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Kai Luo

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### « ANALYSIS AND OPTIMIZATION OF SINGLE AND DUAL SOURCING DECISIONS IN SUPPLY CHAIN »

THESE

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# Abstract

There has been a growing body of evidence that inventory problems in the supply chain are more important than ever. The objective of this research is to develop conceptual, analytical, and managerial models and insights by analyzing a portion of the supply chain made up of a retailer dealing with two suppliers in an uncertain environment. In the first part of this thesis, we consider a single high-end (or perishable) product analyzed over a single period. This first problem has the following characteristics: the product has variable unit price, variable unit production cost, variable unit shortage cost, variable unit salvage value and the product demand is stochastic. For this case, we prove that, under some conditions, the total expected profit function remains concave. Through extensive numerical examples, we show that the optimal solution can be obtained numerically. The product characteristics considered here are particularly useful in the case of reverse logistics. Therefore, we consider a remanufactured product of which the demand is exponentially distributed. We show that by using some relaxed linear parameters, an approximated closed form solution can be obtained.

In a second part of the thesis, we consider settings inspired by the case of large international companies sourcing some of their products from low cost countries. This means very often long delivery lead times and therefore very limited opportunities of replenishments within short time horizons. Here, we focus on the development of models for cases where companies use different strategies with different types of secondary warehouses. Besides the similar product characteristics assumptions of the first part of the thesis, the developed models, developed in this second part, share a common basic structure. This structure is as follows: two products (one sourced locally and the other sourced abroad), a three-period, two-stages, two capacitated suppliers, and a single capacitated retailer. Both analytical and numerical results are provided for this part of the thesis. Important theoretical results and insights are developed for these types of settings. These models can be used as decision-making aid tools in such environments.

Finally, limitations and important open research questions are discussed in the last part of this thesis.

**Keywords:** Inventory management, newsboy problem, stochastic programming, planning, resource allocation, supply chain management, dual-sourcing, two-stage problem, lost sale, periodic-review models, heuristic methods.

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# Chapter 1

## Introduction and Research Problem Statement

There has been a growing body of evidence that inventory problems in the supply chain are more important than ever. We consider settings inspired by the case of large international companies sourcing some of their products from low cost countries. This means very often long delivery lead times and therefore very limited opportunities of replenishments within short time horizons. Here, we focus on the development of models for cases where companies use different strategies with different types of secondary warehouses. The objective of this research is to develop conceptual, analytical, and managerial models and insights by analyzing a portion of the supply chain made up of a retailer dealing with two suppliers in an uncertain environment. In the first part of this thesis, we start with a single product, single period, nonlinear problem.

The single-period inventory management problem, also known as the Newsboy Problem (NB), is used to find the optimal ordering quantity which maximizes the expected profit. For instance, a retailer with order quantities of perishable items for sale in a single season is a typical application of a classical newsboy problem. Here, the retailer purchases the product at a fixed unit cost and sells it at a fixed unit price while the demand during the season is often variable. At the end of the period, all unsold items are salvaged at a fixed unit price much lower than the selling unit price. Clearly, the challenge of the decision maker is to decide the initial stock level before the season in order to maximize expected profit.

A number of extensions to the classical newsboy model are available in the literature, e.g., literature on competitive games, principal-agent, etc. - for a review, see next section. However, restrictive assumptions are still used in these models. i.e., constant unit cost, constant unit price, constant unit salvage value, constant unit shortage cost.

The relaxation of these assumptions are motivated by real-world applications. First, the

unit price decreases often as the observed quantity increases as in the case of agriculture and electronic products. Second, in the case of remanufacturing, the unit production cost increases as the produced quantity increases while the unit price may decrease in the quantity offered.

Work has been done where these assumptions have been relaxed one by one, but we found no work focusing on simultaneous relaxation of all of these assumptions which, of course, increases the complexity of the problem. In this thesis I depart from these practices by relaxing the strong assumptions simultaneously.

In the second part of the thesis, we introduce another supplier and consider the two products, two suppliers, multi-period problem. The developed models, developed in this second part, share a common basic structure. This structure is as follows: two products (one sourced locally and the other sourced abroad), a three-period, two-stages, two capacitated suppliers, and a single capacitated retailer.

In our problem, there are three periods in a selling season. The local supplier is able to deliver the product twice in the selling season, while the far-away supplier can only send the product once due to longer lead time. Details are presented in the next section. For the above two types of companies, the lead time of the far-away produced product is the manufacturing time plus the delivery time and that of the locally produced product is negligible. The lead time is comparable to the selling period. Demand is deterministic for companies using pull strategy while it is stochastic if push strategy is used. In reality, suppliers' production capacities and retailer's warehouse capacity are usually limited. In this case, an uncapacitated secondary warehouse can be rented. It is important to decide the ordering quantity and inventory allocation in each period carefully and intelligently so as to find the best tradeoff between lost sales and inventory costs.

The second part of the thesis is concerned with planning and coordinating the activities of component procurement and end-product assembly, across the supply chain. Motivated by the previous work of Bollapragada and Rao (2006), we consider our decision making for a finite horizon of time, as we believe that high-end and low-end products always exist in a retailer's inventory in the stores. The main idea here is to provide guidance to the management on how many units to order and how much space to allocate to these two classes of products and provide guidance on inventory performance over a period of several months or a year. Our main task is to maximize the total profit for the company, given the

limited inventory capacity, limited suppliers' production capacity and uncertain demand.

In other words, the purpose is to build a framework for this problem so as to (i) provide strategic insights and guiding principles which aid enterprises to manage the procurement of products from two suppliers, and allocate the two products inventory to the existing shelf space (in the second stage problem) (ii) provide insights to the management on the choice of a supplier, based on the customer demand information.

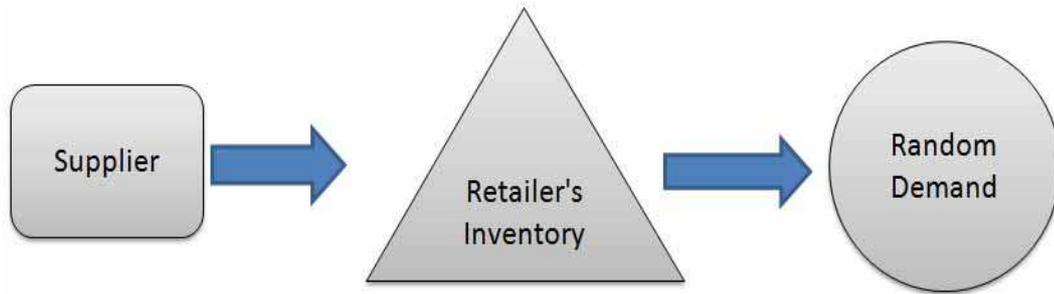
We accomplish the above through (i) researching new optimization models and advanced algorithms to synchronize the procurement of components with inventory allocation and (ii) evaluating performance from multiple perspectives, including a different set of assumptions, and evaluate the different trade-offs involving expected profit, selling price, shortage cost, salvage value, inventory holding cost, inventory capacity, production cost and production capacity.

For the above problem, quantity received in reality could be a part of the sending order due to transportation uncertainty and production uncertainty. For example, shipping delays and quality problems result in the final quantity to be usually lower than the expected quantity due to production uncertainty. Therefore, I extend the model to investigate the case of unreliable suppliers at the end.

## **1.1 Problem I: Single-product Type, Single-supplier, Single-period Non-linear Problem**

The first problem is a single-product type, single-supplier, one-stage nonlinear problem. The firm has only one type of product to sell in a coming season. This product type is assumed to be very expensive product with a very short life cycle time, which is commonly regarded as a Newsboy (News Vendor) problem. An increasingly number of extensions to the classical newsboy model has been developed, e.g. competitive games, principal-agent, etc., being published. More details are also presented in the literature review. However, strong assumptions of the classical newsboy model have been carried to these extensions. Today, the increased complexity of the worldwide operations requires the understanding of the consequences of relaxing the strong assumptions (i.e., constant unit cost, constant unit price, constant unit salvage value, constant unit shortage cost) of the classical newsboy model.

Figure 1.1 The Model of Problem I.



The need for this relaxation of assumptions can be justified by the following examples. First, in reality, unit price often decreases in observed quantity, like for agriculture products and electronic products. Second, in the case of luxury goods, unit production cost could increase in produced quantity. In the case of remanufacturing, the production unit cost is increasing in the quantity produced (see Ferguson et al. [16]). The unit price of products like monitor panels decreases as the offered quantity increases. Third, in cases of perishable goods, unit salvage value decreases both in time and in left-over quantity. Fourth, in cases of several substantial products with different prices, unit shortage penalty for lost sales could not be constant. Finally, unlimited production capacity is an unrealistic and strong assumption. Through these above cases, we see that assumptions of constant unit production cost and constant unit salvage value may not be appropriate.

Moreover, there is a need to understand what happens when all these mentioned classical assumptions are relaxed simultaneously.

Therefore, the first problem is centered around understanding the newsboy and pushing the current restrictions of the classical assumptions to a new level where unit price, unit production cost, unit salvage value and unit shortage cost are assumed to be non-linear. Therefore, the objective is to provide, through better modeling of the problem, optimization techniques that will allow more realistic solutions and better managerial decisions.

## 1.2 Problems II and III: Periodic Review, Two-product Types, Two-supplier, Two-stage Problem

We present in Table 1.1 the relationship between the cases and the models in Problem II and Problem III. For **Retailer 1** and **2**, we build model 1.1, 2.1, 2.2 under deterministic demand assumption. All demand information is known in period 0. We also build model

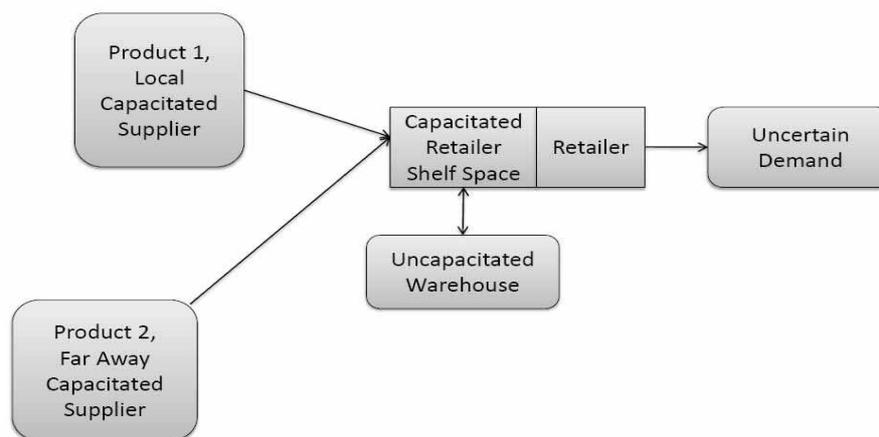
3.1, 3.2, 4.1, 4.2 to investigate cases where the demand is stochastic. We assume that the demands in different periods are independently distributed and are not necessarily identical. This helps us to investigate both stationary and non-stationary demand in our experiments.

Table 1.1 Model List-1

		Deterministic Demand	Stochastic Demand
Cheaper 2 <sup>nd</sup> Warehouse	Retailer 1	Model 1	Model 3.1
	Retailer 2		Model 3.2
Expensive 2 <sup>nd</sup> Warehouse	Retailer 1	Model 2.1 and 2.2	Model 4.1
	Retailer 2		Model 2.1

These models share a common basic model of which the structure is presented in Figure 1.2. It is a two-product three-period two-capacitated suppliers with different fixed lead times single-capacitated-retailer two-stage model.

Figure 1.2 The Basic Models.

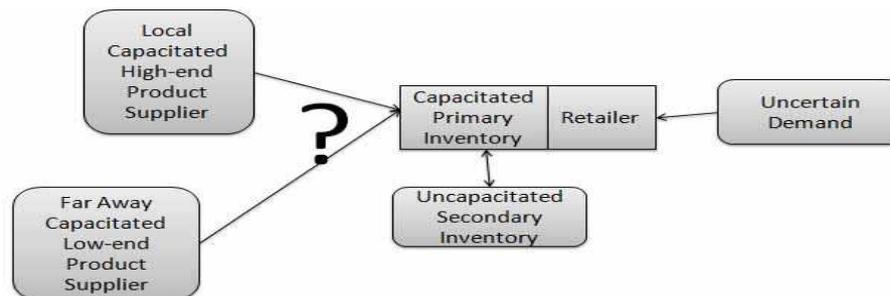


As shown in Figure 1.2, the retailer owns limited shelf space (primary warehouse) with capacity  $K$  and he/she can place any units to a warehouse (rented secondary warehouse). There are three periods in a selling season. In period 0, the retailer orders some units of **product 1** and **product 2** from the capacitated local supplier and the capacitated far away supplier, respectively. The delivery time of product 1 is negligible, but it takes the retailer one period to prepare the product. In period 1, product 2 arrives and are placed to the warehouse(s) in order to satisfy the demand in period 2. Based on the realized demand of the product 1 in period 1, the retailer orders some units of product 1 from the local supplier in

order to satisfy the related demand in period 2. The retailer also has to decide the allocation of the following items between the two warehouses: possible leftovers of product 1, newly arrived product 2 and product 1.

### 1.3 Problem IV: Two-product Types, Two-unreliable-supplier, Two-stage, Three-period Problem

Figure 1.3 The Model of Problem IV.



For the above problem, quantity received in reality could be a part of the sending order due to transportation uncertainty and production uncertainty. For example, shipping delays and quality problems result in the final quantity to be usually lower than the expected quantity due to production uncertainty. On the other hand, however, due to delivery delay of the previous period (ex., a cargo of units), the retailer may receive something much more than scheduled. The retailer pays for the quantity ordered in the case of in-house production while he/she pays for the quantity received in the case of external supply. The quantity received is a proportion (may greater than 1) of the relative ordered quantity. The high-end product has shorter supply lead times with low variability, while the low-end product has longer supply delivery lead times with high variability. The determinant of the expensive product's supply lead times is primarily the manufacturing lead time (as the transportation time is negligible, in comparison). On the other hand, the cheaper product's supply lead time is mainly the transportation lead times (as the manufacturing lead time is negligible in comparison with this). Due to uncertain demand and unreliable diverse suppliers (ex., random lead time, unreliable suppliers, etc), cutting ordering quantity increases shortage.

In this research, we model two significant sources of uncertainty, component supply and end-product demand, as impacted by dynamic competitive factors. Motivated by the previous work of Bollapragada and Rao (2006), the research objective of this problem is to

evaluate the supply chain risks in the context of a two product, two-stage system. One product is an high-end product, while the other is a low-end product. The expensive one is manufactured close to the retail stores, while the cheaper one is made far away.

We consider our decision making for a finite horizon of time, as we believe that high-end and low-end products always exist on a retailer's inventory in the stores. In this part, our goal is to build a frame work to evaluate supply risks for each of the two products, that (i) provide strategic insight and guiding principles that help enterprises manage the procurement of products from diverse suppliers, and the allocation of products to the existing shelf space, subject to both competitive demand and supply uncertainty (ii) provide insights to the management on the choice of a supplier, based on the customer demand information and the associated risk characteristics of the suppliers.

We accomplish the above through (i) researching new optimization models and advanced algorithms to synchronize the procurement of components for inventory allocation. The algorithms offer a new way for resource planning that captures some of the dynamics between uncertain system factors such as workload on the availability of the products from the suppliers and the resulting lead-time. (ii) Define and evaluate diverse performance metrics. We evaluate performance from multiple perspectives, including a different set of assumptions, and evaluate the different trade-offs involving expected cost, supply chain risk, and customer service measures.

## Chapter 2

### Literature Review

#### 2.1 Problem I: Single-product Type, Single-supplier, Single-period Nonlinear Problem

In this part, we focus on a supply chain made up of one supplier and one retailer. The single-period inventory problem has been extensively studied over the last decades. In his pioneering paper, Scarf [36] found the optimal policy structure for the single supplier/single retailer problem (newsboy problem). Via the convexity of the objective function, he showed that it is of a threshold form, by following the research of Arrow et al [1] and Bellman [5]. Veinott[43] proved that the expected cost rate is quasi-convex under the assumption that the holding cost and the shortage cost are linear. Many extensions of the newsboy problem are available (see Khouja [22] for a comprehensive literature review).

More recently, several papers dealt with complex supply chain problems related to the classical newsboy problem, under standard linearity assumptions for the cost and profit functions. For example, Lin et al. [31] developed a multi-location newsboy model with an expected profit criterion. Chen [11] investigated auctioning supply contracts based on the classical newsboy problem, where a buyer and several potential suppliers determine their purchase quantity and price. Koulamas [24] considered a newsboy problem with revenue sharing and channel coordination between a retailer and a manufacturer. Netessine and Rudi [32] analyzed a competition model with a wholesaler and several independent retailers, each of them acting as a classical newsboy.

The above papers, as well as many others, assumed that the unit price, the unit ordering cost, the unit salvage cost and the unit shortage cost are constant. As a consequence, one

can take advantage of the resulting concavity of the profit function to get the global optimal solution. However, these assumptions are often not realistic. To overcome this, some work has been done where some assumptions were relaxed on a one by one basis. Gerchak et al. [17] considered a model including linear shortage cost, linear holding cost and random yield. In their paper, production capacity is random and described by a known probability distribution. They proved that the expected total cost function is unimodal.

Kaj [35] discussed the non-linear shortage cost in two special cases and proved that the objective function remains concave. Bayindir et al. [4] solved a deterministic inventory problem with a linear shortage cost rate, a linear salvage value, a non-linear increasing holding cost and a piecewise linear concave production cost.

Table 2.1 Position of the research problem with respect to the literature review

Model	Unit price	Total Production Cost	Total Salvage Value	Total Shortage cost	Other constraints	Expected Profit Function
News-boy 1955	Constant	Linear	Linear	Linear		Concave
Kaj 2001		Linear	Linear	Two special non-linear functions in shortage		Quasi-Concave
Bayindir 2007	Constant	Piecewise linear Concave	Linear	Linear		Unimodal
Karlin's Model I	constant	Linear	Non-Linear Function in salvage	Non-Linear Function in shortage		concave
Karlin's Model II	constant	Convex	Non-Linear Function in salvage	Non-Linear Function in shortage		Unimodal
Karlin's Model III	constant	Concave	Linear	Linear	Fixed Order Level	Unimodal
Karlin's Model IV			General Non-Linear Function	General Non-Linear Function	Random Ordering Level	Unimodal
Our Model (Theorem I)	Concave-monotony decreasing in ordering quantity	Convex, non-decreasing in ordering quantity	Concave, non-decreasing in salvage quantity	Convex, non-decreasing in shortage quantity		Concave in the interval containing the global optimal solution

Karlin [21] investigated newsboy models combining some non-linear cost/profit mecha-

nisms, but he assumed constant unit price. Chen and Parlar [10] investigated a problem with nonlinear total salvage value. They discussed an extension of the classical newsboy problem where a compensation is made for a proportion of unsold items. A certain quantity is made before the selling season. At the end of the season, if the demand is lower than this quantity, the retailer could sell the difference at a constant unit price as a compensation.

In contrast with the above literature, we propose, a simultaneous relaxation of all the linearity assumptions for the cost and profit functions. TABLE 2.1 presents our assumptions with respect to the above literature review. As a consequence, the structure of the expected profit function is complex and it needs to be theoretically characterized.

To conclude, in the first part of this thesis, we propose simultaneous relaxation of all the above assumptions, thus leading to a complex expected profit function. TABLE 2.1 shows the statement of our sub-problem with respect to the above literature review. We will show that, under certain conditions, the concavity still holds in a interval containing the global optimal solution.

## **2.2 Problems II III and IV: Periodic Review, Two-product Types, Two-supplier, Two-stage Problem.**

There are many researchers who investigate single-product, single-supplier problems. In chapter 7 of his book, Zipkin (2000) presents a single-product single-supplier random demand random lead time problem. In this chapter, he studied both unlimited and limited inventory capacitated supply systems. He states that with Poisson demand and limited capacity, the supply system becomes a bulk queue which is difficult to analyze and provides approximation methods to solve the problem. Zipkin (2000) further suggested that using normal distribution to approximate the inventory order level could be better when capacity is relatively flexible in comparison with the exponential approximation. Levi et al. (2008) followed on this direction and gave computationally efficient policies for a single-product, single-supplier, periodic-review inventory problem with lost sales, capacity constraints and non-crossed stochastic lead times. Their model does not investigate linear unit selling price but minimizes the total expected cost under independent (not necessarily i.i.d.) random demand scenario. Bollapragada et al. (2006) consider a single supplier with limited production capacity and stochastic demand problem. Cheaitou et al. (2009) showed that there exists a unique optimal

solution for a single-product, single capacitated supplier, two period problem, but the unmet demand is assumed to be backlogged.

For researchers, who investigated single-product, multi-supplier problems, Whittemore & Saunders (1977) built a two supplier deterministic lead time model. They assume general random demand, linear cost for each period and demonstrate a unique optimal base stock level for the consecutive lead time model (i.e., the slower supplier delivers only one period later than the faster one with shorter lead time). They also comment that the optimal policy for the two non-consecutive modes is difficult to obtain. Scheller-Wolf & Tayur (1998) studied a markovian state dual-source problem. They assume a non-negative, state-dependent, time stationary discrete demand distribution and provide an optimal base-stock policy. Tomlin (2006) studies a single-product with random demand, two capacitated suppliers, multi-period inventory problem. One supplier is unreliable and the other is reliable and more expensive. They ignore all fixed ordering costs and inventory capacity, and assume equal lead times for both suppliers. They also suggest that the use of multiple suppliers can be beneficial if the suppliers differ in lead times. Veeraraghavan & Scheller-Wolf (2008) investigate a two-supplier problem (differed by fixed lead times) and take their productions capacity constraints into account. There is no inventory constraint and unsatisfied demands are backlogged. They provide a simulation-based optimization procedure and use numerical examples to demonstrate that their method is near optimal (within 1% or 2%) for the majority of cases. They comment that random yield, non-stationary demand, returns, supply disruptions and some random lead times could be introduced into their model as a basis for future research. Feng et al. (2005, 2006) introduce an additional supplier to this problem and investigate a three-supplier, single-product, random-demand, multi-period consecutive-lead time inventory problem. They show that with forecast updates, there exists a base-stock policy for the two-supplier-model, while it may not be optimal for the three-supplier-model (in general even under some strict assumptions). This is due to the fact that the third one is related to the first two. They provide contradictory examples to support their conclusion. In their model, there is neither suppliers' production capacity constraint nor retailer's inventory capacity constraint. Kouvelis & Milner (2002) use random capacity assumption for multiple suppliers in the context of outsourcing. In their model, capacity is the decision variable and they analyze both single and multi-period supply chain problems using dynamic programming.

Besides the above single-product problems, there are researchers that focus on multi-

product cases but only on a single-period. Seyed et al (2010) investigates a multi-product, single capacitated supplier, single capacitated retailer, single period problem with constant demand. The retailer is assumed to have only one warehouse and there is an upper bound on the number of total orders. The unsatisfied demand is assumed to be lost. They provide a genetic algorithm to find a near-optimal solution. Mihai et al (2010) investigate the best strategy of a seller who owns two capacity-constrained resources and markets two products in a single period. Tomlin and Wang (2005) connect the two-stage mix-flexibility and dual-sourcing factors in unreliable multi-products resulting in a newsboy formulation. However, this model is limited to a single period and there is no retailer's inventory capacity constraint. Demand for different products is jointly random distributed and delivery failure rebate is considered. They assume a Bernoulli nature to the supply process and each supplier is independent. In the second stage, the firm allocates production after demands and real ordering quantity is realized. They compare the single source and the dual source strategy through numerical examples.

As presented in Table 2.2, we extend previous studies by considering a two-supplier, production capacity constraints, different lead times, retailer's inventory capacity constraint, lost sales, multi-period and stochastic demand simultaneously.

Table 2.2 Literature Review and Contribution

	Product	Supplier	Supplier Capacity	Lead-time	Retailer's Capacity	Random Demand	Unmet Demand	Multi-period
Zipkin (2000)	1	1	No	Random	Yes	Yes	Lost or Backlogged	Yes
Levi et al (2008)	1	1	Yes	Random	Yes	Yes	Lost	Yes
Bollapragada et al. (2006)	1	1	Yes	Different	No	Yes	Lost	Yes
Cheaitou et al. (2009)	1	1	Yes	Fixed	No	Yes	Backlogged	Yes
Whittemore & Saunders (1977)	1	2	No	Different	No	Yes	Backlogged	Yes
Scheller-Wolf & Tayur (1998)	1	2	Yes	Different	No	Yes	Backlogged	Yes
Tomlin (2006)	1	2	Yes	Equal	No	Yes	Backlogged	Yes
Veeraraghavan & Scheller-Wolf (2008)	1	2	Yes	Defferent	No	Rando m	Backlogged	Yes
Kouvelis & Milner (2002)	1	M	Rando m	Equal	No	Yes	No unmet demand	Yes
Seyed et al (2010)	M	1	Const ant	Equal	Yes	No	Lost	No
Mihai et al (2010)	2	2	Const ant	Equal	Yes	No	No	No
Tomlin & Wang (2005)	M	M	Const ant	Equal	No	Yes	Lost	No
Our Model	2	2	Yes	Different	Yes	Yes	Lost or Backlogged	Yes

## Chapter 3

### Problem I: Single-product Type, Single-supplier, Single-period Nonlinear Problem

#### 3.1 Research Motivation

The single-period inventory management problem, known as the *newsboy* problem, consists of finding the optimal ordering quantity which maximizes the expected profit when facing a stochastic demand. In the classical setting, the retailer purchases the product at a fixed unit cost and sells it at a fixed unit price while the demand during the season is variable. At the end of the period, all unsold items are salvaged at a fixed unit price, usually much lower than the selling unit price. For instance, a typical example would be a retailer considering order quantities of perishable items for sale in a single season.

Extensions to this classical newsboy model are available in the literature, e.g. in competitive games, principal-agent, etc,... (for a review, refer to section 2). However, a wide range of simplifying assumptions are used in these models such as constant unit cost, constant unit price, constant unit salvage value and constant unit shortage cost. Often, these assumptions are restrictive and they do not reflect real-world applications. Indeed, the unit price often decreases as the observed quantity increases as in the case of agricultural and electronic products. For many commodities, the production unit cost decreases as the produced quantity increases due to economies of scale. In the case of luxury goods and remanufacturing, the production unit cost increases with the quantity produced while the unit price decreases as the offered quantity increases. In cases of perishable goods, the salvage unit value decreases both over time and in the remaining quantity.

Of course, work has been done where these assumptions have been relaxed but only one by one. We found no work focusing on simultaneously relaxing all of these assumptions.

This is precisely what we focus on here.

### 3.2 Assumptions and Model Introduction

In this thesis, we consider a supply chain made up of one supplier and one retailer. The retailer issues a single order  $q$  for a single product with uncertain demand based on a given purchasing cost  $C(q)$ . The uncertain demand, denoted by a random variable  $D$ , is drawn from a general density probability distribution function  $f(\cdot)$ . To be specific, we assume that  $C(\cdot)$  is a continuous increasing convex function which is differentiable over  $[0, +\infty)$ . We use such a non-linear function  $C(\cdot)$  to capture situations when the marginal production cost increases as the quantity produced increases, as in [46]. For instance, such situations occur when the supplier has a finite production capacity for the product. If the ordered quantity  $q$  exceeds the supplier's current capacity, the retailer has to pay a higher purchasing unit cost to account for the extra supplier costs. Such extra costs potentially correspond to overtime, additional staffing or extra shifts. This phenomenon also occurs in remanufacturing situations. As mentioned by Ferguson et al. [16], there are typically two steps involved when remanufacturing an item. In the first step, the used cores must be collected, inspected and sorted from the best to worst condition upon arrival. In the second step, a subset of the number of cores collected are processed. Let  $C(q)$  denote the minimum total variable cost to remanufacture  $q$  units, we assume that it is convex increasing in the quantity remanufactured. This assumption is also used by Ferguson et al. [16].

Furthermore, we assume that the unit price function  $r(q)$  is a twice differentiable decreasing concave function in the inventory level  $q$  which corresponds to the order size. The concavity characteristic allows us to represent situations when abundance diminishes the perceived value of a product just like in the case of Ananth Raman [34] who argues that the unit price could be affected by the inventory on hand. This is the case in many industries such as the housing market (Hanson [19]) and monitor panels (Wang et al. [44]) where higher inventory levels usually push down the selling price. Subsequently, we will show that  $r(\cdot)$  could be of some more general decreasing function instead of decreasing and concave.

Given an ordered quantity  $q$  and a realized demand  $D$ , we have two situations to consider:

- The case of  $D \geq q$

Here, the retailer loses  $(D - q)$  units in sales and a shortage penalty cost  $S(D - q)$  incurs.

By using an assumption similar to that of Downs et al. [13], the shortage cost function  $S(\cdot)$  is assumed to be convex and non-decreasing in the number of unsold items.

- The case of  $D < q$

We consider a non-linear function  $V(\cdot)$  associated with the salvage process in case of excess inventory. This salvage value function is assumed to be concave with respect to the leftover items after the sales season. This assumption is similar to that used by Chatwin [8] who assumes a nondecreasing and concave total salvage value. A concave salvage cost function could model situations when the retailer continues to sell leftover items after the sales season but at a discount rate. As in other papers using related assumptions and presented in Table 1.2 (Appendix), we consider that the larger the leftover quantities at the end of the sales season, the smaller will be the unit price obtained by the retailer. Thus, we have for  $D < q$ ,

$$r(q)D + V(q - D) \leq r(q)q. \quad (1)$$

Further, as in classical newsboy models, we assume that there is no setup fixed cost associated with the order.

Then, our problem is to determine the order quantity  $q$  such that the expected profits are maximized. Hence, for a given order size  $q$  and a given demand realization  $D$ , the profit function, denoted as  $\Pi[q, D]$ , is similar to that of the classical newsboy problem,

$$\Pi[q, D] = r(q) \min\{q, D\} - C(q) + V([q - D]^+) - S([D - q]^+) \quad (2)$$

with  $[x]^+ = \text{Max}\{x, 0\}$ .

Let  $E[\cdot]$  to be the expectation operator on the distribution of  $D$ . For the sake of notation simplification, let  $g(q) \equiv E[\Pi[q, D]]$ . The resulting optimization problem is then:

$$q^* = \arg_q \max_q g(q), \quad (3)$$

with

$$\begin{aligned} g(q) &= \int_0^{+\infty} \Pi(q, y) f(y) dy \\ &= \int_0^q [r(q)y - C(q) + V(q - y)] f(y) dy \\ &+ \int_q^{+\infty} [r(q)q - C(q) - S(y - q)] f(y) dy. \end{aligned} \quad (4)$$

**Assumptions.** In order to avoid extreme cases (namely  $q = 0$  or  $q = +\infty$ ), we introduce the following assumptions. First, the system is assumed to be profitable for some decision, i.e.

$$\exists \bar{q} > 0 : g(\bar{q}) > 0. \quad (5)$$

Second, the initial marginal production cost is assumed to be greater than or equal to the initial marginal salvage value, but smaller than initial unit price. This assumption amounts to

$$r(0) > C'(0) \geq V'(0). \quad (6)$$

### 3.3 Theoretical Results

In this section, we provide the main theoretical result for this general newsboy model with generalized cost/profit functions.

#### 3.3.1 Model Notations

We first summarize the previously defined notations:

$q$  : the order quantity; a decision variable,

$D$  : the random demand in the period,

$r(\cdot)$  : the unit price function; it is concavely decreasing,

$C(\cdot)$  : the total cost function; a continuously increasing convex function which is differentiable over  $[0, +\infty)$ ,

$S(\cdot)$  : the shortage cost function; it is convex and non-decreasing in shortage quantity,

$V(\cdot)$  : the salvage value function; it is concave and non-decreasing in leftover inventory.

#### 3.3.2 Theoretical Analysis

We first present two lemmas characterizing the unit price function  $r(\cdot)$ .

**Lemma 1.** Define the function  $m(\cdot)$  as

$$m(q) = r(q)q - C(q). \quad (7)$$

If  $r(q)$  is monotonically decreasing, then either  $m(\cdot)$  has two different roots : 0 and a strictly positive finite value, denoted as  $q_r > 0$ , or  $m(\cdot)$  has the value 0 as single root and  $m(\cdot)$  remains strictly positive over the interval  $(0, +\infty)$ .

*Proof.* Note that, by definition, for  $q = 0$  no cost and no profit are incurred, we have

$$m(0) = 0, \quad (8)$$

which shows that  $q = 0$  is the first root of  $m(\cdot)$ . Furthermore, as

$$\begin{aligned} m'(q) &= r'(q)q + r(q) - C'(q), \\ m''(q) &= r''(q)q + 2r'(q) - C''(q) \end{aligned} \quad (9)$$

and  $r''(q) < 0$ ,  $r'(q) < 0$ ,  $C''(q) > 0$ , we directly have  $m''(q) < 0$  so  $m(q)$  is concave.

By (6), we directly have

$$m'(0) = r(0) - C'(0) > 0 \quad (10)$$

and by the concavity of  $m(q)$ , it implies, either  $m(q)$  has two different roots : 0 and a strictly positive finite value, denoted as  $q_r > 0$ , or  $m(q)$  has the value 0 as single root and  $m(\cdot)$  remains strictly positive over the interval  $(0, +\infty)$ .

□

**Lemma 2.** Let us define  $q_u$  as

$$q_u = \sup\{q \mid m(q) > 0, q \in (0, +\infty)\}. \quad (11)$$

If  $r(\cdot)$  is monotonically decreasing, then  $r(q_u) > 0$ .

*Proof.* First note that with definition (11),  $q_u$  is either  $q_r$  or  $+\infty$ .

According to Lemma 1, two cases have to be considered.

First, if  $r(q)q - C(q) > 0$  over the interval  $(0, +\infty)$ , i.e. if  $m(\cdot)$  has the value 0 as single root, then  $r(\cdot) > 0$  over  $(0, +\infty)$ .

Second, if  $m(\cdot)$  has the pair of roots 0 and  $q_r$ , then one has  $q_u = q_r$ , and

$$r(q_u)q_u - C(q_u) = 0 \tag{12}$$

which implies that  $r(q_u) = \frac{C(q_u)}{q_u} > 0$ . □

The next lemma provides an upper bound for the optimal solution.

**Lemma 3.** *The optimal solution  $q^*$  defined in (3) can only be found in the interval  $(0, q_u)$ .*

*Proof.* As in classical inventory models and according to (1), the profit function (2) is maximized when the order quantity  $q$  and the demand realization  $D$  are equal, i.e. one has

$$\Pi[q, D] \leq \Pi[q, q] = r(q)q - C(q) = m(q). \tag{13}$$

As a direct consequence, by taking the expectation, we have

$$g(q) \leq m(q). \tag{14}$$

First, if  $m(\cdot)$  has the pair of roots 0 and  $q_r$ , we fix  $q_u = q_r$ .

Expression (14) shows that any order size  $q$  above the threshold  $q_u$  will only induce a non-optimal negative expected total profit.

By assumption (5), there exists an order size corresponding to strictly positive expected profit. As a consequence, the optimal solution can only be found in  $(0, q_u)$ .

Second,  $m(\cdot)$  has the single root 0.

We set  $q_u = +\infty$ , which is obviously the upper bound of the optimal solution. To conclude, the optimal solution has to be found in  $(0, q_u)$ .  $\square$

In order to show that the objective function  $g(q)$  is concave over the interval  $(0, q_u)$ , containing the global optimal solution, let us first introduce the following lemma.

