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Contents

Abstract

This thesis is composed of three essays on the economics of innovation. Each chapter is dedicated to a specific question and can therefore be read separately.

The first two chapters are devoted to the study of the recent emergence of new actors in the market for patents, namely, Non-Practicing Entities (NPEs), who acquire patents with no aim to use them to produce a final good. The main focus here is on two types of NPEs that differ according to their acquisition strategies. On the one hand, Patent Assertion Entities (PAEs), often referred to as patent trolls, seek to acquire patents with the intent to monetize them through litigation or the threat of litigation for patent infringement. PAEs usually operate in technology fields (such as ICT) where products encompass numerous overlapping patents. The likelihood of inadvertently infringing a patented technology is particularly high when R&D intensive firms develop technical components in such complex technological industries where several patented inventions enter their final good. Importantly, PAEs' immunity to countersuits gives them a strategic advantage over producing firms through a greater bargaining power when it comes to extracting damage payments from alleged infringers, thereby imposing tremendous costs on producing firms. For instance, Bessen and Meurer (2014) argue that, in 2011, United States business entities incurred $29 billion in direct costs because of them. On the other hand, defensive aggregators (also known as preemptive intermediaries) acquire patents to provide safety from litigation brought by patent trolls to their affiliated firms.

In this respect, the first chapter considers the auction of a patent which, upon enforcement, threatens two producing firms for patent infringement. The patent has common value among bidders, which include both firms and a patent troll. In order to capture the troll's strategic advantage over firms, it is assumed that he holds a private-value advantage so that he enjoys a strictly higher ex-post valuation for the patent. On the other hand, he is completely uninformed

about the common value of the patent. The results of this chapter first suggest that, without intermediation, firms have no means to protect themselves against litigation brought by the troll since the latter always wins the auction.

The baseline model is then extended with the introduction of an intermediary who, in exchange of an up-front membership fee, enables firms to collectively contribute toward the patent purchase. Then the intermediary competes against the troll in the auction on behalf of his affiliated firms. The results state that there is no equilibrium in which the intermediary wins at a positive price due to a collective action issue, and that firms pledge either conservative or aggressive contributions. Finally, the intermediary's probability to win the auction is strictly positive, suggesting that his intervention partially hampers the troll's litigious activity.

This chapter also contributes to the vast literature on second-price sealed-bid common value auctions. While asymmetries in terms of either information or ex-post valuations for the object across bidders (in the so-called almost common value auctions) have been extensively studied, the existing literature does not incorporate both sources of asymmetries with more than two bidders. As such, the model developed in this chapter is therefore at the intersection of these two strands of the literature by considering an almost common value auction with three asymmetrically informed bidders. The results indicate that perturbing the information structure so that the advantaged bidder is also uninformed restores the extreme result of almost common value auctions with two imperfectly informed bidders. Namely, the advantaged bidder always wins in any ex-post equilibrium. Furthermore, it is shown that the participation of an uninformed advantaged bidder substantially raises the seller's expected equilibrium revenue.

In a similar vein, the second chapter theoretically and empirically examines recent business models of patent preemption entities (PPEs) who specialize in the preemptive acquisition of patents that could threaten firms that subscribe to their services, thereby alleviating the risk of litigation brought by PAEs. We develop a simple theoretical model in which a PPE seeks to attract subscribers before competing against a PAE in a patent auction. We show that PPEs can establish a profitable business model by restricting their protection to the most threatening patents and by targeting a subset of highly exposed firms, while maintaining a credible threat of litigation against other potential infringers. Using patent reassignment and litigation data, we then provide evidence supporting the model's prediction. This chapter is a joint project with Henry Delcamp, Aija Leiponen and Yann Ménière.

The last chapter contributes to the literature on non-tournament models of R&D in which firms engage in cost-reducing innovation and then compete à la Cournot in the product market. In such models, it is widely recognized that exogenous knowledge spillovers create distortions in R&D investment decisions. While most of the extant literature on imperfectly appropriable R&D focused on multidirectional spillovers, Amir and Wooders (1999), henceforth AW, instead consider a stochastic directed spillover process whereby know-how may flow only from the more R&D intensive firm to its rival, so that spillovers only admit extreme realizations - full or no spillovers occur with a given probability. The idea underlying the assumption of a unidirectional spillover process is that it may better approximate the potential leakages that occur when the R&D process is either one-dimensional, i.e. there is a single research path to achieve unit cost reductions, or multi-dimensional in which case this spillover structure suggests that there is a more natural path to follow. In this context, the spillover parameter may be interpreted as being related to the length of patent protection, but also to a measure of the imitation lag.

This chapter examines the certainty-equivalent of AW's model in the sense that a fraction of the R&D undertaken by the leader flows to its rival with certainty. While firms are ex-ante identical, one obtains a unique asymmetric equilibrium so that the roles of R&D innovator and imitator are endogenously determined. We analyze the impact of uncertainty about the appropriability of firms' R&D investments and find that both the spread between firms' investments and the industry's total cost reductions are higher in the stochastic framework. Firms are better off when spillovers are uncertain for a wide range of parameters due to increased asymmetries in terms of their unit cost structure in the product market competition. Finally, we provide a welfare analysis, showing that the stochastic spillover process is superior to its deterministic counterpart, and examine the social costs of imposing symmetric R&D invesments among firms. This last chapter is a joint project with Małgorzata Knauff.

Chapter 1

Intermediaries versus Trolls in Contests for Patents

English summary

This paper examines the emergence of non-practicing entities in the market for patents, who acquire patents with no aim to engage into innovative activities. While patent trolls seek to monetize their acquired patents through the threat of litigation against alleged infringers, intermediaries instead intend to provide their affiliated firms with safety to operate from trolls' litigious activity by buying out patents that would otherwise fall in trolls' hands. We develop a model of patent acquisition through an auction incorporating both patent trolls and intermediaries. We highlight trolls' greater ability to preempt patents that represent a threat of infringement as compared to producing firms. We find that firms have no means to protect themselves against threatening patents when individually competing against the troll in the auction, while the seller's revenue sharply increases in response to the participation of a troll in the auction. We then examine the effectiveness of intermediaries to protect firms against the troll's litigious activity by analyzing their patent funding mechanism. Since the patent is collectively financed through voluntary individual contributions, firms tend to free ride on other contributors. While the intermediary's probability to outbid the troll in the auction is strictly positive, the collective action issue inherent to his funding mechanism greatly hampers his performance in the auction and undermines the seller's revenue. Overall, our results nonetheless suggest that the presence of NPEs in the patent acquisition process positively impacts the revenue of sellers of likely infringed patents.

Résumé français

Ce papier examine l'émergence des entreprises non productrices sur le marché des brevets, qui acquièrent des brevets sans intention d'innover. Tandis que les chasseurs de brevets cherchent à acquérir des brevets en vue de les monétiser par la menace d'action en contrefaçon, les intermédiaries quant à eux acquièrent des brevets afin de protéger leurs entreprises clientes contre des litiges initiés par les chasseurs de brevets. Nous développons un modèle d'acquisition de brevet par le biais d'une enchère incorporant les chasseurs de brevets ainsi que les intermédiaires. Nous mettons en évidence l'aptitude supérieure des premiers quant à l'acquisition de brevets représentant une menace d'action en contrefaçon par rapport aux producteurs. Nous montrons que, sans intermédiaire, les firmes n'ont aucun moyen de se protéger contre les actions

initiées par le chasseur de brevets, puisque ce dernier gagne toujours l'enchère. En outre, nos résultats montrent que le revenu du vendeur du brevet augmente considérablement dès lors qu'un chasseur de brevet participe à l'enchère. Nous examinons ensuite l'efficacité des intermédiaires quant à leur protection des producteurs contre les menaces légales initiées par les chasseurs de brevets en analysant leur mécanisme de financement du rachat de brevets. Puisque le brevet est collectivement financé via les contributions individuelles et volontaires des firmes, ces dernières sont incitées à laisser les autres membres contribuer à leur place. Tandis que l'intermédiaire a une probabilité strictement positive de gagner l'enchère, le problème d'action collective généré par son mécanisme de rachat collectif a un impact négatif sur sa performance dans l'enchère, ainsi que sur le revenu du vendeur. Néanmoins, nos résultats suggèrent que les vendeurs de brevets menaçants bénéficient de la présence de ces deux types d'entreprises non productrices.

1.1 Introduction

The last decade has seen a growing activity of patent assertion entities (PAEs), also known as patent trolls. Trolls typically do not produce anything covered by their patents, and are therefore frequently referred to as "non-practicing entities". Such firms instead seek to acquire patents so as to use them as a strategic tool to extort rents from alleged infringers, through either litigation or the threat of litigation. Trolls usually operate in technology fields (such as ICT) where products encompass numerous overlapping patents. The likelihood of inadvertently infringing a patented technology is particularly high when R&D intensive firms develop technical components in such complex technological industries where several patented inventions enter their final good[1].

Since trolls do not engage in innovative activities, their immunity to countersuits gives them a strategic advantage over producing firms through a greater bargaining power when it comes to extracting damage payments from alleged infringers, thereby imposing tremendous costs on producing firms and raising concerns regarding their impact on firms' incentives to innovate. For instance, Bessen et al. (2011) find that over the last four years, defendants incurred over $80 billion per year in lawsuits initiated by patent trolls. On the other hand, their proponents instead argue that trolls may enable inventors lacking resources to either manufacture products embedding their technology, license their technology or even enforce their rights, to earn rents.

The proliferation of trolls' litigious activity further gave rise to a different type of NPE, often called defensive aggregators, such as RPX Corporation and Allied Security Trust. Their primary goal is to provide producing firms with safety to operate by acquiring threatening patents that might otherwise get in the possession of trolls. For an annual membership fee, these intermediaries search for patents that might threaten their members upon litigation for patent infringement. The identified patents are then collectively financed through members' voluntary contributions. More specifically, each member decides whether to contribute toward the patent purchase, and if so, by how much. Importantly, the contributors' identity as well as the amount they pledge is not disclosed. The intermediary then provides contributors with non-exclusive licenses to the acquired patents, thereby annihilating any risk of patent infringement.

This paper develops a model of patent acquisition incorporating both trolls and intermediaries, and focuses on the sale of a patent that threatens two producing firms upon enforcement

[1] The emergence of patent trolls is also closely related to the issue of uncertain patents (see e.g. Amir et al., 2014).

for patent infringement, so as to highlight trolls' greater ability to acquire patents as compared to producing firms, and then study the effectiveness of intermediaries' mechanism to protect firms from litigation brought by trolls.

The existing theoretical literature instead focuses on the effect of patent trolls on incentives to undertake R&D and on litigation. Lemus and Temnyalov (2015) examine PAEs' "patent privateering" strategies, which consist of acquiring patents from operating firms to subsequently enforce them against alledged infringers, usually competitors of the patent seller. They analyze the impact of such strategies on incentives to undertake R&D and to engage in costly litigation. Without PAEs, producing firms are reluctant to enforce their patents against their rivals due to the threat of countersuits. In turn, the authors show that outsourcing patent monetization to PAEs enhances the offensive value of patents due to PAEs' immunity to countersuits but undermines their defensive value. In particular, they find that when the former effect prevails, PAEs spur incentives to innovate and enhance social welfare.

Hovenkamp (2013) instead develops a dynamic model of patent assertion and reputation building in order to study PAEs' strategy of predatory litigation. By aggressively asserting weak patents against alleged infringers, PAEs develop a tough reputation and gain credibility in their litigation threats, so that other firms are more encline to settle on a licensing agreement before reaching the courts. While PAEs experience losses when litigating patents that are likely to be invalidated, the author argues that the prospect of substantial licensing payments through subsequent settlement agreements compensates. In a similar vein, Choi and Gerlach (2015) examine PAEs' litigation strategies and the credibility of their threats. They show that naming multiple defendants using related technologies enhances the credibility of their litigation threat and their bargaining position through information externalities generated across litigation suits.

In contrast to the extant literature, we focus on the strategic behavior of trolls and producing firms in the patent acquisition process, and study how trolls successfully preempt patents as compared to firms. We then examine intermediaries' ability to successfully counter trolls' litigious activity by analyzing their collective funding mechanism through firms' individual contributions.

To address these issues, we first consider the sale of a patent through a second-price sealed-bid auction between a troll and two producing firms, which once bought out, threatens both firms upon enforcement for patent infringement. We assume that firms use the same technology, but that they operate in different markets so that they are not direct competitors. Prior to the auction, each firm privately receives a signal, which captures her exposure (or likelihood of infringement) to the patent for sale. The value of the patent for a firm equals the damage payments she can extract by asserting it against her rival plus the damages she would have had incurred if she were sued for patent infringement. As to the troll, his value for the patent equals the total damages to receive by litigating both firms. As such, the patent for sale has common value among bidders. In order to capture the troll's strategic advantage over firms through his immunity to countersuits, it is assumed that he holds a private-value advantage so that he enjoys a strictly higher ex-post valuation for the patent. On the other hand, he is completely uninformed about the common value of the patent.

We show that the troll adopts an extreme equilibrium bidding behavior depending on the magnitude of his private-value advantage, namely, he bids either very aggressively or very cautiously. In turn, firms do not suffer from ex-post regret and mildly shade their bids down, so that the expected equilibrium revenue of the seller of the patent subtantially increases when the troll participates in the auction. Importantly, the troll always wins the auction in any ex-post equilibrium due to his immunity to countersuits, thereby motivating intermediaries' intervention in the market for patents as an attempt to protect producing firms against litigation brought by trolls.

Therefore, we extend the baseline model by introducing an intermediary who, in exchange of a non-refundable up-front membership fee, offers firms to compete against the troll on their behalf in the auction for patent buyout. Upon acceptance of the intermediary's offer, firms then simultaneously choose whether to contribute and if so, the amount of their contribution. The intermediary's bid then simply aggregates firms' contributions. When winning the auction, the intermediary then provides his members with non-exclusive licenses thereby annihilating any risk of litigation for patent infringement.

We show that the intermediary screens out low-signal firms in order to charge a strictly positive membership fee. Moreover, we identify two necessary conditions for the intermediary to outbid the troll: both firms must contribute and any excess of contributions must be fully refunded to contributing firms. Nevertheless, because the patent is collectively financed through individual contributions, the collective action issue inherent to the intermediary's funding mechanism greatly hampers his performance in the auction and dramatically lowers the seller's revenue. Indeed, we show that there is no equilibrium in which the intermediary wins the auction with a strictly positive price. We highlight two classes of equilibria yielding two opposite outcomes. The first one exhibits a free rider problem whereby each firm has an incentive to lower her contribution so that the other firm incurs a larger share of the patent purchase. As a result, firms' total contributions are too low and the troll always wins. The second instead involves firms pledging aggressive contributions so that the troll always bids zero to ensure losing. Invoking forward induction arguments, we argue that the second class of equilibria is more plausible. Thus, the intermediary wins the patent for sale with a strictly positive ex-ante probability, yet lower than that of the troll, thereby partially overcoming the troll's threat for firms.

This paper also contributes to the vast literature on auctions. In second-price common-value auctions with two bidders, introducing asymmetries among players through a private-value advantage drastically affects the outcome of the auction, namely, the advantaged bidder always wins and the seller's revenue substantially decreases (see Bikhchandani (1988) and Avery and Kagel (1997)). Levin and Kagel (2005) examine whether this extreme result still obtains with more than one regular bidder in an almost common-value second-price auction where each bidder receives a private signal. They show that, in the wallet auction, regular bidders have a positive probability to win the auction, yet lower than that of the advantaged bidder, and that a small private-value advantage only slightly decreases the seller's revenue. However, while asymmetries in terms of either information or ex-post valuations for the object across bidders have been extensively studied, the existing literature does not incorporate both sources of asymmetries with more than two bidders. As such, the model we consider is therefore at the intersection of these two strands of the literature by considering an almost common-value auction with three asymmetrically informed bidders.

In this respect, our results indicate that perturbing the information structure so that the advantaged bidder is also uninformed restores the extreme result of almost common-value auctions with two imperfectly informed bidders. Namely, the advantaged bidder always wins in any ex-post equilibrium. However, we find that the participation of an uninformed advantaged bidder substantially raises the seller's expected equilibrium revenue.

The remainder of the paper is organized as follows. Section 1.2 presents the model and the equilibrium concept. Section 1.3 characterizes the equilibria of the patent auction in which firms compete against the troll and examines players' exposure to ex-post regret. In Section 1.4, we extend the model with the introduction of an intermediary who aggregates firms' contributions toward the patent purchase and competes with the troll in the auction on behalf of firms. Concluding remarks are provided in Section 1.5. Finally, Section 1.6 contains all the proofs.

1.2 The model

We consider the auction of a patent which, once bought out, might threaten two producing firms upon enforcement for patent infringement. We assume that firms use the same technology, but that they operate in different markets so that they are not direct competitors. We do not model product market interaction, rather, we focus on the strategic value of the patent for sale. Bidders include the two producing firms (indexed by $i = 1, 2$) and a patent troll (indexed by T), and we denote by $\mathcal{B}$ this set of risk neutral bidders.

1.2.1 Patent value and information structure

Each firm i is characterized by a different degree of exposure, denoted by x_i, to the patent for sale, which can be thought of as the probability that a court deems the patent valid and infringed by firm i. Throughout the paper, x_i denotes the value of the signal received by firm i. Signals X_1, X_2 are assumed to be independently and identically drawn from the uniform distribution over the support $[0, 1]$. We assume that, prior to the auction, each firm *privately* receives her signal, but remains uninformed about the other firm's degree of exposure.

Acquiring the patent confers its new owner the right to enforce it against potential infringers so as to collect damages $D > 0$. More specifically, the benefits of winning the patent auction for firm i are twofold. First, it allows to save on damages to be paid if the patent is bought out and subsequently enforced by any other bidder. Second, it also entitles firm i to sue the other infringing firm j. As to the troll, the benefit derived from acquiring the patent is to assert it against both firms so as to collect damage fees. Hence, the value of the patent for sale, v, equals the total expected damages that can be extracted from infringers, that is $v(x_1, x_2) = D(x_1 + x_2)$, and is *common* to all bidders. In what follows, we normalize damages D to one so that the common value reduces to $v(x_1, x_2) = x_1 + x_2$, as in the well-known wallet game (see Klemperer, 1998). That is, firm i's ex-post valuation for the patent is given by[2] $V_i(x_1, x_2) = v(x_1, x_2)$ for

[2]Following Milgrom and Roberts (1982), firm i's payoff is normalized to zero in the event where she is prosecuted so that her value for the patent equals her opportunity cost of litigation, x_i, plus damage payments, x_j, that she can extract from firm j.

all $i = 1, 2$.

In contrast to firms, the patent troll is assumed to be completely uninformed[3] about the common value of the patent. However, the troll benefits from a *private-value advantage*, denoted by λ, so that his *ex-post* valuation for the patent is

$$V_T(x_1, x_2) = (1 + \lambda)v(x_1, x_2) \quad \text{with } \lambda \in [0, 1]$$

This private-value advantage[4] captures the troll's strategic advantage over producing firms, such as his immunity to countersuits for infringement, or his better ability to sue due to a greater expertise in patent assertion activities.

Once the patent is awarded to the highest bidder, the degree of exposure of the defendant is assumed to be truly revealed to the plaintiff after the latter incurs an information acquisition cost, which we normalize to zero for computational convenience.

The patent is auctionned through a second-price sealed-bid auction[5], with random tie-breaking rule, where it is assigned to the highest bidder who pays the second highest bid. We are therefore in the context of a second-price *almost* common-value auction with three asymmetrically informed bidders. In the vast literature on second-price sealed-bid common-value auctions, asymmetries in terms of either information or ex-post valuations for the object across bidders have been extensively studied[6]. Nevertheless, the existing literature does not incorporate both sources of asymmetries with more than two bidders. Hence, the model under consideration is at the intersection between these two strands of the literature.

Let $\boldsymbol{x} = (x_1, x_2) \in [0, 1]^2$ denote the vector of signal realizations and $\boldsymbol{b}_{-h}$ the vector of bidding strategies of all players but h. The *ex-post* payoff of bidder $h \in \mathcal{B}$ is then given by:

$$u_h(b_h, \boldsymbol{b}_{-h}, \boldsymbol{x}) = [V_h(\boldsymbol{x}) - \max_{l \neq h}\{b_l\}]\mathbb{1}_{b_h \geq \max\limits_{l \neq h}\{b_l\}}$$

[3] Formally, letting x_T denote the realized signal received by the troll, the common value of the patent is given by $\tilde{v}(x_1, x_2, x_T) = x_1 + x_2 = v(x_1, x_2)$.

[4] While we postulate that the private-value advantage enters the troll's ex-post valuation multiplicatively, it may be easily verified that our qualitative results hold if the private-value advantage instead enters additively, i.e. if the troll's ex-post valuation for the patent is of the form $\tilde{V}_T(x_1, x_2) = v(x_1, x_2) + \kappa$, $\kappa \geq 0$.

[5] We do not endogenize the patent seller's behavior. The seller exogenously sets a reserve price that does not exclude any bidder from participating in the auction.

[6] See for instance Hernando-Veciana (2004) and de Frutos, Pechlivanos (2006) for asymmetries in terms of the information structure; Bikhchandani (1988) and Levin, Kagel (2005) for almost common-value auctions with respectively two and strictly more than two bidders.

where $\mathbb{1}_E$ is equal to one in event E, and zero otherwise (i.e., $\mathbb{1}_E$ is the indicator function of event E).

1.2.2 Timing and equilibrium concept.

The timing of the game is as follows.

t=0 Signals are simultaneously and independently drawn by Nature from the uniform distribution over the unit interval, and each firm privately observes her realized signal.

t=1 The patent is auctionned through a second-price sealed-bid auction and the patent reassignee enforces its rights.

It is well known that second-price common-value auctions are plagued by a plethora of equilibria (Milgrom, 1981). Therefore, we first restrict our attention to (pure-strategy) Bayesian Nash equilibria in *undominated* strategies (or, *undominated* equilibria) to eliminate some trivial equilibria that would not be meaningful in our context. For instance, there is a whole class of equilibria in which one player submits a prohibitively high bid, while its competitors bid more conservatively.

For firm i, bidding $b_i = v(x_i, 0) = x_i$, i.e. the lowest possible value of the patent given her signal realization, weakly dominates any lower bid. To see this, suppose that firm i bids instead according to $b' < x_i$, then the outcome only changes if $b' < \max_{j \neq i}\{b_j, b_T\} < x_i$. In this case, firm i gets

$$u_i(b', \boldsymbol{b}_{-i}, \boldsymbol{x}) = 0 \leq v(\boldsymbol{x}) - x_i < v(\boldsymbol{x}) - \max_{j \neq i}\{b_j, b_T\} = u_i(b_i = x_i, \boldsymbol{b}_{-i}, \boldsymbol{x})$$

Since this holds for all $\boldsymbol{b}_{-i}$, $b_i = x_i$ weakly dominates any $b' < x_i = b_i$. A similar argument shows that bidding $b_i = v(x_i, 1) = x_i + 1$, i.e. the highest possible value of the patent given her signal realization, weakly dominates any higher bid. Thus, the set of undominated strategies of firm i writes $\mathcal{A}_i = [x_i, x_i + 1]$, and an undominated (pure) strategy for firm i is then a function $b_i : [0, 1] \to \mathcal{A}_i$ that maps signals into her set of undominated strategies.

Likewise, because the troll is completely uninformed about the common value of the patent, his undominated bids necessarily lie between his lowest possible *ex-post* valuation (that is, $V_T(\mathbf{0}) = 0$), and his highest possible *ex-post* valuation (namely, $V_T(\mathbf{1}) = 2(1 + \lambda)$), so that his set of undominated strategies is $\mathcal{A}_T = [0, 2(1 + \lambda)]$. Thus, an undominated (pure) strategy for the troll is simply $b_T \in \mathcal{A}_T$. We can now define the equilibrium notion that we will use in this paper.

Definition 1.1. The vector of bids $\boldsymbol{b}^* = (b_T^*, b_1^*, b_2^*)$ is an *undominated equilibrium* of the patent auction if for all $h \in \mathcal{B}$, for all $\boldsymbol{x} \in [0,1]^2$ and all $a_h \in \mathcal{A}_h$,

$$\begin{cases} \mathbb{E}[u_T(\boldsymbol{b}^*(\boldsymbol{X}), \boldsymbol{X})] \geq \mathbb{E}[u_T(a_T, \boldsymbol{b}^*_{-T}(\boldsymbol{X}), \boldsymbol{X})] \\ \mathbb{E}[u_i(\boldsymbol{b}^*(\boldsymbol{X}), \boldsymbol{X})|X_i = x_i] \geq \mathbb{E}[u_i(a_i, \boldsymbol{b}^*_{-i}(X_j), \boldsymbol{X})|X_i = x_i] \quad \forall i \neq j, \ i,j = 1,2 \end{cases}$$

The first inequality says that bidding b_T^* is optimal for the troll against firms' strategies $\boldsymbol{b}^*_{-T}$, and since he does not hold any information about the common value of the patent, the expectation operator is with respect to the random vector $\boldsymbol{X}$. Instead, the second inequality states that bidding b_i^* is optimal for firm i against her competitors' strategies $\boldsymbol{b}^*_{-i}$ when evaluated at the *interim* stage, that is, once she learns her exposure to the patent.

Another natural refinement is to further focus on equilibrium strategies satisfying the no ex-post regret property, defined below. In words, a bidder's strategy is immune to ex-post regret if knowing the vector of realized signals $\boldsymbol{x}$ would not change its bidding behavior, regardless of whether it wins or loses the auction. In the next section, we will see that this desirable property is *always* satisfied by firms' equilibrium strategies if they are symmetric. However, we shall see that this result does not typically carry over to the troll's bidding strategies because of his lack of information about the common value of the patent. Furthermore, his private-value advantage tends to exacerbate his exposure to ex-post regret as it spurs his incentives to bid aggressively.

Definition 1.2. Let $\boldsymbol{b}_{-h}$ be a vector of actions of bidders other than h. An undominated strategy b_h for bidder $h \in \mathcal{B}$ satisfies the *no ex-post regret property* if for all $\boldsymbol{x} \in [0,1]^2$ and all $a_h \in \mathcal{A}_h$, $u_h(b_h, \boldsymbol{b}_{-h}, \boldsymbol{x}) \geq u_h(a_{h,}, \boldsymbol{b}_{-h}, \boldsymbol{x})$.

Finally, we next introduce the stronger concept of ex-post equilibrium generally adopted in common-value auctions, which ensures that the equilibrium vector of bids is immune to ex-post regret for *all* bidders.

Definition 1.3. The vector of bids $\boldsymbol{b}^* = (b_T^*, b_1^*, b_2^*)$ is an *ex-post equilibrium* in undominated strategies of the patent auction if for all $h \in \mathcal{B}$, for all $\boldsymbol{x} \in [0,1]^2$ and all $a_h \in \mathcal{A}_h$,

$$\begin{cases} u_T(\boldsymbol{b}^*(\boldsymbol{x}), \boldsymbol{x}) \geq u_T(a_T, \boldsymbol{b}^*_{-T}(\boldsymbol{x}), \boldsymbol{x}) \\ u_i(\boldsymbol{b}^*(\boldsymbol{x}), \boldsymbol{x}) \geq u_i(a_i, \boldsymbol{b}^*_{-i}(x_j), \boldsymbol{x}) \qquad \forall i \neq j,\ i,j = 1,2 \end{cases}$$

1.3 Equilibrium analysis

In this section, we restrict our attention to symmetric equilibrium strategies among firms, and show that the troll always either bids zero or aggressively. Then, we characterize equilibria in which firms employ symmetric linear strategies, which further allows us to examine whether the troll suffers from ex-post regret due to the combination of a lack of information and a private-value advantage. Throughout, we assume that firms' symmetric bidding strategies are continuous and strictly increasing in their signal. For the reader's convenience, we begin with the special case where a single firm faces the troll alone.

1.3.1 The case of a single firm

In second-price common-value auctions with two bidders, introducing asymmetries among players through a private-value advantage drastically affects the outcome of the auction, namely, the advantaged bidder *always* wins and the seller's revenue substantially decreases (see Bikhchandani (1988) and Avery and Kagel (1997)). This result is still robust when the advantaged bidder is also completely *uninformed* about the common value. To see this in our context, consider the patent auction in which the troll competes with only one firm. The patent value then simply reduces to $v(x) = x$, where x is the realized signal received by the firm. Clearly, the firm is now perfectly informed about the patent value, and standard arguments show that her unique (weakly) dominant strategy is to bid her true value, i.e. $b(x) = x$. In turn, the troll optimally responds by bidding aggressively, i.e. above the patent highest possible value $\bar{v} \equiv v(1) = 1$, and wins the patent with probability one.

Lemma 1.1. *Consider the patent auction with one firm and the troll. There is a continuum of ex-post equilibria in undominated strategies in which the troll always wins the patent for sale with* $b(x) = x,\ b_T \in (1, 1+\lambda]$. *Furthermore, as the troll's private-value advantage vanishes (i.e. as* $\lambda \downarrow 0$*), the strategies* $b(x) = x$ *and any* $b_T \in [0,1]$ *constitute an ex-post equilibrium.*

Note that the troll is indifferent over his whole set of undominated strategies when he does not enjoy a private-value advantage since he gets a zero ex-post payoff regardless of whether he wins or loses the patent auction. Instead, even a tiny private-value advantage over the firm shrinks the set of ex-post equilibria so that the troll *always* gets the patent. Driving this result is the fact that the firm has now "private" values. Given the firm's bidding behavior and the auction format, the troll can always outbid her without fearing to overpay, thereby winning the auction and getting a positive *ex-post* payoff despite his lack of information.

Levin and Kagel (2005) examine whether this extreme result still obtains with more than one regular bidder in an almost common-value second-price auction where *each* bidder receives a private signal. They show that, in the wallet auction, regular bidders have a positive probability to win the auction, yet lower than that of the advantaged bidder, and that a small private-value advantage only slightly decreases the seller's revenue. However, because of the information structure we adopt, this result does not carry over here since the advantaged bidder (namely, the troll) is also uninformed.

1.3.2 Symmetric strategies among firms

It is well known that, with two bidders, the second-price *pure* common-value auction admits a unique[7] symmetric equilibrium where each player bids twice its signal, i.e. $b^S(x_i) = 2x_i$, and neither suffers from ex-post regret (Milgrom, 1981). Unfortunately, this nice result does not carry over to our context because of the asymmetries among bidders. The first type of asymmetry that arises comes from the information structure of our model. Namely, firms are imperfectly informed about the common value of the patent through the signal they receive, while the troll is completely uninformed. Second, bidders differ according to their ex-post valuation for the patent as the troll enjoys a private-value advantage coming from his immunity to countersuits, reflected in λ.

The presence of the troll at the patent auction impacts firms' bidding behavior in two opposite ways. On the one hand, one might expect that the mere participation of the troll in the patent auction will induce firms to bid more cautiously in order to avoid ex-post regret, ceteris paribus. Intuitively, the troll can be thought of as a "noisy bidder" in the sense that the bid he submits does not reflect or contain any relevant information about the patent value. Rather, driving

[7]See Levin and Harstad (1986) for a proof of uniqueness.

the troll's bidding strategy is the magnitude of his private-value advantage. Given the auction format under consideration, if the troll is the second highest bidder, then the winning firm will likely overpay for the patent, and make a loss. In other words, the presence of the troll worsens firms' winner's curse. On the other hand, firms may be incentivized to submit more "aggressive" bids, up to their maximum willingness-to-pay for the patent in the absence of the troll. By doing so, firms exploit their information advantage over the troll so that the latter strictly prefers to lose the auction for low values of his private-value advantage.

The next result describes the troll's equilibrium bidding behavior in the auction for patent buyout and its consequences on firms' exposure to ex-post regret and the sellers's expected revenue.

Theorem 1.1. *Suppose that firms' bidding strategies are symmetric. Then, the following holds in any undominated equilibrium:*

(i) *The troll bids either the upper bound or the lower bound of his set of undominated strategies.*

(ii) *Firms do not suffer from ex-post regret regardless of the outcome of the auction.*

(iii) *The troll's participation in the auction raises the seller's expected revenue.*

Knowing that his participation lowers firms' maximum willingness-to-pay for the patent through a more severe winner's curse, the troll anticipates that firms bid closer to their interim expected value for the patent. It follows that the troll's winner's curse gets milder despite his information disadvantage, which in turn enhances the profitability of winning the auction. Because of his lack of information about firms' exposure to the patent, bidding aggressively, that is, above the highest possible value of the patent, ensures that he always wins and that firms do not regret losing since outbidding the troll would result in a strictly negative ex-post payoff upon winning. Notably, the higher his private-value advantage the more frequently the troll resorts to this strategy since winning becomes more profitable.

Even though the seller benefits from the addition of a bidder in the auction through harsher competition to acquire the patent for sale, the fact that the troll holds a private-value advantage worsens firms' exposure to the winner's curse, and incentivizes them to submit more cautious bids (Levin and Kagel, 2005). While the latter effect usually outweighs the former in almost common-value settings, thereby lowering the seller's revenue, this result does not carry over here because of information asymmetries across bidders. The troll's lack of information about the common value of the patent together with his private-value advantage encourage him to adopt an extreme bidding behavior. Because he bids either above the highest possible patent value or zero, firms respond to the troll's participation in the auction by mildly shading their bids down, so that the competitive effect instead dominates and positively impacts the seller's revenue.

We now offer further insight into Theorem 1.1 by considering explicitely the special case of linear strategies.

Proposition 1.1. *Suppose that firms pledge symmetric linear bids. There is a continuum of undominated equilibria in which the troll always wins the auction. The equilibrium strategies are then*

$$b_T = 2(1+\lambda) \quad \textit{and} \quad b(x_i) = \alpha x_i \ \textit{with} \ \alpha \in [1, \frac{3}{2}]$$

This equilibrium profile is robust when the troll's private-value advantage vanishes, i.e. as λ goes to zero, which suggests that firms' cautious bidding behavior is mainly driven by the troll's lack of information, yet worsened by the latter's private-value advantage.

Conversely, if firms adopt a more aggressive behavior, we shall see below that the troll then strictly prefers to lose the auction when his private-value advantage is too low as the price to pay upon winning exceeds the true value of the patent. Every firm then infers that, upon winning, the price she has to pay will necessarily be coming from the other firm. Thus, firms behave as if the troll did not participate in the auction and one obtains the symmetric equilibrium of the pure common-value auction with two bidders, which guarantees that neither firm suffers from ex-post regret and that the seller's expected revenue remains the same. The next result characterizes an equilibrium for which the outcome of the auction depends on the magnitude of the troll's private-value advantage.

Proposition 1.2. *The following strategies constitute an undominated equilibrium:*

- *if* $\lambda < \frac{1}{3}$, *then* $b_T = 0$ *and* $b(x_i) = 2x_i$.
- *if* $\lambda \geq \frac{1}{3}$, *then* $b_T = 2(1+\lambda)$ *and* $b(x_i) = \beta x_i$, *with* $\beta \in [1, 2]$.

A few comments are in order. First, firms have a strictly positive ex-ante probability of winning the patent auction only if the troll's private-value advantage is low enough ($\lambda < 1/3$).

