in real terms)

in the US than in France, a stylized fact sometimes attributed to the higher flexibility of mortgages in the US where real estate can be used as a collateral for consumption. Some of these rates are closely related (treasury bill, fedfunds and certificates of deposits). Appart from the rate on the Certificate of Deposit which is superimposed to the Fedfunds, I keep all these rates to remain as exhaustive as possible.[5]

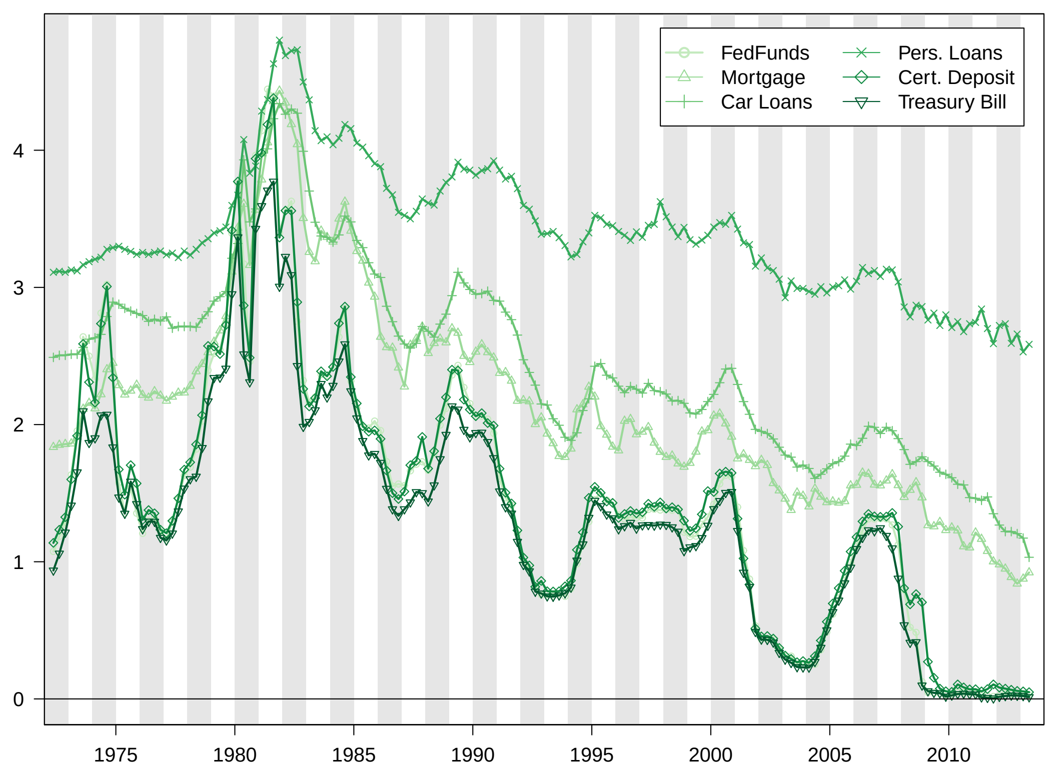


Figure 3.1 – Actual interest rates in nominal terms

Model estimation I estimate equation (3.7) by maximum likelihood over the period 1972Q2 to 2013Q1 and compute both the nominal marginal rate of substitution between

[5] Data are further detailed in appendix 3.A.1

present and future consumption (hereafter *MRS*) denoted $\tilde{i}_t$

$$\tilde{i}_t = \delta + \sigma E_t \Delta c_{t+1} + E_t \pi_{t+1} + \delta + \begin{cases} \alpha \Delta c_t \\ -\sigma \mu \frac{SR_t - SR_{t-1}}{(1-SR_t)(1-SR_{t-1}-\mu)} \end{cases} \tag{3.10}$$

and the optimal interest rate combination denoted $\bar{i}_t$

$$\bar{i}_t = (1 - \sum_{r \neq fed} \gamma^r) i_t^{fed} + \sum_{r \neq fed} \gamma^r i_t^r \tag{3.11}$$

The estimates for the different cases (CES utility function, with habit, with constrained households *à la* (Campbell and Mankiw, 1990) (CM), and both habit and *CM* households) are reported in Table 3.2. The model's constant allows to compute the discount factor.[6] It is equal to 98%. The estimated inverse elasticity of substitution or CRRA parameter is below one as in (Hansen and Singleton, 1982) and in constrast with (Hall, 1988). Habit formation is found either to be at its upper bound (1), which contradicts the model's assumption, or 0.36 when estimated together with a constrained households *à la* Campbell and Mankiw hypothesis. However the share of these households is very small (4% or null) compared to the original paper's result (between 40 and 50%), making a small case in favour of either assumption. As for the optimal interest rate combination, the weights are constrained between 0 and 1. I find a combination of 80% personal loans and 20% Treasury bill throughout the specifications with in particular a null weight on the Fedfunds.

As I estimate the Euler equation under the non standard assumption of the residuals being correlated to the Fedfunds, for comparison, I run the same estimations under the reverse assumption, as in (Hall, 1988). First, these estimations reject the habit formation or constrained households assumptions, the habit parameter is found to be null and the share of constrained households equal to 1%. The CRRA parameter is estimated to be large (around 10). Such a value is sometimes encountered in the equity premium puzzle literature as it increases the *MRS* closer to the average return of risky assets but at the same time it raises another issue as it increases the volatility of the *MRS* and thus of the implied risk free rate (Abel, 1990). Here, it can be seen as the result of an estimation bias as this strategy minimizes the variance of ζ_t/σ so that increasing the value of σ indeed improves the fit of the regression but may well deteriorate the fit of the Euler equation. This may explain (Hall, 1988) findings of a non significant intertemporal elasticity of substitution (i.e. a high risk aversion σ). In line with this high point estimate, the discount factor is found larger than one: with long term growth, this does not jeopardize the model's stability (see (Kocherlakota, 1990) in the RBC case and (Poissonnier, 2015a) in the neo Keynesian case). As for the interest rate, this estimation yields a combination of 20% personal loans rate and 80% car loans rate but again a null weight on the Fedfunds.

[6] The constant verifies $-\delta = ln(\beta) + \frac{\sigma^2}{2} var(\epsilon^c) + \sigma cov(\epsilon^c, \epsilon^\pi) + \frac{1}{2} var(\epsilon^\pi))$, see Appendix 3.B

	CES	+ habit	+ CM	+both
delta	0.02	0.02	0.02	0.02
beta	0.98	0.98	0.98	0.98
sigma	0.27	0.69	0.27	0.52
gamma	0	1	0	0.36
mu	0	0	0.04	0
Treasury Bill	0.22	0.17	0.22	0.2
Mortgage	0	0	0	0
Pers. Loans	0.78	0.79	0.78	0.77
FedFunds	0	0	0	0
Car Loans	0	0.04	0	0.03

Table 3.2 – Euler equation models (3.7) (baseline CES utility function with or without external habit and constraint households *à la* Campbell and Mankiw)

Comparisons of the *MRS* with the optimal rate combination are depicted in Figure 3.2.[7] Table 3.3 displays some comparative statistics for actual and Euler interest rates in real terms.

The estimated *MRS* is not correlated with the Fedfunds ($<11\%$). The correlation with the optimal rate combination is larger (between 20% and 30%) whether deflation is based on expected future inflation (correlations 2) on contemporaneous inflation (correlations 1) and regardless of the model considered. In real terms, the *MRS* is less volatile than the corresponding optimal rate combinations (Table 3.3) but in nominal terms the former exhibits small but rapid fluctuations, while the optimal rate combination and actual rates are much smoother (Figure 3.2).

Finally, the optimal rate combination markedly differs from the monetary policy rate although it replicates similar fluctuations. Considering a weighted average of the rate on personal loans and the treasury bill explains a large share of the spread between the *MRS* and the monetary policy rate but there remains a sizeable discrepancy in the short term volatility which is largely due to the volatility of inflation expectations.

As a partial conclusion: overall fit to the data To measure the fit to the data I compute both the log-likelihood of the full model (3.9) (even though it is estimated in two steps) and the adjusted R^2 of the Euler equation (3.7) (Table 3.4).[8]

Using households' specific rates other than the monetary policy rate improves the overall fit to the data (Euler equation & expectations VAR): the log-likelihood is markedly improved (+30). As for the Euler equation, its adjusted R^2 jumps from -1% to 30% (see

[7] I represent only the *MRS* (resp. optimal rate combination) for *CM* households and habit, but the three other *MRS* (resp. optimal rate combinations) are almost superimposed (correlations are above 99%).

[8] As I estimate the model under non linear and non differentiable constraints (parameters constrained between 0 and 1), it is not possible to perform Fischer or likelihood ratio tests.

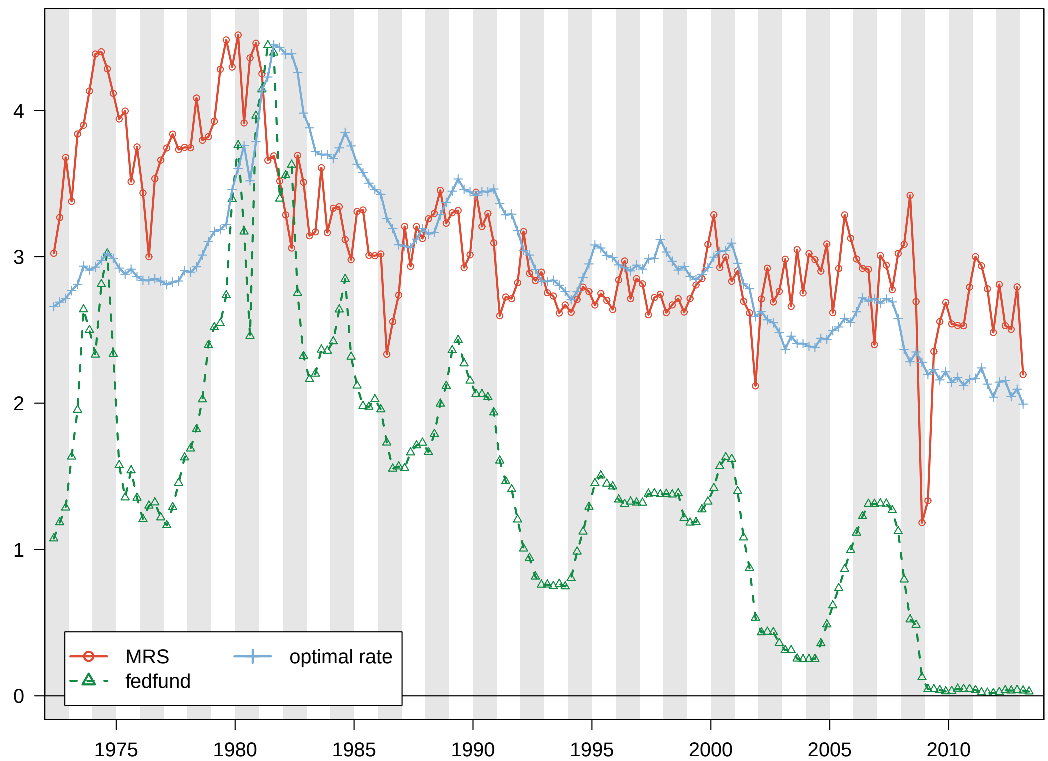


Figure 3.2 – Actual interest rates and *MRS* with a CES utility function

	Fedfunds	optimal combination	MRS	MRS habit	MRS CM	MRS both
Mean	0.46	1.96	2.08	2.15	2.08	2.11
Std deviation	0.71	0.59	0.07	0.09	0.07	0.08
Minimum	-0.89	0.44	1.88	1.67	1.88	1.76
Maximum	2.83	3.38	2.23	2.35	2.24	2.27
Correlation1	1	0.72	0.11	-0.01	0.08	0.04
Correlation1	0.72	1	0.29	0.22	0.26	0.28
Correlation2	1	0.67	0.01	0.02	0.01	0.02
Correlation2	0.67	1	0.21	0.3	0.21	0.29

I test two transformations of actual nominal rates in real terms: correlation 1 corresponds to the rate deflated by contemporaneous inflation $i_t - \pi_t$ and correlation 2 uses expected future inflation $i_t - E_t\pi_{t+1}$.

Table 3.3 – Statistics for the actual and *MRS* (in real terms)

Table 3.4).

	1 rate Fedfunds	5 rates	5 rates habit	5 rates CM	5 rates both
log-lik.	1947.81	1977.37	1979.09	1977.38	1978.79
R2adj (Euler)	-1	32	33	31	32

log-lik. is the log likelihood of the sample of the 3 residuals: from the Euler equation and the consumption and inflation forecasts measuring expectations. R^2adj is the adjusted R^2 for the Euler equation only.

Table 3.4 – Comparison of the fit to the data of the different models (Euler equation+expectation VAR)

3.3 Expectations in the Euler equation

In section 3.1, I exposed how estimating the VARX of expectations *ex ante* biases the test of the Euler equation towards its rejection while sticking to the rational expectations hypothesis. To investigate this bias, I estimate jointly the system (3.9). Doing so I now allow expectations to depart from the rational expectation assumption (i.e. VARX model) so as to improve the fit of the Euler equation.[9] Because of this bias, the outcome of the estimation in terms of expectations must be looked into before concluding to a validation of the Euler equation. Building on section 3.2, I only consider two interest rates, the one on Treasury bills and personal loans.

Estimating the model and expectations jointly I find a zero weight on the Treasury bill, a discount factor equal to 99% and a much larger CRRA parameter (3.3). As for habit formation or constrained households, the corresponding parameters are estimated to be null.

How close to optimality are expectations? As for the expectation VARX parameters, I can test whether they are significantly different from the estimates in section 3.2 (see Table 3.5). Consumption expectations are significantly different from the unconstrained estimate (rational expectations). However, inflation expectations are not.

	Forecast (c)	Forecast (π)
Stat	7.23	0.37
5%	2.07	2.07
p-value	0	0.92

Table 3.5 – Fisher's test on the equality of forecasting equations estimated separately or jointly with the Euler equation

[9] I put the same weight on the Euler equation residual and the expectation errors, but these could be arbitrarily modified. A full bias in favour of the Euler equation would put an infinite weight on the Euler equation relative to the expectation residuals but such a strategy would not yield interpretable expectations for inflation and consumption separately.

Graphically, expectations of inflation are very close to their unconstrained counterparts (Figure 3.4b). For consumption (Figure 3.4a), results show quite the opposite, these expectations estimated under the constraint of the Euler equation only loosely replicate their unconstrained counterpart. There is in particular a strong misalignment of expectations during the recent crisis: expectations boom while actual data plummet. As this crisis entailed an increase in uncertainty faced by households, we may be limited by the assumption of constant second moments and the Euler equation with precautionary savings (Carroll et al., 1992; Kimball, 1990) may better capture this episode.

The economic interpretation of these expectations is that households can optimally forecast inflation but fail to forecast their own consumption: a model I do not find satisfactory enough to conclude with a validation of the Euler equation. Instead, this result may point to a misspecification of the model: as pointed out by (Mankiw, 1982), consumption may better be modelled as durable products with different durabilities than through a dichotomy durables/non durables; a model of durable consumption would imply a flow of consumption services smoothed compared to actual consumption flows similar to Figure 3.4a.

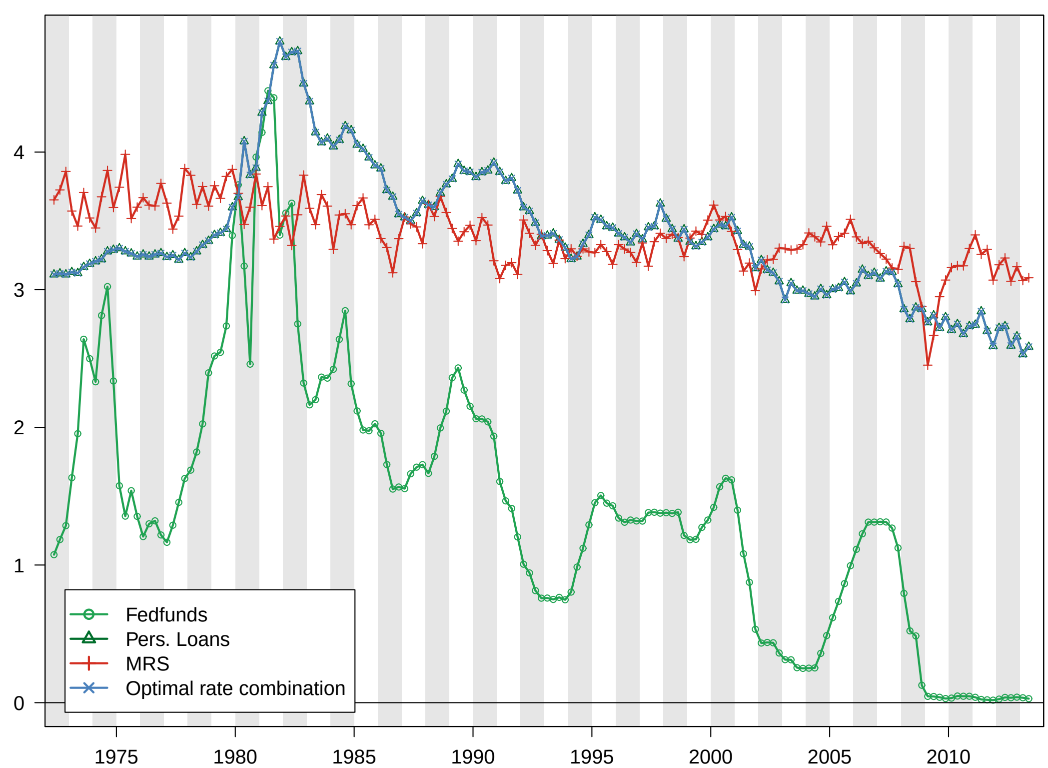


Figure 3.3 – Actual interest rates and *MRS* (joint estimation)

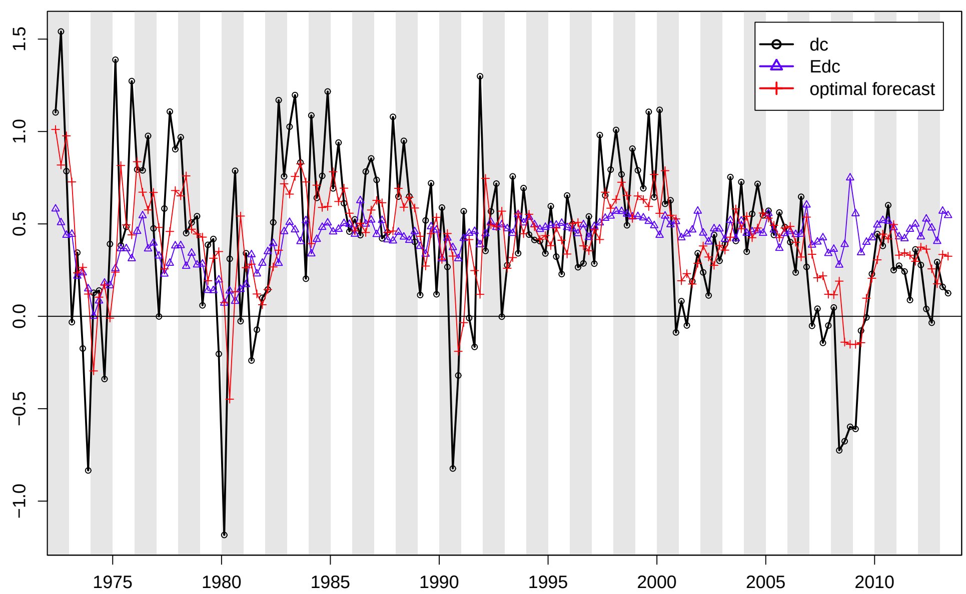
In addition, these very smoothed expectations for consumption imply a quite flat *MRS* which does not replicate the mid frequency fluctuations of the interest rate on personal loans (Figure 3.3). The improved fit of the data (see below) thus seems driven by the elimination of high frequency volatility. As surveys can provide information on households behaviour and expectations (Fuhrer, 1988; Carroll et al., 1994; Biau et al., 2010) in section 3.3.1 I use such surveys to measure expectations in the full model (3.9).

This result may also have to do with the econometrics which by construction penalizes more inflation than consumption forecasting errors on the first half of the sample as inflation was high and volatile. To verify this intuition, I test the impact of high and volatile inflation in the seventies eighties on the estimation result by dividing the sample in two, before and after 1990 (Figure 3.5). I plot the corresponding expectations in dark blue for the corresponding period but also in light blue for the out of estimation sample period.

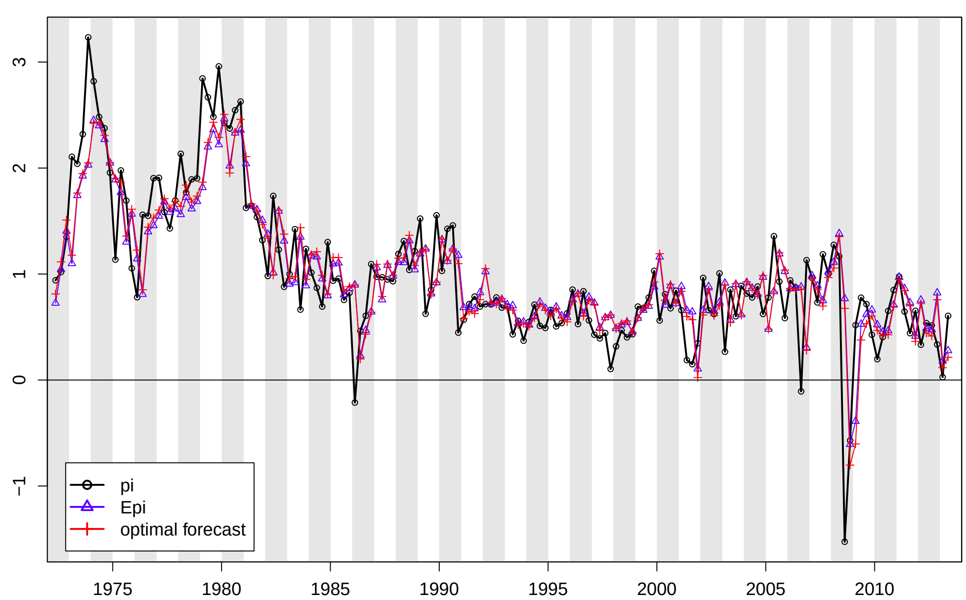
Expectations of inflation when estimated before 1990 closely replicate their optimal counterpart, including on the out of estimation sample period (Figure 3.5b). This is not the case for the post 1990 estimation where I find much smoother inflation expectations over the full sample. For consumption (Figure 3.5a), it is quite the opposite: expectations estimated prior to 1990 do not replicate the post 1990 period but estimation on the post 1990 period replicate the optimal forecast over the full sample.

Thus a poor fit of the Euler equation under the rational expectation hypothesis may hide some modification in households expectations or behaviour between 1972 and 2013. In particular, if there is a cost to collecting and processing information, it can be rational for households to adjust their expectations less frequently (Reis, 2006) which at the aggregate level generates suboptimal expectations. In this framework, households' behaviour should be quite different before and after 1990 since, especially for inflation, volatility (i.e. the cost of inattention) was quite large in the 70s 80s. The contrasted results on the two subperiods can be interpreted in those terms. In a period of high and volatile inflation, households dedicate more attention to forecast inflation than consumption. When inflation is steadily low, a naive forecast of inflation is optimal (Diron and Mojon, 2008) and more attention is given to consumption. This interpretation is not comparable to money illusion as described by Fisher (1928) for the beginning of the twentieth century: households can still assess correctly inflation as they did in the seventies eighties, when it was high.

Overall fit to the data To compare the present one-step estimation procedure with the preceding sequential one, I compute the log likelihood of the full model (3.9) and the adjusted R^2 of the Euler equation. As I have restricted the model to the two interest rates on the Treasury bill and personal loans, I compute the new statistics for the sequential estimation. These are improved compared to Table 3.4 as I no longer use the Fedfunds as

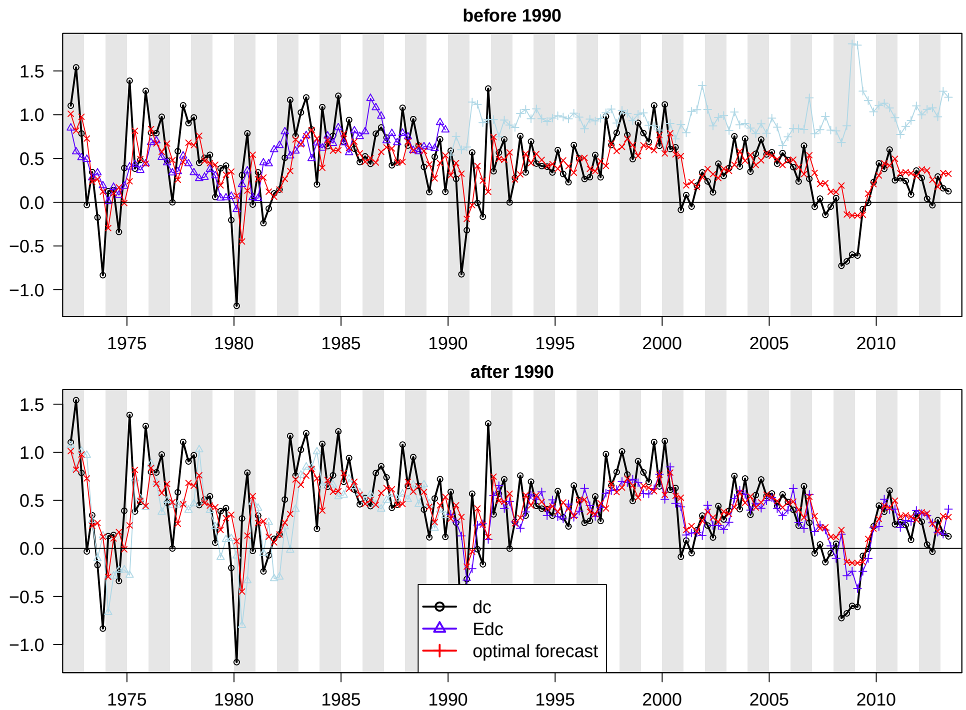


(a) Consumption

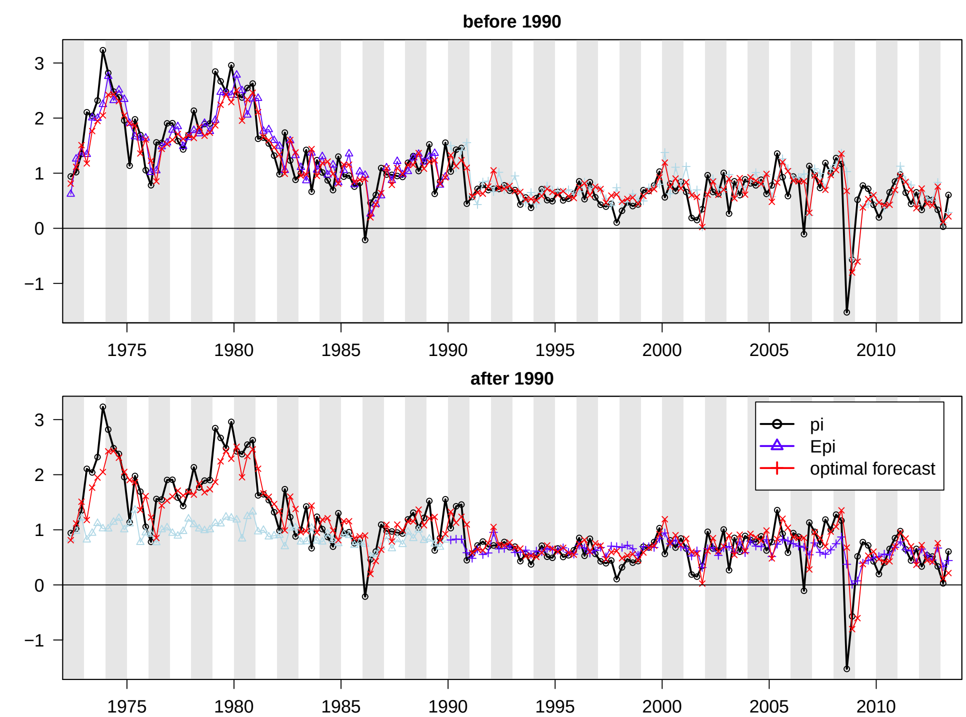


(b) Inflation

Figure 3.4 – Actual, expected consumption and inflation and optimal forecast



(a) Consumption growth



(b) Inflation

Figure 3.5 – Actual data, optimal forecast and expectations derived from system (3.9), for the US before and after 1990

a reference.

As expected, a joint estimation improves the fit to both the full model and the Euler equation: its adjusted R^2 goes from 15% to 46%. The increase in expectation errors is more than compensated by the better fit of the Euler equation so much so that the joint estimation naturally improves the overall fit to the data as well (+30 log likelihood).

	Sequential	Joint max.	
			$\sigma = 2$
log-lik.	1975.2	2004.68	2003.22
R2adj (Euler)	15	46	41

log-lik. is the log likelihood of the sample of the 3 residuals: from the Euler equation and the consumption and inflation forecasts measuring expectations. R^2adj is the adjusted R^2 for the Euler equation only.

Table 3.6 – Comparison of the fit to the data (Euler equation+expectation VAR, joint and sequential estimation of (3.9))

3.3.1 Reported expectations

I use data from the Michigan survey to measure households expectations in equation (3.8). The model is otherwise similar to (3.9). With respect to the previous estimations with macroeconomic data, this method has a slightly weaker fit to the data ($R^2_a dj = 23\%$ and loglikelihood 2002.38), but the outcome in terms of *MRS* and optimal combinations are in between those with sequential and joint estimations with the baseline VARX model: the weight on the treasury Bill is 15% and the CRRA parameter close to one (1.17). It is also noteworthy that the *MRS* reflects more the cyclical fluctuations of actual interest rates than with the previous joint estimation (Figure 3.6).

Expectations optimality Fisher's tests show that expectations for inflation and consumption are not markedly different from what would be estimated in an unconstrained approach (Table 3.7).

	Forecast (c)	Forecast (π)
Stat	2.46	3.29
5%	2.66	2.43
p-value	0.06	0.01

Table 3.7 – Fisher's test on the forecasting equations estimated separately or subject to the Euler equation, with survey data

Although graphically similar for an unconstrained forecast, households surveys estimated jointly with the Euler equation yield much better expectations of consumption growth than the benchmark estimation using macroeconomic data (Figure 3.7a compared

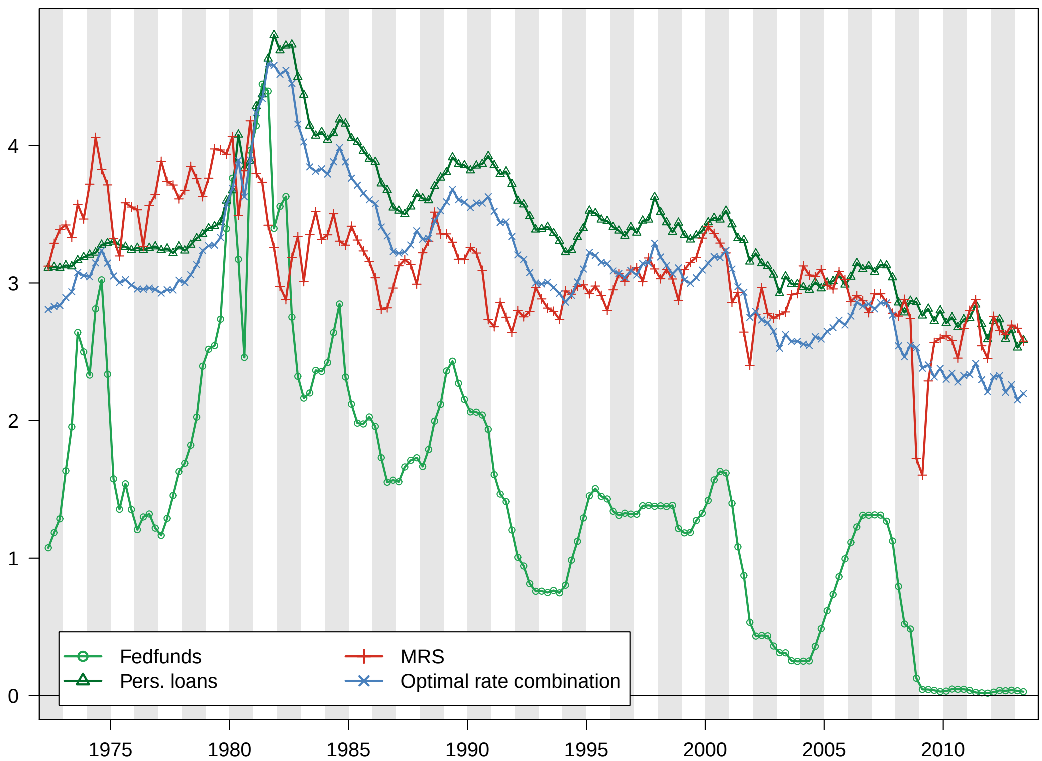
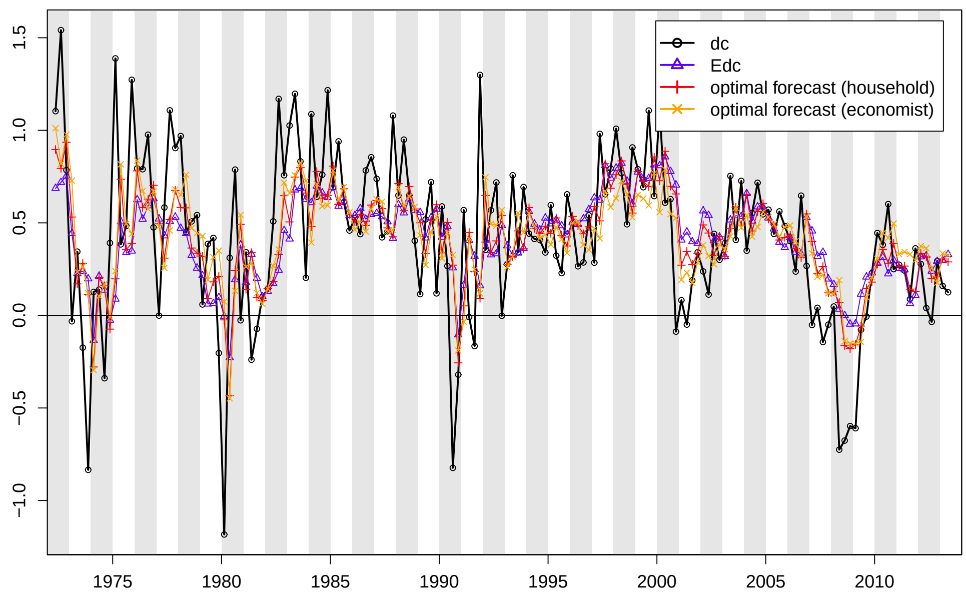


Figure 3.6 – Actual interest rates and *MRS* (Joint estimation with survey data)

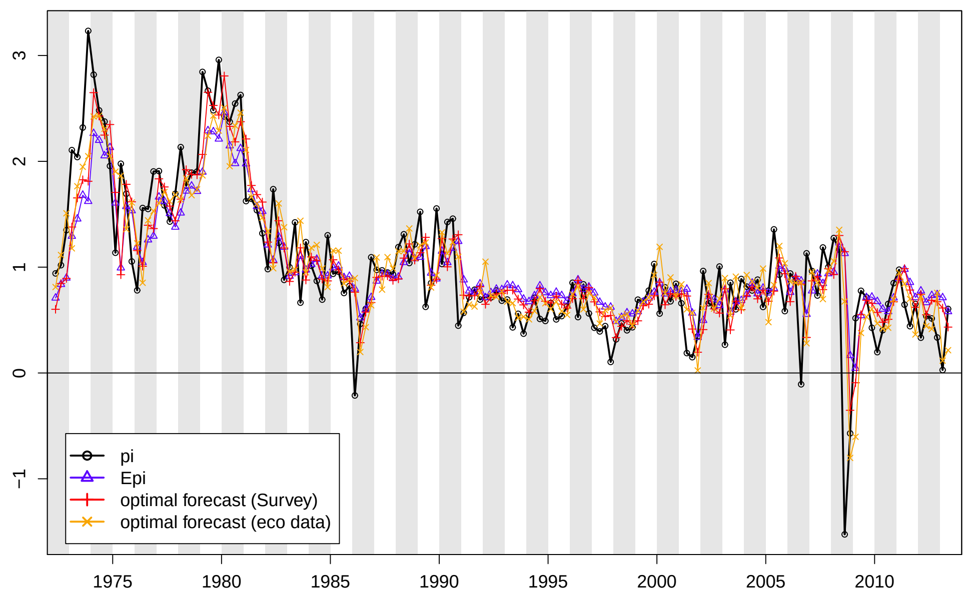
to 3.4a). High frequency fluctuations for both inflation and consumption growth are slightly smoothed but lower frequency fluctuations are well replicated. As a consequence, it seems that a small departure from the plain and simple rational expectation hypothesis would improve the fit of the Euler equation with the data, be consistent with households report and replicate a part of the cyclical fluctuations in household specific interest rates.