**Lemma 4.** *Over the interval  $q \in (0, q_u)$ , the unit price function  $r(\cdot)$  satisfies*

$$r(q) > V'(0). \tag{15}$$

*Proof.* From Lemma 1, for  $q \in (0, q_u)$  we have  $r(q)q > C(q)$ . Since the function  $C(\cdot)$  is convex non-decreasing, we find directly that  $C(q) \geq C'(0)q$ , which, by (6), amounts to  $C'(0)q \geq V'(0)q$  and to (15).  $\square$

**Theorem 1.** *The function  $g(\cdot)$  is strictly concave over the interval  $(0, q_u)$ .*

*Proof.* The objective function  $g(q)$  given in (4) has the following second derivatives,

$$\begin{aligned}
g''(q) &= \int_0^q \left\{ r''(q)y - C''(q) + \frac{d^2V(q-y)}{d^2(q-y)} \right\} f(y)dy \\
&+ \{r'(q)q - C'(q) + V'(0)\} f(q) \\
&+ \int_q^\infty \left\{ r''(q)q + 2r'(q) - C''(q) - \frac{d^2S(y-q)}{d^2(y-q)} \right\} f(y)dy \\
&+ \{-(r'(q)q + r(q) - C'(q) + S'(0))\} f(q)
\end{aligned} \tag{16}$$

By rearranging terms, we find

$$g''(q) = \int_0^q \left\{ r''(q)y - C''(q) + \frac{d^2V(q-y)}{d^2(q-y)} \right\} f(y)dy \tag{17}$$

$$+ \int_q^\infty \left\{ r''(q)q + 2r'(q) - C''(q) - \frac{d^2S(y-q)}{d^2(y-q)} \right\} f(y)dy \tag{18}$$

$$+ \{-(r(q) - V'(0) + S'(0))\} f(q). \tag{19}$$

First, in expression (17), since  $C(q)$  is convex,  $r(\cdot)$  is concavely decreasing over  $[0, q_u]$  and function  $V(\cdot)$  is assumed to be concave, we have

$$r''(q)y - C''(q) + \frac{d^2V(q-y)}{d^2(q-y)} \leq 0 \text{ and the integral (17) is non-positive.}$$

Second, in expression (18), we have by the main assumptions,

$$r''(q)q \leq 0, r'(q) \leq 0, C''(q) \geq 0 \text{ and } -\frac{d^2S(y-q)}{d^2(y-q)} \leq 0,$$

which implies that the integral (18) is non-positive.

In the last expression (19), according to Lemma 4 and the non-increasing property of the shortage function, over the interval  $(0, q_u)$ , we have

$$\{-(r(q) - V'(0) + S'(0))\} f(q) < 0. \quad \square$$

**Corollary 1.** *An important corollary for Theorem 1 is that in fact,  $r(q)$  does not need the concavity property for all  $q \in R^+$ , but it only needs to be concave for  $q \in [0, q_u]$ . In the appendix, we will prove Lemma 1 again without using the concavity property of  $r(q)$ . Given the fact that Lemma 1 and 2 do not require concavity of  $R$ , only monotonicity, thus  $r(q)$  only needs to hold monotonicity for the interval  $[q_u, \infty]$ .*

Clearly, Theorem 1 can be used in the efficient search of the optimal solution. For illustrative purposes, two examples are provided in the next section.

### 3.4 Three Applications of The New Theoretical Results

To illustrate the usefulness of our theoretical results, we present here two examples:

1. An inventory model with an exponentially distributed demand and relaxed linear parameters.
2. An inventory model with a normally distributed demand and non-linear parameters.
3. A Remanufacturing model with gamma distributed demand and non-linear parameters.

#### 3.4.1 An Inventory Model with an Exponentially Distributed Demand and Relaxed Linear Parameters: an Approximated Closed Form Solution

For illustrative purposes, we show here that by using relaxed linear parameters and an exponential distributed demand, we obtain an approximated closed form expression for the optimal solution and we show the concavity of the objective function.

This particular setting is as follow: The demand density probability function is given by

$$f(y) = \lambda e^{-\lambda y}, \lambda \geq 0$$

and the profit and costs functions have the following expressions,

$$r(q) = r_0 - r_1 q, \tag{20}$$

$$C(q) = cq, \tag{21}$$

$$V(q - D) = v(q - D), \tag{22}$$

$$S(D - q) = s(D - q), \tag{23}$$

where  $r_0, r_1, c, s, v \geq 0$  and  $r_0 \geq c \geq v$  according to assumption (6).

Note that the  $r(q)$  is linearly decreasing while in a classical newsboy model, it is often constant.

According to (12) in Lemma 2,

$$q_u = \frac{r_0 - c}{r_1}. \quad (24)$$

According to Lemma 3, the optimal solution defined in (3) can only be found in the interval  $(0, \frac{r_0 - c}{r_1})$ .

From (42) and according to Lemma 4, for any  $q \in (0, \frac{r_0 - c}{r_1})$ , we have

$$r_0 - r_1 q - v > 0 \quad (25)$$

By introducing the above notation, one finds the following expressions for the first and the second derivatives,

$$g'(q) = -r_1 q e^{-\lambda q} + (-v + r_1/\lambda + r_0 + s) e^{-\lambda q} + (-r_1/\lambda - c + v), \quad (26)$$

$$\begin{aligned} g''(q) &= (\lambda r_1 q - 2r_1 + \lambda v - r_0 \lambda - s \lambda) e^{-\lambda q} \\ &= -\lambda(r_0 - r_1 q - v) e^{-\lambda q} - (2r_1 + s \lambda) e^{-\lambda q} \\ &\leq -\lambda(r_0 - r_1 q - v) e^{-\lambda q} \\ &< 0 \end{aligned} \quad (27)$$

where the last inequation comes from (25).

Thus the objective function  $g(\cdot)$  is strictly concave over the interval  $q \in (0, \frac{r_0 - c}{r_1})$ . By this concave property and by Lemma 3, the global optimal solution  $q^*$  should satisfy  $q^* \rightarrow 0$ ,  $q^* \rightarrow \frac{r_0 - c}{r_1}$  or  $g'(q^*) = 0$ .

It is easy to check that  $g(0) < 0$  and  $g(\frac{r_0 - c}{r_1}) < 0$ . Since  $g(q)$  is a continuous function, we have

$$\lim_{q \rightarrow 0} g(q) < 0 \quad (28)$$

$$\lim_{q \rightarrow \frac{r_0 - c}{r_1}} g(q) < 0 \quad (29)$$

As a consequence, the optimal solution can only be found where  $g'(q) = 0$ . Let  $g'(q) = 0$ , we have

$$e^{\lambda q} = \frac{-r_1 q}{\frac{r_1}{\lambda} + c - v} + \frac{\frac{r_1}{\lambda} - v + r_0 + s}{\frac{r_1}{\lambda} + c - v}. \quad (30)$$

There is no closed-form solution for equation (30) but one can search for the solution numerically. However, given the fact that Mathematica software package already have a built-in Lambert W function, we provide a precise version solution based on the W function that makes the search much easier.

We first introduce the Lambert W function (see [26]).

$$z = W(z)e^{W(z)} \quad (31)$$

We solve (30) by using the Lambert W function and get the precise version but not closed-form expression of the optimal solution

$$q^* = \frac{(r_1 + \lambda(r_0 + s - v))\text{Log}[e] - r_1 W \left[ \frac{e^{1 + \frac{\lambda(r_0 + s - v)}{r_1}} (c\lambda + r_1 - \lambda v)\text{Log}[e]}{r_1} \right]}{\lambda r_1 \text{Log}[e]} \quad (32)$$

$$\text{st. } q^* \in \left(0, \frac{r_0 - c}{r_1}\right) \quad (33)$$

Please note that the (32) is multivalued due to the Lambert W function. Therefore, constraint (33) is used to ensure a feasible optimal solution. Another consequence of using the multivalued Lambert W function is that the insights on the solution are not clear. Therefore, we also offer closed-form approximation. The reason is that, in a back of the envelope calculation, one can find an approximation without using the computer. We believe that it is valuable to have an approximation for large scale inventory management problems.

The approximation of  $e^{\lambda q}$  by the first three terms of the associated Maclaurin series (see [6]), we have

$$e^{\lambda q} = \frac{\lambda^2}{2} q^2 + \lambda q + 1 \quad (34)$$

The above condition leads to

$$\frac{\lambda^2}{2} q^2 + \left( \frac{2r_1 + \lambda c - \lambda v}{\frac{r_1}{\lambda} + c - v} \right) q + \frac{c - r_0 - s}{\frac{r_1}{\lambda} + c - v} = 0. \quad (35)$$

Since  $r_0, r_1, c, s, v \geq 0, r_0 \geq c \geq v, \lambda > 0$ , it is easy to get the expression of the approximate optimal solution.

$$q_a^* = \frac{-c\lambda - 2r_1 + \lambda v + \frac{1}{2}\sqrt{-8\lambda(c - r_0 - s)(c\lambda + r_1 - \lambda v) + 4(c\lambda + 2r_1 - \lambda v)^2}}{\lambda(c\lambda + r_1 - \lambda v)} \quad (36)$$

Note that as expected,  $q_a^*$  is strictly increasing in the unit shortage cost  $s$  and the initial unit price  $r_0$ .

### 3.4.2 An Inventory Model with a Normally Distributed Demand and Non-linear Parameters: a Numerical Example

We compare here, via a numerical example, a direct solution approach, exploiting the exhibited concavity property, with a more classical solution technique relying on standard newsboy solution and linear approximations of the costs and profit functions.

We consider the following setting for the numerical example analyzed in this section.

The unit price function form is  $r(q) = r_0 - \frac{q^2}{m}$  (with as numerical parameter values  $r_0 = 20000$  and  $m = 0.1$ ).

The total cost function is given by  $C(q) = a_1q^{a_2} + a_3q$  (with  $a_1 = 10, a_2 = 2$  and  $a_3 = 1$ ).

The salvage value function is  $V(x) = b_1 \log(b_2x + 1)$  (with  $b_1 = 100000, b_2 = 1$ ).

The shortage cost function is chosen as  $S(x) = s_1x^{s_2} + s_3x$  (with  $s_1 = 10, s_2 = 2$  and  $s_3 = 1$ ).

The random demand is assumed to follow Gaussian probability distribution, with mean demand  $\mu = 20$  and standard deviation  $\sigma = 3$ .

Based on **Theorem 1** and via optimization algorithms exploiting concavity of the objective function, a direct solution technique gives the optimal solution  $q^* = 28.5254$  with an associate expected profit  $g(q^*) = 448470$ .

### 3.5 A Remanufacturing Model

In this subsection, we present an application of our model on the remanufacturing industry.

#### 3.5.1 Model Introduction

For most firms, remanufacturing an item typically involves two steps. First, the used cores must be collected, inspected and sorted from the best to worst condition upon arrival; and second, a subset of the number of cores collected are processed (testing, cleaning, and replacing of parts) in the order of their decreasing arrival conditions. Thus, a decision to remanufacture  $q$  units involves an optimization problem where the firm chooses the number of units to collect, with only a proportion of these units being processed. Same as Ferguson et al (2006), Let  $C(q)$  represent the minimum total variable cost to remanufacture  $q$  units.  $C(q)$  is convex increasing in the quantity remanufactured if either the variable collection cost is convex increasing in the collection quantity or the processing cost is convex increasing in the processing quantity, with neither cost being concave decreasing.

There are many cases where the collection cost is convex increasing in quantity. Transportation cost often increases in the number of cores collected since the firm loses economies of scale as it moves from collecting cores in densely populated areas to collecting cores in more rural areas. Agnihotri et al. (1990) describe a refuse collection facility where the cost of collecting is convex increasing in the amount collected. The acquisition cost for the cores may also increase in the number of units as consumers have heterogeneous reservation prices for what it takes to convince them to return their used products. This cost is modeled explicitly in Ray et al. (2005) through the use of trade-in rebates. Many firms also experience a convex increasing processing cost due to the variance in condition of the returned cores and the fact that the firms process the cores in the best condition (upon arrival) first. Ferguson et al. (2004) provide an example where 80% of HP's inkjet printer returns required no significant processing besides cleaning and repackaging. For this reason, most firms sort their returned units and only process a percentage of the total returns with the highest quality. An alternative situation, modeled in Guide et al. (2003), is where the firm pays higher prices for higher quality product returns. Under either of these strategies, the total variable cost of processing is convex increasing in quantity. Same as Ferguson et al. (2006), the total

production cost function is

$$C(q) = 0.005q^2 \quad (37)$$

### 3.5.2 A Remanufacturing Model with an Exponentially Distributed Demand and Relaxed Linear Parameters: an Approximated Closed Form Solution

For remanufactured product of which demand is exponentially distributed, we show here that by using relaxed linear parameters, an approximated closed form solution can be obtained.

This particular setting is as follow: The demand density probability function is given by

$$f(y) = \lambda e^{-\lambda y}, \lambda \geq 0$$

and the profit and costs functions have the following expressions,

$$r(q) = r_0 - r_1 q, \quad (38)$$

$$C(q) = cq^2, \quad (39)$$

$$V(q - D) = v(q - D), \quad (40)$$

$$S(D - q) = s(D - q), \quad (41)$$

where  $r_0, r_1, c, s, v \geq 0$  and  $r_0 > 0 \geq v$  according to assumption (6). The last inequality implies a non-positive unit salvage value. This is true in cases where the remanufactured leftover is either unsalvageable or recycled.

Note that the  $C(q)$  is a general quadratic increasing function in  $q$  which of course, includes the special case (37).

According to (12) in Lemma 2,

$$q_u = \frac{r_0}{r_1 + c}. \quad (42)$$

According to Lemma 3, the optimal solution defined in (3) can only be found in the interval  $(0, \frac{r_0}{r_1 + c})$ .

From (42) and according to Lemma 4, for any  $q \in (0, \frac{r_0}{r_1+c})$ , we have

$$r_0 - r_1q - v > 0 \quad (43)$$

By introducing the above notation, one finds the following expressions for the first and the second derivatives,

$$\begin{aligned} g'(q) &= -r_1qe^{-\lambda q} + (-v + r_1/\lambda + r_0 + s)e^{-\lambda q} + (-r_1/\lambda - 2cq + v), \quad (44) \\ g''(q) &= (\lambda r_1q - 2r_1 + \lambda v - r_0\lambda - s\lambda)e^{-\lambda q} - 2c \\ &= -\lambda(r_0 - r_1q - v)e^{-\lambda q} - (2r_1 + s\lambda)e^{-\lambda q} - 2c \\ &\leq -\lambda(r_0 - r_1q - v)e^{-\lambda q} - 2c \\ &< 0 \end{aligned} \quad (45)$$

where the last inequation comes from (43).

Thus the objective function  $g(\cdot)$  is strictly concave over the interval  $q \in (0, \frac{r_0}{r_1+c})$ . By this concave property and by Lemma 3, the global optimal solution  $q^*$  should satisfy  $q^* \rightarrow 0$ ,  $q^* \rightarrow \frac{r_0}{r_1+c}$  or  $g'(q^*) = 0$ .

It is easy to check that  $g(0) < 0$  and  $g(\frac{r_0}{r_1+c}) < 0$ . Since  $g(q)$  is a continuous function, we have

$$\lim_{q \rightarrow 0} g(q) < 0 \quad (46)$$

$$\lim_{q \rightarrow \frac{r_0}{r_1+c}} g(q) < 0 \quad (47)$$

As a consequence, the optimal solution can only be found where  $g'(q) = 0$ . Let  $g'(q) = 0$ , we have

$$-r_1qe^{-\lambda q} + (-v + r_1/\lambda + r_0 + s)e^{-\lambda q} + (-r_1/\lambda - 2cq + v) = 0. \quad (48)$$

There is no closed-form solution for equation (48) but one can search for the solution numerically. Therefore, we also offer closed-form approximation. The reason is that, in a back of the envelope calculation, one can find an approximation without using the computer. We believe that it is valuable to have an approximation for large scale inventory management problems.

The approximation of  $e^{\lambda q}$  by the first three terms of the associated Maclaurin series (see [6]), we have

$$e^{\lambda q} = \frac{\lambda^2}{2}q^2 + \lambda q + 1 \quad (49)$$

Substituting the above equation in (48), we solve for the approximated optimal solution and get

$$\begin{aligned} q_a^* = & -\frac{1}{6c\lambda^2}(4c\lambda + \lambda r_1 - \lambda^2 v + (\lambda^2(-8c^2 + (r_1 - \lambda v)^2 + 4c(-4r_1 + \lambda v)))/(-80c^3\lambda^3 \\ & -12c^2\lambda^3(11r_1 + 9\lambda(r_0 + s) - 5\lambda v) + \lambda^3(r_1 - \lambda v)^3 - 6c\lambda^3(-4r_1 + \lambda v)(-r_1 + \lambda v) \\ & + \sqrt{(\lambda^6((8c^2 - (r_1 - \lambda v)^2 + 4c(4r_1 - \lambda v))^3 + (80c^3 + 12c^2(11r_1 + 9\lambda(r_0 + s) - 5\lambda v) \\ & - (r_1 - \lambda v)^3 + 6c(r_1 - \lambda v)(4r_1 - \lambda v))^2))^{1/3} + (-80c^3\lambda^3 - 12c^2\lambda^3(11r_1 + 9\lambda(r_0 + s) - 5\lambda v) \\ & + \lambda^3(r_1 - \lambda v)^3 - 6c\lambda^3(-4r_1 + \lambda v)(-r_1 + \lambda v) + \sqrt{(\lambda^6((8c^2 - (r_1 - \lambda v)^2 + 4c(4r_1 - \lambda v))^3 \\ & + (80c^3 + 12c^2(11r_1 + 9\lambda(r_0 + s) - 5\lambda v) - (r_1 - \lambda v)^3 + 6c(r_1 - \lambda v)(4r_1 - \lambda v))^2))^{1/3}}) \quad (50) \end{aligned}$$

### 3.5.3 A Remanufacturing Model with an Exponentially Distributed Demand and Relaxed Linear Parameters: A Numerical Example

In classical linear Newsboy model, the average unit production cost equals to the unit cost of the first production  $c_u$  and the related marginal cost  $c_m$ . Retailers may use  $c_u$  or  $c_m$  to compute the expected total profit. In the remanufacture scenario however, huge profit loss can be expected if the remanufacturer consider either of the above two costs of the first collected unit as the average unit production cost.

In this subsection, not only the optimal solution in different scenarios will be presented, but also an illustration of the influence of the above two costs will be made.

Checking the first derivative yields the marginal total production cost

$$C'(q) = 0.01q^2 \quad (51)$$

According to (37) and (51), the unit cost of the first production is  $c_u = 0.005$  and the related marginal cost is  $c_m = 0.01$ .

We illustrate here, via numerical examples, the error of the total expected profit caused by a remanufacturer using the above two costs as average unit production cost. We assume that the demand follows a gamma distribution  $Gamma(k, \theta)$ .

The parameter settings are summarized as follow:

- $r(q) = 1$
- $C(q) = 0.005q^2$
- $V(x) = b_1 \log(b_2 x + 1)$   
 $b_1 = 1, b_2 = 0.0005$
- $S(x) = s_1 x^{s_2}$   
 $s_1 = 0.001, s_2 = 2$
- $k = 100, \theta = 1$

We move  $k$  from 20 to 100 and compare the optimal expected profit as follow.

Table 3.1 Optimal Solution

k	20	40	60	80	100
$0.005q^2$	23.1321	41.0929	58.2085	74.4789	88.8476
$c_v q$	216.475	108.2875	108.2875	108.2875	121.8109
$c_m q$	40.6703	54.1938	81.2406	101.5258	121.8109

Table 3.2 Expected Total Profit

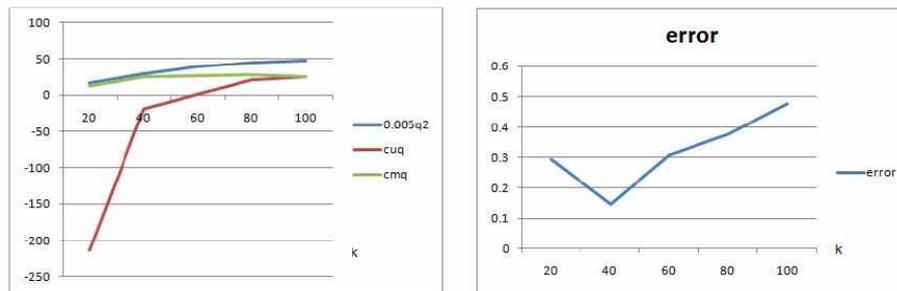
k	20	40	60	80	100
$0.005q^2$	16.6153	29.4869	38.917	45.11	48.3415
$c_uq$	-214.526	-18.7537	1.1584	21.0641	25.2453
$c_mq$	11.7287	25.2273	26.8597	28.1536	25.2453

Table 3.3 Error Comparison

k	20	40	60	80	100
$0.005q^2$	16.6153	29.4869	38.917	45.11	48.3415
$c_mq$	11.7287	25.2273	26.8597	28.1536	25.2453
error	0.294102	0.144457	0.309821	0.37589	0.477772

Results shows that the marginal cost  $c_m = 0.01$  approximate the average unit production cost better than the unit cost of the first production  $c_u = 0.005$ . The error caused by  $c_u = 0.005$  is also illustrated in figure 1.

Figure 3.1 Error Comparison



Results shows that even the minimum error is around 15%. Therefore, it is important for the remanufacturer to choose the real production function  $C(q)$  when making the production decision.

## 3.6 Conclusions

In this chapter, we have considered a newsboy problem with a non-linear cost structure. This is a generalization of the traditional linear structure of such models. The resulting problem is mathematically complex to solve and no closed-form solution is available. In fact, most of the literature review shows that this model has been often simplified by using linear assumptions or by introducing non-linearity on a one by one basis.

Under some much less restrictive assumptions, we show that, even if the different cost/profit functions in the model are non-linear, the expected profit criterion is concave over a certain interval which includes the optimal solution. Therefore, we can develop efficient search methods that can be implemented to compute this solution. Theorem 1 is a valuable tool in this search process. We illustrate our approach through two cases with relaxed assumptions and we show that the optimal solution is within the defined interval.

In the remanufacturing example, results shows that even the minimum error is around 15%. Therefore, it is important for the remanufacturer to choose the real production function  $C(q)$  when making the production decision.

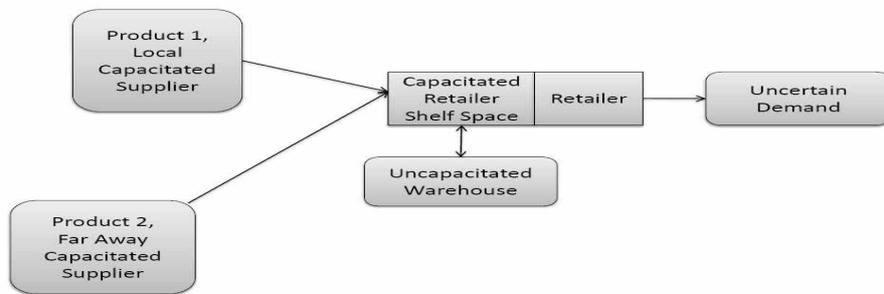
## Chapter 4

# Problem II: Two-product Types, Two-supplier, Two-stage, Three-period Problem — Stochastic Programming

### 4.1 Model Introduction

In Problem II, we start with model 4.1 and investigate the following two-product, two capacitated-suppliers with different fixed lead times, single-capacitated-retailer, two-stage (inventory-allocation), stochastic demand with lost sale assumption, periodic-review supply chain problem. This model is shown in figure 4.1.

Figure 4.1 The Model.



From the supply perspective, the high-end product has short supply lead time (local supplier), while the low-end product has long supply delivery lead time (far away supplier). The determinant of the expensive product's supply lead times is primarily the manufacturing lead time (as the transportation time is negligible, in comparison). On the other hand, the cheaper product's supply lead time is mainly the transportation lead time (as the manufacturing lead time is negligible in comparison with this).

From the demand perspective, demand variability is high for the high-end product, while

it is low for the low-end product. This is due to the fact that the product life cycle of an expensive product is short, and is a function of its performance in the market once it is introduced (e.g. fashion clothing). The customers for these products are highly variable. On the other hand, the product life cycle of cheap products is longer, as customers for the same always exist. In addition, we model demand uncertainties through quasi-concave distributions (e.g. Gamma, Normal).

Let  $T$  be the horizon ( $T=2$ ). We use  $t(0 \leq t \leq T)$ , to indicate a period while presenting the different parameters. In this case, the demand of the high-end and low-end product in period  $t$  is denoted by  $D_t^H$  and  $D_t^L$ , respectively.

We assume that all units must be placed into the retailer's inventory for a *preparation period* before sale. That is, for instance, 10 units of the high-end products arrive at period 1, cannot be sold before period 2. This is true in practice. As an example, products arriving at big retail stores like Nordstrom, Macys and Zara are usually packaged in big boxes which cannot be sold directly to the customers. Retailers need some time to unload them from the trucks, transfer them from the boxes to the shelves for the customers.

In period 0, there's no demand for both products. The following demand assumption is similar to the one in Levi et al. (2008). We assume that demands in different periods are independently distributed and are not necessarily identical. This helps us to investigate both stationary and non-stationary demand in our experiment.

Denote  $r_H$  to be the unit selling price for the high-end products and  $r_L$  for the low-end products. The unit cost for high-end product is  $c_H$  and that of the low-end product is  $c_L$ . Let  $V_H, V_L$  be the unit salvage value for the high-end products and low-end products respectively. Similarly, we set  $S_H, S_L$  to be the unit shortage cost for the high-end and low-end products respectively. The high-end product supplier's production capacity in a period is  $k^H$ , while that of the low-end product is  $k^L$ . We reasonably assume that the retailer has constant inventory capacity  $K$  in each period.

Furthermore, for units that exceed the primary inventory capacity, the decision maker has an option to move them to secondary inventory, costing  $h'_H$  for a high-end unit and  $h'_L$  for a low-end unit.

The other notations involved in this problem are summarized below:

Decision Variables

$X_t^H \geq 0$  : quantity of high-end products ordered in period  $t$  that is to be placed into the primary warehouse when arrived,  $t = 0, 1$

$Y_t^H \geq 0$  : quantity of high-end products ordered in period  $t$  that is to be placed into the secondary warehouse when arrived,  $t = 0, 1$

$X_t^L \geq 0$  : quantity of low-end products ordered in period  $t$  that is to be placed into the primary warehouse when arrived,  $t = 0$

$Y_t^L \geq 0$  : quantity of low-end products ordered in period  $t$  that is to be placed into the secondary warehouse when arrived,  $t = 0$ .

#### State Variables

$I_{H,t}$  : quantity of physical high-end product in the primary warehouse at the end of period  $t$ ,  $t = 0, 1, 2$

$I'_{H,t}$  : quantity of physical high-end product in the secondary warehouse at the end of period  $t$ ,  $t = 0, 1, 2$

$I_{L,t}$  : quantity of physical low-end product in the primary warehouse at the end of period  $t$ ,  $t = 2$

$I'_{L,t}$  : quantity of physical low-end product in the secondary warehouse at the end of period  $t$ ,  $t = 2$ .

$I_{H,t}^-$  : total shortage quantity of high-end product at the end of period  $t$ ,  $t = 0, 1, 2$

$I_{L,t}^-$  : total shortage quantity of low-end product at the end of period  $t$ ,  $t = 1, 2$

#### Demand Parameters

$\mu_t^H$ : mean demand of high-end product in period  $t$  ( $t=1,2$ ).

$\mu_t^L$ : mean demand of low-end product in period  $t$  ( $t=1,2$ ).  $\mu_1^L = 0$ .

$\sigma_t^H$ : demand variance of high-end product in period  $t$  ( $t=1,2$ ).

$\sigma_t^L$ : demand variance of low-end product in period  $t$  ( $t=1,2$ ).  $\sigma_1^L = 0$ .

$C_{v,t}^H$ : coefficient of variation of the demand of high-end product in period  $t$  ( $t=1,2$ ).

$C_{v,2}^L$ : coefficient of variation of the demand of low-end product in period 2.

$\varphi_{H,t}(y_t)$ : demand probability density function of high-end product in period  $t$  ( $t=1,2$ ).

$\varphi_{L,2}(y_3)$ : demand probability density function of low-end product in period 2.

Based on the research of van Delft and Vial (2004), we apply stochastic programming to solve for the optimal solution. Our research differs from their paper by introducing the secondary warehouse and the assumption of lost-sales.

In the first phase of the analysis, we consider a deterministic version of the problem so as to design a mathematical programming model of this situation. It will be the backbone of the deterministic equivalent to be constructed later.

In order to formulate the objective function, we first define revenues and expenses separately. The revenues  $R_s$  have five terms.

$$\begin{aligned}
& R_s(I_H^-, I_L^-, I_H, I_H', I_L, I_L') \\
&= r_H(D_1^H - I_{H,1}^-) + r_H(D_2^H - I_{H,2}^-) + r_L(D_2^L - I_{L,2}^-) \\
&\quad + V_H(I_{H,2} + I_{H,2}') + V_L(I_{L,2} + I_{L,2}')
\end{aligned} \tag{52}$$

The first two terms are the revenues of the high-end product from the effective sales in period  $t = 1, 2$ ; the third term is the revenues of the low-end product from the effective sales and the last two terms pertain to the salvage value of the left-over inventory of the two products at the end of period 2.

The  $E_s$  expenses have six terms,

$$\begin{aligned}
& E_s(I_H, I_H', I_L, I_L', X^H, Y^H, X^L, Y^L) \\
&= \sum_{t=0}^1 c_H(X_t^H + Y_t^H) + c_L(X_0^L + Y_0^L) + \sum_{t=0}^1 h_H I_{H,t} + \sum_{t=0}^1 h_H' I_{H,t}' \\
&\quad + (h_L X_0^L + h_L' Y_0^L) + \left( \sum_{t=1}^2 S_H I_{H,t}^- + S_L I_{L,2}^- \right)
\end{aligned} \tag{53}$$

The first term is the cumulated purchasing cost of the high-end product; the second term is the purchasing cost of the low-end product; the third term and the fourth one pertain to the cumulated holding cost of the high-end product in the primary and secondary warehouse,

respectively; the fifth term is the total holding cost of the low-end product; the last term is the total shortage cost of the high-end and low-end product.

Consequently, the deterministic problem can be reformulated as a linear programming problem as follows:

$$\max (R_s(I_H^-, I_L^-, I_H, I'_H, I_L, I'_L) - E_s(I_H, I'_H, I_L, I'_L, X^H, Y^H, X^L, Y^L)) \quad (54)$$

$$s.t. \quad I_{H,0} = X_0^H \quad (55)$$

$$I'_{H,0} = Y_0^H \quad (56)$$

$$I_{H,1} + I'_{H,1} = I_{H,0} + I'_{H,0} - (D_1^H - I_{H,1}^-) + X_1^H + Y_1^H \quad (57)$$

$$I_{H,2} + I'_{H,2} = I_{H,1} + I'_{H,1} - (D_2^H - I_{H,2}^-) \quad (58)$$

$$I_{L,2} + I'_{L,2} = I_{L,1} + I'_{L,1} - (D_2^L - I_{L,2}^-) \quad (59)$$

$$X_t^H + Y_t^H \leq k^H, \quad t = 0, 1 \quad (60)$$

$$X_0^L + Y_0^L \leq k^L \quad (61)$$

$$I_{H,0} \leq K \quad (62)$$

$$I_{H,1} + I_{L,1} \leq K \quad (63)$$

$$X_t^H, Y_t^H \geq 0, \quad t = 0, 1 \quad (64)$$

$$X_0^L, Y_0^L \geq 0 \quad (65)$$

$$I_{H,t}, I'_{H,t}, I_{H,t}^- \geq 0, \quad t = 1, 2 \quad (66)$$

$$I_{L,t}, I'_{L,t}, I_{L,t}^- \geq 0, \quad t = 2 \quad (67)$$

## 4.2 The event tree and the stochastic demand process

In this subsection we make the previous problem stochastic by considering that the demand follows a stochastic process. In order to write the stochastic model as a mathematical programming problem, we use the same assumptions as van Delft and Vial (2004): the underlying stochastic process is discrete and is independent of the state and decision variables of the deterministic model. To formalize this type of stochastic process, we use the following event tree representation. We first discuss how to construct the two trees and navigate through them. We then show how one can plug the stochastic process value into them.

### 4.2.1 Event tree representation

The nodes of a event tree represent the state of a discrete state stochastic process at a

given period; the (oriented) arcs correspond to the probabilistic transitions from one node at given period to another node at the next period. The main point is that there exists exactly one arc leading to a node, while there may be many arcs emanating from a node. As any tree, an event tree is unequivocally described by a node numbering and a predecessor function that gives the number of the node that immediately precedes the current node. We choose a rather natural node numbering by periods and define the predecessor mapping as a function that can be computed by the modeling language like MATLAB and MATHEMATICA.

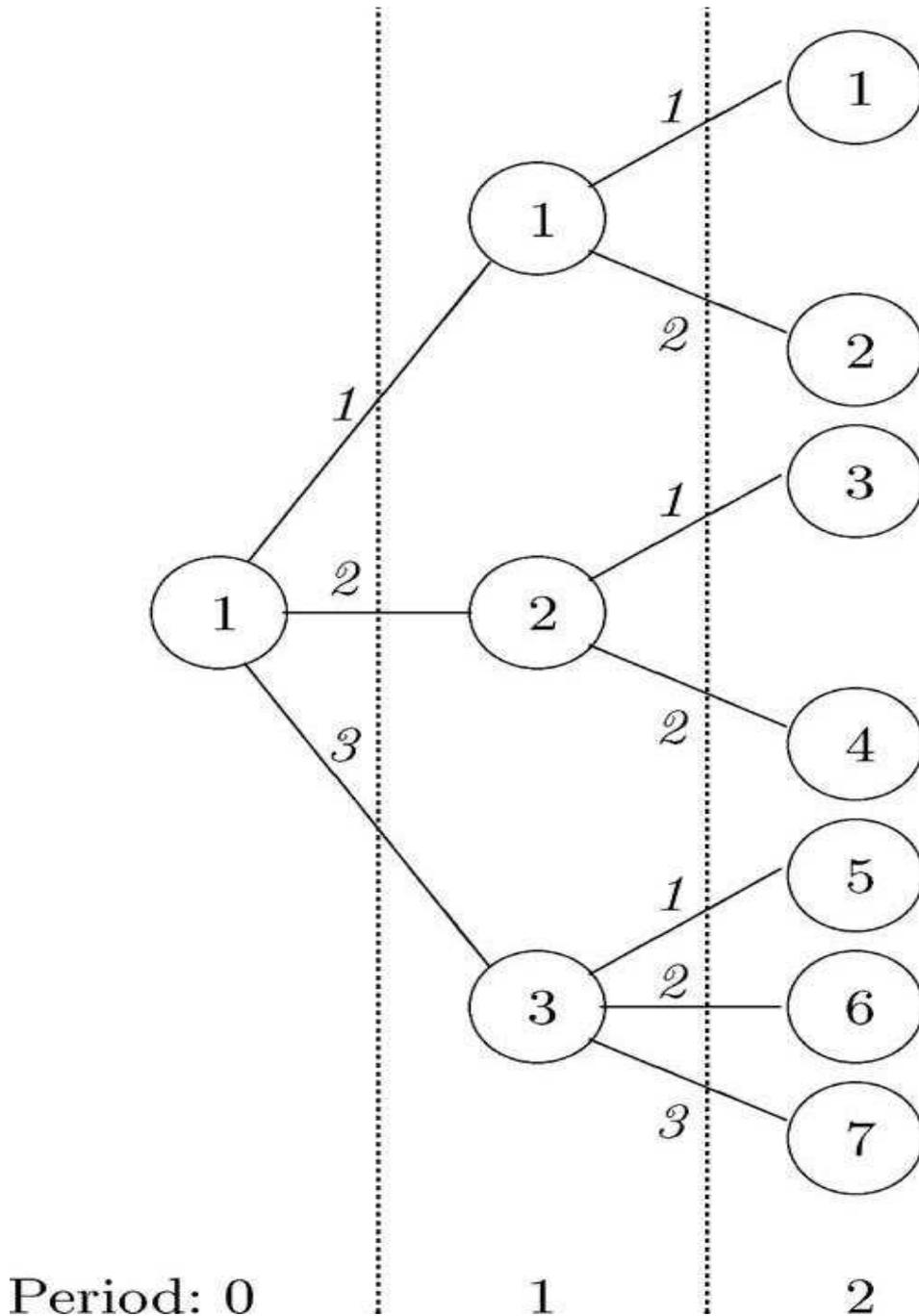
Let  $P \in \{H, L\}$  and subscript  $p \in \{H, L\}$  represent the type of the two products throughout this subsection. We denote  $N(t)$  as the number of nodes at period  $t$ . Nodes on the event tree are indexed with a pair  $(t, n)$  and  $T = 2$  is the last period. At any given period  $t$ , the nodes are numbered from 1 to  $N(t)$  going from top to bottom. Thus, node  $(t, n)$  is the  $n$ th node from the top in period  $t$ . Similarly, the transitions from a node to its immediate successors are numbered from top to bottom. Let also  $f(t, n)$  denote the number of branches emanating from node  $(t, n)$ . The tree is rooted at period  $t = 0$ , but the first realization of takes place at the following period. The number of nodes at period  $t$  is then computed by

$$N(t) = \sum_{k=1}^{N(t-1)} f(t-1, k) \quad (68)$$

with  $N(0) = 1$ .