In this case, the participation of the troll in the auction does not alter the symmetric equilibrium strategies played by firms in the pure common-value case without the troll. Interestingly, if $\beta = 2$, then this result extends to *any* λ. Put differently, in our context, the symmetric equilibrium of the pure common-value auction with two bidders is robust to the introduction of a third uninformed advantaged bidder. Namely, firms behave as if the troll did not take part in the auction for patent buyout. Nevertheless, if $\lambda \geq \frac{1}{3}$, firms' attempt to prevent the troll from getting the patent is in vain as the troll always wins the auction in *any* equilibrium.

1.3.3 On the patent troll's ex-post regret

While firms' symmetric equilibrium bidding strategies are immune to ex-post regret, this property is less likely to be satisfied by the troll's equilibrium bid because of his lack of information about the patent common value. We now restrict our attention to equilibrium profiles of strategies satisfying the no ex-post regret property for *all* bidders, and examine whether the set of ex-post equilibria reduces to a unique outcome of the auction.

Before stating the main results, we first illustrate the issue at hand by focusing on the equilibrium profile in which firms play the symmetric strategies of the pure common-value case in order to grasp some intuition about the impact of the troll's "all-or-nothing" equilibrium behavior on his exposure to ex-post regret upon both winning and losing. Throughout this subsection, we assume w.l.o.g. that signal realizations are such that $x_1 \geq x_2$.

Lemma 1.2. *Consider the following equilibrium profile of strategies:*

$$b^S(x_i) = 2x_i \quad \forall x_i \in [0,1], \quad b_T^S = \begin{cases} 2(1+\lambda) & \text{if } \lambda \geq \frac{1}{3} \\ 0 & \text{if } \lambda < \frac{1}{3} \end{cases}$$

We have that:

- *if* $\lambda \in \left[\frac{1}{3}, \Delta(\boldsymbol{x})\right]$, *then the troll suffers from ex-post regret upon winning*
- *if* $\lambda \in \left[\Delta(\boldsymbol{x}), \frac{1}{3}\right)$, *then the troll suffers from ex-post regret upon losing*

with $\Delta(\boldsymbol{x}) \equiv \frac{x_1 - x_2}{x_1 + x_2}$.

This result is straightforward upon noticing that $\Delta(\boldsymbol{x}) \gtreqless \frac{1}{3}$ is equivalent to $x_2 \lesseqgtr \frac{x_1}{2} = \mathbb{E}(X_2 | X_2 \leq x_1)$. Namely, if the *realized* degree of exposure of firm 2 (that is, the low-signal firm) is lower than its interim expected value, then the troll suffers from ex-post regret upon winning since the patent value is too low compared to the price paid by the troll for acquiring it. Conversely, if the degree of exposure of firm 2 is greater than its interim expected value, then the troll suffers from ex-post regret upon losing since the patent value is higher than expected, and the troll could have extracted a positive surplus by winning the patent. Finally, note that as $\Delta(\boldsymbol{x})$ goes to the troll's cutoff point, or equivalently as x_2 gets closer to its interim expected value, then the troll does not suffer from ex-post regret regardless of the outcome of the auction.

Figure 1.1 provides a partition of the $(\Delta(\boldsymbol{x}), \lambda)$-space showing whether the troll suffers from ex-post regret in equilibrium with the aforementioned strategies. The troll is ex-post indifferent between winning and losing the auction along the 45° line as

$$\lambda = \Delta(\boldsymbol{x}) = \frac{x_1 - x_2}{x_1 + x_2} \Leftrightarrow (1+\lambda)(x_1 + x_2) = 2x_1 \Leftrightarrow V_T(\boldsymbol{x}) = b^S(x_1)$$

The upper-half space characterizes all combinations of signal realizations and private-value advantage that yield a strictly positive ex-post payoff to the troll upon winning the patent, while the lower-half space depicts combinations for which the troll strictly prefers to lose the auction from an ex-post perspective.

As his private-value advantage goes to zero (resp. to one), the troll is ex-post better off losing (resp. winning) the patent auction for any vector of signal realizations $\boldsymbol{x} \in [0,1]^2$, i.e. for any value of the patent. Instead, the troll is vulnerable to ex-post regret when playing according to $\boldsymbol{b}^S$, i.e. either the upper bound or the lower bound of his set of undominated strategies, for any $\lambda \in (0,1)$. This is due to the fact that the he pays the most exposed firm's bid, which does not necessarily capture the patent value. It follows that the troll likely overpays for the patent, thereby getting a strictly negative ex-post payoff, unless his private-value advantage is sufficiently large to compensate the loss associated with his lack of information. For instance, if firms are very heterogeneous in terms of exposure to patent infringement, i.e. for $|x_i - x_j| \to 1$, then we have that $\Delta(\boldsymbol{x}) \to 1$ and the troll *always* suffers from ex-post regret upon winning. As well, if firm 2 faces a very low risk of patent infringement, then the troll gets

$$u_T = \lim_{x_2 \to 0} [(1+\lambda)(x_1 + x_2) - 2x_1] = (1+\lambda)x_1 - 2x_1 \leq 0 \quad \forall \lambda \leq 1$$

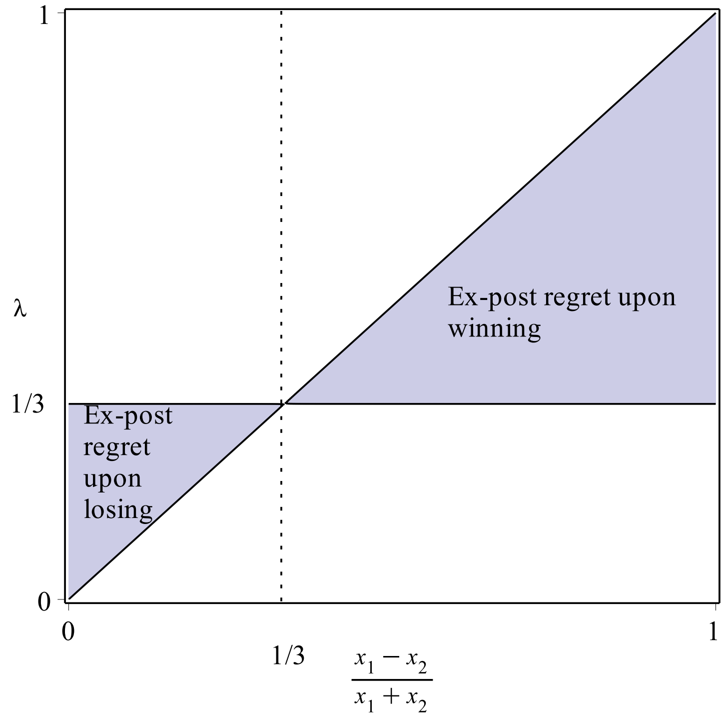

Figure 1.1: Troll's exposure to ex-post regret

Instead, as firms face a similar risk of patent infringement, i.e. as $|x_i - x_j| \to 0$, then the price that the troll would have to pay upon winning tends to the patent *true* value, which in turn would lead to a positive ex-post payoff upon winning as

$$u_T = \lim_{x_2 \to x_1} [(1+\lambda)(x_1 + x_2) - 2x_1] = 2\lambda x_1 \geq 0 \quad \forall \lambda \geq 0$$

In such a case, the troll *always* suffers from ex-post regret upon losing. In fact, our next result states that, if the troll enjoys a strictly positive private-value advantage, then he must win in any ex-post equilibrium in which firms' bidding functions are symmetric.

Theorem 1.2. *Suppose that firms play symmetric strategies. If $\lambda > 0$, then there is no ex-post equilibrium in which firms have a positive ex-ante probability to win the auction.*

Hence, the set of ex-post equilibria yields a unique outcome, namely, the troll always wins, which further motivates intermediaries' intervention since firms have no means to protect themselves against threatening patents when individually competing against the troll in the auction. This result therefore suggests that, with more than two players, perturbing the information structure so that the advantaged bidder is also uninformed restores the extreme result of almost common-value auctions with two imperfectly informed bidders: the advantaged bidder always wins.

We now provide a necessary and sufficient condition on the private-value advantage to support the troll's aggressive bidding strategy as part of an ex-post equilibrium in which firms pledge symmetric and linear bids.

Proposition 1.3. *The strategies* $b_T = 2(1+\lambda)$, $b(x_i) = \gamma x_i$ *with* $\gamma \in [1,2]$ *, form an ex-post equilibrium in undominated strategies if and only if* $\lambda \geq \gamma - 1 \equiv \underline{\lambda}$.

Typically, bidding aggressively makes the troll vulnerable to ex-post regret when firms bid above their signal realization (that is, for $\gamma > 1$). By symmetry and linearity of firms' strategies, the troll pays the bid of the *most* exposed firm upon winning. Yet, the patent value depends on *each* firm's degree of exposure. For instance, if firms are very heterogeneous in terms of exposure to patent infringement, i.e. if $|x_i - x_j|$ is close to one, then the troll will suffer from ex-post regret upon winning, unless his private-value advantage is high enough (namely, such that $\lambda \geq \underline{\lambda}$). In this case, the troll's greater ex-post valuation for the patent compensates his information disadvantage relative to firms when formulating his bid.

It directly follows that the vector of bids as indicated in Proposition 1.1 in which the troll plays aggressively for any $\lambda \in [0,1]$ constitutes an ex-post equilibrium in undominated strategies if and only if $\alpha = 1$. If firms bid as low as their signal realization, the troll is ensured to *never* regret winning since, for any λ, he gets $u_T = (1+\lambda)(x_1 + x_2) - x_1 \geq 0, \forall x_i \in [0,1], \ i = 1,2$. Importantly, this result obtains once one reestablishes symmetry across bidders' ex-post valuations for the patent. Thus, as opposed to the auction with only one firm, there is an ex-post equilibrium in which the troll still wins the auction with probability one as his private-value advantage vanishes when competing with two firms. This is due to the fact that, with only one firm, the information gap between bidders is maximal as the firm is perfectly informed about the patent value. Therefore, given the auction format, the troll cannot extract any surplus upon acquisition of the patent for sale if he does not benefit from a private-value advantage over the firm.

1.4 Intermediation in the patent auction

In this section, we extend the previous model by introducing an intermediary who, in exchange of an up-front membership fee, enables firms to gather their interests by voluntarily contributing toward the patent purchase[8]. The intermediary then aggregates contributions and competes with the troll in the auction for patent buyout on behalf of his members. Upon winning the auction, the intermediary provides his member(s) with non-exclusive license(s) to the acquired patent, thereby annihilating any risk of litigation for patent infringement brought by the troll.

1.4.1 Augmented model setup

The crucial difference with the model specified before is that, whenever (say) firm i accepts the intermediary's offer, she then has "private values" for the patent in the sense that she cannot sue firm j for patent infringement, regardless of whether firm j accepted or rejected the offer. This comes from the fact that holding a non-exclusive license precludes any right of enforcing the patent, rather, this right accrues to the patent owner. Therefore, firm i's valuation for the patent now simply equals her degree of exposure x_i, that is, $\tilde{V}_i(x_i) = x_i$, while the troll's valuation remains unchanged due to his ability to enforce the patent against both firms, regardless of whether they joined the intermediary, i.e. $V_T(x_1, x_2) = (1 + \lambda)(x_1 + x_2)$.

1.4.1.1 Exclusion of low-signal firms

The intermediary is assumed to be uninformed about firms' signals, or equivalently about the true value of the patent, but makes his offer at the *interim* stage, that is, after each firm privately receives her signal. Straightforwardly, he cannot discriminate among firms through signal-contingent membership fees since firms would fail to truthfully self-select within this menu. The intermediary's offer therefore consists of a uniform membership fee, i.e., $t = t_i$ for all $i = 1, 2$, which further implies that proposing a fee targeting the whole set of possible signal realizations, $[0, 1]$, is not profitable since he would then get zero profit.

Hence, the intermediary instead chooses a threshold signal $\hat{x} \in (0, 1)$ and a membership fee $t > 0$ such that firms with a signal in $[\hat{x}, 1]$ accept his offer, while firms with a signal in $[0, \hat{x})$ reject it. Letting $a_i \in \{A, R\}$ denote the decision of firm i to accept or reject the offer, the

[8] See for instance: http://www.alliedsecuritytrust.com/Services/AcquisitionModel.aspx and http://www.rpxcorp.com/rpx-services/rpx-defensive-patent-acquisitions/

intermediary's set of members, $\mathcal{I} \subseteq \{1,2\}$, is then given by

$$\mathcal{I} = \{i \in \{1,2\} \text{ such that } a_i = A\} \quad \text{with } |\mathcal{I}| = \sum_{i \in \{1,2\}} \mathbb{1}_{a_i = A}$$

Furthermore, we suppose that the intermediary's number of members becomes common knowledge once firms made their decisions, that is

Assumption 1. $|\mathcal{I}|$ *is common knowledge.*

This assumption comes from the fact that intermediaries' websites often display their number of members, but do not (usually) disclose their identity[9]. In what follows, we let $\Gamma_{|\mathcal{I}|}$ denote the continuation game after $|\mathcal{I}|$ firms accepted the intermediary's offer.

1.4.1.2 Collective patent purchase through voluntary individual contributions

Upon paying the membership fee, firms then simultaneously choose the amount of their contribution, s_i, toward the patent purchase. The intermediary then competes with the troll in the auction for patent buyout where his bid then simply equals the sum of his members' contributions, that is, $b_I = \sum_{l \in \mathcal{I}} s_l$. If $b_I < b_T$, then the intermediary loses the auction and contributions are fully refunded to firms. Instead, upon winning, we assume that the intermediary uses a *proportional* rebate rule[10] whenever the total contributions exceed the troll's bid. Namely, if $b_I = s_i + s_j \geq b_T$, firm i then retrieves

$$r_i(s_i, s_j, b_T) = \frac{s_i}{s_i + s_j} (s_i + s_j - b_T)$$

Observe that the intermediary's funding mechanism for the patent purchase is similar to the well-known *subscription game*[11] in the literature on the private provision of a discrete public good through voluntary contributions. In such games, agents voluntarily choose the amount of their contribution to the funding of a public good, which is then provided if the sum of contributions exceeds an exogenous threshold cost[12]. However, in our model, the threshold cost

[9] In some cases, they provide the name of some of their members, usually major firms in their technology area. See for instance: http://www.alliedsecuritytrust.com/ASTMembers.aspx

[10] See Spencer et al. (2009) for alternative rebate rules in the context of public good provision.

[11] Following the terminology of Admati and Perry (1991), contributions are fully refunded in *subscription* games whenever insufficient to provide the public good, as opposed to *contribution* games in which they are retained by the collector.

[12] See for instance Menezes et al. (2001).

is *endogenously* determined by the troll's bidding strategy given the auction format.

1.4.1.3 Payoffs

Suppose first that at least one firm decided to join the intermediary. For a given pair $(\hat{x}, t)$, let $\boldsymbol{\sigma} = (\boldsymbol{s}, b_T)$ denote the vector of actions in the continuation game $\Gamma_{|\mathcal{I}|}$, where $\boldsymbol{s}$ is the $|\mathcal{I}|$-dimensional vector of contributions and b_T is the troll's bid. Straightforwardly, since the intermediary's number of members is common knowledge, firm i can then infer firm j's acceptance decision. The ex-post *net* payoff of firm i with signal x_i, when choosing a_i and contributing s_i, is thus given by $\tilde{u}_i(s_i, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(a_i, a_j))$. More specifically, if both firms accept, then firm i, $i = 1, 2$, gets

$$\tilde{u}_i(s_i, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(A, A)) = \begin{cases} x_i - s_i + r_i(s_i, s_j, b_T) - t = x_i - \frac{s_i.b_T}{s_i+s_j} - t & \text{if } s_i + s_j \geq b_T \\ -t & \text{otherwise} \end{cases}$$

Instead, if firm i accepts and firm j rejects, $i \neq j$, ex-post payoffs are then given by

$$\tilde{u}_i(s_i, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(A, R)) = \begin{cases} x_i - b_T - t & \text{if } s_i \geq b_T \\ -t & \text{otherwise} \end{cases}$$

and

$$\tilde{u}_j(s_j, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(R, A)) = \begin{cases} x_j & \text{if } s_i \geq b_T \\ 0 & \text{otherwise} \end{cases}$$

If neither firm accepts the intermediary's offer, then ex-post payoffs are the same as those of the patent auction without the intermediary, namely,

$$\tilde{u}_i(s_i, \boldsymbol{\sigma}_{-i}, \boldsymbol{x}|(R, R)) = u_i(b_i, \boldsymbol{b}_{-i}, \boldsymbol{x}) = [V_i(\boldsymbol{x}) - \max_{l \neq i}\{b_l\}]\mathbb{1}_{b_i \geq \max_{l \neq i}\{b_l\}} \quad \forall i = 1, 2$$

Finally, upon observing $|\mathcal{I}|$, the troll's ex-post payoff is

$$\tilde{u}_T(b_T, \boldsymbol{\sigma}_{-T}(\boldsymbol{x}), \boldsymbol{x}||\mathcal{I}|) = \begin{cases} u_T(b_T, \boldsymbol{b}_{-T}, \boldsymbol{x}) = [V_T(\boldsymbol{x}) - \max_{l \neq T}\{b_l\}]\mathbb{1}_{b_T \geq \max_{l \neq T}\{b_l\}} & \text{if } \mathcal{I} = \emptyset \\ [V_T(\boldsymbol{x}) - b_I]\, \mathbb{1}_{b_T \geq b_I} & \text{otherwise} \end{cases}$$

1.4.1.4 Timing and equilibrium concept

The game now unfolds as follows.

t=0 Signals are simultaneously and independently drawn by Nature from the uniform distribution over the unit interval, and each firm privately observes her realized signal.

t=1 The intermediary chooses a threshold signal and proposes a subscription fee, and firms simultaneously either accept or reject the intermediary's offer[13].

If both firms reject ($\mathcal{I} = \emptyset$), then firms individually participate in the auction and compete against the troll, and the game proceeds as without the intermediary's intervention, namely:

t=2 The patent is auctionned through a second-price sealed-bid auction between firms and the troll, and the patent reassignee enforces its rights.

If instead at least one firm accepts ($\mathcal{I} \neq \emptyset$), then the game proceeds as follows:

t=2' Each member simultaneously submits a voluntary contribution toward the patent purchase.

t=3' The patent is auctionned through a second-price sealed-bid auction between the intermediary and the troll, and the patent reassignee enforces its rights.

A pure strategy for firm i is now a pair (a_i, s_i) where $a_i \in \{A, R\}$ denotes firm i's decision to accept or reject the intermediary's offer, and $s_i : [0,1] \to \mathbb{R}_+$ is the contribution of firm i to the patent purchase, $i = 1, 2$. Since the intermediary's bid is simply the sum of his members' contributions, he can therefore be thought of as a "passive" bidder in the auction. Thus, given that firms simultaneously pledge their contributions, together with the fact that they are not observable by the troll, the continuation game $\Gamma_{|\mathcal{I}|}$ can then be treated as a one-shot game.

Throughout, we will use the concept of perfect Bayesian equilibrium with the additional requirement that any candidate vector of actions $\boldsymbol{\sigma}^*$ forms an ex-post equilibrium of the corresponding continuation game.

[13]Throughout, we adopt the conventional assumption that, when indifferent, firms accept the intermediary's offer.

Definition 1.4. The vector of actions $\boldsymbol{\sigma}^* = (\boldsymbol{s}^*, b_T^*)$ is an ex-post equilibrium of the continuation game $\Gamma_{|\mathcal{I}|}$ if for all $\boldsymbol{x} \in [0,1]^2$,

$$\begin{cases} \tilde{u}_T(\boldsymbol{\sigma}^*(\boldsymbol{x}), \boldsymbol{x}||\mathcal{I}|) \geq \tilde{u}_T(b'_T, \boldsymbol{\sigma}^*_{-T}(\boldsymbol{x}), \boldsymbol{x}||\mathcal{I}|) & \forall b'_T \in \mathbb{R}_+ \\ \tilde{u}_i(\boldsymbol{\sigma}^*(\boldsymbol{x}), \boldsymbol{x}|(a_i, a_j)) \geq \tilde{u}_i(s'_i, \boldsymbol{\sigma}^*_{-i}(x_j), \boldsymbol{x}|(a_i, a_j)) & \forall s'_i \in \mathbb{R}_+,\ \forall i \neq j,\ i,j = 1,2 \end{cases}$$

Definition 1.5. An equilibrium of the intermediated patent auction is a threshold signal $\hat{x}^*$ and a membership fee t^* such that $(\hat{x}^*, t^*)$ is optimal for the intermediary given other players' strategies, together with a vector of decisions $\boldsymbol{a}^* = (a_1^*, a_2^*)$ satisfying

$$\begin{cases} a_i^* = A & \Rightarrow x_i \in [\hat{x}^*, 1] \\ a_i^* = R & \Leftarrow x_i \in [0, \hat{x}^*) \end{cases} \quad \text{for all } i = 1,2$$

and such that:

1. Letting $S_i \subseteq [0,1]$ and $S_j \subseteq [0,1]$, the troll and firms' updated beliefs are compatible with Bayes' rule:

$$\begin{cases} \mu_T\left(\boldsymbol{X} \in S_i \times S_j | \, |\mathcal{I}| = k\right) & = \frac{\Pr(|\mathcal{I}|=k|\boldsymbol{X}\in S_i \times S_j)\Pr(\boldsymbol{X}\in S_i\times S_j)}{\Pr(|\mathcal{I}|=k)} \\ \mu_i\left(X_j \in S_j | \, |\mathcal{I}| = k, X_i = x_i\right) & = \frac{\Pr(|\mathcal{I}|=k, X_i=x_i|X_j\in S_j)\Pr(X_j\in S_j)}{\Pr(|\mathcal{I}|=k)} \end{cases} \quad \forall i,j = 1,2 \quad i \neq j$$

2. The vector of actions $\boldsymbol{\sigma}^* = (\boldsymbol{s}^*, b_T^*)$ satisfies Definition 1.4 given the system of beliefs $\boldsymbol{\mu}$, the intermediary's strategy $(\hat{x}^*, t^*)$ and the vector of firms' decisions $\boldsymbol{a}^*$.

1.4.2 Equilibria of the continuation games

In this subsection, we examine, for a fixed threshold $\hat{x} \in (0,1)$ and membership fee $t > 0$, firms' strategic behavior when contributing toward the patent purchase, and characterize equilibria of the continuation games that begin after either one firm or both of them accepted the intermediary's offer.

Remark. If neither firm accepts the intermediary's offer (that is, if $\mathcal{I} = \emptyset$), then the troll's set of undominated strategies shrinks as he infers that the *true* patent value satisfies $v(\boldsymbol{x}) \leq 2\hat{x}$. In particular, standard arguments show that bidding $b_T = 2(1+\lambda)\hat{x}$ weakly dominates any higher bid so that the troll's set of undominated strategies becomes $\mathcal{A}_T^0 = [0, 2(1+\lambda)\hat{x}] \subset \mathcal{A}_T$ for any threshold $\hat{x} \in (0,1)$. While the troll bids less aggressively than before, it is easy to see that the results of the baseline model without intermediation do not qualitatively change. Namely, the troll wins the auction in any ex-post equilibrium of the continuation game Γ_0 (cf. Theorem 1.2).

In order to carry revenue comparisons, we shall restrict our attention to symmetric linear contributions in every continuation game. Thus, from Proposition 1.3, the continuation game Γ_0 admits a continuum of ex-post equilibria with $b_T = 2\hat{x}(1+\lambda)$, $b(x_i) = \gamma x_i$ with $\lambda \geq \gamma - 1$, $\gamma \in [1,2]$.

1.4.2.1 The case of a single member firm

Consider the continuation game after only one firm accepted the intermediary's offer. Then this firm has private values for the patent upon being the only member, so that asymmetries across bidders are exacerbated. Besides his private-value advantage λ, the troll now also benefits from a greater ex-post valuation for the patent relative to firms coming from his ability to name multiple defendants. Because the intermediary's funding mechanism for the patent purchase relies exclusively on his members' contributions, it follows that the troll always preempts the patent whenever only one firm contributes. To see this, suppose that, say, firm 1 accepted the offer while firm 2 rejected it, so that the intermediary's bid is simply equal to the contribution of firm 1. By pledging a contribution s_1, firm 1's ex-post *net* payoff is

$$\tilde{u}_1(s_1, b_T, \boldsymbol{x}|(A,R)) = \begin{cases} x_1 - b_T - t & \text{if } s_1 \geq b_T \\ -t & \text{otherwise} \end{cases}$$

We now show that firm 1 has a unique weakly dominant strategy, to contribute her true value for the patent, i.e. $s(x_1) = x_1$. Suppose instead that firm 1 contributes any $\bar{s} > x_1$. The outcome only changes if $\bar{s} > b_T > x_1$, in which case firm 1 gets $x_1 - b_T - t < -t$. Similarly, contributing according to $\underline{s} < x_1$ only changes the outcome if $x_1 > b_T > \underline{s}$ and leads to $-t < x_1 - b_T - t$. In turn, the troll's optimal strategy is then to submit a bid that ensures winning with probability one since his ex-post payoff upon winning is then $(1+\lambda)(x_1 + x_2) - x_1 > 0$.

Thus, as the following lemma formalizes, the intermediary cannot preempt the patent for sale with only one contributor. In fact, since the membership fee is non-refundable, firm 1 is instead strictly worse off with the intermediary's intervention relative to section 3 whenever she ends up being the only contributing firm.

Lemma 1.3. *If $|\mathcal{I}| = 1$, then there is a continuum of ex-post equilibria of the continuation game Γ_1 in which the intermediary loses the patent auction with probability one. The equilibrium strategies are then $s(x_1) = x_1$ and $b_T \in (1, (1+\lambda)(1+\hat{x})]$.*

One obtains the same result as in the auction without the intermediary in which the troll competes with only one firm (cf. Lemma 1.1). Namely, the troll always wins in equilibrium and neither player suffers from ex-post regret since the intermediary's bid, $b_I = s(x_1) = x_1$, is lower than the troll' ex-post valuation for the patent, $v(\boldsymbol{x}) = x_1 + x_2$. However, this result is now robust as the troll's private-value advantage vanishes (that is, as $\lambda \downarrow 0$), so that the he *always* outbids the intermediary regardless of the magnitude of his private-value advantage λ. Driving this result is the fact that the troll has a higher ex-post valuation for the patent as compared to firm 1 since he can also extract damage payments from firm 2 upon winning, while firm 1 has now "private values" for the patent due to her inability to enforce the patent when holding a non-exclusive license. Therefore, the troll's ability to extract the whole value of the patent for sale through litigation compensates for his lack of information about firms' likelihood of infringement.

Finally, observe that if winning the patent auction were feasible with only one contributor, firms' incentives to join the intermediary would be greatly undermined. Indeed, by rejecting the offer, a firm could then be *de facto* protected from litigation brought by the troll while saving on the subscription fee if the other firm instead accepted the intermediary's offer. Clearly, the intermediary would then have to offer a lower subscription fee in order to account for such incentives to free ride. Hence, Lemma 1.3 ensures that firms' incentives to accept the intermediary's offer in the first place are preserved.

1.4.2.2 Collective action issue with two contributing firms

We now turn to the case where both firms accepted the intermediary's offer, namely, signal realizations are now such that $x_i \geq \hat{x}\ \forall i = 1, 2$. From Lemma 1.3, a necessary condition for the intermediary to outbid the troll in the patent auction is that both firms accept his offer.

Nevertheless, we now shall see that this is not a sufficient condition for his success in the auction.

While the fact that the intermediary cannot acquire the patent for sale with only one contributor annihilates firms' incentives to free ride when deciding whether to accept his offer, his mechanism to finance the patent purchase creates a collective action problem which potentially undermines his performance in the auction. Because the patent is collectively financed through voluntary individual contributions, this creates a free-rider problem whereby a firm has incentives to slightly lower her contribution so that the other contributing firm incurs a larger share of the patent purchase, ceteris paribus. This is due to the fact that, whenever both firms join the intermediary, the patent becomes a *collective good* in the sense that both firms (i) benefit from its acquisition by the intermediary through the non-exclusive license they receive, (ii) and are entitled to get the license regardless of their contribution (see Olson, 1965). The next result sheds light on the impact of free riding on the outcome of the auction.

Theorem 1.3. *Suppose that firms pledge symmetric contributions. There is no ex-post equilibrium of the continuation game Γ_2 in which the intermediary wins the auction with a strictly positive price.*

Observe that the collective action issue inherent to the intermediary's funding mechanism greatly benefits the troll, as submitting a strictly positive bid triggers firms' free-riding behavior and undermines the intermediary's bid, thereby increasing the profitability of winning the auction despite his lack of information about the patent value.

In what follows, we focus on equilibria involving symmetric linear contributions of the form $s(x_i) = kx_i$, $k \geq 0$, so that the intermediary's bid amounts to $b_I(\boldsymbol{x}) = k(x_1 + x_2) \equiv kv(\boldsymbol{x})$. Importantly, beyond their tractability, linear contributions make the troll immune to ex-post regret regardless of whether he wins or loses the auction since the intermediary's bid aggregates firms' private information about the patent true value. To begin with, we propose a class of ex-post equilibria which illustrates the collective action issue at hand. Formally, any profile of strategies of the form

$$\sigma^w = \{s^w(x_i) = \underline{k}x_i \text{ with } 0 \leq \underline{k} < 1 + \lambda,\ b_T^w = 2(1 + \lambda)\}$$

constitutes an ex-post equilibrium of the continuation game Γ_2, in which the intermediary loses the patent auction for sure. To see this, suppose first that firms play according to σ^w. By bidding $b_T^w = 2(1+\lambda)$, the troll wins the auction for sure as

$$b_T^w = 2(1+\lambda) \geq (1+\lambda)(x_1+x_2) > \underline{k}(x_1+x_2) = b_I$$

and gets $(1+\lambda)(x_1+x_2) - \underline{k}(x_1+x_2) > 0$ for any $\underline{k} < 1+\lambda$ which ensures that he does not regret winning. Given the auction format, bidding any $\bar{b} > b_T^w$ does not improve his payoff, while bidding according to $\underline{b} < b_T^w$ triggers a positive probability to lose the auction if $\underline{b} < b_I$, then resulting in a zero ex-post payoff. Similarly, consider, say, firm i and suppose that the troll and firm $j \neq i$ play according to σ^w. Contributing according to $s^w(x_i) = \underline{k}x_i$ leads to the intermediary's defeat in the auction, with associated ex-post payoff $-t < 0$. Obviously, any lower contribution yields the same auction outcome and ex-post payoff. Pledging instead any $\bar{s} > s^w(x_i) = \underline{k}x_i$ changes the outcome only if $\bar{s} > b_T^w - s^w(x_j)$. In this case, the intermediary wins the auction and firm i's ex-post net payoff is then

$$\begin{aligned}
\tilde{u}_i(\bar{s}, \boldsymbol{\sigma}_{-i}^w, \boldsymbol{x}|(A,A)) &= x_i - \frac{\bar{s}.b_T^w}{\bar{s}+s^w(x_j)} - t \\
&< x_i - \frac{\left(b_T^w - s^w(x_j)\right).b_T^w}{b_T^w} - t \\
&= x_i - 2(1+\lambda) + \underline{k}x_j - t \\
&< x_i - (1+\lambda)(2-x_j) - t \\
&< -t \\
&= \tilde{u}_i(\boldsymbol{\sigma}^w, \boldsymbol{x}|(A,A))
\end{aligned}$$

which ensures that firm i does not regret losing. Finally, since firms are symmetric, a similar reasoning holds for firm $j \neq i$.

Hence, in such equilibria, the intermediary's intervention fails to provide firms with safety from litigation brought by the troll. As compared to the patent auction without intermediation, firms are actually strictly worse off since they end up with a strictly negative payoff due to the fact that the membership fee is non-refundable. On the one hand, if the troll sticks to an aggressive bidding behavior, then the collective patent purchase through firms' contributions is not feasible as the troll's bid exceeds firms' *aggregate* value for the patent: $x_1 + x_2 < 2(1+\lambda) = b_T^w \ \forall \boldsymbol{x} \in [0,1]^2$. It follows that any vector of contributions $\boldsymbol{s}$ such that $b_I > b_T^w$ would make firms strictly worse off. Thus, firms optimally react by shading their contributions down so that the intermediary loses the auction for sure. On the other hand, the free-rider issue inherent to the intermediary's funding mechanism greatly impairs his bid, which in turn spurs the troll's aggressive bidding behavior.

One way to circumvent this severe free-rider problem is for firms to pledge aggressive contributions so that the troll always prefers to lose the auction from an ex-post perspective. Namely, if each firm contributes $s(x_i) > (1+\lambda)x_i$, then the troll finds it optimal to *always* lose the auction since $(1+\lambda)(x_1+x_2) - b_I < 0$. In particular, submitting a nill bid is a best response for the troll. Importantly, even though the intermediary's refund mechanism does not effectively alleviate the free-rider issue inherent to the contribution game, it is necessary for the existence of an equilibrium in which the intermediary outbids the troll with two contributors as it enables firms to play aggressively so as to drive the troll's bid down. The next result formalizes.

Proposition 1.4. *There exists an ex-post equilibrium of the continuation game Γ_2 in which the intermediary always wins the auction only if the excess of contributions is refunded. Firms then pledge aggressive contributions $s^a(x_i) = \bar{k}x_i$ with $\bar{k} > 1+\lambda$, $i = 1, 2$, while the troll bids $b_T^a = 0$.*

By adopting an aggressive behavior, firms are fully protected from any risk of infringement of the patent for sale. While they have no means to win the auction and always face costly litigation when individually competing against the troll in any ex-post equilibrium of the unintermediated auction (see Theorem 1.2), the intermediary's intervention may overturn this negative result by encouraging and aggregating aggressive contributions. Nevertheless, since the price is set by the second highest bid, it follows that the seller's revenue is strongly undermined whenever the intermediary wins in equilibrium since the troll sharply decreases his bid in response to his opponent's aggressiveness.

1.4.3 The intermediary's problem

Observe first that the intermediary finds it optimal to induce aggressive contributions in the continuation game Γ_2. Indeed, since the patent purchase is not feasible through the contribution of a sole firm (cf. Lemma 1.3), firms' perceived probability that the intermediary wins the auction would otherwise be zero so that incurring the non-refundable membership fee would then be strictly unprofitable. Consequently, firms would turn his offer down for *any* threshold signal and strictly positive subscription fee, yielding zero profit to the intermediary.