3.4 One way of reconciliation

Based on the previous estimation I assume that households are subject to a combination of 75% personal loans rate and 25% Treasury bill rate. I also assume that these households are inattentive (Reis, 2006) so that aggregate expectations for inflation are smoothed compared to the VARX forecast. This stylised fact is not only one of the results from joint estimations, but found in consumer surveys and an important aspect of the mismatch between the *MRS* and actual interest rates: volatile expectations feed into a volatile *MRS* while nominal interest rates are quite smooth. I thus measure expectations through a moving average of past VAR forecast to approximate this inattentiveness hypothesis.



(a) Consumption



(b) Inflation

Figure 3.7 – Actual, expected consumption and inflation and optimal forecast, with survey data

In addition I consider habit formation, constrained households *à la* Campbell and Mankiw, precautionary savings (Carroll et al., 1992; Kimball, 1990), Uzawa preferences, households answer to the Michigan survey on changes in the interest rate and durable consumption. Habit formation and constrained households are discussed in section 3.1.

Uzawa preferences In equation (3.7), the discount rate and variance term are assumed to be constant. However with Uzawa type preferences, the discount factor should be augmented with another endogenous variable (consumption level in Uzawa's formulation, but also possibly the stock of wealth). I capture this model by the introduction of consumption or GDI per capita in level in the Euler equation.

Precautionary savings or *animal spirits* Under an *animal spirits* hypothesis the discount factor could also be augmented by a psychological factor such as the sentiment index from households survey. Such a model could explain why Carroll et al. (1994) find that households surveys have a predictive power for consumption beyond the permanent income theory. This same variable could also be seen as a measure of uncertainty faced by households, i.e. a modification of the variance term instead of the discount factor. Indeed, in equation (3.7), the variance term is assumed constant which mutes the cyclical effects of risks on households behaviour. This missing variable shuts down the precautionary saving motive and could explain the steadiness of the *MRS* in some of the previous estimations. It could also explain why estimated expectations greatly depart from actual consumption during the recent crisis in the previous estimations. In addition to households' survey, the unemployment rate is often taken as a measure of households perception of risk (and particularly risk to his income). I test these two additional variables in the Euler equation.

Households' perception of interest rates In the Michigan survey, there is a balance of opinion on expected changes in the borrowing rates. I test this additional variable which I do not include in the *MRS* but in the optimal rate combination.

Durable consumption Mankiw (1982) concludes his test of the permanent income theory with the intuition that *durable goods, non-durable goods and services differ only in their rate of depreciation.*[10] A model where households derive utility from a stock of past consumption would yield a much smoother argument in the Euler equation than expected consumption growth (a moving average), which could account for the too smooth expectations observed in the joint estimation. This would be interpretable in the same way as inattentiveness on the consumption part. For this reason, I use a past moving average of expected consumption as an argument of the Euler equation and leave it to the reader to choose between the two interpretations.

[10]Indeed, sunglasses are classified as durables while shoes are not.

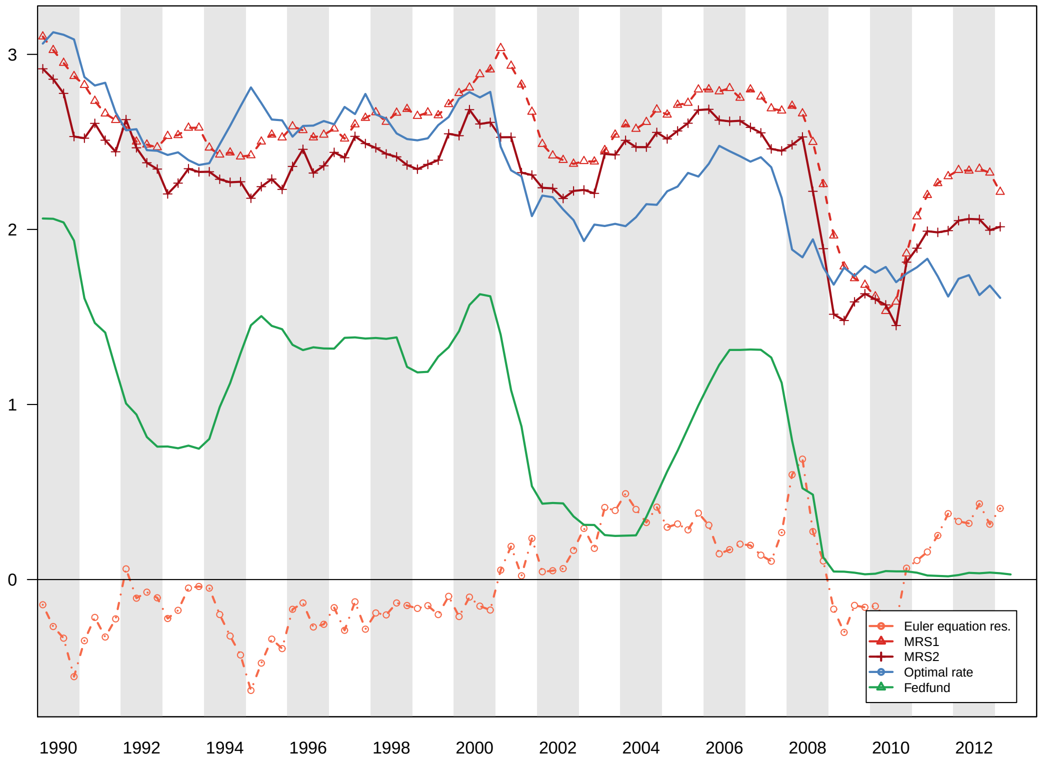


Figure 3.8 – Actual interest rates and *MRS*

Estimating the different models and their combinations from 1990, I select the following model (Table 3.8) combining precautionary savings (unemployment rate) and households perception of changes in interest rates (as reported in the Michigan survey). Habit formation, constrained households or animal spirit assumptions are rejected by the data, but all estimations point to a CRRA parameter of 0.5. The comparison of the *MRS* with the interest rate combination is depicted on Figure 3.8 (MRS1). For comparison I add to this graph the case for durable consumption (or inattentive expectations of consumption) which also replicate the main fluctuations in the interest rate combination (MRS2).

	Estimate	t value
cte	0.02	11.21
Edc	0.53	3.19
Michigan rate	0.26	1.79
unemployment	-0.1	-4.57
Fstat/R2adj	17.56	0.35

Table 3.8 – Euler equation estimation

One result is particularly striking from Figure 3.8: the residual between *MRS* and the actual interest rate shows a small trend. This trend can be well captured by Uzawa preferences (R^2_{adj} = 70% under this assumption), but implies a discount factor which, contrary to Uzawa's intuition, is increasing with consumption/income/wealth. Though unusual this model may require deeper thinking as, at the micro level, it can explain increasing disparities in wealth. More simply this result may also be due to the weights of the two interest rate which are in principle time varying.

3.5 Conclusion

The first result, which is robust throughout the paper, is that households are subject to an interest rate different from the one set by monetary policy makers. This result may seem trivial as it is clear that households can not borrow directly from the Central Banker's desk. What is less trivial however, is that the spread between household-specific interest rates and the monetary policy instrument matters so much and shall not be relegated to the residual of the Euler equation. Models should account for financial market frictions faced by households and consider weaker transmission mechanism from the nominal side to the real side of the economy. Implications in terms of monetary policy are also noteworthy: since most of final demand responds to market rates rather than the monetary policy, unconventional measures of monetary policy targeting households market conditions should prove efficient.

Similar work on French data (Appendix 3.C) provides informative comparisons: while American households react to the interest rate on personal savings; in France, they react to the interest rate on regulated savings accounts (*livret*). These results reflect important differences between countries: in the US, the savings ratio is much lower than in France, American consumers use credit cards while 90% of French residents have a *livret*.

The main innovation of the paper is to estimate households expectations under the constraint of the Euler equation as a compatibility test of this equation with rational expectations. The differences between a joint or sequential estimation of the Euler equation and expectations can be interpreted in two opposite directions: either the theoretical model is correct and expectations are not rational (as approximated by a VARX); or the rational expectation assumption is correct but incorporated in the wrong model. For this reason I do not fully reconcile the Euler equation and rational expectations but identify directions for improving their compatibility in monetary policy models.

Building on the *intuition* from first sections, I test a model where households react to a combination of personal loans and treasury bill rates, where households are inattentive (Reis, 2006), in particular as regards inflation,[11] and where precautionary savings are

[11] When inflation is steadily low it can be optimal to form naive expectations (Diron and Mojon, 2008), in Cochrane's terms *near rationality* would be close enough to perfect rationality.

linked to the risk of unemployment. This set of modelling assumptions yield a close link between the *MRS* and the interest rate to which households are subject ($R^2_{adj} = 35\%$), in sharp contrast with Canzoneri et al.'s rejection.

Appendix

3.A Data and the VAR of expectations

3.A.1 Data

US I use NIPA data for consumption, GDP, disposable income and deflators. I restrict consumption to non-durable goods and services. Real variables are taken per-capita. Interest rates, population and unemployment data are taken from the Fred database. I also use the surveys of consumers by the university of Michigan. I use quarterly data spanning from 1972Q1 to 2013Q2.

France I use Insee's Quarterly National Accounts data for consumption excluding durables, its deflator, GDP, disposable income.[12] Data, including interest rates, are available since the early eighties but I restrict the sample to post 1986 for stationariness reasons as the full sample would include only the end of the period of high and volatile inflation. Interest rate (Pibor-Euribor and households specific rates) are taken from Banque de France and the ECB.
I use the 3 month interbank rate (Pibor-Euribor) and three different rates, all for new contracts for households and individual enterprises: *Deposits with agreed maturity* up to one year, *Deposits redeemable at notice* and *Loans for house purchases excl. bank overdrafts*, total maturity.
Deposits redeemable at notice refer to regulated savings accounts. The oldest one (*livret A*) dates back to the nineteenth century and is the most widespread financial product in France: more than 90% of French residents have such an account nesting with other *livrets* (*livret jeune, livret developement durable, livret d'épargne populaire*), standing for more than 15% of French households financial assets (Noyer and Mérieux, 2012). As shown on Figure 3.C.1 the return on these savings is not linked to the monetary policy rate on most of the sample. More recently, this rate has been indexed on a combination of inflation, Eonia and Euribor with a discretionary component set by the government.[5]

[12] Data are taken from the first publication of base 2010 (2014Q1) but I also estimated with similar results the model on the last publication of base 2005.

For households balances of opinion, I use the European harmonized Households Survey published for France by Insee. Population and unemployment data are also retrieved from Insee's public database. I use quarterly data spanning from 1986Q1 to 2013Q4.

3.A.2 The VAR of expectations

For section 3.2, I estimate a VARX to model households expectations. This method is based on Fuhrer (2000) and Canzoneri et al. (2007). I restricted the specification of the expectation VAR to a VAR in first difference and only two lags (a necessary restriction to limit the number of parameters) while a VAR in level (for consumption and its deflator) with many lags is theoretically preferable. However, I found that results from section 3.3 are robust to the use of a VAR in level with 12 lags and I show in (Poissonnier, 2015b, Appendix) that the rejection of the Euler equation does not depend on the choice of a VAR model.

US I estimate a VAR model corresponding to the log of consumption in non-durables per capita and its deflator. I replicate the VAR of Fuhrer (2000) and Canzoneri et al. (2007) to model households' expectations in the US: additional regressors are the Fedfunds rate, log GDP and log GDI per capita. However, I do not use the Journal of Commerce industrial materials commodity price index for all items and tested some balances of opinion from the Michigan survey without significant modifications.

France I estimate a VAR model corresponding to the log of consumption in non-durables per capita and its deflator. This VAR model is estimated from 1986Q1 to 2013Q4 in level. To this VAR I add the lags of the purchasing power of gross disposable income, of GDP per capita, the Pibor-Euribor and test some balances of opinion from households opinion survey which do not impact the outcome of the VAR significantly.

3.B The conditional log-normality hypothesis

The equation under scrutiny takes the following form:

$$(1+i_t)^{-1} = \beta E_t\left(\frac{U_c(t+1)}{U_c(t)}\frac{1}{\Pi_{t+1}}\right) \tag{3.12}$$

In the most simple case $U_c(t) = C_t^{-\sigma} = e^{-\sigma c_t}$ with $c_t = log(C_t) \quad \pi_t = log(\Pi_t)$

$$E_t\left(\frac{U_c(t+1)}{U_c(t)}\frac{1}{\Pi_{t+1}}\right) = E_t(exp(-\sigma\Delta c_{t+1} - \pi_{t+1})) = E_t(exp(a'y_{t+1})) \tag{3.13}$$

with $a = [-\sigma, -1]'$ and $y_t = [\Delta c_t, \pi_t]'$.

I suppose that households expectations are formed through the previous VAR model with normal errors ν_t and variance Σ, so that $y_{t+1} = E_t(y_{t+1}) + \nu_{t+1}$. I can then decompose $E_t exp(a' y_{t+1}) = exp(a' E_t y_{t+1}) E_t exp(a'\nu)$, in which the second term can be simplified

$$E_t exp(a'\nu) = \int \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(a'\nu - \frac{1}{2}\nu'\Sigma^{-1}\nu\right) d\nu$$

first change of variables

$\Sigma^{-1} = P'DP$ with $P \in \mathcal{O}(n)$ and D diagonal

$P\nu = \mu$

$\nu = P'\mu$

$$E_t exp(a'\nu) = \int \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(a'P'\mu - \frac{1}{2}\mu' D\mu\right) d\mu$$

second change of variables

$$x = \left(\frac{D}{2}\right)^{\frac{1}{2}} \mu - \left(\frac{D}{2}\right)^{\frac{-1}{2}} \frac{Pa}{2}$$

then $x'x = \frac{1}{2}\mu' D\mu - a'P'\mu + \frac{1}{2} a'P'D^{-1}Pa$

we then have $E_t exp(a'\nu) = \int \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-x'x + \frac{a'\Sigma a}{2}\right) |\frac{D}{2}|^{\frac{-1}{2}} dx$

$$E_t exp(a'\nu) = \exp\left(\frac{a'\Sigma a}{2}\right) = \exp(\frac{\sigma^2}{2} var(\epsilon^c) + \sigma cov(\epsilon^c, \epsilon^\pi) + \frac{1}{2} var(\epsilon^\pi))$$

Then I can rewrite

$$E_t\left(\frac{U_c(t+1)}{U_c(t)} \frac{1}{\Pi_{t+1}}\right) = E_t(exp(a' y_{t+1})) = exp\left(a' E_t(y_{t+1})\right) exp\left(\frac{a'\Sigma a}{2}\right) \tag{3.14}$$

In logarithm, this expression is a linear combination of economic variables including the second moments of the errors. Moreover, when the utility function is CES with habit formation under a multiplicative form as in Abel (1990), this result can be easily generalized.

Although it avoids a first order approximation, this method still forms a strong assumption: second moments Σ are constant. In particular, this assumption shuts down one channel for precautionary savings: when the environment is more uncertain, Σ increases, which is equivalent to a decrease in the interest rate. This channel of uncertainty shocks could account for an important share of business cycle fluctuations (Bloom, 2009).

3.C On French data

I performed similar estimations as those presented for the US on French data. These estimations yield similar results: accounting for household-specific interest rates improves the fit of the Euler equation to the data. Estimating expectations jointly with the Euler equation improves this fit further (Table 3.C.1).

In terms of interest rate, household respond to the interest rate on regulated saving (*livret*), a perfectly liquid savings account held by more than 90% of households whose return is set by the government (Figure 3.C.2).

When estimated jointly with the Euler equation, expectations depart significantly from their unconstrained counterparts (3.C.2) but not when expectations are measure through households surveys (Appendix 3.3.1).

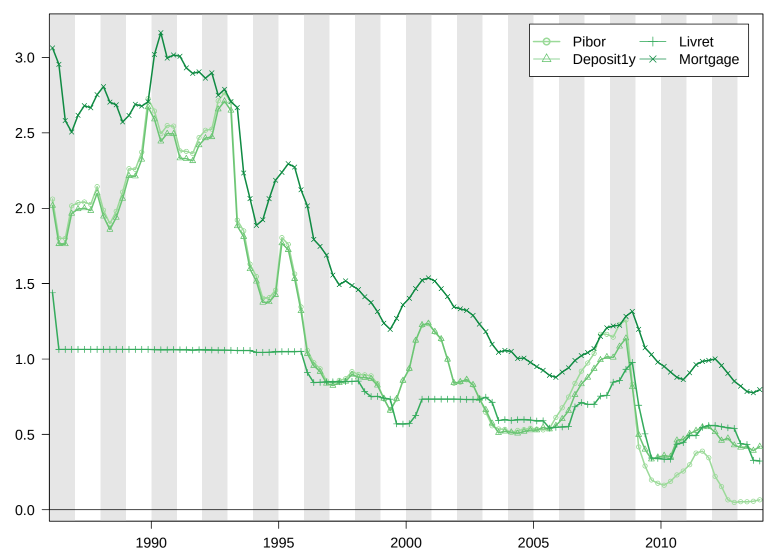


Figure 3.C.1 – French interest rates in nominal terms

	1 rate no habit	1 rate habit	4 rates no habit	4 rates habit
Stdev	0.73	0.73	0.4	0.4
log-lik.	1230	1229.71	1468.54	1470.21
R2 (Euler)	11	10	86	87

(a) VARX estimated *ex ante*

	Sequential	Joint max.		
			$\sigma = 1$	$\forall x \neq x_0 \gamma^x > 0$
Stdev	0.4	0.41	0.49	0.42
log-lik.	1468.54	1553.3	1372.28	1539.23
R2 (Euler)	86	98	92	97

(b) Joint estimation

Stdev is the square root of the quadratic norm of the covariance matrix of the 3 residuals: from the Euler equation and the consumption and inflation forecasts measuring expectations. It is equivalent to the standard deviation of the residual in an univariate regression. *log-lik.* is the log likelihood of the sample of the same 3 residuals. The R^2 for the Euler equation differs from the regression as it is computed from the ex-post real monetary policy rate.

Table 3.C.1 – Comparison of the fit to the data of the different models for France (Euler equation+expectation VAR)

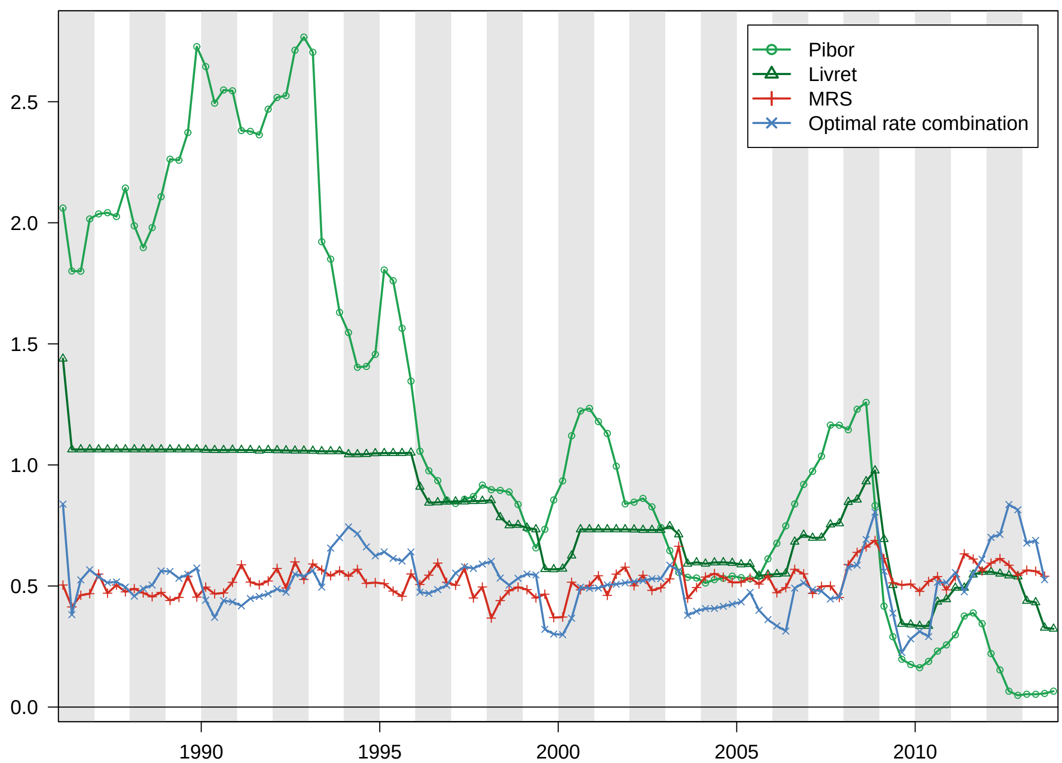


Figure 3.C.2 – Actual interest rates and *MRS* for France (joint estimation)

	Forecast (c)	Forecast (π)
Stat	3.6	2.47
5%	2.1	2.1
p-value	0	0.02

Table 3.C.2 – Fisher's test on the forecasting equations estimated separately or jointly with the Euler equation for France

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Chapter

4 The Taylor principle is valid under wage stickiness: an analytical proof

with Alexis Blasselle

We consider the textbook neo-Keynesian model with staggered prices and wages in discrete time. We prove that the Taylor principle holds in this case, a result usually conjectured but which remained to be established analytically. When both contracts exhibit sluggish adjustment to market conditions, the policy maker faces a trade-off between stabilizing three welfare relevant variables: output, price inflation and wage inflation. We consider a monetary policy rule designed accordingly: the central banker can react to both inflations and the output gap. In addition to generalizing the Taylor principle we show that the frontier of determinacy embeds the frontier derived with staggered prices only, generalizes the frontier of determinacy in the limit case of continuous time and is symmetric in price and wage inflations.

JEL-code: *C62, C68, E52, E58*

Keywords: *Dynamic Stochastic General Equilibrium model, Monetary Policy Rule, Sun Spot Equilibria, Taylor Principle*

4.1 Introduction

In (Taylor, 1993), John Taylor advocates the use of monetary policy rules where the Central Banker reacts to both price inflation and output as a benchmark to be used judgementally.

In the simplest neo-Keynesian model with staggered prices only, this Taylor rule is key in ruling out sun-spot fluctuations. The Taylor principle associated to such rules states that the Central Banker should overreact to inflation to ensure the uniqueness of the solution under rational expectations.

Extensions to models with both staggered prices and wages have emphasized the welfare optimization problem while assuming, backed on numerical simulations, that the Taylor principle still holds in these cases (Erceg et al., 2000; Galí, 2008).

Flaschel et al. (2008) derive analytically the frontier of determinacy in the same model but on the limit case of continuous time using a simpler and more general strategy. Although the authors expect their demonstration in continuous time to be informative of the discrete time case, based on the precept that such a property should not depend on the time length of the period, they acknowledge in their concluding remarks that their intuition is *not unchallenged*. In this paper we prove them right: the Taylor principle does hold in the discrete time version of the model and is a generalization of their result.

We consider the same model as (Galí, 2008, chapter 6) or (Erceg et al., 2000) and a monetary policy rule in line with Erceg et al.'s results: the Central Banker can react to both inflations and the output gap. With straightforward notations, the Taylor rule takes the following form:

$$i_t = \Phi_p \pi_t^p + \Phi_w \pi_t^w + \Phi_y y_t \tag{4.1}$$

We show that the necessary and sufficient condition to rule out sun-spot equilibria is symmetric in inflations:

$$\Phi_p + \Phi_w + \frac{1-\beta}{\tilde{\kappa}} \Phi_y > 1 \tag{4.2}$$

with $\tilde{\kappa}$ a coefficient depending symmetrically on both slopes of the prices and wages Phillips curves and β the discount factor. This frontier, taking the form of a condition on $\Phi_p + \Phi_w$ eased as Φ_y increases, is in line with numerical investigations (Galí, 2008, chap 6).

Though the model's symmetry may not appear straightforward, it is deeply rooted in the model, (Blanchard and Galí, 2007) call it a *divine coincidence*. Similar symmetry arises when studying the optimal monetary policy (see the functional form of the welfare criterion derived by Galí and Erceg et al.) as well as in our demonstration.

Our result generalizes in discrete time that of Flaschel et al. (2008) in the limit case of continuous time. It is also a direct generalization of the frontier derived in the simpler case of staggered prices only (Woodford, 2001). Moreover it has an identical interpretation in the long run: the Central Banker should react more than one for one to permanent changes in inflation (Woodford, 2011, chapter 4).

A majority of authors consider staggered prices only and have investigated both questions of welfare optimization and uniqueness of the solution. These questions are independent of one another: optimal rules do not necessarily avoid sun-spot fluctuations (Clarida et al., 1999) and in both respects, it has been shown that the Taylor rule has appealing properties (Woodford, 2001). The properties of the Taylor go beyond these two questions as it is also key in a model with adaptive learning (Bullard and Mitra, 2002).

When considering both staggered prices and wages, some of the appealing properties of the Taylor rule are weakened. In a model with staggered prices only, the introduction of cost-push shocks induces a short run trade-off between stabilizing inflation and output (Clarida et al., 1999). Allowing for both rigidities generates a trade-off between stabilizing inflation and output even in the absence of these cost-push shocks: in the simple model with staggered prices only, the Phillips curve implies that stabilizing price inflation is equivalent to stabilizing the output gap, a result Blanchard and Galí (2007) present as a *divine coincidence* because it allows the Central Banker to enforce the social optimum. With the addition of staggered wages, this result no longer holds. In a particular case, Erceg et al. (2000) come to the same conclusion. They show that it is not possible for the monetary policy to fully stabilize more than one of the three objectives: price inflation, wage inflation or output, but the variance of each is detrimental to welfare. Using numerical simulations, they also show that sole price or wage inflation targeting is suboptimal in this context, but a policy rule such as suggested by Taylor or with reactions to both price and wage inflations performs nearly as well as the optimal rule. Blanchard and Galí (2007) note a weaker form of the *divine coincidence* in Erceg et al.'s model: combining the two Phillips curves yields that stabilizing the output gap is equivalent to stabilizing a weighted average of price and wage inflation (with the weight on each inflation being the slope of the others Phillips curve). (Galí, 2008, chapter 6) show that targeting this *composite inflation* performs nearly as well as the optimal policy.

In the remainder of this chapter, section 6.2 recalls the model, section 4.3 outlines the proof detailed in appendix 4.A and section 4.4 concludes.

4.2 A monetary model with sticky wages and prices

We study the model exposed in (Galí, 2008, chap 6) and (Erceg et al., 2000). This model extends the standard neo-Keynesian model for monetary policy analysis which consist of an *IS* curve relating the output gap to the expected real interest rate, a Phillips curve relating inflation, expected inflation and output gap and a monetary policy rule describing how the interest rate is set by the Central Banker. The present extension of the model considers wage rigidities under the form of Calvo contracts. It follows from this rigidity that real wages may deviate from their flexible equivalent due to exogenous disturbances.

The model takes the following linear form:

$$\pi_t^p = \beta E(\pi_{t+1}^p|t) + \kappa_p y_t + \lambda_p \omega_t \tag{4.3}$$
$$\pi_t^w = \beta E(\pi_{t+1}^w|t) + \kappa_w y_t - \lambda_w \omega_t \tag{4.4}$$
$$\omega_{t-1} = \omega_t - \pi_t^w + \pi_t^p + \Delta\omega_t^n \tag{4.5}$$
$$y_t = E(y_{t+1}|t) - \frac{1}{\sigma}(i_t - E(\pi_{t+1}^p|t) - r_t^n) \tag{4.6}$$
$$i_t = \Phi_p \pi_t^p + \Phi_w \pi_t^w + \Phi_y y_t + v_t \tag{4.7}$$

The complete derivation of the model is exposed in full details in (Galí, 2008, chap 6) with the same notations. This system describes the dynamic of 5 endogenous variables $(\pi^p,\ \pi^w,\ \omega,\ y,\ i)$. $E(.|t)$, is the rational expectations operator at date t, i.e. the expectation conditional on the values of every variables up to date t and the model itself. Equations (4.3) and (4.4) are the Phillips curves on price inflation (π^p) and wage inflation (π^w). They describe the progressive adjustment of prices and wages to market conditions. Prices may increase with expected inflation or the marginal cost of production. This cost depends positively on the output gap (y_t, defined as the deviation of output from its fully flexible equivalent) and the real wage gap (ω_t, defined as the deviation of real wage from its fully flexible equivalent). Wages may increase with expected wage inflation or decrease with the wage mark-up (taken in deviation from the flexible contracts case). This mark-up depends positively on the real wage gap and negatively on the output gap. Equation (4.5) describes the fact that because of nominal rigidities, real wages depart from their fully flexible counterpart. Exogenous shocks to the economy affecting the *natural* real wage ($\Delta\omega^n$) are not instantaneously transmitted to the actual real wage but only to its flexible counterpart, hence driving a wedge between inflations and the dynamic of the real wage gap. Equation (4.6) describes the evolution of the output gap (y) as a function of interest rate (i) and expected inflation. The implicit assumption here is that output is driven, in the short run, by private demand which verifies the standard Euler equation of households. r_t^n is the natural rate of interest, that is the real interest rate which would prevail under fully flexible contracts. Equation (4.7) describes the interest rate decision of the central banker. It is a Taylor rule modified to account for the central banker's possible reaction to wages inflation as well as price inflation. The higher inflations or output are, the higher the central banker will set the interest rate in order to temper the economic growth. Moreover, the central banker may depart from this rule for reasons exogenous to the model (v).

The parameters of this model are:

- $0 < \beta < 1$, the discount factor of households.

- $\sigma \geq 0$, the inverse intertemporal elasticity of substitution of consumption.
- $\Phi_p > 0$, the central banker's reaction to price inflation.(Taylor, 1993) considers $\Phi_p = 1.5$
- $\Phi_w \geq 0$ the central banker's reaction to wage inflation. In the standard cases the central banker only reacts to price inflation ($\Phi_w = 0$)
- $\Phi_y \geq 0$ the central banker's reaction to the output gap. (Taylor, 1993) considers $\Phi_y = 0.5$.
- $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}$, with
  - $0 < \theta_p < 1$, the Calvo parameter on prices, in other words the stickiness of prices (if 0, prices are fully flexible)
  - $0 < \alpha < 1$, and $1-\alpha$ the elasticity of output with respect to labour
  - $0 < \varepsilon_p \leq 1$, the elasticity of substitution among goods

$\Rightarrow$ $0 < \lambda_p$

- $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varphi\varepsilon_w)}$, with
  - $0 < \theta_w < 1$, the Calvo parameter on wages, in other words the stickiness of wages (if 0, wages are fully flexible)
  - $0 < \varphi$, the Frisch elasticity, in other words the convexity of the cost of labour in terms of welfare.
  - $0 < \varepsilon_w \leq 1$, the elasticity of substitution among labour types

$\Rightarrow$ $0 < \lambda_w$

- $\kappa_p = \frac{\alpha\lambda_p}{1-\alpha}$
- $\kappa_w = \lambda_w(\sigma + \frac{\varphi}{1-\alpha})$

Denoting $x_t = [y_t,\ \pi_t^p,\ \pi_t^w,\ \omega_{t-1}]^T$, the endogenous variables, and $z_t = [r_t^n - v_t,\ \Delta\omega_t^n]^T$, the exogenous variables, the equations (4.3) to (4.7) can be written in the form:

$$x_t = A^{-1}\left(\ E(x_{t+1}|t) + B\ z_t\right) \tag{4.8}$$

In the equation (4.8), the matrix of interest A is:

$$A = \begin{bmatrix} 1 + \dfrac{\kappa_p}{\sigma\beta} + \dfrac{\Phi_y}{\sigma} & \dfrac{\beta\Phi_p - 1 - \lambda_p}{\sigma\beta} & \dfrac{\beta\Phi_w + \lambda_p}{\sigma\beta} & \dfrac{\lambda_p}{\sigma\beta} \\ \dfrac{-\kappa_p}{\beta} & \dfrac{1+\lambda_p}{\beta} & \dfrac{-\lambda_p}{\beta} & \dfrac{-\lambda_p}{\beta} \\ \dfrac{-\kappa_w}{\beta} & \dfrac{-\lambda_w}{\beta} & \dfrac{1+\lambda_w}{\beta} & \dfrac{\lambda_w}{\beta} \\ 0 & -1 & 1 & 1 \end{bmatrix} \tag{4.9}$$

There are three forward looking variables in this model: $([y_t,\ \pi_t^p,\ \pi_t^w])$. According to (Blanchard and Kahn, 1980), the system (4.8) has a unique solution if and only if the matrix A defined by (4.9) has 3 eigenvalues strictly larger than one in modulus and one eigenvalue smaller than one in modulus. In addition, this solution is stable if this last eigenvalue is strictly smaller than one in modulus.

4.3 Outline of the proof

Defining the frontier of indeterminacy is based on the study of the roots of the characteristic polynomial of matrix A, a fourth degree polynomial. Though it is not complex mathematics, it is rather cumbersome. We are particularly grateful to Yvon Maday and other mathematicians at Laboratoire Jacques-Louis Lions for proofreading and comments.[1]

In appendix 4.A.1 we develop the proof in the case $\Phi_y = 0$. We use the intuition that in this case the frontier of determinacy is $\Phi_p + \Phi_w = 1$ and decompose the polynomial as a fourth degree polynomial corresponding to this case plus deviations from this case in both directions (Φ_p, Φ_w). In the limit case $\Phi_p + \Phi_w = 1$, the model is at the limit of solution determinacy: 1 is a root of its characteristic polynomial; its real roots are non-negative and at most one real root is in $]0, 1[$; its complex roots have a modulus strictly greater than one. The deviation from this limit case ensures the uniqueness of the model's solution if and only if the deviation from $\Phi_p + \Phi_w = 1$ is positive: the root 1 moves in the direction ensuring the uniqueness of the model's solution (depending on the existence of another root smaller than one), the other roots are kept outside or inside the unit disk depending on their initial position.

In appendix 4.A.2 we show that the case $\Phi_y \neq 0$ can be treated identically to the case $\Phi_y = 0$. We consider the frontier of indeterminacy under the form $\Phi_p + \Phi_w = 1 - \theta$ and show that setting $\theta = \Phi_y \frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w}$ allows a decomposition of the characteristic polynomial which has the same properties as in the case $\Phi_y = 0$. We can conclude that equation (4.10) generalizes the frontier of indeterminacy.

4.4 Conclusion

In the textbook neo-Keynesian model with both staggered prices and wages, we show that any monetary policy rule satisfying

$$\Phi_p + \Phi_w + \Phi_y \frac{(1-\beta)(\lambda_p + \lambda_w)}{\kappa_w\lambda_p + \kappa_p\lambda_w} > 1 \tag{4.10}$$

[1] The frontier of determinacy coincides with the condition for the uniqueness of the steady state. This necessary but not sufficient condition for our result is ironically very easy to find analytically. The coincidence between these two frontiers implies its interpretation in the terms suggested by (Woodford, 2011, chapter 4) -*c.f.* infra.

rules out sunspot fluctuations.[2] The admissibility of a policy rule symmetrically depends on wage inflation and price inflation: when the central bank does not respond to changes in output, the condition on the monetary policy parameters comes down to $\Phi_p + \Phi_w > 1$ in line with Galí's numerical investigations. Also in line with Galí's numerical investigations, when the central bank reacts to changes in output, doing so relaxes the constraint above, proportionally to Φ_y with a factor $\frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w}$. This coefficient crucially and symmetrically depends on the Phillips curves of prices and wages: more impatient agents (smaller β) or flatter Phillips curves (smaller λ or κ), facilitate the task of the central banker to prevent sun spot fluctuations.

In continuous time, Flaschel et al. (2008) derive an identical result; determinacy holds under the following condition:

$$\Phi_p + \Phi_w + \Phi_y \frac{\rho(\lambda_p + \lambda_w)}{\kappa_w\lambda_p + \kappa_p\lambda_w} > 1 \tag{4.11}$$

with ρ related to the discount factor as follows $1/\beta(t) = 1 + \rho t$. In (4.11) ρ is the limit of $(1 - \beta(t))/t$ when the time step t tends to zero, while the time step is one in (4.10). Hence the combination of these results proves, as expected by Flaschel et al., that the determinacy condition of this model can be taken to the limit of continuous time.

In a model with staggered prices only,the frontier of determinacy is (4.12) (Woodford, 2001), of which our result (4.10) is a direct generalization.

$$\Phi_p + \frac{1-\beta}{\kappa_p}\Phi_y > 1 \tag{4.12}$$

With both rigidities, a permanent shift in price inflation ($\tilde{\pi}$) implies an identical permanent shift in wage inflation (equation (4.5)). The Phillips curves (equations (4.3) and (4.4)) imply a proportional shift in output gap $\tilde{y} = \frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w}\tilde{\pi}$. In turn, the Taylor rule (4.7) implies that the reaction of the central banker is to raise the nominal interest rate by $\tilde{i} = \left[\Phi_p + \Phi_w + \Phi_y \frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w}\right]\tilde{\pi}$. Thus, as in the standard neo-Keynesian model without wage rigidities (Woodford, 2011, chapter 4), our frontier of indeterminacy can also be interpreted as a strict Taylor principle applied to permanent shocks: the *central banker should react more than one for one to permanent changes in inflation.*

Finally, if wage inflation and price inflation play similar roles for the design of the optimal monetary policy (Galí, 2008; Erceg et al., 2000), we show that they also play symmetric roles for eliminating sun-spot fluctuations. This extended conclusion remains *at odds with the practice of most central banks, which seem to attach little weight to wage inflation as a target variable* (Galí, 2008).