To illustrate our point, we consider the case of  $N(t) = 3$  and present it on Figure 4.2. In this representation, the uncertainty unfolds in time from left to right. Nodes appearing in the same vertical slice belong to the same time period. At  $t = 0$ , there are three branches giving rise to three nodes in  $t = 1$  ( $f(0, 1) = 3$  and  $N(1) = 3$ ). At  $t = 1$ , each node has three branches ( $f(1, 1) = f(1, 2) = f(1, 3) = 3$ ). In  $t = 2$ , there are  $N(2) = 9$  nodes. Time  $t = 2$  is the horizon and no branch emanates from those nodes.

Figure 4.2 Example of event tree representation



In stochastic programming most constraints deal with variables linked to different periods. To express those constraints one needs auxiliary functions that help backtracking from a node to its predecessors. Therefore, we use the predecessor function  $a(t, n; k)$  which map the current node  $(t, n)$  to the index of its predecessor node in period  $t - k$  along the unique path that goes from the root to the node  $(t, n)$ . This function is as follows. Let us first introduce the one-period-predecessor function  $a(t, n, 1)$  recursively defined by

$$a(t, n + 1, 1) = \begin{cases} a(t, n, 1) & \text{if } n + 1 \leq \sum_{k=1}^{a(t, n, 1)} f(t - 1, k), \\ a(t, n, 1) + 1 & \text{otherwise,} \end{cases}$$

with  $a(t, 1, 1) = 1$ , for  $0 \leq t \leq 2$ . After that, we define the general  $k$ -period-predecessor functions as

$$a(t, n, 1) = \begin{cases} a(t, n, 1) & \text{if } k = 1, \\ a(t - 1, a(t, n, 1), k - 1) & \text{otherwise,} \end{cases} \quad (69)$$

for all  $1 \leq k \leq t$ .

We then introduce the function  $b(t, n)$  which give the indexes of the node in slice  $t$  that is crossed by the scenario leading to node  $(T, n)$ . The mapping  $b$  is thus

$$b(t, n) = a(T, n, T - t) \quad (70)$$

Therefore, the scenario associated with the terminal nodes  $(T, n)$  can be represented as the sequence of nodes  $\{(0, 1), b(1, n), \dots, (T, b(T, n)), (T, n)\}$ .

After that, we need the auxiliary functions  $l(t, n)$  which is defined as

$$l(t, n) = n - \sum_{k=1}^{a(t, n, 1) - 1} f(t - 1, k) \quad (71)$$

which gives the transition indexed that cause the transitions from  $(t - 1, a(t, n, 1))$  to  $(t, n)$ . These functions are also used by van Delft and Vial (2004).

#### **4.2.2 Stochastic process values on the event tree**

To write formulas generating the process values at the nodes, we need some auxiliary variables. We first define  $f(t, n)$  as the number of transitions from node  $(t, n)$ . Each node represents a realization of the demand pair  $(D_t^H, D_t^L)$ . We will first discuss the structure of the discretization of the two variables without considering the event tree. After that, we provide a mapping from node  $(t, n)$  to the related realized demand.

The related two discretized distributed random variables at time  $t$  can be denoted as  $\epsilon_H$  and  $\epsilon_L$ , where each random variable  $\epsilon_p$  ( $p \in \{H, L\}$ ) is characterized by its discrete

state space  $V_{p,t} = \{\epsilon_p(t, 1), \epsilon_p(t, 2), \dots, \epsilon_p(t, N_p(t))\}$  ( $p \in \{H, L\}$ ), with  $N_p(t)$  as cardinality of demand  $p$  ( $p \in \{H, L\}$ ) at time  $t$ . The associated probability distributions  $Pr_{p,t} = \{pr_p(t_p, 1), pr_p(t_p, 2), \dots, pr_p(t_p, N_p(t))\}$ , ( $p \in \{H, L\}$ ).

As mentioned, each node  $(t, n)$  represents a realization of the demand pair  $(D_t^H, D_t^L)$ . The mapping from  $n$  in any given node  $(t, n)$  to  $(i, j)$  in  $(\epsilon_H(t, n, i), \epsilon_L(t, n, j))$  is as follow:

$$i = \text{ceil}\left(\frac{n}{N_L(t)}\right) \quad (72)$$

$$j = n - (i - 1)N_L(t) \quad (73)$$

where  $i \in \{1, \dots, N_H(t)\}$  and  $j \in \{1, \dots, N_L(t)\}$ .

For a given  $N_p(t)$  value, the selection of the state space  $V_{p,t}$  and the computation of associate probability distribution  $Pr_{p,t}$  is a standard problem and can be solved via simple computations.

Similar to van Delft and Vial (2004), the discretized independent demand at node  $(t, n)$  ( $t = 2, \dots, T$ ) can be then computed by

$$D_H(t, n) = \mu_t^H + \epsilon_H(t - 1, a[t, n, 1], \text{ceil}\left(\frac{l[t, n]}{N_L(t)}\right)) \quad (74)$$

$$D_L(t, n) = \mu_t^L + \epsilon_L(t - 1, a[t, n, 1], l[t, n] - (i - 1)N_L(t)) \quad (75)$$

with

$$D_H(1, n) = \mu_1^H + \epsilon_H(0, 1, l_H(1, n_H))\sigma_1^H \quad (76)$$

$$D_L(1, n) = 0 \quad (77)$$

Since the demand is independent, the conditional transition probability from node  $(t - 1, a[t, n, 1])$  to the current note  $(t, n)$  is

$$pr(t - 1, a[t, n, 1], l[t, n]) = pr_H\left(t, \text{ceil}\left(\frac{l[t, n]}{N_L(t)}\right)\right) pr_L(t, l[t, n] - (i - 1)N_L(t)) \quad (78)$$

The unconditional occurrence probability  $Pr(t, n)$  of node  $(t, n)$  can be recursively computed by

$$Pr(t, n) = pr(t - 1, a[t, n, 1], l[t, n])Pr(t - 1, a(t, n, 1)) \quad (79)$$

with  $Pr(0, 1) = 1$ .

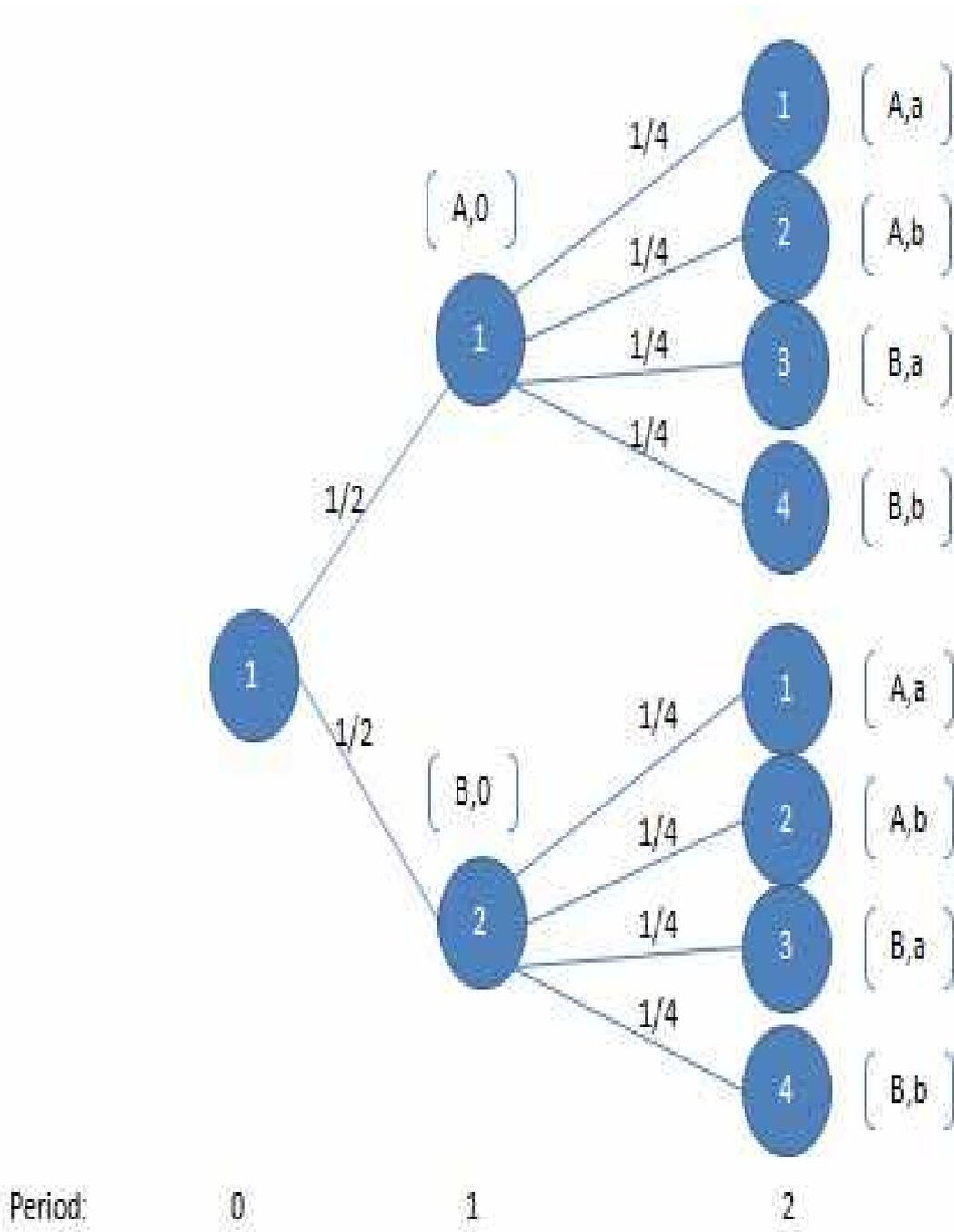
For illustrative purpose, we provide an example as follow. The grid number is 2. As shown in the following table, the demand of the high-end product in both period 1 and 2 is either A or B with equal probability. The demand of the low-end product in period 1 is 0 with probability 1. In period 2, the demand is either a or b with equal probability.

Table 4.1 An example

	t=1	t=2
Demand of the high-end product	A or B	A or B
Demand of the low-end product	0	a or b

The associated event tree is presented in Figure 4.3. The transition probabilities are shown on the arcs and the demand of the associated node is presented beside the node. In period 1, the demand  $(D_1^H, D_1^L)$  is either  $(A, 0)$  in node  $(1, 1)$  or  $(B, 0)$  in node  $(1, 2)$  with equal probability  $1/2$ . In period 2, the transition probability from node  $(1, 1)$  to its first successor  $(2, 1)$  is  $\frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$ . The same methodology applies for the rest of the event tree.

Figure 4.3 An example



### 4.3 The Deterministic Equivalent

In the stochastic version, not only the demands but also the decision and state variables are stochastic. Note that the decision variables  $X_0^H, Y_0^H, X_0^L, Y_0^L$  are fixed at the beginning of the horizon and are thus deterministic. In contrast,  $X_1^H, Y_1^H, I_H, I'_H, I_L$  and  $I'_L$  are random variables depending on the demand pattern. In the stochastic demand case, we formulate

the objective of (54) as an expected value. Formally, the problem can be written as

$$\begin{aligned} & \max E \left[ (R_s(I_H^-, I_L^-, I_H, I'_H, I_L, I'_L) - E_s(I_H, I'_H, I_L, I'_L, X^H, Y^H, X^L, Y^L)) \right] \\ \text{s.t.} \quad & I_{H,0} = X_0^H \end{aligned} \tag{81}$$

$$I'_{H,0} = Y_0^H \tag{82}$$

$$I_{H,1} + I'_{H,1} = I_{H,0} + I'_{H,0} - (D_1^H - I_{H,1}^-) + X_1^H + Y_1^H \tag{83}$$

$$I_{H,2} + I'_{H,2} = I_{H,1} + I'_{H,1} - (D_2^H - I_{H,2}^-) \tag{84}$$

$$I_{L,2} + I'_{L,2} = I_{L,1} + I'_{L,1} - (D_2^L - I_{L,2}^-) \tag{85}$$

$$X_t^H + Y_t^H \leq k^H, \quad t = 0, 1 \tag{86}$$

$$X_0^L + Y_0^L \leq k^L \tag{87}$$

$$I_{H,0} \leq K \tag{88}$$

$$I_{H,1} + I_{L,1} \leq K \tag{89}$$

$$X_t^H, Y_t^H \geq 0, \quad t = 0, 1 \tag{90}$$

$$X_0^L, Y_0^L \geq 0 \tag{91}$$

$$I_{H,t}, I'_{H,t}, I_{H,t}^- \geq 0, \quad t = 1, 2 \tag{92}$$

$$I_{L,t}, I'_{L,t}, I_{L,t}^- \geq 0, \quad t = 2 \tag{93}$$

In the above problem the constraints apply to stochastic variables. They should hold at each outcome of the stochastic process. However, the stochastic problem does not fit the standard scheme of mathematical programming in this format. In order to write the problem in a linear programming format, it suffices to enforce the constraints at each node of the event trees. Actually, constraints and variables must be indexed by the nodes to which they apply. When this operation is performed, the above problem becomes a genuine mathematical programming problem. In case the constraints and the objective are linear as in the present case, the problem is a linear programming problem. The new deterministic equivalent problem has a much larger size than the original deterministic version. However, it is worth pointing out that the stochastic state and decision variables need not be discretized, which is a formidable advantage over dynamic programming formulations.

### 4.3.1 The Event Tree Related Formalism

In this section, we will first index variables, parameters and constraints with respect to the tree nodes. After that, we introduce the probabilistic elements that were defined in

the previous sections: namely, the parameters  $a, b$  and  $l$  defined in (69),(70),(71), and the probabilities  $Pr$  given in (79).

All constraints indexed by  $t$ , should now be indexed by  $t$  and by  $n$ . As an illustration, consider the case of a variable, say  $I_{H,t}$ , with a single time index  $\tau \leq t$  appearing in a constraint associated with the node  $(t, n)$ . The index  $\tau$  specifies the date at which the variable has been fixed. Then  $I_{H,t}$  should be replaced by  $I_H(\tau, a(t, n, t - \tau))$ . A similar treatment is to be applied to all parameters, such as  $D_t^H$ , that are time and node dependent. These notational changes are not necessary for the parameters which are not node dependent, as for example  $r_H, c_L, V_H$  or  $S_H$ .

By introducing the event tree related formalism in the general stochastic linear program, we have the following explicit model. For each scenario, indexed by  $n \in 1, \dots, N(T)$  the revenues of the process ends in node  $(T, n)$  are given by

$$\begin{aligned}
R_s(n) &= r_H(D_H(1, b(1, n)) - I_H^-(1, b(1, n))) + r_H(D_H(2, b(2, n)) - I_H^-(2, b(2, n))) \\
&\quad + r_L(D_L(2, b(2, n)) - I_L^-(2, b(2, n))) + V_H(I_H(2, b(2, n)) + I_H'(2, b(2, n))) \\
&\quad + V_L(I_L(2, b(2, n)) + I_L'(2, b(2, n))) \tag{94}
\end{aligned}$$

The first two terms are the revenues of the high-end product from the effective sales in period  $t = 1, 2$ ; the third term is the revenues of the low-end product from the effective sales and the last two terms pertain to the salvage value of the left-over inventory of the two products at the end of period 2.

The  $E_s$  expenses of the process ends in node  $(T, n)$  can be reformulated as,

$$\begin{aligned}
E_s(n) &= \sum_{t=0}^1 c_H(X^H(t, b(t, n)) + Y^H(t, b(t, n))) + c_L(X^L(0, 1) + Y^L(0, 1)) \\
&\quad + \sum_{t=0}^1 h_H I_H(t, b(t, n)) + \sum_{t=0}^1 h_H' I_H'(t, b(t, n)) + (h_L X^L(0, 1) + h_L' Y^L(0, 1)) \\
&\quad + \left( \sum_{t=1}^2 S_H I_H^-(t, b(t, n)) + S_L I_L^-(2, b(2, n)) \right) \tag{95}
\end{aligned}$$

The first term is the cumulated purchasing cost of the high-end product; the second term is the purchasing cost of the low-end product; the third term and the fourth one pertain to the cumulated holding cost of the high-end product in the primary and secondary warehouse, respectively; the fifth term is the total holding cost of the low-end product in the primary and secondary warehouse, respectively; the last term is the shortage cost of the high-end and low-end product, respectively.

The deterministic equivalent linear program is

$$\max \quad \sum_{n=1}^{N(T)} Pr(T, n) (R_s(n) - E_s(n)) \quad (96)$$

$$\begin{aligned} \text{s.t.} \quad & I_H(t, b(t, n)) = X^H(t, b(t, n)), \\ & t = 0, \quad n = 1, \dots, N(t), \end{aligned} \quad (97)$$

$$\begin{aligned} & I'_H(t, b(t, n)) = Y^H(t, b(t, n)), \\ & t = 0, \quad n = 1, \dots, N(t), \end{aligned} \quad (98)$$

$$\begin{aligned} & I_H(1, b(1, n)) + I'(1, b(1, n)) = I_H(0, b(0, n)) + I'_H(0, b(0, n)) \\ & - (D_H(1, b(1, n)) - I^-_H(1, b(1, n))) + X^H(1, b(1, n)) + Y^H(1, b(1, n)), \\ & n = 1, \dots, N(1), \end{aligned} \quad (99)$$

$$\begin{aligned} & I_H(2, b(2, n)) + I'(2, b(2, n)) = I_H(1, b(1, n)) + I'_H(1, b(1, n)) \\ & - (D_H(2, b(2, n)) - I^-_H(2, b(2, n))), \\ & n = 1, \dots, N(2), \end{aligned} \quad (100)$$

$$\begin{aligned} & I_L(2, b(2, n)) + I'(2, b(2, n)) = X^L(0, 1) + Y^L(0, 1) \\ & - (D_L(2, b(2, n)) - I^-_L(2, b(2, n))), \\ & n = 1, \dots, N(2), \end{aligned} \quad (101)$$

$$\begin{aligned} & X^H(t, b(t, n)) + Y^H(t, b(t, n)) \leq k^H, \\ & t = 0, 1, \quad n = 1, \dots, N(t), \end{aligned} \quad (102)$$

$$X^L(0, 1) + Y^L(0, 1) \leq k^L, \quad (103)$$

$$I_H(0, b(0, 1)) \leq K, \quad (104)$$

$$I_H(1, b(1, n)) + X^L(0, 1) \leq K, \quad (105)$$

$$\begin{aligned} & X^H(t, b(t, n)), \quad Y^H(t, b(t, n)) \geq 0, \\ & t = 0, 1, \quad n = 1, \dots, N(t), \end{aligned} \quad (106)$$

$$X^L(0, 1), Y^L(0, 1) \geq 0, \quad (107)$$

$$\begin{aligned} & I^-_H(t, b(t, n)) \geq 0, \\ & t = 0, 1, 2, \quad n = 1, \dots, N(t), \end{aligned} \quad (108)$$

$$\begin{aligned} & I^-_L(t, n) \geq 0, \\ & t = 1, 2, \quad n = 1, \dots, N(t) \end{aligned} \quad (109)$$

#### 4.4 Numerical Results and Conclusion

In this section, we apply the stochastic programming methodology to search for the optimal solution of the following example:

$$r_H = 10, c_H = 4, V_H = 1, h_H = 1, S_H = 8, h'_H = 5, r_L = 5, c_L = 2, V_L = 1, h_L = 1, S_L = 4, h'_L = 3, k^H = 200, k^L = 1000, K = 1000, C_{v1}^H = C_{v2}^H = 2, C_{v2}^L = 0.1, \mu_1^H = \mu_2^H = 300, \sigma_1^H = \sigma_2^H = 600, \mu_2^L = 700, \sigma_2^L = 70.$$

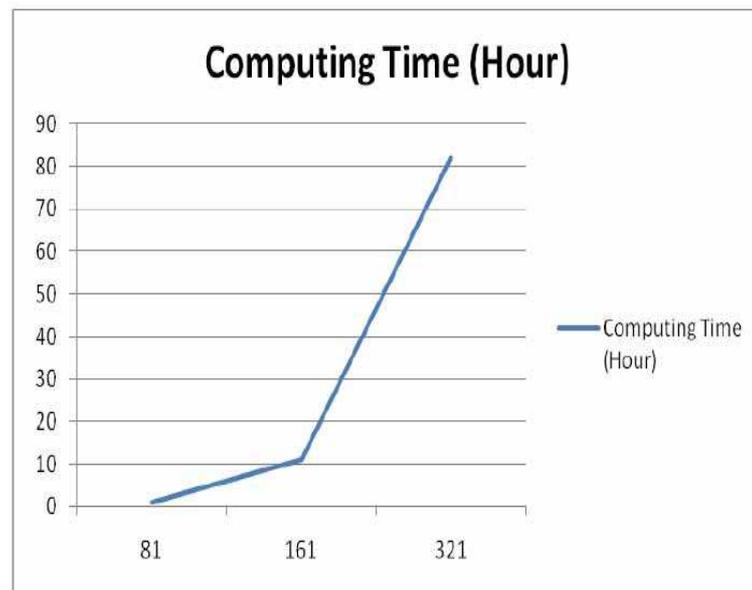
The continuous demands are approximated by the aforementioned discrete values. The idea is to use grids with the same number of points in each period: this base number is 81, yielding a tree with 531522 nodes. As a result, the deterministic equivalent linear programs have 3189132 variables and 2657610 constraints. We perform the computation on a PC with Intel 3.0 GHZ CPU. Solving this two-period problem takes approximately one hour.

The choice of an appropriate approximation of the stochastic process is an important issue in stochastic programming. A finer grid is liable to yield more reliable results, but the size of the deterministic equivalent program increases dramatically with the size of the grid. We have carried out a few experiments to test the impact of the grid size on the objective function and on the decision variables values. The finer grid we considered has 321 points per period. This leads to a tree with 33179523 nodes and a deterministic equivalent linear program with 104332 constraints and 311699 variables. This is the maximal size we could handle on our somewhat limited hardware. The solution required 82 hours. Details are presented in Table 4.2 and Graph 4.4.

Table 4.2 First period decision and related computation time

$X^H_0$	$Y^H_0$	$X^L_0$	$Y^L_0$	Number of nodes per period	Total nodes	Total variables	Total constraints	Computing Time (Hour)
180	0	661	0	81	538083	20339	6892	1
196	0	680	0	161	4199363	79059	26572	11
200	0	694	0	321	33179523	311699	104332	82

Figure 4.4 Computation time - Number of grid



In the above table, we also compute the associated expected profit for different size of grid. The above results show that less grid size results in less order quantities and therefore, lower expected profit. We also notice that the computation time decreases dramatically as the size of the grid decreases. In the following table, we try to gear good first stage solutions by using a looser approximation of the demand in the second stage so as to reduce the computation time. The grid in the first stage remains 321 and is reduced to 161<sup>2</sup> and 81<sup>2</sup> in the second stage.

Table 4.3 Looser approximation in the second stage

$X_0^H$	$Y_0^H$	$X_0^L$	$Y_0^L$	Grid	Total nodes	Total variables	Total constraints	Computing Time (Hour)	Profit*	Profit lost %
198	0	672	0	321 x (81) <sup>2</sup>	2106403	8425616	8425937	5	1485	15
200	0	685	0	321 x (161) <sup>2</sup>	8320963	33283856	33284177	21	1695	3
200	0	694	0	321 x (321) <sup>2</sup>	33076483	132305936	132306257	82	1742	0

As expected, the total computation time decreases dramatically as the grid size in the second period decreases. The solution of the high-end product confirms the idea that a grid size of 161<sup>2</sup> in the second stage is accurate enough. However, the solution of the low-end

product in the first stage is not accurate, which leads to a profit lost of 3%. The associate computation time decreases dramatically from 82 hours to 21 hours, but is still considerable. In the following numerical examples, we use the grid  $321 \times 321^2$  so as to get more accurate results and provide managerial insights.

We consider the following two situations in which both demands are gamma distributed:

Table 4.4 Parameter Settings

	Demand	$\mu$	$\sigma$
Case I	$D_1^H$	200	400
	$D_2^H$	200	400
	$D_2^L$	800	80
Case II	$D_1^H$	800	400
	$D_2^H$	800	400
	$D_2^L$	200	80

In case I, the sale proportion of the high-end product in period 2 is around 20% while it is around 80% in the next case. The other parameter settings is presented as follow:

$$r_H = 10, c_H = 4, V_H = 1, h_H = 1, S_H = 8, h'_H = 5, r_L = 5, c_L = 2, V_L = 1, h_L = 1, S_L = 4, h'_L = 3, k^H = 1500, k^L = 1500, C_{v1}^H = C_{v2}^H = 2, C_{v2}^L = 0.1.$$

In each case, we try different value of K to explore the influence of the primary warehouse capacity. In case it plays an important rule, we want to find out the minimum level of this capacity. The results are presented as follow:

Figure 4.5 Sensitivity Analysis of K in Case I

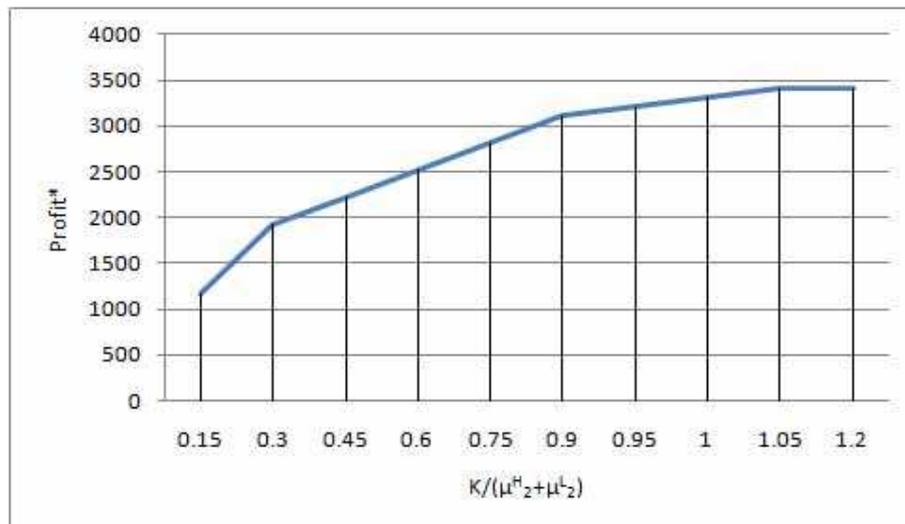
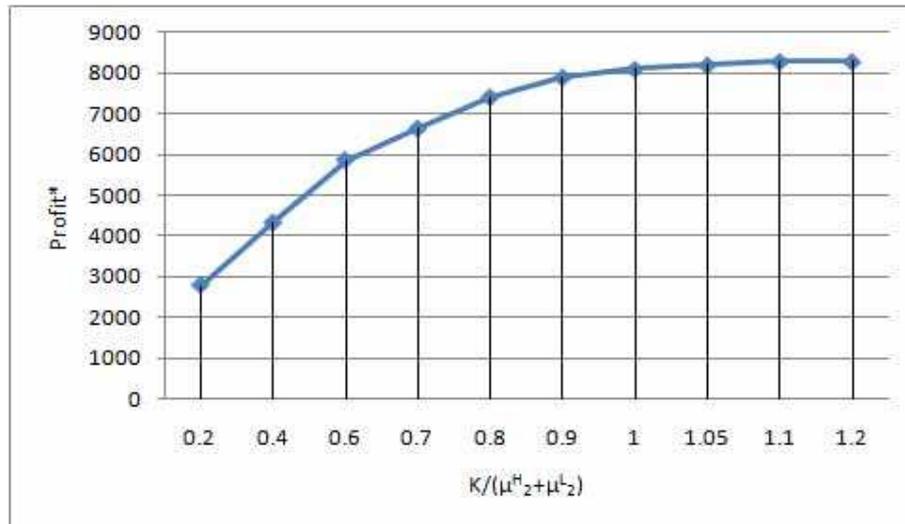


Figure 4.6 Sensitivity Analysis of K in Case II



An observation shows that the retailer can be very beneficial by increasing the primary inventory capacity if it is not enough. Due to demand compensation, we compute the inventory/demand ratio  $\frac{K}{\mu^H_2 + \mu^L_2}$  in each scenario. The above results suggest that the best ratio should be between 1.05 and 1.1. In other words, the primary warehouse capacity should be 5%  $\rightarrow$  10% higher than the summation of the demand mean in the second period.

The average computation time of the above examples is 82 hours. Therefore, we need to develop a more efficient method so as to provide more managerial insights via a large number of numerical examples.

In the next section, we formulate problems II and III in a Newsboy framework and provide a GA-based heuristic method to search for the 'optimal' solution. We will show that it is efficient in comparison to the Steepest Descent Method, the Classical Genetic Algorithm and the Stochastic Programming Method.

# Chapter 5

## Problems II and III: Periodic Review, Two-product Types, Two-supplier, Two-stage Problem in Newsboy Framework

### 5.1 Model Introduction

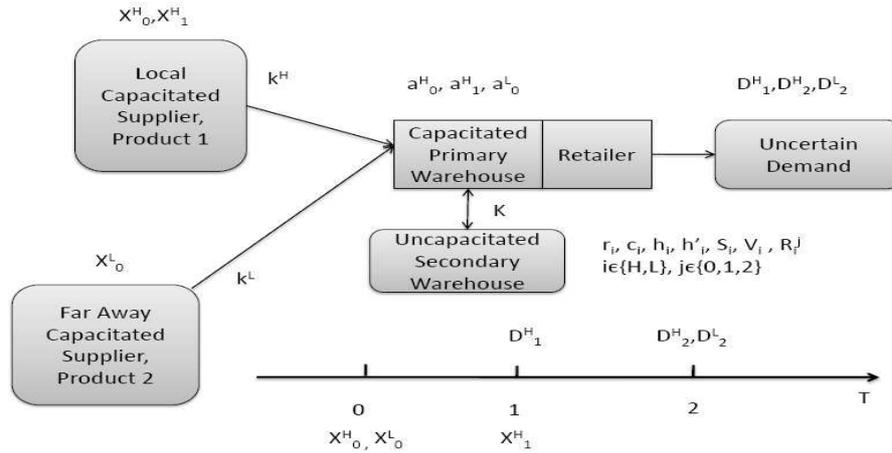
We recall Table 1.1 the relationship between the cases and the models. For companies using pull strategy, we build model 1.1, 1.2, 2.1, 2.2 under deterministic demand assumption. All demand information is known in period 0. We also build model 3.1, 3.2, 4.1, 4.2 to investigate cases where a company uses push strategy and the demand is stochastic. We assume that the demands in different periods are independently distributed and are not necessarily identical. This helps us to investigate both stationary and non-stationary demand in our experiments.

Table 1.1 Model List-1

		Deterministic Demand	Stochastic Demand
Cheaper 2 <sup>nd</sup> Warehouse	Retailer 1	Model 1	Model 3.1
	Retailer 2		Model 3.2
Expensive 2 <sup>nd</sup> Warehouse	Retailer 1	Model 2.1 and 2.2	Model 4.1
	Retailer 2	Model 2.1	Model 4.2

These models share a common basic model of which the structure is presented in Figure 5.1a and 5.1b. It is a two-product three-period two-capacitated suppliers with different fixed lead times single-capacitated-retailer two-stage model. The retailer owns the primary warehouse with capacity  $K$  and it can use additional storage in a rented secondary warehouse. In this research, we investigate the following two types of retailers.

Figure 5.1a The Basic Models of Retailer 1.

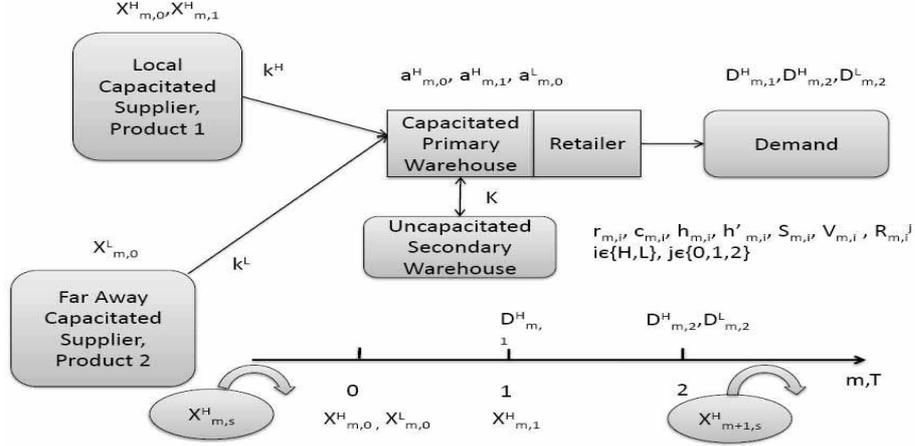


As shown in Figure 5.1a, retailer 1 orders a perishable product from a local supplier and another perishable product from a far away supplier. We use the following 2 types of time units: selling season and period. The selling season is used to capture the feature that products have short life cycle. For short life cycle products such as fashion type products or perishables, leftovers must be salvaged at the end of a selling season. The period is used to capture the fact that suppliers have different delivery times in a selling season. There are three periods in a selling season. We use the notation  $n(0 \leq n \leq 2)$ , to represent a period in our different parameters. In period 0, the retailer orders  $X_0^H$  units of **product 1** from the local supplier and another  $X_0^L$  units of **product 2** from the far away supplier, subject to production capacity  $k^H, k^L$ , respectively. The delivery time of product 1 is negligible, but it takes the retailer one period to prepare the product (the reason for this is explained later). The  $X_0^H$  units of product 1 is available for sale in period 1. In period 1, product 2 arrives and are placed to the warehouse(s) in order to satisfy the demand  $D_2^L$  in period 2. Based on the realized demand of the product 1 in period 1, namely  $D_1^H$ , the retailer orders  $X_1^H$  units of product 1 from the local supplier in order to satisfy the related demand ( $D_2^H$ ) in period 2. The retailer also decides the allocation of the following items between the two warehouses: possible leftovers of product 1, newly arrived product 2 and product 1.

Retailer 2 orders a non-perishable product from a local supplier and another perishable product from a far away supplier. Products which have longer life cycle can be carried over to the next selling season. In this model, notations of the variables and parameters are similar to 'retailer 1' model but with additive subscription  $m$  for a given selling season  $m$  ( $m \in \{1, 2, \dots, M\}$ ). For instance, the decision variable  $X_0^H$  and the unit selling price of the

high-end product  $r_H$  in season  $m$  are replaced by  $X_{m,0}^H$  and  $r_{m,H}$ , respectively. The leftover of product 1 at the end of the selling season  $m-1$  ( $m>1$ ) is kept in the warehouse with unit holding cost  $h_{m-1,1}$  and is considered as a state variable  $X_{m,s}^H$  in the upcoming selling season  $m$ . In period 0 of selling season  $m$ , these units, along with the newly arrived  $X_{m,0}^H$  units, are placed to the secondary warehouse for sale in the next period.

Figure 5.1b The Basic Models of Retailer 2.



We also assume that all units must be placed into the warehouse(s) for a *preparation period* before sale. That is, for instance, 10 units of product 1 arrived in period 1 cannot be sold before period 2. This is true in practice. As an example, products arriving at large retail stores like Nordstrom, Macy's and Zara are usually packaged in big boxes, which cannot be sold directly to the customers. Retailers need some time to unload them from the trucks, place them to the warehouse(s), and transfer them from the boxes to the shelves for the customers. The unit holding cost of the primary and secondary warehouses of product 1 is  $h_H$  and  $h'_H$ , respectively. That of the product 2 is denoted as  $h_L$ ,  $h'_L$ . The secondary warehouse could either be more expensive than the primary retail space, or it can be an off-site warehouse which is cheaper than the primary one. We investigate both cases in this research.