Hence, the intermediary chooses a threshold signal $\hat{x} \in (0,1)$ and a strictly positive membership fee t such that any firm holding a signal above the threshold finds it optimal to become a member. In this respect, a firm finds it profitable to accept the offer if her expected benefit from joining the intermediary, defined as her updated probability that the intermediary wins the auction times her value for the patent, net of the membership fee, exceeds her payoff upon rejecting the offer. Since the intermediary fails to acquire the patent with only one contributor, it follows that firm i's updated probability that the intermediary wins the auction, conditional on holding a signal $x_i \in [\hat{x}, 1]$, simply equals the probability that firm j's signal exceeds the threshold $\hat{x}$ as well by independence, and that her payoff upon rejecting the offer is zero regardless of whether the other firm accepts. Thus, the participation constraint of (say) firm i writes

$$(1-\hat{x})x_i - t \geq 0 \quad \forall x_i \in [\hat{x}, 1]$$

The intermediary therefore chooses a threshold signal $\hat{x}$ and a membership fee t that maximize his ex-ante expected profit, which equals the total expected membership fees, subject to firms' participation constraint. That is, the intermediary's problem writes

$$\max_{(\hat{x},t)\in[0,1]\times\mathbb{R}_+} \quad 2q_2(\hat{x}).t + q_1(\hat{x}).t$$

$$s.t. \quad (1-\hat{x})x_i - t \geq 0 \quad \forall x_i \in [\hat{x}, 1]$$

where $q_k(\hat{x})$ denote the intermediary's prior probability that k firms hold a signal greater than the threshold $\hat{x}$, $k = 1, 2$. The probability that both firms hold a signal greater than $\hat{x}$ is simply given by

$$q_2(\hat{x}) = \Pr\left[(X_1 > \hat{x}) \cap (X_2 > \hat{x})\right] = (1-\hat{x})^2$$

while the probability that only one firm does is computed as follows

$$
\begin{aligned}
q_1(\hat{x}) &= \Pr\{[(X_1 > \hat{x}) \cap (X_2 \le \hat{x})] \cup [(X_1 \le \hat{x}) \cap (X_2 > \hat{x})]\} \\
&= \Pr[(X_1 > \hat{x}) \cap (X_2 \le \hat{x})] + \Pr[(X_1 \le \hat{x}) \cap (X_2 > \hat{x})] \\
&\quad - \Pr\{[(X_1 > \hat{x}) \cap (X_2 \le \hat{x})] \cap [(X_1 \le \hat{x}) \cap (X_2 > \hat{x})]\} \quad \text{(by definition)} \\
&= \Pr[(X_1 > \hat{x}) \cap (X_2 \le \hat{x})] + \Pr[(X_1 \le \hat{x}) \cap (X_2 > \hat{x})] \quad \text{(by mutual exclusivity)} \\
&= (1-\hat{x})\,\hat{x} + \hat{x}\,(1-\hat{x}) \quad \text{(by independence)} \\
&= 2\hat{x}\,(1-\hat{x})
\end{aligned}
$$

Since optimality requires that the participation constraint binds for the threshold signal $\hat{x}$, the membership fee is given by $t = (1-\hat{x})\hat{x}$. Plugging these into the intermediary's objective and rearranging yields $\Pi_I(\hat{x}) = 2\hat{x}(1-\hat{x})^2$. One can easily check that the intermediary's profit function Π_I is strictly quasi-concave in $\hat{x}$, and that the unique optimal threshold signal and membership fee pair is given by

$$(\hat{x}^*, t^*) = \left(\frac{1}{3}, \frac{2}{9}\right)$$

The intermediary finds it optimal to screen out low signal firms by proposing a strictly positive membership fee such that firms holding a signal above the threshold $\hat{x}^*$ find it profitable to become members. Nevertheless, observe that firms are engaged in a coordination game with the troll whenever both of them join. Indeed, our previous analysis characterizes two classes of ex-post equilibria in the continuation game Γ_2 yielding two opposite outcomes (see subsection 1.4.2). While firms get a strictly positive ex-post payoff when they both pledge aggressive contributions and the troll bids zero, firms end up strictly worse off if the equilibrium profile in which they play conservatively while the troll bids aggressively instead prevails since the membership fee is non-refundable, so that it is optimal to accept (resp. reject) the intermediary's offer in the former (resp. latter) case. Therefore, if both firms hold a signal greater than the threshold $\hat{x}^*$, the whole game admits the two following classes of equilibria

$$
\begin{aligned}
\mathcal{E}_w &= ((\hat{x}^*, t^*), (R, R), \boldsymbol{\sigma}^w) \\
\mathcal{E}_a &= ((\hat{x}^*, t^*), (A, A), \boldsymbol{\sigma}^a)
\end{aligned}
$$

These two classes of equilibria yielding two opposite outcomes, we now ask whether either equilibrium constitutes an "unreasonable" prediction when both firms hold a signal above the threshold $\hat{x}^*$. Intuitively, if both firms decide to join the intermediary, they are giving up a certain payoff of zero. Since they get a strictly negative payoff when contributing cautiously, the troll should therefore expect firms to play aggressively, and bid zero himself. Invoked here is the idea of *forward induction* (Kohlberg and Mertens, 1986) which says that the play leading to the continuation game Γ_2 conveys information about firms' intentions to play subsequently. Hence, upon observing $|\mathcal{I}| = 2$, the troll should assign probability zero to firms pledging cautious contributions in equilibrium. In other words, the equilibrium $\mathcal{E}_a$ is robust to forward induction. The next result summarizes these findings.

Proposition 1.5. *The equilibrium of the intermediated auction entails the following:*

1. *The intermediary's optimal pair of threshold signal and membership fee* $(\hat{x}^*, t^*) = (\frac{1}{3}, \frac{2}{9})$ *is unique, and such that a firm accepts his offer if and only if she holds a signal in* $\left[\frac{1}{3}, 1\right]$*, and rejects if and only if her signal instead lies in* $\left[0, \frac{1}{3}\right)$*.*

2. *If* $|\mathcal{I}| = 2$*, then the intermediary always outbids the troll , whereas if* $|\mathcal{I}| < 2$*, then the troll always wins the auction.*

The intermediary's equilibrium ex-ante probability of winning the auction is thus $q_2(\hat{x}^*) = (1 - \hat{x}^*)^2 = \frac{4}{9}$, while the troll's is now $\frac{5}{9}$. From an ex-ante perspective, the intermediary therefore partially hampers the troll's supremacy when it comes to acquiring threatening patents with the intent to engage in litigious activity against firms. The effectiveness of his intervention to protect firms against litigation brought by the troll is nonetheless mitigated by the fact that the collective patent purchase is feasible *only if* both firms contribute. Though his probability of winning the auction is slightly lower than that of the troll, the true value of the patent for sale is higher whenever the intermediary acquires it as compared to the troll. This comes from the fact that the intermediary wins only if both firms hold a signal above the threshold $\hat{x}^*$, whereas the troll wins otherwise.

However, the seller's revenue dramatically falls whenever the intermediary wins the auction since firms' aggressive contributions drive the troll's bid down to zero. But, as stated in the next result, the seller's expected revenue is still higher when the intermediary intervenes as compared to the case where firms are the only participants in the auction. At a first glance, this result might seem surprising because the presence of the intermediary harms the seller through two channels. First, the troll's response to firms' aggressiveness whenever both contribute yields zero revenue to the seller nearly half of the time in equilibrium. Also, because he bids on behalf of his members, the intermediary's intervention softens competition to acquire the patent for sale by decreasing the number of bidders in the auction.

The combination of these two negative effects is nonetheless offset by the fact that, whenever both of them reject the offer, firms mildly decrease their bids in response to the troll's participation in the auction (cf. Proposition 1.3). Likewise, if only one of them joins the intermediary, her high signal compensates for the lower contribution she submits (cf. Lemma 1.3).

Corollary 1.1. *The seller's equilibrium expected revenue ranks as follows*

$$\mathbb{E}(R^T) \geq \mathbb{E}(R^I) \geq \mathbb{E}(R^0)$$

The participation of the troll in the auction substantially raises the seller's expected revenue which continuously increases with the parameter γ. From Proposition 1.3, since $\gamma \leq 1 + \lambda$, it follows that the seller's revenue is higher when the troll benefits from a significantly higher ex-post valuation for the patent over firms through a greater private-value advantage λ. In other words, asymmetries across bidders in terms of ex-post valuation for the patent benefits to the seller. This result also suggests that the intermediary's positive impact on the seller's expected revenue comes from his probability to lose the auction slightly more than half of the time so that the troll's strong positive effect on the seller's revenue dominates.

1.5 Concluding remarks

The aim of this paper is to study the emergence of non-practicing entities in the market for patents, who acquire patents with no aim to engage into innovative activities. While patent trolls seek to monetize their acquired patents through the threat of litigation against alleged infringers, intermediaries instead intend to provide their affiliated firms with safety to operate from trolls' litigious activity by buying out patents that would otherwise fall in trolls' hands.

We develop a model of patent acquisition through an auction incorporating both patent trolls and intermediaries. We highlight trolls' greater ability to preempt patents that represent a threat upon enforcement for patent infringement as compared to producing firms due to their immunity to countersuits. We find that firms have no means to protect themselves against threatening patents when individually competing against the troll in the auction for patent buyout, whereas the seller's revenue substantially increases in response to the participation of a troll in the auction.

We then examine the effectiveness of intermediaries to protect firms against the troll's litigious activity by analyzing their patent funding mechanism. Since the patent is collectively financed through voluntary individual contributions, firms tend to free ride on other contributors. While the intermediary's probability to outbid the troll in the auction is strictly positive, the collective action issue inherent to his funding mechanism greatly hampers his performance in the auction and undermines the seller's revenue. Overall, our results nonetheless suggest that the presence of NPEs in the patent acquisition process positively impact the revenue of sellers of likely infringed patents.

1.6 Proofs

This section provides the proofs for all the results of the paper.

Proof of Lemma 1.1

We first establish that bidding $b(x) = x$ is optimal for the firm. Observe that she always loses against the troll as $b(x) = x \leq 1 < b_T$ and therefore gets a zero ex-post payoff. She does not regret losing since outbidding the troll would instead yield $x - b_T < x - 1 \leq 0$, and pledging instead any $b' < x$ does not alter the outcome of the auction or her payoff. Hence, bidding according to $b(x) = x$ is indeed a best response for the firm. Next, we show that bidding any $b_T \in (1, 1 + \lambda]$ is a best response for the troll. By playing b_T, the troll always wins against the firm as $b(x) = x \leq 1 < b_T$, and does not regret winning since he gets $(1 + \lambda)x - x = \lambda x \geq 0$ for any $\lambda \in [0, 1]$. Given the auction format, submitting a higher bid does not improve his payoff upon winning. Thus, bidding any $b_T \in (1, 1 + \lambda]$ constitutes a best response for the troll. Finally, the equilibrium profile does not involve weakly dominated strategies as $V_T(0) = 0 < 1 < b_T \leq 1 + \lambda = V_T(1)$, and, as is well known, $b(x) = x$ is a weakly dominant strategy for the firm.

Similarly, we now consider the case where $\lambda \downarrow 0$. If $0 < b_T < x$, then the firm wins and does not suffer from ex-post regret since she gets $x - b_T > 0$ upon winning. If $b_T = x$, she gets zero ex-post payoff regardless of the outcome of the tie resolution. Finally, if $x < b_T \leq 1$, then the firm loses for sure and does not regret as $x - b_T < 0$. Clearly, for any $b_T \in [0, 1]$, the troll gets zero ex-post payoff whether he wins or loses the auction, and does not suffer from ex-post regret in either case. □

Proof of Theorem 1.1

Let β_e denote firms' common equilibrium strategy, which is assumed to be continuous and strictly increasing in the signal they receive, and $\phi(b) \equiv \beta_e^{-1}(b)$ firms' equilibrium inverse bidding function, where $\phi : [\beta_e(0), \beta_e(1)] \to [0, 1]$ is continuous and strictly increasing. Throughout, we shall say that the troll bids aggressively whenever he submits any $b > \beta_e(1)$.

Furthermore, we reorder X_1, X_2 and let Y_1, Y_2 denote the rearranged signals so that $Y_1 \geq Y_2$, where Y_k is distributed according to F_k, $k = 1, 2$, given by[14]

$$F_1(y_1) = [F(y_1)]^2 = y_1^2 \quad \text{and} \quad F_2(y_2) = 2F(y_2) - [F(y_2)]^2 = 2y_2 - y_2^2$$

with associated marginal densities $f_1(y_1) = 2y_1$ and $f_2(y_2) = 2(1 - y_2)$, and joint density $f_{1,2}(y_1, y_2) = 2$ if $0 \leq y_2 \leq y_1 \leq 1$ and 0 elsewhere.

(i) We first show that firms' equilibrium bidding strategies are bounded above by $\bar{w}(x_i) = 2x_i$ for all $x_i \in [0, 1]$, $i = 1, 2$. Consider (say) firm i and let us derive her maximum willingness-to-pay for the patent, $w_i(x_i)$, defined as the tying bid at which firm i is indifferent between winning and losing. Two cases need to be considered. If the tying bidder is the troll, then firm i infers that $X_j < x_i$ so that her maximum willingness-to-pay is given by

$$\mathbb{E}[v(X_i, X_j) | X_i = x_i, X_j < x_i] - w_i(x_i) = 0 \quad \Leftrightarrow w_i(x_i) = \frac{3}{2}x_i \equiv \underline{w}(x_i)$$

Instead, if the tying bidder is firm j, then firm i infers that $X_j = x_i$ by symmetry of bidding strategies. Therefore, her maximum willingness-to-pay is given by

$$\mathbb{E}[v(X_i, X_j) | X_i = x_i, X_j = x_i] - w_i(x_i) = 0 \quad \Leftrightarrow w_i(x_i) = 2x_i \equiv \bar{w}(x_i)$$

[14]See for instance Krishna (2009), pp. 281-284.

Thus, firms' equilibrium bids are indeed bounded above by $\bar{w}(x_i) = 2x_i \ \forall x_i \in [0,1]$, $i = 1,2$.

Next, for given firms' equilibrium strategies $\beta_e(x_i)$, let us define $G(\lambda)$ as the troll's expected payoff upon winning when bidding any $b > 0$. Since the troll wins if $b > \beta_e(Y_1) \Leftrightarrow Y_1 < \phi(b)$, we have

$$\begin{aligned} G(\lambda) &\triangleq \mathbb{E}\left[((1+\lambda)(Y_1+Y_2) - \beta_e(Y_1))\,\mathbb{1}_{y_1 \le \phi(b)}\right] \\ &= 2\int_0^{\phi(b)}\int_0^{y_1}[(1+\lambda)(y_1+y_2) - \beta_e(y_1)]\mathrm{d}y_2\mathrm{d}y_1 \end{aligned} \tag{1.1}$$

Consider first the case where firms' equilibrium strategy is such that $x_i \le \beta_e(x_i) \le \frac{3}{2}x_i$ for any $x_i \in [0,1]$, $i = 1,2$, where the first inequality comes from the fact that we focus on equilibria in undominated strategies. The troll's expected payoff upon winning when bidding $b > 0$ is then

$$\begin{aligned} 2\int_0^{\phi(b)}\int_0^{y_1}[(1+\lambda)(y_1+y_2) - \beta_e(y_1)]\mathrm{d}y_2\mathrm{d}y_1 &\ge 2\int_0^{\phi(b)}\int_0^{y_1}[(1+\lambda)(y_1+y_2) - \frac{3}{2}y_1]\mathrm{d}y_2\mathrm{d}y_1 \\ &= \lambda\,[\phi(b)]^3 \\ &\ge 0 \quad \forall \lambda \in [0,1] \end{aligned}$$

Hence, the troll always prefers to win for any $\lambda \in [0,1]$ whenever firms' equilibrium strategy lies in $[x_i, \frac{3}{2}x_i]$ for all $x_i \in [0,1]$, $i = 1,2$, so that bidding aggressively is optimal as it ensures winning with probability one. In particular, $b = 2(1+\lambda)$ is a best response.

Consider now the case where $\frac{3}{2}x_i < \beta_e(x_i) \le 2x_i$. First, we show that winning is no longer profitable for the troll whenever his private-value advantage λ falls below a cutoff $\hat{\lambda} \in (0,1)$ as defined hereafter. From Eq. (1.1), observe that

$$G(0) = \mathbb{E}\left[(Y_1+Y_2-\beta_e(Y_1))\,\mathbb{1}_{y_1\le\phi(b)}\right] < \mathbb{E}\left[\left(Y_1+Y_2-\frac{3}{2}Y_1\right)\mathbb{1}_{y_1\le\phi(b)}\right] = 0$$

and

$$
\begin{aligned}
G(1) = \mathbb{E}\left[(2(Y_1+Y_2)-\beta_e(Y_1))\,\mathbb{1}_{y_1\leq\phi(b)}\right] &\geq \mathbb{E}\left[(2(Y_1+Y_2)-2Y_1)\,\mathbb{1}_{y_1\leq\phi(b)}\right] \\
&= 2\mathbb{E}\left[Y_2\mathbb{1}_{y_1\leq\phi(b)}\right] \\
&= 2\int_0^{\phi(b)}\int_0^{y_1} 2y_2 \mathrm{d}y_2 \mathrm{d}y_1 \\
&= \frac{2}{3}\left[\phi(b)\right]^3 > 0
\end{aligned}
$$

Moreover, we have that $G'(\lambda) = \mathbb{E}\left[Y_1+Y_2|Y_1<\phi(b)\right] > 0$ for any $\phi(b) > 0$. Together with the fact that, since ϕ is continuous, G is continuous, there exists a unique $\hat{\lambda} \in (0,1)$ such that $G(\hat{\lambda}) = 0$. Therefore, it follows that $G(\lambda) \geq 0$ for any $\lambda \in [\hat{\lambda}, 1]$ so that bidding aggressively is optimal for the troll since winning is always profitable whenever his private-value advantage exceeds the cutoff $\hat{\lambda}$. In particular, submitting $b = 2(1+\lambda)$ constitutes a best response. Instead, since $G(\lambda) < 0$ for all $\lambda \in [0, \hat{\lambda})$, the troll strictly prefers to lose the auction which is ensured by bidding zero.

(ii) We first show that if the troll pledges zero in equilibrium, then firms' best response is to bid twice their signal. To see this, consider, say, firm i and suppose that firm $j \neq i$ pledges $\beta_e(x_j) = 2x_j$. By playing $\beta_e(x_i) = 2x_i$, firm i wins if $x_i > x_j$. She gets $x_i + x_j - 2x_j = x_i - x_j > 0$ which ensures that she does not regret winning, and given the auction format, submitting a higher bid does not improve her payoff. Instead, if $x_i < x_j$, then she loses and does not suffer from ex-post regret either as $x_i + x_j - 2x_j = x_i - x_j < 0$. Hence, playing $\beta_e(x_i) = 2x_i$ for all $x_i \in [0,1]$, $i = 1,2$ is optimal and satisfies the no ex-post regret property. Finally, suppose that the troll instead bids $b = 2(1+\lambda)$ in equilibrium so that he always wins. Firms do not regret losing since $x_i + x_j < 2 \leq b$, which completes the proof of the second part.

(iii) Let R^T and R^0 denote respectively the seller's expected revenue with and without the troll's participation in the auction. If the troll does not participate in the auction, then in a symmetric equilibrium, firms play $\beta_e(x_i) = 2x_i$ as derived by Milgrom (1981), in which case the seller's expected revenue is $\mathbb{E}(R^0) = 2\mathbb{E}[Y_2] = \frac{2}{3}$. Clearly, if the troll bids zero in equilibrium, we have that $\mathbb{E}(R^T) = \mathbb{E}(R^0)$ from part (ii). Instead, if the troll bids $b = 2(1+\lambda)$, the seller's expected revenue is then given by

$$\mathbb{E}(R^T) = \mathbb{E}[\beta_e(Y_1)] \geq \mathbb{E}[Y_1] = \frac{2}{3} = 2\mathbb{E}[Y_2] = \mathbb{E}(R^0)$$

where the first inequality follows from the fact that we discard equilibria involving the use of weakly dominated strategies. $\square$

Proof of Proposition 1.1

Let us first consider the troll and suppose that firms play according to b. By bidding b_T, the troll always wins and gets

$$\int_0^1 \int_0^{y_1} 2\left[(1+\lambda)(y_1+y_2) - \alpha y_1\right] \mathrm{d}y_2 \mathrm{d}y_1 = 1 + \lambda - \frac{2}{3}\alpha \geq 0 \quad \forall \lambda \in [0,1],\ \forall \alpha \in [1, \frac{3}{2}]$$

Given the auction format, submitting a higher bid does not improve his payoff, while lowering his bid triggers a positive probability to lose the auction. Hence, bidding aggressively ensures that the troll outbids firms and in particular, $b_T = 2(1+\lambda)$, constitutes a best response.

We now turn to (say) firm i and show that bidding $b(x_i)$ is a best response and satisfies the no ex-post regret property. Suppose that the troll and firm $j \neq i$ play according to the aforementionned strategies. By pledging $b(x_i)$, firm i always loses the auction since $b(x_i) < b_T$, and gets zero payoff. She does not suffer from ex-post regret since winning would instead yield $x_1 + x_2 - b_T = x_1 + x_2 - 2(1+\lambda) \leq 0 \ \ \forall \lambda \in [0,1],\ \forall \boldsymbol{x} \in [0,1]^2$. Obviously, either lowering her bid or bidding any $a_i \in (b(x_i), b_T)$ does not change the outcome or her payoff. Since firms' strategies are symmetric, a similar argument holds for firm j, $j \neq i$.

Finally, note that these equilibrium strategies are undominated since $b_T = 2(1+\lambda) = V_T(\mathbf{1})$ and $V_i(x_i, 0) = x_i \leq b(x_i) < x_i + 1 = V_i(x_i, 1)$ for all $x_i \in [0,1]$, $i = 1, 2$. $\square$

Proof of Proposition 1.2

Consider first the troll, and suppose that firms play linear strategies of the form $b(x_i) = kx_i,\ k \geq 1$, for any $x_i \in [0,1]$, $i = 1, 2$. Following the proof of Theorem 1.1, the troll's expected payoff when bidding an amount $b > 0$ is then

$$2\int_0^{\frac{b}{k}}\int_0^{y_1}\left[(1+\lambda)(y_1+y_2)-ky_1\right]\mathrm{d}y_2\mathrm{d}y_1$$

Simplifying, the troll's problem is therefore given by

$$\max_{b\,\in\,\mathcal{A}_T}\Pi_T(b)=\left[\frac{3}{2}(1+\lambda)-k\right]\frac{b^3}{3k^3}$$

Differentiating the objective with respect to b yields

$$\frac{d\Pi_T}{db}=\left[\frac{3}{2}(1+\lambda)-k\right]\frac{b^2}{k^3}\gtreqless 0\;\Leftrightarrow\;\lambda\gtreqless\frac{2k-3}{3}$$

Therefore, we have that if $k=2$, then $\frac{\partial\Pi_T}{\partial b}<0$ for all $\lambda<\frac{1}{3}$ so that playing $b_T=0$ is optimal. Instead, if $k\in[1,2]$, then $\frac{\partial\Pi_T}{\partial b}\geq 0$ for any $\lambda\geq\frac{1}{3}$ so that playing $b_T=2(1+\lambda)$ is optimal.

We now turn to, say, firm i and suppose that the troll and firm $j\neq i$ play the proposed strategies. If $\lambda<1/3$, then by bidding $b(x_i)=2x_i$, firm i wins against firm j if $x_i>x_j$ and gets $x_i+x_j-2x_j=x_i-x_j>0$, which ensures that she does not regret winning, and given the auction format, increasing her bid does not improve her ex-post payoff. If $\lambda\geq 1/3$, then firm i always loses when bidding $b(x_i)$ and gets a zero ex-post payoff. She does not regret losing as $x_i+x_j\leq 2<2(1+\lambda)=b_T$. By symmetry, a similar argument holds for firm j. Finally, since $b_T\in\mathcal{A}_T$ and $b(x_i)\in\mathcal{A}_i$ for all $i=1,2$, the equilibrium does not involve the use of weakly dominated strategies. $\square$

Proof of Lemma 1.2

Suppose first that $\lambda\geq\frac{1}{3}$. Then the troll always outbids firms and his ex-post payoff upon winning is

$$(1+\lambda)(x_1+x_2)-2x_1\leq 0\Leftrightarrow\lambda\leq\frac{x_1-x_2}{x_1+x_2}$$

Thus, the troll suffers from ex-post regret upon winning whenever $\lambda\in[\frac{1}{3},\frac{x_1-x_2}{x_1+x_2}]$. Similarly, suppose that $\lambda<1/3$ so that the troll loses the auction. His ex-post payoff is then

$$(1+\lambda)(x_1+x_2)-2x_1\geq 0\Leftrightarrow\lambda\geq\frac{x_1-x_2}{x_1+x_2}$$

so that he suffers from ex-post regret upon losing for any $\lambda \in \left[\frac{x_1-x_2}{x_1+x_2}, \frac{1}{3}\right)$. $\square$

Proof of Theorem 1.2

Assume not. Since from Theorem 1.1, the troll bids either zero or $b_T = 2(1+\lambda)$ in any equilibrium in which firms employ symmetric strategies, it is sufficient to restrict our attention to the case where the troll's equilibrium bid is $b_T = 0$. From the proof of Theorem 1, firms' best-response is then to pledge $b(x_i) = 2x_i$, which ensures that they do not suffer from ex-post regret regardless of the outcome of the auction. Thus, for $(b_T, b(x_1), b(x_2))$ to form an ex-post equilibrium, it must be that the troll does not suffer from ex-post regret upon losing. That is, the following inequality must hold for *any* pair $(x_1, x_2) \in [0,1]^2$

$$0 \geq (1+\lambda)(x_1 + x_2) - 2x_1 \tag{1.2}$$

But notice that for $x_2 \to x_1$, then the right-hand-side of Eq. (1.2) goes to $2(1+\lambda)x_1 - 2x_1 > 0$ for any $\lambda > 0$, a contradiction. $\square$

Proof of Proposition 1.3

($\Rightarrow$) Suppose that the strategies b_T and $b(x_i)$ constitute an ex-post equilibrium. Then, it must be that

$$u_T(b_T, b(x_1), b(x_2), \boldsymbol{x}) = (1+\lambda)(x_1+x_2) - \gamma.\max\{x_1, x_2\} \geq u_T(a_T, b(x_1), b(x_2), \boldsymbol{x}) \tag{1.3}$$

for all $\boldsymbol{x} \in [0,1]^2$ and all $a_T \in \mathcal{A}_T = [0, 2(1+\lambda)]$. Suppose that the troll instead bids any $a_T < 2(1+\lambda)$. Given the auction format, the troll's payoff only changes if $a_T < \gamma.\max\{x_1, x_2\}$, in which case the troll loses the auction and gets zero payoff. Suppose w.l.o.g. that $x_1 \geq x_2$, we have that:

$$\begin{aligned}
(1+\lambda)(x_1+x_2) - \gamma.\max\{x_1, x_2\} &= (1+\lambda)(x_1+x_2) - \gamma x_1 \geq 0 \quad \forall \boldsymbol{x} \in [0,1]^2 \\
&\Rightarrow\ (1+\lambda)x_1 \geq \gamma x_1 \quad x_1 \in [0,1] \\
&\Leftrightarrow\ \lambda \geq \gamma - 1 \equiv \underline{\lambda}
\end{aligned}$$

($\Leftarrow$) Suppose that $\lambda \geq \gamma - 1$. We now establish that the proposed strategies constitute an ex-post equilibrium. By bidding $b_T = 2(1+\lambda)$, the troll always wins and gets $(1+\lambda)(x_1+x_2) - \gamma.\max\{x_1, x_2\} \geq \gamma(x_1+x_2) - \gamma.\max\{x_1, x_2\} \geq 0$ for any $\boldsymbol{x} \in [0,1]^2$, which ensures that he does not regret winning. Since the price he pays upon winning is the second highest bid, increasing his bid does not improve his payoff, while a lower bid triggers losing the auction resulting in a zero ex-post payoff. Likewise, firms do not regret losing since outbidding the troll would lead to $x_1 + x_2 - b_T \leq x_1 + x_2 - 2 \leq 0 \quad \forall \boldsymbol{x} \in [0,1]^2$. Finally, note that $b_T = 2(1+\lambda) = V_T(\mathbf{1})$ and $V_i(x_i, 0) = x_i \leq b(x_i) \leq x_i + 1 = V_i(x_i, 1)$ for all $x_i \in [0,1]$, $i = 1, 2$, which ensures that the equilibrium strategies are undominated. $\square$

Proof of Lemma 1.3

The proof closely follows that of Lemma 1.1. We first establish that contributing $s(x_1) = x_1$ is a best response for the firm. Observe that she always loses against the troll as $s(x_1) = x_1 \leq 1 < b_T$ and therefore gets a strictly negative ex-post payoff $-t$. She does not regret losing since outbidding the troll would instead yield $x_1 - b_T - t < x_1 - 1 - t < 0$. Pledging instead any $s' < x_1$ does not alter the outcome of the auction or her payoff. Hence, $s(x_1) = x_1$ is indeed a best response. Next, we show that bidding any $b_T \in (1, (1+\lambda)(1+\hat{x})]$ is a best response for the troll. By playing b_T, the troll always wins against the firm as $s(x_1) = x_1 \leq 1 < b_T$, and does not regret winning since he gets $(1+\lambda)(x_1+x_2) - x_1 \geq 0$ for any $\lambda \in [0,1]$. Given the auction format, submitting a higher bid does not improve is payoff upon winning. Thus, bidding any $b_T \in (1, (1+\lambda)(1+\hat{x})]$ constitutes a best response for the troll. $\square$

Proof of Theorem 1.3

Towards a contradiction, assume first that there exists a profile of strategies $(s^e(x_1), s^e(x_2), b_T^e)$ that constitutes an ex-post equilibrium with $s^e(x_1) + s^e(x_2) > b_T^e > 0$. Consider, say, firm 1. By slightly lowering her contribution to $s_1' = s^e(x_1) - \epsilon$, with $\epsilon > 0$ such that $s_1' + s^e(x_2) \geq b_T^e$, she is strictly better off as

$$x_1 - \frac{s_1'.b_T^e}{s_1' + s^e(x_2)} - t > x_1 - \frac{s^e(x_1).b_T^e}{s^e(x_1) + s^e(x_2)} - t$$

$$\Leftrightarrow s^e(x_1)(s_1' + s^e(x_2)) > s_1'(s^e(x_1) + s^e(x_2))$$

$$\Leftrightarrow s^e(x_1) > s_1' = s^e(x_1) - \epsilon$$

Clearly, she finds it profitable to do so until the sum of contributions meets the troll's equilibrium bid, i.e. up to the point where $\underline{s}_1 + s^e(x_2) = b_T^e$. Tying with the troll then leads to

$$x_1 - \frac{\underline{s}_1 b_T^e}{\underline{s}_1 + s^e(x_2)} - t = x_1 - \frac{\underline{s}_1 b_T^e}{b_T^e} - t = x_1 - \underline{s}_1 - t$$

Two cases need to be considered: (a) if $x_1 - t \geq \underline{s}_1$, then firm 1 is strictly better off by deviating to $\underline{s}_1$, contradicting an equilibrium in symmetric strategies; (b) if $x_1 - t < \underline{s}_1$, then firm 1 strictly prefers to further reduce her contribution so that the troll wins the auction, a contradiction.

Consider now the case where $(s^e(x_1), s^e(x_2), b_T^e)$ forms an ex-post equilibrium with $s^e(x_1) + s^e(x_2) = b_T^e > 0$. By definition, it must be that all players are ex-post indifferent between winning and losing, i.e. $(s^e(x_1), s^e(x_2))$ must satisfy

$$\begin{cases} (1+\lambda)(x_1 + x_2) = s^e(x_1) + s^e(x_2) \\ x_i - t = s^e(x_i) \end{cases} \qquad \forall \boldsymbol{x} \in [0,1]^2,\ i = 1,2$$

Clearly, these two equalities are mutually exclusive. Hence, there is no pair $(s^e(x_1), s^e(x_2))$ such that tying with the troll at a positive price constitutes an ex-post equilibrium. This last argument completes the proof. □

Proof of Proposition 1.4

Suppose that the profile of strategies $(s(x_1), s(x_2), b_T)$ constitutes an ex-post equilibrium of the continuation game Γ_2 for which the intermediary wins the auction. From Theorem 1.3, it must be that $b_T = 0$. By contradiction, assume that excessive contributions are not refunded. Then, in equilibrium, it must be that firms pledge at most their signal, i.e. $s(x_i) \leq x_i$, as any higher contribution $s_i' > x_i$ yields $x_i - s_i' < 0$, making $s_i = 0$ a strictly profitable deviation. By definition, one must have that the troll does not regret bidding zero, that is, the following must hold for any pair $(x_1, x_2) \in [0,1]^2$

$$0 \geq (1+\lambda)(x_1 + x_2) - (s(x_1) + s(x_2))$$

which contradicts $s(x_i) \leq x_i$. Hence, the troll strictly prefers to win the auction so that pledging any $b'_T > 2 \geq x_1 + x_2$ is a profitable deviation, a contradiction.

We now establish that the proposed strategies constitute an ex-post equilibrium of Γ_2. Suppose that firms play according to s^a. By bidding $b^a_T = 0$, the troll always loses and gets

$$0 > (1+\lambda)(x_1 + x_2) - (s^a(x_1) + s^a(x_2))$$

which ensures that he does not regret losing. Consider now firm i, and assume that the troll and firm $j \neq i$ play the aforementioned strategies. By pledging $s^a(x_i)$, the intermediary always wins and firm i gets $x_i - t \geq -t$ so that she does not suffer from ex-post regret. Lowering her contribution to some s'_i only changes the outcome if both $s'_i = 0$ and $x_j = 0$ in which case she gets $-t < 0$. Hence, pledging $s^a(x_i)$ indeed constitutes a best-response for firm i, $i = 1, 2$. $\square$

Proof of Proposition 1.5

Recall that the intermediary's objective writes $\Pi_I(\hat{x}) = 2\hat{x}(1-\hat{x})^2$. We first show that $\Pi_I(\hat{x})$ is strictly quasi-convave in $\hat{x}$. To this end, it is sufficient to show that the second derivative of $\Pi_I(\hat{x})$ is strictly negative whenever the first derivative equals zero.

$$\Pi_I^{'}(\hat{x}) = 2(1-\hat{x})(1-3\hat{x})$$
$$\Pi_I^{''}(\hat{x}) = -8 + 12\hat{x}$$

Observe that the first derivative vanishes at $\hat{x} = \frac{1}{3}$ and $\hat{x} = 1$. However, $\hat{x} = 1$ is not optimal since $\Pi_I(1) = 0$. Since $\Pi_I^{''}(\frac{1}{3}) = -4 < 0$, $\Pi_I(\hat{x})$ is strictly quasi-concave and admits a unique interior argmax $\hat{x}^* = \frac{1}{3}$. Since firms' participation constraint binds at $x_i = \hat{x}$ for all $i = 1, 2$, we have that $t^* = (1-\hat{x}^*)\hat{x}^* = \frac{2}{9}$ and $\Pi_I(\hat{x}^*) = \frac{8}{27} > 0$. The proof of the following equivalences

$$\begin{cases} a_i^* = A & \Leftrightarrow x_i \in [\frac{1}{3}, 1] \\ a_i^* = R & \Leftrightarrow x_i \in [0, \frac{1}{3}) \end{cases} \quad \text{for all } i = 1, 2$$

directly follows from Definition 1.5 and the forward induction criterion, while the proof of the second part directly follows from the proofs of Theorem 1.2, Lemma 1.3 and Proposition 1.4. $\square$

Proof of Corollary 1.