[2]Using Dynare (Adjemian et al., 2011), it is possible to verify numerically frontier (4.10); code available upon request

Appendix

4.A Proof

4.A.1 The case for $\Phi_y = 0$

Based on numerical evidence (Galí, 2008), we study the problem in deviation from the limit case $\Phi_w + \Phi_p = 1$, we introduce a new parameter ϕ_p and use the following parametrization:

$$\Phi_p = \phi_p + \xi \qquad \Phi_w = \phi_w + \gamma = 1 - \phi_p + \gamma \tag{4.13}$$

$$0 < \phi_p < 1 \qquad \xi \text{ s.t } \quad \Phi_p > 0 \qquad \gamma \text{ s.t. } \quad \Phi_w > 0 \tag{4.14}$$

The domain of interest, $D_{p,w}$, is displayed on Figure 4.A.1. This parametrization of $D_{p,w}$ is not injective, as three parameters (ϕ_p, ξ, γ) describe a two dimensional domain, but this choice makes the study easier. In particular, it allows to use deviations of the same sign in both directions to describe the two domains of solution determinacy and indeterminacy.

Let X denote the vector of parameters:

$$X = [\beta, \phi_p, \kappa_p, \kappa_w \lambda_p, \lambda_w, \sigma] \quad \in]0,1[\times]0,1[\times (\mathbb{R}_+^*)^5 = D_X. \tag{4.15}$$

from which γ and ξ are excluded and hereafter treated as special parameters.

The characteristic polynomial of the matrix defined in (4.9) above can be expressed as follows, with implicit dependency on X:

$$P_{\gamma,\xi}(t) = at^4 - bt^3 + c_{\gamma,\xi}t^2 - d_{\gamma,\xi}t + e_{\gamma,\xi}. \tag{4.16}$$

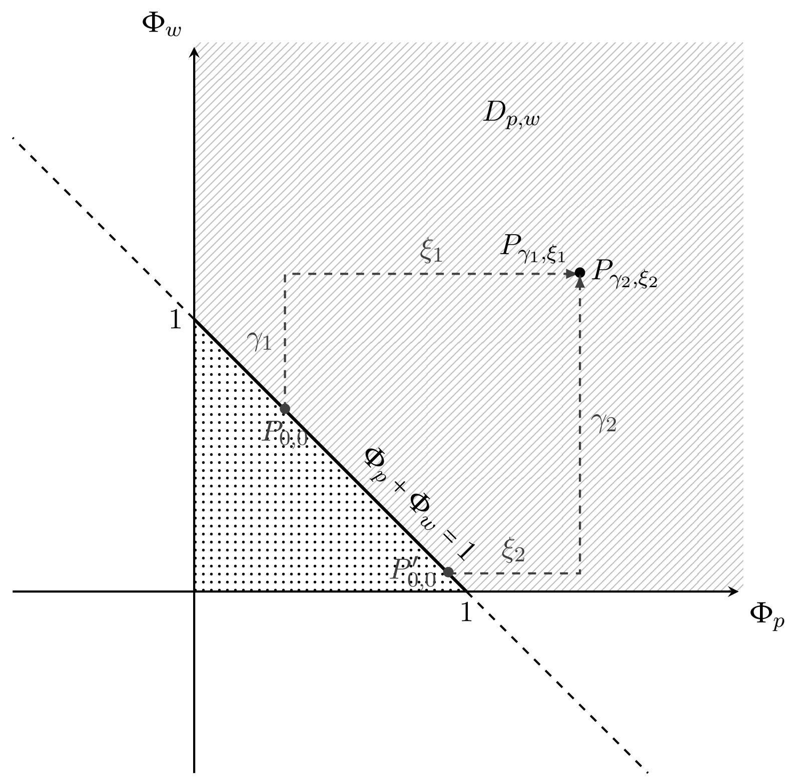

Figure 4.A.1 – Domain of interest (solution determinacy dashed, indeterminacy dotted) and parametrization

with the coefficients:

$$
\begin{aligned}
a &= \sigma\beta^2 \\
b &= \beta[\kappa_p + \sigma(2 + 2\beta + \lambda_p + \lambda_w)] \\
c_{\gamma,\xi} &= \kappa_p[1 + \beta + \lambda_w + \beta\xi] + \kappa_w[\lambda_p + \beta\gamma] \\
&\quad + \sigma[1 + 4\beta + \beta^2 + (\lambda_p + \lambda_w)(1 + \beta)] + \beta\phi_p\kappa_p + \beta(1 - \phi_p)\kappa_w \\
d_{\gamma,\xi} &= \kappa_p[1 + \lambda_w(1 + \gamma + \xi) + (1 + \beta)\xi] + \kappa_w[\lambda_p(1 + \gamma + \xi) + (1 + \beta)\gamma] \\
&\quad + \sigma[2(1 + \beta) + \lambda_p + \lambda_w] + (1 + \beta)\kappa_p\phi_p + (1 + \beta)\kappa_w(1 - \phi_p) \\
e_{\gamma,\xi} &= \sigma + \kappa_p\xi + \kappa_w\gamma + \kappa_p\phi_p + \kappa_w(1 - \phi_p)
\end{aligned}
$$

To study the eigenvalues of this polynomial in deviation from the limit case $\phi_p + \phi_w = 1$, we use the simplifying notations $P_0 = P_{0,0}$, $c = c_{0,0}$, $d = d_{0,0}$ and $e = e_{0,0}$ and the following decomposition:

$$P_{\gamma,\xi}(t) = P_0(t) + \gamma\,\kappa_w\,Q(t) + \xi\,\kappa_p\,S(t) \qquad \text{where} \tag{4.17}$$

$$Q(t) = \beta\,t^2 - [\lambda_w \frac{\kappa_p}{\kappa_w} + (1+\beta+\lambda_p)]t + 1 \tag{4.18}$$

$$S(t) = \beta\,t^2 - [\lambda_p \frac{\kappa_w}{\kappa_p} + (1+\beta+\lambda_w)]t + 1 \tag{4.19}$$

It is noteworthy that the deviation in direction Φ_p ($\xi\,\kappa_p\,S(t)$) is exactly symmetric to the deviation in direction Φ_w ($\gamma\,\kappa_w\,Q(t)$).

Property 4.A.1 *For every vector of parameters $X \in D_X$, $(a,b) \in (\mathbb{R}_+^*)^2$ and $\forall\,\gamma \geq -\phi_p$, $\forall\,\xi \geq \phi_p - 1$, $(c_{\gamma,\xi}, d_{\gamma,\xi}, e_{\gamma,\xi}) \in (\mathbb{R}_+^*)^3$. This implies that $\forall\,t \leq 0$, $P_{\gamma,\xi}(t) > 0$.*

Proof: From their definition and the definition (4.15) of D_X, coefficients a to $e_{\gamma,\xi}$ are all positive. The sign of the polynomial $P_{\gamma,\xi}(t)$ derives from (4.16). ∎

Property 4.A.2 *The limit case polynomial P_0 verifies the following properties:*

(i) *1 is a root of P_0 which can be written $P_0(t) = (t-1)R_0(t)$ with*

$$R_0(t) = at^3 - (b-a)t^2 + (a-b+c)t + a - b + c - d$$

(ii) *P_0 has no negative root,*

(iii) *P_0 has at most two complex roots, these roots are conjugate and their modulus is strictly larger than one*

(iv) *if P_0 has a multiple root in 1, it is a double one and the other roots are real and larger than one*

(v) *P_0 has at most one root in $]0,1[$*

Proof:

(i) From the definition of P_0 and its coefficients, one can check that $a - b + c - d + e = P_0(1) = 0$ ⧫

(ii) This property derives directly from property 4.A.1 ⧫

(iii) P_0 is a fourth degree polynomial with real coefficients and one real root (1). It follows that it has at most two complex roots which are conjugate. These roots are also the roots of polynomial R_0 which we denote z and $\bar{z}$ and let r denote the remaining real root. We can write:

$$\begin{aligned} R_0(t) &= at^3 - (b-a)t^2 + (a-b+c)t + a - b + c - d \\ &= a(t-r)(t-z)(t-\bar{z}) = a(t-r)(t^2+|z|^2) \\ &= at^3 - art^2 + a|z|^2 t - r|z|^2 \end{aligned}$$

$$\text{property 4.A.6} \Rightarrow r = \frac{b-a}{a} > 3 \quad \& \quad |z|^2 = \frac{a-b+c}{a} > 3$$

Thus $z,\bar{z}$ are outside the unit disk. ♦

(iv) We showed above that if P_0 has complex roots, the two remaining real roots are 1 and $r > 3$. So 1 can not be a double root of P_0 in this case. Property 4.A.8 implies that if $P_0'(1) = 0$ then $P_0''(1) > 0$, thus 1 can not be a triple root of P_0. If 1 is a double root of P_0, the two other roots, necessarily real and positive, can not be both smaller than one as the product of the four is larger than one. If we assume that one of them is smaller than one, a table of variation of P_0 shows that it implies that $P_0''(1) < 0$, which contradicts property 4.A.8. ♦

(v) We showed above that if P_0 has complex roots, the two remaining real roots are 1 and $r > 3$.
If P_0 has four real roots, one of them is 1 and their product is $\frac{e}{a} > 1$. Thus at least one of its roots is strictly larger than 1.
Property 4.A.7 also implies if P_0 has four real roots, that $3b^2 > 8ac$ and P_0'' has two real roots, denoted t_- and t_+. As $P_0''(t) = 12at^2 - 6bt + 2c$, we obtain that:

$$t_\pm = \frac{b}{4a} \pm \left(\left(\frac{b}{4a}\right)^2 - \frac{c}{6a} \right)^{\frac{1}{2}}.$$

We proceed by contradiction and assume that P_0 has two roots $\in]0,1[$.A table of variations of P_0 and its derivatives show that this assumption implies $P_0'(1) < 0$ and $t_- < 1$.

$$\begin{aligned} 0 < \frac{b}{4a} - 1 < \left(\left(\frac{b}{4a}\right)^2 - \frac{c}{6a} \right)^{\frac{1}{2}} &\iff 1 - \frac{b}{2a} < -\frac{c}{6a} \\ \iff \quad 6a - 3b + c < 0 \iff & \quad P_0''(1) < 0. \end{aligned}$$

This is in contradiction with property 4.A.8, both the first and second derivatives of P_0 can not be negative in 1. Thus P_0 has at most one root in $]0,1[$. ■

Property 4.A.3 *There exist a neighbourhood of 1 where both $Q(t) < 0$ and $S(t) < 0$*

Proof: From their definition, it follows that both Q and S are strictly negative in 1, so there is a neighbourhood of 1 were each of them is negative. On the intersection of these neighbourhoods they are both negative. ■

Property 4.A.4 $\forall \xi > 0, \gamma \geq 0$ *or* $\forall \gamma > 0, \xi \geq 0$ *(that is on any point in the dashed domain on Figure 4.A.1),* $P_{\gamma,\xi}$ *has only one root within the unit disk. This root is in* $]0,1[$ *and if it has complex roots they remain outside the unit disk.*

Proof: We consider the dynamic of the roots of $P_{\gamma,\xi}$ starting from the limit case P_0. Due to property 4.A.1, the real roots of $P_{\gamma,\xi}$ can not be negative. Property 4.A.2 implies that if one is a single root of P_0, there is at most one root of P_0 in $]0,1[$, 1 is another of its roots and the remaining real roots, if any, are strictly greater than one. The other possible configuration is, 1 is a double root of P_0, the other roots are real and strictly greater than one. Applying lemma 4.A.1 to this configuration shows that:

- if P_0 has a root strictly smaller than one, then $\forall \xi > 0, \gamma \geq 0$ or $\forall \gamma > 0, \xi \geq 0$ the root 1 will shift to the right,
- if P_0 has no root strictly smaller than one, then $\forall \xi > 0, \gamma \geq 0$ or $\forall \gamma > 0, \xi \geq 0$ the root 1 will shift to the left,
- if 1 is a double root of P_0, then $\forall \xi > 0, \gamma \geq 0$ or $\forall \gamma > 0, \xi \geq 0$, the root 1 will *split* on both sides of the unit disk.

Still in application of lemma 4.A.1, 1 is a repulsive point for the dynamic of the roots of $P_{\gamma,\xi}$: with γ and ξ increasing, if a root of $P_{\gamma,\xi}$ enters the neighbourhood of 1 were both Q and S are negative, this root will be pushed back to the left (resp. right) if it is smaller (resp. larger) than one.
Thus $\forall \xi > 0, \gamma \geq 0$ or $\forall \gamma > 0, \xi \geq 0$, $P_{\gamma,\xi}$ has one real root in $]0,1[$ and its other real roots are larger than 1.

If $P_{\gamma,\xi}$ has complex roots, it can have only two complex conjugate roots, since $P_{\gamma,\xi}(0) > 0$, $lim_{t\to+\infty} P_{\gamma,\xi}(t) = +\infty$ and $P_{\gamma,\xi}(1) < 0$. Let $r^{\pm}$ denote the real roots of $P_{\gamma,\xi}$ and z, $\bar{z}$ its complex roots. *A fortiori* $r^- < 1 < r^+$.

$$\begin{aligned} P_{\gamma,\xi}(t) &= a(t-r^+)(t-r^-)(t^2+|z|^2) && (4.20)\\ &= a(t^4-(r^++r^-)t^3+(|z|^2+r^+r^-)t^2-(r^++r^-)|z|^2t+r^+r^-|z|^2) && (4.21)\end{aligned}$$

By identification we get

$$r^+ + r^- = \frac{b}{a} \quad |z|^2 + r^+r^- = \frac{c_{\gamma,\xi}}{a} \quad (r^++r^-)|z|^2 = \frac{d_{\gamma,\xi}}{a} \quad r^+r^-|z|^2 = \frac{e_{\gamma,\xi}}{a}$$

$$\begin{aligned} r^- < 1 \Rightarrow r^+ < \frac{b-a}{a} \Rightarrow |z|^2 + r^+r^- = \frac{c_{\gamma,\xi}}{a} &< \frac{b-a}{a} + |z|^2 \\ \Rightarrow \frac{c_{\gamma,\xi} - b + a}{a} &< |z|^2 \\ \text{Property 4.A.6} \Rightarrow 3 < \frac{c_{\gamma,\xi} - b + a}{a} &< |z|^2 \end{aligned}$$

Hence, if $P_{\gamma,\xi}$ has complex roots, their modulus is strictly larger than one. ■

Property 4.A.5 $\forall \xi < 0$ *or* $\forall \gamma < 0$ *(that is on any point in the dotted domain on Figure 4.A.1),* $P_{\gamma,\xi}$ *has two or zero real roots and two or zero complex roots within the unit disk.*

Proof: Symmetrically to the previous one, this property derives from the application of lemma 4.A.1 with $\xi < 0$ or $\gamma < 0$ in the neighbourhood of one.

- if P_0 has a root strictly smaller than one, then with $\xi < 0$ or $\gamma < 0$ the root 1 shifts to the left,
- if P_0 has no root strictly smaller than one, then with $\xi < 0$ or $\gamma < 0$ the root 1 shifts to the right,
- if 1 is a double root of P_0, then with $\xi < 0$ or $\gamma < 0$ the root 1 becomes two complex conjugate roots.

From this configuration, 1 is a repulsive point for the dynamic of the roots of $P_{\gamma,\xi}$ which implies that this configuration remains with γ or ξ decreasing.
As for the complex roots, they are conjugate thus either both inside or outside the unit disk. ■

Theorem 4.A.1 *For every* $X \in D_X$*, if* $\Phi_y = 0$*, for all* $(\Phi_p, \Phi_w) \in (\mathbb{R}_+^*)^2$ *, model* (4.3) *to* (4.7) *has a unique and stable solution* ***if and only if*** $\Phi_p + \Phi_w > 1$

Proof: Properties 4.A.4 and 4.A.5 imply that for all admissible values of γ and ξ (namely such that $\Phi_p > 0$ and $\Phi_w > 0$), $P_{\gamma,\xi}$ has only one root smaller than one in modulus and three roots larger than one in modulus **if and only if** $\gamma + \xi > 0$, i.e $\Phi_p + \Phi_w > 1$. The Blanchard and Kahn (1980) condition implies that this configuration of the roots of $P_{\gamma,\xi}$ is equivalent to the model's solution determinacy. ■

4.A.2 The case for $\Phi_y \neq 0$

A priori the frontier of indeterminacy in this case can be written $\phi_p + \phi_w = 1 - \theta$, with θ positive, decreasing with Φ_y and to be determined (Galí, 2008). We introduce the following parametrization:

$$\Phi_p = \phi_p + \xi \qquad \Phi_w = 1 - \phi_p - \theta + \gamma \qquad \Phi_y = \phi_y + \zeta \tag{4.22}$$

$$0 < \phi_p < 1 \qquad 0 < \theta < 1 - \phi_p \tag{4.23}$$

$$\xi \text{ s.t } \quad \Phi_p > 0 \qquad \gamma \text{ s.t. } \quad \Phi_w > 0 \qquad \zeta \text{ s.t. } \quad \Phi_y > 0 \tag{4.24}$$

The characteristic polynomial P of A is now defined by:

$$P_{\gamma,\xi,\zeta}(t) = at^4 - b_\zeta t^3 + c_{\gamma,\xi,\zeta} t^2 - d_{\gamma,\xi,\zeta} t + e_{\gamma,\xi,\zeta}.$$

It is equivalent to (4.16) with augmented coefficients:

$$\begin{aligned}
a &= \sigma\beta^2 \\
b_\zeta &= \beta[\kappa_p + \Phi_y\beta + \sigma(2 + 2\beta + \lambda_p + \lambda_w)] \\
c_{\gamma,\xi,\zeta} &= \kappa_p[1 + \beta + \lambda_w + \beta\xi] + \kappa_w[\lambda_p + \beta\gamma] + \sigma[1 + 4\beta + \beta^2 + (\lambda_p + \lambda_w)(1 + \beta)] \\
&\quad + \beta\phi_p\kappa_p + \beta(1 - \theta - \phi_p)\kappa_w + \Phi_y\beta[\lambda_p + \lambda_w + 2 + \beta] \\
d_{\gamma,\xi,\zeta} &= \kappa_p[1 + \lambda_w(1 + \gamma + \xi) + (1 + \beta)\xi] + \kappa_w[\lambda_p(1 + \gamma + \xi) + (1 + \beta)\gamma] \\
&\quad + \sigma[2(1 + \beta) + \lambda_p + \lambda_w] + (1 + \beta)\kappa_p\phi_p + (1 + \beta)\kappa_w(1 - \theta - \phi_p) \\
&\quad + \Phi_y[1 + \lambda_w + \lambda_p + 2\beta] - (\kappa_p\lambda_w + \kappa_w\lambda_p)\theta \\
e_{\gamma,\xi,\zeta} &= \sigma + \kappa_p\xi + \kappa_w\gamma + \kappa_p\phi_p + \kappa_w(1 - \theta - \phi_p) + \Phi_y.
\end{aligned}$$

We define the baseline polynomial by choosing $\{\theta, \phi_y\}$ such that $P_0(1) = 0$, which implies that θ is proportional to ϕ_y:

$$\theta = \phi_y \frac{(1 - \beta)(\lambda_p + \lambda_w)}{\kappa_w\lambda_p + \kappa_p\lambda_w}. \tag{4.25}$$

Our limit case, i.e. with $\zeta = \gamma = \xi = 0$, is:

$$\phi_p + \phi_w + \phi_y \frac{(1 - \beta)(\lambda_p + \lambda_w)}{\kappa_w\lambda_p + \kappa_p\lambda_w} = 1 \tag{4.26}$$

Let now X denote the extended vector of parameters:

$$X = [\beta, \phi_p, \theta, \kappa_p, \kappa_w\lambda_p, \lambda_w, \sigma] \quad \in]0,1[^3 \times (\mathbb{R}_+^*)^5 = D_X. \tag{4.27}$$

We can decompose polynomial $P_{\gamma,\xi,\zeta}$ similarly to (4.17):

$$P_{\gamma,\xi}(t) = P_0(t) + \gamma\,\kappa_w\,Q(t) + \xi\,\kappa_p\,S(t) + \zeta\,T(t) \qquad \text{where} \tag{4.28}$$

$$Q(t) = \beta\,t^2 - [\lambda_w \frac{\kappa_p}{\kappa_w} + (1 + \beta + \lambda_p)]\,t + 1 \tag{4.29}$$

$$S(t) = \beta\,t^2 - [\lambda_p \frac{\kappa_w}{\kappa_p} + (1 + \beta + \lambda_w)]\,t + 1 \tag{4.30}$$

$$T(t) = -\beta\,t^3 + \beta\,(\lambda_p + \lambda_w + 2 + \beta)\,t^2 - (1 + \lambda_p + \lambda_w + 2\beta)\,t + 1 \tag{4.31}$$

(4.32)

From the definition of $P_{\gamma,\xi,\zeta}$ and its coefficients, we can generalize property 4.A.1 to the case $\Phi_y \neq 0$. The limit case polynomial also verifies property 4.A.2 (the proof is identical). Since $T(1) < 0$, we can generalize property 4.A.3 as well. All the developments derived in the case $\Phi_y = 0$ apply and theorem 4.A.1 can be generalized:

Theorem 4.A.2 *For every $X \in D_X$, for all $(\Phi_p, \Phi_w, \Phi_y) \in (\mathbb{R}_+^*)^3$, model (4.3) to (4.7) has a unique and stable solution **if and only if** $\Phi_p + \Phi_w + \Phi_y \frac{(1-\beta)(\lambda_p+\lambda_w)}{\kappa_w\lambda_p+\kappa_p\lambda_w} > 1$*

4.A.3 Some useful properties

On the polynomial coefficients if $\Phi_y = 0$

Property 4.A.6 *The following inequalities hold:*

$$\forall X \in D_X \quad \forall \gamma \geq 0 \quad \forall \xi \geq 0 \qquad b > 4a \qquad c_{\gamma,\xi} > 2a + b > 6a \qquad e_{\gamma,\xi} > a$$

The same inequalities hold for the coefficients of $P_{\gamma,\xi,\zeta}$ when $\Phi_y \neq 0$.

Proof: These inequalities derive directly for the definitions of the coefficients. ■

Property 4.A.7 *$\forall X \in D_X$, $\forall \gamma \geq 0$, $\forall \xi \geq 0$ $P_{\gamma,\xi}$ has two conjugate complex roots if and only if $3b^2 < 8ac_{\gamma,\xi}$*
The same property applies to $P_{\gamma,\xi,\zeta}$ when $\Phi_y \neq 0$.

Proof: The discriminant of the second order derivative $P''_{\gamma,\xi}$ is $\Delta(P''_{\gamma,\xi}) = 12\left[3b^2 - 8ac_{\gamma,\xi}\right]$; if it is negative, this polynomial is positive for every t. In this case, $P'_{\gamma,\xi}$ is strictly increasing and has only one real root, and $P_{\gamma,\xi}$ is strictly decreasing and then strictly increasing. As we already know that it has two real roots, the two others are complex. ■

Property 4.A.8 *Whether $\Phi_y = 0$ or not, with the respective definitions of polynomial P_0 and the parameters domain D_X : for all $X \in D_X$, $P_0'(1) = 4a - 3b + 2c - d$ and $P_0''(1) = 12a - 6b + 2c$ cannot be both negative or null.*

Proof: If $\Phi_y = 0$:

$$\begin{aligned} P_0'(1) =& 4a - 3b + 2c - d \\ =& \kappa_p\lambda_w + \kappa_w\lambda_p - (1-\beta)\left((1-\phi_p)(\kappa_w - \kappa_p) - \sigma(\lambda_p + \lambda_w)\right) \end{aligned}$$

$$\begin{aligned} \frac{1}{2}P_0''(1) =& 6a - 3b + c \\ =& \kappa_p[1 - \beta + \lambda_w] + \kappa_w\lambda_p + \sigma[(1-\beta)^2 + (\lambda_p + \lambda_w)(1-\beta)] \\ &+ \beta\left((1-\phi_p)(\kappa_w - \kappa_p) - \sigma(\lambda_p + \lambda_w)\right) \end{aligned}$$

It is a necessary condition for $P_0'(1)$ to be negative or zero that:

$$(1-\phi_p)(\kappa_w - \kappa_p) - \sigma(\lambda_p + \lambda_w) > 0 \tag{4.33}$$

However, if this is the case, then

$$\frac{1}{2}P_0''(1) > \kappa_p[1 - \beta + \lambda_w] + \kappa_w\lambda_p + \sigma[(1-\beta)^2 + (\lambda_p + \lambda_w)(1-\beta)] > 0 \tag{4.34}$$

If $\Phi_y \neq 0$:

$$\begin{aligned}P_0'(1) =& 4a - 3b + 2c - d \\ =& \kappa_p\lambda_w + \kappa_w\lambda_p - (1-\beta)\left((1-\phi_p)(\kappa_w-\kappa_p) + \phi_y(1-\beta) - \sigma(\lambda_p+\lambda_w) - \theta\kappa_w\right)\end{aligned}$$

$$\begin{aligned}\frac{1}{2}P_0''(1) =& 6a - 3b + c \\ =& \kappa_p[1-\beta+\lambda_w] + \kappa_w\lambda_p + \sigma[(1-\beta)^2 + (\lambda_p+\lambda_w)(1-\beta)] + \phi_y\beta(\lambda_p+\lambda_w) \\ &+ \beta\left((1-\phi_p)(\kappa_w-\kappa_p) + \phi_y(1-\beta) - \sigma(\lambda_p+\lambda_w) - \theta\kappa_w\right)\end{aligned}$$

It is a necessary condition for $P_0'(1)$ to be negative or zero that:

$$(1-\phi_p)(\kappa_w-\kappa_p) + \phi_y(1-\beta) - \sigma(\lambda_p+\lambda_w) - \theta\kappa_w > 0 \tag{4.35}$$

However, if this is the case, then

$$\frac{1}{2}P_0''(1) > \kappa_p[1-\beta+\lambda_w] + \kappa_w\lambda_p + \sigma[(1-\beta)^2 + (\lambda_p+\lambda_w)(1-\beta)] + \phi_y\beta(\lambda_p+\lambda_w) > 0 \tag{4.36}$$

■

Real roots shift

Lemma 4.A.1 *Let P be a polynomial and λ a simple or double root of this polynomial. If the root is simple ($P(\lambda) = 0$ but $P'(\lambda) \neq 0$) and we add a real quantity q sufficiently small to the polynomial, it translates the root λ in the direction defined by the sign of $-P'(\lambda)\,q$, + defining a translation to the right, - to the left.*
If the root is double ($P(\lambda) = 0$, $P'(\lambda) = 0$ but $P''(\lambda) \neq 0$) and we add a real quantity q sufficiently small to the polynomial, the root becomes two distinctive roots on both sides of λ if $-P'(\lambda)\,q > 0$ and become complex roots otherwise.

Proof If $P(\lambda) = 0$ but $P'(\lambda) \neq 0$, the property is a direct application of the implicit function theorem to $f(t,q) = P(t) + q$ in the neighbourhood of $(t,q) = (\lambda, 0)$.
Let's assume that $P(\lambda) = 0$, $P'(\lambda) = 0$ and $P''(\lambda) > 0$. For regularity, we consider a neighbourhood V of λ where $P'' > 0$. In V, a table of variation of P and its derivatives shows that P is strictly positive except in λ. If we add a positive q, arbitrarily small, to polynomial P, $P+q$ becomes strictly positive on V. By a continuity argument the double root λ became two conjugate complex roots.
Also, there exists $\delta > 0$, arbitrarily small, and $\lambda^- < \lambda < \lambda^+$ such that $P(\lambda^-) = P(\lambda^+) = \delta$. If we subtract q, arbitrarily small, to polynomial P, $P(\lambda) - q$ is strictly negative while $P(\lambda^-) - q = P(\lambda^+) - q = \delta - q$ can be set strictly positive. Since P' is strictly monotonous on V, $P - q$ has two roots in V, one on each side of the original root λ.
All in all, if $P''(\lambda) > 0$, the double root become complex if we add a positive quantity, arbitrarily small, to P and two real roots on each side of the initial one if this quantity is negative. A symmetric argument completes the proof for the case if $P''(\lambda) < 0$. ■

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Chapter

5

Households Satellite Account for France. Methodological issues on the assessment of domestic production

with Delphine Roy

We estimate in a household satellite account (HHSA) the value of French domestic production in 2010 and 1998, using the input method and following Eurostat's recommendations. In line with previous studies, we find that extending the system of national accounts (SNA) frontier of production to domestic activities (house chores, cooking, care...) has a sizeable effect on key macroeconomic indicators (+33% GDP, -5 p.p. GDP growth, +50% disposable income, +58% consumption, and -10 p.p. of purchasing power growth).
We conduct a sensitivity analysis to various methodological issues which have not yet been settled by an international benchmark. Quantitatively, the two most important issues are the boundary of household production -we favour a relatively narrow definition- and the use of a gross or a net wage -we prefer gross wage-. However, estimates are much less sensitive to otherwise greatly debated issues such as which substitute wage to use.

JEL-code: *D13, E01*

Keywords: *unpaid work, domestic production, household satellite account, time-use survey*

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5.1 Introduction

From their daily domestic work, households produce services they directly consume. No monetary transaction takes place to record this process. Whether a family has dinner at home or in a restaurant, they consume a meal which has been cooked. The same goes for the shirts they clean and iron themselves instead of taking them to the dry cleaner: in both cases, a service is produced and consumed, thus participating to the material well-being of households. But in one case, there is a market transaction, and the consumption is recorded by system of national accounts (SNA), whereas in the other case, it goes unrecorded because of a lack of market transaction.

As the Sen-Stiglitz-Fitoussi report on the measurement of economic progress recently pointed out (Stiglitz et al., 2009), this is not without consequence for international comparisons of households' consumption across countries that differ in their reliance on the market for the provision of household services (the US vs European countries vs developing countries, for instance). This caveat is not new to economists: some of them have worked on the valuation of hours of unpaid work recorded by time use surveys (TUS) in the last decades and pioneer works date back to the thirties. It is not new to national accountants either: the SNA in its 1993 edition (IMF et al., 1993) exposes the limitations of its production boundary, in particular with respect to household production, but for conceptual and technical reasons it consigned the measurement of domestic production to a satellite account.

An attempt to overcome such limitations of the SNA is the recent development of households satellite accounts (HHSA). It consists in additional tables, compatible with the SNA framework, describing the economic transactions (monetary or not) related to domestic production. The HHSA affects households' production and consumption, but also their income. It also marginally impacts their investment. These modifications may have marked consequences on their savings ratio and the purchasing power of their disposable income. Also, care must be taken not to disrupt the fragile balance of the SNA; in particular not to create monetary counterparts to non-monetary transactions or not to record only one side of a transfer between two categories of agents.

The present paper is an attempt at implementing such principles in the design of a households satellite account for France in 2010 and 1998. Quantitatively, our estimates confirm previous works on the magnitude of domestic production with regard to GDP and consumption. In line with previous studies in many developed countries, we find that remodelling the frontier of production to include domestic activities (house chores, cooking, care...) has a sizeable effect on key macroeconomic indicators (GDP +33%, disposable income +50%, consumption +58%, savings ratio −4 percentage points in 2010 and respectively +31%, +49%, +56% and −4 p.p. in 1998). The proportion of the three inputs of household production and its distribution by functions (housing, food, care...) are very similar in 2010 and 1998 and these household productions are much larger than expenditure on market equivalents. In addition, macroeconomic evolutions are markedly impacted: the purchasing power of gross disposable income is much less dynamic in HHSA than in SNA (+17% against +27%) and GDP growth is scaled down by 5 points (from 20% in SNA to 15% in HHSA).

The estimation of HHSA aggregates is very sensitive to methodological choices. A vast strand of literature exists on these issues. In the wake of work done in the 1990s, a European task force made a first set of recommendations in 2003 (Eurostat, 2003). Still, a consensus has yet to emerge on several points. These points, and their relative importance for international comparisons of HHSA, are the object of this paper.

A crucial matter for harmonisation is the frontier between domestic work (a productive activity but excluded from the SNA) and leisure (non productive). In its broadest definition, domestic work can be twice as large as it is in a more restrictive sense. Because its components are more consensual and less subject to an overestimation of productivity, we favour the narrowest definition of domestic production (hereafter *core perimeter*).

A greatly debated question is the choice of the wage to value time spent on various domestic activities. The *generalist substitute* method levels out any composition effect of domestic productions. In practice, such a composition effect seems marginal and hard to disentangle from the statistical noise between successive time use surveys. We therefore believe that this does not constitute an obstacle to the use of the generalist substitute method. Also, using the *specialised* or *generalist* substitutes has a secondary effect on the estimates. In both cases, whether the wage is net of taxes and social contributions or gross is a key issue. Their inclusion raises other difficulties, in particular for the interpretation of the savings ratio and thus is a key issue for harmonisation.

The evaluation of services provided by households durables is also a difficult issue. First it raises the question of identifying domestic capital among consumption in durable goods; and then measuring the services of this new capital. A simplified perpetual inventory method (PIM) can be used, but remains highly conventional. This aspect of HHSA is to be improved in the future.

Some other accounting questions can be solved by weighing up the pros and the cons of sophistication: a parsimonious HHSA should not modify taxes and subsidies on production or changes in inventories, nor arbitrarily distribute ancillary functions of domestic production to principal functions. It can however easily avoid double counts of *production for own final use* already recorded by the SNA.

The remainder of this paper is organized as follows: section 5.2 reviews the literature on HHSA; section 5.3 deals with the definition and valuation of hours of unpaid work, and quantifies the effects of related methodological choices, for 2010; section 5.4 deals with several issues specific to the national accounts and the HHSA, exemplified on the year 2010 as well; finally section 5.5 presents our estimation of the HHSA for France in 2010 and 1998.

5.2 Domestic production amounts to 30 to 50% of GDP in most studies

An old debated question The measurement of domestic work and domestic production is an old debated question in national accounts as recalled by Vanoli (2002). In the related literature, one may find references dating back to the 19th century (Charlotte Perkins Gilman (Women and Economics 1898)), or to the 1930s and 1940s such as Margaret Reid (Economics of Household Production, 1934), Wassily Leontief (The Structure of the American Economy, 1941), cited in (Ironmonger, 2000) or S. Kuznets, L. Epstein and W. I. King, H. Kirk, W. C. Mitchell cited by Chadeau and Fouquet (1981), Alfred Marshall (Principles of economics: An introductory volume, 1920), Arthur Pigou (The Economics of Welfare, 1932) in (Abraham and Mackie, 2006) or Lindahl *et al* (1937), Wesley C Mitchell *et al* (1921), Kuznets (1941) in (Vanoli, 2002). This question is also a matter of history (Folbre and Wagman, 1993) since the prevalence of market over informal economy is relatively recent in economic history.

Hawrylyshyn (1976) reviews some of the early quantitative studies on domestic work, from the second half of the 20th century. They mostly deal with the US, but pioneer quantifications were performed in Nordic countries as well (Denmark, Norway, Sweden). Over the last 30 years, many authors have investigated this issue, mainly through time use surveys and the valuation of hours worked for domestic production. We found references to 27 national or regional economies[1] where at least hours of domestic work have been converted to monetary equivalents. Chadeau (1992) reviews such work in 7 countries; Goldschmidt-Clermont and Pagnossin-Aligisakis (1995) and Goldschmidt-Clermont and Pagnossin-Aligiasakis (1999) do so in 14 countries.

Over the last 15 years or so, the focus has shifted from the valuation of productive time to the construction of households satellite accounts as suggested by the SNA (IMF et al., 1993). In addition to the long debated questions that have already been raised by hours worked and their valuation, the production of HHSA yields specific issues. The interested reader will find them expounded in (Eurostat, 2003), (Varjonen and Aalto, 2006) for the European Input approach, in (Abraham and Mackie, 2006), (Nordhaus, 2006) for the US Input approach and (Holloway et al., 2002) for the UK's Output approach.

Similarities of the estimates despite methodological differences The valuation of time in different countries and at different dates usually concurs to the same (blurry) picture of domestic work.

[1]Australia, Austria, Basque Country, Bulgaria, Canada, Finland, France, Germany, Hungary, Italy, Israel, Japan, Luxembourg, Madrid, Mexico, Nepal, Netherlands, New-Zealand, Norway, Portugal, Russia, Slovenia, South-Africa, Sweden, Switzerland, United Kingdom, United States, but unfortunately we could find less than half of the referenced papers.

Hours of unpaid work are at least equal to hours of paid work (Chadeau, 1992), (Goldschmidt-Clermont and Pagnossin-Aligisakis, 1995), and (Roy, 2012). The value of this time can be estimated using various sets of assumptions and methodologies. These choices account for an important share of the estimates' dispersion in the literature (Chadeau, 1985). In his review, Hawrylyshyn (1976) corrects such methodological differences and finds that "housework is about a third of GNP". Chadeau (1985), Chadeau (1992) or Goldschmidt-Clermont and Pagnossin-Aligiasakis (1999) find ratios closer to 40%.