Denote  $r_H$  to be the unit selling price for product 1 and  $r_L$  for product 2. The unit cost for product 1 is  $c_H$  and that of the low-end product is  $c_L$ . Let  $V_H, V_L$  be the unit salvage values for product 1 and product 2, respectively. To avoid trivial solutions, we assume that:

$$r_H \geq V_H \quad (110)$$

$$r_L \geq V_L \quad (111)$$

Similarly, we set  $S_H, S_L$  to be the unit shortage costs for the product 1 and 2, respectively. We also set  $R_n^H(\bullet), R_n^L(\bullet)$  to be the profits for product 1 and 2 in period  $n$ , respectively. The local supplier's production capacity in a period is  $k^H$ , while that of the far-away supplier is  $k^L$ . Inventory holding cost in period  $n$  is marked as  $I_n(\bullet)$ .

Without loss of generality, we assume that the upper bound of the profit margin for units in both warehouses is non-negative, as storing units will never be profitable for the retailer. These assumptions are shown below:

$$r_H - c_H - \max(h_H, h'_H) \geq 0 \quad (112)$$

$$r_L - c_L - \max(h_L, h'_L) \geq 0 \quad (113)$$

Denote the fraction factor  $a_n^H$  to be the proportion of the  $n$ 'th period's leftover of product 1 placed in the primary warehouse for sale in the next period. Thus  $1 - a_n^H$  percent of product 1 is assigned to the rented secondary inventory. We make similar settings to  $a_n^L, 1 - a_n^L$  for product 2 at the end of period  $n$ .

The differences of the models are shown in Table 5.1. For instance, model 1 and model 2.1 are built for retailer 1, but we will show that the optimal solution for retailer 1 is also the best solution for retailer 2. The problems of retailer 1 facing deterministic demand are formulated in models 3.1 and 4.1. Both products have short life cycle, thus the leftovers have to be salvaged at the end of the selling season. Therefore, we formulate them as two single selling season (three periods) problems. For retailer 2, the locally produced product has longer life cycle and can be carried to the next selling season. In cases when the demand is stochastic, we formulated the problems as a general  $M$  season ( $3M$  periods) problem. For companies facing deterministic demand, suppliers are less likely to be capacitated due to precise forecast. We assume uncapacitated suppliers but capacitated cases are also studied via numerical examples to provide managerial insights.

Table 5.1 Model List-2

Model	Local Supplier	Stochastic Demand	Unmet Demand	# of Period	Suppliers' Capacity	Cheaper 2 <sup>nd</sup> Warehouse	Salvage Low-end
1	Perishable	No	Lost	3	No	Yes	Yes
	Non-perishable	No	Lost	3	No	Yes	Yes
2.1	Perishable	No	Lost	3	No	No	Yes
	Non-perishable	No	Lost	3	No	No	Yes
2.2	Perishable	No	Lost	3	Yes	No	Yes
3.1	Perishable	Yes	Backlogged	3	Yes	Yes	Yes
3.2	Non-perishable	Yes	Backlogged	Multiple	Yes	Yes	No
4.1	Perishable	Yes	Lost	3	Yes	No	Yes
4.2	Non-perishable	Yes	Lost	Multiple	Yes	No	No

## 5.2 Optimization Problems

### 5.2.1 Model 1

In case the uncapacitated secondary warehouse is cheaper, apparently all units should be placed into the secondary one. Since the demand is deterministic and meeting a unit of demand is profitable (assumption (112)(113)), the optimal solution is  $(X_0^H, X_1^H, X_0^L) = (D_1^H, D_2^H, D_2^L)$ . This model will not be further discussed here.

### 5.2.2 Model 2

#### Model 2.1

The total revenue in period 0 is:

$$R_0^H(\bullet) + R_0^L(\bullet) = -c_H X_0^H - c_L X_0^L - I_0(X_0^H, a_0^H) \quad (114)$$

The last item is determined in the second stage. The second stage problem arises due to the primary (main) inventory constraint. In case the total primary inventory amount exceeds the capacity  $K$ , the decision maker rents outside secondary inventory space, and thus faces the following allocation problem:

$$\text{Min} I_0(X_0^H, a_0^H) = \text{Min} (h_H a_0^H X_0^H + h'_H (1 - a_0^H) X_0^H) \quad (115)$$

subject to

$$a_0^H X_0^H \leq K \quad (116)$$

In period 1,  $X_1^H$  units of product 1 are ordered with cost  $c_H X_1^H$  and arrive with the  $X_0^L$  units of product 2 that are ordered one period ahead. Both products are placed in the inventory for sale in the next period. At the end of period 1, there are  $(X_0^H - D_1^H)^+ + X_1^H$  units of product 1 and  $X_0^L$  units of product 2 left in the inventory for period 2. The total inventory cost at the end of period 1,  $I_1(\bullet)$ , is a function of the quantity for sale in period 2 and the allocation, which we analyze in the second stage. The total profit of period 1 is:

$$R_1^H(\bullet) + R_1^L(\bullet) = r_H \min(D_1^H, X_0^H) - c_H X_1^H - S_H (D_1^H - X_0^H)^+ - I_1(X_0^H, X_1^H, X_0^L, a_1^H, a_1^L) \quad (117)$$

The second stage problem in period 1 can be written as:

$$\begin{aligned}
Min I_1(X_0^H, X_1^H, X_0^L, a_1^H, a_1^L) &= Min(h_H a_1^H ((X_0^H - D_1^H)^+ + X_1^H) \\
&+ h'_H (1 - a_1^H) ((X_0^H - D_1^H)^+ + X_1^H) \\
&+ h_L a_1^L X_0^L + h'_L (1 - a_1^L) X_0^L) \quad (118)
\end{aligned}$$

In period 2, we have similar analysis as in period 1, except for the leftovers. At the end of this last period, these units are sold at salvaged prices differed by types. At the end of period 2, product 1 that is leftover and is salvaged is  $\left(\left((X_0^H - D_1^H)^+ + X_1^H\right) - D_2^H\right)^+$ , while that of the product 2 is  $(X_0^L - D_2^L)^+$ . Similarly, the shortage amount of product 1 is  $\left(D_2^H - \left((X_0^H - D_1^H)^+ + X_1^H\right)\right)^+$ , while that of product 2 is  $(D_2^L - X_0^L)^+$ . The total profit of period 2 is:

$$\begin{aligned}
R_2^H(\bullet) + R_2^L(\bullet) &= r_H min(D_2^H, (X_0^H - D_1^H)^+ + X_1^H) \\
&- S_H \left(D_2^H - \left((X_0^H - D_1^H)^+ + X_1^H\right)\right)^+ \\
&+ V_H \left(\left((X_0^H - D_1^H)^+ + X_1^H\right) - D_2^H\right)^+ \\
&+ r_L min(D_2^L, X_0^L) - S_L (D_2^L - X_0^L)^+ + V_L (X_0^L - D_2^L)^+ \quad (119)
\end{aligned}$$

To conclude, the formal original problem in model 2 is:

$$\max \pi(X_0^H, X_1^H, X_0^L, a_0^H, a_1^H, a_1^L) = \max_{X_0^H, X_1^H, X_0^L, a_0^H, a_1^H, a_1^L} \sum_{i=0}^2 (R_i^H(\bullet) + R_i^L(\bullet)) \quad (120)$$

where

$$\begin{aligned}
\sum_{i=0}^2 (R_i^H(\bullet) + R_i^L(\bullet)) &= -c_H X_0^H - c_L X_0^L - h_H a_0^H X_0^H - h'_H (1 - a_0^H) X_0^H \\
&+ r_H \min(D_1^H, X_0^H) - c_H X_1^H - S_H (D_1^H - X_0^H)^+ \\
&- h_H a_1^H ((X_0^H - D_1^H)^+ + X_1^H) \\
&- h'_H (1 - a_1^H) ((X_0^H - D_1^H)^+ + X_1^H) \\
&- h_L a_1^L X_0^L - h'_L (1 - a_1^L) X_0^L \\
&+ r_H \min(D_2^H, (X_0^H - D_1^H)^+ + X_1^H) \\
&- S_H \left( D_2^H - \left( (X_0^H - D_1^H)^+ + X_1^H \right) \right)^+ \\
&+ V_H \left( \left( (X_0^H - D_1^H)^+ + X_1^H \right) - D_2^H \right)^+ \\
&+ r_L \min(D_2^L, X_0^L) - S_L (D_2^L - X_0^L)^+ + V_L (X_0^L - D_2^L)^+
\end{aligned} \tag{121}$$

subject to the following constraints:

$$a_0^H X_0^H \leq K \tag{122}$$

$$a_1^H ((X_0^H - D_1^H)^+ + X_1^H) + a_1^L X_0^L \leq K \tag{123}$$

$$0 \leq a_0^H, a_1^H, a_1^L \leq 1 \tag{124}$$

$$X_0^H, X_1^H, X_0^L \geq 0 \tag{125}$$

## Model 2.2

This model is an extension of Model 2.1 with the following two extra constraints of the suppliers:

$$X_0^H, X_1^H \leq k^H \tag{126}$$

$$X_0^L \leq k^L \tag{127}$$

### 5.2.3 Model 3

#### Model 3.1

For retailer 1 facing stochastic demand, we reasonably assume that demands are independently distributed random variables with the following notations:

$\mu_i^H$ : mean demand of product 1 in period  $i$  ( $i=1,2$ ).

$\mu_2^L$ : mean demand of product 2 in period 2.

$\sigma_i^H$ : demand variance of product 1 in period  $i$  ( $i=1,2$ ).

$\sigma_2^L$ : demand variance of product 2 in period 2.

$C_{vi}^H$ : coefficient of variation of the demand of product 1 in period  $i$  ( $i=1,2$ ).

$C_{v2}^L$ : coefficient of variation of the demand of product 2 in period 2.

$\varphi_{Hi}(y_i)$ : demand probability density function of product 1 in period  $i$  ( $i=1,2$ ).

$\varphi_{L2}(y_3)$ : demand probability density function of product 2 in period 2.

In cases where the secondary warehouse is cheaper than the primary one, all units will be placed to the uncapacitated secondary warehouse and the profit of the two products can be maximized independently. In this case, the unit holding cost of product 1 and 2 are  $h_1 = h'_H$  and  $h_2 = h'_L$ . The expected profit of product 1 is:

$$\begin{aligned}
& R_{UH}(X_0^H, X_1^H) \\
= & \int_0^{X_0^H} (r_H y_1 - (c_H + h_1)X_0^H) \varphi_{H1}(y_1) dy_1 \\
& + \int_0^{X_0^H} \left( \int_0^{X_1^H + X_0^H - y_1} (r_H y_2 - c_H X_1^H - h_1(X_1^H + X_0^H - y_1)) \right. \\
& + V_H(X_1^H + X_0^H - y_1 - y_2) \varphi_{H2}(y_2) dy_2 \\
& + \int_{X_1^H + X_0^H - y_1}^{+\infty} (r_H(X_1^H + X_0^H - y_1) - c_H X_1^H - h_1(X_1^H + X_0^H - y_1)) \\
& - S_H(y_2 - X_1^H - X_0^H + y_1) \varphi_{H2}(y_2) dy_2 \varphi_{H1}(y_1) dy_1 \\
& + \int_{X_0^H}^{+\infty} (r_H y_1 - (c_H + h_1)X_0^H - S_H(y_1 - X_0^H)) \varphi_{H1}(y_1) dy_1 \\
& + \int_{X_0^H}^{+\infty} \left( \int_0^{X_1^H + X_0^H - y_1} (r_H y_2 - (c_H + h_1)X_1^H + V_H(X_1^H - y_2)) \varphi_{H2}(y_2) dy_2 \right. \\
& + \int_{X_1^H + X_0^H - y_1}^{+\infty} (r_H X_1^H - (c_H + h_1)X_1^H - S_H(y_2 - X_1^H)) \varphi_{H2}(y_2) dy_2 \varphi_{H1}(y_1) dy_1 \\
& \left. \right) \varphi_{H1}(y_1) dy_1
\end{aligned} \tag{128}$$

The sub-problem of product 2 is equivalent to a Newsboy problem with unit cost  $c_L + h_2$ . The expected profit is shown as follows:

$$\begin{aligned}
& R_{UL}(X_0^L) \\
= & -(c_L + h_2)X_0^L + r_L \left( \int_0^{X_0^L} y_3 \varphi_{L2}(y_3) dy_3 + X_0^L \int_{X_0^L}^{\infty} \varphi_{L2}(y_3) dy_3 \right) - S_L \int_{X_0^L}^{\infty} (y_3 - X_0^L) \varphi_{L2}(y_3) dy_3 \\
& + V_L \int_0^{X_0^L} (X_0^L - y_3) \varphi_{L2}(y_3) dy_3
\end{aligned} \tag{129}$$

The global problem of Model 3.1 is presented as follows:

$$\begin{aligned}
& \max R_{UHL}(X_0^H, X_1^H, X_0^L) \tag{130} \\
& \text{st. } 0 \leq X_0^H, X_1^H \leq k^H \\
& \quad 0 \leq X_0^L \leq k^L \\
& \text{where } R_{UHL}(X_0^H, X_1^H, X_0^L) = R_{UH}(X_0^H, X_1^H) + R_{UL}(X_0^L) \tag{131}
\end{aligned}$$



$$\begin{aligned}
& R_{m,UL}(X_{m,0}^L) \\
&= -(c_{m,L} + h_{m,2})X_{m,0}^L + r_{m,L} \left( \int_0^{X_{m,0}^L} y_{m,3} \varphi_{m,L2}(y_{m,3}) dy_{m,3} + X_{m,0}^L \int_{X_{m,0}^L}^{\infty} \varphi_{L2}(y_{m,3}) dy_{m,3} \right) \\
&- S_{m,L} \int_{X_{m,0}^L}^{\infty} (y_{m,3} - X_{m,0}^L) \varphi_{m,L2}(y_{m,3}) dy_{m,3} + V_{m,L} \int_0^{X_{m,0}^L} (X_{m,0}^L - y_{m,3}) \varphi_{m,L2}(y_{m,3}) dy_{m,3}
\end{aligned} \tag{133}$$

The total expected profit in period m is presented as follows:

$$\begin{aligned}
& R_{m,UHL}(X_{m,0}^H, X_{m,1}^H, X_{m,0}^L) = R_{m,UH}(X_0^H, X_1^H) + R_{m,UL}(X_0^L) \\
st. \quad & 0 \leq X_0^H, X_1^H \leq k^H \\
& 0 \leq X_0^L \leq k^L
\end{aligned} \tag{134}$$

For the M-season problem with  $1 \leq M < \infty$ , let  $V_m(X_{m,s}^H)$  denote the optimal discounted profit of product 1 when the initial inventory is  $X_{m,s}^H$  and there are m seasons remaining in the decision horizon. Given the inventory information  $X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L$  and the demand information  $D_{m,1}^H, D_{m,2}^H, D_{m,2}^L$  in selling season m, the initial inventory (product 1) in season m-1 is  $X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - D_{m,1}^H - D_{m,2}^H$ . Let  $\beta$  ( $0 \leq \beta \leq 1$ ) denote the discount factor. At the end of the horizon, any unmet demand of product 1 is assumed to be satisfied by a special production or outsourcing at a cost of  $e$  per unit; and any unused inventory has salvage value of  $v$  per unit. Without loss of generality, we assume that  $e \geq v$ . This assumption is previously used by Li et al (2006). From the standard theory of Markov Decision Programming (MDP) (see Li et al (2006) for example), we have:

$$\begin{aligned}
V_m(X_{m,s}^H) &= \max_{X_{m,0}^H, X_{m,1}^H, X_{m,0}^L} J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L), \quad m = M, \dots, 1, \text{ with} \\
J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L) &= R_{m,UHL}(X_{m,0}^H, X_{m,1}^H, X_{m,0}^L) \\
&\quad + \beta E[V_{m-1}(X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - D_{m,1}^H - D_{m,2}^H)], \text{ and} \\
V_m(X_{0,s}^H) &= vX_{0,s}^{H+} - eX_{0,s}^{H-} \\
st. \quad & 0 \leq X_{m,0}^H, X_{m,1}^H \leq k_m^H \\
& 0 \leq X_{m,0}^L \leq k_m^L
\end{aligned} \tag{135}$$

## 5.2.4 Model 4

### Model 4.1

In this subsection, we formulate the problem of retailer 1 as a three-period problem, since both product 1 and 2 have short life cycle. Compared to Model 3.1, we here focus on the case that the secondary warehouse is more expensive than the primary one. The allocation problem (second stage) arises due to the limited primary warehouse capacity. The expected total profit of this problem is shown below and is derived in detail in Appendix section 1.

$$\begin{aligned}
& E_D[\pi(X_0^H, X_1^H, X_0^L, a_0^H, a_1^H, a_1^L)] \\
&= -c_H X_0^H - c_L X_0^L - h_H a_0^H X_0^H - h'_H (1 - a_0^H) X_0^H \\
&+ r_H \left( \int_{X_0^H + X_1^H - \frac{\kappa - a_1^L X_0^L}{a_1^H + \epsilon}}^{X_0^H} y_1 \varphi_{H1}(y_1) dy_1 + X_0^H \int_{X_0^H}^{\infty} \varphi_{H1}(y_1) dy_1 \right) - c_H X_1^H \\
&- S_H \int_{X_0^H}^{\infty} (y_1 - X_0^H) \varphi_{H1}(y_1) dy - h_H a_1^H \left( \int_{X_0^H + X_1^H - \frac{\kappa - a_1^L X_0^L}{a_1^H + \epsilon}}^{X_0^H} (X_0^H - y_1) \varphi_{H1}(y_1) dy_1 + X_1^H \right) \\
&- h'_H (1 - a_1^H) \left( \int_{X_0^H + X_1^H - \frac{\kappa - a_1^L X_0^L}{a_1^H + \epsilon}}^{X_0^H} (X_0^H - y_1) \varphi_{H1}(y_1) dy_1 + X_1^H \right) \\
&- h_L a_1^L X_0^L - h'_L (1 - a_1^L) X_0^L \\
&+ r_H \int_{X_0^H + X_1^H - \frac{\kappa - a_1^L X_0^L}{a_1^H + \epsilon}}^{X_0^H} \left( \int_0^{X_0^H - y_1 + X_1^H} y_2 \varphi_{H2}(y_2) dy_2 \right. \\
&+ (X_0^H - y_1 + X_1^H) \int_{X_0^H - y_1 + X_1^H}^{\infty} \varphi_{H2}(y_2) dy_2 \left. \right) \varphi_{H1}(y_1) dy_1 \\
&+ r_H \int_{X_0^H}^{\infty} \left( \int_0^{X_1^H} y_2 \varphi_{H2}(y_2) dy_2 + X_1^H \int_{X_1^H}^{\infty} \varphi_{H2}(y_2) dy_2 \right) \varphi_{H1}(y_1) dy_1 \\
&- S_H \int_{X_0^H + X_1^H - \frac{\kappa - a_1^L X_0^L}{a_1^H + \epsilon}}^{X_0^H} \left( \int_{X_0^H - y_1 + X_1^H}^{\infty} (y_2 - (X_0^H - y_1 + X_1^H)) \varphi_{H2}(y_2) dy_2 \right) \varphi_{H1}(y_1) dy_1 \\
&- S_H \int_{X_0^H}^{\infty} \left( \int_{X_1^H}^{\infty} (y_2 - X_1^H) \varphi_{H2}(y_2) dy_2 \right) \varphi_{H1}(y_1) dy_1 \\
&+ V_H \int_{X_0^H + X_1^H - \frac{\kappa - a_1^L X_0^L}{a_1^H + \epsilon}}^{X_0^H} \left( \int_0^{X_0^H - y_1 + X_1^H} (X_0^H - y_1 + X_1^H - y_2) \varphi_{H2}(y_2) dy_2 \right) \varphi_{H1}(y_1) dy_1 \\
&+ V_H \int_{X_0^H}^{\infty} \left( \int_0^{X_1^H} (X_1^H - y_2) \varphi_{H2}(y_2) dy_2 \right) \varphi_{H1}(y_1) dy_1 \\
&+ r_L \left( \int_0^{X_0^L} y_3 \varphi_{L2}(y_3) dy_3 + X_0^L \int_{X_0^L}^{\infty} \varphi_{L2}(y_3) dy_3 \right) - S_L \int_{X_0^L}^{\infty} (y_3 - X_0^L) \varphi_{L2}(y_3) dy_3 \\
&+ V_L \int_0^{X_0^L} (X_0^L - y_3) \varphi_{L2}(y_3) dy_3
\end{aligned} \tag{136}$$

Given the decision variables  $X_0^H, X_1^H, X_1^L, a_0^H, a_1^H, a_1^L$ , the problem of the retailer is:

$$\max E_D[\pi(X_0^H, X_1^H, X_0^L, a_0^H, a_1^H, a_1^L)] \quad (137)$$

$$st. \quad a_0^H X_0^H \leq K, \quad (138)$$

$$a_1^H X_1^H + a_1^L X_0^L \leq K, \quad (139)$$

$$0 \leq a_0^H, a_1^H, a_1^L \leq 1, \quad (140)$$

$$0 \leq X_0^H, X_1^H \leq k^H, \quad (141)$$

$$0 \leq X_1^L \leq k^L. \quad (142)$$

## Model 4.2

We formulate the problem of retailer 2 facing stochastic demand with an expensive secondary warehouse as a M-season ( $1 \leq M < \infty$ ), two-stage problem. Notations are similar to the M-season problem in Model 3.2. Unmet demand is assumed to be lost; and the leftover of product 1 at the end of a selling season  $m$  ( $m < M$ ) is held in the warehouses. Due to the capacitated cheaper primary warehouse, we have a second stage problem with three new decision variables  $a_{m,0}^H, a_{m,1}^H, a_{m,1}^L$  for a given selling season  $m$ .

Similar to Model 3.2, given the inventory information  $X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L$  and the demand information  $D_{m,1}^H, D_{m,2}^H, D_{m,2}^L$  in selling season  $m$ , the initial inventory (product 1) in season  $m-1$  is  $\left( (X_{m,s}^H + X_{m,0}^H - D_{m,1}^H)^+ + X_{m,1}^H - D_{m,2}^H \right)^+$ . This is different from Model 3.2 since the unmet demand is assumed to be lost. The expected profit of selling season  $m$  is:

$$\begin{aligned}
& R_{m,UHL}[(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L, a_{m,0}^H, a_{m,1}^H, a_{m,1}^L)] \\
&= -c_{m,H}(X_{m,s}^H + X_{m,0}^H) - c_{m,L}X_{m,0}^L - h_{m,H}a_{m,0}^H(X_{m,s}^H + X_{m,0}^H) - h'_{m,H}(1 - a_{m,0}^H)(X_{m,s}^H + X_{m,0}^H) \\
&+ r_{m,H} \left( \int_{X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - \frac{\kappa_m - a_{m,1}^L X_{m,0}^L}{a_{m,1}^H + \epsilon}}^{X_{m,s}^H + X_{m,0}^H} y_{m,1} \varphi_{m,H1}(y_{m,1}) dy_{m,1} \right. \\
&+ (X_{m,s}^H + X_{m,0}^H) \int_{X_{m,s}^H + X_{m,0}^H}^{\infty} \varphi_{H1}(y_{m,1}) dy_{m,1} \left. - c_{m,H} X_{m,1}^H \right. \\
&- S_{m,H} \int_{X_{m,s}^H + X_{m,0}^H}^{\infty} (y_{m,1} - X_{m,s}^H + X_{m,0}^H) \varphi_{m,H1}(y_{m,1}) dy \\
&- h_{m,H} a_{m,1}^H \left( \int_{X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - \frac{\kappa_m - a_{m,1}^L X_{m,0}^L}{a_{m,1}^H + \epsilon}}^{X_{m,s}^H + X_{m,0}^H} (X_{m,s}^H + X_{m,0}^H - y_{m,1}) \varphi_{m,H1}(y_{m,1}) dy_{m,1} + X_{m,1}^H \right) \\
&- h'_{m,H} (1 - a_{m,1}^H) \left( \int_{X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - \frac{\kappa_m - a_{m,1}^L X_{m,0}^L}{a_{m,1}^H + \epsilon}}^{X_{m,s}^H + X_{m,0}^H} (X_{m,s}^H + X_{m,0}^H - y_{m,1}) \varphi_{m,H1}(y_{m,1}) dy_{m,1} + X_{m,1}^H \right) \\
&- h_{m,L} a_{m,1}^L X_{m,0}^L - h'_{m,L} (1 - a_{m,1}^L) X_{m,0}^L \\
&+ r_{m,H} \int_{X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - \frac{\kappa_m - a_{m,1}^L X_{m,0}^L}{a_{m,1}^H + \epsilon}}^{X_{m,s}^H + X_{m,0}^H} \left( \int_0^{X_{m,s}^H + X_{m,0}^H - y_{m,1} + X_{m,1}^H} y_{m,2} \varphi_{m,H2}(y_{m,2}) dy_{m,2} \right. \\
&+ (X_{m,s}^H + X_{m,0}^H - y_{m,1} + X_{m,1}^H) \int_{X_{m,s}^H + X_{m,0}^H - y_{m,1} + X_{m,1}^H}^{\infty} \varphi_{m,H2}(y_{m,2}) dy_{m,2} \left. \varphi_{m,H1}(y_{m,1}) dy_{m,1} \right. \\
&+ r_{m,H} \int_{X_{m,s}^H + X_{m,0}^H}^{\infty} \left( \int_0^{X_{m,1}^H} y_{m,2} \varphi_{m,H2}(y_{m,2}) dy_{m,2} + X_{m,1}^H \int_{X_{m,1}^H}^{\infty} \varphi_{m,H2}(y_{m,2}) dy_{m,2} \right) \varphi_{m,H1}(y_{m,1}) dy_{m,1} \\
&- S_{m,H} \int_{X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - \frac{\kappa_m - a_{m,1}^L X_{m,0}^L}{a_{m,1}^H + \epsilon}}^{X_{m,s}^H + X_{m,0}^H} \left( \int_{X_{m,s}^H + X_{m,0}^H - y_{m,1} + X_{m,1}^H}^{\infty} (y_{m,2} - (X_{m,s}^H + X_{m,0}^H - y_{m,1} \right. \\
&+ X_{m,1}^H)) \varphi_{m,H2}(y_{m,2}) dy_{m,2} \left. \varphi_{m,H1}(y_{m,1}) dy_{m,1} \right. \\
&- S_{m,H} \int_{X_{m,s}^H + X_{m,0}^H}^{\infty} \left( \int_{X_{m,1}^H}^{\infty} (y_{m,2} - X_{m,1}^H) \varphi_{m,H2}(y_{m,2}) dy_{m,2} \right) \varphi_{m,H1}(y_{m,1}) dy_{m,1} \\
&- h_{m,H} \int_{X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - \frac{\kappa_m - a_{m,1}^L X_{m,0}^L}{a_{m,1}^H + \epsilon}}^{X_{m,s}^H + X_{m,0}^H} \left( \int_0^{X_{m,s}^H + X_{m,0}^H - y_{m,1} + X_{m,1}^H} (X_{m,s}^H + X_{m,0}^H - y_{m,1} + X_{m,1}^H \right. \\
&- y_{m,2}) \varphi_{m,H2}(y_{m,2}) dy_{m,2} \left. \varphi_{m,H1}(y_{m,1}) dy_{m,1} \right. \\
&- h_{m,H} \int_{X_{m,s}^H + X_{m,0}^H}^{\infty} \left( \int_0^{X_{m,1}^H} (X_{m,1}^H - y_{m,2}) \varphi_{m,H2}(y_{m,2}) dy_{m,2} \right) \varphi_{m,H1}(y_{m,1}) dy_{m,1} \\
&+ r_{m,L} \left( \int_0^{X_{m,0}^L} y_{m,3} \varphi_{m,L2}(y_{m,3}) dy_{m,3} + X_{m,0}^L \int_{X_{m,0}^L}^{\infty} \varphi_{L2}(y_{m,3}) dy_{m,3} \right) \\
&- S_{m,L} \int_{X_{m,0}^L}^{\infty} (y_{m,3} - X_{m,0}^L) \varphi_{m,L2}(y_{m,3}) dy_{m,3} + V_L \int_0^{X_{m,0}^L} (X_{m,0}^L - y_{m,3}) \varphi_{m,L2}(y_{m,3}) dy_{m,3} \tag{143}
\end{aligned}$$

Then, from the standard theory of Markov Decision Programming (MDP) (see Li et al (2006) for example), we have:

$$V_m(X_{m,s}^H) = \max_{X_{m,0}^H, X_{m,1}^H, X_{m,0}^L, a_{m,0}^H, a_{m,1}^H, a_{m,1}^L} J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L, a_{m,0}^H, a_{m,1}^H, a_{m,1}^L), m = M, \dots, 1,$$

with

$$J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L, a_{m,0}^H, a_{m,1}^H, a_{m,1}^L) = R_{m,UHL}(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L, a_{m,0}^H, a_{m,1}^H, a_{m,1}^L) + \beta E[V_{m-1}(\left((X_{m,s}^H + X_{m,0}^H - D_{m,1}^H)^+ + X_{m,1}^H - D_{m,2}^H\right)^+)], \text{ and}$$

$$V_m(X_{0,s}^H) = vX_{0,s}^{H+} - eX_{0,s}^{H-}$$

$$\text{st. } a_{m,0}^H(X_{m,s}^H + X_{m,0}^H) \leq K_m, m = M, \dots, 1, \quad (144)$$

$$a_{m,1}^H X_{m,1}^H + a_{m,1}^L X_{m,0}^L \leq K_m, m = M, \dots, 1, \quad (145)$$

$$0 \leq a_{m,0}^H, a_{m,1}^H, a_{m,1}^L \leq 1, m = M, \dots, 1, \quad (146)$$

$$0 \leq X_{m,s}^H + X_{m,0}^H, X_{m,1}^H \leq k_m^H, m = M, \dots, 1, \quad (147)$$

$$0 \leq X_1^L \leq k_m^L, m = M, \dots, 1, \quad (148)$$

## 5.3 Analytical Results

### 5.3.1 Model 2.1

**Theorem 2.** *The optimal solution of Model 2 is as follows:*

$$(X_0^{H*}, X_1^{H*}, X_0^{L*}) = (D_1^H, D_2^H, D_2^L),$$

$$a_0^{H*} = \min(1, \frac{K}{D_1^H}), a_1^{H*} = a_1^{L*} = 1 \text{ if } D_2^H + D_2^L \leq K.$$

In case,  $D_2^H + D_2^L > K$ ,  $(a_1^{H*}, a_1^{L*}) = \left(\max(\frac{K-D_2^L}{D_2^H}, 0), \min(1, \frac{K}{D_2^L})\right)$  if  $(h_H - h_L) - (h'_H - h'_L) \geq 0$ . Else  $(a_1^{H*}, a_1^{L*}) = \left(\min(1, \frac{K}{D_2^H}), \max(\frac{K-D_2^H}{D_2^L}, 0)\right)$  if  $(h_H - h_L) - (h'_H - h'_L) < 0$ .

*Proof.* In period 0, with the demand information  $(D_1^H, D_2^H, D_2^L)$  and the assumption of equation (112)(113), the optimal ordering quantity should make the available units equal to the current demand, this results in neither shortage nor leftovers for the retailer. That is:

$$(X_0^{H*}, X_1^{H*}, X_0^{L*}) = (D_1^H, D_2^H, D_2^L) \quad (149)$$

Given the above optimal ordering quantity, we search for the optimal allocation decision, which will also be used to build an efficient heuristic method later in Model 4.1.

By knowing the optimal ordering quantity, the inventory cost is a function of the allocation made in the second stage. The total revenue in period 0 is:

$$\begin{aligned} R_0^H(\bullet) + R_0^L(\bullet) &= -c_H X_0^H - c_L X_0^L - I_0(a_0^H) \\ &\leq -c_H D_1^H - c_L D_2^L - I_0(a_0^{H*}) \end{aligned} \quad (150)$$

The last item is determined in the second stage. As mentioned earlier, the allocation decision has to be made for the  $D_1^H$  units in order to minimize the total inventory cost of period 0. We have:

$$\begin{aligned} \text{Min} I_0(a_0^H) &= \text{Min} (h_H a_0^H D_1^H + h'_H (1 - a_0^H) D_1^H) \\ &= \text{Min} ((h_H - h'_H) D_1^H a_0^H + h'_H D_1^H) \end{aligned} \quad (151)$$

subject to

$$\begin{aligned} a_0^H D_1^H &\leq K \\ I_0(a_0^H) &= (h_H - h'_H) D_1^H a_0^H + h'_H D_1^H \end{aligned} \quad (152)$$

Since the secondary warehouse is more expensive ( $h_H - h'_H < 0$ ), function (151) is strictly decreasing in  $a_0^H$ . According to constraint (152), we have:

where

$$a_0^{H*} = \min\left(1, \frac{K}{D_1^H}\right) \quad (153)$$

This result is intuitive. Since the units of product 1 are the only ones that need consideration, they are not sent to an expensive secondary inventory until the cheaper shelf space (the retailer's inventory) is filled.

In period 1, we have:

$$\begin{aligned} R_1^H(\bullet) + R_1^L(\bullet) &= r_H \min(D_1^H, X_0^H) - c_H X_1^H - S_H(D_1^H - X_0^H)^+ - I_1(X_0^H, X_1^H, X_0^L, a_1^H, a_1^L) \\ &\leq r_H D_1^H - c_H D_2^H - I_1(a_1^{H*}, a_1^{L*}) \end{aligned} \quad (154)$$

The last inequation comes from the fact that the optimal ordering solution is the one where available units just fit the demand. The second stage problem in period 1 can be written as:

$$\text{Min}I_1(a_1^H, a_1^L) = \text{Min} (h_H a_1^H D_2^H + h'_H (1 - a_1^H) D_2^H + h_L a_1^L D_2^L + h'_L (1 - a_1^L) D_2^L) \quad (155)$$

Subject to

$$a_1^H D_2^H + a_1^L D_2^L \leq K \quad (156)$$

and it is rewritten as:

$$\text{Min}I_1(a_1^H, a_1^L) = \text{Min} ((h_H - h'_H) D_2^H a_1^H + (h_L - h'_L) D_2^L a_1^L + h'_H D_2^H + h'_L D_2^L) \quad (157)$$

subject to

$$D_2^H a_1^H + D_2^L a_1^L \leq K \quad (158)$$

In case  $D_2^H + D_2^L \leq K$ , where the total amount of leftover is less than the primary inventory capacity, the optimal solution is to place all units in the cheaper primary inventory in comparison to the secondary inventory cost. That is:

If  $D_2^H + D_2^L \leq K$ ,

$$a_1^{H*} = a_1^{L*} = 1 \quad (159)$$

If  $D_2^H + D_2^L > K$ , the primary inventory is first fulfilled according to its lower holding costs and the rest of the leftovers are allocated to the secondary inventory. This implies that constraint (158) is tight and:

$$D_2^H a_1^H + D_2^L a_1^L = K \quad (160)$$

This can also be rewritten as:

$$a_1^L = \frac{1}{D_2^L} (K - D_2^H a_1^H) \quad (161)$$

By substituting (161) into (157), we rewrite the second stage objective function as:

$$\text{Min}I_1(a_1^H, a_1^L) = \text{Min} ((h_H - h_L) - (h'_H - h'_L)) D_2^H a_1^H + (h_L - h'_L) K + h'_H D_2^H + h'_L D_2^L \quad (162)$$

The only variable in the function is  $a_1^H$ .