From the proof of Theorem 1.1, we have that $\mathbb{E}(R^0) = \frac{2}{3}$. When the troll participates in the auction, the seller instead gets

$$\mathbb{E}(R^T) = \gamma\mathbb{E}(Y_1) = \gamma.\frac{2}{3} \in \left[\frac{2}{3}, \frac{4}{3}\right]$$

In turn, the seller's expected revenue in the intermediated auction, $\mathbb{E}[R^I]$, is given by

$$\begin{aligned}\mathbb{E}[R^I] =& \Pr\left[X_1 \geq \hat{x}, X_2 \geq \hat{x}\right].0 + \Pr\left\{\left[(X_1 > \hat{x}) \cap (X_2 \leq \hat{x})\right] \cup \left[(X_1 \leq \hat{x}) \cap (X_2 > \hat{x})\right]\right\}.\mathbb{E}\left[Y_1 | Y_1 \geq \hat{x}, Y_2 < \hat{x}\right] \\ &+ \Pr\left[X_1 < \hat{x}, X_2 < \hat{x}\right].\gamma\mathbb{E}\left[Y_1 | Y_1 < \hat{x}\right] \\ =& 2\hat{x}(1-\hat{x})\mathbb{E}\left[Y_1 | Y_1 \geq \hat{x}, Y_2 < \hat{x}\right] + \hat{x}^2\gamma\mathbb{E}\left[Y_1 | Y_1 < \hat{x}\right]\end{aligned}$$

Letting $F_{1,2}(y_1, y_2)$ denote the joint distribution of (Y_1, Y_2), we have that

$$\begin{aligned}\Pr\left[(Y_1, Y_2) \in [\hat{x}, 1] \times [0, \hat{x})\right] &= F_{1,2}(1, \hat{x}) - F_{1,2}(\hat{x}, \hat{x}) - F_{1,2}(1, 0) + F_{1,2}(\hat{x}, 0) \\ &= \int_0^1 \int_0^{\hat{x}} 2\mathrm{d}y_2\mathrm{d}y_1 - \int_0^{\hat{x}} \int_0^{\hat{x}} 2\mathrm{d}y_2\mathrm{d}y_1 - \int_0^1 \int_0^0 2\mathrm{d}y_2\mathrm{d}y_1 + \int_0^{\hat{x}} \int_0^0 2\mathrm{d}y_2\mathrm{d}y_1 \\ &= 2\hat{x}(1-\hat{x}) \\ &= \Pr\left\{\left[(X_1 > \hat{x}) \cap (X_2 \leq \hat{x})\right] \cup \left[(X_1 \leq \hat{x}) \cap (X_2 > \hat{x})\right]\right\}\end{aligned}$$

Thus, the density of Y_1 conditional on the event $(Y_1, Y_2) \in [\hat{x}, 1] \times [0, \hat{x})$ is

$$f_1\left(y_1 | (Y_1, Y_2) \in [\hat{x}, 1] \times [0, \hat{x})\right) = \frac{2y_1}{2\hat{x}(1-\hat{x})}\mathbb{1}_{y_1 \geq \hat{x}}$$

which leads to

$$\mathbb{E}\left[Y_1 | Y_1 \geq \hat{x}, Y_2 < \hat{x}\right] = \int_{\hat{x}}^1 \frac{y_1^2}{\hat{x}(1-\hat{x})}\mathrm{d}y_1$$

Similarly, from the proof of Proposition 1.1, we have that $\mathbb{E}\left[Y_1 | Y_1 < \hat{x}\right] = \int_0^{\hat{x}} \frac{2y_1^2}{\hat{x}^2}\mathrm{d}y_1$. Plugging these into the seller's expected revenue and rearranging yields

$$\mathbb{E}[R^I] = \int_{\hat{x}}^1 2y_1^2\mathrm{d}y_1 + \gamma\int_0^{\hat{x}} 2y_1^2\mathrm{d}y_1 = \frac{2}{3}\left(1 + \hat{x}^3(\gamma - 1)\right)$$

Since $\gamma \in [1, 2]$, evaluating at $\hat{x}^* = \frac{1}{3}$ finally gives us

$$\mathbb{E}[R^I] = \frac{2}{3}.\left(\frac{26+\gamma}{27}\right) \in \left[\frac{2}{3}, \frac{56}{81}\right]$$

Hence, we indeed have that $\mathbb{E}(R^T) \geq \mathbb{E}(R^I) \geq \mathbb{E}(R^0)$. $\square$

Chapter 2

Preemptive Intermediaries in the Market for Patents

English summary

We theoretically and empirically examine business models of "patent preemption entities" (PPEs), a type of non-practicing entity in markets for technology. PPEs specialize in the preemptive acquisition of patents that could legally threaten their clients that subscribe to their services, thereby preventing the risk of litigation brought by patent assertion entities (PAEs). We develop a theoretical model where a PPE seeks to attract clients before competing against a PAE in an auction for patent buyout. We show that PPEs can establish a profitable business model by restricting their protection to the most threatening patents and to the most highly exposed firms, while maintaining a credible threat of litigation against other potential infringers. Using patent reassignment and litigation data, we then provide evidence supporting the model's predictions.

Résumé français

Nous examinons théoriquement et empiriquement les modèles d'affaire des "entités de préemption de brevets" (EPB), un type d'entreprise non productrice dans les marchés de technologie. Les EPB sont spécialisées dans l'acquisition de brevets pouvant représenter une menace légale pour leurs clients, anéantissant ainsi tout risque de litiges initiés par les entités de revendication de brevets (ERB). Nous développons un modèle théorique dans lequel une EPB cherche à attirer des clients avant d'entrer en concurrence avec une ERB dans une enchère de brevet. Nous montrons que les EPB peuvent établir un modèle d'affaire profitable en restreignant leur offre de protection aux firmes les plus exposées contre les brevets les plus menaçants, ainsi qu'en maintenant une menace de litige crédible à l'encontre d'autres contrevenants présumés. Enfin, s'appuyant sur des données de transferts de brevets et d'actions en contrefaçon, nos résultats empiriques corroborent les prédictions du modèle.

2.1 Introduction

This paper examines the business model of patent preemption entities that have emerged as a competitive force in markets for technology. As the trade of patents has substantially increased since the strengthening of the patent rights in the United States (see Branstetter et al., 2006; Maskus, 2000), new types of intermediaries have emerged to exploit the legal and technological arbitrage opportunities (Hagiu & Yoffie, 2013; Delcamp and Leiponen, 2015). These non-practicing entities do not sell products or perform R&D. They identify and buy dormant, unutilized patents in order to license or sell (reassign) them to operating companies that develop and commercialize products using these technologies. There are three types of non-practicing entities. Patent assertion entities intend to profit from their acquired patents by legally asserting them against infringing operating companies. Patent marketplaces are pure intermediaries that enable patent trading without necessarily taking ownership of the portfolios for sale. Instead, patent preemption entities, the focus of this study, intend to profit from their acquired patents by licensing them to operating companies to defend against the attacks of patent assertion entities. Acting on behalf of their operating company clients, patent preemption entities attempt to acquire and license patents before patent assertion entities, also called patent trolls, get a hold of them.

The business model of patent assertion entities is relatively well understood: they identify patents that can later be asserted against operating companies in order to extract settlement fees or court-awarded damages. Similarly, patent market makers such as auction houses play a reasonably simple role in this market by facilitating reassignment. By contrast, the business model of patent preemption entities has not been analyzed in depth. Delcamp and Leiponen (2015) provide a descriptive analysis of their activities. The purpose of this paper is to model and empirically test the theoretical predictions regarding the interplay between patent assertion entities and patent preemption entities, thus highlighting the economic mechanisms underpinning the business model of patent preemption entities. After describing the differences between the two patent entities, we develop a theoretical model of patent acquisition explaining when either entity is likely to win a patent that is for sale. We then provide empirical evidence supporting our theoretical results. Our results suggest that preemption entities tend to restrict their protection to the most threatening patents and to the most highly exposed firms, while maintaining a credible threat of litigation against other potential infringers.

We consider a model of patent acquisition, whereby a preemption entity first attracts client firms that seek protection in a particular technology field and then competes against an assertion entity in an auction for a patent (or a patent portfolio) in that field. The patent threatens a continuum of operating firms with heterogeneous degrees of exposure to infringement suits in the field. Firms privately know their degree of exposure, while both non-practicing entities only know the distribution. The infringement fee that the patent holder can extract depends on the individual degree of exposure of operating firms and on the intrinsic value of the patent (e.g., its legal strength, or the value of the patented invention). While this value is known when the auction starts, it is still uncertain when operating firms must decide whether to become a client of the patent preemption entity.

The value of the patent for the patent assertion entity is based on the total expected infringement fees that it can collect, net of the cost of extracting these fees from each targeted infringer. We posit that this cost has both a fixed and variable part. The variable part captures the costs of bilateral litigation and/or negotiations with each targeted operating firm. It implies that a part of the fees eventually paid by infringers is dissipated in the enforcement process. The fixed part instead corresponds to the upfront cost of setting up an enforcement strategy, such as e.g. the cost of searching for potential infringers. It implies that the assertive entity cannot profitably enforce the patent if the intrinsic value fo the patent is too low.

The preemptive entity's bidding strategy is contingent on the contract established with operating firms in the first stage, when the value of the patent for sale was still uncertain. We consider a contract whereby, upon payment of a subscription fee, the preemption entity commits to protect client firms against litigation for patents within a particular range of intrinsic values. Importantly, subscription fees are not the only source of revenue for the preemption entity. If a subset of firms reject its offer, it also has the option to sue these non-client firms upon buying the patent - the so-called "Catch-and-Release" strategy.

We find that the catch-and-release strategy is a critical element in the PPE's business model. Without maintaining a credible threat of litigation against non-clients, operating firms would have an incentive to free ride on the preemptive entity's intervention: they are better off saving on the subscription fee while benefitting from the removal of the patent threat once the preemptive entity has purchased the patent. As a consequence, resorting to the catch-and-release strategy must be a credible threat, and the PPE cannot afford to preempt patents that it cannot profitably assert against non client firms afterwards. This imposes a joint restriction on the mass of client firms to target and on the set of patents that can be preempted. In this respect, we find that the PPE offers protection against a limited set of high value patents to a subset of highly exposed operating firms.

We build a database of 2,608 U.S. patents reassigned to preemption and assertion entities to empirically test the prediction that preemption entities are more likely to purchase patents that are of higher value in the technology fields where they are active. Using citation, litigation, and other patent information to identify the importance and strength of the patent, and the demand for the patent, our results are consistent with the model: preemption entities tend to acquire more valuable patents. Moreover, this result is stronger when the value coincides with a technical field of interest for preemption entities, as proxied by high exposure of their clients.

The paper is organized as follows. We next review the recent literature on assertive and preemptive entities. We introduce the theoretical model in Section 2.3. We present our empirical evidence in Section 2.4 and conclude in Section 2.5. Finally, proofs are relegated to Section 2.6.

2.2 NPEs in the market for patents: a review

2.2.1 Patent assertion entities

Patent assertion entities (PAEs) - NPEs with an explicit strategy to enforce patents through litigation - have been fast developing in the last decade. According to industry estimates, there were 550 IP lawsuits in the United States in 2010 against 3000 defendants, that is, over 2000 unique companies (some of which were sued more than once)[1]. Many of these legal cases were concentrated in the communication technology industry, particularly smartphones, and about 17% of lawsuits were brought by NPEs in 2008.

Steiner and Guth (2005) observe that PAEs often buy patents and then wait until the associated product market takes off. Then, PAEs are able to obtain compensations that are higher than what potential licensees would have been willing to pay ex ante (Reitzig et al., 2007). Having no R&D or production activity, PAEs are unexposed to patent suits, which deprives the defendant from wielding the threat of counter-suit as a bargaining argument (Shapiro, 2001;

[1]RPX 2011 Annual Report, retrieved from www.rpx.com on May 7, 2012.

Galasso, 2012). Levko et al. (2009) also suggest that PAEs differ from practicing entities in terms of litigation strategies. For instance, they tend to name multiple defendants to maximize settlement revenues and minimize legal costs. PAEs also seem to be less successful in their litigation than practicing entities (29% rate of success compared with 41% for practicing entities, ibid.).

There are relatively few and contrasted empirical studies of PAEs' acquisition and litigation strategies. Fischer and Henkel (2012) suggest that the probability that a traded patent is acquired by an NPE rather than a practicing entity (operating company) increases in the scope of the patent, in the patent density of its technology held, and in the patent's technological quality. A number of other empirical analyses confirm these findings (e.g., Shrestha, 2012; Risch, 2012). By contrast, a recent empirical study by Cohen et al. (2014) finds evidence that PAEs usually target firms that are flush with cash, irrespective of the closeness of those firms' patents to the PAEs'.

2.2.2 Patent preemption entities

Our main focus in this paper is on so-called patent preemption entities (PPEs), also known as defensive patent aggregators (McDonough, 2006; Wang, 2010; Hagiu and Yoffie, 2013). Their activity consists in acquiring patents so that they do not end up in the hands of parties that are likely to assert them. PPEs then provide freedom of operation and safety from litigation for their operating company members or partners.

The two most advanced PPEs are companies called RPX and AST (Allied Security Trust). Whereas certain PAEs, such as Intellectual Ventures or Mosaid, also provide comparable services, RPX and AST are the most purely preemptive in their stated objectives. Their stated foci are on pooling risks, costs, and transaction activities related to acquiring or licensing problematic patents in high-technology industries. For example, RPX may negotiate licenses with external PAEs to license or acquire their IP that is alleged to be infringed by RPX members. Thus, RPX pools the bargaining power of its members to obtain licenses to relevant IPRs. This may reduce the licensing or acquisition prices paid to IP sellers.

However, RPX and AST are structured rather differently from one another. AST is a non-profit company that attempts to return as much of the value back to its clients. In a stark contrast, RPX as a publicly-traded entity attempts to capture as much of the created value as possible and utilizes it to grow its businesses and at the same time generate reasonable benefits for members. AST engages in patent acquisition based on (confidential) ex-ante bids by its individual members. Meanwhile, RPX also pools information from its clients regarding patent threats and litigation exposures, but its clients do not necessarily need to commit to bidding ex-ante. AST buys, licenses, and sells patents but does not enforce them directly. In contrast, and aligned with PAEs, RPX aggregates and enforces patents, but is committed to litigation only indirectly through holding companies.

2.3 A model of patent acquisition

2.3.1 Setup

We consider a continuum of mass one of operating firms exposed to patent infringement in a given technology field. Firms are heterogeneous in terms of their exposure to patents in the field, denoted by θ, which is distributed according to the twice differentiable distribution F over the unit interval. Throughout, we assume that the probability density function $f = F'$ is strictly positive everywhere and the following:

(A1) The probability density function f is logconcave on $[0, 1]$.

Firms expect that a patent assertion entity (henceforth, PAE) might acquire and assert a patent against them, whereby a type θ firm has to pay a damage fee equal to $\theta\pi$, where $\pi \in [0, 1]$ denotes the intrinsic value of the patent. At the beginning of the game, the value of the patent for sale, π, is unknown to all players. Rather, they all share the same prior that π is distributed according to the twice differentiable distribution G over [0,1], with strictly positive density $g = G'$.

Prior to the patent being for sale in the market, firms can subscribe to a patent preemption entity (henceforth, PPE). In exchange for an up-front subscription fee, the PPE contractually commits to protecting clients against litigation brought by the PAE for a range of patent values. We posit that firms privately know their degree of exposure to the patent, while both NPEs only know the distribution F. It directly follows that the PPE cannot discriminate among firms as proposing a menu of type-contingent subscription fees is not incentive compatible: it fails to induce firms to select properly within this menu. Put differently, the optimal incentive feasible contract bunches types. In particular, proposing a contract targeting the whole set of types yields zero profit to the PPE.

We shall therefore restrict our attention to a contract with shut-down of a subset of types. Namely, the PPE chooses a threshold type $\hat{\theta} \in (0, 1]$ and proposes a contract that consists of:

(i) a patent value threshold $\hat{\pi}$ such that it commits to preempt the patent for sale whenever its realized value π^r satisfies $\pi^r \geq \hat{\pi}$, and provides freedom to operate for its clients,

(ii) and a uniform non-negative type-contingent subscription fee $T \geq 0$, i.e., $T(\theta) = T(\theta')$ $\forall\,(\theta, \theta') \in [\hat{\theta}, 1]^2$.

Furthermore, we adopt the usual assumption that the contract is perfectly enforceable, which notably presupposes that there is a penalty, denoted by ρ, to be incurred by the PPE upon breaching the contract. That is, if the value of the patent for sale π^r exceeds the contractually defined threshold $\hat{\pi}$, then the PPE incurs ρ whenever losing the auction.

The patent is sold through a second-price sealed-bid auction (without reserve price) between the PAE and the PPE, where it is assigned to the highest bidder who pays the second highest bid. In case of a tie, we assume that the patent is reassigned to either NPE with equal probability. Importantly, we asssume that the *realized* value π^r of the patent for sale becomes common knowledge before the auction starts, but after contracting takes place.

The PAE's expected benefit of buying the patent is to assert it against operating firms so as to collect damage fees. The cost of enforcing the patent consists of a fixed cost $C \in (0, 1)$ of setting up a licensing program for the patent, plus a fraction $(1 - \alpha)$, with $\alpha \in (0, 1)$, of the total damages $\theta\pi$ extracted from each exposed firm that is dissipated through litigation and transaction costs. Upon acquiring the patent, the PAE's expected payoff is therefore equal to

$$\alpha\pi^r \int_0^1 \theta f(\theta)\,\mathrm{d}\theta - C$$

which is positive if and only if

$$\pi^r \geq \frac{C}{\alpha \int_0^1 \theta f(\theta)\,\mathrm{d}\theta} \equiv \pi_0$$

That is, the PAE finds it profitable to enforce the patent if its value is not too low so that operating firms are not threatened by litigation when the patent for sale is of too limited value. In order to ensure that $\pi_0 \in (0, 1)$, we further assume that the parameters $(C, \alpha) \in (0, 1)^2$ satisfy the following

(A2) $0 < C \ll \alpha < 1$

Hence, the PAE's valuation for the patent $v_A(\pi^r)$ is given by

$$v_A(\pi^r) = \begin{cases} \alpha\pi^r \int_0^1 \theta f(\theta)\,\mathrm{d}\theta - C & \text{if } \pi^r \geq \pi_0 \\ 0 & \text{otherwise} \end{cases}$$

Finally, the game unfolds as follows.

t=1 The PPE proposes a contract $(\hat{\pi}, T)$ to firms which provides protection against a contractually defined subset of potentially threatening patents.

t=2 Operating firms either accept or reject the PPE's offer[2].

t=3 The value of the patent for sale is drawn by Nature from the distribution G over the unit interval, and it becomes common knowledge among players.

t=4 The PPE and the PAE compete in an auction to purchase the patent, and the winner may enforce its rights.

2.3.2 Auction for patent buyout

Let us first characterize the equilibrium of the auction for patent buyout, for a given patent value threshold $\hat{\pi} \in (0, 1)$. Throughout, we focus on equilibria in which the PAE's bidding strategy is undominated.

Prior to formulating their bids, both NPEs learn the realized value of the patent for sale π^r. Given its contractual commitment to preempt patents above the value threshold $\hat{\pi}$, the PPE must bid aggressively so as to outbid the PAE whenever the value of the patent for sale is such that $\pi^r \geq \hat{\pi}$. An obvious way to fulfill this commitment is to bid according to $\bar{b}_P > v_A(\pi^r)$, that is, strictly above the PAE's valuation for the patent, whenever $\pi^r \in [\hat{\pi}, 1]$. Indeed, the PAE then strictly prefers to lose the auction since it would otherwise get $u_A = v_A(\pi^r) - \bar{b}_P < 0$ upon winning. However, the PPE's incentives to honor its contract might be overturned if the penalty to be incurred upon breaching the contract, ρ, is too low. Namely, letting v_P^1 denote the PPE's *gross* payoff derived from subscription fees, and $v_P^2 \in [0, v_A(\pi^r))$ denote the PPE's

[2] Throughout, we adopt the conventional assumption that, when indifferent, firms accept the offer.

expected gain from asserting the patent against non-clients, breaching the contract is strictly preferred if and only if

$$v_P^1 - \rho > v_P^1 + v_P^2 - b_A(\pi^r) \quad \forall \pi^r \geq \hat{\pi}$$

where $b_A(\pi^r)$ denotes the PAE's equilibrium bid. Therefore, we need the following assumption to ensure that the PPE's penalty is high enough so that deviating from its contractual commitment is strictly unprofitable.

(A3) $\rho > b_A(\pi^r) - v_P^2$

Conversely, whenever the patent value π^r lies below the threshold $\hat{\pi}$, then the PPE always prefers to lose the auction since it gets $v_P^1 \geq v_P^1 + v_P^2 - v_A(\pi^r)$. In this respect, pledging any $\underline{b}_P < v_A(\pi^r)$ ensures such an outcome since the PAE then strictly prefers to win as it gets $u_A = v_A(\pi^r) - \underline{b}_P > 0$. The next result characterizes the equilibrium of the patent auction which honors the PPE's contract[3].

Proposition 2.1. *If (A3) holds, then the following bidding strategies constitute an equilibrium of the patent auction:*

$$b_P^*(\pi^r) = \begin{cases} \bar{b}_P & \text{if } \pi^r \in [\hat{\pi}, 1] \\ \underline{b}_P & \text{if } \pi^r \in [\pi_0, \hat{\pi}) \\ 0 & \text{if } \pi^r \in [0, \pi_0) \end{cases}, \quad b_A^*(\pi^r) = v_A(\pi^r) = \begin{cases} \alpha\pi^r \int_0^1 \theta f(\theta)\, d\theta - C & \text{if } \pi^r \geq \pi_0 \\ 0 & \text{otherwise} \end{cases}$$

with $0 \leq \underline{b}_P < \alpha\pi^r \int_0^1 \theta f(\theta)\, d\theta - C < \bar{b}_P$.

When the value of the patent for sale is such that it is strictly unprofitable to litigate operating firms, that is when $\pi^r < \pi_0$, both NPEs bid zero since they cannot extract any revenue from

[3]The patent auction actually admits myriad of equilibria supporting the PPE's contractual commitment, provided that the penalty ρ is adjusted accordingly. Formally, one obtains a continuum of equilibria in which the PAE bids according to

$$\beta_A(\pi^r) = \begin{cases} \underline{b}_A & \text{if } \pi^r \in [\hat{\pi}, 1] \\ \bar{b}_A & \text{if } \pi^r \in [\pi_0, \hat{\pi}) \\ 0 & \text{if } \pi^r \in [0, \pi_0) \end{cases} \quad \text{with } 0 \leq \underline{b}_A \leq v_A(\pi^r) \leq \bar{b}_A$$

while the PPE's pledges $b_P(\pi^r)$ as specified in Proposition 1. Nevertheless, given the auction format, such equilibrium profiles of strategies are problematic in our context since they fail to provide us with a *unique* prediction of the PPE's expected cost of patent acquisition. Instead, refining the set of equilibria by discarding the use of weakly dominated strategies by the PAE overcomes this issue, which will prove useful when solving for the PPE's problem.

such patents and each of them has a probability of 1/2 to win. The PAE always preempts lower value patents such that enforcement is profitable, while the PPE instead always wins higher value patents, thereby fulfilling its contractual commitment.

2.3.3 The PPE's problem

We now turn to the characterization of the PPE's optimal contract. The PPE chooses a patent value threshold $\hat{\pi}$, a threshold type $\hat{\theta}$ and a subscription fee T that maximize its ex ante expected profit and satisfy firms' participation constraint for all types in $[\hat{\theta}, 1]$, and such that:

- types in $[\hat{\theta}, 1]$ accept the PPE's offer and are protected from litigation for any patent such that $\pi^r \geq \hat{\pi}$,
- types in $[0, \hat{\theta})$ do not subscribe and face litigation regardless of the patent reassignee's identity, provided that the value of the patent for sale satisfies $\pi^r \geq \pi_0$.

Moreover, the PPE may enforce the patent vis-à-vis non-client firms or, equivalently, sell the patent encumbered with a free license to its clients to a PAE - the so-called catch-and-release strategy. Whether the PPE can credibly use this strategy affects firms' incentives to accept its contract. Indeed, enforcing a patent requires incurring the fixed cost C, and may therefore not be profitable if damages can only be collected from the subset of non-clients. Formally, upon winning the auction, the PPE finds it profitable to assert its patent against non-clients if the expected damages it receives from the mass of firms comprised in $[0, \hat{\theta})$ outweigh the fixed cost of enforcement C. That is,

$$\alpha\pi \int_0^{\hat{\theta}} \theta f(\theta)\, \mathrm{d}\theta \geq C$$

Provided that $\hat{\theta} > 0$, this condition rewrites as

$$\pi \geq \frac{C}{\alpha \int_0^{\hat{\theta}} \theta f(\theta)\, \mathrm{d}\theta} \equiv \underline{\pi}(\hat{\theta}) \tag{CC}$$

Observe that $\underline{\pi}(\hat{\theta}) \geq \pi_0$ since $\hat{\theta} \leq 1$, meaning that the critical patent value so that enforcing the patent against non-clients becomes profitable exceeds that of the profitability to litigate *all* operating firms. Therefore, the catch-and-release strategy is profitable for the PPE if the value of the patent for sale is high enough to compensate for the lower expected damage fees it receives upon litigation. Hence, the use of this strategy is a credible threat if the PPE chooses $(\hat{\pi}, \hat{\theta})$ such that $\hat{\pi} \geq \underline{\pi}(\hat{\theta})$. In particular, notice that the critical patent value $\underline{\pi}(\hat{\theta})$ is decreasing with the threshold type $\hat{\theta}$. That is, the PPE faces a trade off between the subset of firms to attract and the range of patents that it commits to preempt in order to maintain a credible threat of litigation against non-clients.

Clearly, the PPE *never* offers to protect its clients against a range of patent values such that enforcement is not profitable. Indeed, by choosing $(\hat{\pi}, \hat{\theta})$ such that $\pi_0 \leq \hat{\pi} < \underline{\pi}(\hat{\theta})$, the PPE cannot profitably litigate non-clients when it preempts patents of intermediate value $\hat{\pi} \leq \pi^r < \underline{\pi}(\hat{\theta})$. In this case, any firm that did not subscribe to the PPE is thus de facto protected from the patent once the PPE has purchased it. Firms' incentives to accept the PPE's offer are therefore undermined as they can freely benefit from the PPE's intervention with probability $G(\underline{\pi}(\hat{\theta})) - G(\hat{\pi}) \geq 0$. Since the patent value is unknown when deciding whether to accept the PPE's offer, a type θ firm subscribes if the expected damages to be paid when not subscribing exceed its expected cost of subscribing, which consists of the subscription fee plus the expected damages to pay to the PAE when the latter wins the auction, i.e.

$$\theta \int_{\pi_0}^{\hat{\pi}} \pi g(\pi)\,\mathrm{d}\pi + \theta \int_{\underline{\pi}(\hat{\theta})}^{1} \pi g(\pi)\,\mathrm{d}\pi \geq T + \theta \int_{\pi_0}^{\hat{\pi}} \pi g(\pi)\,\mathrm{d}\pi$$

$$\Leftrightarrow T \leq \theta \int_{\underline{\pi}(\hat{\theta})}^{1} \pi g(\pi)\,\mathrm{d}\pi \quad \forall \theta \in [\hat{\theta}, 1] \tag{2.1}$$

Notice that firms' maximum willingness to pay for the PPE's services, as given by the right-hand side of (2.1), does not depend on the threshold patent value $\hat{\pi}$ since litigation brought by the PPE is not a credible threat for any patent such that $\pi^r \in [\hat{\pi}, \underline{\pi}(\hat{\theta}))$. In other words, highly exposed firms find it profitable to accept the PPE's offer if the subscription fee is lower than the expected damage fees to be paid to the PPE, provided that the patent can be profitably enforced.

Proposition 2.2. *It is not optimal for the PPE to commit to preempt patents that cannot be subsequently asserted against non-client firms.*

Intuitively, because of its contractual commitment, setting the patent value threshold $\hat{\pi}$ below the critical value to profitably enforce its rights obliges the PPE to bear the cost of preempting a larger set of patents without being able to profitably enforce all of them against non-clients. Hence, ex-ante, the PPE solves

$$\max_{\{\hat{\pi},\hat{\theta},T(.)\}} \underbrace{\left[1-F(\hat{\theta})\right].T}_{\text{Expected subscription fees}} + \underbrace{\int_{\hat{\pi}}^{1}\left[\alpha\pi\int_{0}^{\hat{\theta}}\theta f(\theta)\,\mathrm{d}\theta - C\right]g(\pi)\,\mathrm{d}\pi}_{\text{Expected revenue from catch-and-release}} - \underbrace{\int_{\hat{\pi}}^{1} v_A(\pi)\,g(\pi)\,\mathrm{d}\pi}_{\text{Expected price for the patent}}$$

subject to

$$T+\theta\int_{\pi_0}^{\hat{\pi}}\pi g(\pi)\,\mathrm{d}\pi \leq \theta\int_{\pi_0}^{1}\pi g(\pi)\mathrm{d}\pi \quad \forall\theta\in[\hat{\theta},1] \tag{PC}$$

$$\hat{\pi} \geq \underline{\pi}(\hat{\theta}) \tag{CC}$$

(PC) states that, in order to induce a type θ firm to accept its offer, the PPE's subscription fee plus the expected damages to be paid to the PAE if it wins the auction must be lower than the expected damages she has to pay when rejecting the PPE's offer. Straightforwardly, optimality requires that (PC) binds for the threshold type $\hat{\theta}$ so that the resulting tariff is given by

$$T^{*} = \hat{\theta}\int_{\hat{\pi}}^{1}\pi g(\pi)\mathrm{d}\pi \tag{2.2}$$

Moreover, since $\underline{\pi}(\hat{\theta}) \geq \pi_0$ for any $\hat{\theta} \leq 1$, the expected price to pay for preempting the patent can be rearranged as

$$\int_{\hat{\pi}}^{1} v_A(\pi)\,g(\pi)\,\mathrm{d}\pi = \underbrace{\int_{\hat{\pi}}^{1}\left[\alpha\pi\int_{\hat{\theta}}^{1}\theta f(\theta)\,\mathrm{d}\theta - C\right]g(\pi)\,\mathrm{d}\pi}_{\text{PAE's expected revenue from the PPE's clients}}$$

$$+\underbrace{\int_{\hat{\pi}}^{1}\left[\alpha\pi\int_{0}^{\hat{\theta}}\theta f(\theta)\,\mathrm{d}\theta - C\right]g(\pi)\,\mathrm{d}\pi}_{\text{PAE's expected revenue from other firms}} \tag{2.3}$$

Observe that the PAE's expected revenue from firms that are not clients of the PPE coincides with the PPE's expected revenue from the catch-and-release strategy. Thus, net of the benefit derived from catch-and-release, the expected cost of preempting the patent reduces to the PAE's expected revenue derived from litigating the PPE's clients. Plugging (2.2) and (2.3) into the

PPE's objective and rearranging then yields

$$\max_{\hat{\theta},\hat{\pi}} V_P = \left[\int_{\hat{\pi}}^{1} \pi g(\pi)\, \mathrm{d}\pi\right] \cdot \left[\int_{\hat{\theta}}^{1} \left(\hat{\theta} - \alpha\theta\right) f(\theta)\, \mathrm{d}\theta\right]$$

$$\text{s.t.} \quad \hat{\pi} \geq \underline{\pi}(\hat{\theta}) \tag{CC}$$

The PPE's objective now appears as a simple product of the two terms in brackets, capturing respectively the expected value of the patents that the PPE commits to preempt, and the differential valuation for the PPE's clients by the PAE and the PPE. Note that this latter term may be negative if its mass of client firms is too large (i.e. for low values of $\hat{\theta}$), which suggests that the PPE needs to severely screen out firms by setting a high threshold type in order to make a positive profit.

The next result provides a sufficient condition on the probability density functions f and g for a unique pair $(\hat{\theta}, \hat{\pi})$ maximizing the PPE's problem. Nevertheless, this is not a necessary condition since most generic optimization problems admit a unique solution (see Kenderov, 1984).

Lemma 2.1. *The PPE's objective monotonically decreases (resp. increases) with the patent value threshold $\hat{\pi}$ whenever the threshold type $\hat{\theta}$ satisfies*

$$\hat{\theta} \geq (\textit{resp.} \ \leq) \alpha\mathbb{E}\left(\theta | \theta > \hat{\theta}\right)$$

Furthermore, if the densities f and g are everywhere non-decreasing, then the PPE's problem admits a unique argmax.

It directly follows that setting a threshold type such that $\hat{\theta} < \alpha\mathbb{E}\left(\theta|\theta > \hat{\theta}\right)$ holds cannot be optimal as it would then yield $\hat{\pi} = 1$, and therefore leads to zero profit for the PPE. Instead, the PPE must screen out low type firms by choosing a high threshold type so that enforcing the patent against the set of non-clients becomes profitable. Hence, the PPE instead chooses a threshold type satisfying $\hat{\theta} \geq \alpha\mathbb{E}\left(\theta|\theta > \hat{\theta}\right)$ so that the catch-and-release credibility condition binds, that is, $\hat{\pi} = \underline{\pi}(\hat{\theta})$. The PPE's reduced problem is then

$$\max_{\hat{\theta}} \left[\int_{\underline{\pi}(\hat{\theta})}^{1} \pi g(\pi)\, \mathrm{d}\pi\right] \cdot \left[\int_{\hat{\theta}}^{1} \left(\hat{\theta} - \alpha\theta\right) f(\theta)\, \mathrm{d}\theta\right]$$

$$\text{s.t.} \quad \hat{\theta} \geq \alpha \mathbb{E}\left(\theta | \theta > \hat{\theta}\right)$$

and the threshold type becomes the only choice variable left. By imposing a further restriction on its mass of clients, this new constraint ensures that the PPE maintains a credible threat of litigation against non-clients and gets a positive profit. Winning the auction indeed requires that the fees collected by the PPE (and defined according to the threshold type $\hat{\theta}$) are high enough to outweigh the PAE's valuation for the PPE's clients. As an immediate consequence, this constraint is also a sufficient condition to guarantee the profitability of the PPE's business model. Choosing the optimal threshold type $\hat{\theta}$ is then a problem of balancing, on the one hand, the benefit of restricting its mass of clients through a higher subscription fee, and on the other hand, the benefit of collecting fees from a larger mass of clients. Proposition 3 establishes the existence of an interior solution to this problem.

Proposition 2.3. *There exists an interior threshold $\hat{\theta}^*$ satisfying*

$$0 < \alpha \mathbb{E}\left(\theta | \theta > \hat{\theta}^*\right) \leq \hat{\theta}^* < 1$$

such that the PPE finds it optimal optimal to shut down the least exposed firms. The optimal incentive feasible contract bunches types in $[\hat{\theta}^*, 1]$ and entails:

$$\hat{\pi}^* = \frac{C}{\alpha \int_0^{\hat{\theta}^*} \theta f\left(\theta\right) d\theta} > 0 \quad \textit{and} \quad T^* = \hat{\theta}^* \int_{\hat{\pi}^*}^1 \pi g(\pi) d\pi$$

Hence, the PPE can always make positive profits by offering protection against a subset of patents with the highest value to a subset of operating firms with the highest exposure. This suggests that this type of patent intermediary provides only a partial solution to the legal threat created by PAEs. Indeed, it leaves its clients unprotected against the milder threat of lower-value patents and does not address the exposure to litigation of non-client firms. Rather, the PPE needs to maintain a credible threat of catch-and-release against them in order to sustain its business model.