Beyond working time, domestic production has been estimated in several countries. Accounting for this production in households' consumption (as households are both producers and consumers of domestic production) substantially modifies the national accounts figures. For 6 countries, in the seventies-eighties, using the specialist substitute method, Chadeau (1992) finds that, prior to any adjustment, domestic work alone would increase households' consumption by 57% to 83%. Goldschmidt-Clermont and Pagnossin-Aligisakis (1995) find for Finland, Germany and Bulgaria in the late eighties-early nineties, that once domestic production is accounted for it is equal to 60% of total consumption.

Using the HHSA framework, Landefeld and McCulla (2000), Landefeld et al. (2009), Varjonen and Aalto (2006), Hamunen et al. (2012), Eustat (2004), Holloway et al. (2002) and Ruger and Varjonen (2008) have estimated domestic production for the US, Finland, the Basque Country, the UK and Germany, respectively. Table 5.1 gathers some of their results showing the impact of HHSA on key macroeconomic aggregates. Accounting for all the inputs of domestic production has a sizeable effect on major macroeconomic aggregates. For the US and the Basque Country, this effect diminishes with time. According to the authors, this can be explained by a greater access of women to the labour market. Also, accounting for domestic production can have a marked impact on savings ratios in both directions: British households' savings ratio would be negative, while Finnish one's would be closer to zero.

Inputs for domestic production as described by the HHSA come in quite similar shares across countries (see Table 5.A.1 in Appendix 5.A). Net value added is equal to approximately three fourth of production while capital consumption is the smallest of the three components. However, the comparison between (Ruger and Varjonen, 2008) and (Varjonen and Aalto, 2006) for Finland in 2001 shows that international comparisons going beyond orders of magnitude are fragile: Ruger and Varjonen (2008) revise initial estimates of the German and Finish HHSA so as to make them comparable, which significantly modifies the estimates both in absolute and relative terms.

Gross Value Added by function of domestic production are broadly similar in the UK, the Basque Country and Finland (see Table 5.A.2 in Appendix 5.A), but it is impossible to say which part of the differences stems from national specificities or from the methodology. In particular, with their output method, Holloway et al. (2002) have an extensive approach of child care and accommodation for the UK. However, the accommodation function is

Country	Source	Year	Revisions GDP	Revisions Cons.	Revisions GFCF	Income and Savings ratio Income	SNA$^{\triangle}$	non-SNA$^{\triangle}$
USA	a	1946	+50%	+63%	+50%	+59%	8.3%	10.8%
USA	b	1965	+39%	+49%	+50%	+49%	8.6%	11.5%
USA	a	1997	+36%	+34%	+54%	+38%	1.8%	8.5%
USA	b	2004	+27%	+26%	+48%	+32%	1.8%	4.2%
Finland	c	2001	+40%	+59%	+60%	+81%	-1.2%	0.2%
Finland	d	2006	+39%	+55%	+47%	+77%	-1.8%	-0.3%
Basque Country	e	1993	+49%	+74%*	-	-	-	-
Basque Country	e	1998	+39%	+64%*	-	-	-	-
Basque Country	e	2003	+33%	+56%*	-	-	-	-
UK	f	2000	+63%*	+95%*	+98%*	+93%*	4.2%	-6.9%*
Finland	g	2001	+36%	-	-	-	-	-
Germany	g	2001	+43%	-	-	-	-	-

*: our calculations, -: not available

$^{\triangle}$: SNA refers to production and other concepts as defined by the System of National Accounts, while non-SNA refers to the extension of these concepts for the purpose of the households satellite account.

a: (Landefeld and McCulla, 2000), b:(Landefeld et al., 2009), c:(Varjonen and Aalto, 2006), d:(Hamunen et al., 2012), e:(Eustat, 2004), f:(Holloway et al., 2002), g: (Ruger and Varjonen, 2008)

GFCF: household gross fixed capital formation, Cons.: household final consumption includes individual consumption except for Basque Country

Table 5.1 – HHSA estimates in 5 countries: effects on key macroeconomic aggregates

similar across countries which hides the fact that for the UK, figures cover both SNA (imputed rents) and non-SNA[2] production for own use. Care covers twice as much gross value added than in the other two countries. Also, the small fraction of food production in UK households could reflect the stereotypical lack of interest in food of the British, but can also be explained by the way ancillary functions (transportation in particular) are reallocated to other principal functions.

From these comparisons we can draw the following conclusion: our estimation should be broadly in line with others, but in details, comparisons can not be made without specific adjustments. The corollary conclusion is that there is a great need for a benchmarking method from which historical and international comparisons can be made.

5.3 The accounting and valuation of hours of domestic work

5.3.1 Time-use surveys: towards harmonisation

We use the latest two French time use surveys, whose fieldwork took place over the years 1998 and 2010 respectively. They are representative of the non-institutional population

[2] Non-SNA refers to the concepts of production and other operations for the purpose of households satellite accounts as opposed to SNA referring to the definitions in the system of national accounts.

of mainland France, and in 2010, of 3 overseas *départements*. In 1998, all members of a household above 14 were interviewed and each of them had to fill in one time-use diary. This yielded 15450 diaries, filled by just as many individuals belonging to 7950 households. In 2010, one individual was selected in each sampled household, among its members aged 11 and above. His/her spouse or partner, if there was one, was also interviewed. This time, respondents were given two diaries to fill in, one for a weekday and one for a weekend day. The 2010 sample eventually consisted of 12 000 households, 18 500 interviewed individuals, 27 900 diaries. In 2010, the activity list comprises roughly one hundred basic activities, in compliance with Eurostat's Guidelines on Harmonised Time Use Survey (Eurostat, 2008). The diary and activity list were improved between 1998 and 2010. In particular, the location/means of transportation was added to the description of each activity. This makes some distinctions possible in 2010 but not in 1998. For example, car use can be measured directly in 2010, but hypotheses have to be made to estimate it for 1998. Other issues for the comparison of 1998 and 2010 are addressed in section 5.5.

Household surveys and National Accounts have different scopes In 2010, the sum of the weights of the respondents with a diary is 54,4 millions, when the total population of France was 64,6 on Jan 1st, 2010. The difference consists of: inhabitants of French Guyana; children under 11; residents of institutions such as care homes, boarding schools, prisons; and students on campuses. The amount of unpaid domestic work made in France over the year 2010 estimated from time use data will therefore exclude Guyana, and using it as our estimate implies making 2 additional hypotheses:

- The amount of domestic work made by children under 11 is negligible, which seems to be a realistic assumption, in a developed country like France at least.

- Residents of institutions do little domestic work. This assumption is standard in this literature and seems acceptable since by definition, most of these institutions provide cooking, cleaning, etc. for their residents.

When computing growth rates between 1998 and 2010, we do so on the scope covered by the 1998 survey: mainland France, and people aged 15 and above. Results in level are based on 2010 on the corresponding scope: mainland France and three overseas *départements*, and people aged 11 and above.

Since their onset, time use surveys have been at the heart of an international community of researchers, and they are fairly comparable across countries, as regards their activities coding list in particular. At the European level, most countries follow Eurostat's Guidelines (Eurostat, 2008). Thus the data exist to compute comparable estimates of hours of unpaid work. The crux of the problem is to agree on which activities to include in domestic work.

5.3.2 Defining domestic work

The definition is debatable, we test three possibilities

The question is not so much to give a theoretical definition of domestic work, as it is to decide where to set the boundary between productive and unproductive activities. Our view is that a consensus can not be reached solely from any set of criteria. Yet, estimates are very sensitive to the definition of domestic work, so that the agreement on a boundary is one of the keys to make international comparisons possible.

We favour a restrictive definition (*core* perimeter) of domestic work for three main reasons: all its elements are commonly accepted as productive, it is *a priori* the easiest to measure across countries, and it is less subject to an overestimation of productivity (see 5.3.3 and 5.4.2), a key issue for the input method.

Drawing the frontier of production across the *grey zone* The *third party criterion* is usually the cornerstone of the definition of domestic production: "If an activity is of such character that it might be delegated to a paid worker, then that activity shall be deemed productive" (Reid, 1934, p.11) cited by Ironmonger (2000). Being too inclusive, this criterion has been completed with the reference to social norms: "the third party criterion comes up against borderline cases which must be resolved by reference to normal social practice and standards" (Chadeau, 1992).

However, there may remain ambiguous cases and these criteria should be seen as general guidelines more than golden rules. If sexual intercourse is identified as an important activity for well being (Stiglitz et al., 2009), it is chastely eluded in the literature on HHSA. However, it meets both criteria suggested by Reid and Chadeau. It can be delegated to a third party outside the household (sometimes to the detriment of the institution of marriage). Prostitution also exists (legal or tolerated) in most countries. However, we find it hard to argue that unpaid sexual intercourse within the household should be deemed productive.

Here, national accounting encounters anthropological issues that we are bound to leave unresolved within the scope of this paper. Following the third party criterion to the letter, we could have included sexual intercourse in our most extensive perimeter since it is part of the "grey zone". In practice, we totally lack the data to conduct this thought experiment.

The point of this far-fetched counterexample is: the SNA frontier of production is conventional and imperfect, the frontier of domestic production will be just as much. Comparability comes at this cost: somehow arbitrary, but unified conventions.

Including the *grey zone* could double the duration of domestic work House chores, cooking, taking care of a dependent adult, driving children to their football class...

are commonly accepted as *productive* activities. On the other hand, breathing, sleeping and eating are undebated examples of *non-productive* activities. But beyond these core physiological activities lies a wide *grey zone* of daily actions that can be considered productive or not. The literature on domestic work traditionally relies on two criteria to sort productive and non productive activities, without solving all the conflictual cases. In order to highlight the impact of methodological choices on estimates of household production, we define 3 possible perimeters of domestic work, from the most restrictive (the *core* definition) to the most inclusive (the *extensive* definition) (see Table 5.1). The *extensive* perimeter is almost twice as large as the *core* perimeter, both in terms of duration and imputed value. The choice of a particular perimeter also has implications for the other inputs of domestic production (see 5.4.3 and Appendix 5.C for a synthesis of these implications).

Perimeter	core (I)	intermediate (II)	extensive (III)
Included Activities	cooking, dish washing, household upkeep, cleaning, child and adult care, laundry, household management, driving children or others	(I) and shopping, home repair, gardening, playing with children	(II) and driving oneself, walking the dog
Daily	2 h 07	3 h 04	3 h 53
Weekly	14 h 50	21 h 30	27 h 14
Share of volunteer work	3.7%	3.8%	5%
Women's Share	72%	64%	60%

Coverage: individuals aged 11 and over, France (excl. French Guyana and Mayotte).
Source: Insee, Time Use Survey 2010.

Table 5.1 – Average working time per person for three possible perimeters of domestic work in 2010

1. The *core* perimeter consists of only those activities that every study in the literature agrees to be productive: chores such as cleaning, doing the laundry, the dishes, etc.; cooking; material care to and driving children and disabled persons; household management. All these routine tasks can be delegated and many households use market substitutes for them.

2. The *median* perimeter adds to the first list a number of activities that belong to the *grey zone*, either because they border on leisure (*productive leisure* such as gardening, home repairs and decoration, fishing and hunting, picking berries...) and are probably performed less efficiently than in a professional context, or because their utility lies (at least partly) in the process itself and their delegability can be questioned

(productive leisure, playing with children). Shopping is also classified here because in our data, we cannot distinguish everyday grocery shopping, a productive chore, from *window shopping* or shopping for pleasure.

3. The *extensive* perimeter furthermore contains travelling by car for oneself and walking the dog.

The activities included in the three perimeters, and the issues raised by their inclusion within the production boundary, are further discussed in Appendix 5.B.

5.3.3 The valuation of time

In the literature, three methods for valuing domestic work coexist: the *generalist substitute*, the *specialised substitute*, and the *opportunity cost* methods.

In the *generalist substitute* method, hours worked are valued using the hourly wage of a worker performing all tasks indifferently (e.g. *housekeeper*). In the *specialised substitute* method, each hour worked is valued using the hourly wage of a worker performing that task specifically (resp. *cook*, *housecleaner*, *handyman*...). In the *opportunity cost* method, hours worked are valued using the market hourly wage of the person performing the task (e.g. dentist wage when he is cooking).

We adapt the *specialised substitute* to avoid an overestimation of productivity. We also test 2 *generalist substitute* methods for a sensitivity analysis: the minimum wage and the housekeeper wage. We disregard the *opportunity cost* method. Disregarding it is standard in the literature on households satellite accounts, but we have an additional reason to do so: this method is a welfare economics method, while we perform a national accounting exercise. Also, we assess the sensitivity of time valuation to the treatment of imputed taxes and social contributions.

We use the least qualified specialist wage We value the time spent on each activity at the wage of the *specialised substitute* one would have to hire to do the job. This method is one of the methods suggested by Eurostat (2003). Of course, very few people have all the skills of a cook, a plumber, a childminder and a teacher at the same time, so this valuation might somewhat over-estimate the productivity of household work. But two elements allow us to mitigate this criticism. First, people tend to self-select out of the tasks they are very unproductive at. Very few economists do their own plumbing at home, for example. Second, the tasks that make up the greater part of unpaid work are not the most skilled ones: food preparation, housecleaning, child care. Nevertheless, there may remain some differences due to capital intensity and increasing returns to scale. In order to account for this, we choose the least qualified and least capital intensive job as our specialist substitute every time we have a choice: we value cooking time at the wage of a kitchen aide, not at that of a chef, and cleaning at the wage of a domestic cleaner, not an industrial one.

We favour the *core perimeter* which is less subject to an overestimation of productivity As mentioned in 5.3.2, we favour the core perimeter of domestic work for three main reasons: all its elements are commonly accepted as productive, it is *a priori* the most easily measurable across countries and it is less subject to an overestimation of productivity. Indeed, contrary to the core perimeter, the median perimeter includes many productive activities which can be performed for their own sake: gardening, sewing, knitting, handy-work... When considered as leisure by the households, using a market wage to value the hours worked will most surely overestimate the value of the output. An example of this overestimation is given by the *production for own final use* already accounted for in the national accounts, with an output method. In 5.4.2, we compare these figures with those derived from the input method using the TUS (see Table 5.1). The output method might be somewhat imprecise and conventional since it is not accurately measured each year, but the input method clearly overestimates the productivity of households in their kitchen garden or when they fish, hunt, pick-up mushrooms... For this reason, we favour the most restrictive perimeter of domestic work which contains mostly off-putting tasks that a majority of people would consider chores (apart from cooking in some countries): dish washing, house cleaning, laundry...

We do not consider the valuation of time through the opportunity cost method The *opportunity cost* method is fraught with well known difficulties, it implies imputing a potential market wage to all individuals outside of the labour market, e.g. at-home parents, retired persons... The usual argument to disqualify this method is the following: if one values domestic productive time with the market wage of the person performing the house chore then a dentist would implicitly be a much better cook than a bus driver: there is *a priori* no reason for this outcome to be right. However, it does not suffice to disqualify the opportunity cost method. Essentially, the market wage represents the opportunity cost only in the simplest microeconomic allocation of time model, where workers can freely allocate marginal amounts of time between market work, domestic work and unproductive activities (leisure). One could then argue that a refined model could allow to more appropriately measure the opportunity cost than equalizing it to the market wage. Beyond the difficulty of building such a model, our argument is more straightforward: the frontier between national accounts and welfare economics is drawn in such a way that the opportunity cost method is beyond the scope of the present exercise[3] (see also Landefeld and McCulla for more details).

[3]This should not be seen as an endorsement of the current distinction from our part; we leave to more experienced national accountants/economists the task of remodelling it, if need be.

	core perimeter	median perimeter	extensive perimeter	generalist substitute	minimum wage
Super gross	15.84	15.80	16,04	14.53	10.43
Net	9.57	9.55	9.65	9.01	6.95

Source: Insee, Time Use Survey 2010, DADS 2010 - our calculations

Table 5.2 – Substitute wage in € per hour in 2010

Wage sensitivity analysis

Generalist or specialised substitute (−8%) The generalist wage method consists in valuing all the hours of domestic work at the same rate. It is one of the methods suggested by Eurostat (2003): using the wage of a *generalist housekeeper*. This method might be preferred for an international benchmark since the data for the *specialised substitute* method are not available in every country (see Varjonen et al. Appendix 2). It is also the method used by Landefeld and McCulla (2000) and Landefeld et al. (2009) on US data.

However, the generalist wage method does not solve the issue of international comparability: even where the data do exist, the choice of the reference wage is not clear-cut and references are made, depending on the publication, to ISCO-88 categories 3221 (Medical assistants), 3231 (Nursing associate professionals), 512 (Housekeeping and restaurant services workers), 5121 (Housekeepers and related workers), 513 (Personal care and related workers), 9131 (Domestic cleaners and helpers), ISCO-08 category 5322 (Home-based personal care workers) or simply 51 (Personal services workers). With our data set, based on the French PCS classification, we would use *domestic services and cleaning workers* for the generalist substitute (PCS 563c *Employés de maison et personnels de ménage chez des particuliers*, which includes ISCO-08 5152 *Domestic housekeepers* and 9111 *Domestic cleaners and helpers*). Their hourly wage is equal to 14.5€ while the average wage of the specialised substitute lies between 15.8 and 16.0€ depending on the perimeter (see Table 5.2). The method of the generalist substitute scales down by less than 1.5€ the hourly wage, i.e. by 8%, the valuation of domestic work. The choice of a substitute wage is thus not the first priority to limit methodological discrepancies in comparison with the definition of domestic work and the treatment of social contribution and working time, at least in the French context.

Gross or net wages: a critical choice (−40%) Whether or not one should include all taxes and social contributions within the valuation of hours of unpaid work is a matter of perspective. If the main interest is household production, the imputed wage would include taxes and social contributions. If the main interest is household income, the imputed wage would be net of taxes and social contribution.

We choose to use *super gross* hourly wages (i.e., including all taxes and social contributions whether paid by the employer or the employee). The main reason for this choice is that it is coherent with the national accounts concept of *compensation of employees* (the total remuneration, in cash or in kind, payable by an enterprise to an employee in return for work done by the latter during the accounting period).

Using net wages (before income tax) would induce a 40% decrease in the valuation of domestic work. Thus, the decision to use one wage or the other is not marginal quantitatively, and it should be a priority in the agenda towards international harmonisation. It is also not benign for the interpretation of the savings ratio (see 5.4.9).

Moreover, the conversion from net to super gross wage raises its own conceptual issues. In France, paid domestic work is subject to tax rebates and subsidies. In particular, compensations paid by households to domestic personnel are partly subsidized by the general government for some specific domestic work (help for the elderly, handicapped people but also care of young children). Under some conditions, up to 50% of the compensations paid can be deducted from the employers' income tax. This tax rebate is treated as imputed subsidies by the ESA 2010. Should we include specific subsidies and tax rebates in imputed wages? For sake of simplicity, we chose not to.

The minimum wage (−34%) France has an hourly minimum wage (SMIC) below which workers can not legally be paid. By construction, this wage is lower than average wages recorded by our administrative wage data (DADS). Although the SMIC is almost a reflex for a French economist, using it to value hours of unpaid work will not allow for international comparisons as not all countries have a minimum wage, and existing ones are not even comparable. In some countries, the minimum wage is very low compared to the mean wage, and very few people are actually paid at the minimum wage. In other countries such as France, the minimum wage is set at a level closer to the median wage, it can be considered as a *living wage*, and a significant proportion of the workforce actually earns it (around 15%).

In 2010, the minimum wage was 10.43€/h super gross and 6.95 €/h net. It is noteworthy that due to regressive social contribution rebates on small wages, the difference between the valuation with the minimum wage and the specialist substitute is smaller in net (−28%) than in super gross (−34%, see Table 5.2).

In both cases, using this wage for the valuation of domestic work does not seem relevant, at least not for the purpose of international comparisons.

The generalist substitute method levels out any composition effect across time (< 1%) With the specialized substitute method, wage differences should reflect differences in the productivity of each task. The generalist wage method has the major drawback of levelling out the various skills required for different domestic tasks and may be disqualified for this reason.

For instance, if women now spend less time doing the laundry and more time helping their children with homework, we would like for the hourly wage of domestic production to account for such a shift in productivity. Using the same wage for all domestic tasks would prevent us from registering any composition effect of this kind.

An evaluation of this structural effect can be performed for 2010 by computing the average hourly wage using times from both 1998 and 2010. The structural effect is as low as 0.4% for the core perimeter as a whole but can be larger (close to 2%) for some functions.

Thus this is not a strong case against the generalist substitute method.

Working time vs. paid time (+22%) Our source for hourly wage (DADS) uses paid time as a reference but TUS record worked time. The difference between these durations are paid holidays, sick leaves, maternity leaves, national holidays... periods during which workers are paid but not productive. As a consequence, our hourly wage from DADS is equal to $w = \frac{Annualwage}{Annualpaidtime}$ which we multiply by TUS's worked time, a duration conceptually shorter. On aggregate for the market industries there is a 22% wedge between these two durations in 2010: 22% of paid time is actually not worked. As a first approximation, we could thus assume that our valuation of domestic work underestimates by approximately 22% the true value of domestic work because it ignores non worked paid time. However, we can not assess whether there are some specificities linked to the particular occupations we consider (the working time and paid time of domestic cleaners paid by the hour are probably closer to one another, for instance). Before engaging in complex correction, the choice of an international benchmark on this matter could be guided by the available data.

5.4 From TUS to HHSA

5.4.1 The output and input approaches are two polar ways of measuring an in-between reality

Alternatively to the input method used here, the output method has been used for domestic production. As the UK's experience shows (Holloway et al., 2002), it is quite complex, whereas the input approach, based on previous experience on Time-Use Surveys, seems more practical to implement (both in terms of method and available data). Previous experiences and TUS availability are the main reasons for our choice of the input method. This choice is thus open to criticism and orientates the scope of domestic production we consider, in particular when it comes to capital (see 5.4.7).

However, there seems to be a consensus on the fact that the output method would, theoretically, be the first-best estimation procedure. Yet one may argue otherwise: the output and input approaches are two polar ways of measuring an in-between reality.

When a market exists, prices theoretically measure the willingness to pay of the marginal buyer for a good or service. The price embeds information beyond the cost of producing the said good or service. Typically, when the right logo is printed on a T-shirt, the value of the product increases by much more than the printing cost. What is relevant from the national accounts perspective is that prices are public and allow for a better description of the transactions.

On the one hand, using the output method and applying market prices to domestically produced goods and services implies that the willingness to acquire these products does not depend on their producers: market and domestic products are essentially identical. It is thus implicitly assumed that households are constrained to produce domestically in some way (financially, because of time, through social norms...) but otherwise they would purchase their domestic production from the market.

On the other hand, the input method implies that the decision to domestically produce is deliberate so that market and domestic products are essentially different. In the absence of any price to measure the specific value of these products, the best value we can (objectively) impute to domestic production is the valuation of its inputs.

Both methods are subject to a problem of quality evaluation. The output method raises the difficulty that market prices embed characteristics which do not apply to domestic production (allowing firms to price with a mark-up). The input method poses similar difficulties on wages as a measure of productivity.

In both cases, one may be tempted to go beyond these objective measures to capture the true willingness to pay for domestic production. Indeed, one can argue that nothing compares with dad's chocolate cake, while nothing is worse than wearing grandma's hand-knit pullover at school, implying that their value is neither the price of their market equivalent nor their production cost. For the present exercise we did resist this temptation, which is quite consensual and justified by similar reasons as those invoked for not considering the *opportunity cost method*: we are working within the theoretical framework of national accounting, not welfare economics.

5.4.2 Avoiding double counts of *Output for own final use* (5% of production)

There are some double counts between the SNA household account and the TUS estimates (see also Appendix 5.B). Specifically, food products, either grown, picked, hunted, fished, bred, milked, vinified, distilled or brewed are already counted, both in *Output for own final use* (P12) and *final consumption expenditure*. Also, major construction work and maintenance of dwellings are counted both in *Output for own final use* and *GFCF*.

HHSA	Activity	Gardening ($\frac{1}{2}$)	Breeding	Fishing, hunting	Gathering	Construction
	w*H	26.6	6.3	11.9	0.8	2.9
SNA	B1			2.5		2.3
	P12			3.4		4.9
	P2			0.9		2.6

For construction (incl. major maintenance of dwellings), the SNA figures are close to the imputed value of time from the TUS. For domestic production of food products, valuing time with low qualified specialists' wages is 20 times larger than the output estimation from the SNA account.
Source: Insee, National Accounts - Base 2005, Time Use Survey 2010, DADS 2010 - our calculations

Table 5.1 – Labour input from TUS and Output for own final use in billion (€)

In these cases we favour the existing national accounts estimation of *P12* and we do not count the corresponding time from the TUS data. This choice has a limited impact for construction work (see Table 5.1), but the input method appears to overestimate the productivity of households in their kitchen garden or when they fish, hunt, pick-up mushrooms... probably because these are both productive and leisure activities. We choose to trust the output-based estimation, in spite of its own limitations. Provided that the output method was perfectly accurate, the overestimation avoided by not valuing the agricultural production of households with the input method would amount to 49 billion €, i.e. 5% of domestic production in the *core perimeter*.

5.4.3 Reclassified consumption expenditure (25%)

In France, household consumption expenditure is built using the *Nace rev2* nomenclature for the balancing of the Supply and Use Table (SUT). It is also published using the COICOP classification. We use the Nace to isolate intermediate consumption and gross fixed capital formation (GFCF) because it is available in greater detail.

... into intermediate consumption (23%) The value of intermediate consumption ranges from 255 billion € to 315 billion €, depending on the chosen perimeter (Table 5.C.4 in Appendix 5.C). The definition of domestic production therefore has a limited impact on the value of intermediate consumption : ±60 billion €. It amounts to one tenth of the effect of the choice of the perimeter on the valuation of time (Table 5.C.3 in Appendix 5.C). From the *core* to the *intermediate* perimeters, productive leisure activities such as gardening, sewing and knitting account for 12 billion € of additional intermediate consumption. From the *intermediate* to the *extensive* perimeters, the fraction of car use counted as domestic production jumps from 11% to 97%. Consequently, the proportion of car-related expenditures that falls into intermediate consumption dramatically increases (+47 billion €), explaining most of the difference between perimeters as regards intermediate consumption.

... into GFCF (2%) The list of goods and services we reallocate into GFCF is shorter than the one used with the input method in Finland (Varjonen and Aalto, 2006) or the US (Landefeld et al., 2009). The reason for this is explained in 5.4.7:

we assume that durables which do not take part in an active production process are not productive. In the same way as for intermediate consumption, capital is also impacted by the perimeter of domestic production, mainly through the way car use is counted as productive. GFCF ranges from 18 billion € for the core perimeter to 84 billion € in the extensive perimeter (see Table 5.C.4 in appendix 5.C) This effect is however small, compared to that of the valuation of time (Table 5.C.3 in Appendix 5.C).

Because CFC for each function is a moving average of the corresponding GFCF with specific weights defined by the perpetual inventory method (see section 5.4.8) and as the trends in GFCFs are small, CFCs have the same order of magnitude as GFCFs.

5.4.4 No change in taxes and subsidies on production

We do not change the taxes and subsidies on production as they currently appear in the SNA household account. The reason for this is twofold: first, we are reluctant to reclassify transactions which have a counterpart outside the households account (here in the general government account); second, it would not be significant. Luckily for us, there is no longer a tax on car use in France, taxes on dwellings are already properly accounted for and there are only marginal subsidies, if any, that are conditional on engaging in domestic production of some kind (childcare for instance). We could have reallocated some individual consumption of general government to subsidies, when it was on products used as intermediate consumption for domestic production (e.g. food bank). In addition to representing only a small amount, this choice would raise similar issues as volunteer work: everything else being unchanged, this reclassification from *transfers in kind* to *subsidies* would modify the gross disposable income without changing household final consumption expenditure. Savings would be impacted, but hardly in link with domestic production, consumption, or actual saving behaviour.

5.4.5 Household production as its own intermediate consumption (neutral on value added and final consumption but +4% on production)

We could call it the *driving to the shop to purchase food to cook dinner* problem. The question is how much of a specific domestic production do you engage in, not for its own sake, but as means to another one. Our convention on this matter is chosen for the sake of simplicity.

Eurostat (2003) suggests estimating domestic production in five principal functions: housing, food, clothing, care and volunteer work. Ancillary functions (transportation, shopping, management) should be allocated to their true final purpose (driving to the shop to purchase food to cook diner = food preparation).

Unfortunately, allocating ancillary work to principal functions is not always possible with our data (*shopping* and *transportation*). Since any judgemental breakdown from our part would have a sizeable impact on the relative sizes of domestic production functions, we choose to treat these two ancillary functions as if they were an end to themselves.

This convention is neutral on the total value added and final consumption. It also enables better international comparisons than when allocations are made differently across countries, and allows others to use their own breakdown, when more data is available. Moreover, counting the ancillary functions *transportation* and *shopping* as intermediate consumption in the other functions would increase domestic production by 4% in the *core perimeter* and 33% in the *extensive perimeter*.

5.4.6 No changes in inventories

The standard framework already accounts for changes in inventories of households as users. We see no reason to modify this estimation even though some goods were reclassified from final to intermediate consumption.

Productive households may also generate *other inventories* and *work in progress*. Under this category, work in progress for construction and other major maintenance of dwellings are already accounted for. In the remaining possibilities, as most of domestic production falls into the services category, we could only think of such things as unfinished knitting by December 31st and jars of jam. Hopefully, our judgemental estimation of such changes in inventories (0) is not too far from reality.

5.4.7 New Frontier - on the capital side (+9% production)

Defining the frontier of domestic production from the sole point of view of time use surveys could be misleading (Ironmonger, 2000). Indeed, dwellings produce rents (real or imputed) without any hours worked. Similarly, one could consider that owning (or more restrictively using) any durable is similar to producing a rental service for oneself. Instead, we do not include production resulting from capital alone in our estimate of domestic production, that is when sole ownership or use for recreational purposes could be said productive (TV, sofa...). We only consider durables which take part in a deliberate production (cooking, cleaning, driving...).

This choice is open to debate and made mainly to stress that we think the alternative to estimate capital services for all durables through the perpetual inventory method (PIM) would be unsatisfactory. In the present estimation, consumption of fixed capital (CFC) is by far the smallest of the three inputs so that defined as it is, domestic production is satisfactorily estimated despite the flaws of the PIM. Considering all the durables as productive capital would scale-up GFCF and CFC to approximately 100 billion € resulting in an 9% increase (respectively 2%) of domestic production in its *core perimeter* (resp.

extensive perimeter).

In the principal function *housing*, we mainly consider house chores, decorative gardening and small house repairs. The services provided by a *fully furnished* dwelling are not included. The output method does not raise this kind of issues because it does not require the identification of productive capital (Holloway et al., 2002).

5.4.8 The robustness of the Perpetual Inventory Method (PIM) (±0.1% of production)

Our approach to capital depreciation is in-between that of Landefeld and McCulla (2000), who break down the total services provided by durables in proportion of hours of unpaid work, and Fraumeni (1997) or Jalava and Kavonius (2009), who specify depreciation rates for each durable. We do not develop a complete set of depreciation factors for each durable reclassified in GFCF, but borrow from the capital accounts 3 sets of such factors, which are compatible with the PIM:

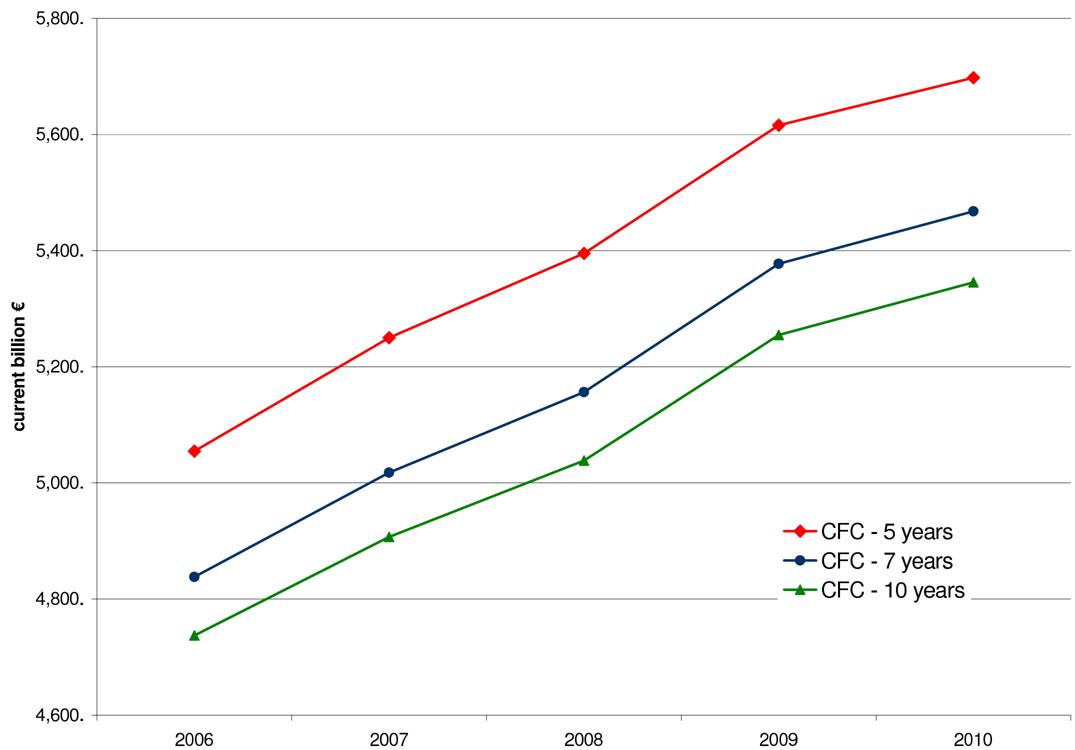
- (AN.11131) *transportation equipment*: average duration 7 years, maximum 21 years
- (AN.111321) *computers*: average duration 5 years, maximum 10 years
- (AN.111322) *communication equipment*: average duration 10 years, maximum 20 years

Investment is grouped for each of the domestic production functions and as a sensitivity test, the three sets of coefficients are applied.

Depreciation factors are not chosen for the similarity of the assets with reclassified durables, but because the average and maximum durations seem reasonable assumptions for domestic appliances nowadays. However, for cars and other transportation-related investment, coefficients for transportation equipment should be the favoured assumption. They should be chosen from an industry which uses mainly cars to avoid other transportation equipment.

Figure 5.1 shows the sensitivity of the estimated *CFC* for food services to the choice of a depreciation rate of durables. It is the function with the highest sensitivity to the depreciation factor: in 2010 when the average duration of capital goods is assumed to be 5 years, CFC is 6.2% below its value when the assumed duration is 10 years. Our estimation using the PIM is not very precise, however an uncertainty of 5% on CFC weights only 1 billion € whereas the total value of domestic production is larger than 900 billions. It is only a fraction of the suspected uncertainty on the valuation of time spent in productive leisure such as gardening (see 5.4.2).

If all durables were reclassified into investment, a 5% uncertainty on CFC would still weight only 5 billion € on total domestic production. Hence, our choice not to consider



Source: Insee, National Accounts - Base 2005 - our calculations

Figure 5.1 – *Consumption of Fixed Capital (CFC)* of durables used for domestic production of food services

capital production more extensively may be thought drastic. However, we wish to make it clear that if a lot has been done empirically and conceptually in the treatment and harmonization of TUS, for a complete input approach of domestic production, more needs to be done on the treatment of capital. Valuing the service of capital through time of use multiplied by rental cost would *a priori* be a good start but many alternatives are available (OECD, 2009).

5.4.9 Implications for the interpretation of the savings ratio and purchasing power of disposable income

The key figures for economic analysis extracted from households accounts are the savings ratio and the purchasing power of disposable income. The construction of a HHSA raises questions on both concepts and their value must be handled with great caution.

Purchasing power of disposable income This figure is directly impacted by all the choices made to value domestic work: perimeter, substitute wage, imputed taxes and contributions, worked time or paid time (see 5.3.3). These methodological choices can greatly affect the imputed disposable income derived from domestic production (equal to the value added derived from this activity). This additional income is not marginal (648 billion €) in the total disposable income (SNA plus domestic production). It is almost as high as households' *gross wages and salaries* (768 billion €).

In addition, to estimate the purchasing power of this total income (SNA+non SNA), it is possible to measure price inflation of domestic production from consumption prices and wages. On the core perimeter, from 1998 to 2010, the purchasing power of gross disposable income grew by 27.2% according to the SNA definition, but by only 17.2% in the HHSA. This growth differential is due to the fast increase of hourly wages (+51% over the period against +20% for consumption prices).

In this respect, the input or output method for the construction of a HHSA will provide quite different results. The output method will mechanically limit the price differential between market and domestic production. However, without a clear model in mind for households preferences and constraints (see section 5.4.1), it is impossible to say whether our quantitative results are an argument in favour of one or the other method.

Minor effects can be expected in these figures deriving from the chain-linking of the HHSA over 12 years while SNA accounts are chain-linked on an annual basis.