If  $(h_H - h_L) - (h'_H - h'_L) \geq 0$ , we try to lower  $a_1^H$  and fulfill the primary inventory first with the units of product 2. From (161), we get:

$$a_1^{L*} = \min(1, \frac{K}{D_2^L}), a_1^{H*} = \max(\frac{K - D_2^L}{D_2^H}, 0). \quad (163)$$

Else, if  $(h_H - h_L) - (h'_H - h'_L) < 0$ , we try to place product 1 in the primary inventory as much as possible, so from (161), we get:

$$a_1^{H*} = \min(1, \frac{K}{D_2^H}), a_1^{L*} = \max(\frac{K - D_2^H}{D_2^L}, 0). \quad (164)$$

The total profit for both products in period 2 is written as:

$$\begin{aligned} R_2^H(\bullet) &= r_H \min(D_2^H, (X_0^H - D_1^H)^+ + X_1^H) - S_H \left( D_2^H - \left( (X_0^H - D_1^H)^+ + X_1^H \right) \right)^+ \\ &\quad + V_H \left( \left( (X_0^H - D_1^H)^+ + X_1^H \right) - D_2^H \right)^+ \\ &\leq r_H D_2^H \end{aligned} \quad (165)$$

$$\begin{aligned} R_2^L(\bullet) &= r_L \min(D_2^L, X_0^L) - S_L (D_2^L - X_0^L)^+ + V_L (X_0^L - D_2^L)^+ \\ &\leq r_L D_2^L \end{aligned} \quad (166)$$

The last two inequations follow from the optimal ordering quantity, where  $X_1^{H*} = D_2^H, X_0^{L*} = D_2^L$ .

According to (149)(153)(159)(163)(164), the optimal total profit is:

$$\begin{aligned} \pi^* &= -c_H D_1^H - c_L D_2^L - I_0(a_0^{H*}) \\ &\quad + r_H D_1^H - c_H D_2^H - I_1(a_1^{H*}, a_1^{L*}) \\ &\quad + r_H D_2^H + r_L D_2^L \end{aligned} \quad (167)$$

where the closed form of the optimal solution is:

$$I_0^*(a_0^{H*}) = (h_H - h'_H) D_1^H a_0^{H*} + h'_H D_1^H \quad (168)$$

$$I_1^*(a_1^{H*}, a_1^{L*}) = (h_H - h'_H) D_2^H a_1^{H*} + (h_L - h'_L) D_2^L a_1^{L*} + h'_H D_2^H + h'_L D_2^L \quad (169)$$

$$(X_0^{H*}, X_1^{H*}, X_0^{L*}) = (D_1^H, D_2^H, D_2^L), a_0^{H*} = \min(1, \frac{K}{D_1^H}), a_1^{H*} = a_1^{L*} = 1 \text{ if } D_2^H + D_2^L \leq K.$$

In case,  $D_2^H + D_2^L > K$ ,  $(a_1^{H*}, a_1^{L*}) = \left( \max(\frac{K - D_2^L}{D_2^H}, 0), \min(1, \frac{K}{D_2^L}) \right)$  if  $(h_H - h_L) - (h'_H - h'_L) \geq 0$ . Else  $(a_1^{H*}, a_1^{L*}) = \left( \min(1, \frac{K}{D_2^H}), \max(\frac{K - D_2^H}{D_2^L}, 0) \right)$  if  $(h_H - h_L) - (h'_H - h'_L) < 0$ . The proof is completed. □

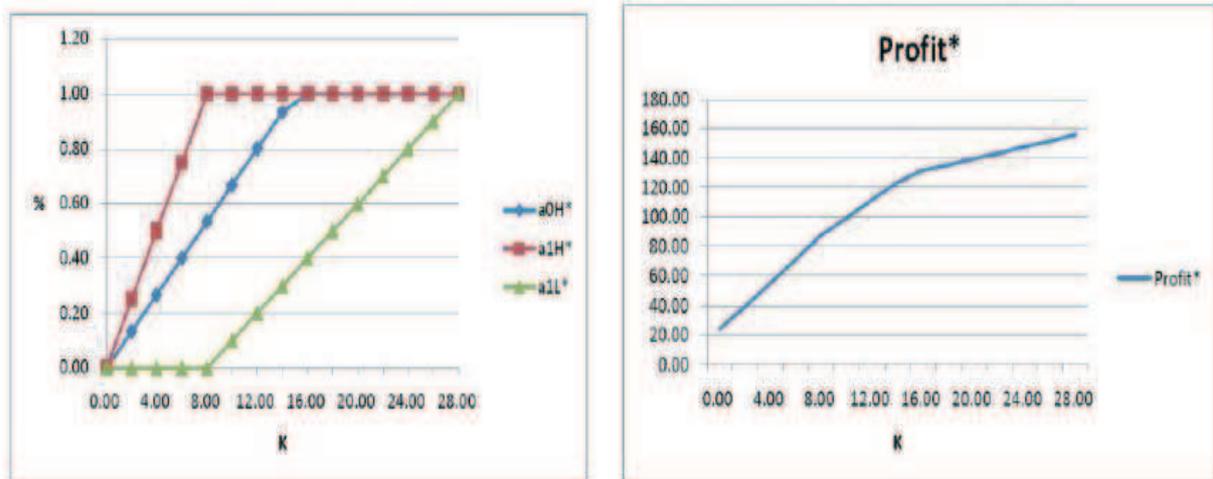
We then present numerical examples for **Retailer 1**. The parameters are given as follows:

Product 1, High-end products:  $r_H = 10, c_H = 4, V_H = 1, h_H = 1, S_H = 8, D_1^H = 15, D_2^H = 8$ ;

Product 2, Low-end products:  $r_L = 5, c_L = 2, V_L = 1, h_L = 1, S_L = 4, D_2^L = 20$ ;

and the primary inventory capacity  $K = 10$ . We set  $h'_H = 5, h'_L = 3$  for the secondary inventory, so that it is more expensive than the primary one.

Graph 5.1 Allocation decision changes with primary inventory capacity K Graph 5.2 Total profit changes with primary inventory capacity K



According to the closed form, by noticing the condition, where  $D_2^H > K, D_2^H + D_2^L > K, (h_H - h'_H) - (h_L - h'_L) < 0$ , the optimal solution is:

$(X_0^{H*}, X_1^{H*}, X_0^{L*}) = (D_1^H, D_2^H, D_2^L) = (15, 8, 20), (a_0^{H*}, a_1^{H*}, a_1^{L*}) = (\frac{2}{3}, 1, \frac{1}{10})$ . This result indicates that one should place  $\frac{2}{3}$  of the high-end units in the primary inventory in period 0, and the rest in the secondary inventory. In period 1, all the 8 high-end units should be placed in the primary inventory in addition to the 2 units (10%) of the low-end units. The remaining items are stored in the secondary inventory. By placing the optimal solution back into the objective function (167), the optimal total revenue is 99.

To perform the sensitivity analysis for the primary capacity  $K$ , we use the same data, but change  $K$  from 0 to 20. Results are presented in graph 5.1 and graph 5.2. In graph 5.2,  $a_0H^*, a_1H^*, a_1L^*$  represent  $a_0^{H*}, a_1^{H*}, a_1^{L*}$ , respectively.

The results show that, as  $K$  increases, the retailer should first place the high-end product in the primary inventory. They also show that the profit increases as  $K$  increases.

### 5.3.2 Model 2.2

In the previous model, the optimal ordering quantity is easy to capture since demand is deterministic and suppliers' production capacity is unlimited. However, since suppliers may be capacitated in reality, there is a need to study the effect of the suppliers' capacity constraints.

In this model, both the objective function and the constraints are nonlinear. Since the global optimal ordering quantity in Model 2.1 cannot be guaranteed due to suppliers' capacity constraints, we find it difficult to get analytical solutions, so we provide managerial insights through numerical examples for **Retailer 1**.

When the suppliers are capacitated, the retailer can only order up to the production capacity value, when the demand is high. Furthermore, there are instances when the retailer will order up to the capacity, even when the demand in the current period is low because the upcoming demand could be high.

Table 5.2 The profit changes with  $k^H$  and  $k^L$

$k^H$	$k^L$	$x_0^{H*}$	$x_1^{H*}$	$x_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*
0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	-264.00
0.00	5.00	0.00	0.00	5.00	0.29	1.00	1.00	-234.00
0.00	10.00	0.00	0.00	10.00	0.04	1.00	1.00	-204.00
0.00	15.00	0.00	0.00	15.00	0.82	1.00	0.67	-184.00
0.00	20.00	0.00	0.00	20.00	0.43	1.00	0.50	-164.00
5.00	0.00	5.00	5.00	0.00	1.00	1.00	0.41	-134.00
5.00	5.00	5.00	5.00	5.00	1.00	1.00	1.00	-104.00
5.00	10.00	5.00	5.00	10.00	1.00	1.00	0.50	-84.00
5.00	15.00	5.00	5.00	15.00	1.00	1.00	0.33	-64.00
5.00	20.00	5.00	5.00	20.00	1.00	1.00	0.25	-20.00
10.00	0.00	10.00	8.00	0.00	1.00	1.00	0.49	-30.00
10.00	5.00	10.00	8.00	5.00	1.00	1.00	0.40	-6.00
10.00	10.00	10.00	8.00	10.00	1.00	1.00	0.20	14.00
10.00	15.00	10.00	8.00	15.00	1.00	1.00	0.13	34.00
10.00	20.00	10.00	8.00	20.00	1.00	1.00	0.10	54.00
15.00	0.00	10.00	8.00	0.00	0.67	1.00	0.80	15.00
15.00	5.00	15.00	8.00	5.00	0.67	1.00	0.40	39.00
15.00	10.00	15.00	8.00	10.00	0.67	1.00	0.20	59.00
15.00	15.00	15.00	8.00	15.00	0.67	1.00	0.13	79.00
15.00	20.00	15.00	8.00	20.00	0.67	1.00	0.10	99.00

To get an intuition of the suppliers' production capacity sensitivity, we use the data of the last example (in the previous section) for comparison, we use the parameter settings in the previous subsection and set  $k^H : 0 \rightarrow 15$ . For each  $k^H$ , we set  $k^L : 0 \rightarrow 20$  and search for the optimal solutions. From the results in Table 5.2, we find that both  $k^H$  and  $k^L$  have significant influence on the retailer's decision.

Therefore, we investigate this problem through the following four cases. In **Case I**, only  $k^H$  is limited. This case is very common, so we investigate it thoroughly. In reality, the supplier of the low-end product usually has enough production capacity ( $k^L$  is unlimited), while that of the high-end product is usually limited. In the deterministic demand cases, the retailer knows the demand information in each period so he usually has enough (unlimited) primary inventory capacity. However, there are many real cases, where the retailer does have limited primary inventory capacity, so we also investigate **Case II** (where only  $K$  and  $k^H$  are limited) and **Case III** (where only  $K$  is limited). The closed form expression for the optimal solution in **Case III** is provided in the previous subsection.

Furthermore, in reality, there are cases, when the supplier of the low-end product does have limited production capacity. With the help of our model, for illustrative purpose, we provide **Case IV** where  $k^H, k^L, and K$  are all limited.

For the general case with no capacity constraints (both suppliers' production capacity constraints and retailer's primary inventory capacity constraint), the model can be regarded as two independent periodic review Newsboy models, and results can be found in the existing literature.

Details of the aforementioned cases are shown below, and conclusions are presented after that.

### **Case I. Limited high-end product's production capacity case.**

As mentioned before, this case is the most important in this sub-problem, so we conduct sensitivity analysis through the following four dimensions with unlimited  $K$ , unlimited  $k^L$  and limited  $k^H$ .

Dimension I. Demand sensitivity analysis.

We let the high-end demand to be identical in the two periods  $D_1^H = D_2^H = D^H$ , and set the retailer's primary inventory capacity equal to the sum of the total mean demand in the second period, where,  $K = D_2^H + D_2^L = 10$ .

Based on these settings, we analyze three combinations of the demand  $(D_1^H, D_2^H, D_2^L)$  where  $D^H/D_2^L$  is increasing. Step settings in Dimension I and Dimension II are presented in Table 5.3.

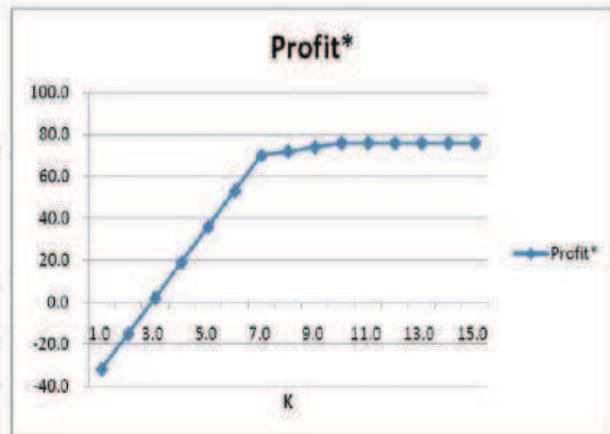
Dimension II. Suppliers' capacity sensitivity analysis.

Since the high-end product provides higher profit margin, in this dimension, we focus on the sensitivity analysis of the production capacity constraint of the high-end product by changing the high-end product production capacity  $k^H$  in steps presented in Table 5.3.

Table 5.3 Parameter settings

Graph 5.3 Total profit changes with K

Dimension I \ Dimension II	Step I ( $D_1^H, D_2^H, D_2^L$ ) =(3,3,7)	Step II ( $D_1^H, D_2^H, D_2^L$ ) =(5,5,5)	Step III ( $D_1^H, D_2^H, D_2^L$ ) =(7,7,3)
Step I	$k^H=1$	$k^H=1$	$k^H=1$
Step II	$k^H=3$	$k^H=5$	$k^H=5$
Step III	$k^H=5$	$k^H=10$	$k^H=10$



Dimension III. Initial Bottleneck sensitivity analysis.

We initially use the same parameter settings from the previous section and conduct sensitivity analysis on the following parameters:  $r_H, V_H, r_L, V_L, k^H, k^L, K$  are increased while  $c_H, h_H, h'_H, S_H, c_L, h_L, h'_L, S_L$  are decreased. In **step I**, we change the above parameters one unit at a time to see the effect on the optimal solution and the related profit. We increase the value of the parameters in the input group and reduce those in the output group in order to compare the difference of the positive effect of the parameters to the profit. This process is repeated in **step II** by changing all the parameters by the same average percentage (the average of the changed percentage of the parameters in **step I**).

Dimension IV. Extended Bottleneck sensitivity analysis.

In order to do further sensitivity analysis, we make **Elite** investigation in a similar way on the most important parameters from Dimension III: we continue to relax the most active constraints (bottlenecks) one unit /same percentage at a time and compare the improvement in the total profit.

## Case II. Limited high-end product's production capacity and Limited primary

### inventory capacity case.

As mentioned before, the previous results are obtained through the above four dimensions, where the retailer's primary inventory capacity constraint is binding (see Dimension II). In our problem, the two kinds of products are produced by different suppliers and the products' demands are independent of each other. The two products are competing with each other for the limited inventory capacity of the retailer. Thus, there is a need to investigate the sensitivity performance of the primary inventory capacity  $K$ .

For instance, we let  $(k^H, k^L, D_1^H, D_2^H, D_2^L) = (1, 10, 7, 7, 3)$ . Graph 5.3 shows that as  $K$  increases from 1 to  $D_2^H$ , the ordering quantity of the high-end units in period 1 ( $X_1^H$ ) increases linearly to the related demand  $D_2^H$ . After that, as  $K$  increases from  $D_2^H$  to  $D_2^H + D_2^L$ , the profit increases at a lower rate due to lower low-end units holding cost ( $h_L < h'_L$ ). This result shows that the retailer can concavely increase the total profit by increasing  $K$ .

In **Case II**, all capacities are limited ( $K = k^H = k^L = 3, D_1^H = D_2^H = D_2^L = 5$ ). In subcase I, the other parameters remain the same, and we have  $(h_H - h_L) - (h'_H - h'_L) < 0$ . In subcase II, we set  $h_H = 2, h'_L = 5$  so as to make  $(h_H - h_L) - (h'_H - h'_L) > 0$ . According to constraint (113), we set  $r_L = 8$ .

### Case III. Limited primary inventory capacity case.

In this case, we set  $k^H = k^L = 5$  so that only the retailer's primary inventory capacity is limited. All the other parameters in the two subcases (differed by the sign of  $(h_H - h_L) - (h'_H - h'_L)$ ) are the same as **Case II**.

The production capacity constraint of the low-end product is binding in all the above three cases. To provide managerial insights, the following case (Case IV) is provided, where all capacities are limited.

### Case IV. Limited primary inventory capacity, limited high-end product's production capacity and limited low-end product's production capacity case.

In this case, all capacities are limited by setting  $K = k^H = k^L = 3, D_1^H = D_2^H = D_2^L = 5$ . Similar to the previous two cases, we provide two sub-cases differed by the sign of  $(h_H - h_L) - (h'_H - h'_L)$ .

## Subsection Numerical Results Summary

The most important parameters in **Case I** are summarized in Figure 5.2 and Figure 5.3 (we have  $D_1^H = D_2^H = D^H, D_2^L = D^L$ ). The way to interpret Figure 5.2 and Figure 5.3 is as follows: as an example,  $r_L, c_L, r_H, c_H$  in the foremost left hand corner indicate that they relate to step I, Dimension I ( $(D_1^H, D_2^H, D_2^L) = (3, 3, 7)$ ) and step III Dimension II ( $k^H = 5$ ). Further,  $r_L, c_L, r_H, c_H$  are presented from the most important to the least important. Similarly, the most important parameters in **Case I, II, III, IV** are summarized in Table 5.4 (from the most important to the least important).

Figure 5.2 Change 1 unit at a time

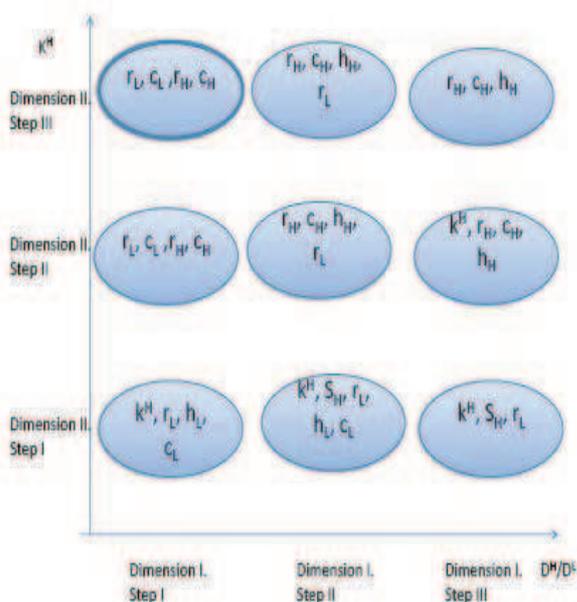


Figure 5.3 Change % at a time

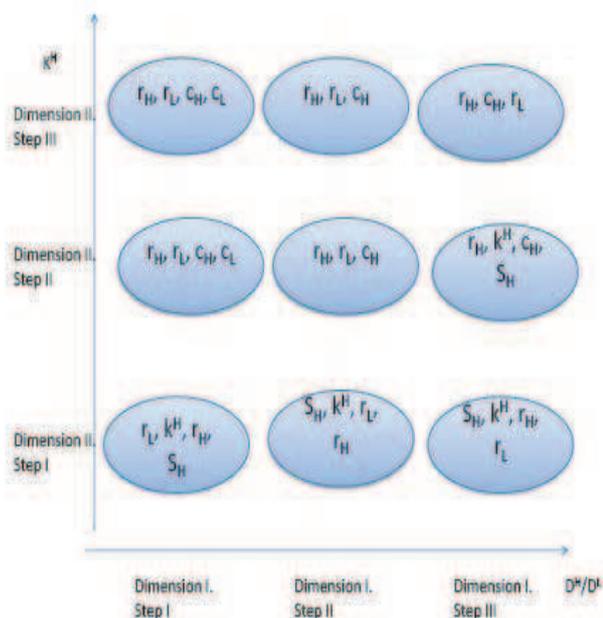


Table 5.4 The most important parameters in **Case I, II, III, IV**

	Case I	Case II Subcase I	Case II Subcase II	Case III Subcase I	Case III Subcase II	Case IV Subcase I	Case IV Subcase II
Important parameters	$r_H, r_L, k, S_H, c_H$	$r_H, S_H, k^H, r_L, c_H$	$r_H, k^H, r_L, S_H, c_H$	$K, r_H, c_H, h_H$	$r_H, r_L, c_H, h'_H$	$k^H, r_H, c_H, h_H$	$k^H, r_H, c_H, h'_H$

Counter-intuitively, if  $(h_H - h_L) - (h'_H - h'_L) > 0$ , the retailer should place the low-end product into the primary inventory and the expensive high-end product into the secondary

inventory with a much higher holding cost. The results also show that the profit is more sensitive to the selling price than to the unit cost and the holding cost.

In cases, where only the production capacity of the high-end product is limited, the retailer should negotiate with the supplier to increase this capacity and in the meantime, reduce the opportunity cost of the high-end product. In cases, where only the primary inventory capacity is limited and the high-end units are placed in the primary inventory, the retailer should increase the capacity, since the secondary inventory holding cost of the high-end product is high. In cases, where both the primary inventory and the production capacities of the high-end product are limited, the production capacity should be increased first. Increasing the primary inventory capacity is less important here, since the extra units can always be placed into the secondary inventory.

In cases, where both the two suppliers' production and the retailer's primary inventory capacities are limited, the production capacity of the high-end product is of the most interest to the retailer. In this case the high-end units are placed into the secondary inventory with unlimited capacity (the priority for the production capacity of the high-end product is only lower than the selling price). The production capacity of the low-end product is not as sensitive as that of the high-end product, but it is still much more important than the primary inventory capacity.

### 5.3.3 Model 3.1

In cases, where the uncapacitated secondary warehouse is cheaper than the primary one; all units will be placed into the secondary warehouse. The profit of the two products can thus be maximized independently.

In this case, the unit holding costs of the high-end and the low-end product are  $h_1 = h'_H$  and  $h_2 = h'_L$ , respectively.

The unmet demand is assumed to be backlogged in this case. The expected profit of the high-end profit is:

$$\begin{aligned}
& R_{UH}(X_0^H, X_1^H) \\
= & \int_0^{X_0^H} (r_H y_1 - (c_H + h_1)X_0^H) \varphi_{H1}(y_1) dy_1 \\
& + \int_0^{X_0^H} \left( \int_0^{X_1^H + X_0^H - y_1} (r_H y_2 - c_H X_1^H - h_1(X_1^H + X_0^H - y_1)) \right. \\
& + V_H(X_1^H + X_0^H - y_1 - y_2) \varphi_{H2}(y_2) dy_2 \\
& + \int_{X_1^H + X_0^H - y_1}^{+\infty} (r_H(X_1^H + X_0^H - y_1) - c_H X_1^H - h_1(X_1^H + X_0^H - y_1)) \\
& - S_H(y_2 - X_1^H - X_0^H + y_1) \varphi_{H2}(y_2) dy_2 \varphi_{H1}(y_1) dy_1 \\
& + \int_{X_0^H}^{+\infty} (r_H y_1 - (c_H + h_1)X_0^H - S_H(y_1 - X_0^H)) \varphi_{H1}(y_1) dy_1 \\
& + \int_{X_0^H}^{+\infty} \left( \int_0^{X_1^H + X_0^H - y_1} (r_H y_2 - (c_H + h_1)X_1^H + V_H(X_1^H - y_2)) \varphi_{H2}(y_2) dy_2 \right. \\
& + \int_{X_1^H + X_0^H - y_1}^{+\infty} (r_H X_1^H - (c_H + h_1)X_1^H - S_H(y_2 - X_1^H)) \varphi_{H2}(y_2) dy_2 \varphi_{H1}(y_1) dy_1 \\
& \left. \right) \varphi_{H1}(y_1) dy_1
\end{aligned} \tag{170}$$

The optimization sub-problem of the high-end product with uncapacitated warehouse (sub-problem **UH**) is then written as follows:

$$\begin{aligned}
& \max_{X_0^H, X_1^H} R_{UH}(X_0^H, X_1^H) \\
st. & \quad 0 \leq X_0^H, X_1^H \leq k^H
\end{aligned} \tag{171}$$

**Lemma 5.** (170) is jointly concave and there exists a unique optimal solution.

*Proof.* Cheaitou et al. (2009) investigate a single product; two-period periodic review inventory problem with initial units. In that paper, delivery time is negligible and product is available for sale once arrived. Only the supplier is capacitated and the unmet demand is backlogged. At the beginning of the second period, a part of the leftovers can be salvaged. They proved that the objective function is jointly concave.

In our model, the delivery time is also assumed to be negligible but arrived units take the retailer one period time to prepare, so the actual unit cost of a unit before sale is the

unit cost  $c_H$  plus the unit holding cost  $h_1$ . Sub-problem **UH** is mathematically equivalent to the problem investigated by Cheaitou et al. (2009) through the following translations:

1. Orders of the high-end product are placed in periods 1 and 2 instead of period 0 and 1, respectively,
2. Newly arrived units are available for sale and the unit cost  $c_H$  is replaced by the actual unit cost  $c_H + h_1$  in both periods 0 and 1,
3. No salvage option at the end of period 1.

Therefore, objective function (170) is jointly concave in  $(X_0^H, X_1^H)$ , and thus there exists a unique optimal solution.  $\square$

The sub-problem of the low-end product (**UL**) is actually a single period problem. The expected profit is shown as follows:

$$\begin{aligned}
& R_{UL}(X_0^L) \\
&= -(c_L + h_2)X_0^L + r_L \left( \int_0^{X_0^L} y_3 \varphi_{L2}(y_3) dy_3 + X_0^L \int_{X_0^L}^{\infty} \varphi_{L2}(y_3) dy_3 \right) - S_L \int_{X_0^L}^{\infty} (y_3 - X_0^L) \varphi_{L2}(y_3) dy_3 \\
&+ V_L \int_0^{X_0^L} (X_0^L - y_3) \varphi_{L2}(y_3) dy_3
\end{aligned} \tag{172}$$

subject to the following constraint:

$$0 \leq X_0^L \leq k^L \tag{173}$$

It is equivalent to a classical Newsboy problem with unit cost  $c_L + h_2$  and production capacity constraint, therefore,  $R_{UL}(X_0^L)$  is concave in  $X_0^L$  and sub-problem **UL** has a unique optimal solution.

For the global problem in this model:

$$\begin{aligned}
& \max R_{UHL}(X_0^H, X_1^H, X_0^L) & (174) \\
\text{st. } & 0 \leq X_0^H, X_1^H \leq k^H \\
& 0 \leq X_0^L \leq k^L \\
\text{where } & R_{UHL}(X_0^H, X_1^H, X_0^L) = R_{UH}(X_0^H, X_1^H) + R_{UL}(X_0^L), & (175)
\end{aligned}$$

we have the following theorem:

**Theorem 3.** *The objective function (175) is jointly concave, and thus problem (174) has a unique optimal solution.*

*Proof.* Function (175) is jointly concave in  $(X_0^H, X_1^H, X_0^L)$  due to the following reasons:

1. According to Lemma 5, the first item on the right side of function (175) is jointly concave in  $(X_0^H, X_1^H)$ ;
2. The second item on the right side of function (175) is concave in  $X_0^L$  since sub-problem **UL** is equivalent to a classical capacitated Newsboy Problem;
3. The items on the right side of function (175) are independent.

Therefore, problem (174) has a unique optimal solution. □

However, there exists no closed form of the optimal solution in problem (174). One can take advantage of concavity to search for the optimal solution numerically.

### 5.3.4 Model 3.2

For Retailer 2, the locally produced low-end product (product 1) will not be salvaged at the end of a selling season and can be sold in the next season. We will show in this section, the joint concavity of the objective function of the global problem 3.2.

We first recall the problem as follow:

$$\begin{aligned}
V_m(X_{m,s}^H) &= \max_{X_{m,0}^H, X_{m,1}^H, X_{m,0}^L} J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L), \quad m = M, \dots, 1, \text{ with} \\
J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L) &= R_{m,UHL}(X_{m,0}^H, X_{m,1}^H, X_{m,0}^L) \\
&\quad + \beta E[V_{m-1}(X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - D_{m,1}^H - D_{m,2}^H)], \text{ and} \\
V_m(X_{0,s}^H) &= vX_{0,s}^{H+} - eX_{0,s}^{H-} \\
\text{st.} \quad &0 \leq X_{m,0}^H, X_{m,1}^H \leq k_m^H \\
&0 \leq X_{m,0}^L \leq k_m^L
\end{aligned} \tag{176}$$

**Lemma 6.**  $R_{m,UHL}(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L)$  is jointly concave in  $(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L)$ .

*Proof.* With a similar proof in Lemma 5, we can show that  $R_{m,UH}(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H)$  is jointly concave in  $(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H)$ . The problem of product 2 is equivalent to a Newsboy Problem with unit cost  $c_{m,L} + h_{m,2}$ , so the objective function  $R_{m,UL}(X_{m,0}^L)$  is concave in  $X_{m,0}^L$ . Since the two functions,  $R_{m,UH}(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H)$  and  $R_{m,UL}(X_{m,0}^L)$ , are independent, it is straight forward that the summation is jointly concave in  $(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L)$ . Proof is completed.  $\square$

We then have the next theorem.

**Theorem 4.** For  $m = M, \dots, 1$ ,  $J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L)$  is jointly concave in  $(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L)$  and  $V_m(X_{m,s}^H)$  is concave in  $V_m(X_{m,s}^H)$ .

*Proof.* Since  $e \geq v$ ,  $V_0(X_{0,s}^H)$  is concave. It is then sufficient to show that, if  $V_{m-1}(X_{m-1,s}^H)$  is concave,  $J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L)$  is jointly concave and  $V_m(X_{m,s}^H)$  is concave.

Suppose that  $V_{m-1}(X_{m-1,s}^H)$  is concave, then  $E[V_{m-1}(X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - D_{m,1}^H - D_{m,2}^H)]$  is jointly concave in  $(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H)$ , since  $X_{m,s}^H + X_{m,0}^H + X_{m,1}^H - D_{m,1}^H - D_{m,2}^H$  is linear in  $(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H)$ . According to Lemma 6,  $J_m(X_{m,s}^H, X_{m,0}^H, X_{m,1}^H, X_{m,0}^L)$  is also jointly concave. This implies that  $V_m(X_{m,s}^H)$  is concave in  $X_{m,s}^H$ , since concavity is preserved under maximization (Heymal et al. (1984)).  $\square$

One can then take the advantage of concavity to search for the optimal solution numerically.

### 5.3.5 Model 4.1

Problem (137) is a stochastic, nonlinear program with 6 variables. Both the objective function and some of the constraints are nonlinear. We find it difficult to prove the joint concavity or unimodality of the objective function. In fact, we find through numerical examples that it has several local optimal solutions. Therefore, we provide the following heuristic method to search for the optimal solution in order to get managerial insights. We will show through numerical examples that our method is more efficient compared to the Steep Descendant Method and Classical Genetic Algorithm.

#### 5.3.5.1 GA Based Heuristic Method

Based on the closed form expression we get in the deterministic demand model, the heuristic program starts with known inventory allocation priority. The heuristic method pseudocode is presented as follows:

1. The sign of  $(h_H - h'_H) - (h_L - h'_L)$  determines which product is placed into the primary inventory first;
2. Choose the initial population of individuals based on step 1;
3. Start Genetic Algorithm;
4. Stop when termination condition is achieved.

#### 5.3.5.2 Numerical Experiments: Computation Efficiency

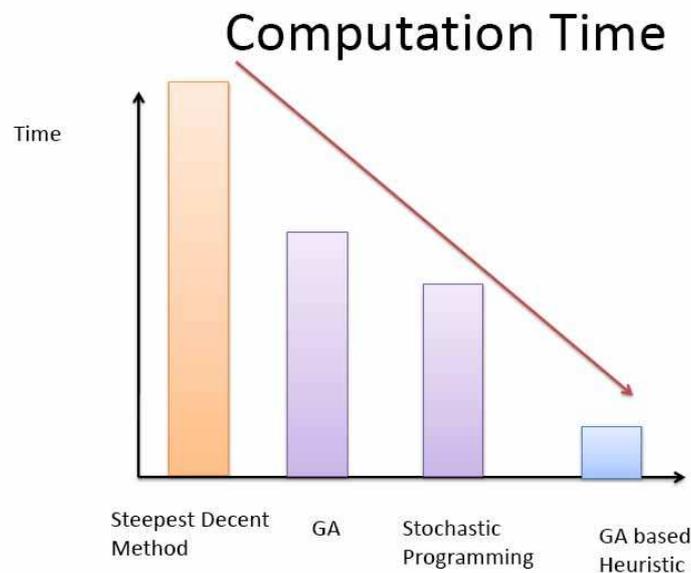
We choose the parameter settings in the last part of Chapter 4 and use the steepest descent method, the classical genetic algorithm and our GA based heuristic method to search for the optimal solution. The Steepest Descent method gets results far from optimal since the objective function has many local optimal solutions. Our GA based heuristic method is on average 10 times faster than the Classical Genetic Algorithm and 9 times faster than

the Stochastic Programming Method, so we hereafter use the GA based heuristic method to search for the optimal solutions.

Table 5.5 Computation Efficiency

Method	Average Computation Time	Result
Steepest Decent	2 minutes	Far from optimal
Genetic Algorithm	90 hours	Near-optimal
Stochastic Programming	82 hours	Near-optimal
GA based Heuristic	9 hours	Near-optimal

Graph 5.4 Computation Time Illustration



### 5.3.5.3 Numerical Experiments: Parameter Sensitivity Analysis

In Zara's case, the demand variability is high for the high-end products, while it is low for the low-end product. This is due to the fact that the product life cycle of expensive products is short and is a function of its performance in the market once it is introduced (e.g. fashion clothing). The customers for these products are highly variable. On the other hand, the product life cycle of cheap products is longer, as customers for the same product always exist. In addition, we model demand uncertainties through quasi-concave distributions

(e.g. Gamma, Normal). We assume that all demands are independently gamma distributed (since it represents many commonly used distributions, ex., exponential distribution, normal distribution, etc):  $\varphi_{H1}(y_1) \rightarrow \text{Gamma}(gama1, gamb1)$ ,  $\varphi_{H2}(y_2) \rightarrow \text{Gamma}(gama2, gamb2)$ ,  $\varphi_{L2}(y_3) \rightarrow \text{Gamma}(gama3, gamb3)$ . We also assume that the low-end products have relatively lower demand variance. Without loss of generality, we assume  $r_H \geq r_L, c_H \geq c_L, r_H \geq c_H, r_L \geq c_L$ . As Zara's warehouses are close to each other, when the secondary warehouse is more expensive, we reasonably assume that the secondary unit holding cost of both products are higher than the primary warehouse. (i.e.,  $h_H < h'_H, h_L < h'_L$ ).

We perform numerical experiments in a similar way as in the previous subsection of Case I. Given the following initial parameter settings (same as the previous section)  $r_H = 10, c_H = 4, V_H = 1, h_H = 1, S_H = 8, h'_H = 5, r_L = 5, c_L = 2, V_L = 1, h_L = 1, S_L = 4, h'_L = 3$ , we analyze the parameter sensitivity in the following four dimensions sequentially.

New Dimension I. Demand sensitivity analysis.

Given the demand distribution of the two products in each period, the related coefficient of variation  $C_{v1}^H, C_{v2}^H, C_{v2}^L$  and the mean demand  $(\mu_1^H, \mu_2^H, \mu_2^L)$  are determined, and vice versa. Without loss of generality, we assume that  $C_{v2}^H > C_{v2}^L$  since the demand of the high-end product is more unpredictable than that of the low-end product. In the following examples we use  $C_{v1}^H = C_{v2}^H = 2, C_{v2}^L = 0.1, \mu_1^H = \mu_2^H = \mu^H, \text{ and } \sigma_1^H = \sigma_2^H = \sigma^H$ . According to the single-period single-product problem (Newsboy problem), the best ordering quantity is around or even higher than the mean demand. We start with  $K = \mu_2^H + \mu_2^L$ . The parameter settings of the steps in New Dimension I and New Dimension II are presented in Table 4.1.

New Dimension II. Suppliers' capacity sensitivity analysis.

In this dimension, we set  $K = k^L = 1000$  and adjust  $k^H$ . We investigate all combinations where  $k^H$  is higher/equal/lower than (to) different mean demands. Details are shown in Table 5.6.

Dimension III. Initial bottleneck sensitivity analysis.

The settings here are the same as they are in the previous subsection.

Table 5.6 Parameter settings

New Dimension I \ New Dimension II	Step I	Step II	Step III
I	$(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2)$	$(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2)$	$(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2)$
New Dimension II	$= (300, 300, 700, 600, 600, 70)$	$= (500, 500, 500, 1000, 1000, 50)$	$= (700, 700, 300, 1400, 1400, 30)$
Step I	$k^H = 1$	$k^H = 1$	$k^H = 1$
Step II	$k^H = 200$	$k^H = 500$	$k^H = 500$
Step III	$k^H = 400$	$k^H = 1000$	$k^H = 3000$

Dimension IV. Extended bottleneck sensitivity analysis.