2.4 Quantitative evidence of NPEs' business models

In this section we empirically compare the acquisition strategies of assertive and preemptive intermediaries engaged in patent acquisition through regression analyses of patent reassignment data. We first describe our data on the different patent trading models. We then test the predictions that the likelihood that the preemption entity acquires a patent depends on the intrinsic value of the patent (as measured by the legal strength of and demand for the patent), controlling for the legal exposure of the clients of the entity.

2.4.1 Data

We gathered data on patents reassigned to assertion and preemption entities using the U.S patent reassignment database. In total, our dataset contains 2608 patents that were reassigned to at least one of the identified intermediaries between 1988 and 2012. 865 of these were bought by the preemption entities Allied Security Trust and RPX Corporation, and the rest by assertion entities including 1st Technology, Acacia Patent Acquisition, Arrival Star, Cheetah Omni, Innovation Management, Innovative Sonic Limited, Intellectual Ventures, IPG Healthcare 501, Mosaid Technologies, Papst Licensing, Rembrandt IP Management, Scenera Research, Tessera Technologies, Trontech Licensing, Wi-Lan Inc., and Wisconsin Alumni Research[4].

2.4.2 Characteristics of reassigned patents

Table 1.1 summarizes the main characteristics of the patents in our two samples. There are a few differences between the patents reassigned to preemption versus assertion entities. Preemption entities tend to acquire patents that are slightly older and more highly cited than those of assertive entities. Although the average ages of patents reassigned to preemption and assertion entities differ by less than a year, this statistically significant age difference suggests that preemption organizations acquire patents that are already known to be problematic or valuable, whereas assertion organizations may acquire patents on a somewhat more speculative basis.

Preemption entities tend to buy significantly more-cited patents than do assertion entities. The likelihood of litigation is also higher for patents reassigned to preemption entities, as com-

[4]However, we were unable to reliably distinguish the reassignments to Intellectual Ventures, because the company operates through so many different funds, subsidiaries, and limited liability companies that this would require substantial amount of detective work to compile (cf. Avancept 2011. The Intellectual Ventures Report. Second Edition, http://avancept.com/iv-report2Ed.html).

pared with patents reassigned to assertion ones. However, the differences in litigation rates are not statistically significant.

	Preemptive entities	Assertive entities
Number of patents	865	1,743
Mean application year (SD)	1,996.51 (4.83)	1,997.28 (5.42)
Mean forward citations (SD)	17.07 (28.49)	14.96 (23.05)
Likelihood of litigation (SD)	0.060 (0.24)	0.050 (0.22)

Table 2.1: Characteristics of the reassigned patents

2.4.3 "Catch-and-release" strategies

One of the key insights of the theoretical model is that the preemptive entities' ability to acquire the most valuable patents depends on their use of catch-and-release strategies whereby they buy patents, negotiate licenses to the patents with potential buyers on behalf of their clients, and then sell the patents "encumbered" with the licenses. Then the members of the entity will enjoy the licensing terms possibly for the life of the patent, even though the patent may be further reassigned (sold) ex post. Catch-and-release provides incentives for legally exposed operating companies to subscribe to the preemption entity because only their clients will benefit from the licensing of the acquired patents.

Catch-and-release strategies are not easy to confirm empirically. Most patent reassignments involving preemption entities have taken place rather recently, and, therefore, our dataset may not yet capture many of the follow-on trades. Moreover, whereas some preemption entities (e.g. AST) buy and sell large numbers of patents, this is not necessary for catch-and-release to be effective. A credible commitment to catch-and-release might require selling just a handful of key patents to assertion entities. Therefore, in order to find evidence about the existence of such strategies, we examine whether (1) some of the patents reassigned to RPX or AST were subsequently reassigned to another entity, and whether (2) some of the patents reassigned to RPX or AST were subsequently litigated. If we find evidence of both, it is highly likely that these firms use catch-and-release as a credible threat to incentivize membership.

Our sample suggests that 32% of patents reassigned to RPX Corporation were subsequently reassigned to another entity. However, we are not able to precisely identify, for all reassignments, if the second reassignee was a client of RPX at the time of reassignment[5]. The percentage of

[5]The majority of patents reassigned to RPX or AST were reassigned in 2009 or later. 10 See

secondary reassignments is much higher for AST, around 80%. However, as AST organizes itself through subsidiaries, we do not have comprehensive data on the history of its subsequent reassignments. However, AST explicitly states that its goal is to sell all the patents ex post[6].

Furthermore, according to our sample, out of those 100 patents that were reassigned to RPX or AST and litigated, 22 patents were part of a lawsuit that was filed after the reassignment to RPX or AST[7]. We have not been able to identify the parties involved in these lawsuits or to confirm whether these patents were part of a litigation initiated by RPX, AST or by subsequent reassignees. However, these cases imply subsequent monetization of the patent through enforcement, which is consistent with a broad definition of the catch-and-release strategy. Moreover, going to court may not be necessary in all cases before reaching a settlement in the dispute. Accordingly, the number of litigated patents should be seen as a lower bound for the actual deployment of catch-and-release. In summary, we observe at least selective litigation of patents after reassignment to preemptive entities, which is aligned with the prediction of our model that without such strategies the preemptive business model will not work.

2.4.4 Regression analyses of reassignments to assertion versus preemption entities

Proposition 2.3 of our theoretical model predicts that whenever a patent is for sale in its technological field of activity, the likelihood that a preemption entity acquires this patent increases with the value of the patent, broadly construed. In order to test this proposition, we measure the technical importance of the patent in terms of its forward citations; its legal strength in terms of past litigation; and its potential demand in terms of the number of companies among the holders of the citing patents. This last measure of value reflects the fragmentation of the follow-on invention (cf. Ziedonis, 2004), which is likely to strengthen the bargaining position and expected returns of the focal patent holder.

According to our model, only firms that are substantially legally exposed to infringement in the technical fields targeted by preemptive entities will subscribe to their protection services. Therefore, we use the clients' exposure to infringement to each traded patent to identify whether this patent may be a potential target for the preemptive entity. For this purpose, we collect

http://www.alliedsecuritytrust.com/services/divestitureprocess.aspx, retrieved on Oct 15, 2015.

[6]See http://www.alliedsecuritytrust.com/services/divestitureprocess.aspx, retrieved on Oct 15, 2015.

[7]11 of these patents were initially reassigned to RPX and 11 to AST.

information about the clients of each patent preemption entity and their prior art citations to each traded patent. We assume that citations to the focal patent indicate that the citing patents are closely related to the cited patent. Citations are often interpreted to delimit and define the boundaries of invention claims presented in each patent (Hall et al. 2001; Lampe, 2012). When a patent cites another patent, its inventors (or examiners) acknowledge that the earlier patent reduces the extent and originality of the subsequent patent. Therefore, citations are indicators of closeness of the inventions in the technology space.

Using the number of forward citations to indicate legal exposure to a patent presents some empirical challenges. The number of citations is correlated with the age of the patent, and citations and reassignments could be subject to reverse causality, with reassignments causing subsequent citations. Thus, differences in the number of citations between assertion and preemption entities could be generated by differences in the numbers of reassignments or variation in the timing of reassignments of patents to each type of entity. To overcome these issues, we only use the number of forward citations that occurred during the first five years of the patent's life excluding all patents that were reassigned during these initial years (345 out of the 2,608 patents). For each reassigned patent, we used the list of the preemption entity's clients to distinguish the numbers of citations by clients and non-clients.

In estimating the auction outcome we control for observable characteristics of the reassigned patent including its age and technological class, and estimate the simple cross-sectional linear regression of the likelihood that a patent preemption entity (PPE) wins the auction for patent p:

$$\begin{aligned} Preemptive_p = \alpha_0 + \alpha_1 Importance_p + \alpha_2 Strength_p + \alpha_3 Demand_p \\ + \alpha_4 GrantYear_p + \alpha_5 TechnologyClass_p + \alpha_6 ReassignmentYear_p + \epsilon_p \end{aligned}$$

with:

$Preemptive_p$: Likelihood, for the patent p, to be reassigned to a PPE,

$Importance_p$: Total forward citations to patent p, or alternatively,

$CitationsbyClients_p$: Number of citations to patent p in patents held by PPE clients and

$CitationsbyNonClients_p$: Number of citations to patent p in patents held by firms that are not clients of the PPE

$Strength_p$: Past litigation of patent p,

$Demand_p$: Number of companies citing patent p,

$GrantYear_p$: Set of dummies for grant years of the patent

$TechnologyClass_p$: Set of dummies for technological classes of the patent

$ReassignmentYear_p$: Dummies for the year of reassignment

α_p= Error term

In Tables 2.2 and 2.3, we provide the descriptive statistics and correlations among the estimation variables. Approximately one third of the patents in our sample were reassigned to a PPE. These organizations thus acquire a significant share of patents traded by intermediaries. An average traded patent in the sample receives almost 16 citations, three of which from clients of PPEs, and 12 from non-clients. Six percent of these patents have been litigated, and there are usually about five companies (assignees) who generate the forward citations.

Table 2.2: Descriptive statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Reassignment to a preemptive entity (binary)	1187	0.33	0.47	0	1
Number of forward citations received by the focal patent	1187	15.77	18.41	0	151
Focal patent has been litigated before reassignment (binary)	1187	0.06	0.24	0	1
Focal patent has been litigated before and the litigation started and ended before reassignment (binary)	1187	0.01	0.15	0	1
Focal patent has been litigated before and the litigation started but did not end before reassignment (binary)	1187	0.04	0.19	0	1
Number of assignees in patents citing the focal patent	1187	5.01	5.85	1	48
Number of forward citations by clients of the preemptive entity	1187	3.40	6.77	0	88
Number of forward citations by non-clients of the preemptive entity	1187	12.40	14.69	0	126
Number of technological classes in patents citing the focal patent	1187	2.65	4.46	0	68
Focal patent has been declared standard essential	1187	0.03	0.17	0	1

Table 2.4 presents the linear probability regression results. We also estimated the models with logit maximum likelihood and the results were qualitatively aligned. We prefer the linear regression for easy interpretation of the coefficients and interaction effects. In every specification we include dummies for technology classes, patent age and reassignment year, as there may be systematic differences in the probability or reassignment across these categories.

Additionally we control for potentially confounding factors related to the generality of the patent (number of technology classes among the citing patents) and the standard-essentiality of the patent (whether it has been declared essential in a standard-setting organization). Each of

	Likelihood to be reassigned to a defensive	Number of forward citations by clients of a preemptive entity	Number of forward citations by a non-client	Focal patent has been litigated before	Number of assignees in patents citing the focal patent	Number of technological classes in patents citing the focal patent	Focal patent has been declared standard essential
Likelihood to be reassigned to a defensive	1.0000						
Number of forward citations by clients of a preemptive entity	0.1666	1.0000					
	(0.0000)						
Number of forward citations by a non-client	-0.0199	0.3889	1.0000				
	(0.4941)	(0.0000)					
Focal patent has been litigated before	0.0938	0.1401	0.1070	1.0000			
	(0.0012)	(0.0000)	(0.0002)				
Number of assignees in patents citing the focal patent	-0.0254	0.4049	0.5969	0.1861	1.0000		
	(0.3812)	(0.0000)	(0.0000)	(0.1312)			
Number of technological classes in patents citing the focal patent	0.0317	0.1169	0.0415	0.1316	0.2575	1.0000	
	(0.2475)	(0.0000)	(0.0013)	(0.2494)	(0.0204)		
Focal patent has been declared standard essential	-0.0254	0.0124	0.0108	0.0425	0.0130	-0.0072	1.0000
	(0.0412)	(0.6517)	(0.6930)	(0.0647)	(0.6343)	(0.7932)	

Table 2.3: Correlation matrix

these factors could conceivably influence the reassignment value of the patent. We add the main explanatory variables of interest one by one and in specification (5) we include them all. In this specification we also control for the generality and standard-essentiality of the patent.

The regressions presented in Table 2.4 suggest that the probability of reassignment of a patent to a PPE increases with the value of the patent measured by its importance, strength, and potential demand. We find that the importance of the patent measured as the number of forward citations only matters to the PPE clients—the number of citations to the patent by non-clients of PPEs is insignificant. In other words, preemption entities are likely to acquire patents that are important for their clients, rather than for the general industry. The legal strength of the patent in terms of past litigation also significantly increases the likelihood of PPE acquisition. Past litigation validates the patent's claims. Similarly the demand for the patent in terms of the number of assignees (unique companies) in patents citing the focal patent for sale significantly increases the likelihood that a PPE acquires it.

As the variables are measured in units and we estimate the linear probability model, the coefficients directly indicate the marginal effects of the estimates. For example, past litigation increases the probability of PPE acquisition of the patent by 9–11 percentage points; an additional assignee citing the patent by up to six percentage points; and an additional citation by

Table 2.4: Linear probability regression results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable:	Coef.	Coef.	Coef.	Coef.	Coef.	Coef.	Coef.	Coef.
Reassignment to a pre-emptive entity	(SD)	(SD)	(SD)	(SD)	(SD)	(SD)	(SD)	(SD)
Number of forward citations received by the focal patent	0.004*							
	(0.002)							
Focal patent has been litigated before		0.091*			0.092*	0.108*	0.092***	0.113**
		(0.028)			(0.020)	(0.061)	(0.045)	(0.058)
Focal patent has been litigated before and the litigation started and ended before reassignment							-0.122*	-0.088
							(0.054)	(0.061)
Focal patent has been litigated before and the litigation started but did not end before reassignment							0.105	0.108
							(0.078)	(0.078)
Number of assignees in patents citing the focal patent			0.013*		0.061*	0.003*		0.001
			(0.003)		(0.022)	(0.001)		(0.001)
Number of forward citations by clients of a preemptive entity				0.015**	0.026**	0.027**	0.016*	0.015
				(0.004)	(0.008)	(0.009)	(0.083)	(0.008)
Number of citations by non-clients				-0.001	-0.009	-0.008	-0.015***	-0.016***
				(0.004)	(0.004)	(0.003)	(0.004)	(0.004)
Forward citations by PPE clients *Number of assignees in citing patents						0.001*		0.002*
						(0.0003)		(0.0002)
Forward citations by PPE clients *Focal patent has been litigated before						-0.0001	0.089***	0.073***
						(0.013)	(0.016)	(0.016)
Forward citations by PPE clients *Focal patent has been litigated before and the litigation started and ended before reassignment							-0.037	-0.045
							(0.026)	(0.028)
Forward citations by PPE clients *Focal patent has been litigated before and the litigation started but did not end before reassignment							0.053*	0.056**
							(0.026)	(0.019)
Number of technological classes in patents citing the focal patent					0.001			
					(0.001)			
Focal patent has been declared standard-essential (binary)					-0.117			
					(0.062)			
Constant	0.248**	0.445***	0.289***	0.841***	0.272***	0.430**	0.564***	0.492**
	(0.062)	(0.052)	(0.047)	(0.042)	(0.024)	(0.028)	(0.062)	(0.043)
Technology class dummies	Y	Y	Y	Y	Y	Y	Y	Y
Focal patent age dummies	Y	Y	Y	Y	Y	Y	Y	Y
Reassignment year dummies	Y	Y	Y	Y	Y	Y	Y	Y
Observations	1187	1187	1187	1187	1187	1187	1187	1187
Adjusted R^2	0.1122	0.311	0.098	0.273	0.309	0.309	0.434	0.435

Legend: * p<0.05; ** p<0.01; *** p<0.001. Robust standard errors in parentheses.

PPE clients by 2–3 percentage points. These variables thus have economically significant effects on the probability of acquisition by a PPE.

Citations from PPE clients reflect the clients' legal exposure to the technical claims made by the patent in question. In the last three specifications, we interact this variable with the other measures of value: litigation and potential demand. These specifications most directly assess the implications of our theoretical model. Our propositions suggest that the most exposed firms will join the PPE and, conditional on the client relationship, the intrinsic value of the patent determines who wins the auction. A positive interaction term in the estimation model would then indicate that the value of the patent (measured as its legal strength and demand) matters significantly more for the legally exposed firms.

Indeed, in specification (6) we find that a greater demand for patents to which many PPE clients are exposed significantly drives the PPE's bidding strategies in the auction. In other words, the demand for patents in terms of a large number of citing companies drives PPE acquisition strategies particularly strongly if the PPE clients are highly exposed to that patent. Economically, this effect is significant at the higher end of the number of citations by PPE clients, i.e., for the highest levels of PPE client exposure. At the mean value of citations by PPE clients (3.40), the combined effect of the direct and interaction effects of the number of assignees is only 0.54 percentage points. However, the number of citations by PPE clients ranges from 0 to 88 and at the midpoint of 44, an additional citing company would increase the probability of PPE acquisition by 3.3 percentage points. In other words, the estimated effect of demand for the patent measured as the number of citing companies (unique assignees of citing patents) is quite substantial for the patents to which PPE clients are the most exposed.

In contrast, the litigation variable does not significantly interact with the exposure of PPE clients. Litigation of a patent significantly increases its likelihood of being acquired by a PPE, but this does not depend on the exposure of PPE clients to the patent. One possible interpretation for this is that litigated patents may be acquired out of active lawsuits based on demand by a smaller number of PPE clients, whereas the demand indicator (the number of companies citing the patent in their own patents) suggests there is a large number of companies pursuing the technology area in question, and if those companies are also highly exposed, then the PPE is very likely to respond by acquiring the patent. There thus appear to be two distinct acquisition strategies that PPEs may follow to provide legal protection to their clients.

To disentangle these two acquisition strategies, we add two binary litigation variables in the last two specifications, one for patents that have been litigated in the past and which litigation has been concluded prior to the acquisition in question, and one for patents that were litigated during the transaction in question. It turns out that almost all litigated patents with some forward citations by the PPE's clients were more likely to be acquired by PPEs. In particular, patents concurrently involved in litigation during the time of reassignment were very likely to be acquired by a PPE, and much more so if they were highly cited by patents of PPE clients. Although not all these direct and interaction effects are statistically significant, the results roughly indicate that patents that were cited at the mean rate of 3.4 by PPE clients' patents and were litigated in the past were 15% more likely to be acquired by a PPE, whereas similar patents that were involved in ongoing litigation were 67% more likely to be acquired by a PPE. Thus, although the signal of value from past litigation alone does not necessarily induce acquisition by a PPE, any connection to the technology areas on which PPE clients are working strongly enhances the probability of reassignment to a PPE, particularly if the patent is concurrently subject to litigation concurrently. However, some of the coefficients are only marginally significant, possibly because there are very few cases of litigation that have already ended before reassignment.

2.5 Conclusion

This paper studies how preemptive and assertive non-practicing entities (NPEs) operate in the patent market. We theoretically examine competition between an patent assertion entity and a patent preemption entity in a patent auction, and find that the latter can prevail by attracting the most exposed firms to become members and using their collective demand to preempt the most valuable patents. Our analysis also shows that the profitability of PPEs' business model relies on preserving a credible threat of enforcement of the preempted patent against non-client firms (Catch-and-Release strategy), thereby obliging the preemption entity to commit to preempt a subset of high value patents.

We test the validity of these claims with analyses of patent reassignment and litigation data. We use citation data to identify legally exposed member firms, and find that our empirical evidence is consistent with the model. Specifically, the PPE's probability of buying a patent increases with its intrinsic value in terms of technical importance, strength and demand. Furthermore, the value of the patent has a greater impact on firms that are legally exposed to the patent claims. We find that patent value significantly interacts with legal exposure.

Our analyses highlight key features of preemptive and assertive strategies in the patent marketplace by focusing on the information and bidding strategies of different types of NPEs. Our empirical analyses, however, are based on the behavior of just two preemption entities, RPX Corporation and Allied Security Trust. Although they are major participants in this market, they are the only companies thus far known to provide substantial preemption services in their stated missions and observed operations.

We find that these two companies indeed provide services to preempt patent assertion entities on behalf of their clients. They also subsequently monetize their patents by selling them to their own clients, other practicing entities, or patent assertion entities for further monetization and possibly litigation. Thus, what makes preemptive entities "defensive", as they call themselves, is the fact that they enable sharing the risks, costs, and information related to patent threats among their members. The defensive business model does not mean that patents are left "unmonetized" on the shelf. As of 2015, AST and RPX have not engaged in extensive litigation themselves, but it would not be surprising if RPX decided to do so in the future, as it is holding a large and rapidly growing portfolio of unmonetized assets. Its limited catch-and-release of the acquired patents may prevent it from acquiring some valuable targets, and hence from providing the best defensive services.

2.6 Proofs

Proof of Proposition 2.1.

Throughout the proof, we will use the following notation: $u_P(b_P) = v_P^1 + \left(v_P^2 - b_A\right) \mathbb{1}_{b_P \geq b_A} \mathbb{1}_{\pi^r \geq \pi_0}$ denotes the PPE's net payoff upon bidding $b_P \geq 0$, where v_P^1 is the revenue it derives from subscription fees, while $v_P^2 \in [0, v(\pi^r))$ is the PPE's gain from litigating the acquired patent against non-clients. As well, we let $u_A(b_A) = (v_A(\pi^r) - b_P) \mathbb{1}_{b_A \geq b_P}$ denote the PAE's net payoff when bidding $b_A \geq 0$.

We first show that bidding according to

$$b_A^*(\pi^r) = v_A(\pi^r) = \begin{cases} \alpha\pi^r \int_0^1 \theta f(\theta)\,\mathrm{d}\theta - C & \text{if } \pi^r \geq \pi_0 \\ 0 & \text{if } \pi^r < \pi_0 \end{cases}$$

is a weakly dominant strategy for the PAE. If $\pi^r < \pi_0$, then bidding zero yields $u_A(0) = 0$, while bidding any $b' > 0$ only changes the outcome if $b' > b_P > 0$, in which case the PAE then gets $u_A(b') = -b_P < 0 = u_A(0)$. If $\pi^r \geq \pi_0$, then bidding $b_A = \alpha\pi^r \int_0^1 \theta f(\theta)\,\mathrm{d}\theta - C$ weakly dominates any lower bid. To see this, suppose instead that the PAE bids $\underline{b} < b_A$, then the outcome only changes if $\underline{b} < b_P < b_A$. In this case, the PAE loses and gets

$$u_A(\underline{b}) = 0 < b_A - b_P = v_A(\pi^r) - b_P = u_A(b_A)$$

Similarly, bidding b_A weakly dominates any higher bid. Asssume that the PAE bids according to $\bar{b} > b_A$, then the outcome only changes if $b_A < b_P < \bar{b}$. In this case, the PAE wins and gets

$$u_A(\bar{b}) = v_A - \bar{b} = b_A - b_P < 0 = u_A(b_A)$$

Hence, bidding its true value, *i.e.* according to $b_A^*(\pi^r) = v_A(\pi^r)$ is indeed a weakly dominant strategy for the PAE.

We now turn to the PPE and show that, under (A3), playing according to

$$b_P^*(\pi^r) = \begin{cases} \bar{b}_P & \text{if } \pi^r \in [\hat{\pi}, 1] \\ \underline{b}_P & \text{if } \pi^r \in [\pi_0, \hat{\pi}) \\ 0 & \text{if } \pi^r \in [0, \pi_0) \end{cases} \quad \textit{with} \quad 0 \leq \underline{b}_P < \alpha\pi^r \int_0^1 \theta f(\theta)\,\mathrm{d}\theta - C < \bar{b}_P$$

is a best response to the PAE's bid. Suppose that the PAE plays according to $b_A^*(\pi^r)$. If $\pi^r < \pi_0$, then the PPE is indifferent between winning and losing as it gets $u_P = v_P^1 \geq 0$ in both cases. Thus, bidding any $b \geq 0$ yields the same payoff to the PPE and in particular, pledging $b_P = 0$ is a best response. Now, if $\pi^r \in [\pi_0, \hat{\pi})$, the PPE strictly prefers to lose as it gets

$$u_P = v_P^1 > v_P^1 + v_P^2 - v_A(\pi^r) = v_P^1 + v_P^2 - b_A(\pi^r)$$

Hence, bidding any $\underline{b}_P$ such that $0 \leq \underline{b}_P < \alpha\pi^r \int_0^1 \theta f(\theta)\,\mathrm{d}\theta - C = b_A^*(\pi^r)$ is indeed optimal for the PPE. Finally, consider the case where $\pi^r \geq \hat{\pi}$. Due to its contracting commitment, the PPE strictly prefers to win the auction since

$$u_P = v_P^1 + v_P^2 - b_A(\pi^r) > v_P^1 - \rho$$

$$\Leftrightarrow \rho > b_A(\pi^r) - v_P^2$$

which holds by assumption (A3). Hence, bidding any $\bar{b}_P > \alpha\pi^r \int_0^1 \theta f(\theta)\,\mathrm{d}\theta - C = b_A^*(\pi^r)$ constitutes a best response, which completes the proof. □

Proof of Proposition 2.2.

Towards a contradiction, suppose that the PPE chooses $(\hat{\pi}, \hat{\theta}, T)$ such that

$$\hat{\pi} < \frac{C}{\alpha \int_0^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta} \equiv \underline{\pi}(\hat{\theta})$$

That is, the PPE's problem is

$$\max_{(\hat{\pi},\hat{\theta},T)} \underbrace{\left[1 - F(\hat{\theta})\right].T}_{\text{Expected subscription fees}} + \underbrace{\int_{\underline{\pi}(\hat{\theta})}^1 \left[\alpha\pi \int_0^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta - C\right] g(\pi)\,\mathrm{d}\pi}_{\text{Expected revenue from catch-and-release}} - \underbrace{\int_{\hat{\pi}}^1 v_A(\pi)\, g(\pi)\,\mathrm{d}\pi}_{\text{Expected price for the patent}}$$

subject to

$$T + \theta \int_{\pi_0}^{\hat{\pi}} \pi g(\pi)\,\mathrm{d}\pi \leq \theta \int_{\pi_0}^{\hat{\pi}} \pi g(\pi)\,\mathrm{d}\pi + \theta \int_{\underline{\pi}(\hat{\theta})}^1 \pi g(\pi)\,\mathrm{d}\pi \quad \forall \theta \in [\hat{\theta}, 1] \tag{PC}$$

$$\hat{\pi} \leq \underline{\pi}(\hat{\theta}) \tag{$\bar{CC}$}$$

Since optimality requires that (PC) binds for the lowest type, one obtains

$$T^o = \hat{\theta} \int_{\underline{\pi}(\hat{\theta})}^1 \pi g(\pi)\,\mathrm{d}\pi$$

Plugging T^o and $v_A(\pi)$ into the PPE's objective and rearranging yields

$$\max_{(\hat{\pi},\hat{\theta})} V^o(\hat{\pi},\hat{\theta}) = \left[\int_{\underline{\pi}(\hat{\theta})}^{1} \pi g\left(\pi\right) \mathrm{d}\pi\right] \cdot \left[\int_{\hat{\theta}}^{1} \left(\hat{\theta} - \alpha\theta\right) f\left(\theta\right) \mathrm{d}\theta\right] - \alpha \left[\int_{0}^{1} \theta f(\theta) \mathrm{d}\theta\right] \cdot \left[\int_{\hat{\pi}}^{\underline{\pi}(\hat{\theta})} \pi g\left(\pi\right) \mathrm{d}\pi\right]$$

$$\text{s.t.} \quad \hat{\pi} \leq \underline{\pi}(\hat{\theta}) \qquad (\bar{C}C)$$

Differentiating the PPE's objective with respect to $\hat{\pi}$ yields

$$\frac{\partial V^o}{\partial \hat{\pi}} = \alpha \hat{\pi} g(\hat{\pi}) \int_0^1 \theta f(\theta) \mathrm{d}\theta > 0$$

using the fact that the density g is strictly positive everywhere. Therefore, it must be that

$$\hat{\pi} = \frac{C}{\alpha \int_0^{\hat{\theta}} \theta f\left(\theta\right) \mathrm{d}\theta} \equiv \underline{\pi}(\hat{\theta})$$

which is a contradiction. □

Proof of Lemma 2.1.

Let $V(\hat{\pi},\hat{\theta})$ denote the PPE's objective, that is,

$$V(\hat{\pi},\hat{\theta}) = \left[\int_{\hat{\pi}}^{1} \pi g\left(\pi\right) \mathrm{d}\pi\right] \cdot \left[\int_{\hat{\theta}}^{1} \left(\hat{\theta} - \alpha\theta\right) f\left(\theta\right) \mathrm{d}\theta\right]$$

and let V_i denote the first derivative of V with respect to its i-th argument, andV_{ij} be the cross-partial derivative of V with respect to its i-th and j-th argument, $i, j = 1, 2$.

We first establish that V is either monotonically decreasing or increasing with $\hat{\pi}$. Differentiating V with respect to $\hat{\pi}$ yields

$$V_1 = -\hat{\pi} g(\hat{\pi}) \int_{\hat{\theta}}^{1} \left(\hat{\theta} - \alpha\theta\right) f\left(\theta\right) \mathrm{d}\theta = -\hat{\pi} g(\hat{\pi}) \left[\hat{\theta}\left(1 - F(\hat{\theta})\right) - \alpha \int_{\hat{\theta}}^{1} \theta f(\theta) \mathrm{d}\theta\right]$$

Since the density function g is strictly positive everywhere, we have that

$$V_1 \leq (\geq) \, 0 \Leftrightarrow \hat{\theta} \geq (\leq) \, \frac{\alpha}{1 - F(\hat{\theta})} \int_{\hat{\theta}}^{1} \theta f(\theta) \mathrm{d}\theta = \alpha \mathbb{E}\left(\theta | \theta > \hat{\theta}\right)$$

Let us now define

$$h(\hat{\theta}) = \hat{\theta} - \alpha \mathbb{E}\left(\theta | \theta > \hat{\theta}\right)$$

Since h is continuous and $h(0) = -\alpha\mathbb{E}(\theta) < 0$ while $h(1) = 1 - \lim_{\hat{\theta}\to 1} \alpha\mathbb{E}\left(\theta|\theta > \hat{\theta}\right) = 1 - \alpha > 0$, there exists $\theta_0 \in (0,1)$ such that $h(\theta_0) = 0$, that is, $\theta_0 = \alpha\mathbb{E}\left(\theta|\theta > \theta_0\right)$. Furthermore, θ_0 is unique as, letting $m(\theta, \hat{\theta}) \equiv \mathbb{E}\left(\theta|\theta > \hat{\theta}\right)$, we have that

$$h'(\hat{\theta}) = 1 - \alpha \frac{\partial m}{\partial \hat{\theta}} \geq 1 - \alpha > 0$$

where the first inequality follows from the fact that $\frac{\partial m}{\partial \hat{\theta}} \in [0,1]$ under the logconcavity assumption of θ (see Heckman and Honoré, 1990, Proposition 1 p. 1128).

Suppose that f and g are everywhere non-decreasing. We now show that V is strictly quasi-concave in $(\hat{\pi}, \hat{\theta})$ on the region $[\pi_0, 1] \times (\theta_0, 1]$. To this end, it is sufficient to show that the determinant of the Bordered Hessian of V is strictly positive. We have that

$$B = \begin{bmatrix} V_{11} & V_{12} & V_1 \\ V_{21} & V_{22} & V_2 \\ V_1 & V_2 & 0 \end{bmatrix}$$

with

$$\begin{aligned}
V_1 =& -\hat{\pi}g(\hat{\pi}) \int_{\hat{\theta}}^{1} \left(\hat{\theta} - \alpha\theta\right) f(\theta)\,\mathrm{d}\theta \\
V_2 =& \left[-\hat{\theta}(1-\alpha)f(\hat{\theta}) + 1 - F(\hat{\theta})\right] \cdot \int_{\hat{\pi}}^{1} \pi g(\pi)\,\mathrm{d}\pi \\
V_{12} =& V_{21} = -\hat{\pi}g(\hat{\pi})\left[-\hat{\theta}(1-\alpha)f(\hat{\theta}) + 1 - F(\hat{\theta})\right] \\
V_{11} =& -\left[g(\hat{\pi}) + \hat{\pi}g'(\hat{\pi})\right] \cdot \int_{\hat{\theta}}^{1} \left(\hat{\theta} - \alpha\theta\right) f(\theta)\,\mathrm{d}\theta \\
V_{22} =& \left[-(1-\alpha)\left(f(\hat{\theta}) + \hat{\theta}f'(\hat{\theta})\right)\right] \cdot \int_{\hat{\pi}}^{1} \pi g(\pi)\,\mathrm{d}\pi
\end{aligned}$$

Thus, $det(B) > 0$ if $2V_1V_2V_{12} - (V_1)^2V_{22} - (V_2)^2V_{11} > 0$. Since g is strictly positive everywhere, it follows that $V_1 < 0$ and $V_{11} < 0$ for all $\hat{\theta} \in (\theta_0, 1]$. Furthermore, we have that $sign(V_2) = -sign(V_{12})$ so that $2V_1V_2V_{12} \geq 0$. Likewise, $V_{22} < 0$. Hence, $det(B) > 0$.