Savings ratio (11.5% or 13.2%) The SNA's estimate of the savings ratio is 15.9% in 2010. Our estimate in the HHSA is 11.5%. The way the treatment of imputed taxes and social contributions affects the savings ratio is not straightforward. Let C^{HHSA},GDI^{HHSA} denote total consumption and gross disposable income as we measure them, that is SNA plus non-SNA, including imputed taxes and social contributions. Let $\tau^{imputed}$ denote these taxes. If one is interested in the value of domestic production/consumption, taxes and contributions should be included in both concepts as they are included in the value of market production. However, if one is interested in the potential income from domestic production, one may consider net wages $GDI^{HHSA} - \tau^{imputed}$, that is subtract imputed taxes from the *mixed income* in the distribution of income accounts but keep production and consumption as estimated with gross wages.

In this case, the savings ratio would be negative: -1.6% $(\frac{GDI^{HHSA} - \tau^{imputed} - C^{HHSA}}{GDI^{HHSA} - \tau^{imputed}})$. Although this savings ratio seems the most economically relevant, it yields severe accounting and communication issues. Subtracting the imputed taxes and contributions from the mixed income would modify the net lending/net borrowing of households while no monetary transaction is recorded. It is then necessary to create a specific correction to make the HHSA neutral on the financial accounts. This correction is mandatory because the financial account describes the detention of money, stocks and financial assets in general, and they can not be affected by transactions in kind: one can not *save* domestic production. But, this correction leaves the door open to abusive policy recommendations as part of the production simply vanishes in the sequence of accounts.[4]

In line with national accounting practice and for sake of simplicity, we choose not to make such a correction and leave imputed taxes and contributions in the mixed income. This convention is somehow related to the accounting of imputed rents: imputed income equals avoided expenditures. With our choice to use super gross wages the savings ratio is equal to 11.5% ($\frac{GDI^{HHSA} - C^{HHSA}}{GDI^{HHSA}}$). The alternative is to use net wages in the production account, in which case the savings ratio is equal to 13.2% ($\frac{(GDI^{HHSA} - \tau^{imputed}) - (C^{HHSA} - \tau^{imputed})}{GDI^{HHSA} - \tau^{imputed}}$).

The invisibility of volunteer work (+3% of production) Introducing the value of volunteer work into the HHSA is not straightforward. The value of this work can logically be added to the production of NPISHs. It does not appear in household consumption expenditure but only in household final consumption: household savings are neutral to NPISHs production. However, if imputed wages from NPISHs to households were counted, they would have no counterpart in consumption and be added to both *gross saving* and *net lending or net borrowing*. This would have to be corrected as there is no monetary transaction in domestic production which could justify a modification of the *net lending or net borrowing* of the original households account. To correct for this inconsistency, one would have to assume that part of NPISH production (the amount corresponding to household wages due to volunteer work) is in fact consumed directly as household consumption expenditure.

Given the small share of volunteer work in total domestic work, we found it less confusing not to include it in the HHSA but to value this time separately.

5.5 A households satellite account for France in 1998 and 2010

This section primarily comments the results for the core perimeter in 2010 and evolutions since 1998. Tables for both years and the three perimeters are displayed in Appendix 5.C.

5.5.1 The production account (+33% of GDP) and its distribution by functions

Following the input approach, we add up the three inputs of home production (labour, intermediate consumption and *consumption of fixed capital* (CFC)) to obtain an estimated value of this production.

[4] We published a prior version of this work in French (Poissonnier and Roy, 2013). Although we did not subtract imputed taxes from the mixed income, some reactions, both from journalists and the general public, were that we implied a recommendation to tax domestic production.

We value this production at 904 billion € in 2010 and 590 billion € in 1998. The corresponding gross value added amounts to 649 and 416 billion € which corresponds to a revaluation of GDP (in current prices) of +33% and +31% respectively.

This result is in line with other estimates ranging from 27% in the USA in 2004 to 63% in the UK in 2000 (see also Table 5.1).

From 1998 to 2010, domestic production has increased by 54%, of which 42 p.p. are attributable to a price effect. This amounts to a 3% annual inflation, mainly driven by wages and similar to the growth rate of the hourly minimum wage over the same period.[5] Over this period, GDP grew by 20% in volume. Once domestic production is accounted for, this growth is five points smaller. This result is in line with the expectations of early promoters of household production estimations (Vanoli, 2002) but striking as one would expect the transfers from non market to market activities to be less sizeable in the recent years. The difference in the growth rate in volume term of SNA (+20% GDP) and non SNA production (+4% gross valued added of household production) suggest that these transfers are still massive.

For both years, the production function of households is quite similar to that in other countries (see also Table 5.A.1 in Appendix 5.A): labour (or net value added) accounts for 70% of total production while CFC is the smallest of the three inputs (2%).

Household production can be broken down into 4 principal functions and 2 ancillary functions, plus volunteer work.[6] The ventilation of domestic production in functions is quite similar in 1998 and 2010. Two noteworthy changes to the TUS between 1998 and 2010 impact our results. In the 1998 survey, transportation by car was not distinguished from other means of transportation. Hence the change in volume growth of this function must be interpreted with care as we have assumed the share of travelling by car in total travelling to be constant. Significant changes were also made in the coding of volunteer work and some ambiguities were corrected in 2010. The results regarding volunteer work are also to be treated with caution.

The distribution by functions can be compared with satellite accounts in other countries (see Table 5.A.2 in Appendix 5.A). As for the Basque Country (Eustat, 2004) and Finland (Hamunen et al., 2012), *food* and *housing* account for the bulk of domestic production while *clothing* accounts for less than 10% of the total. Compared to Holloway et al. (2002) for the UK, we define *care* and *transportation* more restrictively, which can explain the smaller share of these functions in domestic production. Indeed, with our *extensive* definition of domestic production, transportation accounts for a much larger share of production (27% instead of 5% initially, larger than the UK's estimate of 17.7%) as

[5] http://www.insee.fr/fr/themes/tableau.asp?ref_id=natnon04145

[6] We do not distribute ancillary functions to principal functions (see 5.4.5 for a discussion of this choice), and *volunteer work* is treated separately (see 5.4.9)

almost all car journeys are assumed to be productive in this perimeter (see 5.3.2). Besides, volunteer work is a minor function: 3%, in-between the figures for the UK and Finland.

5.5.2 Consumption is increased by 58%, income by 50% and the savings ratio is lower by 4 percentage points

The inclusion of home production substantially changes the picture of the economic activity of households. Indeed, it implies a 631 billion € net increase in consumption, which can be compared with individual consumption expenditure as it is currently computed in national accounts (1,085 billion €): incorporating home production raises final consumption by 56%. As gross disposable income also increases (by 50%), correlatively, the savings ratio (savings on gross disposable income) goes down from 15.9% to 11.5%.

In 1998, the situation is very similar despite the opposite position in the business cycle.[7] Consumption is scaled up by 56%, gross disposable income by 49% and the savings ratio scaled down by 4 points from 15.2% to 11.1%.

As a consequence, growth rates in current prices between 1998 and 2010 are rather similar whether measured with SNA conventions or with our HHSA. Consumption increased by 51% in the SNA (respectively 53% in the HHSA) and gross disposable income by 19% (respectively 13% in the HHSA). However, price differentials between domestic and market consumptions are sizeable and reflect the differentiated growth of nominal wages and consumption prices: SNA consumption prices increased by 20% over the period while domestic production prices increased by twice as much (42%). Consequently, the purchasing power of gross disposable income grew by 27% according to the SNA but only 17% in the HHSA.

5.5.3 Home made consumptions are much larger than their market equivalents

Even within the *core perimeter*, consumption of home-produced services dwarfs its market equivalents for every function considered both in 2010 and 1998. Home *food* production represents 459 billion € in 2010, 8 times the consumption of meals in restaurants and eateries (59 billion €). The gap is even wider for *household upkeep* - 253 billion € vs. only 6 billion € for the employment of gardeners, cleaners and housekeepers -, and for *clothing* - 61 billion € of home production (laundry, ironing, mending...) vs. less than 2 billion € of corresponding market services (dry cleaning). Finally, one could think that France having a lot of public transportation, a relatively high level of female labour force participation and an active policy of childcare, market consumption of transportation and *care* could be large relative to the amount of household production of these services. Yet we find that the value of *transportation* provided by households (within the *core* perimeter, i.e. excluding self-transportation) is more than 40% higher than consumption of transportation

[7] 1998 within the boom prior to the burst of the dotcom bubble, 2010 in the middle of the great recession.

services whether it is by plane, train, taxi, bus... (38 vs. 27 billion €). As regards *care*, home production (92 billion €) tops household final consumption of *Social work activities* (67 billion €) as recorded by the SNA, i.e. including general government and NPISHs contributions. Comparisons in 1998 provide an identical picture.

It would be interesting to compare these results across countries with various levels of public transportation, of women labour force participation and with different levels of socialization of care. However, since transportation and childcare are precisely two major *grey zones* in the definition of home production, such comparisons cannot be made until an agreement is found over a common delineation of their boundaries. Being able to compare the relative contribution of the private sector, the public one and households to the provision of transportation and care, across economies that are organized differently, would be a particularly valuable outcome of such an agreement.

5.6 Conclusion

One century ago, economists were estimating the value of a housewife using the cost of a housekeeper. Time use surveys then allowed for more precise and less sexist appraisals of domestic work. We are now linking these estimations with national accounts. On the one hand, a lot has been said and done in this literature, on the other hand more harmonisation is needed before we can enlighten policy makers with estimates of domestic production following a shared methodology reproduced every 5 or 10 years.

Our estimates on France in 2010 show that these methodological issues can be ranked: agreeing on a frontier of domestic production and on a net or gross wage rate are the two decisions that have the greater quantitative impact on the results. We chose the most restrictive perimeter of domestic production because we believe it to be the less disputable, and gross wages in accordance with the SNA definition of employee compensation, but these choices are obviously still open to debate.

As for the other methodological choices (specialist or generalist wage, capital depreciation rate...), they seem quantitatively less urgent, at least for comparisons between industrialised countries. We tried to advocate that they should be made under two main guiding principles. The first one is a practical argument of simplicity: whenever possible, the most parsimonious solutions and the ones that imply the least changes in the existing SNA figures should be favoured. The second one is that when working within the framework of national accounting, one should rely on pre-existing national accounting concepts and principles.

Appendix

5.A Additional tables for international comparisons

	Basque Country			Finland				Germany	UK
Source	e	e	e	c	c	d	g	g	f
Reference Year	1993	1998	2003	1990	2001	2006	2001◊	2001◊	2000◊
Production (10^6 €, £)	11.7	13.3	15.9	-	68.9 10^3	86.5 10^3	76.0 10^3	1.3 10^6	879.2 10^3
Net Value Added (%)	77.4	75.0	71.4	76.4	75.1	71.5	67.5	71.5	73.0
Intermediate Cons. (%)	21.6	23.9	27.3	21.4	20.8	24.0	24.7	23.0	21.1
Capital Cons. (%)	1.1	1.1	1.2	2.2	4.1	4.5	7.8	5.5	5.9

*: our calculations, -: not available ◊: SNA + non SNA
c:(Varjonen and Aalto, 2006), d:(Hamunen et al., 2012), e:(Eustat, 2004), f:(Holloway et al., 2002), g: (Ruger and Varjonen, 2008)

Table 5.A.1 – HHSA in 4 countries: inputs of non SNA domestic production

5.B Activity, time and wage

Four main types of activities that explain the differences between the three perimeters require further discussion: personal care, transportation, productive leisure and childcare.

Personal care Men used to go to the barber to get shaved, but no longer do. So, is shaving oneself productive? The textbook example is that of the nobleman of the 18th century who had servants to dress him, comb his hair, read books to him, etc. This would seem anecdotal except that today, more and more dependent elderly people receive paid help to wash, dress, eat, etc. So, washing can be delegated in some cases, and there exists a market substitute for it. Washing one's handicapped spouse is then productive work, but washing him or her if he or she has no disability isn't (and neither is washing oneself), which might seem paradoxical since the exact same task is performed, the same service rendered.

Country	Basque	UK*◊	Finland
Year	2003	2000	2006
Source	e	f	d
Food	46.6	9.6	31.6
Housing	30.3	32.1	34.6
Care	14.8	33.3	13.3
Clothing	8.4	5.4	11.1
Transportation	-	17.7	-
Shopping	-	-	-
HH management	-	-	-
Volunteer work	-	1.9	9.4

*: our calculations, -: not available ◊: SNA + non SNA
d:(Hamunen et al., 2012), e:(Eustat, 2004), f:(Holloway et al., 2002)

Table 5.A.2 – HHSA in 3 countries: ventilation of non SNA domestic Gross Value Added by functions (in %)

In this line of thought, Alesina and Ichino (2009), using the MTUS (Multinational Time Use Survey) database, include the *AV13: dress/personal care* category in their definition of unpaid domestic work. Doing this with our French data would add almost an hour to the average daily estimate of unpaid work. This would mean a dramatic increase: +50% over the core definition of domestic work, +33% over the intermediate one which currently totals 3 hours a day, +25% with the most extensive definition (currently 4 daily hours, see Table 5.1).

The issue is the same with medical care: if a person does their own injections, massages or bandages (as people with chronic diseases often do), should it be counted as production since these acts are usually delegated to nurses and physiotherapists? In theory, the answer should be positive, but we choose not to include medical care done to oneself within our production boundary, mainly for measurement reasons: in our data, it is impossible to distinguish *serious* medical care (injections, strapping...) from everyday benign care (putting a band-aid on your child's finger). The latter is not delegable (you don't call the doctor for that) but constitutes most of the time households spend on *medical care*.

Travel time There is no consensus in the literature on how travel time should be treated (see for example (Eustat, 2004)). Some studies include travel time into the time devoted to the activity to which the travel is leading, for example travel to the store is incorporated into shopping time, travel to work into paid working time, etc., but this is not entirely satisfactory, since at least part of the travel time could be delegated. Clearly, driving someone else, a child or a relative for example, can be delegated and is productive. But what does it mean to delegate self-transportation? If I drive to work, I could delegate the driving to a chauffeur or use public transportation, and use that time to do something else: write work-related emails, read a book, etc. But the time that becomes available cannot be used totally freely, since I would still need to be in a car or a bus. If I have to go somewhere, my travelling can only be partially delegated. And I can pay someone to drive me to work, but what if I'm walking 10 minutes to work? Here, the means of transportation enters into play: only driving can be delegated, and only partially so.

The 2010 TUS data for France includes information on the means of transportation / location for every 10-minute interval. We use this information to include only travelling by car (*travelling to work* or *other travelling*) into unpaid work, and only in the most extensive definition of domestic work. *Accompanying a child* and *travelling for another household* (mostly driving other people) are included in core productive activities, since they are entirely delegable to any trustworthy driver.

Gardening, home repairs, fishing and hunting: the frontier with leisure There is another *grey zone* where domestic work and leisure overlap. If the person mainly derives utility not from the output of the activity (the good or service produced), but from performing the activity, from the process itself, then it can no longer be delegated without losing all its value. Amateur pianists do not play the piano in order to be able to hear some (probably poorly performed) music, but for the sake of playing. So, it is generally agreed that unpaid artistic endeavours (music, painting, photography, making films) should not be counted as productive.

The case of what is, precisely, often called *productive leisure* is less clear cut. It includes gardening; home or vehicle repairs; sewing and knitting; fishing; hunting; picking plants, berries or mushrooms. Producing vegetables, fishing and repairing the car are most often delegated in our society, so one might think that people who engage in these activities do so because they enjoy it. But then, from a national accounting perspective (as opposed to a welfare economics perspective), the question of whether one is enjoying the activity is not necessarily relevant. Indeed, to measure market production, we do not take enjoyment into consideration to measure the value of paid work: the same wage is counted, whether one enjoys the job or not. Most of the literature thus includes gardening, home repairs, knitting and sewing in domestic work, because these are productive, delegable activities. Actually, the current SNA definition of production, used to measure GDP, includes the goods produced by households for themselves, thus recognizing the productive potential of households. In industrialized countries however, it is admitted that only agricultural goods, alcohol, game and fish require counting, because the production of other goods (clothes, furniture, etc.) is too small to be worth the measurement effort.

This means that if counting hours of unpaid domestic work was to become the basis for an input-based valuation of household production to be recorded aside GDP as a complement, goods for own consumption would potentially cause double-counting. The way we correct for double counts is detailed in 5.4.2. Yet the amounts at stake are small relative to the overall value of unpaid domestic work. For France in 2010, the value of household own production of goods included in GDP was 3.18 billion €, a very small figure compared to the 1900 billion € of GDP.

Productive leisure is a case in point regarding the issue of productivity that one necessarily encounters when measuring production through inputs and not outputs. Since gardening, home repairs or fishing are often done for pleasure, we can suspect that people take their time to do it, and productivity is lower than if it was done for pay. On this basis, together with the fact that they are probably done for themselves (as hobbies) as much as for their output, we do not include gardening and DIY in the core definition of domestic work, but only in the intermediate and extensive ones. On the contrary, unpleasant activities such as vacuuming or doing the laundry are less suspect of such bias, and their productivity is probably closer to that of their market equivalent.

The same goes with walking the dog: it can be delegated and *dog-sitters* are beginning to appear in France, but most often, when reading the diaries, one feels that walking the dog and taking a pleasure walk are one and the same activity, and it generally takes much longer than the necessary time for the dog to be walked. This is why we have included walking the dog only in the most extensive definition of domestic work, whereas material care of pets is included in the median definition and care of productive animals in the core definition.

Is all time spent with children productive? Childcare is the last major issue that needs to be tackled if one is to agree on a definition of unpaid domestic work. First, social norms as to what can be delegated are variable over time and place. Breastfeeding is no longer physiologically delegated to another woman in industrialized countries, but feeding a newborn can be delegated thanks to bottle feeding. Some people have therefore argued that breastfeeding is productive, while others who oppose bottle feeding argue that it is not, because it can not be delegated. At the other end of the spectrum, can *playing with one's child* or *having a conversation with one's child* be delegated without losing its (emotional) value?

Within the Eurostat Task Force (Eurostat, 2003), no consensus could be reached on the question of what constitutes productive childcare. In time use data, it is possible to consider as childcare (in descending intensity of care):

- only time spent with an explicit activity of material childcare as primary activity
- time spent engaging into various activities for or involving children, but less material and more leisure-like: games, conversations... as primary activity
- time spent on these two types of activities, either as primary or secondary activity
- all time spent in the presence of a child, even if it is not involved in the activities described in the diary. Even sleeping when a child is present could potentially be counted as childcare, since an adult needs to be there and the task of "being there" could be delegated to a babysitter.

As an illustration, one can compute this latest, extreme figure: the amount of time adult respondents spend alone with a child or several children (so we can assume that the respondent is in charge of supervising the children). It amounts to 46mn a day on average, 138mn for a mother living in a couple, 57mn for a father living in a couple, and 272mn for a single mother. This is twice the time spent with childcare as a primary activity (23mn on average), and the figure would be even larger if time spent with both children and other adults was included.

This shows that, with the French TUS data at least, deciding whether or not to count the time spent with children in passive childcare as productive would have a major impact on the measurement of domestic work. This impact is much stronger than that of secondary activities.

In what follows, we choose to include only active childcare done as a primary activity, and we distinguish between core childcare (material care and supervision), which we include in the *core* definition, and leisurely childcare (playing, discussion with the child) which we include in the *intermediate* and *extensive* definitions only.

5.C Comparisons on the main issue: the frontier of production

in billion euros

		Core Perimeter		Intermediate Perimeter		Extensive Perimeter	
2010	Labour	632	70%	877	75%	1 130	74%
	Growth (vol & price)	3%	51%	-2%	52%	6%	52%
	Intermediate Consumption	255	28%	268	23%	315	21%
	Growth (vol & price)	16%	27%	16%	26%	13%	29%
	Consumption of Fixed Capital	17	2%	19	2%	84	6%
	Growth (vol & price)	47%	-2%	46%	-1%	28%	17%
	Domestic Production	904	100%	1164	100%	1529	100%
	Growth (vol & price)	8%	42%	3%	44%	9%	44%
	Gross Value Added	649	72%	896	77%	1214	79%
	Growth (vol & price) 4%	49%	2%	45%	7%	48%	
1998	Labour	404	69%	592	75%	707	72%
	Intermediate Consumption	174	29%	183	23%	215	22%
	Consumption of Fixed Capital	12	2%	13	2%	57	6%
	Domestic Production	590	100%	788	100%	978	100%
	Gross Value Added	416	71%	605	77%	763	78%

Note: the details of each input by function are displayed in Table 5.C.3 for labour, Table 5.C.4 for intermediate consumption and consumption of fixed capital.
Source: Insee, National Accounts - Base 2005, Time Use Survey 2010 and 1998, DADS 2010 and 1998 - our calculations

Table 5.C.1 – Houshehold domestic production account for three definitions of production in France

in billion euros

		Housing	Food	Clothing	Care	*Transp.*	*Shopping*	**Total**◊	*Volunteer*
2010	Core	253.6	459.5	60.7	92.5	37.8	0.0	904.1	28.3
		28%	51%	7%	10%	4%	0%	100%	3%
	Inter.	352.3	459.5	74.8	129.9	45.6	101.6	1163.7	58.9
		30%	39%	6%	11%	4%	9%	100%	5%
	Ext.	352.3	459.5	74.8	144.2	397.1	101.6	1529.5	59.6
		23%	30%	5%	9%	26%	7%	100%	4%
1998	Core	173.0	307.0	41.2	48.0	20.9	0.0	589.9	16.6
		29%	52%	7%	8%	4%	0%	100%	3%
	Inter.	244.2	307.0	59.0	68.2	27.5	82.0	787.9	39.8
		31%	39%	7%	9%	3%	10%	68%	3%
	Ext.	244.2	307.0	59.0	75.9	210.2	82.0	978.3	40.2
		25%	31%	6%	8%	21%	8%	100%	4%

◊: This total is excluding volunteer work
Source: Insee, National Accounts - Base 2005, Time Use Survey 2010 and 1998, DADS 2010 and 1998 - our calculations

Table 5.C.2 – Three definitions of domestic production in France (by function)

		Perimeter					
		core		intermediate		extensive	
		10^9 h	10^9 €	10^9 h	10^9 €	10^9 h	10^9 €
Housing	2010	13	189	18	275	18	275
	1998	13	133	19	196	19	196
Food	2010	17	278	17	278	17	278
	1998	16	179	16	179	16	179
Clothing	2010	4	54	4	66	4	66
	1998	4	36	5	52	5	52
Care	2010	6	88	8	125	9	140
	1998	4	45	6	65	7	73
Transp.	2010	1	23	2	31	16	270
	1998	1	11	2	18	11	125
Shopping	2010	0	0	7	102	7	102
	1998	0	0	9	82	9	82
Total excl.	2010	40	632	56	877	70	1130
volunteer	1998	39	404	57	592	67	707
Volunteer	2010	2	28	3	59	3	60
	1998	2	17	3	40	3	40

Source: Insee, Time Use Survey 2010 and 1998, DADS 2010 and 1998 - our calculations

Table 5.C.3 – Three definitions of domestic work and their valuation using the least qualified specialised substitute in France

in billion euros

	Perimeter	Housing	Food	Clothing	Care	*Transportation*	**Total**
			Intermediate consumption				
	Core	64	175	5	5	6	255
2010	Intermediate	75	175	7	5	6	268
	Extensive	75	175	7	5	53	315
	Core	39	124	4	3	4	174
1998	Intermediate	47	124	5	3	4	183
	Extensive	47	124	5	3	36	215
			Gross fixed capital formation				
	Core	1	6	2	0	8	18
2010	Intermediate	3	6	2	0	8	19
	Extensive	3	6	2	0	73	84
	Core	1	4	2	0	6	12
1998	Intermediate	2	4	2	0	6	14
	Extensive	2	4	2	0	51	59
			Consumption of fixed capital				
	Core	1	6	2	0	8	17
2010	Intermediate	3	6	2	0	8	19
	Extensive	3	6	2	0	74	84
	Core	1	4	2	0	6	12
1998	Intermediate	2	4	2	0	6	13
	Extensive	2	4	2	0	49	57

Note: There is no intermediate consumption for the ancillary functions *Shopping* and *Volunteer Work* and no capital for the functions *Care*, *Shopping* and *Volunteer Work*
Source: Insee, National Accounts - Base 2005 - our calculations

Table 5.C.4 – Three definitions of intermediate consumption and capital (GFCF and CFC) for domestic production in France

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Chapter

6

Structural reforms in DSGE models: a plead for sensitivity analyses

with Benoît Campagne

We study the effect of structural reforms in large scale institutional models. To decompose the effects at play we focus on their common core, the Smets and Wouters model. The long term output gains and transitional costs of these models is well documented in the literature. We provide a detailed analysis of the underlying mechanisms. The cornerstone of these reforms is households' consumption and leisure arbitrage. A simple change in households' utility parameters can lead to additional gains or losses of a few percentage points in output following goods or labour markets deregulations. In addition, policy recommendations based on welfare are less clear-cut than those solely based on output gains. Introducing non Ricardian agents allows stylized yet informative inequality analyses showing that goods market reforms reduce inequalities while labour market reforms are neutral. In all, our results question the use of such models for structural analysis and advocate for the extensive use of sensitivity analyses for policy purposes.

JEL-code: *E10, E60, E20*

Keywords: *structural reforms, DSGE model, mark-ups, sensitivity, welfare*

6.1 Introduction

The evaluation of structural reforms has become a standard exercise in the DSGE literature and in policy-making publications and reports. Institutions such as the IMF, the European Commission, the OECD, the ECB, and many central banks have now developed and refined their own tools and are capable of conducting such analyses in different contexts. All call for such reforms as a solution to the imbalances and loss of competitiveness in Europe.

Indeed, there is a large consensus on the ability of such reforms to foster output. However, in these large scale models it is difficult to disentangle the core mechanisms through which increased competition affects the economy. Furthermore, despite this consensus numerical differences remain, calling for a thorough sensitivity analysis to identify sources of divergence, as well as their quantitative influence.

These institutional models share a common core, the Smets and Wouters model. As such, these are business cycle models and the question arises of their ability to inform on structural reforms. Therefore, we study the impact of mark-up reforms in the labour and goods markets both in the long run and in the transition in this neo-Keynesian framework. In particular, we provide a detailed analysis of the underlying mechanisms showing the central role of the consumption-leisure arbitrage.

In line with the existing literature, we indeed find gains from pro competitive reforms in the long run but losses in the transition. However, long term effects are particularly sensitive to the modelling of households, their consumption-leisure substitution elasticity, the introduction of habits on consumption and labour, and the introduction of financially constrained households. For the same reform, output gains can differ by as much as a few percentage points of output.

Besides, such output-enhancing reforms can have a negative welfare impact with standard calibrations. Also, when financially constrained households are included in the model, goods market reforms lead to a decrease in inequalities both in terms of consumption and utility, whereas inequalities stagnate following labour market reforms.

In this paper, the underlying model builds on (Christiano et al., 2005) and (Smets and Wouters, 2003, 2005, 2007). Within this model, firms and consumers maximize their objective (utility or profit) by interacting on the goods, labour and capital markets with both prices and wages rigidities introducing neo-Keynesian features in the model à la (Erceg et al., 2000). The model also integrates risk free assets to ensure an intertemporal trade-off and real rigidities on the capital market. This model is the core of many large scale institutional models (ECB, IMF, etc.).

This article follows the wide literature on the effects of structural reforms. D'Auria et al. (2009), Roeger et al. (2008) or Varga et al. (2014) conduct evaluations for EU member states in the R&D version of the Quest III model whereas Annicchiarico et al. (2013) do the same for Italy. The IMF, the OECD or some central banks have also conducted their own evaluations (Bayoumi et al., 2004; Everaert and Schule, 2006, 2008; Cacciatore et al., 2012; Forni et al., 2010). Their simulations concur to output gains of 5 to 10% following an average 15 points decrease in mark-ups. However, Jonsson (2007) and Matheron and Maury (2004) note that these long terms gains are partially offset by transitory losses.

The rest of the paper is organised as follows: in Section 6.2, we briefly recall the *Smets and Wouters* model and give a short presentation of its calibration. In Section 6.3 we analyse the mechanisms at work in mark-up reforms in the labour and goods market both in the long run and in the transition. In Section 6.4 we explicate the dependency of these mechanisms to the behaviour of households (calibration of utility, liquidity constraint), as well as their impact in terms of welfare.

6.2 Model

This section gives a short presentation of the *Smets and Wouters*' model. Namely, it is a neo-Keynesian model in closed economy. Households operate on goods and labour markets to maximize their utility. Firms use capital and labour to produce partially substitutable goods. Nominal rigidities are added on price and wages in a Calvo manner, and real rigidities are introduced on labour and goods with monopolistic competition, and on capital through adjustment costs.

6.2.1 Households

The euro area is populated by a continuum of households of size $\mathbb{N}$.

Consumption and investment

Households arbitrage between consumption and savings, capital and financial assets, as well as between consumption and leisure today. Each household τ maximises his intertemporal utility, non separable in private consumption and labour[1].

$$\begin{aligned} &\max\ E_t \sum_{T=t}^{\infty} \beta^{T-t} \mathcal{U}_T(\tau)\mathcal{V}_T(\tau) \\ &\text{with}\ \ \mathcal{U}_t(\tau) = \frac{1}{1-\sigma_c}\left[C(\tau,t)\left(\frac{C_{t-1}}{\mathbb{N}}\right)^{-h_c}\right]^{1-\sigma_c} \\ &\qquad \mathcal{V}_t(\tau) = \left[1-\kappa(1-\sigma_c)\left(l(\tau,t)\left(\frac{L_{t-1}}{\mathbb{N}}\right)^{-h_l}\right)^{1+\sigma_l}\right]^{\sigma_c} \end{aligned} \tag{6.1}$$

[1] We follow the non-separable utility form advocated by (King et al., 2002) in the presence of growth in the model. The choice of a specific non separable utility function follows (Trabandt and Uhlig, 2011), under this form, σ_l is the constant inverse Frisch elasticity.

subject to the budget constraint:

$$\begin{aligned} FA_T(\tau) =& R_{T-1}FA_{T-1}(\tau) + w_T(\tau)l_T(\tau) \\ & - P_T C_T(\tau) + D_T(\tau) \\ & + \Phi_T(\tau) + P_T r_T^K K_{T-1}(\tau) - P_T I_T(\tau) \end{aligned} \tag{6.2}$$

$$K_T(\tau) = (1-\delta)K_{T-1}(\tau) + \epsilon_T^I \left[1 - \mathcal{S}\left(\frac{I_T(\tau)}{I_{T-1}(\tau)}\right)\right] I_T(\tau) \tag{6.3}$$

where E_t, β are respectively the expectation at time t operator and the discount factor; σ_c is the inverse intertemporal elasticity of substitution; κ a weight assigned to labour in the utility function and σ_l the inverse of the Frisch elasticity. h_c, h_l are the external habit formation (on per capita level) parameters on consumption and labour. $C_t(\tau)$ and $l_t(\tau)$ are respectively the consumption and labour supply of household τ while C_t and L_t are the aggregate counterparts; $w_t(\tau)$ correspond to the wage. $FA_t(\tau)$ is the financial asset holdings at the end of period t while FA_t is its aggregate counterpart; R_t is the gross interest rate set by the monetary authority. $D_t(\tau)$ are the dividends paid by the firm to its owners. $\Phi_t(\tau)$ is a lump-sum transfer to households. P_t is the price level for both consumption and investment goods which are identical.

In the capital accumulation equation, $I_t(\tau)$ is the investment level with an adjustment cost[2] $\mathcal{S}\left(\frac{I_t(\tau)}{I_{t-1}(\tau)}\right)$ depending on previous period level of investment, and $K_t(\tau)$ is the capital stock of households depreciating at rate δ. As a result, households pay for the full investment allotment $I_t(\tau)$ and a share $\mathcal{S}\left(\frac{I_t(\tau)}{I_{t-1}(\tau)}\right)$ is lost in the installation process. ϵ_t^I represents an exogenous shock to this cost. r_t^K is the return paid on capital.

The first order conditions yield the following Euler equation, investment decision and Tobin's Q:

$$\beta E_t \left\{ \frac{\mathcal{U}'_{t+1}(\tau)\mathcal{V}_{t+1}(\tau)}{\mathcal{U}'_t(\tau)\mathcal{V}_t(\tau)} \frac{R_t}{\Pi_{t+1}} \right\} = 1 \tag{6.4}$$

$$\begin{aligned} 1 =& q_t(\tau)\epsilon_t^I \left(1 - \mathcal{S}\left(\frac{I_t(\tau)}{I_{t-1}(\tau)}\right) - \mathcal{S}'\left(\frac{I_t(\tau)}{I_{t-1}(\tau)}\right)\frac{I_t(\tau)}{I_{t-1}(\tau)}\right) \\ & + \beta E_t \left\{ \frac{\mathcal{U}'_{t+1}(\tau)\mathcal{V}_{t+1}(\tau)}{\mathcal{U}'_t(\tau)\mathcal{V}_t(\tau)} q_{t+1}(\tau)\epsilon_{t+1}^I \mathcal{S}'\left(\frac{I_{t+1}(\tau)}{I_t(\tau)}\right)\left(\frac{I_{t+1}(\tau)}{I_t(\tau)}\right)^2 \right\} \end{aligned} \tag{6.5}$$

$$q_t(\tau) = \beta E_t \left\{ \frac{\mathcal{U}'_{t+1}(\tau)\mathcal{V}_{t+1}(\tau)}{\mathcal{U}'_t(\tau)\mathcal{V}_t(\tau)} \left(q_{t+1}(\tau)(1-\delta) + r_{t+1}^k\right) \right\} \tag{6.6}$$

with Π_{t+1} price inflation between t and $t+1$.

[2] See (Christiano et al., 2005; Smets and Wouters, 2003, 2005, 2007). We follow these authors and assume $\mathcal{S} = 0$, $\mathcal{S}' = 0$ and $\mathcal{S}'' > 0$ at steady state.

Labour supply and wage setting

Households provide labour on a monopolistically competitive market. An employment agency aggregates labour supplied and provides firms with an homogeneous labour bundle. The relationship between total demand for labour and each household's supply is a function of the demanded wage ($w_t(\tau)$) over aggregate wage (W_t, Equation 6.7). In this context, households are paid with a mark-up over their marginal disutility of labour which depends on the elasticity of substitution between workers θ_w.

$$l_t(\tau) = \left(\frac{w_t(\tau)}{W_t}\right)^{-\theta_w} \frac{L_t}{\mathbb{N}} \tag{6.7}$$

In addition, wage stickiness is introduced through a Calvo wage setting, each household resetting its wage with an exogenous probability $(1-\xi_w)$.

Linearising the first order condition of households' utility maximisation on the labour market around the steady state yields the following wage Phillips curve:

$$\begin{aligned} &\widehat{RW}_t - \widehat{RW}_{t-1} + (\hat{\Pi}_t - \gamma_w \hat{\Pi}_{t-1}) = \\ &\tilde{\beta}(1+g)\left(\widehat{RW}_{t+1} - \widehat{RW}_t + \left(\hat{\Pi}_{t+1} - \gamma_w \hat{\Pi}_t\right)\right) \\ &+ \frac{(1-\tilde{\beta}\xi_w(1+g))(1-\xi_w)}{\xi_w(1+\theta_w((1+\sigma_l)(1+\mathcal{B})-1))}\left[-\widehat{RW}_t - \hat{L}_t + (1+\sigma_l)(1+\mathcal{B})(\hat{L}_t - h_l\hat{L}_{t-1}) + \hat{C}_t\right]. \end{aligned} \tag{6.8}$$

with $\mathcal{B}$ a function of the parameters of the model, $\tilde{\beta}$ a function of β and g the exogenous growth rate of TFP.[3] RW_t corresponds to the real wage defined as W_t/P_t. γ_w is the degree of indexation of non reset wages on past inflation.

6.2.2 Firms

Demand for production factors

Firms (a continuum of size $\mathbb{P}$) produce partially substitutable goods from labour and capital. They hire domestic labour at cost W_t. In addition, firms rent capital $K_t^d(\varepsilon)$ from households at cost r^k.[4] We assume installation delays so that at market equilibrium and on aggregate $K_t^d = K_{t-1}$.

[3] $\mathcal{B} = \frac{\theta_w - 1}{\theta_w}\frac{\theta - 1}{\theta}\frac{(1-\alpha)(1-\sigma_c)(1-\eta)}{(1+\sigma_l)\sigma_c c y}$ and $\tilde{\beta} = \beta(1+g)^{(1-\sigma_c)(1-h_c)-1}$

[4] The price of capital is by construction the same as investment, which is identical to the price of consumption as we assume that both goods are identical. This is also equivalent to assume a perfectly competitive investment good sector with a one-to-one technology from consumption goods to investment goods. This implies that in nominal terms the rental cost of capital equals $r_t^k K_t^d(\varepsilon) P_t$.