The settings here are the same as they are in the previous subsection.

Due to large computational time involved, we skip calculating Dimension I step II (identical demand type). This is also reasonable in reality, when the high-end product and the low-end product do not usually have the same type of the demand.

The most important parameters are presented in Figure 5.4 and Figure 5.5 in a similar way as in Figure 5.2 and Figure 5.3.

Figure 5.4 Change 1 unit at a time

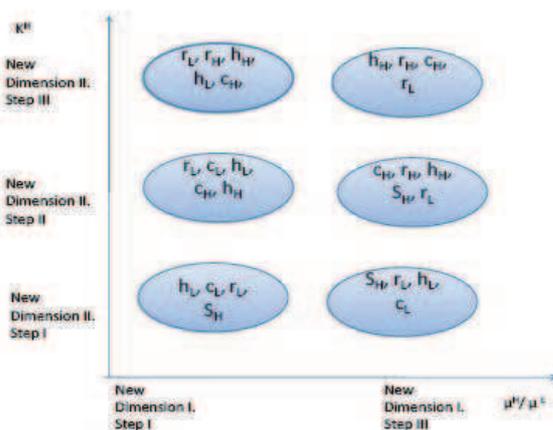
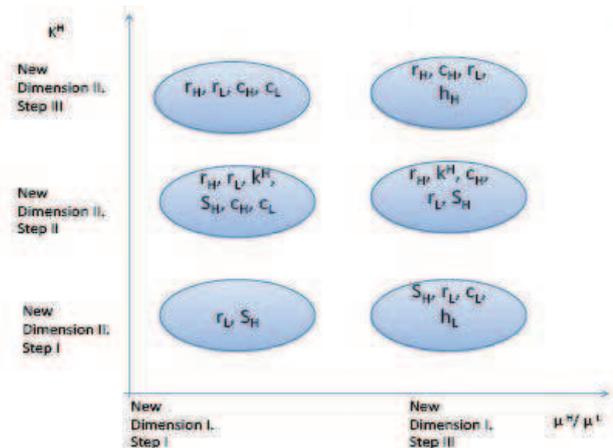


Figure 5.5 Change % at a time



Counter-intuitively, if  $(h_H - h_L) - (h'_H - h'_L) > 0$ , the retailer should place the low-end product into the primary inventory and the expensive high-end product into the secondary inventory with a much higher holding cost. The results also show that, if the actual selling

quantity of a product is high (the selling quantity is not limited by suppliers' production capacity when demand is high), the retailer should increase the selling price and reduce the unit cost and holding cost, because the profit is more sensitive to the selling price than to the unit cost and the holding cost.

In cases, when only the production capacity of the high-end product is low, the retailer should negotiate with the supplier to increase this capacity and, in the meantime, reduce the opportunity cost of the high-end product.

In cases, when only the primary inventory capacity is low and the high-end units are placed into the primary inventory, the retailer should increase the capacity since the secondary inventory holding cost of the high-end product is high. However, if the high-end units are placed to the secondary inventory ( $(h_H - h_L) - (h'_H - h'_L) > 0$ ), the profit is not sensitive to the primary inventory capacity since the extra units can always be placed into the secondary inventory.

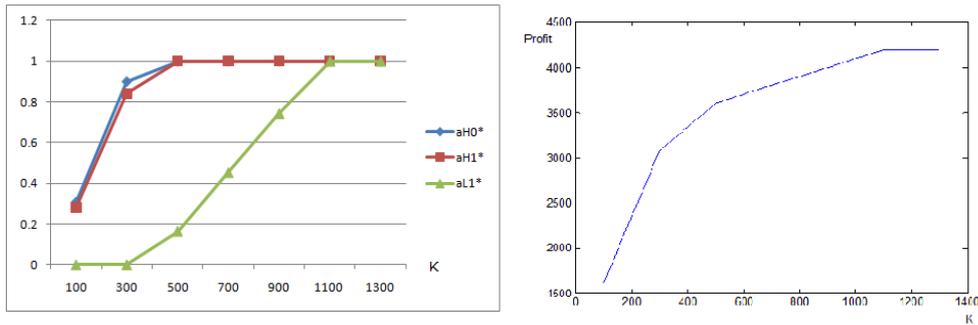
In cases, when both the primary inventory and the production capacities of the high-end product are low, the production capacity should be increased. The expected profit is not sensitive to the primary inventory capacity, since the extra units can always be placed into the secondary inventory.

### 5.3.5.4 Numerical Experiments: Effect of the Primary Warehouse Capacity

With the help of our capacitated model, we show the effect of the primary warehouse capacity through numerical examples.

We use the parameter settings  $r_H = 10, c_H = 4, V_H = 1, h_H = 1, S_H = 8, h'_H = 5, r_L = 5, c_L = 2, V_L = 1, h_L = 1, S_L = 4, h'_L = 2$  and the demand settings in new dimension I step I, where the high-end product has lower demand mean but higher variance. We set  $k^H = 500$  and  $k^L = 1000$  so that only the primary (cheaper) warehouse capacity is limited. We then check the effect of the primary warehouse capacity  $K$  by moving it from 100 to 1300.

Graph 5.5 Allocation decision changes with K      Graph 5.6 Total profit changes with K



In Graph 5.5,  $aH0^*$ ,  $aH1^*$ ,  $aL1^*$  represents  $a_0^{H^*}$ ,  $a_1^{H^*}$ ,  $a_1^{L^*}$ , respectively.  $a_n^i(i\{H, L\})$  is the proportion of the  $n$ 'th period's leftover of high-end and low-end product placed in the primary warehouse for sale in the next period.

The results show that the primary warehouse capacity has significant impact on the retailer's expected profit when  $K \leq 1100$ . Note that the summation of the demand mean in the second period is 1000. The result suggests that the primary warehouse space should be 10% higher than the summation of the demand mean. As K decreases, the retailer loses more than 50% of the profit. This is because more units are placed into the expensive secondary warehouse, and the average unit holding cost increases as a result. It is also confirmed by the results that, in cases, where  $(h_H - h'_H) - (h_L - h'_L) < 0$ , the retailer should first place the high-end product in the primary inventory, as K increases.

### 5.3.6 Model 4.2

We find it difficult to get analytic results since the nonlinear stochastic objective function has 6M variables and the constraints are also nonlinear. Similar to Model 4.1, one can use the GA based method to search for an 'optimal' solution efficiently in any given season  $m(m = 1, \dots, M)$  and then summarize to obtain the solution for the M season problem.

As mentioned before, we do not have any other assumptions on the two products besides (110)(111)(112)(113). In the case  $M = 1$ , the conclusions of model 4.1 hold, if the same parameter settings are used. Complete numerical study is considered as a future research.

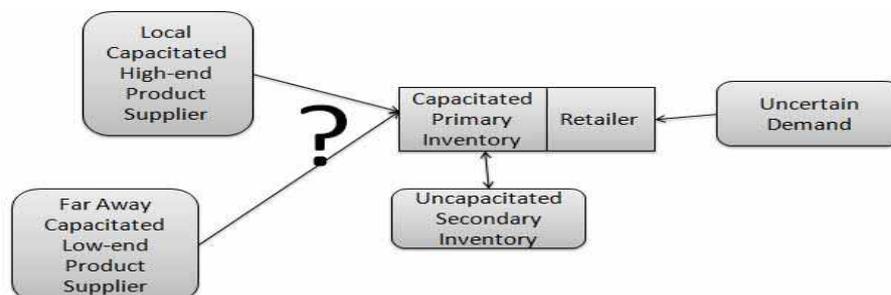
# Chapter 6

## Problem IV: Two-product Types, Two-unreliable-supplier, Two-stage, Three-period Problem

### 6.1 Model Instruction

#### 6.1.1 Notations and Assumptions

Figure 6.1 The Model.



We study a two-stage, capacitated supplier, three period manufacturing problem with random yield and random demand.

We use  $n$  to indicate a period while presenting the different parameters. This makes the parameters suitable for more complicated  $n$  period model which would be investigated later. In this case, the retailer orders  $X_n^H$  high-end products and  $X_n^L$  low-end products at period  $n$  before realization of relative demand  $D_n^H$  and  $D_n^L$ . The following demand assumption is similar to the one in Levi et al. (2008). We assume that demands in different periods are independently distributed and are not necessarily identical. At the beginning of a period  $n$  there is an observed information set denoted by  $\varphi_n^H$  for the high-end products and another

is denoted by  $\varphi_n^L$  for the low-end products. These sets consists of all the realized demands in the passed periods. In general,  $\varphi_n^H, \varphi_n^L$  can contain additional information that is available by time period  $n$ . The information set is:

$$F_n^H = (D_1^H, \dots, D_n^H) \quad (177)$$

$$F_n^L = (D_1^L, \dots, D_n^L) \quad (178)$$

The unit cost for high-end product is  $c_H$  and that of the low-end product is  $c_L$ . The high-end-product supplier's production capacity in a period is  $k^H$ , while that of the low-end product is  $k^L$ . The expected profit of the high-end and the low-end product in period  $n$  ( $n = 0, 1, 2$ ) is denoted by  $R_n^H$  and  $R_n^L$ , respectively.

The retailer pays for the quantity ordered in the case of in-house production while he/she pays for the quantity received in the case of external supply. The quantity received is a proportion (may greater than 1) of the relative ordered quantity:  $\nu_0^H X_0^H, \nu_1^H X_1^H, \nu_0^L X_0^L$ , where  $\nu_0^H, \nu_1^H, \nu_0^L$  are random variables with known density function  $\varphi_{\nu_0^H}, \varphi_{\nu_1^H}, \varphi_{\nu_0^L}$ . Respectively, the mean is  $\mu_{\nu_0^H}, \mu_{\nu_1^H}, \mu_{\nu_0^L}$ .

In this model, the retailer pays for the quantity ordered, however, the selling quantity depends on the quantity received. In each period, the 'worst case' must be taken into account where all ordered units are received.

### 6.1.2 The Model

In external supply case, given the decision variables  $X_0^H, X_1^H, X_1^L, a_0^H, a_1^H, a_1^L$ , the profit of the high-end product in period 0 is

$$R_0^H(\bullet) = -c_H X_0^H - h_H a_0^H \mu_{\nu_0^H} X_0^H - h'_H (1 - a_0^H) \mu_{\nu_0^H} X_0^H \quad (179)$$

and the profit of the low-end product in period 0 is

$$R_0^L(\bullet) = -c_L X_0^L \quad (180)$$

In period 1, the profit of the high-end product is

$$\begin{aligned}
R_1^H(\bullet) = & r_H \int_0^1 \int_0^1 \int_0^1 \int_{\nu_0^H X_0^H + \nu_1^H X_1^H - \frac{\kappa - a_1^L \nu_0^L X_0^L}{a_1^H + \epsilon}}^{\nu_0^H X_0^H} y_1 \varphi_{H1}(y_1) dy_1 \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \\
& + r_H \int_0^1 \nu_0^H X_0^H \int_{\nu_0^H X_0^H}^{\infty} \varphi_{H1}(y_1) dy_1 \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H - c_H X_1^H \\
& - S_H \int_0^1 \int_{\nu_0^H X_0^H}^{\infty} (y_1 - \nu_0^H X_0^H) \varphi_{H1}(y_1) dy \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \\
& - h_H a_1^H \int_0^1 \int_0^1 \int_0^1 \left( \int_{\nu_0^H X_0^H + \nu_1^H X_1^H - \frac{\kappa - a_1^L \nu_0^L X_0^L}{a_1^H + \epsilon}}^{\nu_0^H X_0^H} (\nu_0^H X_0^H - y_1) \varphi_{H1}(y_1) dy_1 \right. \\
& \left. + \nu_1^H X_1^H \right) \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \\
& - h'_H (1 - a_1^H) \int_0^1 \int_0^1 \int_0^1 \left( \int_{\nu_0^H X_0^H + \nu_1^H X_1^H - \frac{\kappa - a_1^L \nu_0^L X_0^L}{a_1^H + \epsilon}}^{\nu_0^H X_0^H} (\nu_0^H X_0^H - y_1) \varphi_{H1}(y_1) dy_1 \right. \\
& \left. + \nu_1^H X_1^H \right) \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \\
& - h_L a_1^L \mu_{\nu_0^L} X_0^L - h'_L (1 - a_1^L) \mu_{\nu_0^L} X_0^L
\end{aligned} \quad (181)$$

In period 1, the profit of the low-end product is

$$R_1^L(\bullet) = -h_L a_1^L \mu_{\nu_0^L} X_0^L - h'_L (1 - a_1^L) \mu_{\nu_0^L} X_0^L \quad (182)$$

In period 2, the profit of the high-end product is

$$\begin{aligned}
& R_2^H(\bullet) \\
&= r_H \int_0^1 \int_0^1 \int_0^1 \int_{\nu_0^H X_0^H + \nu_1^H X_1^H - \frac{\kappa - a_1^L \nu_0^L X_0^L}{a_1^H + \epsilon}}^{\nu_0^H X_0^H} ( \int_0^{\nu_0^H X_0^H - y_1 + \nu_1^H X_1^H} y_2 \varphi_{H2}(y_2) dy_2 \\
&+ (\nu_0^H X_0^H - y_1 + \nu_1^H X_1^H) \int_{\nu_0^H X_0^H - y_1 + \nu_1^H X_1^H}^{\infty} \varphi_{H2}(y_2) dy_2 ) \varphi_{H1}(y_1) dy_1 \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \\
&+ r_H \int_0^1 \int_0^1 ( \int_{\nu_0^H X_0^H}^{\infty} ( \int_0^{\nu_1^H X_1^H} y_2 \varphi_{H2}(y_2) dy_2 \\
&+ \nu_1^H X_1^H \int_{\nu_1^H X_1^H}^{\infty} \varphi_{H2}(y_2) dy_2 ) \varphi_{H1}(y_1) dy_1 ) \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \\
&- S_H \int_0^1 \int_0^1 \int_0^1 \int_{\nu_0^H X_0^H + \nu_1^H X_1^H - \frac{\kappa - a_1^L \nu_0^L X_0^L}{a_1^H + \epsilon}}^{\nu_0^H X_0^H} ( \int_{\nu_0^H X_0^H - y_1 + \nu_1^H X_1^H}^{\infty} (y_2 \\
&- (\nu_0^H X_0^H - y_1 + \nu_1^H X_1^H)) \varphi_{H2}(y_2) dy_2 ) \varphi_{H1}(y_1) dy_1 \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \\
&- S_H \int_0^1 \int_0^1 \int_{X_0^H}^{\infty} ( \int_{X_1^H}^{\infty} (y_2 - X_1^H) \varphi_{H2}(y_2) dy_2 ) \varphi_{H1}(y_1) dy_1 \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \\
&+ V_H \int_0^1 \int_0^1 \int_0^1 \int_{\nu_0^H X_0^H + \nu_1^H X_1^H - \frac{\kappa - a_1^L \nu_0^L X_0^L}{a_1^H + \epsilon}}^{\nu_0^H X_0^H} ( \int_0^{\nu_0^H X_0^H - y_1 + \nu_1^H X_1^H} (\nu_0^H X_0^H - y_1 \\
&+ \nu_1^H X_1^H - y_2) \varphi_{H2}(y_2) dy_2 ) \varphi_{H1}(y_1) dy_1 \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \\
&+ V_H \int_0^1 \int_0^1 \int_{\nu_0^H X_0^H}^{\infty} ( \int_0^{\nu_1^H X_1^H} (\nu_1^H X_1^H - y_2) \varphi_{H2}(y_2) dy_2 ) \varphi_{H1}(y_1) dy_1 \varphi_{\nu_1^H}(\nu_1^H) d\nu_1^H \varphi_{\nu_0^H}(\nu_0^H) d\nu_0^H \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \quad (18)
\end{aligned}$$

In period 2, the profit of the low-end product is

$$\begin{aligned}
R_2^L(\bullet) &= r_L \int_0^1 \left( \int_0^{\nu_0^L X_0^L} y_3 \varphi_{L2}(y_3) dy_3 + \nu_0^L X_0^L \int_{\nu_0^L X_0^L}^{\infty} \varphi_{L2}(y_3) dy_3 \right) \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \\
&- S_L \int_0^1 \int_{\nu_0^L X_0^L}^{\infty} (y_3 - \nu_0^L X_0^L) \varphi_{L2}(y_3) dy_3 \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \\
&+ V_L \int_0^1 \int_{\nu_0^L X_0^L}^{\infty} (\nu_0^L X_0^L - y_3) \varphi_{L2}(y_3) dy_3 \varphi_{\nu_0^L}(\nu_0^L) d\nu_0^L \quad (184)
\end{aligned}$$

To conclude, given the decision variables  $X_0^H, X_1^H, X_1^L, a_0^H, a_1^H, a_1^L$ , the objective of the retailer with respect to the stochastic demand and the random yield is thus:

$$\max \sum_{n=0}^2 (R_n^H(\bullet) + R_n^L(\bullet)) \quad (185)$$

subject to the following constraints:

$$\begin{aligned}
a_0^H X_0^H &\leq K \\
a_1^H X_1^H + a_1^L X_0^L &\leq K \\
0 &\leq a_0^H, a_1^H, a_1^L \leq 1 \\
X_0^H, X_1^H, X_0^L &\geq 0 \\
X_0^H &\leq k^H \\
X_1^H &\leq k^H \\
X_1^L &\leq k^L
\end{aligned} \tag{186}$$

## 6.2 Preliminary Analytical Results

In the previous problem where suppliers are assumed to be reliable, we show that the optimal solution can be obtained by applying the stochastic programming methodology. However, the computation time increases dramatically as the number of the grid increases. In the unreliable supplier version, it can be expected that the number of the nodes in the event tree will raise to a new level. As a result, it takes more time to get the result.

However, there exists a easier way to search for the optimal solution. We will first formulate the problem and show that in comparison to the previous problem, the number of the integral dimension is doubled due to the uncertainty of the suppliers. We then use Monte Carlo integration (R.E.Caflisch, 1998) to calculate the multidimensional integral in the objective function. The convergence rate of the Monte Carlo integration is independent of the dimension of the integral, which is a formidable advantage here. In order to reduce the error due to the randomness of the method, long length of the random vectors  $n$  are need which may take a long time or even make the computer run out of memory. We set  $n = 1000000$  to keep the objective function error under 1 at reasonable computational cost (one hour calculation for each example on a PC). We will show that this method is efficient in comparison to the Steepest Descent method and the Classical Genetic Algorithm.

### 6.2.1 Montecarlo-GA Based Heuristic Method

Based on the analytical result of the deterministic model in the previous problem, the heuristic method pseudocode is presented as follow:

1. The sign of  $(h_H - h'_H) - (h_L - h'_L)$  determines which product is placed into the primary inventory first;
2. Choose the initial population of individuals based on step 1;
3. Start Genetic Algorithm, using Monte Carlo integration;
4. Stop when termination condition is achieved.

### 6.2.2 Preliminary Numerical Results

We use the following parameter settings in the numerical examples:

$r_H = 10, r_L = 8, V_H = 1, V_L = 1, k^H = 200, k^L = 500, K = 800, h_H = 2, h'_H = 5, h_L = 1, h'_L = 5, S_H = 8, S_L = 4, c_H = 4, c_L = 2$ . In the numerical examples, we assume that all demand are independently gamma distributed (since it represents many commonly used distributions, ex., exponential distribution, normal distribution, etc):  $\varphi_{H1}(y_1) \rightarrow \text{Gamma}(gama1, gamb1)$ ,  $\varphi_{H2}(y_2) \rightarrow \text{Gamma}(gama2, gamb2)$ ,  $\varphi_{L2}(y_3) \rightarrow \text{Gamma}(gama3, gamb3)$  with initial parameter settings  $gama1 = gama2 = 150, gamb1 = gamb2 = 2, gama3 = 7000, gamb3 = 0.1$ . We also assume that the random yields are normal distributed with initial parameter settings  $\mu_{\nu_0^H} = \mu_{\nu_1^H} = \mu_{\nu_0^L} = 1, \sigma_{\nu_0^H} = \sigma_{\nu_1^H} = 0.1, \sigma_{\nu_0^L} = 0.4$ .

We explore the sensitivity of the parameters on the following three dimensions: the reliability of the suppliers, the demand pattern and the production capacity constraint of the high-end product. Detail are presented in table 6.1.

Table 6.1 Parameter Settings of the Sensitivity Analysis

Dimension I	Dimension II			Dimension III
$\sigma_{\nu_0^H}, \sigma_{\nu_1^H}, \sigma_{\nu_0^L}$	<b>gama1, gamb1</b>	<b>gama2, gamb2</b>	<b>gama3, gamb3</b>	$k^H$
0.1,0.1,0.4	150,2	150,2	7000,0.1	200.00
0.2,0.2,0.4	250,2	250,2	5000,0.1	300.00
0.2,0.2,0.2	350,2	350,2	3000,0.1	1000.00

# Chapter 7: Summary, Conclusions and Future Research

In this thesis, we first investigated a single-period, single-product, single-supplier, single-retailer problem (Newsboy Problem) with nonlinear parameters such as unit price, total production cost, total shortage cost, total salvage value. The resulting problem is mathematically complex to solve. In fact, most of the literature review shows that this model has been often simplified by using linear assumptions. In our case, we considered that the parameters are nonlinear simultaneously and we showed that the expected profit function is concave within a certain interval where the optimal ordering quantity can be determined. For that, efficient search methods can be applied to determine the optimal solution to this problem.

We showed that linear assumptions may lead to a solution far from the optimal one and therefore this causes huge profit loss. Results in the remanufacturing example show that even the minimum error is around 15%. Therefore, it is important for the remanufacturer to choose the real production function  $C(q)$  when making the production decision.

Furthermore, we showed that by using relaxed linear parameter functions and exponential demand distribution assumption, we obtain a precise version but not closed form expression for the optimal solution.

Besides the above single-product single-period problem, we also investigate a periodic-review, two-product, two-capacitated-supplier, two-stage, single-capacitated-retailer, random demand supply chain problem. Any unsatisfied demand is assumed to be lost.

We first apply the stochastic programming methodology to search for the optimal solution. As the size of the grid increases, the solution converges to the optimal solution. However, the computation time also increases dramatically as the size of the grid increases. The choice of an appropriate approximation of the stochastic process is an important issue in stochastic programming. A finer grid is liable to yield more reliable results, but the

size of the deterministic equivalent program increases dramatically with the size of the grid. We have carried out a few experiments to test the impact of the grid size on the objective function and on the decision variables values. The finer grid we considered has 321 points per period. This leads to a tree with 33179523 nodes and a deterministic equivalent linear program with 104332 constraints and 311699 variables. This is the maximal size we could handle on our somewhat limited hardware. The solution required 82 hours.

It is difficult to check if the solution is accurate or not by increasing the size of the grid from 321. Therefore, more efficient method should be developed to search for the optimal solution and to provide managerial insights via a number of numerical examples.

Then, we formulate Problem II and III in a Newsboy framework and provide a GA-based heuristic method to search for the 'optimal' solution. We also evaluate the performance from multiple perspectives, including different sets of assumptions, and evaluate the different trade-offs involving expected profit, selling price, shortage cost, salvage value, inventory holding cost, inventory capacity, production cost and production capacity.

For retailers facing deterministic demand, we provide closed form expression for the optimal solution. The closed form expression shows that under certain conditions, the retailer should place the cheaper product into the primary inventory and the other one to the secondary inventory with a higher holding cost. This is counterintuitive (it is the case where the gap between expensive and cheap primary and secondary inventory holding costs is positive). The reason is that the allocation decision does not depend on the unit holding costs of the two warehouses, but the marginal savings caused by shifting products from one warehouse to the other.

As for retailers facing stochastic demand and renting cheaper secondary warehouse, we prove that the objective functions in both cases are jointly concave. One can take the advantage of concavity to search for the optimal solution. In cases, where the rented secondary warehouse is more expensive than the primary one, and the unmet demand is lost, the objective function has several local optimal solutions. We provide Genetic Algorithm based heuristic to find 'optimal' solutions. We show through numerical examples that this method is more efficient compared to the Steepest Descent Method (far from optimal) and the Classical Genetic Algorithm Method (ten times faster on average).

Furthermore, we provide managerial insights via numerical examples. To conclude, the

total expected profit is more sensitive to the unit price and unit (holding) cost than the capacity constraints. Both, suppliers' limited production and retailer's limited inventory capacities, have major impact on the performance of the supply chain and may result in lowered profits and higher costs. The order of importance of the capacity constraints is: production capacity of the expensive product, production capacity of the cheaper product and the primary warehouse capacity. Numerical results also confirm that, if the gap between the expensive and cheap primary and secondary inventory holding costs is positive, the retailer should place the cheap product into the primary inventory and the expensive product into the secondary inventory with a higher holding cost. In Model 4.1, we show through numerical examples that the primary warehouse space should be 10% higher than the summation of the demand mean in the second period. As  $K$  decreases, the retailer loses more than 50% of the profit. This is because more units are placed into the expensive secondary warehouse, and thus the average unit holding cost increases.

As mentioned before, we do not have any other assumptions on the two products besides (110)(111)(112)(113). Take **Retailer 1**, for example, the selling price and the unit cost of the high-end product can be lower than the low-end one. Our model is built for retail industry problems, but it is also suitable for internal manufacturing problems, where one of the products is produced far-away. We believe that our model is applicable to short life cycle products as fashion goods type products and perishable products, without ruling out other possible applications.

By applying the stochastic programming methodology, the computation time increases dramatically as the number of grid increases. We try to gear good first stage solutions by using a looser approximation of the demand in the second stage so as to reduce the computation time. The grid in the first stage remains 321 and is reduced to  $161^2$  in the second stage. The solution of the high-end product confirms the idea that a grid size of  $161^2$  in the second stage is accurate enough. However, the solution of the low-end product in the first stage is not accurate, which leads to a profit lost of 3%. The associate computation time decreases dramatically from 82 hours to 21 hours, but is still considerable. In a future work, we plan to find a better way to reduce the computation time.

In the case of cheaper secondary warehouse in both Problem II and Problem III, the lost

sale case is not discussed in this thesis and is considered as an area for future research. In Model 3.1 and 3.2, we prove the concavity of the objective function. In a future work, we will take the advantage of the concavity to solve for the optimal solutions of a set of cases so as to provide managerial insights. We will also use the heuristic method mentioned in Model 4.2 to do numerical analysis and provide managerial insights.

In this thesis, we also investigate cases where suppliers are unreliable and we apply our Montecarlo-GA Based Heuristic Method to search for the 'optimal' solution. Further investigations in this direction are also considered.

# Chapter 8

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# Chapter 9

## Appendices

### Appendix I for Problem I

#### 1. Another Proof of Lemma 1

**Lemma 1.** *Define the function  $m(\cdot)$  as*

$$m(q) = r(q)q - C(q). \quad (1)$$

*If  $r(q)$  is monotonically decreasing, then either  $m(\cdot)$  has two different roots : 0 and a strictly positive finite value, denoted as  $q_r > 0$ , or  $m(\cdot)$  has the value 0 as single root and  $m(\cdot)$  remains strictly positive over the interval  $(0, +\infty)$ .*

*Proof.* Note that, by definition, for  $q = 0$  no cost and no profit are incurred, we have

$$m(0) = 0, \quad (2)$$

which shows that  $q = 0$  is the first root of  $m(\cdot)$ . Furthermore, as

$$m'(q) = r'(q)q + r(q) - C'(q), \quad (3)$$

by (6), we directly have

$$m'(0) = r(0) - C'(0) > 0 \quad (4)$$

and by (2), it implies, by continuity, the existence of some interval  $(0, x)$ , with  $x > 0$ , with

$$m(q) > 0 \quad \text{for } q \in (0, x). \quad (5)$$

Since  $C(q)$  is assumed to be a convex increasing function, with  $C(0) = 0$ , we have

$$C(q) \leq qC'(q). \quad (6)$$

For  $q > 0$ , we can write

$$m(q) = q \left( r(q) - \frac{C(q)}{q} \right), \quad (7)$$

and from (6), a straight calculation gives

$$\left( r(q) - \frac{C(q)}{q} \right)' = r'(q) - \frac{C'(q) - \frac{C(q)}{q}}{q}. \quad (8)$$

As  $r'(q) \leq 0$ , by (6), we find

$$\left( r(q) - \frac{C(q)}{q} \right)' \leq 0, \quad (9)$$

which shows that the term  $\left( r(q) - \frac{C(q)}{q} \right)$  is strictly decreasing in  $q$ .

By (5), one has also  $\left( r(q) - \frac{C(q)}{q} \right) > 0$  for  $q \in (0, x)$ .

As a consequence, two cases have to be considered.

First,  $\left( r(q) - \frac{C(q)}{q} \right)$  is strictly positive in  $(0, +\infty)$  and  $m(q) > 0$  for  $q \in (0, +\infty)$ .

In this case, the single root for  $m(\cdot)$  is 0.

Second, there exists a finite  $q_r > 0$  with  $\left( r(q_r) - \frac{C(q_r)}{q_r} \right) = 0$  and for  $q < q_r$  one has  $m(q) > 0$ .

In this situation, there exists a pair of roots for  $m(\cdot)$ , namely 0 and  $q_r$ . □

Please note that Lemma 1 requires neither the twice differentiability nor the concavity of the function  $r(q)$ .

## 2. Tables for Problem I

Table 1.1. Position of our research problem with respect to the literature review.

Model	Unit price	Total Production Cost	Total Salvage Value	Total Shortage cost	Other constraints	Expected Profit Function
News-boy 1955	Constant	Linear	Linear	Linear		Concave
Kaj 2001		Linear	Linear	Two special non-linear functions in shortage		Quasi-Concave
Bayindir 2007	Constant	Piecewise linear Concave	Linear	Linear		Unimodal
Karlin's Model I	constant	Linear	Non-Linear Function in salvage	Non-Linear Function in shortage		concave
Karlin's Model II	constant	Convex	Non-Linear Function in salvage	Non-Linear Function in shortage		Unimodal
Karlin's Model III	constant	Concave	Linear	Linear	Fixed Order Level	Unimodal
Karlin's Model IV			General Non-Linear Function	General Non-Linear Function	Random Ordering Level	Unimodal
Our Model (Theorem I)	Concave-monotony decreasing in ordering quantity	Convex, non-decreasing in ordering quantity	Concave, non-decreasing in salvage quantity	Convex, non-decreasing in shortage quantity		Concave in the interval containing the global optimal solution

Table 1.2. Summary of papers using similar assumptions.

Parameter Assumptions	Unit price	Total Production Cost	Total Salvage Value	Total Shortage cost
Our Model (Theorem I)	Concave-monotony decreasing in ordering quantity	Convex, non-decreasing in ordering quantity	Concave, non-decreasing in salvage quantity	Convex, non-decreasing in shortage quantity
Paper using same/similar assumptions	Raman 1999, Hanson 2009, Tom/s Hardware 2006	Ferguson et al. 2006, Bayindir 2007, Yang 2004, Karlin 1958	Downs et al. 2001	Chatwin 2000

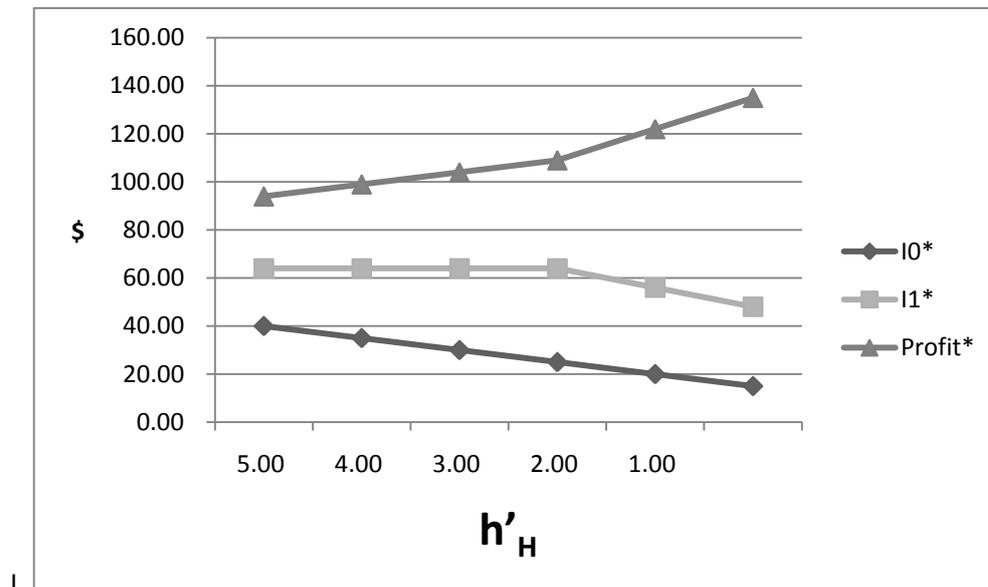
## Appendix II for Problem II

### Model 2.1.

Model 2.1, Table I. Show how holding cost in both period 0 and 1 ( $I_0^*$ ,  $I_1^*$ ) and the total profit changes with high-end product's secondary inventory unit holding cost.

$h'_H$	$X_0^{H^*}$	$X_1^{H^*}$	$X_0^{L^*}$	$a_0^{H^*}$	$a_1^{H^*}$	$a_1^{L^*}$	$I_0^*$	$I_1^*$	Profit*
6.00	15.00	8.00	20.00	0.67	1.00	0.10	40.00	64.00	94.00
5.00	15.00	8.00	20.00	0.67	1.00	0.10	35.00	64.00	99.00
4.00	15.00	8.00	20.00	0.67	1.00	0.10	30.00	64.00	104.00
3.00	15.00	8.00	20.00	0.67	0.00	0.50	25.00	64.00	109.00
2.00	15.00	8.00	20.00	0.67	0.00	0.50	20.00	56.00	122.00
1.00	15.00	8.00	20.00	0.67	0.00	0.50	15.00	48.00	135.00

Model 2.1, Graph I. Show how holding cost in both period 0 and 1 ( $I_0^*$ ,  $I_1^*$ ) and the total profit changes with high-end product's secondary inventory unit holding cost.

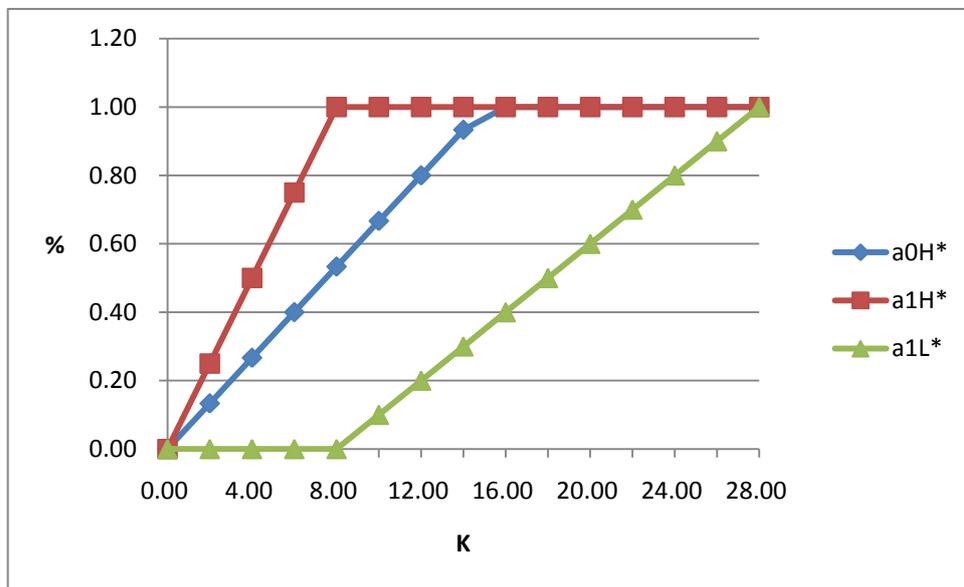


The profit decreases and the holding costs increase as  $h'_H$  increases. High-end products are placed to the primary inventory as the secondary inventory unit holding cost  $h'_H$  increases.