Finally, it may be easily verified that the constraint set

$$\mathcal{C} = \left\{ (\hat{\pi}, \hat{\theta}) \in [\pi_0, 1] \times (\theta_0, 1] : \hat{\pi} \int_0^{\hat{\theta}} \theta f(\theta)\, \mathrm{d}\theta \geq \frac{C}{\alpha} \right\}$$

is non-empty under assumption (A2). We now show that it is convex. Let $(\hat{\pi}_1, \hat{\theta}_1) \in \mathcal{C}$ and $(\hat{\pi}_2, \hat{\theta}_2) \in \mathcal{C}$, and observe that

$$\lambda\hat{\pi}_1 \int_0^{\lambda\hat{\theta}_1} \theta f(\theta) \mathrm{d}\theta = \lambda\hat{\pi}_1 \left\{ \int_0^{\lambda\hat{\theta}_1+(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta - \int_{\lambda\hat{\theta}_1}^{\lambda\hat{\theta}_1+(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta \right\} \geq \lambda \frac{C}{\alpha}$$

and

$$(1-\lambda)\hat{\pi}_2 \int_0^{(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta = (1-\lambda)\hat{\pi}_2 \left\{ \int_0^{\lambda\hat{\theta}_1+(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta - \int_{(1-\lambda)\hat{\theta}_2}^{\lambda\hat{\theta}_1+(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta \right\} \geq (1-\lambda) \frac{C}{\alpha}$$

for any $\lambda \in (0, 1)$. Adding both inequalities yields

$$[\lambda\hat{\pi}_1 + (1-\lambda)\hat{\pi}_2] \int_0^{\lambda\hat{\theta}_1+(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta - \lambda\hat{\pi}_1 \int_{\lambda\hat{\theta}_1}^{\lambda\hat{\theta}_1+(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta - (1-\lambda)\hat{\pi}_2 \int_{(1-\lambda)\hat{\theta}_2}^{\lambda\hat{\theta}_1+(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta \geq \frac{C}{\alpha}$$

$$\Rightarrow [\lambda\hat{\pi}_1 + (1-\lambda)\hat{\pi}_2] \int_0^{\lambda\hat{\theta}_1+(1-\lambda)\hat{\theta}_2} \theta f(\theta) \mathrm{d}\theta \geq \frac{C}{\alpha} \Leftrightarrow \lambda(\hat{\pi}_1, \hat{\theta}_1) + (1-\lambda)(\hat{\pi}_2, \hat{\theta}_2) \in \mathcal{C}$$

Hence, the constraint set is indeed convex. The problem of the PPE therefore admits a unique global constrained maximizer. This last argument completes the proof. □

Proof of Proposition 2.3.

$$\tilde{V}(\hat{\theta}) = \underbrace{\left[\int_{\frac{C}{\alpha \int_0^{\hat{\theta}} \theta f(\theta) \mathrm{d}\theta}}^1 \pi g(\pi) \mathrm{d}\pi \right]}_{\mathrm{A}(\hat{\theta})} . \underbrace{\left[\int_{\hat{\theta}}^1 \left(\hat{\theta} - \alpha\theta \right) f(\theta)\, \mathrm{d}\theta \right]}_{\mathrm{B}(\hat{\theta})}$$

We have that

$$\frac{d\tilde{V}}{d\hat{\theta}} = A'(\hat{\theta})B(\hat{\theta}) + A(\hat{\theta})B'(\hat{\theta})$$

where

$$A'(\hat{\theta}) = \frac{\hat{\theta} C^2 f(\hat{\theta})}{\alpha^2 \left(\int_0^{\hat{\theta}} \theta f(\theta) \mathrm{d}\theta \right)^3} . g\left(\frac{C}{\alpha \int_0^{\hat{\theta}} \theta f(\theta) \mathrm{d}\theta} \right) \quad \text{and} \quad B'(\hat{\theta}) = 1 - F(\hat{\theta}) - \hat{\theta}(1-\alpha) f(\hat{\theta})$$

Evaluating at $\hat{\theta} = 1$, one obtains

$$\left. \frac{d\tilde{V}}{d\hat{\theta}} \right|_{\hat{\theta}=1} = -(1-\alpha) f(1) \int_{\frac{C}{\alpha \int_0^1 \theta f(\theta) \mathrm{d}\theta}}^{1} \pi g(\pi) \mathrm{d}\pi < 0$$

from assumption (A2) and the fact that the density f is strictly positive everywhere. In turn, from the proof of Lemma 1, we know that there exists a unique interior θ_0 implicitely defined by $\theta_0 = \mathbb{E}\left(\theta | \theta > \theta_0\right)$ such that $B(\theta_0) = 0$, and that

$$B(\hat{\theta}) < 0 \quad \text{if } \hat{\theta} \in [0, \theta_0) \quad \text{and} \quad B(\hat{\theta}) > 0 \quad \text{if } \hat{\theta} \in (\theta_0, 1)$$

Hence, it must be that $B'(\theta_0) \geq 0$. Therefore, it implies that

$$\left. \frac{d\tilde{V}}{d\hat{\theta}} \right|_{\hat{\theta}=\theta_0} = A(\theta_0) B'(\theta_0) \geq 0 \Leftrightarrow A(\theta_0) = \int_{\frac{C}{\alpha \int_0^{\theta_0} \theta f(\theta) \mathrm{d}\theta}}^{1} \pi g(\pi) \mathrm{d}\pi \geq 0$$

which strictly holds by (A2). Since $\tilde{V}'(\hat{\theta})$ is continuous, there exists a threshold $\hat{\theta}^*$ satisfying

$$0 < \alpha \mathbb{E}\left(\theta | \theta > \hat{\theta}^* \right) \leq \hat{\theta}^* < 1$$

and such that the first order condition for an interior solution is satisfied at $\hat{\theta} = \hat{\theta}^*$:

$$\left. \frac{d\tilde{V}}{d\hat{\theta}} \right|_{\hat{\theta}=\hat{\theta}^*} = 0$$

□

Chapter 3

Duopoly with Deterministic One-way R&D Spillovers

English summary

We examine the standard symmetric two-period R&D model with a deterministic one-way spillover structure. Though firms are ex-ante identical, one obtains a unique asymmetric equilibrium of R&D investments, leading to inter-firms heterogeneity in the industry. We analyze the impact of a change in the spillover parameter and R&D costs on firms' levels of R&D and profits. We consider R&D cooperation and provide a welfare analysis in which we examine the social costs of imposing symmetric R&D invesments among firms. Finally, we discuss the impact of uncertainty about the appropriability of firms' R&D investments and find that the spread between firms' investments and the industry's total cost reductions are higher in the stochastic framework. Firms sometimes prefer uncertain spillovers due to increased asymmetries in terms of their unit cost structure.

Résumé français

Nous examinons le modèle standard de R&D à deux périodes avec des spillovers unidirectionnels et déterministes. Bien que les firmes soient a priori identiques, l'unique équilibre d'investissements en R&D est asymétrique et induit donc de l'hétérogénéité entre les firmes dans l'industrie. Nous analysons l'impact d'un changement dans le paramètre de spillover et dans les coûts de R&D sur les niveaux de R&D des firmes ainsi que sur leur profit. La coopération en R&D est également étudiée et nous menons une analyse du bien-être social au sein de laquelle nous examinons le coût social associé à la contrainte d'investissements symétriques entre les firmes. Enfin, nous analysons l'impact de l'incertitude quant à l'appropriation des investissements des firmes en R&D. Nous montrons que l'écart entre leurs niveaux d'effort ainsi que les reductions de coût totales sont supérieurs dans un environnement incertain. Le profit des firmes est plus élevé dès lors que les spillovers sont stochastiques du fait d'une plus grande asymétrie dans la structure des coûts unitaires.

3.1 Introduction

In the context of non-tournament models of R&D in which firms engage in cost-reducing innovation and then compete à la Cournot in the product market, it is widely recognized that exogenous knowledge spillovers create distortions in R&D investment decisions. While most of the extant literature on imperfectly appropriable R&D focuses on multidirectional spillovers, Amir and Wooders (1999), henceforth AW, instead consider a stochastic directed spillover process whereby know-how may flow only from the more R&D intensive firm to its rival. In their model, spillovers are stochastic and admit only extreme realizations - either full or no spillovers occur with a given probability.

As argued by AW, the idea underlying the assumption of a unidirectional spillover process is that it may better approximate the potential leakages that occur when the R&D process is either one-dimensional, i.e. there is a single research path to achieve unit cost reductions, or multi-dimensional in which case this spillover structure suggests that there is a more natural path to follow. In this context, the spillover parameter may be interpreted as being related to the length of patent protection, but also to a measure of the imitation lag.

The purpose of the present paper is to examine the certainty-equivalent of AW's model in the sense that a fraction of the R&D undertaken by the leader flows to its rival with certainty. Namely, we consider the standard two-period model of process R&D and product market competition with deterministic one-way spillovers. We adopt the common specification of linear market demand and identical linear firms' cost functions. Though firms are ex-ante identical, one obtains a unique pair of asymmetric equilibria so that the roles of R&D innovator (the more R&D intensive firm) and imitator (the less R&D intensive firm) are endogenously determined. That is, a firm always either spends less than its rival so as to free ride on the latter's R&D investment through spillovers, or spends more if the other firm's investment is too low in order to benefit from a competitive advantage over its rival in the product market, thereby leading to asymmetries in terms of the unit cost structure in the product market competition, and thus unequal market shares. This conclusion establishes a simple link between the nature of the R&D process in an industry –including the associated spillover –and the emergence of inter-firm heterogeneity in that industry.

We examine how R&D investments and firms' profits vary with both the spillover parameter and the R&D cost parameter. In particular, although both firms' reaction curves shift down as the spillover rate increases, we identify separate conditions for each firm's R&D levels to increase in the spillover parameter. Nevertheless, if either demand is high enough relative to initial unit costs or R&D costs are sufficiently convex, R&D expenditures globally decrease with the spillover parameter. Furthermore, we find that an increase in the spillover rate sometimes raises the innovator's profit and lowers that of the imitator. The industry is nonetheless better off with some degree of imperfect appropriability of R&D as compared to no spillovers when R&D costs are high.

Next, we study R&D cooperation among firms by means of a joint lab formation, thereby allowing firms to jointly appropriate the outcome of R&D investments, while sharing the associated costs. It has been shown that, when the spillover process is multidirectional and deterministic, cooperating through a joint lab is superior to R&D competition in terms of levels of investments, industry profit and consumer surplus (see d'Aspremont and Jacquemin, 1988; Kamien et al., 1992). In the context of one-way stochastic spillovers, AW find that, under R&D competition, the innovator sometimes invests more in R&D than the joint lab, and the industry's total profit is sometimes higher than under the joint lab. Clearly, since spillovers vanish under this type of cooperation, the same results obtain with deterministic one-way R&D spillovers.

Then, we consider a benevolent central planner with a second best mandate, i.e. one that can decide on R&D investments without intervening into the market competition. While the second-best optimal symmetric investments coincide with those of the joint lab, social welfare achieved under the joint lab formation is superior since R&D costs are shared among firms. Furthermore, since imposing symmetric R&D investments yield symmetric unit costs in the product market competition, social welfare under R&D competition dominates that under the intervention of the central planner when R&D costs are low enough. This is because social welfare tends to be higher when firms are asymmetric in terms of unit costs (see Salant and Shaffer, 1998). In this respect, relaxing the assumption that the central planner imposes equal treatment among firms, we find that social welfare induced by the second-best welfare maximizing asymmetric R&D investments dominate that of the joint lab if either the spillover parameter or the cost of performing R&D are low enough. Therefore, the well known result that the market typically delivers lower levels of R&D than a (second-best) social planner continues to hold in our setting, despite the resulting asymmetry among firms.

Finally, we discuss the impact of uncertainty about the appropriability of firms' R&D investments by comparing our results with those of AW. This is a meaningful comparison despite the difference in the spillover processes of the two papers, since ours constitutes the certainty-equivalent version of AW's. In other words, the issue at hand here is to investigate the role played by uncertainty in the spillover process. In particular, we find that the spread between firms' investments is higher when the unidirectional spillover process is stochastic. Interestingly, a stochastic spillover structure seems to be more efficient than its deterministic counterpart since the industry needs to invest less to achieve larger total cost reductions for a significant range of parameters. Notably, we show that firms are better off when spillovers are uncertain for a wide range of parameters due to increased asymmetries in terms of their unit cost structure in the product market competition. We provide some economic intuition for this comparison in terms of the curvature of firms' profit functions. Since these are convex in the firms' unit costs, firms behave as risk-loving entities, preferring the stochastic spillover process of AW to the certainty-equivalent version of the present paper.

The rest of the paper is organized as follows. Section 3.2 describes the model and the assumptions. Section 3.3 characterizes the equilibrium under R&D competition. Section 3.4 studies the effect of a change in the spillover parameter and the R&D costs on firms' propensity to invest and profits. R&D cooperation by means of a joint lab is examined in Section 3.5. A welfare analysis is provided in section 3.6. In Section 3.7, we compare R&D competition under both deterministic and stochastic one-way spillovers. Concluding remarks are provided in Section 3.8. Finally, a brief description of the model developed by AW and the proofs of our results is relegated to the Appendix in Section 3.9.

3.2 The model

Consider an industry with two firms producing a homogenous good with the same initial unit cost c, playing the following two-stage game. In the first stage, firms simultaneously choose their autonomous cost reduction level x_1 and x_2, with $x_i \in [0, c]$, $i = 1, 2$. The R&D cost to firm i associated with the cost reduction x_i is $\frac{\gamma}{2}x_i^2$, $\forall i = 1, 2$. We assume, following AW, that the innovation only flows from the more R&D intensive firm (the innovator) to its rival (the imitator). But contrary to AW, we assume that the spillover process is deterministic. Namely, if autonomic cost reductions are x_1 and x_2 with, say, $x_1 \geq x_2$, then the effective cost reductions are $X_1 = x_1$ and $X_2 = x_2 + \beta(x_1 - x_2)$, where the parameter $\beta \in [0,1]$ is the fraction of the cost reduction undertaken by firm 1 that spills over firm 2 with certainty.

In the second stage, upon observing the new unit costs, firms compete in the product market by choosing quantities, with linear inverse demand $P(q_1 + q_2) = a - (q_1 + q_2)$. A pure strategy for firm i is thus a pair (x_i, q_i), where $x_i \in [0, c]$ and $q_i : [0, c]^2 \to \mathbb{R}_+$. Throughout, we use the standard concept of subgame perfect equilibrium.

We assume that demand is high enough relative to the initial unit cost to ensure that the second-stage game admits a *unique* pure strategy Nash equilibrium (PSNE) where both firms are active in the product market, that is,

(A1) $a > 2c$

Cournot equilibrium profit of firm i in the second stage, given the actual unit costs c_i, c_j, is thus given by $\Pi(c_i, c_j) = \frac{(a-2c_i+c_j)^2}{9}$. Firms' *net* profits F_1, F_2, defined as the difference between the second stage profit and the first stage R&D investment, can then be expressed as functions of the autonomous cost reductions x_1 and x_2. Since the game is symmetric, we have that $F_1(x_1, x_2) = F_2(x_2, x_1)$. Therefore, throughout the paper, we omit the subscripts and write $F(x_i, x_j)$ to denote the net profit of firm i, where

$$F(x_i, x_j) = \begin{cases} \frac{(a-c+x_i(2-\beta)-x_j(1-\beta))^2}{9} - \frac{\gamma}{2}x_i^2 & \hat{=} U(x_i, x_j) \quad \text{if } x_i \geq x_j \\ \frac{(a-c+2x_i(1-\beta)+x_j(2\beta-1))^2}{9} - \frac{\gamma}{2}x_i^2 & \hat{=} L(x_i, x_j) \quad \text{if } x_i \leq x_j \end{cases} \tag{3.1}$$

One can easily check that F is continuous and nonconcave along the diagonal. Amir et al. (2010) show that such a payoff function can guarantee the existence of an equilibrium if U and L are globally submodular in (x_i, x_j). Unfortunately, this condition is not satisfied in our model for all values of the spillover parameter β. Indeed, for $\beta \leq \frac{1}{2}$, both U and L are submodular in (x_i, x_j), i.e., $\frac{\partial^2 U(x_i,x_j)}{\partial x_i \partial x_j} < 0$ and $\frac{\partial^2 L(x_i,x_j)}{\partial x_i \partial x_j} < 0$. On the other hand, for $\beta > \frac{1}{2}$, U is submodular but L is supermodular in (x_i, x_j), i.e., $\frac{\partial^2 U(x_i,x_j)}{\partial x_i \partial x_j} < 0$ and $\frac{\partial^2 L(x_i,x_j)}{\partial x_i \partial x_j} > 0$.

Furthermore, we assume the following.

(A2) $9\gamma - 2(2-\beta)^2 > 0$

(A3) $4\frac{a}{c}(1-\beta) < 9\gamma$

(A2) guarantees that U and L are strictly concave with respect to the first variable, while (A3) ensures that firm i's reaction function satisfies $r_i(c) < c$, where $r_i(x_j) \in \text{argmax}\,\{F(x_i, x_j) : x_i \in [0, c]\}$. This enables us to derive the reaction function of, say, firm i as

$$r_i(x_j) = \begin{cases} 2\frac{(2-\beta)(a-c+x_j(\beta-1))}{9\gamma-2(\beta-2)^2} & \text{if } x_i \geq x_j \\ 4\frac{(1-\beta)(a-c+x_j(2\beta-1))}{9\gamma-8(\beta-1)^2} & \text{if } x_i \leq x_j \end{cases} \tag{3.2}$$

and since the game is symmetric, we have that $r_i(x_j) = r_j(x_i)$.

3.3 Equilibrium analysis

Before characterizing the equilibrium investments of the first-stage R&D game, our first result states that firms' reaction functions are not continuous.

Lemma 3.1. *Reaction functions admit a unique downward jump.*

Figure 1a (resp. 1b) depicts firms' reaction curves for $\beta \leq \frac{1}{2}$ (resp. $\beta > \frac{1}{2}$). As was previously mentionned, the upper payoff function U is globally submodular in own and rival's decisions so that it gives rise to a reaction function segment that shifts down as rival's investment increases. As for the lower payoff function L, it is also submodular in own and rival's decision for $\beta \leq \frac{1}{2}$, but supermodular for $\beta > \frac{1}{2}$, so that its reaction function segment shifts up as rival's investment increases for this range of the spillover parameter. In contrast to our model, each player's payoff function in AW[1] is instead globally submodular in (x_i, x_j) so that reaction curves have a similar character to those depicted in Figure 1a.

Given firms' best-response functions as derived in the previous section, straightforward computations establish that reaction curves cross at $(\overline{x}, \underline{x})$ and $(\underline{x}, \overline{x})$, where

$$\overline{x} = \frac{2(a-c)(2-\beta)\left(3\gamma - 4(\beta-1)^2\right)}{27\gamma^2 - 6\gamma(5\beta^2 - 12\beta + 8) + 8(2-\beta)(1-\beta)^2} \tag{3.3}$$

$$\underline{x} = \frac{4(a-c)(1-\beta)(3\gamma - 2(1-\beta)(2-\beta))}{27\gamma^2 - 6\gamma(5\beta^2 - 12\beta + 8) + 8(2-\beta)(1-\beta)^2} \tag{3.4}$$

and $\overline{x} > \underline{x}$ for any $\beta \in (0, 1)$. To guarantee that these two points are in the area of interest

[1] See Appendix 1.

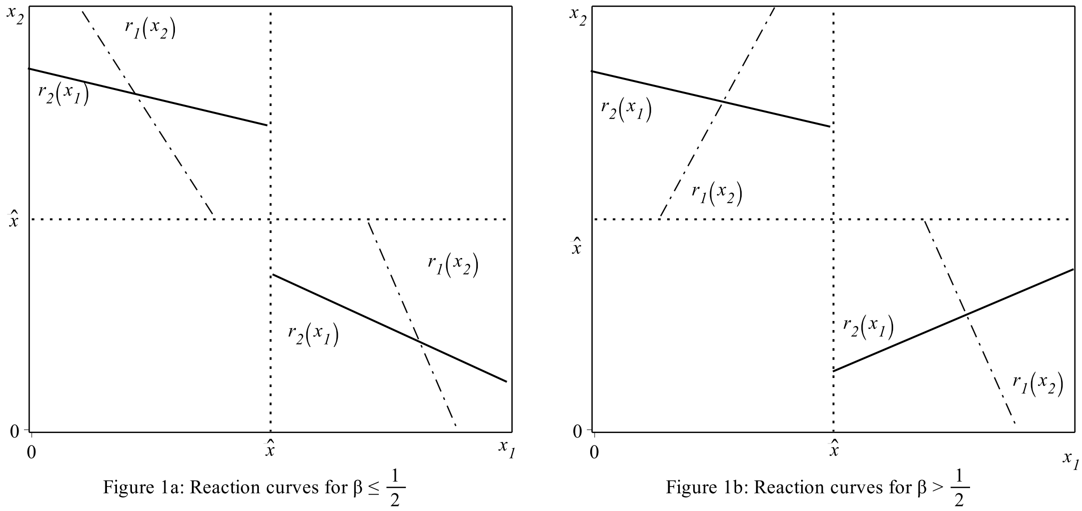

Figure 3.1: Reaction curves for different values of β

$(0, c) \times (0, c)$, we need the following additional assumption

(A4) $9\gamma > I(\beta)$

where

$$
\begin{aligned}
I(\beta) = &\left(\frac{a}{c} - 1\right)(2-\beta) + \left(5\beta^2 - 12\beta + 8\right) \\
&+ \sqrt{\left(\left(\frac{a}{c} - 1\right)(2-\beta) + (5\beta^2 - 12\beta + 8)\right)^2 - 24\frac{a}{c}(2-\beta)(1-\beta)^2}
\end{aligned}
$$

Though firms are ex-ante identical, the next proposition establishes the existence of only asymmetric equilibrium pairs of R&D investments. This gives rise endogenously to a high R&D firm (called the innovator) and a low R&D firm (called the imitator).

Proposition 3.1. *Suppose that (A1) through (A4) hold. The R&D game admits a unique pair of PSNE of the form* $(\overline{x}, \underline{x})$ *and* $(\underline{x}, \overline{x})$.

As in the stochastic version of the model, the equilibrium levels of R&D investments are asymmetric due to the nonconcavity of the net profit function F along the 45° line. By Lemma 3.1, reaction curves jump downward over the diagonal at $\widehat{x}$ as indicated on Figure 3.1 so that, in equilibrium, a firm will always either spend less than its rival so as to free ride on the latter's R&D investment through spillovers, or spend more if the other firm's investment is too low in order to benefit from a competitive advantage over its rival in the product market. Notice that (A4) ensures that the two equilibrium pairs $(\overline{x}, \underline{x})$ and $(\underline{x}, \overline{x})$ are interior solutions. Instead, if

(A1) through (A3) are satisfied, but (A4) is not, we have a boundary equilibrium of the form $\left(\overline{x}^B, \underline{x}^B\right)$ and $\left(\underline{x}^B, \overline{x}^B\right)$ where $\overline{x}^B = c$ and $\underline{x}^B = 4\frac{(a-2c(1-\beta))(1-\beta)}{9\gamma-8(\beta-1)^2}$. Figure 3.2 graphs assumptions (A2) through (A4) in the parameter space $(\beta, 9\gamma)$ and shows whether an interior or a boundary equilibrium prevails.

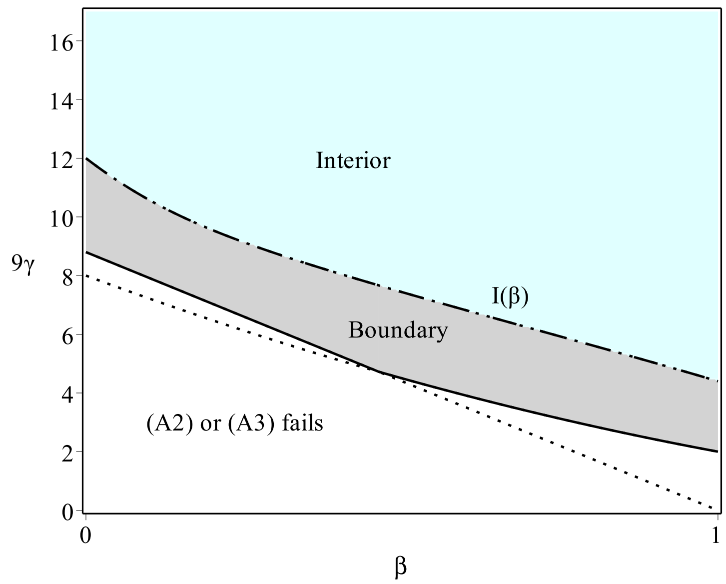

Figure 3.2

3.4 Comparative statics

In this section, we examine how firms' R&D levels of investments and net profits vary with the spillover parameter and the cost of performing R&D.

3.4.1 Autonomous cost reductions

An increase in the spillover parameter impacts firms' R&D expenditures through two channels. Observe first that each payoff is submodular in own R&D level and β.[2] Hence, the reaction functions shift down as β increases, i.e., each firm lowers its R&D investment as its rival's R&D level is held constant (this is the direct effect). Intuitively, greater spillovers lower the innovator's benefit from undertaking R&D as its investment becomes less appropriable, thus enhancing its rival's efficiency in the product market. Likewise, the imitator's incentives to invest in R&D are

[2] In other words, the cross-partial derivative of each payoff with respect to own R&D level and β is ≤ 0.

undermined since its benefit from free riding over the innovator's R&D investment is enhanced through greater spillovers.

An increase in the spillover parameter also indirectly impacts firms' investment decisions. Namely, for $\beta \leq \frac{1}{2}$, both firms' payoff functions are submodular in (x_i, x_j) so that R&D investments are strategic substitutes. Thus, in this range of the spillover parameter, a decrease in one firm's R&D makes its rival optimally react by increasing its own level of investment so that it enjoys a competitive advantage over its rival in the product market. This indirect (or strategic) effect works against the direct effect.

Instead, for $\beta > \frac{1}{2}$, the imitator's payoff function is supermodular in the two R&D levels, so the imitator optimally responds to a decrease in the innovator's level of R&D by lowering its own investment as well. Hence the indirect effect reinforces the direct effect. Intuitively, the larger fraction of R&D that flows to the imitator compensates for the innovator's lower level of investment so that free riding over the innovator's investment is more profitable than undertaking its own R&D.

The next result provides regions of parameters that determine which effect dominates.

Proposition 3.2. *Suppose that the equilibrium pair of R&D decisions $(\overline{x}, \underline{x})$ is interior and let $\bar{\Gamma}$ and $\underline{\Gamma}$ be as depicted in Figure 3.3. Then the following holds:*

(i) *The innovator's propensity to invest in R&D is increasing in β if $9\gamma \in \bar{\Gamma}$, otherwise it is decreasing.*

(ii) *The imitator's propensity to invest is increasing in β if $9\gamma \in \underline{\Gamma}$, otherwise it is decreasing.*

(iii) *Total R&D investments are decreasing in β.*

Not surprisingly, total R&D expenditures are nonetheless decreasing in β, suggesting that the eventual rise in one firm's level of R&D does not compensate for its competitor's lower investment in response to a change in the spillover parameter. Figure 3.3 displays the regions of parameters $\bar{\Gamma}, \underline{\Gamma}$ for which firms' equilibrium levels of R&D increase with the spillover rate.

Nevertheless, if either demand is high enough relative to initial unit costs or R&D cost is sufficiently convex, then the direct effect dominates for both firms: R&D expenditures globally decrease with the spillover rate. In particular, if $\frac{a}{c} > 4$ or $9\gamma > 16$ then the innovator's

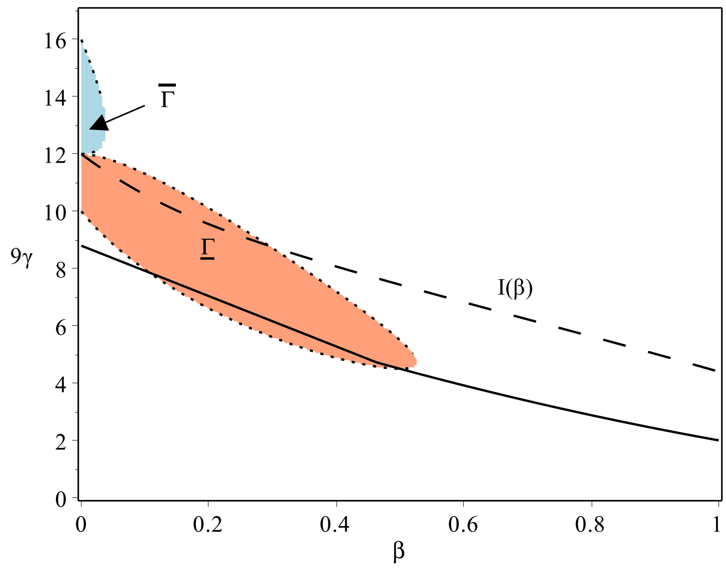

Figure 3.3

equilibrium level of R&D is always decreasing in β. Likewise, if $\frac{a}{c} > 2.7$ or $9\gamma > 12$, then the imitator's investment in equilibrium $\underline{x}$ is always decreasing in β.

3.4.2 Profits

Next, we examine how firms' overall profits vary in both the magnitude of spillovers and the cost of R&D. Again, two opposite effects need to be considered. On the one hand, as said earlier, greater spillovers lower the imitator's incentives to invest in R&D in the first stage so as to freely benefit from the innovator's effort. But on the other hand, decreasing its R&D expenditures exacerbates its production cost disadvantage relative to the innovator at the competition stage, and thus negatively impacts its second-stage profit. Similarly, an increase in β alters the innovator's willingness to conduct R&D as its investment becomes less appropriable. However, a decrease in R&D reduces its competitive advantage over its rival, which in turn undermines its profit at the second stage. The next result characterizes regions of parameters for which either effect dominates.

Proposition 3.3. *Let $\bar{\Gamma}'$ and $\underline{\Gamma}'$ be as indicated in Figure 3.4. Assume that the equilibrium pair of R&D decisions $(\overline{x}, \underline{x})$ is interior. The innovator's profit is increasing in β for $\beta < 2/3$ and $9\gamma \in \bar{\Gamma}'$, while the imitator's profit is decreasing in β if $9\gamma \in \underline{\Gamma}'$.*

Hence, the innovator's overall profit is increasing with the spillover rate as long as $\beta < 2/3$ and R&D costs are convex enough, meaning that the benefit derived from its competitive advantage over the imitator outweighs the cost of performing R&D. Intuitively, since the imitator lowers its R&D expenditures, asymmetries across firms in terms of production costs increase at the competition stage and strengthen the innovator's competitive advantage. However, when $\beta \geq 2/3$, the innovator's overall profit falls as β increases. As the imitator freely benefits from a larger share of the innovator's investment, the efficiency gap between them reduces and negatively impacts the innovator's second-stage profit. As for the imitator, greater spillovers reduce its R&D investment which exacerbate its lack of efficiency relative to the innovator in the product market competition. Notably, its profit decreases with β when the cost of conducting R&D is such that $9\gamma \in \underline{\Gamma}'$. Otherwise, its benefit from free riding over its rival's investment compensates for its competitive disadvantage.

In particular, observe that if $9\gamma > 16$, that is, when the cost of R&D is sufficiently convex, *both* firms are better off with positive spillovers as compared to no spillovers. This is due to the fact that the imitator reduces its investment and the benefit from free riding over its rival's R&D expenditures outweighs the decrease in profit associated with its efficiency loss. As for the innovator, while its R&D effort becomes less appropriable, its competitive advantage in the product market compensates. Figure 3.4 provides a partition of the parameter space which fully illustrates these comparative statics.

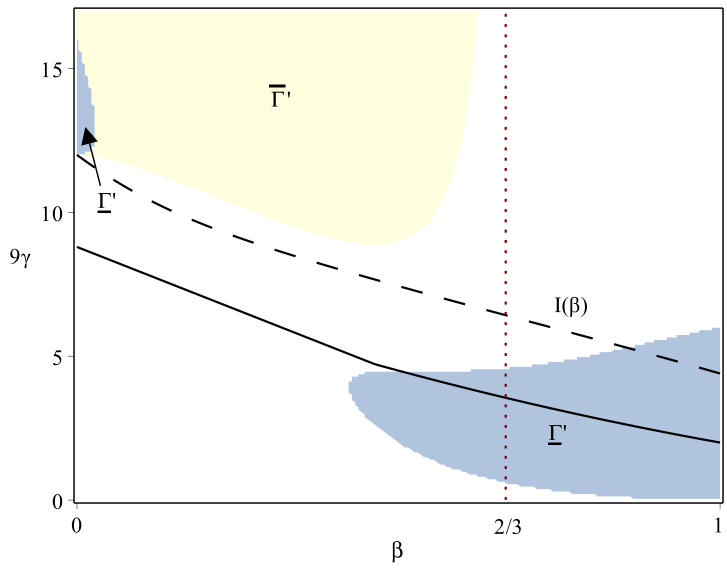

Figure 3.4

Finally, we consider how profits vary with the cost of R&D. Let $\pi(c)$ denote each firm's profit without any cost reduction, which simply equals per firm Cournot equilibrium profit with symmetric unit costs, i.e. $\pi(c) = \frac{1}{9}(a-c)^2$. Straightforwardly, both the innovator's and the imitator's profits tend to $\pi(c)$ as $\gamma \to \infty$. The next two figures illustrate how each firm's equilibrium profit varies as the cost of R&D increases for different values of the spillover rate β.

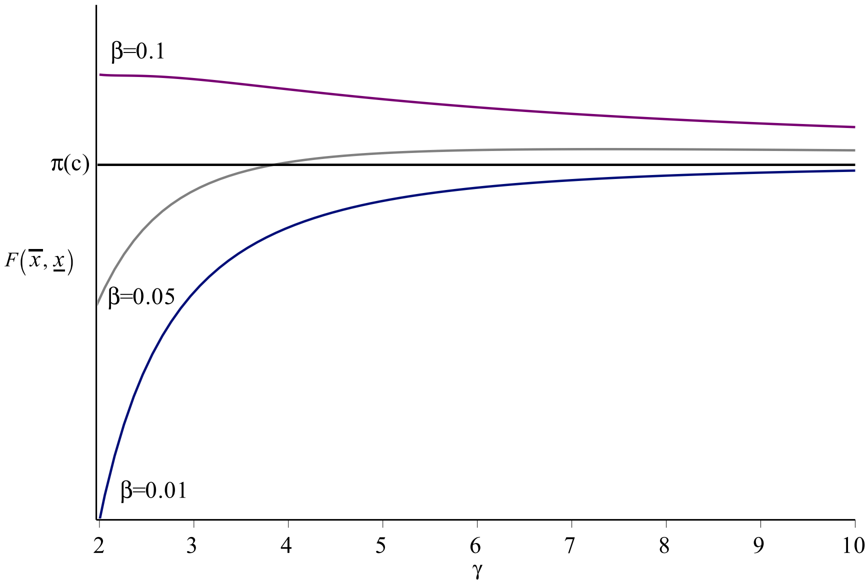

Figure 3.5a

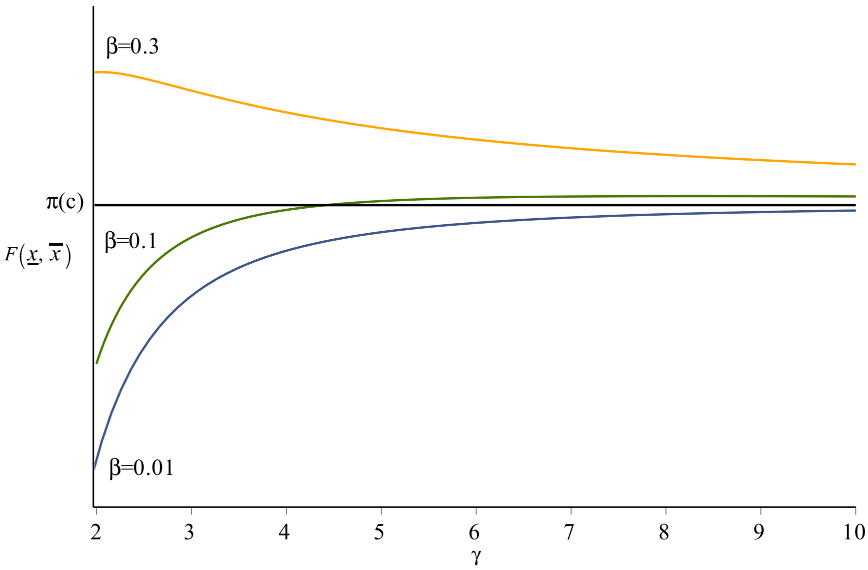

Figure 3.5b

3.5 R&D cooperation

In this section, we examine R&D cooperation by means of a joint lab formation, which allows firms to jointly appropriate the outcome of R&D investments, while sharing the associated cost.Under this configuration, the joint lab therefore chooses a level of R&D that maximizes the sum of firms' profits, net of its cost. That is, the problem of the joint lab is

$$\max_{x\in[0,c]} \tilde{F}(x) = \frac{2}{9}(a-c+x)^2 - \frac{\gamma}{2}x^2$$

which yields the following per firm level of investment

$$x_J = \begin{cases} 4\frac{a-c}{9\gamma-4} & \text{if } 9\gamma > 4\frac{a}{c} \\ c & \text{otherwise} \end{cases}$$

Figure 3.6 shows whether an interior or a boundary equilibrium prevails under the joint lab formation.