Each firm ε produces $y_t(\varepsilon)$ from a standard constant returns to scale production function :

$$y_t(\varepsilon) = (\zeta_t L_t(\varepsilon))^{1-\alpha} \left(K_t^d(\varepsilon)\right)^\alpha \tag{6.9}$$

$$\text{with cost } W_t L_t(\varepsilon) + r_t^k P_t K_t^d(\varepsilon), \tag{6.10}$$

where ζ is the exogenous labour productivity whose deterministic trend grows at rate g and α is the share of capital in value added. The arbitrage condition between labour and capital demand yields:

$$\frac{1-\alpha}{\alpha} = \frac{W_t L_t(\varepsilon)}{r_t^k K_t^d(\varepsilon) P_t} \quad \text{and on aggregate} \quad \frac{1-\alpha}{\alpha} = \frac{W_t L_t}{r_t^k K_{t-1} P_t} \tag{6.11}$$

and the real marginal cost of production:

$$RMC_t = \frac{MC_t}{P_t} = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \left(\frac{RW_t}{\zeta_t}\right)^{1-\alpha} \left(r_t^k\right)^\alpha \tag{6.12}$$

Price setting

Partial substitutability (with elasticity θ) allows firm to price a mark-up over their marginal cost. We assume a *Calvo* price setting. Firm ε resets its price $p_t(\varepsilon)$ with an exogenous probability $(1-\xi)$ and maximises its expected profit until the next price setting possibility, subject to the production factor optimization, the production function, as well as the demand function (Equation 6.13) and a price indexation rule (with parameter γ).[5]

$$y_t(\varepsilon) = \left(\frac{p_t(\varepsilon)}{P_t}\right)^{-\theta} \frac{Y_t}{\mathbb{P}} \tag{6.13}$$

After linearisation of the first order condition, we obtain a standard new-Keynesian price Phillips curve:

$$\hat{\Pi}_t - \gamma\hat{\Pi}_{t-1} = \tilde{\beta}(1+g)\left(\hat{\Pi}_{t+1} - \gamma\hat{\Pi}_t\right) + \frac{(1-\tilde{\beta}\xi(1+g))(1-\xi)}{\xi}\widehat{RMC}_t \tag{6.14}$$

where inflation depends positively on past inflation, future expected inflation but also on real wages, capital returns and negatively on productivity shocks through the real marginal cost of production.

6.2.3 Central bank and government

The central bank sets the nominal interest rate R_t in deviation from its target R^* through a Taylor rule (Taylor, 1993), where it reacts smoothly to both inflation in deviation from its target Π^* (with smoothing parameter ρ) and the output gap (defined as output in

[5] This price indexation is necessary in a model with steady state inflation.

deviation from its steady state $\bar{Y}$).

$$R_t = R_{t-1}^{\rho} \left(R^* \left(\frac{\Pi_t}{\Pi^*} \right)^{r_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{r_y} \right)^{1-\rho} \tag{6.15}$$

Public consumption G_t is assumed exogenous, and financed by lump-sum taxes Φ.

6.2.4 Steady state and calibration

In the present model, growth is exogenous. In the long run, all real variables grow at the same rate as TFP. At steady state, inflation equals the central bank's target.

We calibrate the model based on (Smets and Wouters, 2005).[6] We marginally depart from this estimation by adjusting the mark-ups to better compare our simulations with other market reform evaluations, in particular (Everaert and Schule, 2008). Table 6.1 compares the model at steady state with actual data for the euro area (12 countries). Table 6.2 presents our calibration of structural parameters.

	DATA EA (12)	MODEL EA
Output in 2000 (GDP)*	6943	6943
Output per capita growth rate**	1.2%	1.2%
Working age population in 2000 ***	135.9	135.9
Hours worked per week and working age capita (since 2000)	34.3	34.3
Gross Op. Surplus to VA	46%	42%
Gross wages to VA	53%	58%
Nominal 3 month Euribor**	3.8%	4.0%
Inflation**	2.0%	2.0%
Private consumption to GDP ratio	56.3%	56.9%
Public consumption to GDP ratio	19.7%	19.7%
Investment to GDP ratio	21.9%	23.5%

Sources: Eurostat (National accounts, inflations, Euribor, population, Labour Force Survey -incl. Secondary job)
Data are averaged from 1995 to 2007 to exclude the crisis. Depending on availability, samples may start after 1995 and/or exclude some countries from the euro area (12).
* in billion € in current prices
** annualised
*** aged from 15 to 64, in millions

Table 6.1 – Actual data for the euro area and the corresponding values at steady state with our calibration

[6] The calibration constraints are further detailed in (Campagne and Poissonnier, 2015).

Technology			
Technology parameter	α	0.3	(Smets and Wouters, 2005)
Depreciation rate	δ	0.025	*idem*
Capital rigidity	S	6.17	*idem*
TFP growth rate	g	0.003	Eurostat
Population size	$\mathbb{N}$	135 922 100	Eurostat
Monetary policy			
Inflation	Π^*	1.005	Consensus, ECB
Smoothing parameter	ρ	0.85	(Smets and Wouters, 2005)
Weight on inflation	r_π	1.4	*idem*
Weight on output gap	r_y	0.11	*idem*
Prices and Wages			
Substitutability between goods	θ	6	QUEST III, GEM, NAWM, EAGLE $^\sharp$
Substitutability between workers	θ_w	4	*idem*
Price rigidity	ξ	0.90	(Smets and Wouters, 2005)
Wage rigidity	ξ_w	0.92	*idem*
Price indexation	γ_p	0.29	*idem*
Wage indexation	γ_w	0.90	*idem*
Preferences			
Households discount factor	β	0.9983	steady state constraint
Risk aversion	σ_c	1.13	(Smets and Wouters, 2005)
Inverse Frisch elasticity	σ_l	2	*idem*
Consumption habits	h_c	0.61	*idem*
Labour habits	h_l	0	*idem*
Weight on labour disutility	κ	5812.38	steady state constraint on hours worked

$^\sharp$(Roeger et al., 2008; Bayoumi et al., 2004; Everaert and Schule, 2008; Coenen et al., 2008; Gomes et al., 2013)

Table 6.2 – Structural parameters

6.3 Understanding the mechanisms of structural reforms on the goods and labour markets

6.3.1 Long term effect in the *Smets and Wouters* model

In the *Smets and Wouters* model, as in most other DSGE models, long term growth is exogenous. However, in the long term, the model is equivalent to a standard RBC model with monopolistic competition and GDP as well as other real variables in level depend on structural parameters in a way we can compute.

The long term equilibrium is the result of supply for production factors by households and demand for these factors by the firms.

Firms' profit maximization implies the following demands

$$\text{for capital} \quad \frac{\theta-1}{\theta}\alpha\zeta^{1-\alpha}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha-1} = \bar{r}^k \tag{6.16}$$

$$\text{for labour} \quad \frac{\theta-1}{\theta}(1-\alpha)\zeta^{1-\alpha}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha} = \overline{RW} \tag{6.17}$$

On households' side, the supply for capital is perfectly elastic because households can substitute financial savings for investment. They provide capital as long as its remuneration is equivalent to that of financial savings $\bar{r}^k = \overline{rr} + \delta$, with $\overline{rr}$ the real return on the risk free asset. As for labour, their supply is given by their individual consumption-leisure arbitrage:

$$\overline{RW} = \frac{\theta_w}{\theta_w - 1}(1+\sigma_l)\bar{c}\frac{\sigma_c}{1-\sigma_c}\frac{\kappa(1-\sigma_c)\bar{l}^{(1+\sigma_l)(1-h_l)-1}}{1-\kappa(1-\sigma_c)\bar{l}^{(1+\sigma_l)(1-h_l)}} = \frac{\theta_w}{\theta_w - 1}(1+\sigma_l)\bar{c}f(\bar{l}) \tag{6.18}$$

with $\bar{c}, \bar{l}$ individual consumption and labour supply respectively and f an increasing function of labour supply.

Reform on the goods market A decrease in firms' market power on the goods market ($\theta \uparrow$) fosters output in the long run through different channels. First, as the distortion from the perfectly competitive equilibrium is reduced, firms capture less mark-ups in the production process and distribute more to factor remuneration at given output, hence higher costs (Figure 6.3). Firms then try to compensate this drop by an increase in production : the demand for production factors is then scaled-up (direct effect of θ in Equations (6.16) and (6.17), the demand curves on Figure 6.1 shift from D1 to D').

In a second round effect, the demand for both factor increases (from D' to D2) as the productivity of each factor is fostered by the other (effect of L in Equation (6.16) and K in Equation (6.17)).

Finally, the supply curve for capital is unchanged (investment crowds out financial savings to meet firms increased demand for capital), but the supply curve for labour is shifted downwards by the increase in consumption consecutive to higher income (from S1 to S2 on Figure 6.1b), Equation 6.18).[7]

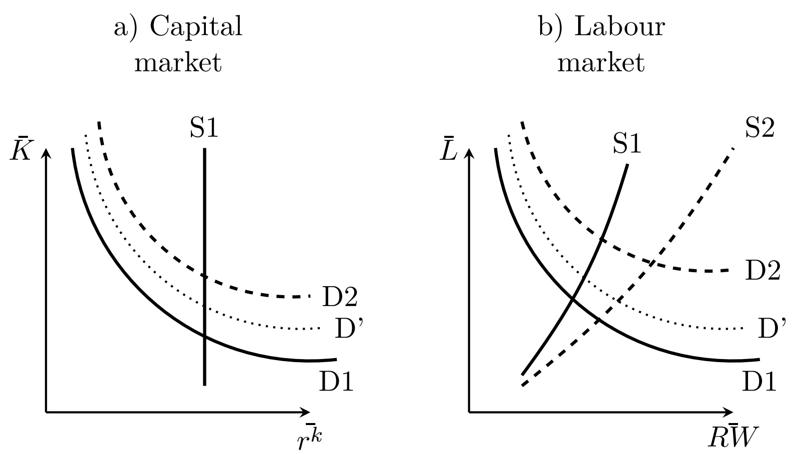


Figure 6.1 – Effect of lower firms' market power ($\theta \uparrow$) on the capital and labour market

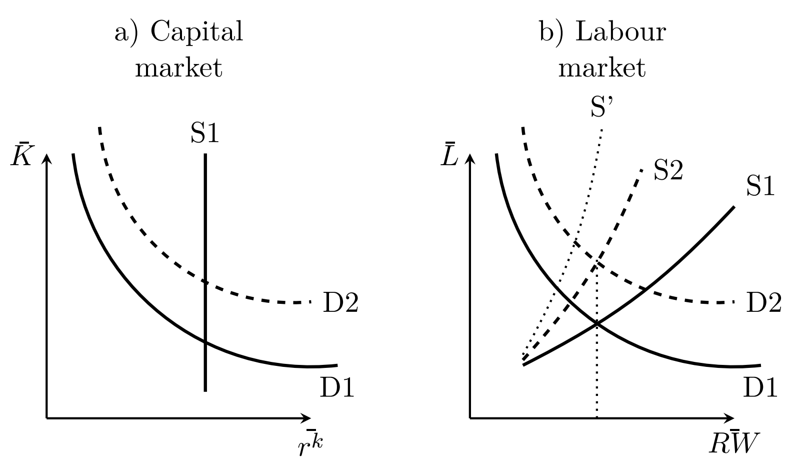


Figure 6.2 – Effect of lower workers' market power ($\theta_w \uparrow$) on the goods and labour market

Reform on the labour market A decrease in households' market power on the labour market directly affects the labour supply curve (Equation 6.18). As they lose market power,

[7] These results are in particular compatible with (Blanchard and Giavazzi, 2003). A permanent increase in the elasticity of substitution across goods eventually leads to higher real wages and labour.

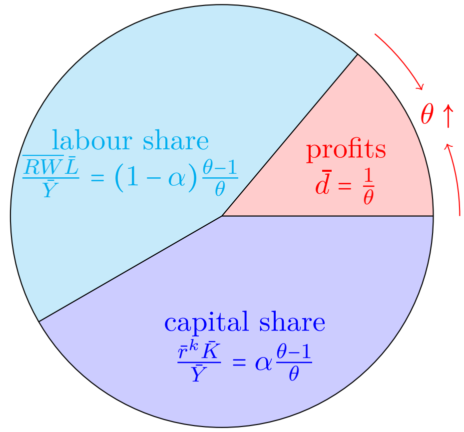


Figure 6.3 – Decomposition of production

households can not withhold their supply as much for a given wage rate (S1 shifts to S' on Figure 6.2b).

As labour supply increases, so does capital productivity and in turn firms' demand for this production factor. Symmetrically this increased capital demand fosters labour productivity and firms demand for this second production factor. In all, demand curves shift upwards from D1 to D2 on Figure 6.2.

Following a reform on the labour market, the relative use of capital and labour is fixed as the return on capital is anchored by the monetary policy rate (Equation 6.16). Consequently the real wage is also unchanged by the reform (Equation 6.17).[8] As a consequence, households adjust their consumption-leisure arbitrage by substituting labour for consumption (labour supply shifts from S' to S2 on Figure 6.2b).

In all, capital and labour increase in the same proportion as output. As a consequence, so does investment and because of market clearing so does consumption. Labour market reforms thus result in an homothetic transformation of real quantities.

Long term elasticities Having clarified the underlying key mechanisms leading to an increase in output, we compute the long term elasticities of output to structural reforms to better understand the determinants of the scale of gains. In (Campagne and Poissonnier, 2015), we solve for the steady state equilibrium in an extended version of the model. These equations can be simplified according to the *Smets and Wouters* set-up (Appendix 6.B).

[8]These results are also in line with (Blanchard and Giavazzi, 2003). A decrease in workers market power eventually leads to higher labour but has no effect on real wages.

From the production function we have :

$$\bar{Y} = \bar{\zeta}\left(\alpha\frac{\theta-1}{\theta}(\frac{1}{\tilde{\beta}}-1+\delta)^{-1}\right)^{\frac{\alpha}{1-\alpha}}\bar{L} \tag{6.19}$$

From households consumption-leisure arbitrage:

$$\bar{L} = \left(\frac{1}{\tilde{\kappa}}\frac{\mathcal{B}}{\mathcal{B}+1}\right)^{\frac{1}{(1+\sigma_l)(1-h_l)}} \tag{6.20}$$

where$cy = \bar{C}/\bar{Y}$ and $\tilde{\kappa} = \kappa(1-\sigma_c)\left(\mathbb{N}\right)^{-(1-h_l)(1+\sigma_l)}$.

Also, from the market clearing the shares of final demands in total output verify:

$$1 = cy + iy + gy \tag{6.21}$$

with the ratio of government expenditure to output gy being exogenous and the ratio of investment to output iy given by the equilibrium on the capital market:

$$\alpha\frac{\theta-1}{\theta} = \frac{\overline{rr}+\delta}{g+\delta}iy \tag{6.22}$$

Finally, in equilibrium the real wage verifies:

$$\overline{RW} = \bar{\zeta}\left(\alpha\frac{\theta-1}{\theta}(\frac{1}{\tilde{\beta}}-1+\delta)^{-1}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)\frac{\theta-1}{\theta} \tag{6.23}$$

Differentiating these equilibrium equations with respect to θ or θ_w we get the following elasticity of output to goods market reforms:

$$\frac{d\bar{Y}}{\bar{Y}} = \left(\frac{\alpha}{1-\alpha}+\frac{1}{(1+\sigma_l)(1-h_l)(1+\mathcal{B})}(1+\frac{iy}{cy})\right)\frac{\frac{d\theta}{\theta}}{\theta-1} \tag{6.24}$$

and to labour market reforms:

$$\frac{d\bar{Y}}{\bar{Y}} = \frac{1}{(1+\sigma_l)(1-h_l)(1+\mathcal{B})}\frac{\frac{d\theta_w}{\theta_w}}{\theta_w-1} \tag{6.25}$$

We also compute the long term elasticity of households' utility to structural reforms on the goods market:

$$\frac{d\bar{U}}{\bar{U}} = (1-\sigma_c)(1-h_c)\left[\left(\frac{\alpha}{1-\alpha}-\frac{iy}{cy}\right)+\frac{1-\frac{\theta_w-1}{\theta_w}\frac{\theta-1}{\theta}\frac{(1-\alpha)(1-h_l)}{(1-h_c)cy}}{(1+\sigma_l)(1-h_l)(1+\mathcal{B})}(1+\frac{iy}{cy})\right]\frac{\frac{d\theta}{\theta}}{\theta-1} \tag{6.26}$$

and on the labour market:

$$\frac{d\bar{U}}{\bar{U}} = (1-\sigma_c)(1-h_c)\frac{1-\frac{\theta_w-1}{\theta_w}\frac{\theta-1}{\theta}\frac{(1-\alpha)(1-h_l)}{(1-h_c)cy}}{(1+\sigma_l)(1-h_l)(1+\mathcal{B})}\frac{\frac{d\theta_w}{\theta_w}}{\theta_w-1} \tag{6.27}$$

In all four cases, the elasticity crucially depends on households utility parametrisation (σ_l, σ_c, $\mathcal{B}$),[9] which calls for a detailled sensitivity analysis (Section 6.4).

The effect of both types of reforms on output and other real variables is positive (Figure 6.4). Replicating a standard exercise in this literature, decreasing product and labour mark-ups to the level of Europe's best performers would imply roughly 10% increases in such variables.

The sign of the elasticity of utility is however ambiguous. This ambiguity does not stem from the term $1-\sigma_c$ whose sign is the same as the steady state utility $\bar{U}$ but in a more complicated way from the other terms.

With our calibration, the effect on utility of an output augmenting structural reform is negative (disutility increases, Figure 6.5).[10] Following either reform, both consumption and labour supply increase, contributing in opposite ways to households utility. In addition, habits (as in Abel (1990); Fuhrer (2000)) add a negative externality to welfare: the general increase in consumption mitigates the gains from each households' higher consumption (Figure 6.5b and 6.5c).

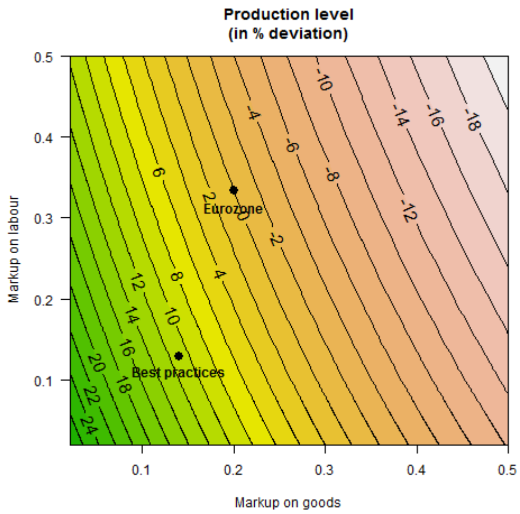
6.3.2 Transition dynamics

We simulate labour and goods market reforms comparable to the scenarios proposed by (Everaert and Schule, 2006). They consider an increase in competition up to the average level of the three best European performers, being Denmark, Sweden and the United Kingdom. The reform corresponds to a shift from 33 to 13% mark-up on labour and 20 to 14% mark-up on goods. Figure 6.6 and 6.7 present the transition of the economy to the new steady state after such reforms. Quantitative aspects are commented in the next section on sensitivity analysis.

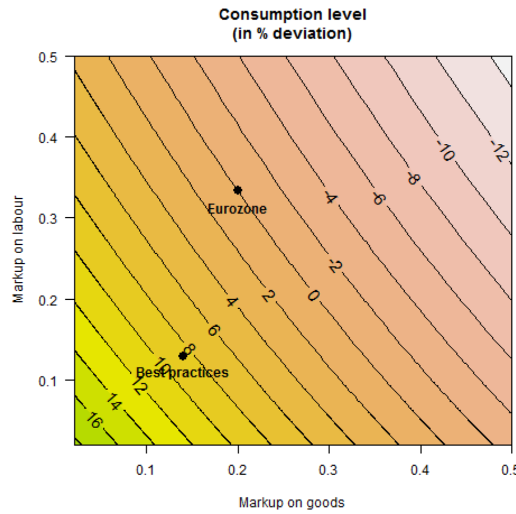
Increased competition on the goods market - Figure 6.6 Increasing competition on the goods market ($\theta\uparrow$ or price markup $= \theta/(\theta-1)\downarrow$) induces an immediate change in the distribution of production factors remuneration as explicated in the previous section. As θ increases, the share of profits in production mechanically diminishes, and the shares paid to capital and labour increase, stirring up the production cost in the short term. On

[9] Note that the sign of $\mathcal{B}$ is the sign of $1-\sigma_c$ and equation (6.44) implies that $1+\mathcal{B}>0$ for the labour level to be well-defined, as $\mathcal{B}/\tilde{\kappa}>0$. Moreover, $\bar{U}$ is also of the sign of $1-\sigma_c$.

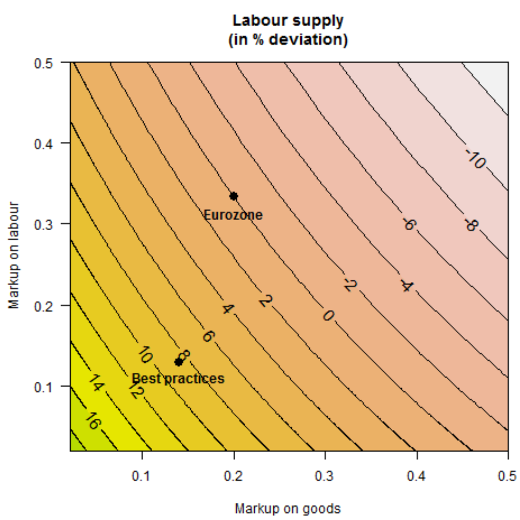
[10] We choose to refer to the disutility has the steady state level of utility is negative. As such, a negative p.p. differential of utility indicates a lesser negative utility level, that is an improvement in utility. An interpretation in terms of disutility is more convenient and intuitive as a negative p.p. differential of disutility is indeed a decrease in the disutility.



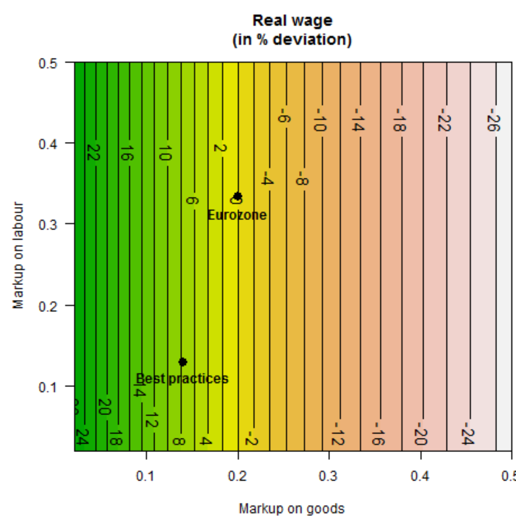
(a) Production



(b) Consumption



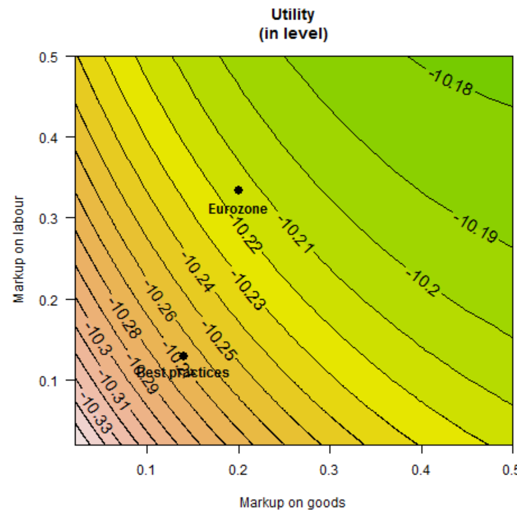
(c) Labour



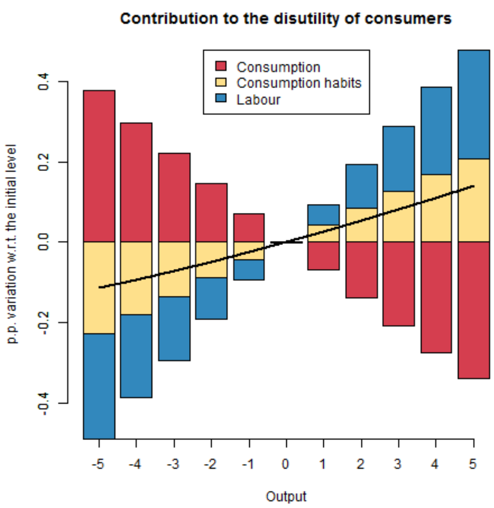
(d) Real wage

The x-axis represents the level of mark-up on goods (i.e. $1/\theta - 1$) whereas the y-axis represents the mark-up on wages (i.e. $1/\theta_w - 1$). The point named *Eurozone* corresponds to the standard calibration of our model and the point *Best practices* to the level of mark-ups in the three best European practices, namely Denmark, Sweden and the United Kingdom, as defined in (Everaert and Schule, 2006).

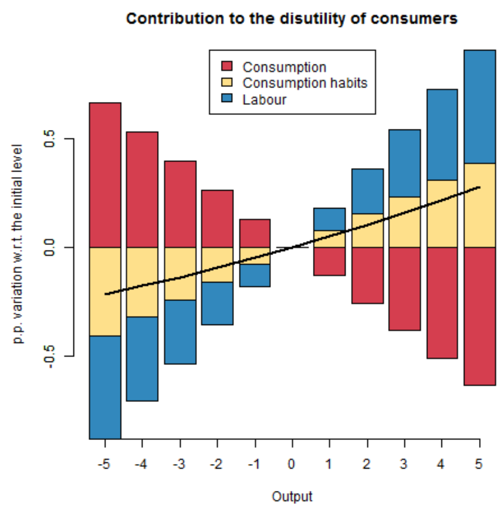
Figure 6.4 – Steady state variations upon reforms in p.p. with respect to the initial calibration



(a) Total utility



(b) Goods market deregulation



(c) Labour market deregulation

Upper figures: The x-axis represents the level of mark-up on goods (i.e. $^1/_{\theta - 1}$) whereas the y-axis represents the mark-up on wages (i.e. $^1/_{\theta_w - 1}$). The point named *Eurozone* corresponds to the standard calibration of our model and the point *Best practices* to the level of mark-ups in the three best European practices, namely Denmark, Sweden and the United Kingdom, as defined in (Everaert and Schule, 2006).
Lower figures: Decomposition of changes in disutility following pro-competitive reforms against the output increase induced by the reform. We represent the disutility of households. A increase in the disutility is therefore detrimental to the households.
The x-axis indexes structural reforms (θ or θ_w) by their impact on output.

Figure 6.5 – Steady state utility levels and decomposition upon reforms

the capital market, adjustment through quantities being sluggish, the return on capital temporarily increases. On the labour market, the real wages only gradually increase, the labour demand overshoots in the short run.[11]

In nominal terms, the reform will eventually imply a decrease in prices. This expected deflation prevails in the Phillips curve compared to the increase in the real marginal cost. As inflation temporarily decreases, so does the nominal interest rate.

On households' side, the consumption-leisure-investment arbitrage is modified through a mix of substitution and wealth effects, consequence of the particular choice of the utility function as well as its calibration. In all, investment immediately increases upon reform to take advantage of the favourable return on capital, this investment is financed through increased labour supply and crowding out of financial savings. Consumption rapidly increases (without overshooting) in line with the increase in the permanent income (wealth effect).

Eventually, the return on capital returns to steady state (unchanged by the reform) as the real interest rate returns to its initial steady state as well (no arbitrage condition). The long term increase in the real marginal cost therefore fully passes through to real wages. In all, production increases permanently due to both an increase in investment (i.e. capital) during the transition and to increased real wages and labour supply.

In terms of utility (Figure 6.7), as labour supply overshoots in the transition to finance an early increase in investment and consumption, the disutility of labour outweighs the gains from higher consumption: there is a transition cost to the reform (Jonsson, 2007; Matheron, 2002; Matheron and Maury, 2004). In the long run, goods market reforms can be detrimental to welfare as well (Equation 6.26). We show how this result depends on the specification of utility in a sensitivity analysis (Section 6.4).

Increased competition on the labour market - Figure 6.6 Following an increase in competition on the labour market ($\theta_w \uparrow$ or wage markup $= \theta_w/(\theta_w - 1) \downarrow$), output, labour, consumption will eventually increase in the same proportion (cf. *supra*).

As for the goods market reform but in a lesser extend, in the transition, labour supply overshoots to finance the increase in consumption and investment, in line with higher capital demand and higher permanent income.

The labour supply overshoot increases capital returns, which in turns causes investment to overshoot. It also generates a small transitory decrease in real wages.[12] In all,

[11]This increase of both real wages and labour in the transition is in line with the short term effect of a goods market reform in (Blanchard and Giavazzi, 2003).

[12]As mentioned by (Blanchard and Giavazzi, 2003) there is a trade-off between lower real wages in the short run against higher employment in the long run. However, as wages are sticky in our model, rapid and large gains in labour supply outweigh the small decrease in real wages in terms of income.

the real marginal cost temporarily increases which puts upward pressure on prices. Consequently, the monetary policy rate also adjusts upward.

In the long run, and as in the previous reform case, the nominal interest rate returns to the initial steady state as inflation converges to the central banker's target. The return on capital follows (no arbitrage condition). Eventually the adjustment of prices offsets the drop of wages so that the real wage returns to the initial steady state as well.

In terms of utility (Figure 6.7), there is also a cost to the reform in the transition and in the long run. We analyse this result more specifically in Section 6.4.

For both types of reforms, within a year most of the output gains are achieved, and convergence is obtained within 15 years.

6.4 A sensitivity analysis

In order to ease comparison, transitions presented on Figure 6.6 correspond, as mentioned above, to the implementation of structural reforms as described in (Everaert and Schule, 2006). In our baseline, product (resp. labour) market reforms lead to a permanent increase in production of 4.8 p.p (resp. 5.8 p.p.), whereas this increase is of 1.6 p.p (resp. 6.1 p.p.) in the GEM model.

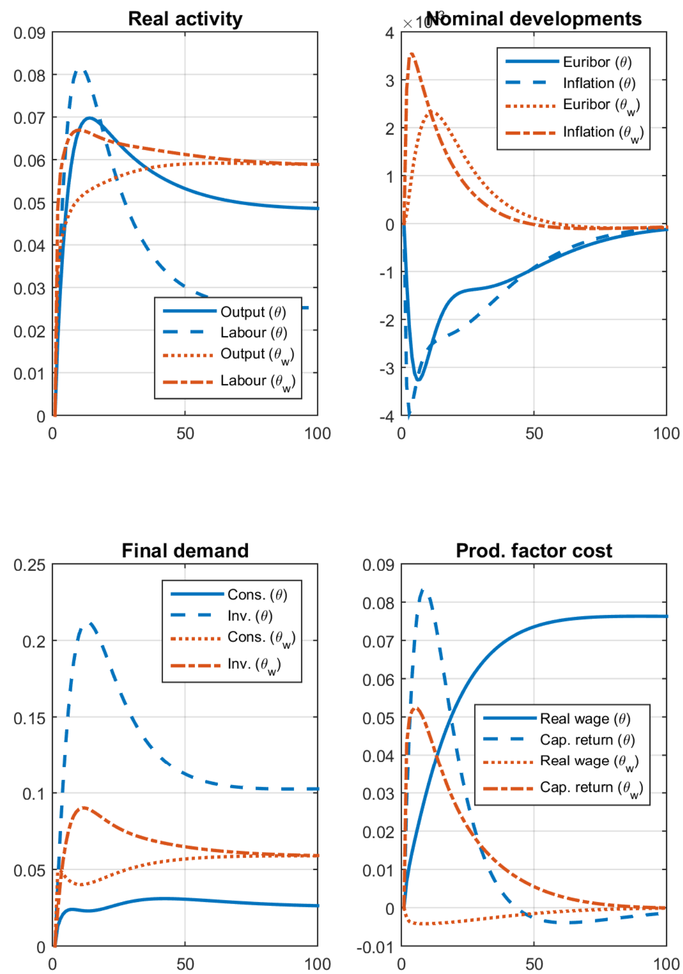
At first sight, structural reforms are output-enhancing in both models. Whereas magnitudes are close for labour market reforms, still with a long term difference of 0.6 p.p in production, this gap widens to more than 3 p.p. of production in the case of product market reforms.

Indeed, the present model remains a medium scale DSGE model *à la Smets and Wouters*, without the additional rigidities often incorporated in larger institutional DSGE models, and without a distinction between tradable and non tradable goods: results should obviously differ. However, and even though directly comparing results might be partially misleading, one should be cautious to carefully understand the size of production gains. In particular, we showed in Equations (6.24) and (6.25) that the impact of reforms crucially depends on the behaviour of households through their utility function.

Therefore, in this section, we focus on the impact of the specification of households' utility on the strength of structural reforms. In addition, we will take a deeper look at the implications in terms of welfare, and to the changes occurring when introducing hand-to-mouth households as it is common in institutional DSGE models.

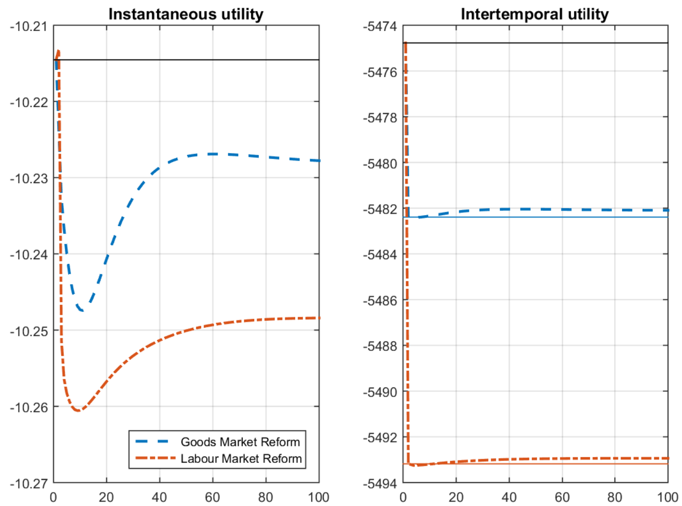
6.4.1 Calibration of households' utility

Range of values As highlighted in (Everaert and Schule, 2006), the estimation and identification of the utility function parameters, and in particular the (Frisch) elasticity



Structural reforms implemented here are the ones of (Everaert and Schule, 2006), that is of an alignment of markups on the level of the three best European performers.

Figure 6.6 – Transition following a decrease in the price or wage mark-up



Structural reforms implemented here are the ones of (Everaert and Schule, 2006), that is of an alignment of mark-ups on the level of the three best European performers. Solid black lines correspond to utilities without reforms, i.e. initial steady state. Coloured solid lines represent the intertemporal utility once the reform is implemented. The difference between this line and the final steady state is the transition cost to the reform.

Figure 6.7 – Utility in the transition following a decrease in the price or wage markup

of labour supply, is very sensitive to the methodology (micro or macro) and the sample considered. As a result, it is important to have a critical eye on the results with respect to these parameters.

(Trabandt and Uhlig, 2011) calibrate their model with an inverse Frisch elasticity of $\sigma_l = 1$ in line with (Kimball and Shapiro, 2008). They also consider an alternative based on (Cooley and Prescott, 1995) with $\sigma_l = 0.33$. These values are in line with the business cycle literature and close to values estimated by Bayesian methods, as for instance the different versions of Smets and Wouters' model with $\sigma_l = 2.4$ (Smets and Wouters, 2003), $\sigma_l = 2.0$ (Smets and Wouters, 2005) and $\sigma_l = 1.9$ (Smets and Wouters, 2007). However, micro and macro evidences are not easily reconciled and lead to very different values of the Frisch elasticity. (Bayoumi et al., 2004) mention that micro studies give a range for σ_l from 3 to as large as 20. In alternative scenarios for the GEM model, (Bayoumi et al., 2004; Everaert and Schule, 2006) set $\sigma_l = 6$ or 7.

For the inverse of the intertemporal elasticity of substitution of consumption σ_c, the debate is less fierce and values range from 0.5 (Bayoumi et al., 2004) to 2 (Trabandt and Uhlig, 2011). The different versions of *Smets and Wouters* give $\sigma_c = 1.3$ in (Smets and Wouters, 2003), $\sigma_c = 1.1$ in (Smets and Wouters, 2005) and $\sigma_c = 1.4$ in (Smets and Wouters, 2007).

Mechanism Recalling long term elasticities (Equations (6.24) and (6.25)), we can anticipate that both σ_c and σ_l will play a key role in the size of the impact of pro-competitive reforms. In particular, the inverse Frisch elasticity σ_l will be a crucial determinant.

Intuitively, we can expect weaker effects of reforms following an increase in both the inverse intertemporal elasticity of consumption and the inverse Frisch elasticity, as they go in the same direction of more *rigid* households.