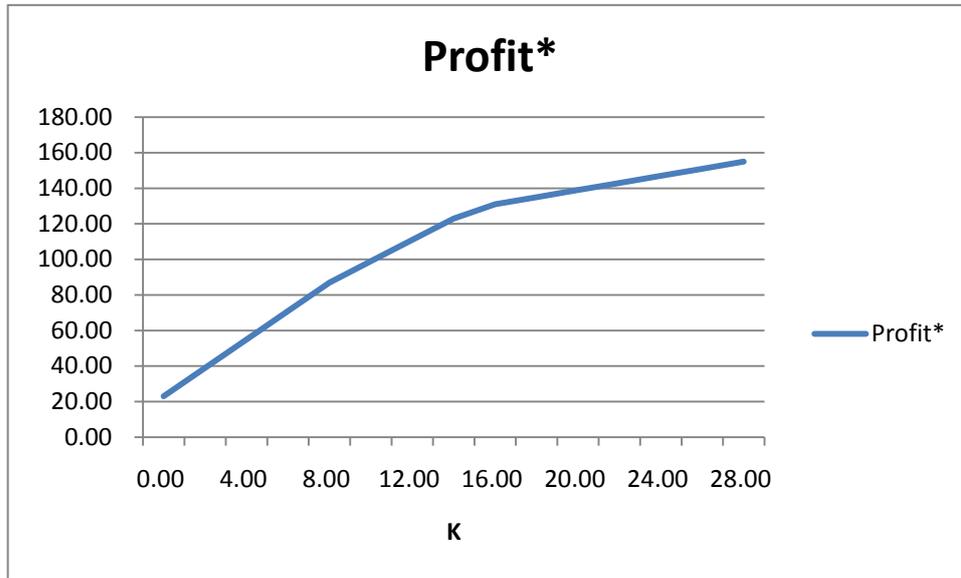
Model 2.1, Table 2.1 Total profit and allocation decision changes with primary inventory capacity K.

K	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*
0.00	15.00	8.00	20.00	0.00	0.00	0.00	23.00
2.00	15.00	8.00	20.00	0.13	0.25	0.00	39.00
4.00	15.00	8.00	20.00	0.27	0.50	0.00	55.00
6.00	15.00	8.00	20.00	0.40	0.75	0.00	71.00
8.00	15.00	8.00	20.00	0.53	1.00	0.00	87.00
10.00	15.00	8.00	20.00	0.67	1.00	0.10	99.00
12.00	15.00	8.00	20.00	0.80	1.00	0.20	111.00
14.00	15.00	8.00	20.00	0.93	1.00	0.30	123.00
16.00	15.00	8.00	20.00	1.00	1.00	0.40	131.00
18.00	15.00	8.00	20.00	1.00	1.00	0.50	135.00
20.00	15.00	8.00	20.00	1.00	1.00	0.60	139.00
22.00	15.00	8.00	20.00	1.00	1.00	0.70	143.00
24.00	15.00	8.00	20.00	1.00	1.00	0.80	147.00
26.00	15.00	8.00	20.00	1.00	1.00	0.90	151.00
28.00	15.00	8.00	20.00	1.00	1.00	1.00	155.00

Model 2.1, Graph 2.1 Allocation decision changes with primary inventory capacity K ( $a_0^{H*}$ ,  $a_1^{H*}$ ,  $a_1^{L*}$  represents  $a_0^{H*}$ ,  $a_1^{H*}$ ,  $a_1^{L*}$ , respectively).



Model 2.1, Graph 2.2 Total profit changes with primary inventory capacity K.

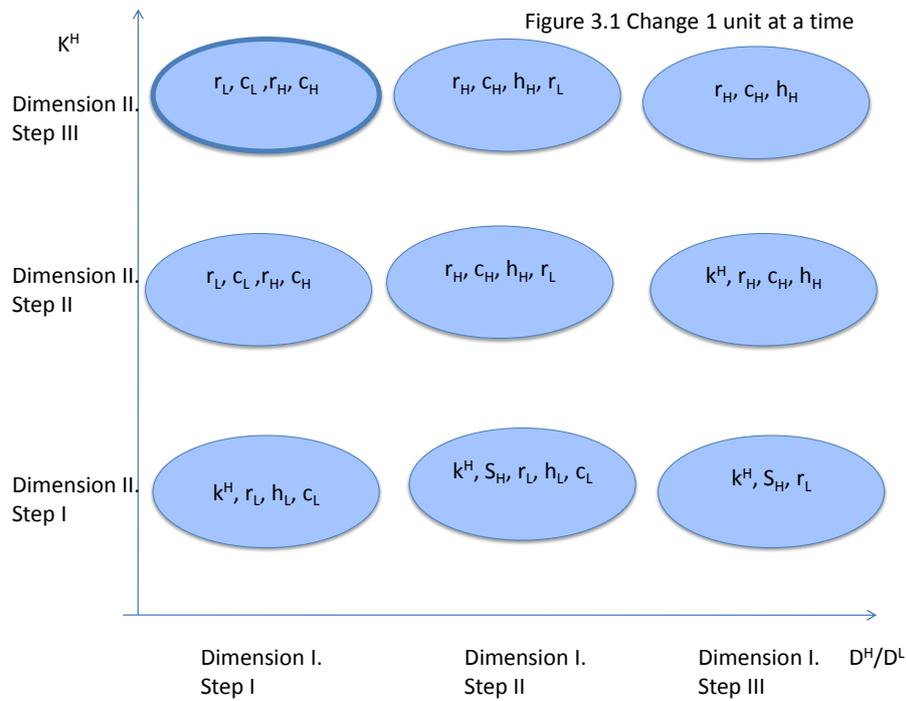


Profit increases as primary inventory capacity K increases. Since  $(h_H - h'_H) - (h_L - h'_L) < 0$ , high-end products are placed to the primary inventory first.

### Model 3.1.

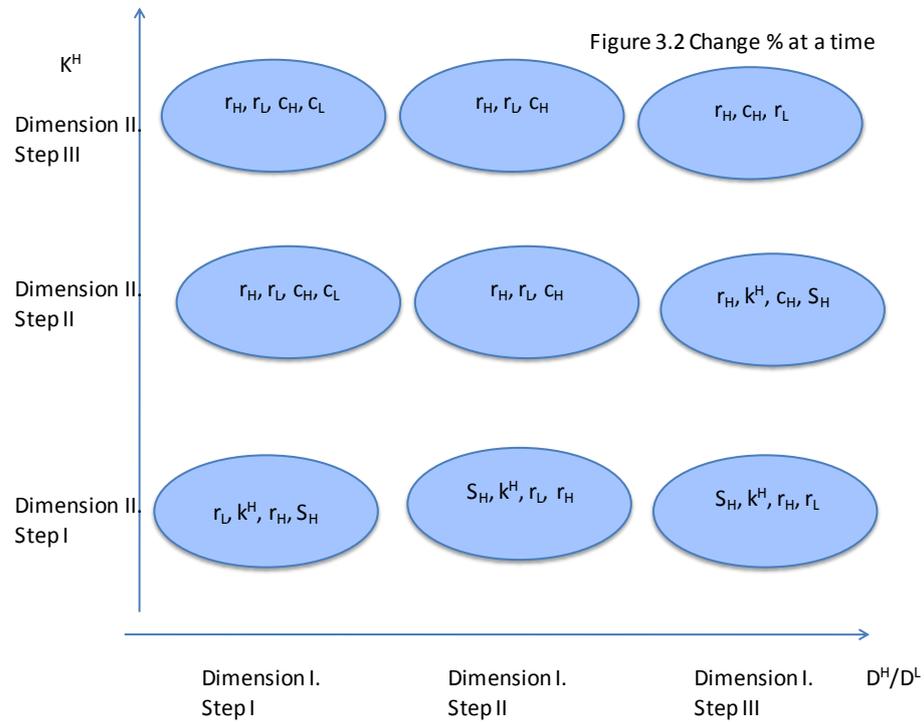
Model 3.1, Figure 3.1.

Show the most important parameters (when changed one unit at a time) in each combination of the steps in Dimension I and the steps in Dimension II in Case I where only  $k^H$  is limited.



No parameters can always be the most important one. We indicate the most important parameters in each combination of the steps in Dimension I and the steps in Dimension II in Case I where only  $k^H$  is limited.

Model 3.1, Figure 3.2. Show the most important parameters (when changed by the same percentage at a time) in each combination of the steps in Dimension I and the steps in Dimension II in Case I where only  $k^H$  is limited.



No parameters can always be the most important one. We indicate the most important parameters in Case I (only  $k^H$  is limited), Case II (both  $K$  and  $k^H$  are limited) and Case III (only  $K$  is limited).

Model 3.1, Table 3.4. Results summaries for Model 3.1.

	Case I	Case II Subcase I	Case II Subcase II	Case III Subcase I	Case III Subcase II	Case IV Subcase I	Case IV Subcase II
Important parameters	$r_H, r_L, k,$ $S_H, c_H$	$r_H, S_H, k^H,$ $r_L, c_H$	$r_H, k^H, r_L,$ $S_H, c_H$	$K, r_H, c_H,$ $h_H$	$r_H, r_L, c_H,$ $h'_H$	$k^H, r_H,$ $c_H, h_H$	$k^H, r_H,$ $c_H, h'_H$

It shows the most important parameters (from the most important to the least important in each cell) in Case I (only  $k^H$  is limited), Case II (both  $K$  and  $k^H$  are limited), Case III (only  $K$  is limited) and Case IV (all capacities are limited). No parameters can always be the most important. We indicate the most important parameters in each case (from the most important to the least important in each cell).

Model 3.1, Table 3.1 The profit changes with  $k^H$  and  $k^L$ .

$k^H$	$k^L$	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*
<b>0.00</b>	<b>0.00</b>	0.00	0.00	0.00	0.00	1.00	1.00	<b>-264.00</b>
<b>0.00</b>	<b>5.00</b>	0.00	0.00	5.00	0.29	1.00	1.00	<b>-234.00</b>
<b>0.00</b>	<b>10.00</b>	0.00	0.00	10.00	0.04	1.00	1.00	<b>-204.00</b>
<b>0.00</b>	<b>15.00</b>	0.00	0.00	15.00	0.82	1.00	0.67	<b>-184.00</b>
<b>0.00</b>	<b>20.00</b>	0.00	0.00	20.00	0.43	1.00	0.50	<b>-164.00</b>
<b>5.00</b>	<b>0.00</b>	5.00	5.00	0.00	1.00	1.00	0.41	<b>-134.00</b>
<b>5.00</b>	<b>5.00</b>	5.00	5.00	5.00	1.00	1.00	1.00	<b>-104.00</b>
<b>5.00</b>	<b>10.00</b>	5.00	5.00	10.00	1.00	1.00	0.50	<b>-84.00</b>
<b>5.00</b>	<b>15.00</b>	5.00	5.00	15.00	1.00	1.00	0.33	<b>-64.00</b>
<b>5.00</b>	<b>20.00</b>	5.00	5.00	20.00	1.00	1.00	0.25	<b>-20.00</b>
<b>10.00</b>	<b>0.00</b>	10.00	8.00	0.00	1.00	1.00	0.49	<b>-30.00</b>
<b>10.00</b>	<b>5.00</b>	10.00	8.00	5.00	1.00	1.00	0.40	<b>-6.00</b>
<b>10.00</b>	<b>10.00</b>	10.00	8.00	10.00	1.00	1.00	0.20	<b>14.00</b>
<b>10.00</b>	<b>15.00</b>	10.00	8.00	15.00	1.00	1.00	0.13	<b>34.00</b>
<b>10.00</b>	<b>20.00</b>	10.00	8.00	20.00	1.00	1.00	0.10	<b>54.00</b>
<b>15.00</b>	<b>0.00</b>	10.00	8.00	0.00	0.67	1.00	0.80	<b>15.00</b>
<b>15.00</b>	<b>5.00</b>	15.00	8.00	5.00	0.67	1.00	0.40	<b>39.00</b>
<b>15.00</b>	<b>10.00</b>	15.00	8.00	10.00	0.67	1.00	0.20	<b>59.00</b>
<b>15.00</b>	<b>15.00</b>	15.00	8.00	15.00	0.67	1.00	0.13	<b>79.00</b>
<b>15.00</b>	<b>20.00</b>	15.00	8.00	20.00	0.67	1.00	0.10	<b>99.00</b>

Both  $k^H$  and  $k^L$  have significant influence on the profit

Model 3.1, Table 3.2 Parameter settings.

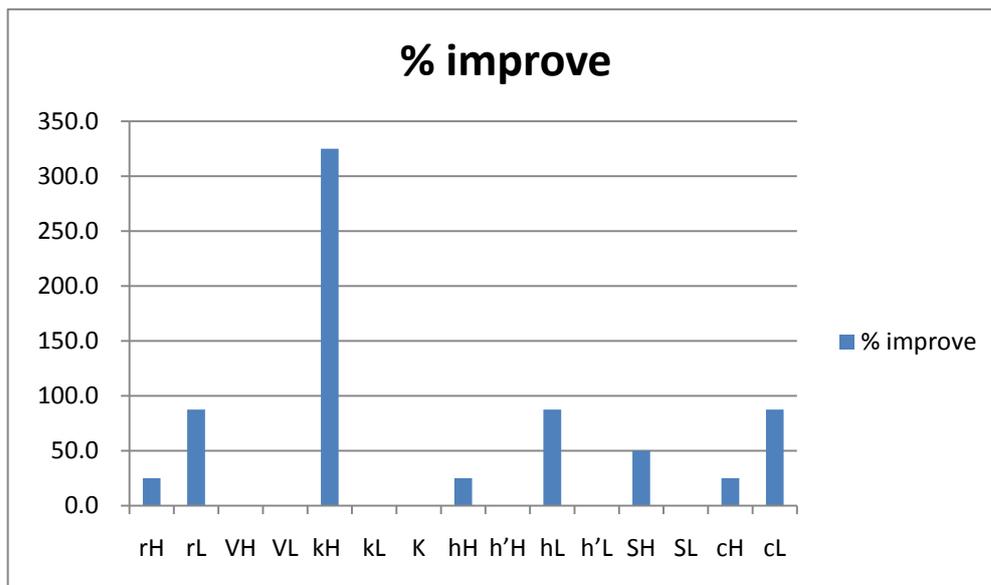
Dimension I Dimension II	Step I ( $D^H_1, D^H_2, D^L_2$ ) =(3,3,7)	Step II ( $D^H_1, D^H_2, D^L_2$ ) =(5,5,5)	Step III ( $D^H_1, D^H_2, D^L_2$ ) =(7,7,3)
Step I	$k^H=1$	$k^H=1$	$k^H=1$
Step II	$k^H=3$	$k^H=5$	$k^H=5$
Step III	$k^H=5$	$k^H=10$	$k^H=10$

Parameter settings of  $k^H$  in Dimension II, including all combinations where  $k^H$  is higher/equal/lower in comparison to any given demand from Dimension I.

Model 3.1, Table I-I-I Case I Sensitivity analysis with parameter settings in step I (Dimension I), step I (Dimension II), step I (Dimension III) where  $(D^H_1, D^H_2, D^L_2, k^H)=(3,3,7,1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			1.0	1.0	7.0	1.0	1.0	1.0	-8.0			
$r_H$	10.0	11.0	1.0	1.0	7.0	1.0	1.0	1.0	-6.0	-8.0	<b>25.0</b>	10.0
$r_L$	5.0	6.0	1.0	1.0	7.0	1.0	1.0	1.0	-1.0	-8.0	<b>87.5</b>	20.0
$V_H$	1.0	2.0	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>	100
$V_L$	1.0	2.0	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>	100
$k^H$	1.0	2.0	1.0	1.0	7.0	1.0	1.0	1.0	18.0	-8.0	<b>325.0</b>	100
$k^L$	10.0	11.0	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>	10.0
K	10.0	11.0	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	1.0	1.0	7.0	1.0	1.0	1.0	-6.0	-8.0	<b>25.0</b>	100.
$h'_H$	5.0	4.0	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	1.0	1.0	7.0	1.0	1.0	1.0	-1.0	-8.0	<b>87.5</b>	100
$h'_L$	3.0	2.0	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	1.0	1.0	7.0	1.0	1.0	1.0	-4.0	-8.0	<b>50.0</b>	12.5
$S_L$	4.0	3.0	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	1.0	1.0	7.0	1.0	1.0	1.0	-6.0	-8.0	<b>25.0</b>	25.0
$c_L$	2.0	1.0	1.0	1.0	7.0	1.0	1.0	1.0	-1.0	-8.0	<b>87.5</b>	50.0
average %												47.7

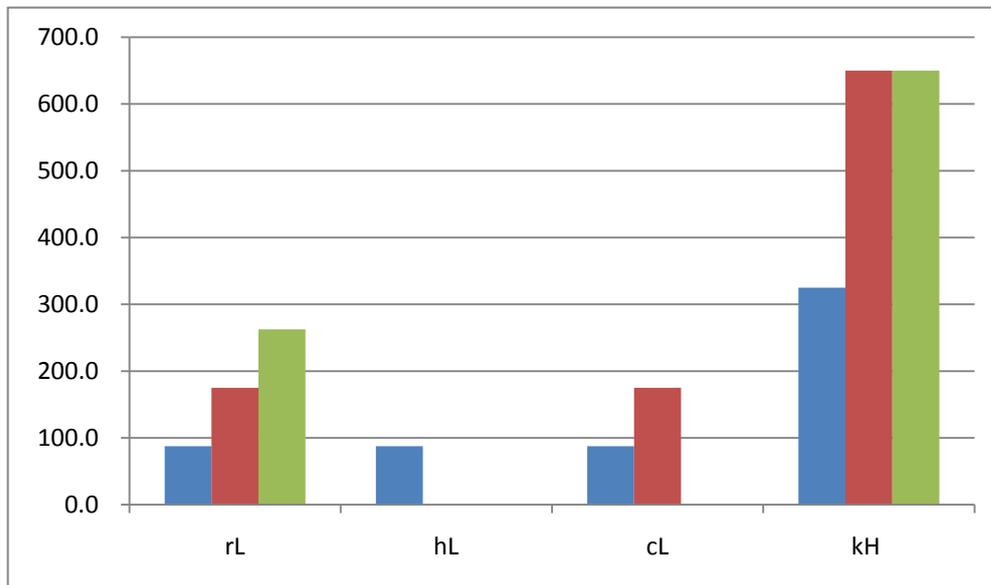
Graph I-I-I. Case I Sensitivity analysis with parameter settings in step I (Dimension I), step I (Dimension II), step I (Dimension III) where  $(D^H_1, D^H_2, D^L_2, k^H)=(3,3,7,1)$ .



Model 3.1, Table I-I-I-Elite Case I Sensitivity analysis with parameter settings in step I (Dimension I), step I (Dimension II), step I (Dimension III), Dimension where  $(D^H_1, D^H_2, D^L_2, k^H)=(3,3,7,1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			1	1	7	1	1	1	- 8		
$r_L$	5	6	1	1	7	1	1	1	-1	-8	<b>87.5</b>
		7	1	1	7	1	1	1	6	-8	<b>175</b>
		8	1	1	7	1	1	1	13	-8	<b>262.5</b>
$h_L$	1	0	1	1	7	1	1	1	-1	-8	<b>87.5</b>
$c_L$	2	1	1	1	7	1	1	1	-1	-8	<b>87.5</b>
		0	1	1	7	1	1	1	6	-8	<b>175</b>
$k^H$	1	2	1	1	7	1	1	1	18	-8	<b>325</b>
		3	3	3	7	1	1	1	44	-8	<b>650</b>
		4	3	3	7	1	1	1	44	-8	<b>650</b>

Graph I-I-I-Elite. Case I Sensitivity analysis with parameter settings in step I (Dimension I), step I (Dimension II), step I (Dimension III), Dimension where  $(D^H_1, D^H_2, D^L_2, k^H)=(3,3,7,1)$ .

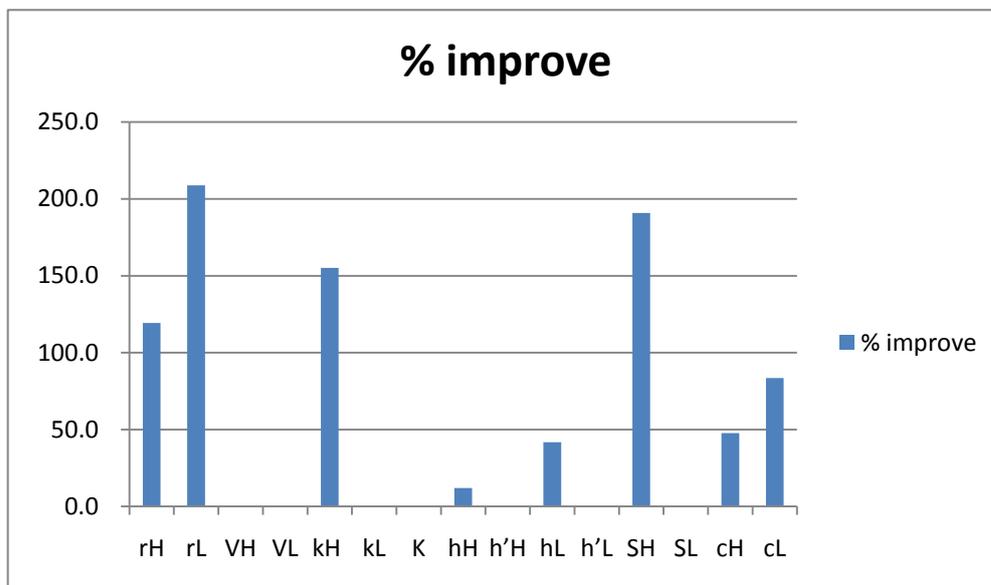


$k^H$  is the most important parameter when it is less than the demand and  $r_L, c_L, h_L$  are the second important ones.

Model 3.1, Table I-I-II Case I Sensitivity analysis with parameter settings in step I (Dimension I), step I (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,1)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			1.0	1.0	7.0	1.0	1.0	1.0	-8.0		
$r_H$	10.0	14.8	1.0	1.0	7.0	1.0	1.0	1.0	1.5	-8.0	<b>119.3</b>
$r_L$	5.0	7.4	1.0	1.0	7.0	1.0	1.0	1.0	8.7	-8.0	<b>208.8</b>
$V_H$	1	1.5	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>
$V_L$	1.0	1.5	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>
$k^H$	1.0	1.5	1.5	1.5	7.0	1.0	1.0	1.0	4.4	-8.0	<b>155.1</b>
$k^L$	10.0	14.8	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>
K	10.0	14.8	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>
$h_H$	1.0	0.5	1.0	1.0	7.0	1.0	1.0	1.0	-7.0	-8.0	<b>11.9</b>
$h'_H$	5.0	2.6	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>
$h_L$	1.0	0.5	1.0	1.0	7.0	1.0	1.0	1.0	-4.7	-8.0	<b>41.8</b>
$h'_L$	3.0	1.6	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>
$S_H$	8.0	4.2	1.0	1.0	7.0	1.0	1.0	1.0	7.3	-8.0	<b>190.9</b>
$S_L$	4.0	2.1	1.0	1.0	7.0	1.0	1.0	1.0	-8.0	-8.0	<b>0.0</b>
$c_H$	4.0	2.1	1.0	1.0	7.0	1.0	1.0	1.0	-4.2	-8.0	<b>47.7</b>
$c_L$	2.0	1.0	1.0	1.0	7.0	1.0	1.0	1.0	-1.3	-8.0	<b>83.5</b>

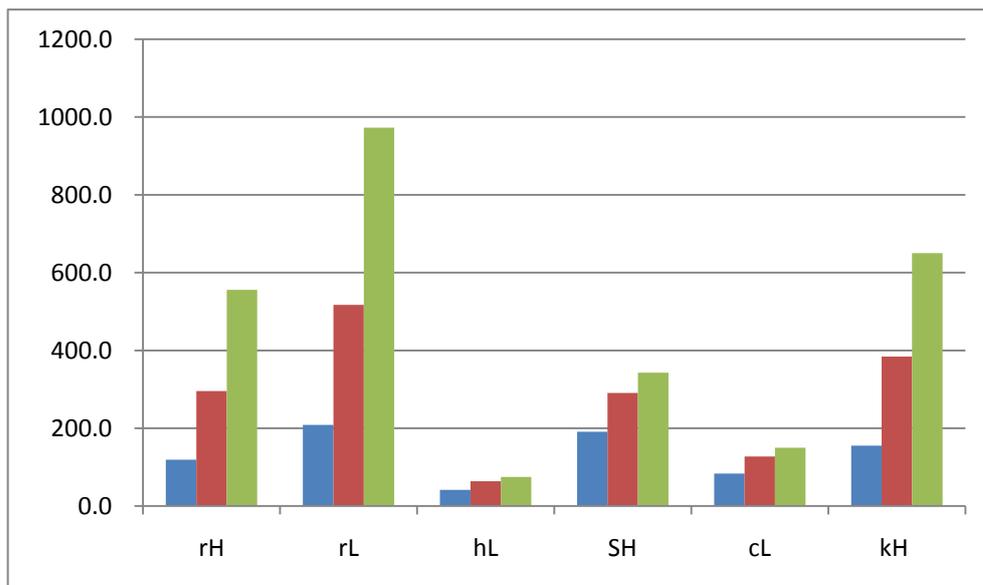
Graph I-I-II. (Average parameter change: 47.7222%). Case I Sensitivity analysis with parameter settings in step I (Dimension I), step I (Dimension II), step II (Dimension III).



Model 3.1, Table I-II-Elite Case I Sensitivity analysis with parameter settings in step I (Dimension I), step I (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,1)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			1.0	1.0	7.0	1.0	1.0	1.0	-8.0		
$r_H$	10.0	14.8	1.0	1.0	7.0	1.0	1.0	1.0	1.5	-8.0	<b>119.3</b>
		21.8	1.0	1.0	7.0	1.0	1.0	1.0	15.6	-8.0	<b>295.5</b>
		32.2	1.0	1.0	7.0	1.0	1.0	1.0	36.5	-8.0	<b>555.9</b>
$r_L$	5.0	7.4	1.0	1.0	7.0	1.0	1.0	1.0	8.7	-8.0	<b>208.8</b>
		10.9	1.0	1.0	7.0	1.0	1.0	1.0	33.4	-8.0	<b>517.2</b>
		16.1	1.0	1.0	7.0	1.0	1.0	1.0	69.8	-8.0	<b>972.8</b>
$h_L$	1.0	0.5	1.0	1.0	7.0	1.0	1.0	1.0	-4.7	-8.0	<b>41.8</b>
		0.3	1.0	1.0	7.0	1.0	1.0	1.0	-2.9	-8.0	<b>63.6</b>
		0.1	1.0	1.0	7.0	1.0	1.0	1.0	-2.0	-8.0	<b>75.0</b>
$S_H$	8.0	4.2	1.0	1.0	7.0	1.0	1.0	1.0	7.3	-8.0	<b>190.9</b>
		2.2	1.0	1.0	7.0	1.0	1.0	1.0	15.3	-8.0	<b>290.7</b>
		1.1	1.0	1.0	7.0	1.0	1.0	1.0	19.4	-8.0	<b>342.9</b>
$c_L$	2.0	1.0	1.0	1.0	7.0	1.0	1.0	1.0	-1.3	-8.0	<b>83.5</b>
		0.5	1.0	1.0	7.0	1.0	1.0	1.0	2.2	-8.0	<b>127.2</b>
		0.3	1.0	1.0	7.0	1.0	1.0	1.0	4.0	-8.0	<b>150.0</b>
$k^H$	1.0	1.5	1.5	1.5	7.0	1.0	1.0	1.0	4.4	-8.0	<b>155.1</b>
		2.2	2.2	2.2	7.0	1.0	1.0	1.0	22.7	-8.0	<b>384.2</b>
		3.2	3.0	3.0	7.0	1.0	1.0	1.0	44.0	-8.0	<b>650.0</b>

Graph I-II-Elite. (Change 47.7222% per step). Case I Sensitivity analysis with parameter settings in step I (Dimension I), step I (Dimension II), step II (Dimension III), Dimension IV.

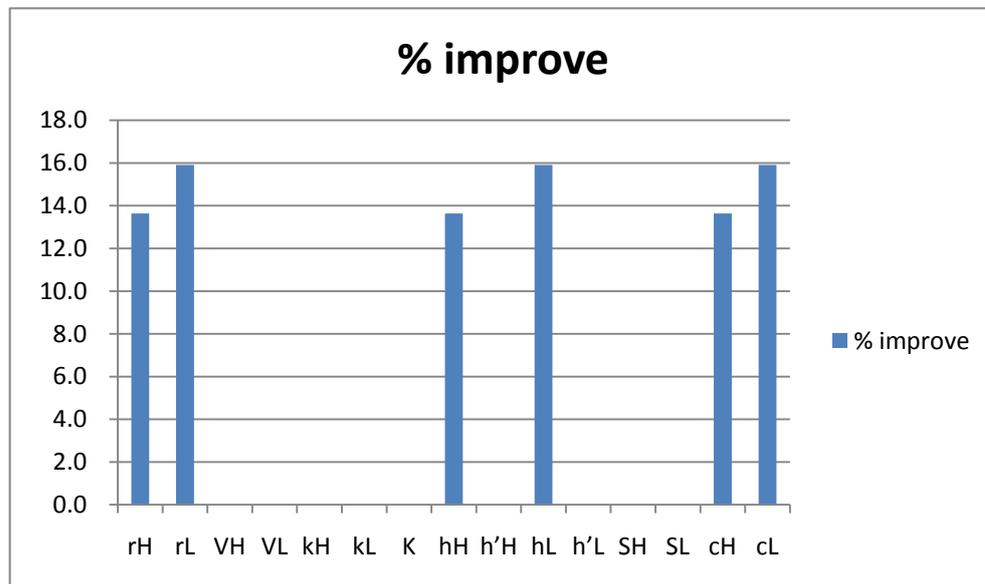


$r_L, k^H, r_H, S_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table I-II-I Case I Sensitivity analysis with parameter settings in step I (Dimension I), step II (Dimension II), step I (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,3)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			3.0	3.0	7.0	1.0	1.0	1.0	44.0			
$r_H$	10.0	11.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>	10.0
$r_L$	5.0	6.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>	20.0
$V_H$	1.0	2.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	100.0
$V_L$	1.0	2.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	100.0
$k^H$	3.0	4.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	33.3
$k^L$	10.0	11.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	10.0
K	10.0	11.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>	100.0
$h'_H$	5.0	4.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>	100.0
$h'_L$	3.0	2.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	12.5
$S_L$	4.0	3.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>	25.0
$c_L$	2.0	1.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>	50.0
average %												43.3

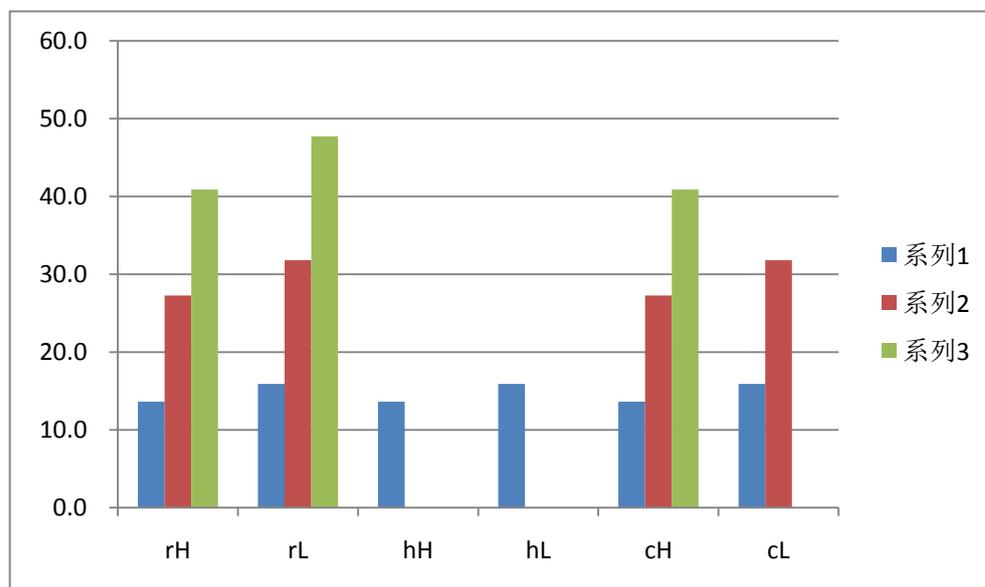
Graph I-II-I



Model 3.1, Table I-II-I-Elite Case I Sensitivity analysis with parameter settings in step I (Dimension I), step II (Dimension II), step I (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,3)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3.0	3.0	7.0	1.0	1.0	1.0	44.0		
$r_H$	10.0	11.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>
		12.0	3.0	3.0	7.0	1.0	1.0	1.0	56.0	44.0	<b>27.3</b>
		13.0	3.0	3.0	7.0	1.0	1.0	1.0	62.0	44.0	<b>40.9</b>
$r_L$	5.0	6.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>
		7.0	3.0	3.0	7.0	1.0	1.0	1.0	58.0	44.0	<b>31.8</b>
		8.0	3.0	3.0	7.0	1.0	1.0	1.0	65.0	44.0	<b>47.7</b>
$h_H$	1.0	0.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>
$h_L$	1.0	0.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>
$c_H$	4.0	3.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>
		2.0	3.0	3.0	7.0	1.0	1.0	1.0	56.0	44.0	<b>27.3</b>
		1.0	3.0	3.0	7.0	1.0	1.0	1.0	62.0	44.0	<b>40.9</b>
$c_L$	2.0	1.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>
		0.0	3.0	3.0	7.0	1.0	1.0	1.0	58.0	44.0	<b>31.8</b>

Graph I-II-I-Elite

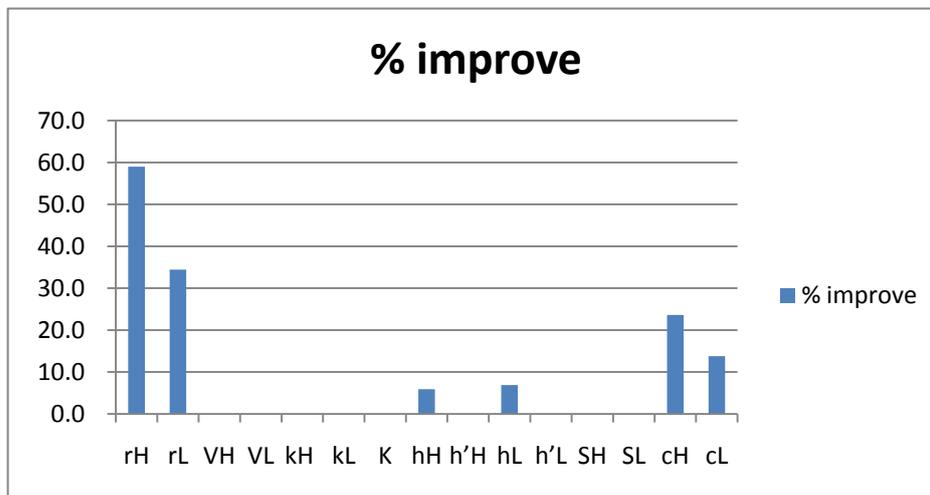


$r_L$ ,  $c_L$ ,  $r_H$ ,  $c_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table I-II-II Case I Sensitivity analysis with parameter settings in step I (Dimension I), step II (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,3)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3.0	3.0	7.0	1.0	1.0	1.0	44.0		
$r_H$	10.0	14.3	3.0	3.0	7.0	1.0	1.0	1.0	70.0	44.0	<b>59.0</b>
$r_L$	5.0	7.2	3.0	3.0	7.0	1.0	1.0	1.0	59.1	44.0	<b>34.4</b>
$V_H$	1.0	1.4	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$V_L$	1.0	1.4	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$k^H$	3.0	4.3	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$k^L$	10.0	14.3	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
K	10.0	14.3	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$h_H$	1.0	0.6	3.0	3.0	7.0	1.0	1.0	1.0	46.6	44.0	<b>5.9</b>
$h'_H$	5.0	2.8	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$h_L$	1.0	0.6	3.0	3.0	7.0	1.0	1.0	1.0	47.0	44.0	<b>6.9</b>
$h'_L$	3.0	1.7	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$S_H$	8.0	4.5	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$S_L$	4.0	2.3	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$c_H$	4.0	2.3	3.0	3.0	7.0	1.0	1.0	1.0	54.4	44.0	<b>23.6</b>
$c_L$	2.0	1.1	3.0	3.0	7.0	1.0	1.0	1.0	50.1	44.0	<b>13.8</b>