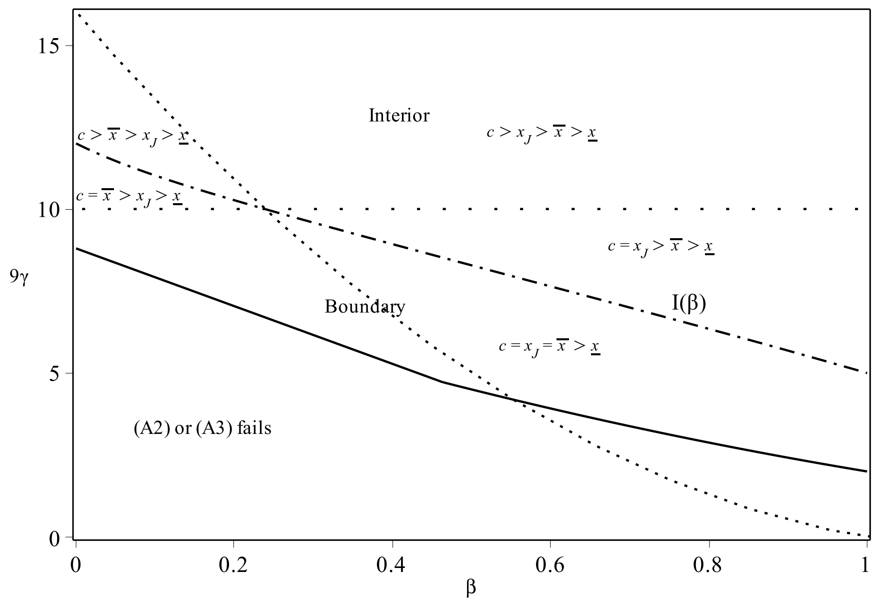

Figure 3.6

Straightforwardly, the joint lab's optimal cost reduction coincides in both the deterministic and the stochastic models as spillovers vanish under this type of R&D cooperation. The next proposition provides a comparison of the joint lab's R&D investment with those of the noncooperative game (as depicted on Figure 3.6).

Proposition 3.4.

(i) $x_J < \bar{x}$ *if* $9\gamma < 4(1-\beta)(4-3\beta)$ *and* $x_J > \bar{x}$ *otherwise*

(ii) $x_J > \underline{x}$

This result firstly says that the innovator invests more in R&D than the joint lab if the spillover parameter and the R&D costs are low enough. Intuitively, in the noncooperative setting, the prospect of efficiency gains when competing in the product market with a weaker rival boosts R&D investments in the first stage if these two conditions are satisfied. Conversely, its incentives to exert R&D effort are undermined if either the associated cost is large or the fraction of its cost reduction that spills over the imitator is high. Consequently, in this case, the joint lab reaches a higher level of cost reduction by both splitting the cost of undertaking R&D among firms and annihilating the free-rider issue.

Not surprinsingly, the level of R&D performed by the imitator in the noncooperative case is instead strictly lower than the joint lab's optimal cost reduction for any R&D cost and any spillover rate since R&D competition leaves scope for free riding over the innovator's investment through the existence of spillovers. Our next result states that, for interior solutions, the total effective cost reduction achieved by means of cooperation via a joint lab is greater than in the noncooperative case.

Proposition 3.5. *If both $(\overline{x}, \underline{x})$ and x_J are interior, then total effective cost reductions achieved under the joint lab formation dominate those of the noncooperative setting.*

Next, we examine the impact of R&D cooperation on firms' equilibrium profit. The joint lab's profit in equilibrium is given by

$$\tilde{F}(x_J) = \begin{cases} 2\frac{(a-c)^2\gamma}{(9\gamma-4)} & \text{if } 9\gamma > 4\frac{a}{c} \\ \frac{4a^2-9c^2\gamma}{18} & \text{otherwise} \end{cases}$$

so that each firm gets $\frac{1}{2}\tilde{F}(x_J)$. The following table provides a full comparison of the innovator's equilibrium profit for interior and boundary equilibria in both settings.

		R&D cooperation through a joint lab	
		Interior $(9\gamma > 4\frac{a}{c})$	Boundary $(9\gamma < 4\frac{a}{c})$
R&D competition	Interior $(9\gamma > I(\beta))$	$\frac{1}{2}\tilde{F}(x_J) > F(\overline{x}, \underline{x})$	$\frac{1}{2}\tilde{F}(c) > F(\overline{x}, \underline{x})$
	Boundary $(9\gamma \leq I(\beta))$	$\frac{1}{2}\tilde{F}(x_J) \gtreqless F\left(\overline{x}^B, \underline{x}^B\right)$	$\frac{1}{2}\tilde{F}(c) > F\left(\overline{x}^B, \underline{x}^B\right)$

Observe first that the innovator is strictly better off cooperating with its rival whenever the interior equilibrium would otherwise prevail in the noncooperative setting since the joint lab formation allows to share both the cost and the results of R&D investments, while suppressing spillovers. The superiority of cooperation is nonetheless jeopardized whenever $4\frac{a}{c} < 9\gamma \leq I(\beta)$. In this region of parameters, R&D competition enables the innovator to stand out from its rival in terms of efficiency gains in the product market, so that its higher profit at the competition stage outweighs both the cost of undertaking R&D and the losses associated with a lower appropriability of its investment. Notably, the innovator strictly prefers to incur substantial R&D expenditures the lower the spillover parameter, so that asymmetries in terms of unit costs are exacerbated at the competition stage. For instance, letting $a = 2.2$, $c = 1$, $\beta = 0.1$ and $\gamma = 1.16$, we have that $\frac{1}{2}\tilde{F}(x_J) = 0.259 < 0.274 = F\left(\overline{x}^B, \underline{x}^B\right)$.

Likewise, the next table compares the imitator's equilibrium profit under both regimes.

		R&D cooperation through a joint lab	
		Interior ($9\gamma > 4\frac{a}{c}$)	Boundary ($9\gamma < 4\frac{a}{c}$)
R&D competition	Interior ($9\gamma > I(\beta)$)	$\frac{1}{2}\tilde{F}(x_J) > F(\underline{x}, \overline{x})$	$\frac{1}{2}\tilde{F}(c) \gtreqless F(\underline{x}, \overline{x})$
	Boundary ($9\gamma \leq I(\beta)$)	$\frac{1}{2}\tilde{F}(x_J) > F\left(\underline{x}^B, \overline{x}^B\right)$	$\frac{1}{2}\tilde{F}(c) > F\left(\underline{x}^B, \overline{x}^B\right)$

The imitator strictly prefers cooperating with the innovator unless $I(\beta) < 9\gamma < 4\frac{a}{c}$, in which case R&D competition may be superior as it enables the imitator to freely benefit from the innovator's investment through spillovers. More specifically, free riding becomes more profitable than sharing the cost of undertaking R&D with the innovator for large values of the spillover parameter. To see this, let $a = 5$, $c = 1$, $\beta = 0.95$ and $\gamma = 1.38$. Then, we have that $\frac{1}{2}\tilde{F}(c) = 2.43$, while $F(\underline{x}, \overline{x}) = 2.5$.

Even though firms inherit the same cost structure under a joint lab, whereby strengthening competition and disseminating firms' profits in the product market, cooperation through a joint lab allows firms to share R&D costs thereby avoiding the inefficiencies associated with the free-rider issue inherent to the noncooperative setting. The next result establishes that the latter effect dominates the former, whereby making the industry strictly better off when cooperating through a joint lab.

Proposition 3.6. *$\tilde{F}(c) \geq F(\overline{x}, \underline{x}) + F(\underline{x}, \overline{x})$ and $\tilde{F}(x_J) \geq F\left(\overline{x}^B, \underline{x}^B\right) + F\left(\underline{x}^B, \overline{x}^B\right)$.*

3.6 Welfare analysis

In this section, we characterize the second-best welfare maximizing solution and examine the social costs of imposing equal treatment among firms. Assume, w.l.o.g., that $x_1 \geq x_2$, and define social welfare in the usual way as the sum of firms' profit and consumer surplus, i.e.

$$
\begin{aligned}
W(x_1, x_2) =& \frac{1}{18}\left[8(a-c)\left((a-c)+x_1(\beta+1)+x_2(1-\beta)\right)\right. \\
& \left. - x_1^2\left(9\gamma+14\beta-11\beta^2-11\right) - x_2^2\left(9\gamma-11(\beta-1)^2\right) + 2x_1x_2(11\beta-7)(1-\beta)\right]
\end{aligned}
\tag{3.5}
$$

Consider a benevolent central planner with a second best mandate, i.e., one that can decide on R&D investments without intervening into the market competition. We first examine the case where the planner imposes symmetric R&D expenditures across firms. In this case, the optimal symmetric per firm investment level is given by $x_W = 4\frac{(a-c)}{(9\gamma-4)}$, which surprisingly coincides with that of the joint lab, that is, $x_W = x_J$. However, since the cost of undertaking R&D is no longer split among firms, it directly follows that the total welfare achieved through cooperation by means of a joint lab is strictly higher.

Whether total welfare achieved with the central planner's intervention is superior to that of the noncooperative setting instead depends on the magnitude of R&D costs. Two conflicting effects need to be considered. On the one hand, imposing symmetric R&D investments implies that firms face the same unit cost when competing in the product market, which in turn leads to profit dissemination. On the other hand, since $x_W = x_J$, total effective cost reductions are higher under the intervention of the central planner (see Proposition 3.5) so that consumers benefit from a lower price. The following result characterizes regions of parameters for which either effect dominates.

Proposition 3.7. *Total welfare ranks as follows:*

$$
W(jointlab) \geq \begin{cases} W(\bar{x}, \underline{x}) > W(x_W, x_W) & \textit{if}\ 9\gamma < K_4 \\ W(x_W, x_W) > W(\bar{x}, \underline{x}) & \textit{otherwise} \end{cases}
$$

where

$$
K_4 = \frac{1}{2}\left(43\beta^2 - 102\beta + 55\right) + \frac{1}{2}\sqrt{1057 - 4212\beta + 5870\beta^2 - 3396\beta^3 + 697\beta^4}
$$

Not surprisingly, total welfare when firms cooperate through a joint lab exceeds that of the noncooperative setting. Indeed, both the innovator and the imitator get a strictly higher profit when cooperating (cf. section 3.5). Moreover, since the second-best welfare maximizing symmetric R&D investments coincide with those of the joint lab, it follows that Proposition 3.5 applies. Namely, total effective cost reductions are higher than those achieved in the noncooperative setting so that aggregate production costs are lower whenever a central planner intervenes in the R&D game. It directly follows that firms charge a lower price, thereby increasing consumer surplus.

Corollary 3.1. $CS\left(x_W, x_W\right) \geq CS(\bar{x}, \underline{x})$

Next, we relax the assumption that the social planner imposes equal treatment across firms. Namely, the problem of the social planner is now to choose a pair of (possibly asymmetric) R&D investments that maximizes total welfare, as given by Eq. (3.5), that is

$$(x_1^W, x_2^W) \in \underset{(x_1,x_2)\in[0,c]^2}{\operatorname{argmax}} W(x_1, x_2) \tag{3.6}$$

Intuitively, one would expect the global argmax of social welfare to be asymmetric, as a result of the well-known fact that Cournot equilibrium industry profit is convex in firms unit costs. In other words, industry profit tends to be higher when firms are asymmetric in terms of unit costs, and this property is inherited by social welfare (Salant and Shaffer, 1998 and 2001).

Indeed, this intuition is confirmed by the solution, as it may be easily verified that the optimal investment levels are given by

$$\begin{aligned} x_1^W &= \frac{4\left(a-c\right)\left(\gamma\left(\beta+1\right)-2\left(1-\beta\right)^2\right)}{\left(9\gamma^2-2\gamma\left(11\beta^2-18\beta+11\right)+8\left(1-\beta\right)^2\right)}, \\ {x_2}^W &= 4\frac{\left(a-c\right)\left(\gamma-2\left(1-\beta\right)\right)\left(1-\beta\right)}{\left(9\gamma^2-2\gamma\left(11\beta^2-18\beta+11\right)+8\left(1-\beta\right)^2\right)}. \end{aligned}$$

and that they satisfy $0 < x_2^W < x_1^W < c$ if $9\gamma > \max\{18(1+\beta), Z_1\}$, where

$$\begin{aligned} Z_1 =& 2\frac{a}{c}\left(1+\beta\right)-\left(11\beta-9\right)\left(1-\beta\right) \\ &+\frac{1}{c}\sqrt{\left(11\beta-9\right)^2\left(\beta-1\right)^2+4(\frac{a}{c})^2\left(\beta+1\right)^2+4\frac{a}{c}\left(\beta-1\right)\left(11\beta^2-16\beta+9\right)} \end{aligned}$$

Since symmetric choices of R&D levels are one option that the social planner has in the optimization problem (3.6), it is clear that $W(x_1^W, x_2^W) > W(x_W, x_W)$, as is indeed easily verified by direct calculation. Nevertheless, despite its sub-optimality, the constrained-symmetric solution may well be of substantial real-life interest, since implementing an asymmetric solution on a priori identical firms may well be politically infeasible. It would be akin to forging a national champion out of two equally efficient firms.

Our next result compares the second-best welfare maximizing asymmetric R&D investments with those of the noncooperative setting, as well as the associated cost reductions.

Proposition 3.8. *The second-best welfare maximizing asymmetric R&D investments satisfy the following:*

(i) $x_1^W > \bar{x}$ *and* $x_2^W > \underline{x}$ *if* $\beta > \frac{2}{3}$, *while* $x_2^W < \underline{x}$ *if both* $\beta \leq \frac{2}{3}$ *and* $9\gamma > \frac{2(1-\beta)(23\beta-11-11\beta^2)}{(3\beta-2)}$

(ii) $(1+\beta)x_1^W + (1-\beta)x_2^W > (1+\beta)\bar{x} + (1-\beta)\underline{x}$

Part (ii) of this result is not surprising, since it simply confirms for the particular setting at hand a well-known general fact about innovation in general: That the market typically undersupplies R&D, due to well-established market failures, in particular to the imperfectly appropriable nature of process R&D here. Thus even a second-best social planner would typically choose to generate higher levels of effective R&D.

Nevertheless, it is noteworthy that for small spillover rates, the social planner would actually dictate a lower R&D level for the imitator. The intuition for this finding is that the social planner is more apt than the non-cooperative solution to take advantage of the aforementioned asymmetry premium for social welfare, and thus more prone to a higher dispersion in R&D levels.

Furthermore, while total welfare achieved under symmetric regulation is inferior to that induced by the joint lab formation, the next result instead states that asymmetric regulation is superior from a welfare point of view whenever the spillover parameter is high enough.

Proposition 3.9. *Total welfare induced by the asymmetric second-best welfare maximizing R&D investments satisfies the following:*

(i) $W(x_1^W, x_2^W) > W(x_W, x_W)$

(ii) $W(x_1^W, x_2^W) > W(jointlab)$ *if either* $\beta \geq \frac{1}{\sqrt{2}}$ *or* $\beta < \frac{1}{\sqrt{2}}$ *and* $9\gamma < Z_2$ *where*

$$Z_2 = (1-\beta)\,\frac{(7\beta-11)-\sqrt{193\beta^2-154\beta+49}}{(2\beta^2-1)}$$

3.7 Deterministic versus stochastic spillover process

In this section, we provide a comparison of our results with those of the stochastic version of the model by Amir and Wooders (1999) in terms of equilibrium levels of R&D investments, profits and welfare. A brief description of their model is provided in Appendix 1. Importantly, while the spillover parameter β denotes the portion of the innovator's R&D investment that flows over to the imitator with certainty in our model, β (resp. $1-\beta$) instead denotes the probability that full (resp. no) spillovers occur in AW. As such, our framework can *a priori* be thought of as the certainty equivalent of the model presented in AW.

3.7.1 Assumptions

To begin with, let us briefly recall assumptions made by AW.

(AW1) $a > 2c$

(AW2) (i) $9\gamma > 4\frac{a}{c}(1-\beta)$, (ii) $9\gamma > 8-6\beta$

Note that (AW1) and (AW2) (i) are identical to (A1) and (A3) in our model, while (AW2) (ii) implies our assumption (A2) for any $\beta \in [0,1]$. Furthermore, they assume that

(AW4) $9\gamma > I_{AW}(\beta)$ where

$$I_{AW}(\beta) = 6(1-\beta)+\frac{a}{c}(2-\beta)+\sqrt{\left(6(1-\beta)-\frac{a}{c}(2-3\beta)\right)^2+8\beta\frac{a}{c}\left(\frac{a}{c}-2\right)(1-\beta)}$$

(AW4) (resp. (A4)) ensures the interiority of the equilibrium pairs of R&D investments in the stochastic (resp. deterministic) model. Figure 3.7 depicts these two interiority conditions in the parameter space and shows that an interior equilibrium obtains more often when the spillover process is deterministic.

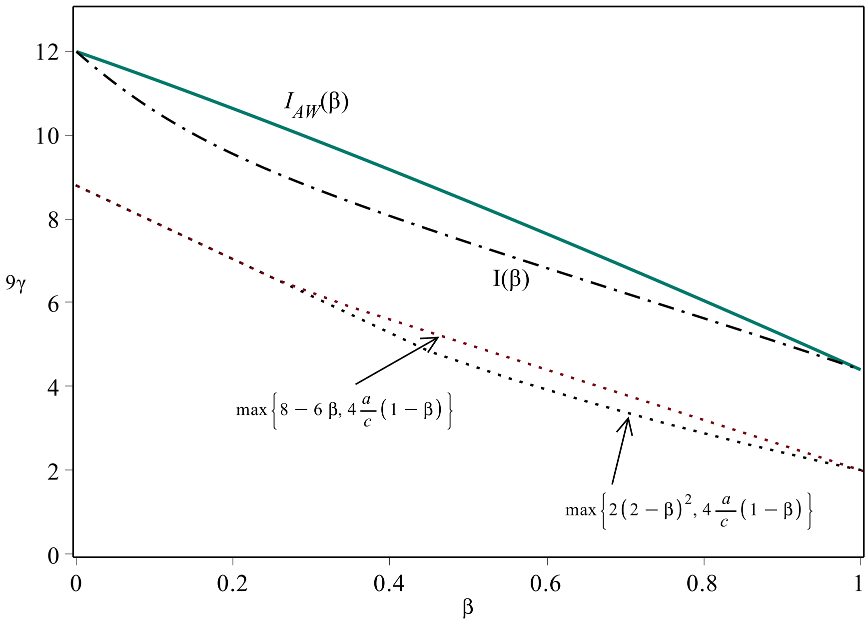

Figure 3.7

3.7.2 R&D investments and input shares

We first examine the role of uncertainty in the spillover process on both firms' equilibrium levels of R&D expenditures. In this respect, the next result states that the innovator (resp. imitator) invests more (resp. less) in AW so that the spread between firms' equilibrium levels of R&D investments is larger when the spillover process is stochastic.

Proposition 3.10. *Equilibrium R&D investments rank as follows*

$$\underline{x}_{AW} < \underline{x} < \overline{x} < \overline{x}_{AW}$$

The intuition underlying this result is that, because of uncertainty about whether spillovers will occur, the innovator has stronger incentives to invest in R&D since it will enjoy an important competitive advantage over its rival at the competition stage in the event where there is no spillover. Instead, in a deterministic environment, the innovator knows that a fraction of its investment will freely benefit to its rival with certainty, whereby reducing asymmetries in terms of unit costs between them.

Similarly, in a stochastic environment, there is a positive probability that the imitator can appropriate the *full* cost reduction of its rival. Thus, in hope of free-riding over the innovator, its incentives to exert costly R&D effort are weakened as compared to the deterministic setting.

Another interesting aspect in which the deterministic and the stochastic models differ is in the total level of R&D performed by the industry, and in the associated total cost reductions.

Proposition 3.11.

(i) *Total R&D investments when the spillover process is deterministic dominate those of the stochastic model.*

(ii) *Total effective cost reductions are higher in the stochastic model if either $\beta > \frac{3}{5}$ or $9\gamma < \frac{12(1-\beta)(3-\beta)}{3-5\beta}$.*

This result states that the stochastic spillover process is more efficient than the deterministic one since the industry needs to invest less to achieve larger total cost reductions for a significant range of parameters. This comes from the dispersion between the equilibrium R&D inputs of the innovator and the imitator which is much higher in the stochastic model from Proposition 3.10. In particular, for large values of β, total cost reductions under uncertainty substantially dominate those achieved in the deterministic setting for *any* γ. Furthermore, this domination still holds for lower values of the spillover parameter provided that the cost of R&D is not too high.

Next, we examine how a change in the spillover parameter and R&D costs affect the innovator's share of R&D investment, and then contrast these comparative statics in both the deterministic and the stochastic environments. Assuming that the equilibrium pair of R&D decisions $(\overline{x}, \underline{x})$ is interior, the innovator's R&D input share is given by

$$\bar{s} \hat{=} \frac{\overline{x}^2}{\underline{x}^2 + \overline{x}^2} = \frac{\left(3\gamma - 4(\beta-1)^2\right)^2 (2-\beta)^2}{9\gamma^2(5\beta^2 - 12\beta + 8) - 24\gamma(3\beta-4)(\beta-2)(\beta-1)^2 + 32(\beta-2)^2(\beta-1)^4}$$

In the special case of no spillover (*i.e.*, $\beta = 0$), the innovator's R&D input share simply reduces to $\frac{1}{2}$. Otherwise, it is decreasing with the cost of R&D γ, and as γ tends to infinity, the innovator's R&D input share goes to $\frac{(2-\beta)^2}{((2-\beta)^2+4(1-\beta)^2)}$, which is the same as in AW. It is instead increasing in β if $9\gamma > I_4(\beta)$ where

$$I_4(\beta) = 12(1-\beta)\left(1-\beta^2+\beta\right)$$

In the stochastic version of the model, the innovator's R&D input share is instead given by

$$\frac{\overline{x}_{AW}^2}{\underline{x}_{AW}^2+\overline{x}_{AW}^2}=\frac{\left(8\left(1-\beta\right)\left(3-2\beta\right)-9\gamma\left(2-\beta\right)\right)^2}{128\left(\beta-1\right)^2\left(2\beta-3\right)^2+\gamma^2 81\left(5\beta^2-12\beta+8\right)+\gamma 144\left(\beta-1\right)\left(3\beta-4\right)\left(2\beta-3\right)}$$

and is larger than in the deterministic case if $9\gamma > I_{AW}(\beta)$. The idea is that, because there is a positive probability that no spillover occurs, the innovator has stronger incentives to provide a larger share of R&D investment so as to become more efficient when competing in the product market, via increased asymmetry across firms' cost structure. Instead, the deterministic nature of spillovers in our model makes the innovator's investment to account for a lower share of total R&D in the industry as a fraction of its investment will freely benefit to the imitator with certainty, whereby making its rival more competitive in the second stage relative to the stochastic model. Finally, note that the range of R&D costs for which the innovator's R&D input share is decreasing in β is smaller in the deterministic case than in the stochastic version of AW.

3.7.3 Profit comparison

We finally turn to the comparison of firms' overall profits in the two versions of the model. In this respect, our next result states that the innovator is weakly better off when the spillover process is stochastic.[3]

Proposition 3.12. *The innovator's profit in AW is strictly higher than in the deterministic case for any $\beta \in (0,1)$.*

This result can be understood in terms of two effects that reinforce each other. Observe first that each firm's overall profit function is strictly convex in its rival's unit cost. Since the imitator's actual unit cost in our model *a priori* equals its expected unit cost in AW, it follows that the innovator strictly prefers a lottery on the imitator's unit cost over its certaintly equivalent. Hence, the innovator is better off when the spillover process is stochastic. Furthermore, in the event where no spillover takes place, the innovator's large profit associated with its substantial production cost advantage over its rival more than compensates for a lower profit when full spillovers occur due to a perfectly symmetric cost structure. Figure 3.8 below illustrates the

[3]It is easy to see that, if either $\beta = 0$ or $\beta = 1$, the two models become equivalent, and there is no meaningful comparison to perform.

innovator's profit in both models as a function of β for given parameters' values.

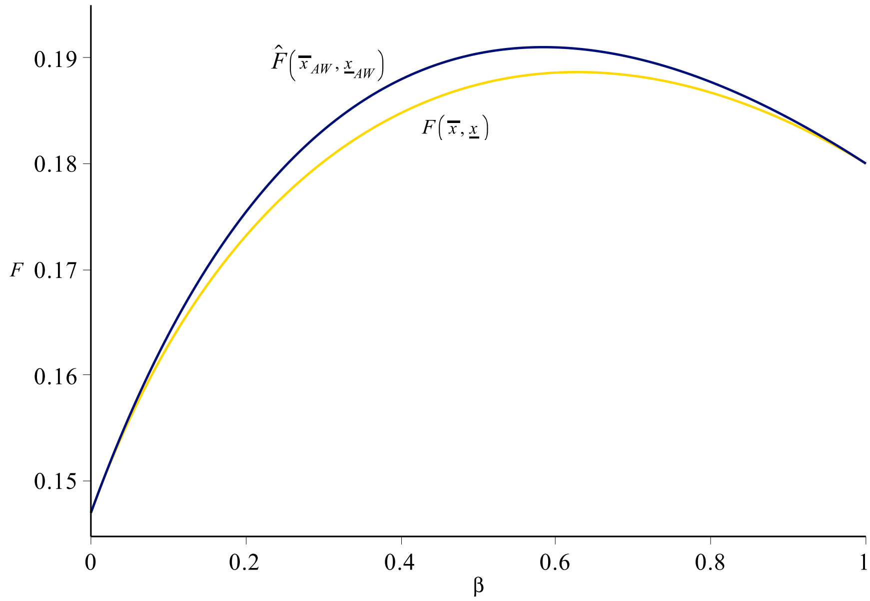

Figure 3.8

Regarding the imitator, the result is not as clear-cut and notably depends on the probability that spillovers occur. Notice that the imitator's *expected* effective cost reduction in the stochastic model amounts to

$$\beta \overline{x}_{AW} + (1-\beta)\underline{x}_{AW} = \underline{x}_{AW} + \beta(\overline{x}_{AW} - \underline{x}_{AW})$$

whereas in the deterministic setting, its effective cost reduction is equal to $\underline{x} + \beta(\overline{x} - \underline{x})$. From Proposition 3.10, we have that $\underline{x}_{AW} < \underline{x}$ and $\overline{x}_{AW} - \underline{x}_{AW} > \overline{x} - \underline{x}$. Thus, when the probability that spillovers occur is small, the imitator's expected effective cost reduction in AW is lower than the effective cost reduction in the deterministic setting. The imitator's competitive disadvantage relative to the innovator being exacerbated, its expected overall profit is therefore lower in the stochastic framework. Conversely, when the probability that full spillovers occur is large enough, the imitator's expected overall profit is larger in the stochastic model as it invests less but still achieves a greater expected effective cost reduction. The following example illustrates the imitator's profit comparison in both models for different values of β.

Example. Let $a = 2.2$, $c = 1$ and $\gamma = 2$. If $\beta = 1/4$, we have that $F(\underline{x}_{AW}, \overline{x}_{AW}) = 0.162\,77 < F(\underline{x}, \overline{x}) = 0.163\,00$. Instead, if $\beta = 3/4$, then $F(\underline{x}_{AW}, \overline{x}_{AW}) = 0.195\,03 > F(\underline{x}, \overline{x}) = 0.192\,93$.

We conclude this section by illustrating firms' joint profit for both models as a function of β

for given parameters' values. Figure 3.9 shows that the industry's total expected profit is greater in the stochastic model than in the deterministic one for any β.

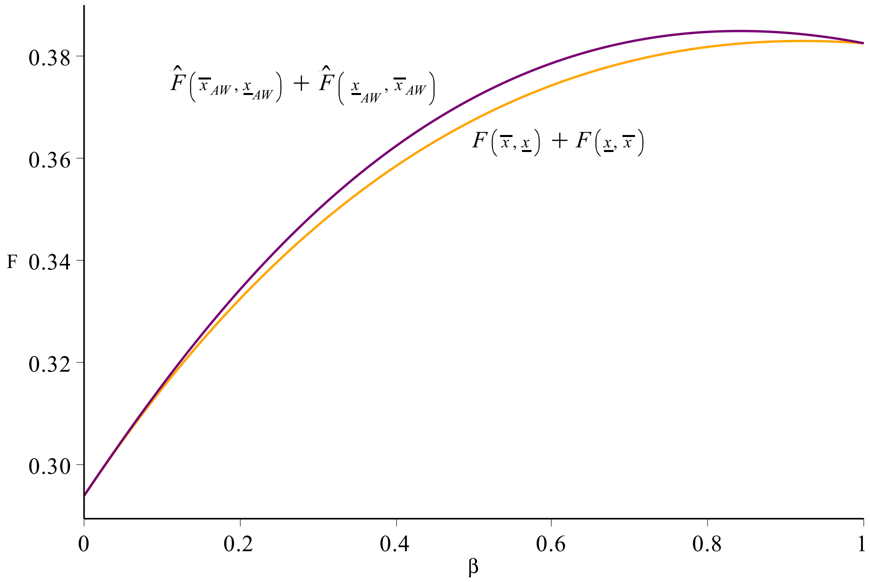

Figure 3.9

3.7.4 Welfare comparison of the noncooperative solution

Throughout this subsection, we suppose that assumptions (A1) through (A4) hold so that the equilibrium pairs of R&D investments are interior. Our next result concerns the impact of a rise in the spillover parameter and the cost of performing R&D onto consumer surplus.

Proposition 3.13. *Consumer surplus is decreasing in both β and γ.*

Since R&D investments and the resulting total effective cost reductions decrease in both the spillover parameter and the cost of undertaking R&D (see Proposition 3.2), it follows that aggregate production costs increase with these two parameters, thereby harming consumers through a higher price.

Therefore, we shall now examine how the nature of the spillover process impacts consumers. Since consumer surplus increases as aggregate production costs decline, it directly follows from Proposition 3.11. that consumers are better off in AW whenever the probability that full spillovers occur is high enough. In fact, it may be verified that $CS(\overline{x}_{AW}, \underline{x}_{AW}) < CS(\overline{x}, \underline{x})$ for small values of β , whereas $CS(\overline{x}_{AW}, \underline{x}_{AW}) > CS(\overline{x}, \underline{x})$ for larger values of β. For instance, suppose that $a = 2.2, c = 1, \gamma = 2$. If $\beta = 1/4$, we have that $CS(\overline{x}_{AW}, \underline{x}_{AW}) = 0.483\,93 <$

$CS(\overline{x}, \underline{x}) = 0.484\,30$, while if $\beta = 3/4$, then $CS(\overline{x}_{AW}, \underline{x}_{AW}) = 0.426\,52 > CS(\overline{x}, \underline{x}) = 0.425\,06$. Nevertheless, the next result states that total welfare in AW is still higher than in our model.

Proposition 3.14. $W(\overline{x}_{AW}, \underline{x}_{AW}) \geq W(\bar{x}, \underline{x})$

This result can be understood as follows. Observe first that since the spread between R&D investments in AW is higher than in our model, total surplus is strictly higher, were the sum of unit costs constant across both models (see Salant and Shaffer, 1999). From Proposition 3.11, total effective cost reduction achieved through the stochastic spillover process dominates that of the deterministic one if either β is large or R&D costs are low enough so that aggregate production costs are lower in AW, which further accentuates the superiority of the stochastic version of the model in terms of total surplus. However, if either of these two conditions fails, the deterministic model exhibits lower aggregate production cost. Proposition 3.14 therefore suggests that the gains associated with a higher heterogeneity across firms' cost structure outweighs the losses in terms of higher aggregate production costs.

3.8 Conclusion

We examine the standard symmetric two-period R&D model with a deterministic one-way spillover structure. Though firms are ex-ante identical, one obtains a unique asymmetric equilibrium of R&D investments, so that the roles of R&D innovator and imitator are endogenously determined. This establishes a simple link between the nature of the R&D process in an industry –including the associated spillover –and the emergence of inter-firm heterogeneity in that industry. We analyze the impact of a change in the spillover parameter and R&D costs on firms' levels of R&D and profits. R&D cooperation through a joint lab is also examined, and we find that the innovator sometimes invests more in R&D than the joint lab, and the industry's total profit is sometimes higher than under the joint lab.

We then provide a welfare analysis in which we examine the social costs of imposing symmetric R&D invesments among firms. Finally, the impact of uncertainty about the appropriability of firms' R&D investments is analyzed and our results suggest that both the spread between firms' investments and the industry's total cost reductions are higher in the stochastic framework. Firms are better off when spillovers are uncertain for a wide range of parameters due to increased asymmetries in terms of their unit cost structure in the product market competition.

3.9 Appendix

3.9.1 The AW model

Here, we briefly describe the model of Amir and Wooders (1999). For more details, the reader is referred to the original paper.

In the first stage, firms simultaneously choose their unit-cost reduction $x_i \in [0, c]$, $i = 1, 2$, with associated R&D cost $\frac{\gamma}{2}x_i^2$. Know-how only flows from the more R&D intensive firm to its rival, namely, spillovers are unidirectional. In contrast to our model, the spillover process in AW is stochastic in the sense that, with probability β, *full* spillover occurs, while with probability $1-\beta$, there is no spillover. Assuming that, say, $x_i \geq x_j$, *expected* effective cost reductions are then given by

$$X_i = x_i \quad \text{and} \quad X_j = \beta x_i + (1-\beta)x_j$$

Firms then compete à la Cournot in the second stage with linear inverse demand function $P(q_1, q_2) = a - q_1 - q_2$.