In more specific details, an increase in σ_l directly translates into flatter labour supply curves on Figures 6.1 and 6.2, that is a lower response of labour to variations in real wages (there is less consumption-leisure arbitrage). Recall that the consumption-leisure arbitrage rewrites as follows from Equation 6.18

$$\frac{\theta_w - 1}{\theta_w}\overline{RW} = (1 + \sigma_c)\bar{C}f(\bar{L}) \tag{6.28}$$

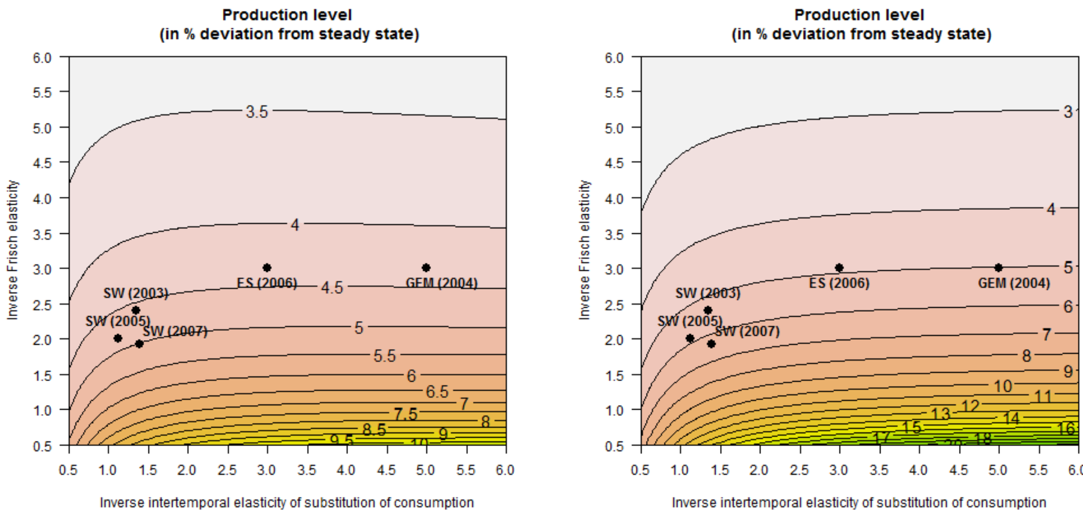
When conducting a pro-competitive reform on the labour market (Figure 6.2), the left-hand side term increases at any given $(\overline{RW}, \bar{L})$ through the increase in ${}^{(\theta_w - 1)}/_{\theta_w}$. As a result, we showed that the supply curve was shifting upwards, the amplitude of this shift being, by definition of the elasticity, negatively related to σ_l. As a result, the combination of both a flatter supply curve and of a lower shift, as σ_l is higher, implies a lower increase in labour and therefore in output.

In the case of a product market reform (Figure 6.1), the increase in output mainly stems from a movement along the labour supply curve S1 when the labour demand curve shifts upwards. In the case of a flatter supply curve, the induced increase in labour (and therefore output) will be lower.

Numerical applications Conducting identical reforms as in the previous section for different calibrations of the utility function, Figure 6.1 shows that this modification of the behaviour of households can lead to a significant change of the gains from increased competition (for extreme values of σ_l closer to 1). Changes in the elasticity of consumption give weak and ambiguous results on these gains.

With respect to the effect on the transition dynamics, Figure 6.A.1 to 6.A.3 in Appendix show that even though these parameters indeed appear in the households' first order conditions (Euler equation, wage Phillips curve, Tobins' Q and investment decision equations), their influence is very minor. They mainly scale the effects up or down in line with the impact on the steady state, but do not modify the *shape* of the dynamics.

Similarly for $h_c = 0$, Figures 6.A.4 and 6.A.8 in Appendix, show that the role of habits on the transition is non-existent, and negligible in the long-run for consumption habits. For labour habits ($h_l = 0.5$) (Figures 6.A.5 and 6.A.8), the effect is also a scale effect, the existence of habits scales the *without habits equivalent* inverse Frisch elasticity down.[13] In the long-run, the introduction of labour habits can boost up the impact of reforms of up to 3 p.p. of production (resp. 6 p.p.) for product (resp. labour) market reforms, in comparison to Figure 6.1.



(a) Goods market reforms - standard

(b) Labour market reforms - standard

The inverse intertemporal elasticity of substitution of consumption corresponds to σ_c, and the inverse Frisch elasticity to σ_l. Structural reforms implemented here are the ones of (Everaert and Schule, 2006), that is of an alignment of markups on the level of the three best European performers.
SW (YYYY) indicates the calibration of the model following Smets and Wouters (YYYY) for the Euro area. Our standard calibration relies on (Smets and Wouters, 2005). *GEM (2004)* corresponds to (Bayoumi et al., 2004) and *ES (2006)* to (Everaert and Schule, 2006).

Figure 6.1 – Impact of structural reforms in p.p. depending on the calibration of the utility function

6.4.2 Welfare costs - welfare gains

Output gains to structural reforms are positive. This result, we found unchallenged in the literature, is the consequence of diminishing market power and reducing the shortage organized by monopolistic competitors. A negative effect on welfare is rather unusual in the literature. Indeed, Jonsson (2007), Matheron (2002), or Matheron and Maury (2004) insist on the welfare cost of imperfect competition, whereas Everaert and Schule (2006), Gomes et al. (2013) and Forni et al. (2010) show based on institutional model simulations that structural reforms are welfare enhancing.

[13]Indeed, terms in $(1+\sigma_l)$ in the equations are now replaced by $(1+\sigma_l\prime) = (1+\sigma_l)(1-h_l)$, so for a given σ_l, the equivalent $\sigma_l\prime$ is lower.

Mechanism The main mechanism behind the positive welfare impact of structural reforms is as follows: as output increases so does households' revenues, consumption and therefore welfare. Indeed, the increase in output also translates into an increase in the disutility incurred from labour. However, in most standard calibrations, this detrimental effect is dominated by the beneficial consumption effect.

Nevertheless, there exists calibrations for which welfare is negatively impacted by the pro-competitive reforms in our model. In particular, recalling the analytical expressions for the long term elasticities (Equations 6.26 and 6.27), we observe that this conclusion can arise in two cases.

First, considering no habit formation ($h_c = h_l = 0$), the sign of long-term utility elasticities for both goods and labour markets reforms partly depends on the sign of $\mathcal{A} = 1 - ({}^{\theta_w - 1}/_{\theta_w})({}^{\theta - 1}/_{\theta})({}^{(1-\alpha)}/_{cy})({}^{1-h_l}/_{1-h_c})$. If negative, structural reforms can imply welfare losses. This can be the case when θ_w or θ are high, that is when there is a high degree of competition.

Second, and for goods market reforms only (Equation (6.26)), the elasticity depends on the sign of $({}^{\alpha}/_{1-\alpha}) - ({}^{iy}/_{cy})$. This term is positive, meaning welfare improvements from pro-competitive reforms, if and only if $iy < \alpha(1 - gy)$. In others words, as long as the government spending share remains low, there is no crowding out of private consumption by public consumption, households can consume the gains from increased revenues and therefore stimulate welfare.

Habit formation In addition to the previous mechanism, habit formation plays a crucial role through the term ${}^{1-h_l}/_{1-h_c}$.

First, the higher h_c, i.e. the more habits on consumption, the closer to zero the $\mathcal{A}$ term is. For strong consumption habits, this $\mathcal{A}$ term might even turn negative inducing welfare losses in the long-term. This decline in welfare logically comes from the fact that an increased aggregate consumption level also means stronger negative externalities at the household level, as they compare to the aggregate level.

Similarly, habits on labour (Bayoumi et al., 2004; Ratto et al., 2009) work in a similar fashion as consumption habits, that is as a *social norm* to which households compare. Following a general increase in labour supply, each household incurs a lesser disutility from working longer hours. However, while consumption habits are detrimental to welfare, labour habits are therefore welfare enhancing. Therefore, the higher h_l is, i.e. the more habits on labour, the further away from zero $\mathcal{A}$ is.

Note that we introduce habit formation in a multiplicative manner (a choice also made by Abel (1990); Gali (1994); Carroll et al. (2000); Fuhrer (2000)) and only consider external habits (i.e. *Catching up with the Joneses*).[14] However, (Carroll, 2000) shows that in the business cycle (i.e. in the linearised equations), multiplicative and additive habits can not be distinguished. However, habit modelled in a multiplicative way affect the long-term elasticity of utility whereas additive habits would be neutral on the long term elasticity of utility to a reform.

Numerical applications As shown on Figure 6.5, utility decreases upon reform in our baseline model. This stems from the introduction of consumption habits that mitigate (and actually offset) the gains from increased consumption. Indeed, considering a recalibration of the model without consumption habits ($h_c = 0$), we find a positive welfare impact of pro-competitive reforms (disutility decreases, Figure 6.2a and 6.2b). The transitions to the post reform steady state are almost identical (Figure 6.A.4 in the Appendix), and transitional costs remain (Figure 6.A.7). These costs (the difference between the final value of intertemporal utility and its value once the reform is effective) is however small relative to the long term gains (the difference between the final value of intertemporal utility and its value before the reform is effective).

Considering labour habits ($h_l = 0.5$) in our model yields very similar shape of transitions (yet different levels, see Figure 6.A.5 in the Appendix) but greatly impacts long term effect of the reforms as it is equivalent to a large reduction of the inverse Frisch elasticity at steady state. In terms of utility, the positive externality from labour habits offsets the otherwise overall negative effect on welfare from the reforms (Figure 6.2c and 6.2d).

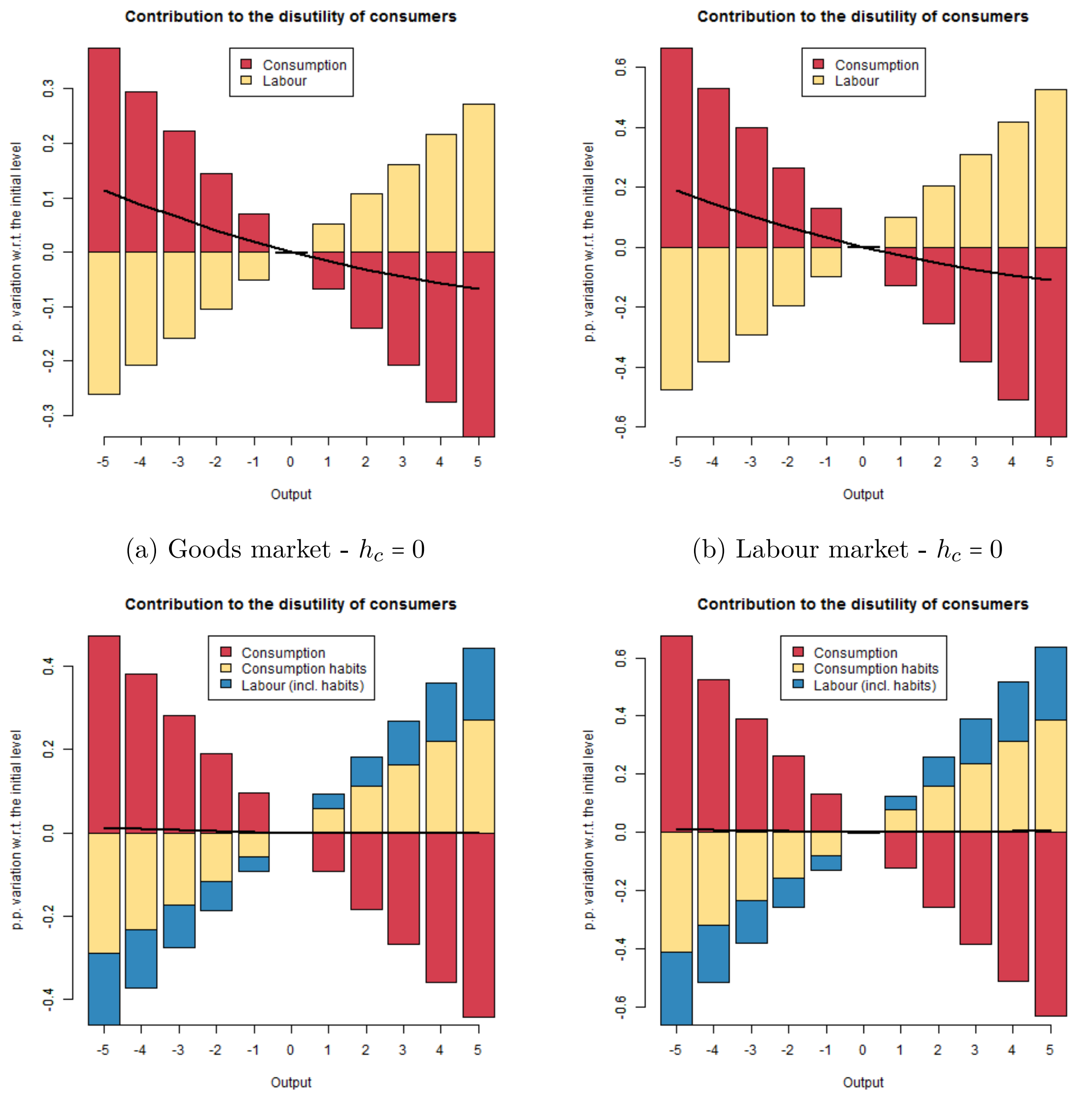
6.4.3 Adding *non-Ricardian* agents

As an additional standard mechanism, we introduce a fraction of hand-to-mouth consumers (Campbell and Mankiw, 1990). This element is often included in institutional neo-Keynesian models and has a sizeable impact on the size of production gains.

Range of values The estimation or calibration of the share of *non-Ricardian* agents in the economy μ is subject to debate. Actually, this parameter is often estimated using Bayesian methods or simply calibrated with *expert* insights as in (Everaert and Schule, 2006). For instance, this share is estimated to be 35% in France and 45% in the Euro area in GEM, and 40% for both in QUEST III.

However, micro-studies highlight that these estimated shares might be over-evaluated as only a few agents are strictly banned from financial markets. Indeed, a large number of agents, designated as *wealthy hand-to-mouth*, do possess a large illiquid wealth, such as housing, so that their short-term consumption is highly correlated to their current income. However, in the long-term, this conclusion might differ as assets can be traded. (Kaplan et al., 2014) compute values for the share of *wealthy hand-to-mouth* agents around 20% for

[14] Internal habit formation would therefore require further sensitivity tests.



(a) Goods market - $h_c = 0$

(b) Labour market - $h_c = 0$

(c) Goods market - $h_l = 0.5$

(d) Labour market - $h_l = 0.5$

Decomposition of changes in disutility following pro-competitive reforms against the output increase induced by the reform. We represent the disutility of households. A increase in the disutility is therefore detrimental to the households.
The x-axis indexes structural reforms (θ or θ_w) by their impact on output.

Figure 6.2 – Steady state utility levels and decomposition upon reforms

France. Close to (Kaplan et al., 2014), (Martin and Philippon, 2014) focus on the fraction of households with liquid assets representing less than 2 months of total gross disposable income and calibrate their model to a 46.6% share of *non-Ricardian* agents in France.

Moreover (Fève and Sahuc, 2013) show that once government spending is accounted for in the utility function, the estimated share of *non-Ricardian* agents in a model *à la* (Smets and Wouters, 2007) drops to 7% only.

We calibrate this share to estimated values in QUEST III, that is $\mu = 40\%$, a value close to the upper bound but rather standard in institutional DSGE models.

Utility and inequalities between households The introduction of heterogeneous households allows to study the effect of increased competition on inequalities. Figure 6.3 presents the long-term responses of *Ricardian* and *non-Ricardian* consumption, labour supply and real wages to both types of pro-competitive reforms. Transitional dynamics and level ratios between both types of households are presented on Figure 6.A.6 and 6.A.9 in the Appendix.

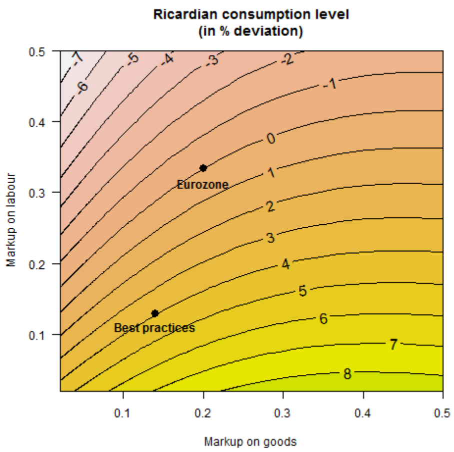
Deregulating the product market leads to a decrease in consumption and welfare disparities. Indeed both types of households are affected by a wealth effect, positive for *non-Ricardians* as the real wage goes up; negative for *Ricardians* as dividends shrink. Therefore, *non-Ricardians'* consumption markedly increases while *Ricardians'* consumption decreases slightly. On the labour market, *Ricardian* agents supply more labour as opposed to constrained households who decrease their supply. In addition, *Ricardians* are the only suppliers for the increased demand in capital which further crowds out their consumption.

Conversely, reforming the labour market leaves firms' profits untouched. The reform affects both types of households labour supply curve in a similar fashion: both work more. The increase in capital income finances investment, so much so that both types' consumption increases alike. All in all consumption, labour and utility inequalities stagnate.

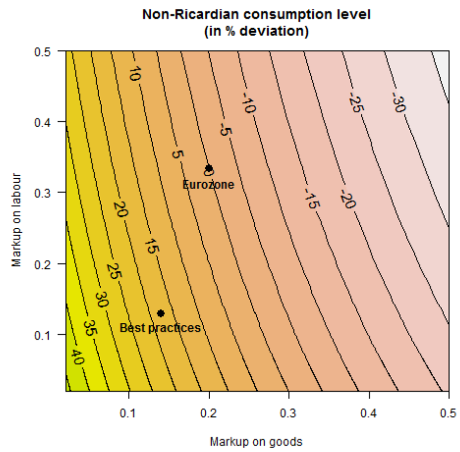
Increased competition on both markets also leads to a decrease in wages inequalities.

In addition, Figure 6.4 presents the steady state variation in the disutility of agents following a mark-up reform (indexed by the corresponding variations in production), as well as the contribution of each variable to this disutility: namely individual consumption, external consumption habits, and labour.

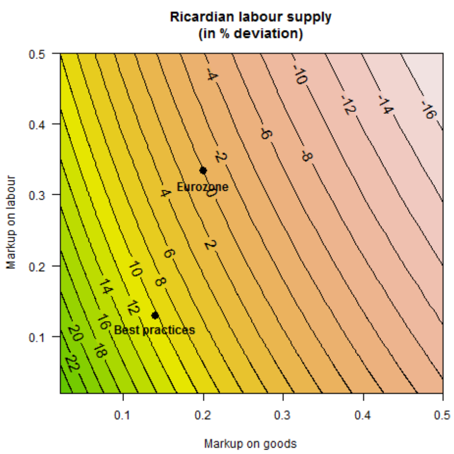
In the case of a product market deregulation (left column), disutility of the representative consumer decreases slightly (the utility therefore increases) with increasing output gains. Therefore, in the long term, the economic desirability of an increased output goes along with an increased utility. However, implications are very different across households. Indeed, *Ricardian* households are adversely affected by the reform as labour, individual



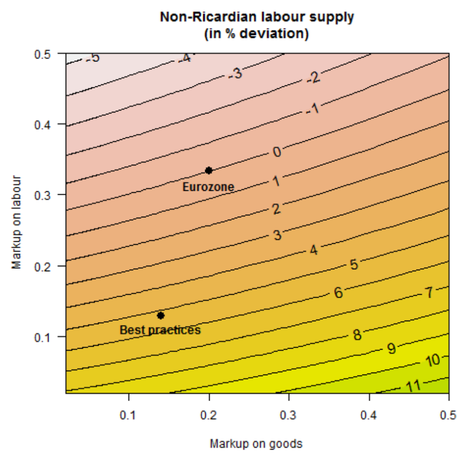
(a) Ricardian consumption



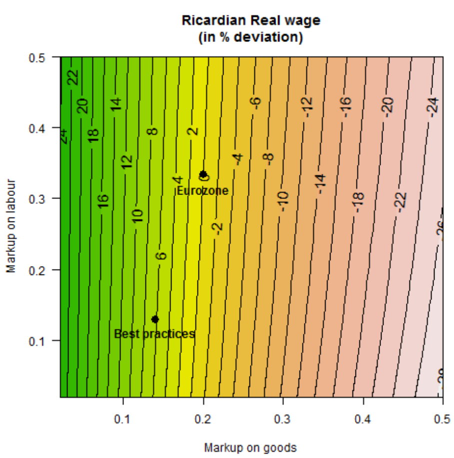
(b) non-Ricardian consumption



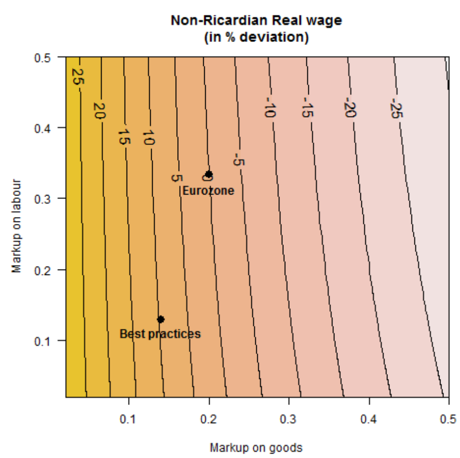
(c) Ricardian labour



(d) non-Ricardian labour



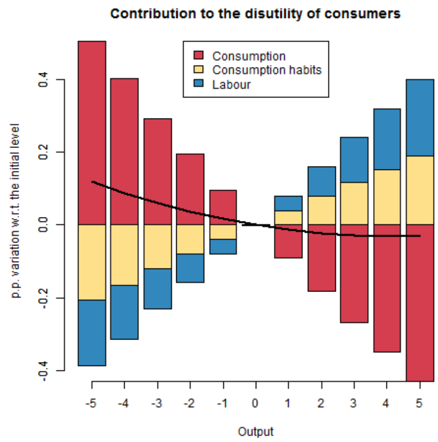
(e) Ricardian real wage



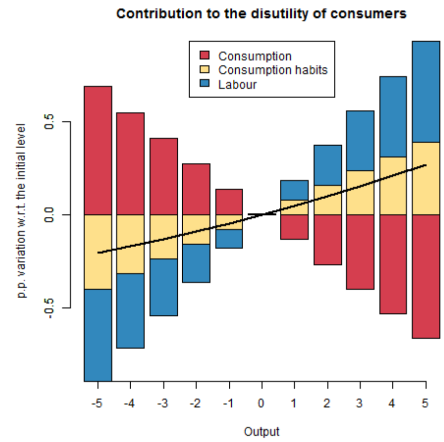
(f) non-Ricardian real wage

The x-axis represents the level of mark-up on goods (i.e. $1/\theta - 1$) whereas the y-axis represents the mark-up on wages (i.e. $1/\theta_w - 1$). The point named *Eurozone* corresponds to the standard calibration of our model and the point *Best practices* to the level of mark-ups in the three best European practices, namely Denmark, Sweden and the United Kingdom, as defined in (Everaert and Schule, 2006).

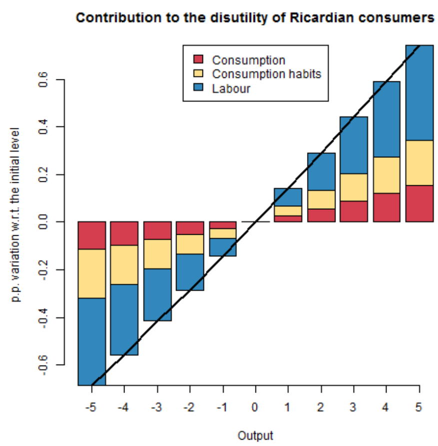
Figure 6.3 – Steady state variations upon reforms in p.p. with respect to the initial calibration the presence of non-Ricardian households



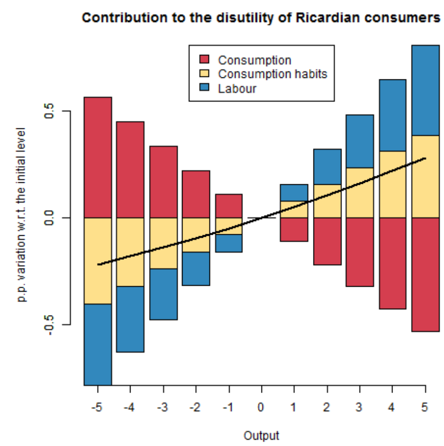
(a) Goods market reform - Representative



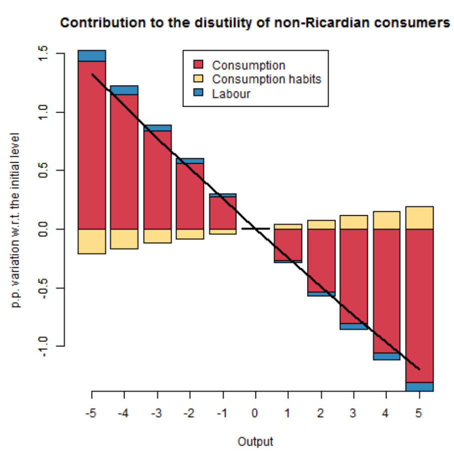
(b) Labour market reform - Representative



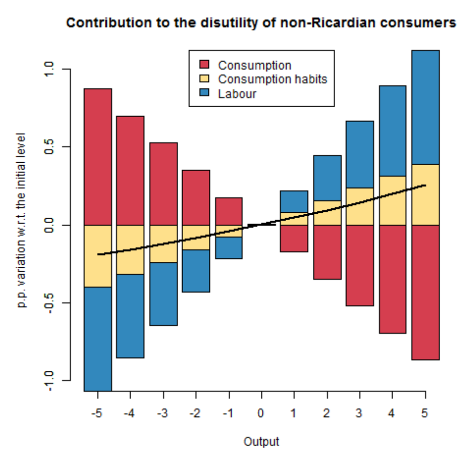
(c) Goods market reform - Ricardian



(d) Labour market reform - Ricardian



(e) Goods market reform - non-Ricardian



(f) Labour market reform - non-Ricardian

Decomposition of changes in disutility following pro-competitive reforms against the output increase induced by the reform. We represent the disutility of households. A increase in the disutility is therefore detrimental to the households.
The x-axis indexes structural reforms (θ or θ_w) by their impact on output.

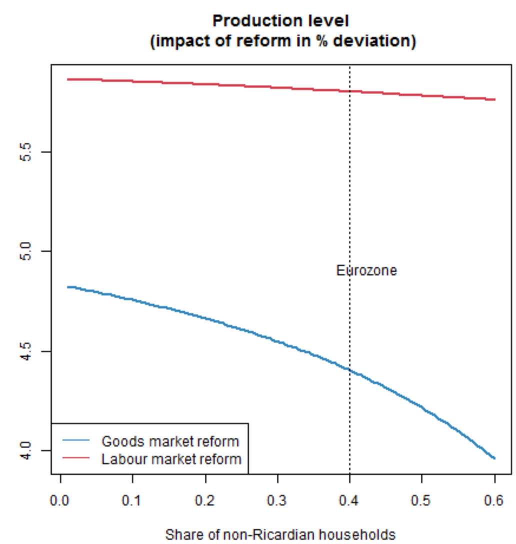
Figure 6.4 – Steady state utility levels and decomposition upon reforms in the presence of constrained households

consumption and consumption habits work in the same way of an increased disutility.

On the contrary, *non-Ricardian* households strongly benefit from the reform as their consumption increases faster than aggregate consumption does (as *Ricardian* consumption decreases upon reform). All in all, the net effect of consumption (individual consumption minus consumption habits) is strongly welfare improving. The decrease in labour supply, following the positive permanent income effect of the reform, also contributes mildly to the decrease in disutility.

For labour market reforms, aggregate disutility is increasing with output gains. Individual consumption increases and contributes positively to the utility. Nevertheless, the total effect of consumption, net of habits, is welfare enhancing, while the increase in labour supply results in a long-term decrease in the utility level. This decomposition is quasi identical across households.

Numerical implications for reforms' impact Increasing competition on both the labour and goods markets, in the presence of an increasing share of constrained households (Figure 6.5) goes for both types of reforms towards weaker long-term production gains. In the presence of liquidity-constrained agents, the effect of product market reforms can be mitigated by more than 0.5 p.p., whereas this effect is negligible for labour market reforms.



The share of *non-Ricardian* households corresponds to μ.
Structural reforms implemented here are the ones of (Everaert and Schule, 2006), that is of an alignment of markups on the level of the three best European performers.

Figure 6.5 – Impact of reforms in p.p. with respect to the initial production level depending on the share of non-Ricardian households

6.5 Conclusion

In a standard neo-Keynesian model which is at the core of most large scale institutional models, we evaluate the impact of structural reforms.

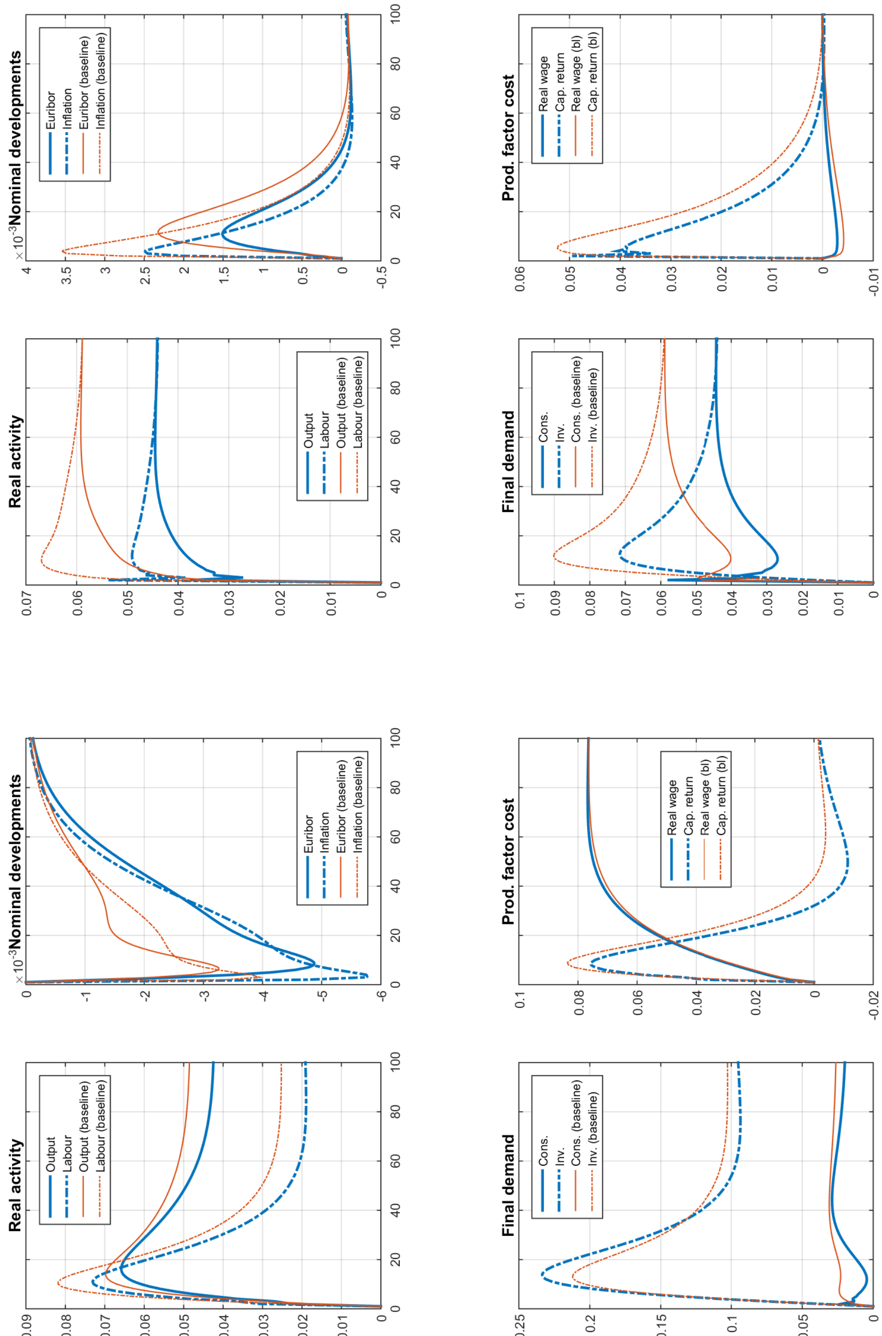
The long term gains in output following pro-competitive reforms are well documented in this class of model. Qualitatively, these models provide results in line with stylized facts obtained in deregulation-oriented models such as (Blanchard and Giavazzi, 2003). However, these similarities hide quite different economic mechanisms. We explicate these mechanisms, stressing in particular the importance of the households' leisure-consumption arbitrage. Quantitatively, we proceed to a sensitivity analysis: whereas qualitative results, including in the transition, are robust to changes in the specification of the model, quantitative results differ across specifications. The simple recalibration of households' utility can lead to additional gains or losses of a few percentage points in output following pro-competitive goods or labour markets reforms.

Morevover, policy recommendations based on welfare analyses are less clear-cut in the long run than recommendations solely based on output analyses. In the transition, welfare losses can be large despite the increase in permanent income. Introducing *non-Ricardian* agents allows stylized yet informative inequality analyses showing that goods market reforms reduce inequalities while labour market reforms are neutral.

These results imply two main recommendations: first, and at least, an extensive use of sensitivity analyses when building policy recommendations from these models, second, and more fundamentally, a rethink of the mechanisms at play in the long run (and in particular on the labour market) and of differences in elasticities (Frisch and intertemporal substitution of consumption) between the long run and over the business cycle.