To avoid confusions, we denote by $\hat{F}(x_i, x_j)$ firm i's overall profit function in AW when its own investment is x_i and its rival's is x_j, i.e.

$$\hat{F}(x_i, x_j) = \begin{cases} \frac{\beta}{9}\left(a - c + x_i\right)^2 + \frac{1-\beta}{9}\left(a - c + 2x_i - x_j\right)^2 - \frac{\gamma}{2}x_i^2 & \text{if} \quad x_i \geq x_j \\ \frac{\beta}{9}\left(a - c + x_j\right)^2 + \frac{1-\beta}{9}\left(a - c + 2x_i - x_j\right)^2 - \frac{\gamma}{2}x_i^2 & \text{if} \quad x_i \leq x_j \end{cases} \tag{3.7}$$

and it may be easily verified that $\hat{F}$ is globally submodular in (x_i, x_j). The equilibrium pair of interior PSNE is then given by

$$\bar{x}_{AW} = (a-c)\frac{4(1-\beta)(9\gamma - 12 + 8\beta) + 18\beta\gamma}{[9\gamma - 8(1-\beta)].(9\gamma - 8 + 6\beta) - 16(1-\beta)^2} \tag{3.8}$$

$$\underline{x}_{AW} = (a-c)\frac{4(1-\beta)(9\gamma - 12 + 8\beta)}{[9\gamma - 8(1-\beta)].(9\gamma - 8 + 6\beta) - 16(1-\beta)^2} \tag{3.9}$$

3.9.2 Proofs

Proof of Lemma 3.1

The reaction function r as given by Eq. (3.2) is not continuous since, letting $x^{S1} = r_1(x^{S1})$ for $x_1 \geq x_2$ and $x^{S2} = r_1(x^{S2})$ for $x_1 \leq x_2$, one obtains

$$\begin{aligned} x^{S1} &= \frac{2(a-c)(2-\beta)}{(9\gamma - 2(2-\beta))}, \\ x^{S2} &= \frac{4(a-c)(1-\beta)}{(9\gamma - 4(1-\beta))} \end{aligned}$$

with $x^{S1} > x^{S2}$. Hence, the reaction function has a downward jump, and letting $\widehat{x}$ be the solution to $U(r_1(\widehat{x}), \widehat{x}) = L(r_1(\widehat{x}), \widehat{x})$, we have that

$$\widehat{x} = \frac{(a-c)\left(\sqrt{1 + \frac{2\beta(4-3\beta)}{\left(9\gamma - 2(\beta-2)^2\right)}} - 1\right)}{\left(2\beta - 1 + (1-\beta)\sqrt{1 + \frac{2\beta(4-3\beta)}{\left(9\gamma - 2(\beta-2)^2\right)}}\right)}$$

Furthermore, $\hat{x}$ is unique since both U and L are monotonic in x_2. U is decreasing in x_2 for all $\beta \in [0, 1]$, while L either increases with x_2 for $\beta > 1/2$ or decreases with x_2 slower than U. $\square$

Proof of Proposition 3.1

A lengthy but simple computation establishes that $\bar{x}, \underline{x}$ as given by Eq. (3.3) and (3.4) satisfy $\overline{x} > \widehat{x}$ if $9\gamma > I_1$ and $\underline{x} < \widehat{x}$ if $9\gamma > I_2$, where

$$\begin{aligned} I_1 &= \left(5\beta^2 - 12\beta + 8\right) + \sqrt{13\beta^4 - 48\beta^3 + 68\beta^2 - 48\beta + 16}, \\ I_2 &= \left(5\beta^2 - 12\beta + 8\right) + \sqrt{73\beta^4 + 224\beta^2 - 216\beta^3 - 96\beta + 16}, \end{aligned}$$

Straightforward computations then establish that $I(\beta) > I_1$ and $I(\beta) > I_2$. Hence, if assumptions (A1) through (A4) hold, the pair of PSNE $(\overline{x}, \underline{x})$ and $(\underline{x}, \overline{x})$, with $\bar{x}, \underline{x}$ as given by Eq. (3.3) and (3.4), is unique. $\square$

Proof of Proposition 3.2

(i) Differentiating $\bar{x}$ with respect to β yields

$$\frac{\mathrm{d}\bar{x}}{\mathrm{d}\beta} = -\frac{6\,(a-c)\,\gamma L_1}{\left(3\gamma\,[9\gamma - 2\,(5\beta^2 - 12\beta + 8)] + 8\,(2-\beta)\,(\beta-1)^2\right)^2}$$

where $L_1 = 27\gamma^2 - 6\gamma\left(13\beta^2 - 28\beta + 14\right) + 8\left(5\beta^2 - 12\beta + 8\right)(\beta-1)^2$. Thus, $\frac{\mathrm{d}\bar{x}}{\mathrm{d}\beta} > 0$ if and only if $L_1 < 0$, which holds if $(\beta, 9\gamma)$ is such that $9\gamma \in \bar{\Gamma} = (\gamma_1(\beta), \gamma_2(\beta))$, where

$$\begin{aligned}\gamma_1(\beta) =& 13\beta^2 - 28\beta - \sqrt{260\beta^2 - 112\beta - 200\beta^3 + 49\beta^4 + 4} + 14\\ \gamma_2(\beta) =& 13\beta^2 - 28\beta + \sqrt{260\beta^2 - 112\beta - 200\beta^3 + 49\beta^4 + 4} + 14\end{aligned}$$

(ii) In a similar fashion, differentiating $\underline{x}$ with respect to β yields

$$\frac{\mathrm{d}\underline{x}}{\mathrm{d}\beta} = -\frac{12\,(a-c)\,\gamma L_2}{\left(3\gamma\,[9\gamma - 2\,(5\beta^2 - 12\beta + 8)] + 8\,(2-\beta)\,(\beta-1)^2\right)^2},$$

where $L_2 = 27\gamma^2 - 6\gamma\left(4\beta^2 - 14\beta + 11\right) + 4\,(\beta-1)\left(18\beta - 15\beta^2 + 5\beta^3 - 10\right)$. Thus, $\frac{\mathrm{d}\underline{x}}{\mathrm{d}\beta} > 0$ if and only if $L_2 < 0$, which holds if $(\beta, 9\gamma)$ is such that $9\gamma \in \underline{\Gamma} = (\gamma_3(\beta), \gamma_4(\beta))$, where

$$\begin{aligned}\gamma_3(\beta) &= \left(4\beta^2 - 14\beta + 11\right) - \sqrt{28\beta - 112\beta^2 + 128\beta^3 - 44\beta^4 + 1}\\ \gamma_4(\beta) &= \left(4\beta^2 - 14\beta + 11\right) + \sqrt{28\beta - 112\beta^2 + 128\beta^3 - 44\beta^4 + 1}\end{aligned}$$

(iii) The sum of autonomous cost reductions is decreasing in β since

$$\frac{\mathrm{d}}{\mathrm{d}\beta}(\overline{x} + \underline{x}) = \frac{6\gamma\,(a-c)\,L_3}{\left(3\gamma\,[9\gamma - 2\,(5\beta^2 - 12\beta + 8)] + 8\,(2-\beta)\,(\beta-1)^2\right)^2}$$

where $L_3 = -81\gamma^2 + 6\gamma\left(21\beta^2 - 56\beta + 36\right) + 16\,(1-\beta)\left(19\beta - 16\beta^2 + 5\beta^3 - 9\right) < 0$ for all $\beta \in [0, 1]$. $\square$

Proof of Proposition 3.3

(i) From Eq. (3.1), the innovator's equilibrium profit is given by

$$F(\overline{x}, \underline{x}) = \frac{\left(a - c + \overline{x}\,(2-\beta) - \underline{x}\,(1-\beta)\right)^2}{9} - \frac{\gamma}{2}\overline{x}^2$$

Differentiating totally $F(\overline{x}, \underline{x})$ with respect to β yields

$$\frac{\mathrm{d}}{\mathrm{d}\beta} F(\overline{x}, \underline{x}) = -\frac{12\left(3\gamma - 4\,(1-\beta)^2\right)\gamma^2\,(a-c)^2\,L_4}{\left(3\gamma\left[9\gamma - 2\left(5\beta^2 - 12\beta + 8\right)\right] + 8\,(2-\beta)\,(\beta-1)^2\right)^3}$$

where $L_4 = 27\gamma^2\,(3\beta - 2) - 6\gamma\left(13\beta^3 - 48\beta^2 + 58\beta - 22\right) + 8\left(5\beta^3 - 16\beta^2 + 20\beta - 10\right)(1-\beta)^2$. Under assumption (A4), the denominator is strictly positive and $3\gamma > 4\,(1-\beta)^2$ for any $\beta \in [0,1]$. Therefore, we have that $\frac{\mathrm{d}}{\mathrm{d}\beta} F(\overline{x}, \underline{x}) > 0$ if and only if $L_4 > 0$, which holds for $\beta < 2/3$ and $9\gamma \in \bar{\Gamma}' = (\gamma_5(\beta), \infty)$, where

$$\gamma_5(\beta) = \frac{22 - 58\beta + 48\beta^2 - 13\beta^3 + \sqrt{4 + 88\beta - 572\beta^2 + 1348\beta^3 - 1540\beta^4 + 864\beta^5 - 191\beta^6}}{(2-3\beta)}.$$

(ii) Likewise, the imitator's equilibrium profit is given by

$$F(\underline{x}, \bar{x}) = \frac{\left(a - c + 2\underline{x}\,(1-\beta) + \bar{x}\,(2\beta - 1)\right)^2}{9} - \frac{\gamma}{2}\underline{x}^2$$

Differentiating totally $F(\underline{x}, \bar{x})$ with respect to β yields

$$\frac{\mathrm{d}}{\mathrm{d}\beta} F(\underline{x}, \bar{x}) = \frac{12\,(a-c)^2\left(3\gamma - 2\,(1-\beta)\,(2-\beta)\right)\gamma^2 L_5}{\left(3\gamma\left[9\gamma - 2\left(5\beta^2 - 12\beta + 8\right)\right] + 8\,(2-\beta)\,(\beta-1)^2\right)^3},$$

where $L_5 = 27\gamma^2 + 6\gamma\left(-14 + 40\beta - 45\beta^2 + 16\beta^3\right) - 8\left(-8 + 24\beta - 27\beta^2 + 10\beta^3\right)(1-\beta)^2$. Again, if (A4) holds, the denominator is strictly positive and $3\gamma > 2\,(1-\beta)\,(2-\beta)$ for any $\beta \in [0,1]$. Hence, the imitator's profit is strictly decreasing in β if and only if $L_5 < 0$, which holds for $9\gamma \in \underline{\Gamma}' = (\gamma_6(\beta), \gamma_7(\beta))$, where

$$\begin{aligned}
\gamma_6(\beta) &= \left(45\beta^2 - 40\beta - 16\beta^3 + 14 - \sqrt{868\beta^2 - 160\beta - 1936\beta^3 + 2177\beta^4 - 1200\beta^5 + 256\beta^6 + 4}\right) \\
\gamma_7(\beta) &= \left(45\beta^2 - 40\beta - 16\beta^3 + 14 + \sqrt{868\beta^2 - 160\beta - 1936\beta^3 + 2177\beta^4 - 1200\beta^5 + 256\beta^6 + 4}\right)
\end{aligned}$$

$\square$

Proof of Proposition 3.4

(i) We have that

$$x_J - \bar{x} = \frac{4(a-c)}{9\gamma-4} - \frac{2\,(a-c)\,(2-\beta)\left(3\gamma - 4\,(\beta-1)^2\right)}{27\gamma^2 - 6\gamma\,(5\beta^2 - 12\beta + 8) + 8\,(2-\beta)\,(1-\beta)^2}$$

Simplifying and rearranging then leads to $sign(x_J - \bar{x}) = sign\left(28\beta + 9\gamma - 12\beta^2 - 16\right) < 0$ if and only if $9\gamma < 4\,(1-\beta)\,(4-3\beta)$.

(ii) Similarly,

$$x_J - \underline{x} = \frac{4(a-c)}{9\gamma-4} - \frac{4\,(a-c)\,(1-\beta)\,(3\gamma - 2\,(1-\beta)\,(2-\beta))}{27\gamma^2 - 6\gamma\,(5\beta^2 - 12\beta + 8) + 8\,(2-\beta)\,(1-\beta)^2}$$

so that $sign(x_J - \underline{x}) = sign\left(9\gamma - 2\left(3\beta^2 - 7\beta + 5\right)\right) > 0$ for $9\gamma > I(\beta) > 2\left(3\beta^2 - 7\beta + 5\right)$.

$\square$

Proof of Proposition 3.5

Total cost reductions achieved under cooperation through a joint lab formation dominate those of the noncooperative regime if

$$\begin{aligned}\frac{8(a-c)}{9\gamma-4} >& (1+\beta)\frac{2\,(a-c)\,(2-\beta)\left(3\gamma - 4\,(\beta-1)^2\right)}{27\gamma^2 - 8\,(\beta-2)\,(\beta-1)^2 - 6\gamma\,(5\beta^2 - 12\beta + 8)} \\ &+ (1-\beta)\frac{4\,(a-c)\,(1-\beta)\,(3\gamma - 2\,(\beta-1)\,(\beta-2))}{27\gamma^2 - 8\,(\beta-2)\,(\beta-1)^2 - 6\gamma\,(5\beta^2 - 12\beta + 8)}\end{aligned}$$

$$\Leftrightarrow 9\gamma > \frac{12\,(1-\beta)\,(3-2\beta)}{(3-\beta)}$$

which holds at an interior equilibrium since $I(\beta) > \frac{12(1-\beta)(3-2\beta)}{(3-\beta)}$.

$\square$

Proof of Proposition 3.6

We first check that $\tilde{F}(x_J) > F(\overline{x}_B, \underline{x}_B) + F(\underline{x}_B, \overline{x}_B)$ holds for $4\frac{a}{c} < 9\gamma \leq I(\beta)$. It may be verified that the difference $\tilde{F}(x_J) - F(\overline{x}_B, \underline{x}_B) - F(\underline{x}_B, \overline{x}_B)$ is positive if and only if

$$
\begin{aligned}
&-729c^2\gamma^4 + 162c\gamma^3\left(13c\beta^2 + (2a-26c)\beta + 2a + 13c\right) \\
&-72\left(\left(21-32\beta+16\beta^2\right)(1-\beta)^2c^2 - 2a(1-\beta)\left(-6+6\beta+\beta^2\right)c + 2a^2\right)\gamma^2 \\
&+32(1-\beta)^2\left(8(1-\beta)^2c^2 + 2a(7-8\beta)(1-\beta)c + a^2\left(9-2\beta+\beta^2\right)\right)\gamma - 128a^2(1-\beta)^4 < 0
\end{aligned}
$$

Numerical computations then establish that this inequality holds for $9\gamma \in [4\frac{a}{c}, I(\beta)]$.

In a similar fashion, we now shall show that $\tilde{F}(c) > F(\overline{x}, \underline{x}) + F(\underline{x}, \overline{x})$ for $I(\beta) < 9\gamma < 4\frac{a}{c}$. A lengthy computation establishes that the sign of the difference $\tilde{F}(c) - F(\overline{x}, \underline{x}) - F(\underline{x}, \overline{x})$ is the same as that of L_6, where

$$
\begin{aligned}
L_6 = &-6561c^2\gamma^5 + \left(14\,580c^2\beta^2 - 34\,992c^2\beta + 20\,412c^2 + 5832ac\right)\gamma^4 \\
&+ (972a^2\beta^2 - 1944a^2\beta - 14\,904ac\beta^2 + 34\,992ac\beta c^2 \\
&- 20\,736ac - 8100c^2\beta^4 + 42\,768c^2\beta^3 - 80\,676c^2\beta^2 + 64\,152c^2\beta - 18\,144)\gamma^3 \\
&+ (9072a^2\beta^3 - 2232a^2\beta^4 - 11\,592a^2\beta^2 + 4320a^2\beta + 576a^2 + 11\,664ac\beta^4 - 56\,160ac\beta^3 + 101\,520ac\beta^2 \\
&- 81\,216ac\beta + 24\,192ac - 4320c^2\beta^5 + 21\,816c^2\beta^4 - 41\,904c^2\beta^3 + 37\,368c^2\beta^2 - 14\,688c^2\beta + 1728c^2)\gamma^2 \\
&+ (1152a^2\beta^6 - 7296a^2\beta^5 + 17\,664a^2\beta^4 - 19\,584a^2\beta^3 + 8064a^2\beta^2 + 1536a^2\beta - 1536a^2 - 2304ac\beta^6 \\
&+ 18\,432ac\beta^5 - 59\,904ac\beta^4 + 101\,376ac\beta^3 - 94\,464ac\beta^2 + 46\,080ac\beta - 9216ac + 576c^2\beta^6 - 4608c^2\beta^5 \\
&+ 14\,976c^2\beta^4 - 25\,344c^2\beta^3 + 23\,616c^2\beta^2 - 11\,520c^2\beta + 2304 + c^2)\gamma \\
&+ \left(256a^2\beta^6 - 2048a^2\beta^5 + 6656a^2\beta^4 - 11\,264a^2\beta^3 + 10\,496a^2\beta^2 - 5120a^2\beta + 1024a^2\right)
\end{aligned}
$$

Numerical computations then demonstrate that $L_6 > 0$ for $9\gamma \in [I(\beta), 4\frac{a}{c}]$. $\square$

Proof of Proposition 3.7

We first shall show that $W(jointlab) > W\left(x_W, x_W\right)$. We have that

$$4\frac{(9\gamma-2)\,(a-c)^2\,\gamma}{(9\gamma-4)^2} > 4\frac{(a-c)^2\,\gamma}{(9\gamma-4)}$$

$$\Leftrightarrow 9\gamma > 3$$

which holds from assumption (A2) and the fact that $9\gamma > 4\frac{a}{c}$.

Next, we establish that $W(jointlab) \geq W(\bar{x}, \underline{x})$. From section 3.5, we have that $\frac{1}{2}F(x_J) > F(\underline{x}, \bar{x})$ and $\frac{1}{2}F(x_J) > F(\bar{x}, \underline{x})$ for $9\gamma > \max\{4\frac{a}{c}, I(\beta)\}$. Hence, it directly follows that the industry's profit under the joint lab formation exceeds that of the noncooperative setting, i.e.

$$F(x_J) > F(\underline{x}, \bar{x}) + F(\bar{x}, \underline{x}) \tag{3.10}$$

Therefore, a sufficient condition for $W(jointlab) \geq W(\bar{x}, \underline{x})$ to hold is that consumer surplus when firms cooperate through a joint lab is higher. The difference $CS\left(x_J, x_J\right) - CS(\bar{x}, \underline{x})$ is given by

$$2\left(\frac{3\gamma\,(a-c)}{(9\gamma-4)}\right)^2 - \frac{18\left(3\gamma+(1-\beta)\,(3\beta-4)\right)^2(a-c)^2\,\gamma^2}{\left(27\gamma^2-6\gamma\left(5\beta^2-12\beta+8\right)-8\,(\beta-2)\,(\beta-1)^2\right)^2}$$

Straightforward computations then establish that $CS\left(x_J, x_J\right) - CS(\bar{x}, \underline{x}) \geq 0$ if and only if $K_6.K_7 \geq 0$, where

$$\begin{aligned} K_6 &= \left(4\,(1-\beta)\left(2\beta^2-9\beta+8\right)+54\gamma^2-3\gamma\left(19\beta^2-45\beta+32\right)\right) \\ K_7 &= \left(3\gamma\,(3-\beta)-4\,(1-\beta)\,(3-2\beta)\right). \end{aligned}$$

Both K_6 and K_7 are positive for all $9\gamma > I(\beta)$. Thus, we have that

$$CS\left(x_J, x_J\right) \geq CS(\bar{x}, \underline{x}) \tag{3.11}$$

Hence, (3.10) together with (3.11) establish the superiority of the joint lab in terms of welfare.

Finally, the difference $W(\bar{x},\underline{x}) - W(x_W, x_W)$ is given by

$$\frac{2\gamma (a-c)^2 \left[162\gamma^3 - -9\gamma^2 \left(41\beta^2 - 96\beta + 56\right) + 3\gamma \left(81\beta^2 - 224\beta + 160\right)(1-\beta)^2 - 32(2-\beta)^2(1-\beta)^4\right]}{\left(27\gamma^2 - 6\gamma(5\beta^2 - 12\beta + 8) - 8(\beta-2)(\beta-1)^2\right)^2}$$
$$- 4\frac{(9\gamma-2)(a-c)^2\gamma}{(9\gamma-4)^2}$$

Simplifying and rearranging, we have that

$$W(\bar{x},\underline{x}) > W(x_W, x_W) \Leftrightarrow 9\gamma \in (K_5, K_4)$$

with K_4 as indicated in the proposition and

$$K_5 = \frac{1}{2}\left(43\beta^2 - 102\beta + 55\right) - \frac{1}{2}\sqrt{1057 - 4212\beta + 5870\beta^2 - 3396\beta^3 + 697\beta^4}$$

where $K_5 < I(\beta)$. □

Proof of Corollary 3.1

The proof is given in that of Proposition 3.7. □

Proof of Proposition 3.8

(i) Upon simplification, we have that the sign of the difference $x_1^W - \overline{x}$ is the same as that of $8(16-11\beta)(1-\beta)^3 + 81\gamma^2 - 18\gamma(1-\beta)(11-9\beta)$, which is strictly positive for $9\gamma > 18(1+\beta)$.

Likewise, it may be easily verified that the sign of $x_2^W - \underline{x}$ is the same as that of $9\gamma(3\beta-2) - 2(1-\beta)\left(23\beta - 11 - 11\beta^2\right)$. This expression is strictly positive if both $\beta > \frac{2}{3}$ and $9\gamma > 18(1+\beta)$, so that $x_2^W > \underline{x}$. Instead, if either $\beta = \frac{2}{3}$, or $\beta < \frac{2}{3}$ and $9\gamma > \frac{2(1-\beta)\left(23\beta-11-11\beta^2\right)}{(3\beta-2)}$, then $x_2^W < \underline{x}$.

(ii) As for total effective cost reductions, straightforward computations establish that $(1+\beta)\overline{x} + (1-\beta)\underline{x} - (1+\beta)x_1^W - (1-\beta)x_2^W < 0$ if $-9(1+\beta)\gamma^2 - 2(1-\beta)\left(-15 + 8\beta + 3\beta^2\right)\gamma + 8(2\beta-3)(1-\beta)^3 < 0$, which holds for any $\beta \in [0,1]$ and $9\gamma > Z_1$. □

Proof of Proposition 3.9

(i) We have that

$$\begin{aligned} W(x_1^W, x_2^W) - W(x_W, x_W) =& \frac{4\left(\gamma - 2(1-\beta)^2\right)(a-c)^2\gamma}{\left(9\gamma^2 - 2\gamma(11\beta^2 - 18\beta + 11) + 8(1-\beta)^2\right)} - 4\frac{(a-c)^2\gamma}{(9\gamma - 4)} \\ =& \frac{16(a-c)^2\beta^2\gamma^2}{(9\gamma-4)\left(9\gamma^2 - 2\gamma(11\beta^2 - 18\beta + 11) + 8(1-\beta)^2\right)} \\ >& 0 \end{aligned}$$

(ii) The difference $W(x_1^W, x_2^W) - W(jointlab)$ is given by

$$\frac{4\left(\gamma - 2(1-\beta)^2\right)(a-c)^2\gamma}{\left(9\gamma^2 - 2\gamma(11\beta^2 - 18\beta + 11) + 8(1-\beta)^2\right)} - 4\frac{(9\gamma-2)(a-c)^2\gamma}{(9\gamma-4)^2}$$

so that $W(x_1^W, x_2^W) - W(jointlab) > 0$ if $16(1-\beta)^2 + 18\gamma^2\left(1 - 2\beta^2\right) - 4\gamma(1-\beta)(11-7\beta) > 0$, which holds if either $\beta < \frac{1}{2}\sqrt{2}$ and $9\gamma < Z_2$, or $\beta \geq \frac{1}{2}\sqrt{2}$ provided that $9\gamma > 18(1+\beta)$. □

Proof of Proposition 3.10

The difference in the innovator's equilibrium R&D investment in both models is

$$\begin{aligned} \bar{x} - \overline{x}_{AW} =& \frac{2(a-c)(2-\beta)\left(3\gamma - 4(\beta-1)^2\right)}{27\gamma^2 - 6\gamma(5\beta^2 - 12\beta + 8) + 8(2-\beta)(1-\beta)^2} \\ & - \frac{4(a-c)(1-\beta)(9\gamma - 12 + 8\beta) + 18\beta\gamma}{[9\gamma - 8(1-\beta)](9\gamma - 8 + 6\beta) - 16(1-\beta)^2} \end{aligned}$$

Simplifying and rearranging yields

$$\bar{x} - \overline{x}_{AW} < 0 \Leftrightarrow -81\gamma^2 + 18\gamma\left(8 - 11\beta + 4\beta^2\right) + 8(5\beta - 6)(1-\beta)^2 < 0$$

which holds for $9\gamma > I(\beta)$ (assumption (A4)). Similarly, the difference in the imitator's equilibrium R&D investment in both models is

$$\begin{aligned}\underline{x} - \underline{x}_{AW} =& \frac{4\,(a-c)\,(1-\beta)\,(3\gamma - 2\,(1-\beta)\,(2-\beta))}{27\gamma^2 - 6\gamma\,(5\beta^2 - 12\beta + 8) + 8\,(2-\beta)\,(1-\beta)^2} \\ &- \frac{4(a-c)(1-\beta)(9\gamma - 12 + 8\beta)}{[9\gamma - 8(1-\beta)](9\gamma - 8 + 6\beta) - 16(1-\beta)^2}\end{aligned}$$

Straightforward calculations then establish that $\underline{x} - \underline{x}_{AW} > 0$ if and only if $9\gamma + 5\beta - 7 > 0$, which is satisfied for $9\gamma > I(\beta)$. □

Proof of Proposition 3.11

(i) The difference $(\overline{x} + \underline{x}) - (\overline{x}_{AW} + \underline{x}_{AW})$ is given by

$$\frac{2(a-c)\left(8\beta^3 - 32\beta^2 - 9\beta\gamma + 40\beta + 12\gamma - 16\right)}{27\gamma^2 - 6\gamma\,(5\beta^2 - 12\beta + 8) + 8\,(2-\beta)\,(1-\beta)^2} - \frac{(a-c)\,[8(1-\beta)(9\gamma - 12 + 8\beta) + 18\beta\gamma]}{[9\gamma - 8(1-\beta)](9\gamma - 8 + 6\beta) - 16(1-\beta)^2}$$

which is positive if and only if $L_8 = 27\gamma + 4\,(7\beta - 9)$ is positive, which holds for $9\gamma > I(\beta)$.

(ii) The difference $(1+\beta)\overline{x} + (1-\beta)\underline{x} - (1+\beta)\overline{x}_{AW} - (1-\beta)\underline{x}_{AW}$ is given by

$$\begin{aligned}&(1+\beta)\frac{2\,(a-c)\,(2-\beta)\left(3\gamma - 4\,(\beta-1)^2\right)}{27\gamma^2 - 8\,(\beta-2)\,(\beta-1)^2 - 6\gamma\,(5\beta^2 - 12\beta + 8)} \\ &+ (1-\beta)\frac{4\,(a-c)\,(1-\beta)\,(3\gamma - 2\,(\beta-1)\,(\beta-2))}{27\gamma^2 - 8\,(\beta-2)\,(\beta-1)^2 - 6\gamma\,(5\beta^2 - 12\beta + 8)} \\ &- (1+\beta)\frac{4(a-c)(1-\beta)(9\gamma - 12 + 8\beta) + 18\beta\gamma}{[9\gamma - 8(1-\beta)](9\gamma - 8 + 6\beta) - 16(1-\beta)^2} \\ &- (1-\beta)\frac{4(a-c)(1-\beta)(9\gamma - 12 + 8\beta)}{[9\gamma - 8(1-\beta)](9\gamma - 8 + 6\beta) - 16(1-\beta)^2}\end{aligned}$$

Simplifying and rearranging, we have that $(1+\beta)\overline{x} + (1-\beta)\underline{x} - (1+\beta)\overline{x}_{AW} - (1-\beta)\underline{x}_{AW} < 0$ if and only if $L_9 = -3\,(3-5\beta)\,\gamma + 4\,(1-\beta)\,(3-\beta) < 0$, which holds if either $\beta > \frac{3}{5}$ or $\beta \leq \frac{3}{5}$ and $9\gamma < 3\frac{4(1-\beta)(3-\beta)}{(3-5\beta)}$. □

Proof of Proposition 3.12

We want to prove that $F(\overline{x}, \underline{x}) < \hat{F}(\overline{x}_{AW}, \underline{x}_{AW})$. To do so, we proceed in two steps: we first show that $F(\overline{x}, \underline{x}) - F(\overline{x}_{AW}, \underline{x}_{AW}) < 0$, and then show that $F(\overline{x}_{AW}, \underline{x}_{AW}) - \hat{F}(\overline{x}_{AW}, \underline{x}_{AW}) < 0$.

1. The difference $F(\overline{x}, \underline{x}) - F(\overline{x}_{AW}, \underline{x}_{AW})$ is given by

$$\frac{(a-c+\overline{x}(2-\beta)-\underline{x}(1-\beta))^2}{9} - \frac{\gamma}{2}\overline{x}^2 < \frac{(a-c+\overline{x}_{AW}(2-\beta)-\underline{x}_{AW}(1-\beta))^2}{9} - \frac{\gamma}{2}\overline{x}_{AW}^2$$

$$\begin{aligned}&\Leftrightarrow 8748\gamma^4 - 243\left(140-172\beta+49\beta^2\right)\gamma^3 + 54\left(872-2076\beta+1696\beta^2-508\beta^3+25\beta^4\right)\gamma^2(1-\beta)^3 \\ &\quad > 48(1-\beta)\left(564-1370\beta+1142\beta^2-343\beta^3+15\beta^4\right)\gamma + 32\left(-168+232\beta-83\beta^2+3\beta^3\right)(1-\beta)^3\end{aligned}$$

which holds from numerical computations.

2. Next, we have that

$$\begin{aligned}F(\overline{x}_{AW}, \underline{x}_{AW}) - \hat{F}(\overline{x}_{AW}, \underline{x}_{AW}) =& \frac{(a-c+\overline{x}_{AW}(2-\beta)-\underline{x}_{AW}(1-\beta))^2}{9} - \frac{\gamma}{2}\overline{x}_{AW}^2 - \frac{\beta}{9}\left(a-c+\overline{x}_{AW}\right)^2 \\ &- \frac{1-\beta}{9}\left(a-c+2\overline{x}_{AW}-\underline{x}_{AW}\right)^2 - \frac{\gamma}{2}\overline{x}_{AW}^2 \\ =& \frac{-1}{9}\beta\left(\underline{x}_{AW}-\overline{x}_{AW}\right)^2(1-\beta) \\ <& 0\end{aligned}$$

Hence, we indeed have that $F(\overline{x}, \underline{x}) < \hat{F}(\overline{x}_{AW}, \underline{x}_{AW})$. □

Proof of Proposition 3.13

Consumer surplus at the noncooperative interior equilibrium is given by

$$CS(\bar{x}, \underline{x}) = \frac{18\left(3\gamma+(1-\beta)(3\beta-4)\right)^2(a-c)^2\gamma^2}{\left(27\gamma^2-6\gamma\left(5\beta^2-12\beta+8\right)-8(\beta-2)(\beta-1)^2\right)^2}$$

Differentiating $CS(\bar{x}, \underline{x})$ with respect to β yields

$$\frac{\mathrm{d}}{\mathrm{d}\beta}CS(\bar{x}, \underline{x}) = \frac{36\gamma^2(a-c)^2K_1K_2}{\left(27\gamma^2-6\gamma\left(5\beta^2-12\beta+8\right)-8(\beta-2)(\beta-1)^2\right)^3}$$

where $K_1 = \left(9\gamma^2(2\beta-3)+6\gamma\left(11\beta^2-24\beta+12\right)-8\left(3\beta^2-8\beta+6\right)(\beta-1)^2\right)$ and

$K_2 = (3\gamma-(1-\beta)(4-3\beta))$. We have that $K_1.K_2 < 0$ since $K_1 < 0$ and $K_2 > 0$ for all $9\gamma > I(\beta)$. Since the denominator is strictly positive, it follows that $\frac{\mathrm{d}}{\mathrm{d}\beta}CS(\bar{x}, \underline{x}) < 0$.

Likewise, we have that

$$\frac{\mathrm{d}}{\mathrm{d}\gamma}CS(\bar{x},\underline{x}) = -\frac{36\,(a-c)^2\,\gamma K_2 K_3}{\left(27\gamma^2 - 6\gamma\,(5\beta^2 - 12\beta + 8) - 8\,(\beta-2)\,(\beta-1)^2\right)^3},$$

where $K_3 = 9\gamma^2\left(\beta^2 - 3\beta + 4\right) - 48\gamma\,(2-\beta)\,(1-\beta)^2 - 8\,(3\beta-4)\,(2-\beta)\,(1-\beta)^3$. Observe that $K_2.K_3 > 0$ since $K_2 > 0$ and $K_3 > 0$ for all $9\gamma > I(\beta)$. Hence, $\frac{\mathrm{d}}{\mathrm{d}\gamma}CS(\bar{x},\underline{x}) < 0$. $\square$

Proof of Proposition 3.14

The sign of $W(\overline{x}_{AW},\underline{x}_{AW}) - W(\bar{x},\underline{x})$ is the same as that of

$$\begin{gathered}50\,301\gamma^4 - 972\left(133 - 163\beta + 47\beta^2\right)\gamma^3 - 108\left(-716 + 1691\beta - 1407\beta^2 + 421\beta^3\right)\gamma^2 \\ -96\,(1-\beta)\left(-186 + 538\beta - 507\beta^2 + 163\beta^3\right)\gamma + 64\left(216 - 325\beta + 113\beta^2\right)(\beta-1)^3\end{gathered}$$

It directly follows from computations that this expression is strictly positive whenever $9\gamma > I(\beta)$. Hence, we indeed have that $W(\overline{x}_{AW},\underline{x}_{AW}) > W(\bar{x},\underline{x})$. $\square$

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Résumé

La première partie de cette thèse est consacrée à l'étude de l'émergence récente de nouveaux acteurs sur le marché des brevets, à savoir les entreprises non productrices, qui acquièrent des brevets sans intention de produire un bien final. D'une part, les chasseurs de brevets cherchent à acquérir des brevets en vue de les monétiser par la menace d'action en contrefaçon de brevet. D'autre part, les agrégateurs défensifs acquièrent des brevets afin de protéger leurs entreprises clientes contre des litiges initiés par les chasseurs de brevets. Nous analysons le comportement stratégique de ces nouveaux intermédiaires dans le processus d'acquisition de brevets et mettons en évidence l'aptitude supérieure des chasseurs de brevets quant au rachat de brevets par rapport aux producteurs. Ensuite, nous examinons l'efficacité du mécanisme de protection proposé par les agrégateurs défensifs contre la menace des chasseurs de brevets envers les producteurs.
Enfin, la dernière partie étudie l'impact des spillovers unidirectionnels dans le contexte des modèles de R&D dans lesquels des firmes à priori identiques investissent dans une innovation réductrice de coût puis sont en concurrence à la Cournot sur le marché des produits. Nous analysons comment les spillovers unidirectionnels et deterministes induisent de l'hétérogénéité entre les firmes dans la concurrence sur le marché des produits, et examinons l'effet de l'incertitude quant à l'appropriation des investissements en R&D sur les incitations des firmes à investir en R&D.

Mots Clés

Innovation, Brevets, Recherche et Développement, Spillovers, Intermédiaires, Litiges

Abstract

The first part of this thesis studies the recent emergence of new actors in the market for patents, namely, non-practicing entities, who acquire patents with no aim to use them to produce a final good. On the one hand, patent assertion entities seek to acquire patents so as to monetize them through the threat of litigation for patent infringement. On the other hand, defensive aggregators acquire patents to provide safety from litigation brought by patent trolls to their affiliated firms. We analyze the strategic behavior of non-practicing entities in the patent acquisition process and highlight patent assertion entities' greater ability to preempt patents as compared to producing firms. Then, we examine the effectiveness of defensive aggregators to protect firms against litigation brought by patent assertion entities.
Finally, the last part instead studies the effects of one-way spillovers in the context of non-tournament models of R&D in which ex-ante identical firms engage in cost-reducing innovation and then compete à la Cournot in the product market. We analyze how a deterministic unidirectional spillover process induces heterogeneity across firms in the product market competition, and examine the impact of uncertainty about the appropriability of R&D investments on incentives to undertake R&D.

Keywords

Innovation, Patents, Research and Development, Spillovers, Intermediaries, Litigation

Numero national de thèse :