Appendix

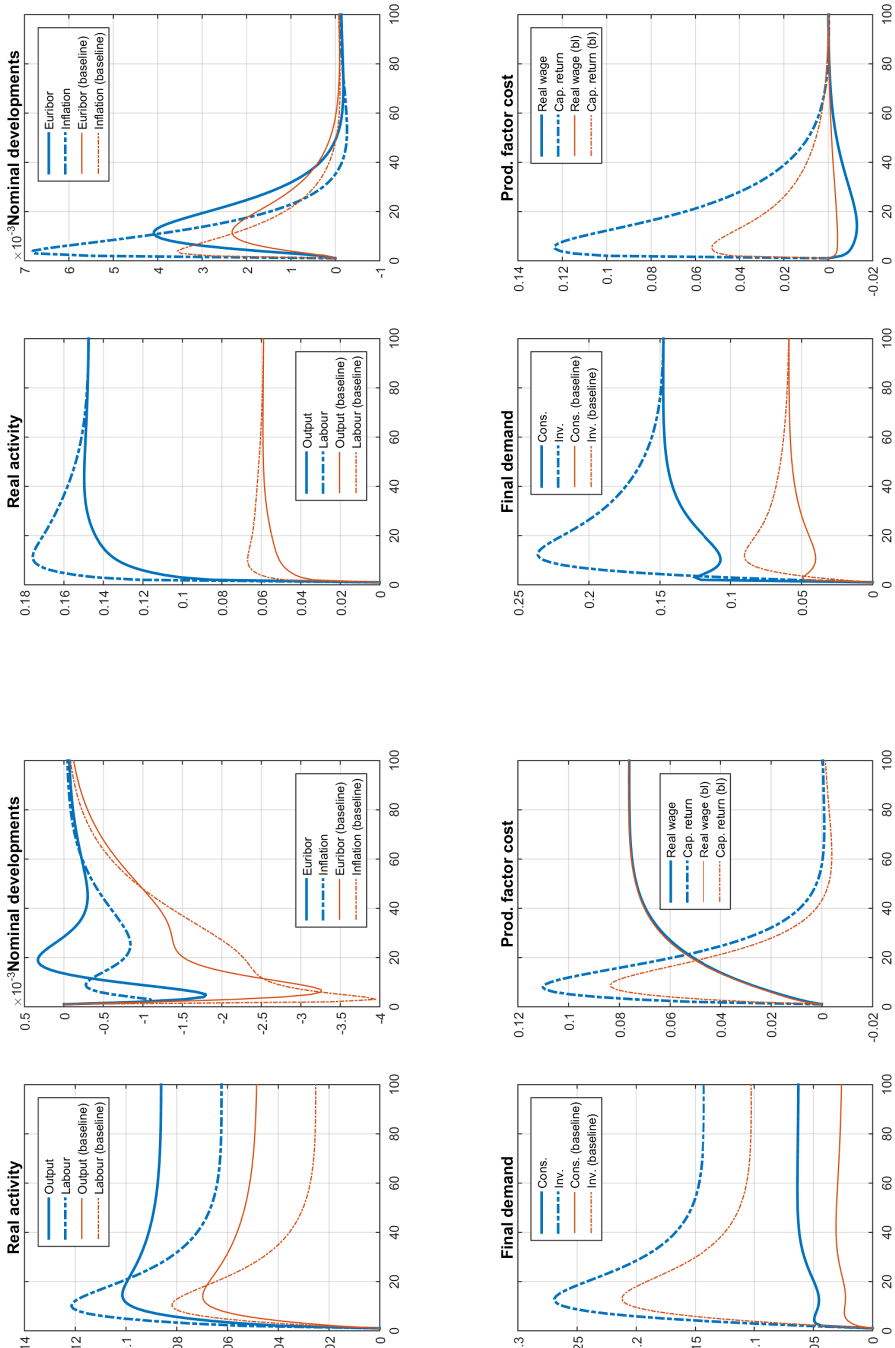
6.A Sensitivity analysis



(a) Goods market reform

(b) Labour market reform

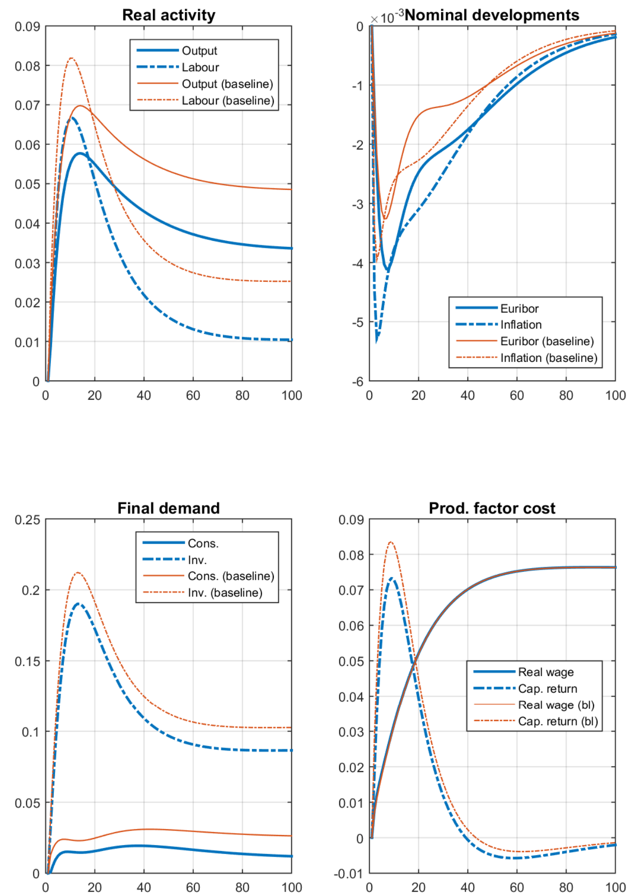
Figure 6.A.1 – Sensitivity of the transition following mark-up reforms to the calibration ($\sigma_c = 0.5$ compared to baseline)



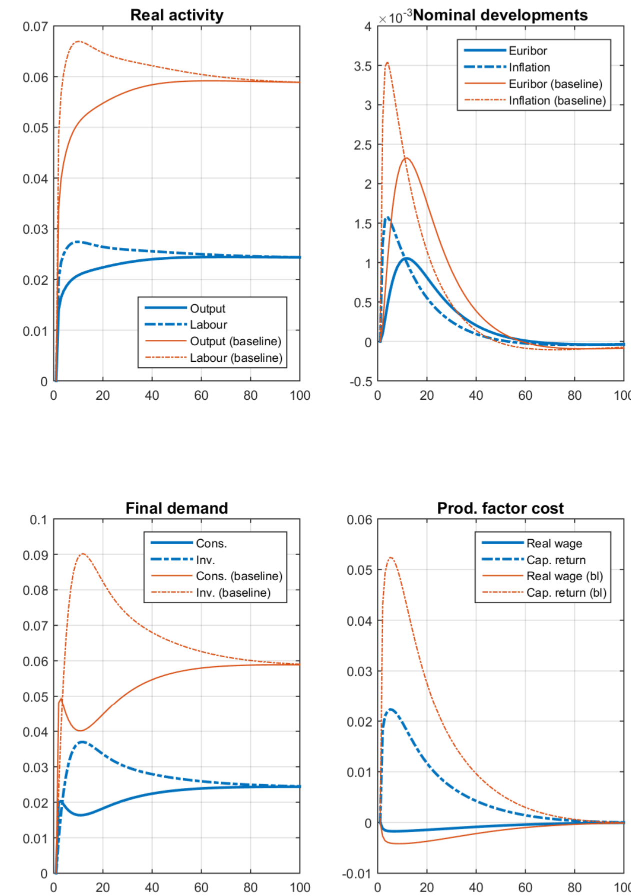
(a) Goods market reform

(b) Labour market reform

Figure 6.A.2 – Sensitivity of the transition following mark-up reforms to the calibration ($\sigma_l = 0.3$ compared to baseline)

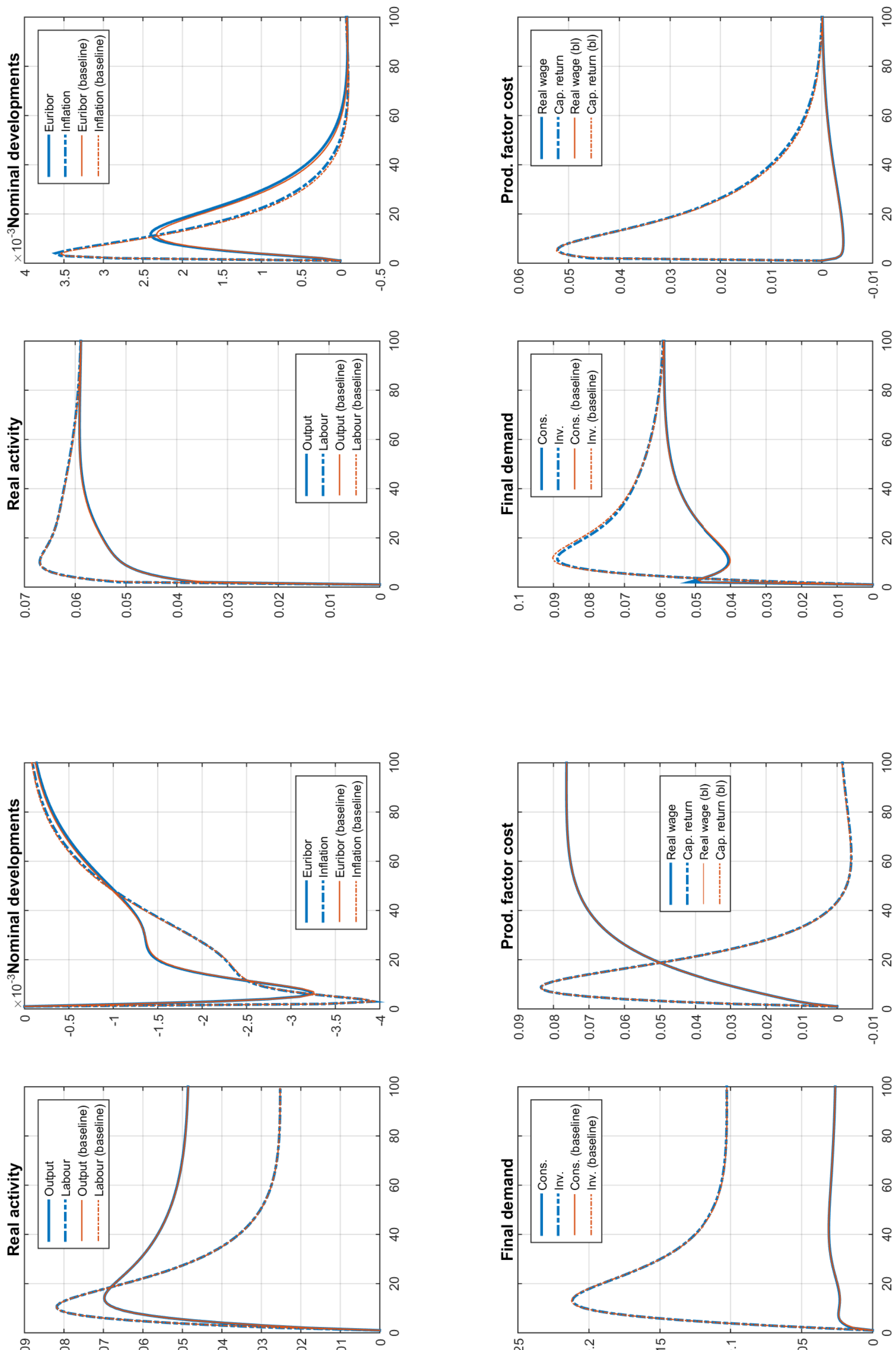


(a) Goods market reform



(b) Labour market reform

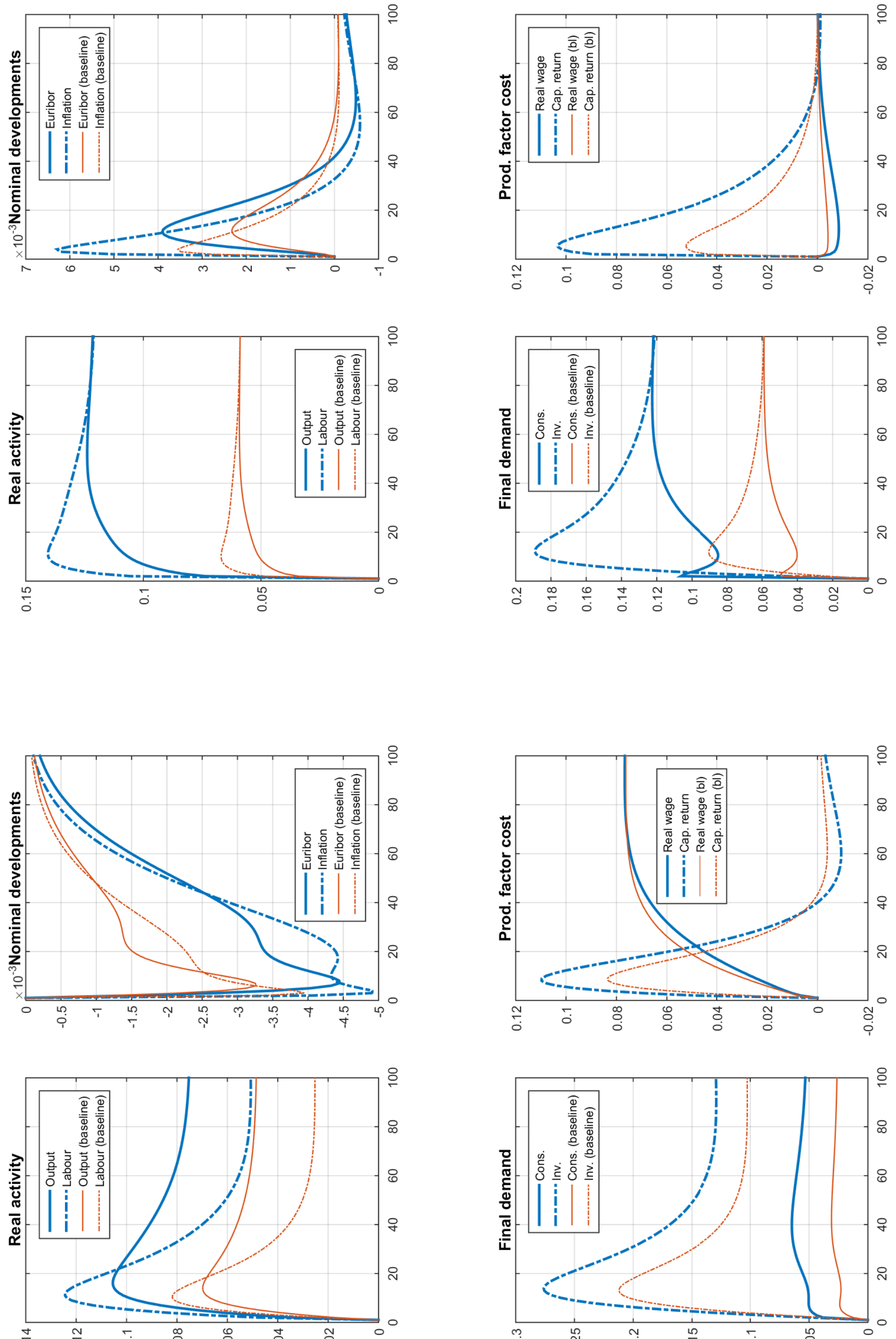
Figure 6.A.3 – Sensitivity of the transition following mark-up reforms to the calibration ($\sigma_l = 6$ compared to baseline)



(a) Goods market reform

(b) Labour market reform

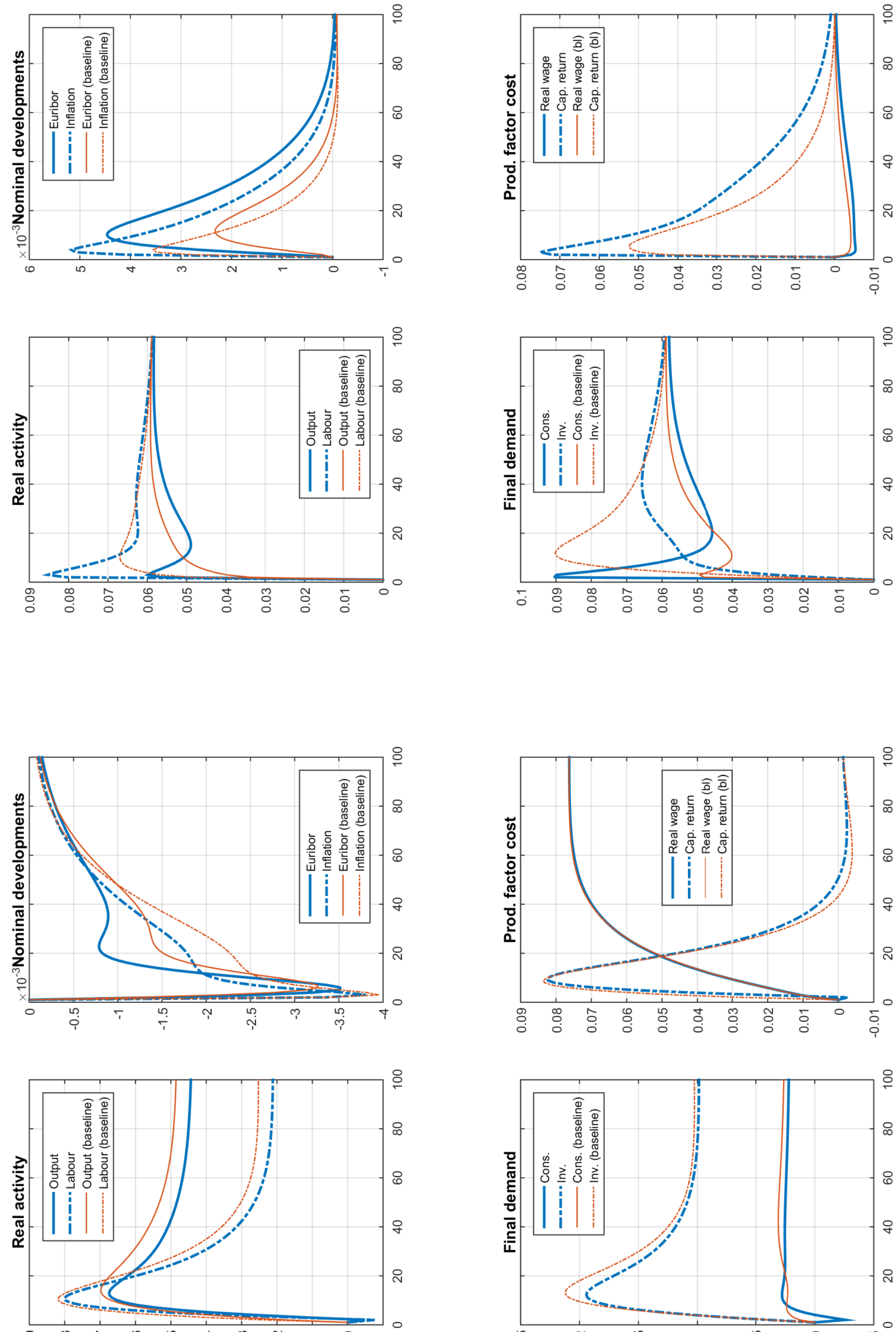
Figure 6.A.4 – Sensitivity of the transition following mark-up reforms to the calibration ($h_c = 0$ compared to baseline)



(a) Goods market reform

(b) Labour market reform

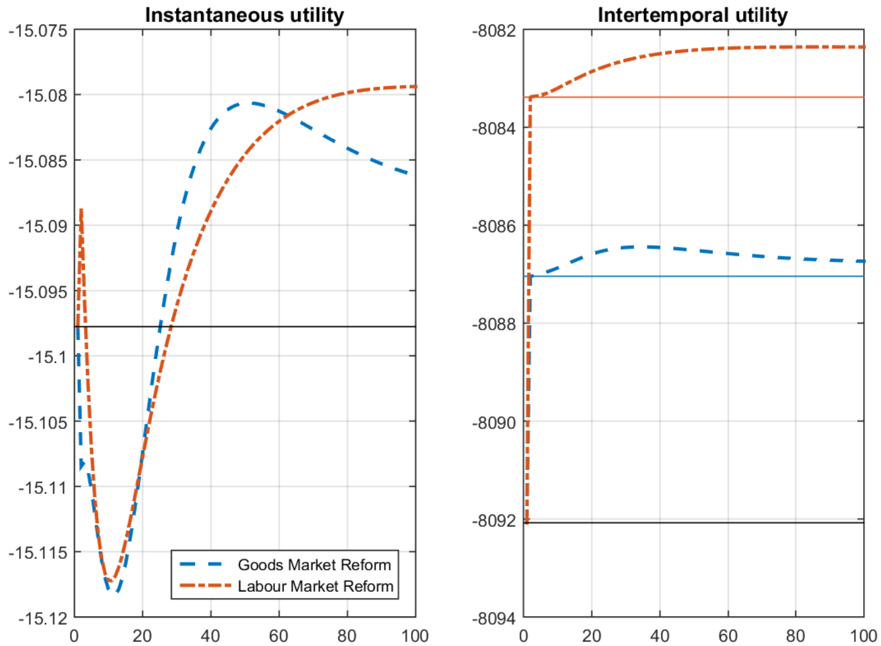
Figure 6.A.5 – Sensitivity of the transition following mark-up reforms to the calibration ($h_l = 0.5$ compared to baseline)



(a) Goods market reform

(b) Labour market reform

Figure 6.A.6 – Sensitivity of the transition following mark-up reforms to the introduction of *non-Ricardian* households ($\mu = 0.4$ compared to baseline)



Structural reforms implemented here are the ones of (Everaert and Schule, 2006), that is of an alignment of mark-ups on the level of the three best European performers. Solid black lines correspond to utilities without reforms, i.e. initial steady state. Coloured solid lines represent the intertemporal utility once the reform is implemented. The difference between this line and the final steady state is the transition cost to the reform.

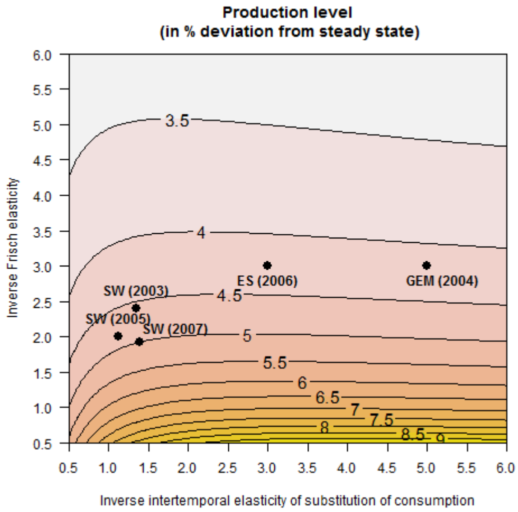
Figure 6.A.7 – Utility in the transition following a decrease in the price or wage mark-up without consumption habits

6.B Some calculations

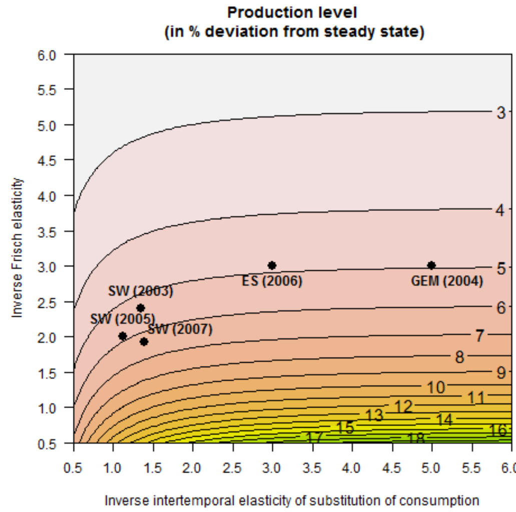
6.B.1 Steady state

In the present model, growth is exogenous. In the long run, all real variables grow at the rate of TFP and prices grow at the steady state inflation rate. Taking into account all steady state equations, it is possible to define all endogenous variables not only in ratio to GDP but also in level.

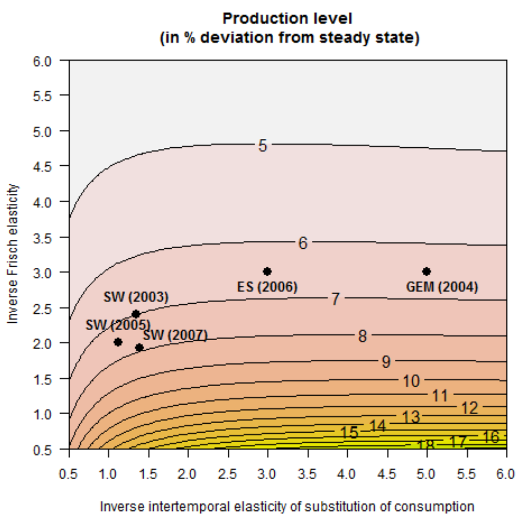
Conveniently, for policy analysis purposes, this allows to assess how this steady state in level depends on structural, technological and preference and by derivation how changes in these parameters would influence the level of our economies.



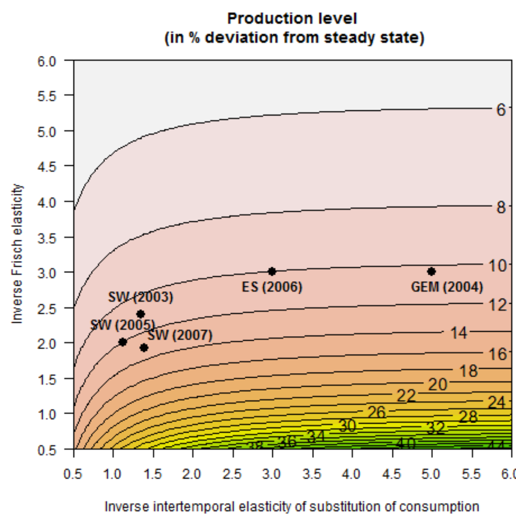
(a) Goods market reforms - $h_c = 0$



(b) Labour market reforms - $h_c = 0$



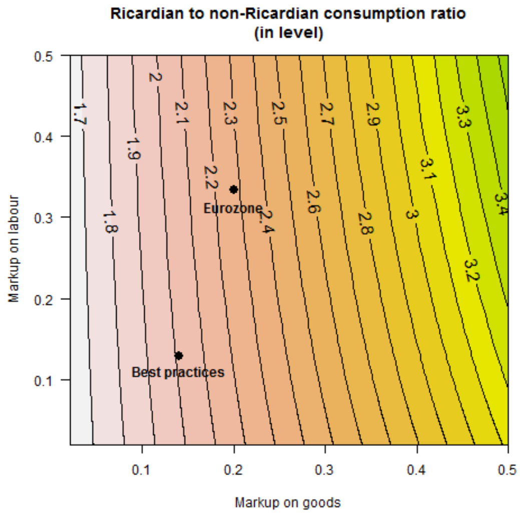
(c) Goods market reforms - $h_l = 0.5$



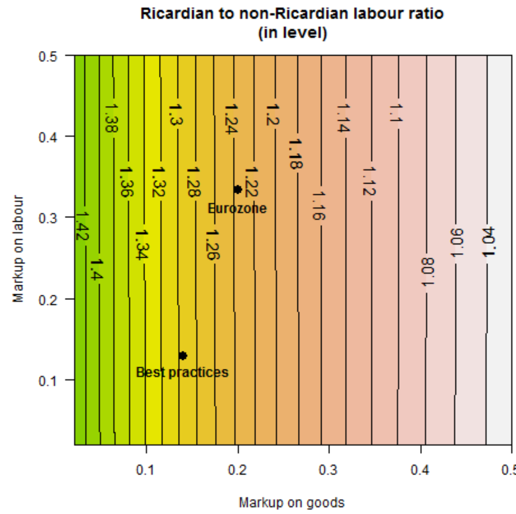
(d) Labour market reforms - $h_l = 0.5$

The inverse intertemporal elasticity of substitution of consumption corresponds to σ_c, and the inverse Frisch elasticity to σ_l. Structural reforms implemented here are the ones of (Everaert and Schule, 2006), that is of an alignment of markups on the level of the three best European performers.
SW (YYYY) indicates the calibration of the model following Smets and Wouters (YYYY) for the Euro area, expect for 2005 focusing on the US. Our standard calibration relies on (Smets and Wouters, 2005). *GEM (2004)* corresponds to (Bayoumi et al., 2004) and *ES (2006)* to (Everaert and Schule, 2006).

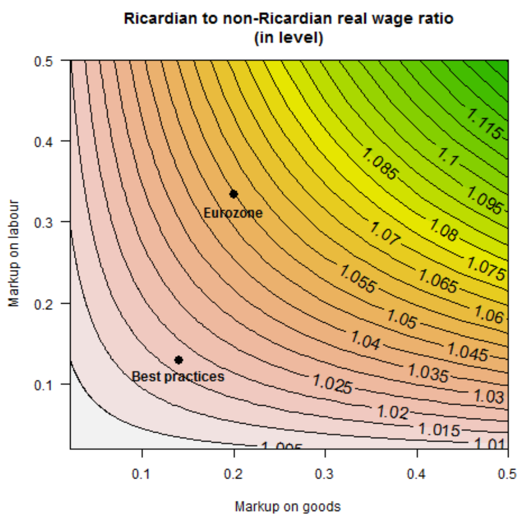
Figure 6.A.8 – Impact of structural reforms in p.p. depending on the calibration of the utility function for alternative calibrations of the model



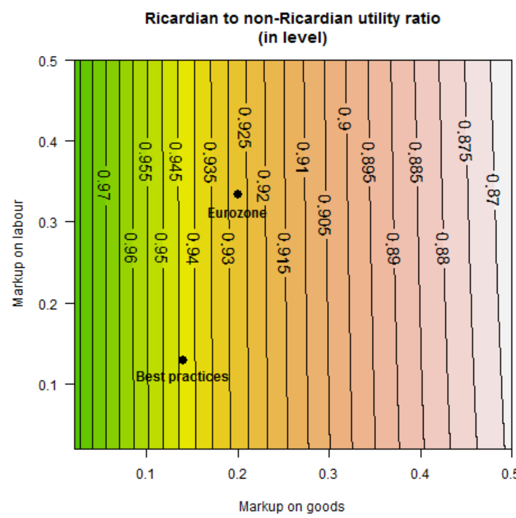
(a) Ricardian to non-Ricardian consumption ratio



(b) Ricardian to non-Ricardian labour ratio



(c) Ricardian to non-Ricardian real wage ratio



(d) Ricardian to non-Ricardian utility ratio

The x-axis represents the level of mark-up on goods (i.e. $1/\theta - 1$) whereas the y-axis represents the mark-up on wages (i.e. $1/\theta_w - 1$). The point named *Eurozone* corresponds to the standard calibration of our model and the point *Best practices* to the level of mark-ups in the three best European practices, namely Denmark, Sweden and the United Kingdom, as defined in (Everaert and Schule, 2006).
For the utility ratio, it is crucial to know that the steady state utility level is negative. As such, a ratio below one indicates a higher utility level for Ricardian households. The closer it is to one, the less inequality.

Figure 6.A.9 – Steady state Ricardian to non-Ricardian level ratios

Starting from the *Ricardian* households' consumption-leisure trade-off, we get:

$$\frac{\overline{RW}^R \bar{L}^R}{\bar{C}^R} = \frac{\theta_w}{\theta_w - 1} \frac{(1+\sigma_l)\sigma_c}{(1-\sigma_c)(1-\eta)} \frac{\kappa(1-\sigma_c)\left[\frac{\bar{L}^R}{(1-\mu)\mathbb{N}}\left(\frac{\bar{L}}{\mathbb{N}}\right)^{-h_l}\right]^{1+\sigma_l}}{1-\kappa(1-\sigma_c)\left[\frac{\bar{L}^R}{(1-\mu)\mathbb{N}}\left(\frac{\bar{L}}{\mathbb{N}}\right)^{-h_l}\right]^{1+\sigma_l}} \tag{6.29}$$

$$\begin{aligned} \frac{1-s_{wl}}{1-s_c}\frac{\overline{RW}\bar{L}}{\bar{C}} &= \frac{\theta_w}{\theta_w - 1} \frac{(1+\sigma_l)\sigma_c}{(1-\sigma_c)(1-\eta)} \frac{\tilde{\kappa}\left[\frac{1-s_{wl}}{1-\mu}\right]^{\frac{\theta_w(1+\sigma_l)}{\theta_w-1}}(\bar{L})^{(1+\sigma_l)(1-h_l)}}{1-\tilde{\kappa}\left[\frac{1-s_{wl}}{1-\mu}\right]^{\frac{\theta_w(1+\sigma_l)}{\theta_w-1}}(\bar{L})^{(1+\sigma_l)(1-h_l)}} \\ &= \frac{1-s_{wl}}{1-s_c}\frac{\theta-1}{\theta}\frac{1-\alpha}{cy} \end{aligned} \tag{6.30}$$

that is

$$\bar{L} = \left(\frac{1}{\tilde{\kappa}}\left[\frac{1-\mu}{1-s_{wl}}\right]^{\frac{\theta_w(1+\sigma_l)}{\theta_w-1}} \frac{\mathcal{B}^R}{\mathcal{B}^R+1}\right)^{\frac{1}{(1+\sigma_l)(1-h_l)}} \tag{6.31}$$

with $\mathcal{B}^R = \frac{1-s_{wl}}{1-s_c}\frac{\theta_w-1}{\theta_w}\frac{\theta-1}{\theta}\frac{(1-\alpha)(1-\sigma_c)(1-\eta)}{(1+\sigma_l)\sigma_c cy}$ and $\tilde{\kappa} = \kappa(1-\sigma_c)(\mathbb{N})^{-(1-h_l)(1+\sigma_l)}$

To determine the level of output we write from the Cobb-Douglas function, simplifying with (6.22):

$$\begin{aligned} \bar{Y} = (\bar{K})^\alpha(\bar{\zeta}\bar{L})^{1-\alpha} = \bar{\zeta}(\frac{\bar{K}}{\bar{Y}})^{\frac{\alpha}{1-\alpha}}\bar{L} &= \bar{\zeta}\left(\frac{iy}{\delta+g}\right)^{\frac{\alpha}{1-\alpha}}\bar{L} \\ &= \bar{\zeta}\left(\alpha\frac{\theta-1}{\theta}(\frac{1}{\tilde{\beta}}-1+\delta)^{-1}\right)^{\frac{\alpha}{1-\alpha}}\bar{L} \end{aligned} \tag{6.32}$$

which we simplify in:

$$\bar{Y} = \mathcal{A}\bar{L} \tag{6.33}$$

From equations (6.33) and (6.44) we also have the following purchasing power of wages:

$$\overline{RW} = \mathcal{A}(1-\alpha)\frac{\theta-1}{\theta} \tag{6.34}$$

Finally, *non Ricardians* and *Ricardians* shares in the population and solving for their shares in consumption and payroll allow to compute the steady state level of consumption, labour supply and wage of both types of households. Similarly, from equation (6.33) the steady state levels of consumption, government spending, investment and capital are directly found by multiplying by cy, gy, iy, and $\frac{iy}{\delta}$ respectively.

6.B.2 Separable utility case

From *Ricardian* households consumption-leisure trade-off, we get:

$$\overline{RW}^R = \frac{\kappa\theta_w}{\theta_w - 1}\frac{\left(\frac{\bar{L}^R}{1-\mu}\right)^{\sigma_l}(\bar{L})^{-h_l(1+\sigma_l)}}{\left(\frac{\bar{C}^R}{1-\mu}\right)^{-\sigma_c}(\bar{C})^{-h_c(1-\sigma_c)}}(\mathbb{N})^{h_l(1+\sigma_l)-\sigma_l-h_c(1-\sigma_c)-\sigma_c} \tag{6.35}$$

$$\frac{\overline{RW}^R\bar{L}^R}{\bar{Y}} = (1-s_{wl})(1-\alpha)\frac{\theta-1}{\theta} \tag{6.36}$$

$$= \frac{\tilde{\kappa}\theta_w}{\theta_w - 1}\frac{\left(\frac{\bar{L}^R}{(1-\mu)\bar{L}}\right)^{\sigma_l}(\bar{L})^{\sigma_l-h_l(1+\sigma_l)}}{\left(\frac{\bar{C}^R}{(1-\mu)\bar{C}}\right)^{-\sigma_c}(\bar{C})^{-(\sigma_c+h_c(1-\sigma_c))}}\frac{\bar{L}^R}{\bar{Y}} \tag{6.37}$$

$$= \frac{\tilde{\kappa}\theta_w}{\theta_w - 1}\frac{(1-\mu)cy\left(\frac{\bar{L}^R}{(1-\mu)\bar{L}}\right)^{1+\sigma_l}(\bar{L})^{(1+\sigma_l)(1-h_l)}}{\left(\frac{\bar{C}^R}{(1-\mu)\bar{C}}\right)^{-\sigma_c}(\bar{C})^{(1-\sigma_c)(1-h_c)}} \tag{6.38}$$

$$= \frac{\tilde{\kappa}\theta_w}{\theta_w - 1}\frac{(1-\mu)cy\left(\frac{1-s_{wl}}{1-\mu}\right)^{(1+\sigma_l)\frac{\theta_w}{\theta_w-1}}(\bar{L})^{(1+\sigma_l)(1-h_l)}}{\left(\frac{1-s_c}{1-\mu}\right)^{-\sigma_c}(\bar{C})^{(1-\sigma_c)(1-h_c)}} \tag{6.39}$$

$$\tag{6.40}$$

$$(\bar{Y})^{(1-\sigma_c)(1-h_c)} = \frac{\tilde{\kappa}\theta_w\theta cy}{(1-\alpha)(\theta_w-1)(\theta-1)\,(cy)^{(1-\sigma_c)(1-h_c)}}\frac{\left(\frac{1-s_{wl}}{1-\mu}\right)^{(1+\sigma_l)\frac{\theta_w}{\theta_w-1}-1}}{\left(\frac{1-s_c}{1-\mu}\right)^{-\sigma_c}}(\bar{L})^{(1+\sigma_l)(1-h_l)} \tag{6.41}$$

which we simplify in:

$$(\bar{Y})^a = \mathcal{B}(\bar{L})^b \tag{6.42}$$

Combined with (6.33), we can conclude that the steady state level of output is equal to

$$\bar{Y} = \left(\frac{\mathcal{A}^b}{\mathcal{B}}\right)^{\frac{1}{b-a}} \tag{6.43}$$

From equations (6.33) and (6.42) we also have:

$$\bar{L} = \left(\frac{\mathcal{A}^a}{\mathcal{B}}\right)^{\frac{1}{b-a}} \tag{6.44}$$

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Thèse pour l'obtention du grade de docteur de l'École Polytechnique en sciences économiques, soutenue le 4 décembre 2015 par Aurélien Poissonnier. Directeur de thèse : Édouard Challe. Rapporteurs: Patrick Fève et Xavier Ragot. Suffrageants : Behzad Diba, Jean-Baptiste Michau, Jean-Guillaume Sahuc.

Le premier chapitre de cette thèse traite de l'arbitrage consommation-épargne des ménages dans les modèles de politique monétaire. L'adéquation entre le modèle et les données peut être notablement améliorée d'une part en prenant en compte les taux d'intérêts auxquels les ménages sont soumis et d'autre part en modélisant ces derniers comme des agents inattentifs.
Le second chapitre démontre analytiquement la validité du principe de Taylor dans un modèle à salaires et prix rigides. D'après le principe de Taylor, lorsque le banquier central surréagit aux chocs inflationnistes, ce dernier élimine les équilibres à tâches solaires, c'est-à-dire réduit les fluctuations économiques. Cette propriété est bien établie pour un modèle où les salaires sont flexibles, mais restait à préciser analytiquement lorsqu'ils sont rigides.
Le troisième chapitre quantifie la production domestique (ménage, cuisine, bricolage...) réalisées en dehors du circuit économique. Valoriser ces activités amène à réviser l'activité économique à la hausse mais sa croissance à la baisse. Les questions méthodologiques autour de cette mesure de l'activité domestique sont discutées en détail.
Le quatrième chapitre aborde l'évaluation des réformes structurelles dans les modèles institutionnels (FMI, Commission Européenne, BCE). L'analyse des mécanismes au coeur de ces modèles et de leur sensibilité à la calibration amène à deux recommendations, d'une part un recours systématique à l'analyse de sensibilité et plus fondamentalement à repenser leurs mécanismes de long terme (notamment d'offre de travail).

The first chapter of this thesis deals with households' consumption-savings trade off in monetary policy models. The model's fit with the data can be markedly improved first by the use of households specific interest rates, second by modelling households as inattentive agents.
The second chapter proves analytically that the Taylor principle is valid in a model with both staggered prices and wages. According to the Taylor principle, when the central banker overreacts to inflationary shocks, he rules out sun spot equilibria and thus reduces economic fluctuations. This property is well known in models with staggered prices only, it had to be established analytically when wages are staggered.
The third chapter quantifies domestic production (house-chores, cooking, handy work...) taking place out of the market. Valuing these activities induces an upward revision of economic activity but a downward revision of its growth. Methodological questions linked to the valuation of domestic work are discussed in details.
The fourth chapter deals with the evaluation of structural reforms in institutional models (IMF, European Commission, ECB). The analysis of the core mechanisms of these models and their sensitivity to the calibration bring about two recommendations, first a systematic use of sensitivity analysis and more fundamentally a rethink of their long term mechanisms (in particular for labour supply).

Mots clés : consommation, équation d'Euler, politique monétaire, principe de Taylor, travail domestique, compte satellite des ménages, réformes structurelles, modèles DSGE

Keywords: consumption, Euler equation, monetary policy, Taylor principle, domestic work, households satellite account, structural reforms, DSGE models