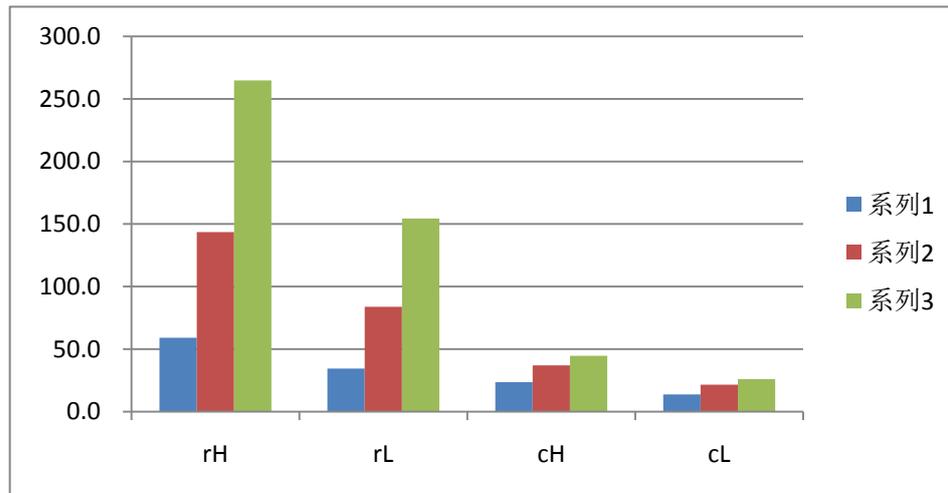
Graph I-II-II



Model 3.1, Table I-II-II-Elite Case I Sensitivity analysis with parameter settings in step I (Dimension I), step II (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,3)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3.0	3.0	7.0	1.0	1.0	1.0	44.0		
$r_H$	10.0	14.3	3.0	3.0	7.0	1.0	1.0	1.0	70.0	44.0	<b>59.0</b>
		20.5	3.0	3.0	7.0	1.0	1.0	1.0	107.2	44.0	<b>143.6</b>
		29.4	3.0	3.0	7.0	1.0	1.0	1.0	160.5	44.0	<b>264.7</b>
$r_L$	5.0	7.2	3.0	3.0	7.0	1.0	1.0	1.0	59.1	44.0	<b>34.4</b>
		10.3	3.0	3.0	7.0	1.0	1.0	1.0	80.8	44.0	<b>83.7</b>
		14.7	3.0	3.0	7.0	1.0	1.0	1.0	111.9	44.0	<b>154.4</b>
$c_H$	4.0	2.3	3.0	3.0	7.0	1.0	1.0	1.0	54.4	44.0	<b>23.6</b>
		1.3	3.0	3.0	7.0	1.0	1.0	1.0	60.3	44.0	<b>37.0</b>
		0.7	3.0	3.0	7.0	1.0	1.0	1.0	63.6	44.0	<b>44.6</b>
$c_L$	2.0	1.1	3.0	3.0	7.0	1.0	1.0	1.0	50.1	44.0	<b>13.8</b>
		0.6	3.0	3.0	7.0	1.0	1.0	1.0	53.5	44.0	<b>21.6</b>
		0.4	3.0	3.0	7.0	1.0	1.0	1.0	55.4	44.0	<b>26.0</b>

Graph I-II-II-Elite

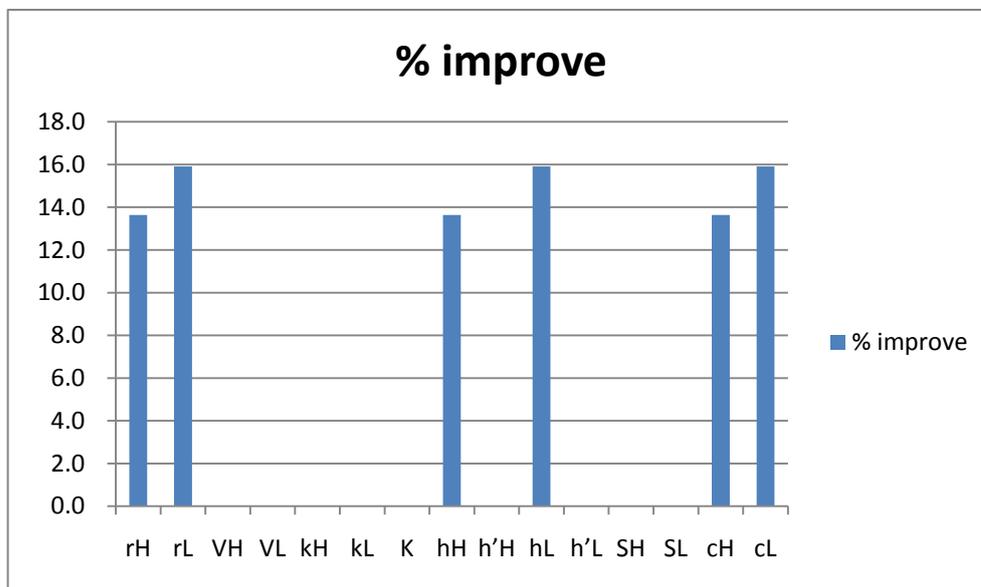


$r_H$ ,  $r_L$ ,  $c_H$ ,  $c_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table I-III-I Case I Sensitivity analysis with parameter settings in step I (Dimension I), step III(Dimension II), step I (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,5)$ .

parameter	original	change I	$X_0^{H*}$	$X_1^{H*}$	$X_1^{L*}$	$t_1^{H*}$	$t_1^{L*}$	$a_1^{L*}$	$t_{11}^{H*}$	$t_{11}^{L*}$	% improve	% change
original			3.0	3.0	7.0	1.0	1.0	1.0	44.0			
$r_H$	10.0	11.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>	10.0
$r_L$	5.0	6.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>	20.0
$V_H$	1.0	2.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	100.0
$V_L$	1.0	2.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	100.0
$k^H$	5.0	6.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	20.0
$k^L$	10.0	11.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	10.0
K	10.0	11.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>	100.0
$h'_H$	5.0	4.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>	100.0
$h'_L$	3.0	2.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	12.5
$S_L$	4.0	3.0	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>	25.0
$c_L$	2.0	1.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>	50.0
average %												42.4

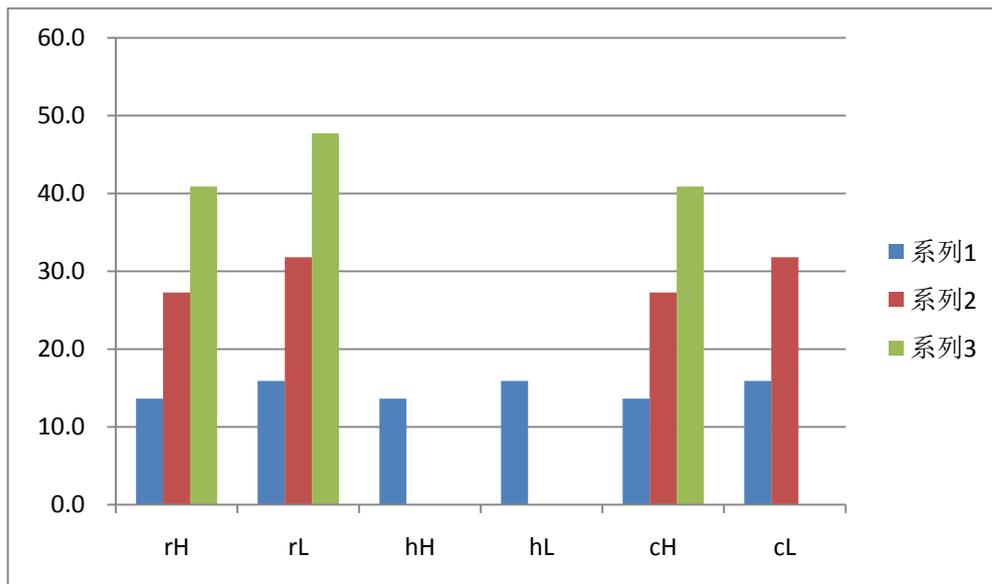
Graph I-III-I



Model 3.1, Table I-III-I-Elite Case I Sensitivity analysis with parameter settings in step I (Dimension I), step III (Dimension II), step I (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,5)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3.0	3.0	7.0	1.0	1.0	1.0	44.0		
rH	10.0	11.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>
		12.0	3.0	3.0	7.0	1.0	1.0	1.0	56.0	44.0	<b>27.3</b>
		13.0	3.0	3.0	7.0	1.0	1.0	1.0	62.0	44.0	<b>40.9</b>
rL	5.0	6.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>
		7.0	3.0	3.0	7.0	1.0	1.0	1.0	58.0	44.0	<b>31.8</b>
		8.0	3.0	3.0	7.0	1.0	1.0	1.0	65.0	44.0	<b>47.7</b>
hH	1.0	0.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>
hL	1.0	0.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>
cH	4.0	3.0	3.0	3.0	7.0	1.0	1.0	1.0	50.0	44.0	<b>13.6</b>
		2.0	3.0	3.0	7.0	1.0	1.0	1.0	56.0	44.0	<b>27.3</b>
		1.0	3.0	3.0	7.0	1.0	1.0	1.0	62.0	44.0	<b>40.9</b>
cL	2.0	1.0	3.0	3.0	7.0	1.0	1.0	1.0	51.0	44.0	<b>15.9</b>
		0.0	3.0	3.0	7.0	1.0	1.0	1.0	58.0	44.0	<b>31.8</b>

Graph I-III-I-Elite

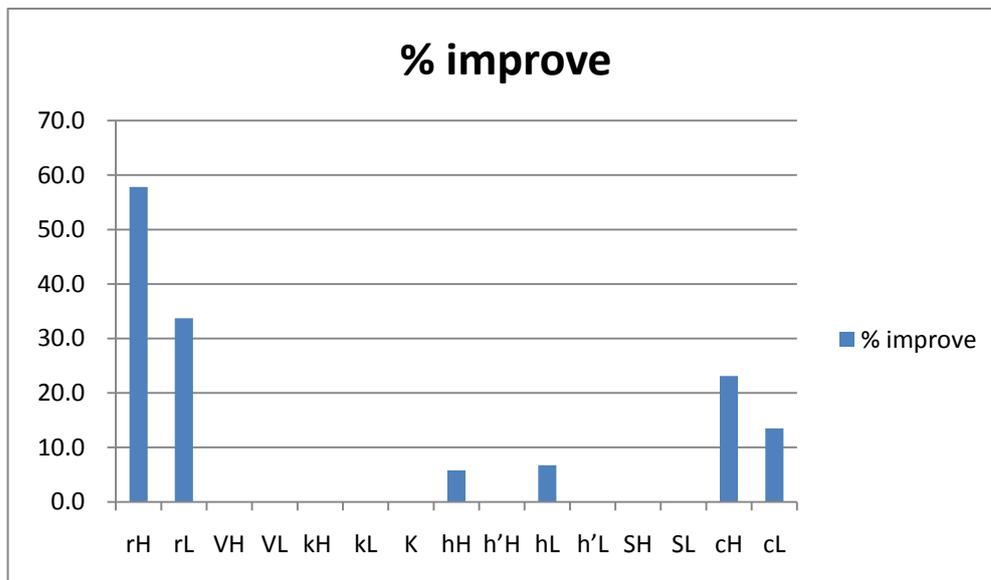


$r_L, c_L, h_L, r_H, c_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table I-III-II Case I Sensitivity analysis with parameter settings in step I (Dimension I), step III (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,5)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3.0	3.0	7.0	1.0	1.0	1.0	44.0		
$r_H$	10.0	14.2	3.0	3.0	7.0	1.0	1.0	1.0	69.4	44.0	<b>57.8</b>
$r_L$	5.0	7.1	3.0	3.0	7.0	1.0	1.0	1.0	58.8	44.0	<b>33.7</b>
$V_H$	1.0	1.4	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$V_L$	1.0	1.4	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$k^H$	5.0	7.1	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$k^L$	10.0	14.2	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
K	10.0	14.2	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$h_H$	1.0	0.6	3.0	3.0	7.0	1.0	1.0	1.0	46.5	44.0	<b>5.8</b>
$h'_H$	5.0	2.9	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$h_L$	1.0	0.6	3.0	3.0	7.0	1.0	1.0	1.0	47.0	44.0	<b>6.7</b>
$h'_L$	3.0	1.7	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$S_H$	8.0	4.6	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$S_L$	4.0	2.3	3.0	3.0	7.0	1.0	1.0	1.0	44.0	44.0	<b>0.0</b>
$c_H$	4.0	2.3	3.0	3.0	7.0	1.0	1.0	1.0	54.2	44.0	<b>23.1</b>
$c_L$	2.0	1.2	3.0	3.0	7.0	1.0	1.0	1.0	49.9	44.0	<b>13.5</b>

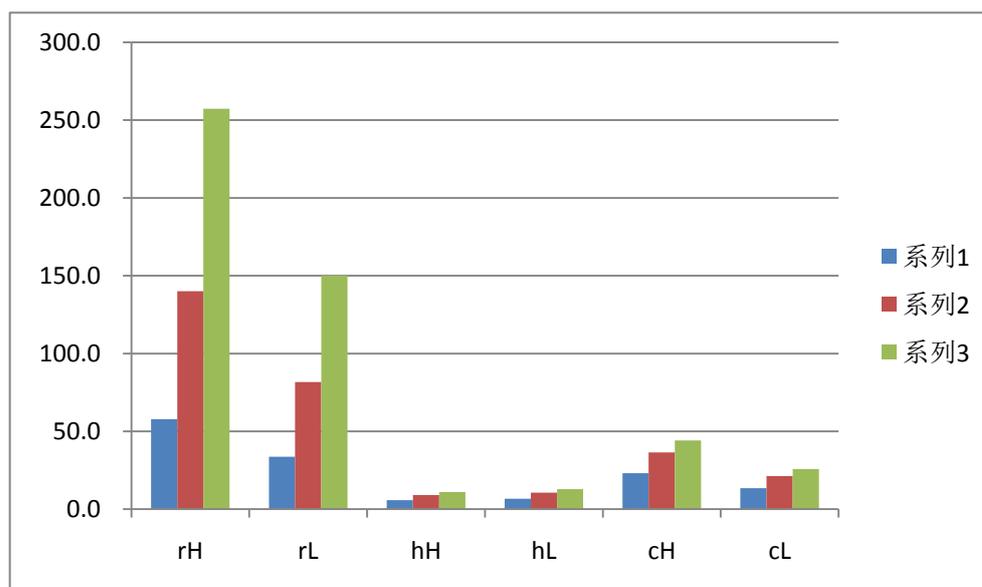
Graph I-III-II



Model 3.1, Table I-III-II-Elite Case I Sensitivity analysis with parameter settings in step I (Dimension I), step III (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(3,3,7,5)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			3.0	3.0	7.0	1.0	1.0	1.0	44.0		
$r_H$	10.0	14.2	3.0	3.0	7.0	1.0	1.0	1.0	69.4	44.0	<b>57.8</b>
		20.3	3.0	3.0	7.0	1.0	1.0	1.0	105.6	44.0	<b>140.1</b>
		28.9	3.0	3.0	7.0	1.0	1.0	1.0	157.2	44.0	<b>257.3</b>
$r_L$	5.0	7.1	3.0	3.0	7.0	1.0	1.0	1.0	58.8	44.0	<b>33.7</b>
		10.1	3.0	3.0	7.0	1.0	1.0	1.0	80.0	44.0	<b>81.7</b>
		14.4	3.0	3.0	7.0	1.0	1.0	1.0	110.0	44.0	<b>150.1</b>
$h_H$	1.0	0.6	3.0	3.0	7.0	1.0	1.0	1.0	46.5	44.0	<b>5.8</b>
		0.3	3.0	3.0	7.0	1.0	1.0	1.0	48.0	44.0	<b>9.1</b>
		0.2	3.0	3.0	7.0	1.0	1.0	1.0	48.9	44.0	<b>11.0</b>
$h_L$	1.0	0.6	3.0	3.0	7.0	1.0	1.0	1.0	47.0	44.0	<b>6.7</b>
		0.3	3.0	3.0	7.0	1.0	1.0	1.0	48.7	44.0	<b>10.6</b>
		0.2	3.0	3.0	7.0	1.0	1.0	1.0	49.7	44.0	<b>12.9</b>
$c_H$	4.0	2.3	3.0	3.0	7.0	1.0	1.0	1.0	54.2	44.0	<b>23.1</b>
		1.3	3.0	3.0	7.0	1.0	1.0	1.0	60.0	44.0	<b>36.4</b>
		0.8	3.0	3.0	7.0	1.0	1.0	1.0	63.4	44.0	<b>44.1</b>
$c_L$	2.0	1.2	3.0	3.0	7.0	1.0	1.0	1.0	49.9	44.0	<b>13.5</b>
		0.7	3.0	3.0	7.0	1.0	1.0	1.0	53.4	44.0	<b>21.3</b>
		0.4	3.0	3.0	7.0	1.0	1.0	1.0	55.3	44.0	<b>25.7</b>

Graph I-III-II-Elite

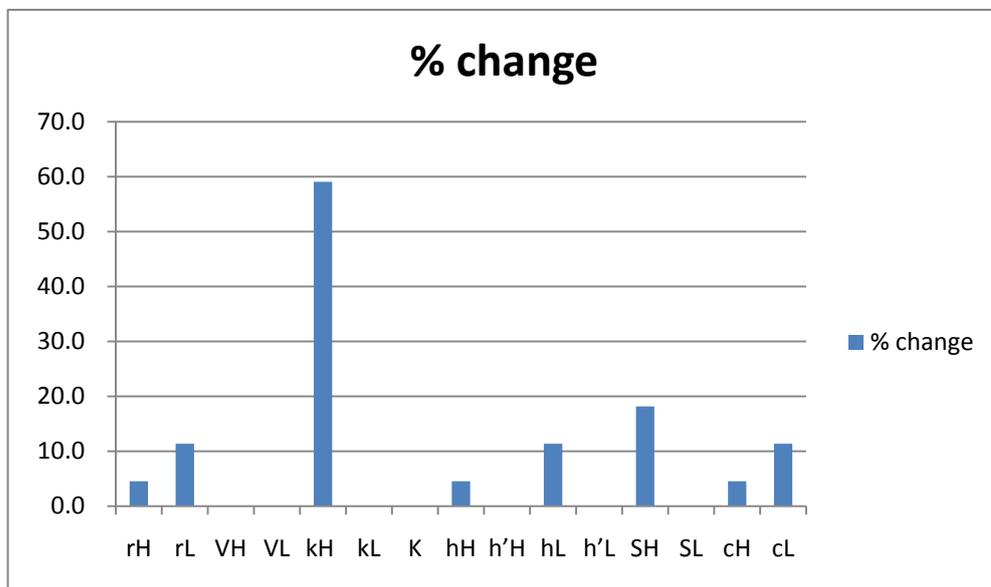


$r_H, r_L, c_H, c_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table II-I-I Case I Sensitivity analysis with parameter settings in step II (Dimension I), step II (Dimension I), step I (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			1.0	1.0	5.0	1.0	1.0	1.0	-44.0			
$r_H$	10.0	11.0	1.0	1.0	5.0	1.0	1.0	1.0	-42.0	-44.0	<b>4.5</b>	10.0
$r_L$	5.0	6.0	1.0	1.0	5.0	1.0	1.0	1.0	-39.0	-44.0	<b>11.4</b>	20.0
$V_H$	1.0	2.0	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>	100.0
$V_L$	1.0	2.0	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>	100.0
$k^H$	1.0	2.0	2.0	2.0	5.0	1.0	1.0	1.0	-18.0	-44.0	<b>59.1</b>	100.0
$k^L$	10.0	11.0	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>	10.0
K	10.0	11.0	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	1.0	1.0	5.0	1.0	1.0	1.0	-42.0	-44.0	<b>4.5</b>	100.0
$h'_H$	5.0	4.0	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	1.0	1.0	5.0	1.0	1.0	1.0	-39.0	-44.0	<b>11.4</b>	100.0
$h'_L$	3.0	2.0	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	1.0	1.0	5.0	1.0	1.0	1.0	-36.0	-44.0	<b>18.2</b>	12.5
$S_L$	4.0	3.0	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	1.0	1.0	5.0	1.0	1.0	1.0	-42.0	-44.0	<b>4.5</b>	25.0
$c_L$	2.0	1.0	1.0	1.0	5.0	1.0	1.0	1.0	-39.0	-44.0	<b>11.4</b>	50.0
average %												47.7

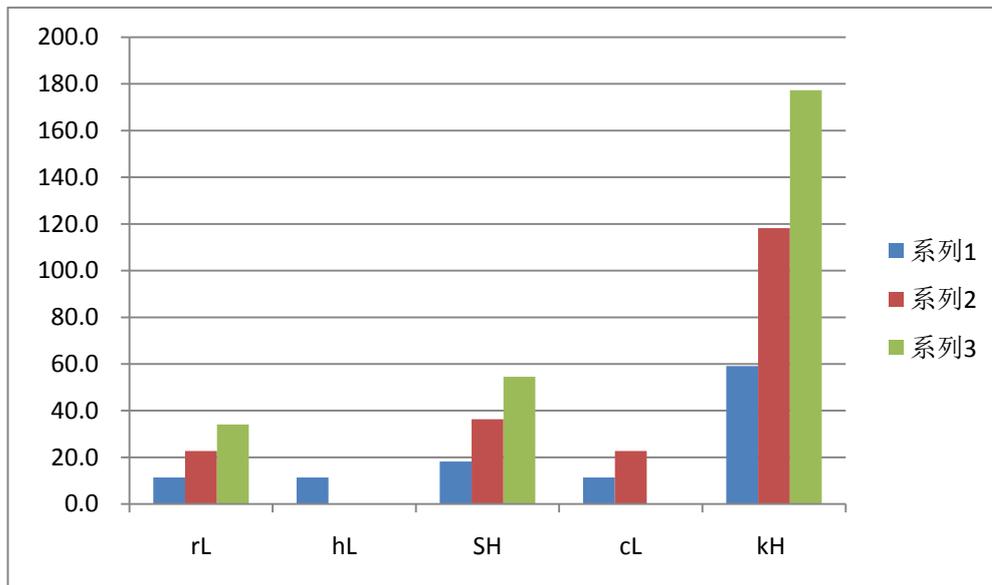
Graph II-I-I



Model 3.1, Table II-I-I-Elite Case I Sensitivity analysis with parameter settings in step II (Dimension I), step I (Dimension II), step I (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			1.0	1.0	5.0	1.0	1.0	1.0	-44.0		
$r_L$	5.0	6.0	1.0	1.0	5.0	1.0	1.0	1.0	-39.0	-44.0	<b>11.4</b>
		7.0	1.0	1.0	5.0	1.0	1.0	1.0	-34.0	-44.0	<b>22.7</b>
		8.0	1.0	1.0	5.0	1.0	1.0	1.0	-29.0	-44.0	<b>34.1</b>
$h_L$	1.0	0.0	1.0	1.0	5.0	1.0	1.0	1.0	-39.0	-44.0	<b>11.4</b>
$S_H$	8.0	7.0	1.0	1.0	5.0	1.0	1.0	1.0	-36.0	-44.0	<b>18.2</b>
		6.0	1.0	1.0	5.0	1.0	1.0	1.0	-28.0	-44.0	<b>36.4</b>
		5.0	1.0	1.0	5.0	1.0	1.0	1.0	-20.0	-44.0	<b>54.5</b>
$c_L$	2.0	1.0	1.0	1.0	5.0	1.0	1.0	1.0	-39.0	-44.0	<b>11.4</b>
		0.0	1.0	1.0	5.0	1.0	1.0	1.0	-34.0	-44.0	<b>22.7</b>
$k^H$	1.0	2.0	2.0	2.0	5.0	1.0	1.0	1.0	-18.0	-44.0	<b>59.1</b>
		3.0	3.0	3.0	5.0	1.0	1.0	1.0	8.0	-44.0	<b>118.2</b>
		4.0	4.0	4.0	5.0	1.0	1.0	1.0	34.0	-44.0	<b>177.3</b>

Graph II-I-I-Elite

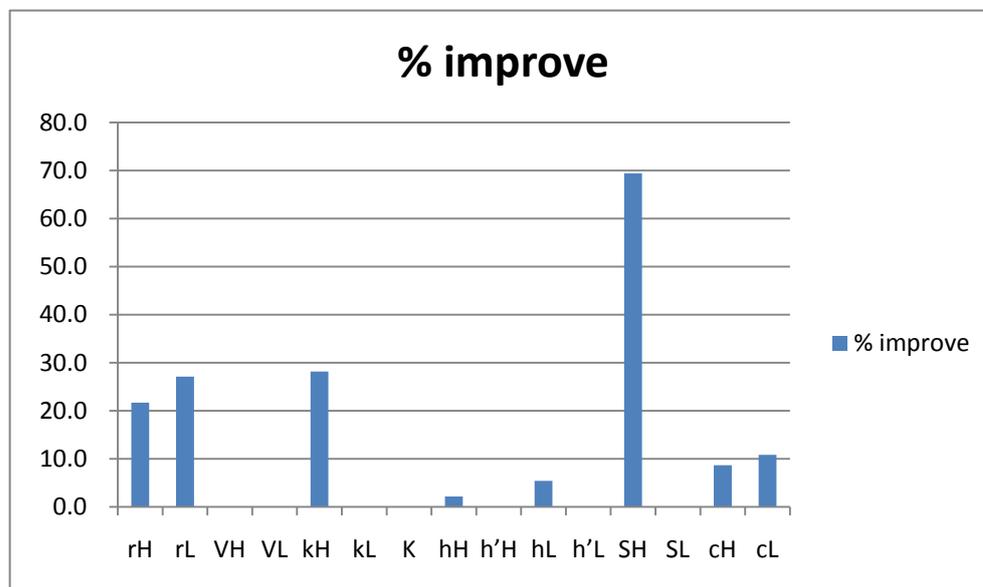


$k^H, S_H, r_L, h_L, c_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table II-I-II Case I Sensitivity analysis with parameter settings in step II (Dimension I), step I (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,1)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			1.0	1.0	5.0	1.0	1.0	1.0	-44.0		
$r_H$	10.0	14.8	1.0	1.0	5.0	1.0	1.0	1.0	-34.5	-44.0	<b>21.7</b>
$r_L$	5.0	7.4	1.0	1.0	5.0	1.0	1.0	1.0	-32.1	-44.0	<b>27.1</b>
$V_H$	1.0	1.5	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>
$V_L$	1.0	1.5	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>
$k^H$	1.0	1.5	1.5	1.5	5.0	1.0	1.0	1.0	-31.6	-44.0	<b>28.2</b>
$k^L$	10.0	14.8	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>
K	10.0	14.8	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>
$h_H$	1.0	0.5	1.0	1.0	5.0	1.0	1.0	1.0	-43.0	-44.0	<b>2.2</b>
$h'_H$	5.0	2.6	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>
$h_L$	1.0	0.5	1.0	1.0	5.0	1.0	1.0	1.0	-41.6	-44.0	<b>5.4</b>
$h'_L$	3.0	1.6	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>
$S_H$	8.0	4.2	1.0	1.0	5.0	1.0	1.0	1.0	-13.5	-44.0	<b>69.4</b>
$S_L$	4.0	2.1	1.0	1.0	5.0	1.0	1.0	1.0	-44.0	-44.0	<b>0.0</b>
$c_H$	4.0	2.1	1.0	1.0	5.0	1.0	1.0	1.0	-40.2	-44.0	<b>8.7</b>
$c_L$	2.0	1.0	1.0	1.0	5.0	1.0	1.0	1.0	-39.2	-44.0	<b>10.8</b>

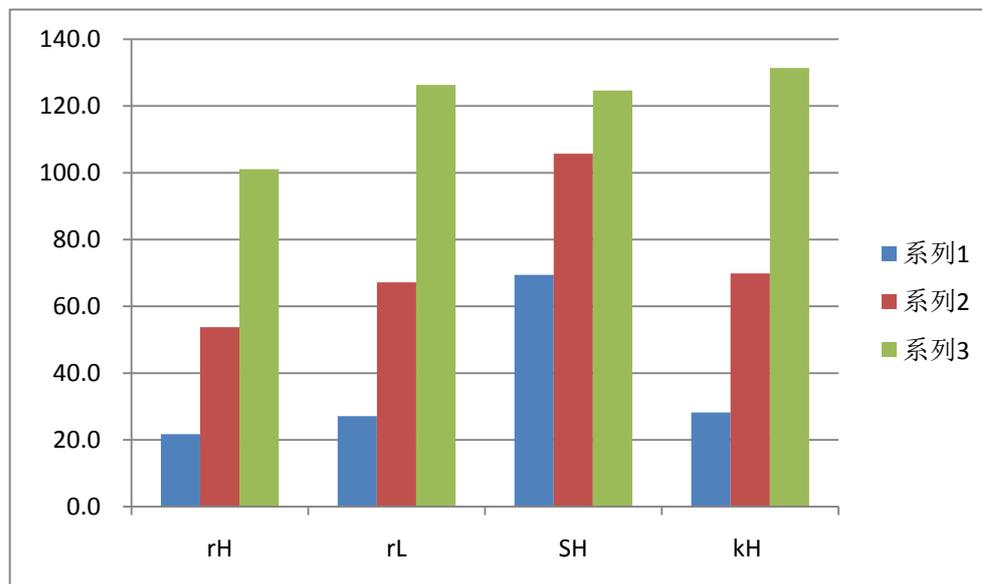
Graph II-I-II



Model 3.1, Table II-I-II-Elite Case I Sensitivity analysis with parameter settings in step II (Dimension I), step I (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,1)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			1.0	1.0	5.0	1.0	1.0	1.0	-44.0		
$r_H$	10.0	14.8	1.0	1.0	5.0	1.0	1.0	1.0	-34.5	-44.0	<b>21.7</b>
		21.8	1.0	1.0	5.0	1.0	1.0	1.0	-20.4	-44.0	<b>53.7</b>
		32.2	1.0	1.0	5.0	1.0	1.0	1.0	0.5	-44.0	<b>101.1</b>
$r_L$	5.0	7.4	1.0	1.0	5.0	1.0	1.0	1.0	-32.1	-44.0	<b>27.1</b>
		10.9	1.0	1.0	5.0	1.0	1.0	1.0	-14.4	-44.0	<b>67.2</b>
		16.1	1.0	1.0	5.0	1.0	1.0	1.0	11.6	-44.0	<b>126.3</b>
$S_H$	8.0	4.2	1.0	1.0	5.0	1.0	1.0	1.0	-13.5	-44.0	<b>69.4</b>
		2.2	1.0	1.0	5.0	1.0	1.0	1.0	2.5	-44.0	<b>105.7</b>
		1.1	1.0	1.0	5.0	1.0	1.0	1.0	10.9	-44.0	<b>124.7</b>
$k^H$	1.0	1.5	1.5	1.5	5.0	1.0	1.0	1.0	-31.6	-44.0	<b>28.2</b>
		2.2	2.2	2.2	5.0	1.0	1.0	1.0	-13.3	-44.0	<b>69.9</b>
		3.2	3.2	3.2	5.0	1.0	1.0	1.0	13.8	-44.0	<b>131.4</b>

Graph II-I-II-Elite

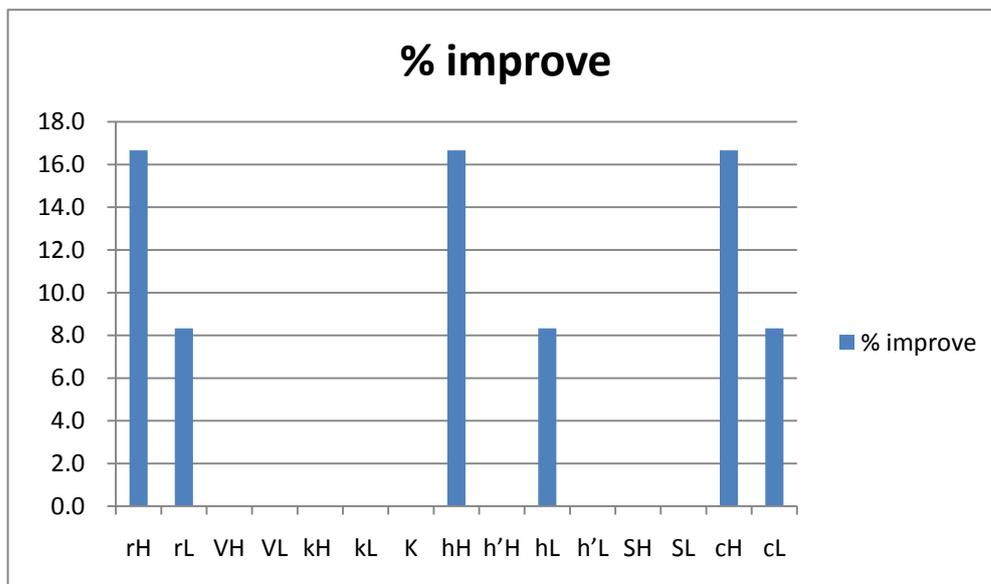


$S_H, r_L, k^H, r_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table II-II-I Case I Sensitivity analysis with parameter settings in step II (Dimension I), step II (Dimension II), step I (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,5)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			5.0	5.0	5.0	1.0	1.0	1.0	60.0			
$r_H$	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>	10.0
$r_L$	5.0	6.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>	20.0
$V_H$	1.0	2.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	100.0
$V_L$	1.0	2.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	100.0
$k^H$	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	10.0
$k^L$	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	10.0
K	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>	100.0
$h'_H$	5.0	4.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>	100.0
$h'_L$	3.0	2.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	12.5
$S_L$	4.0	3.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>	25.0
$c_L$	2.0	1.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>	50.0
average												41.7

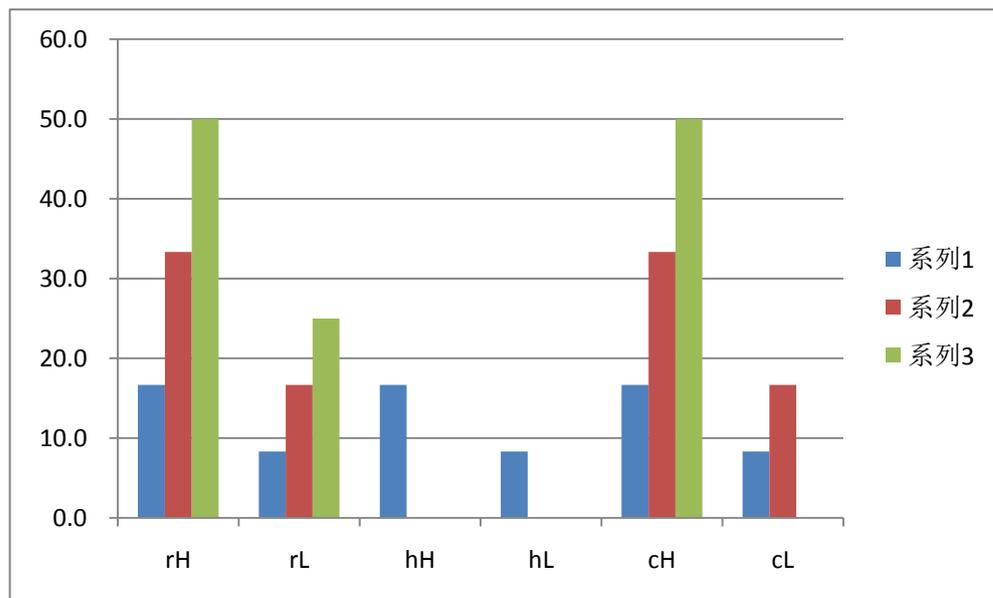
Graph II-II-I



Model 3.1, Table II-II-I-Elite Case I Sensitivity analysis with parameter settings in step II (Dimension I), step II (Dimension II), step I (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,5)$

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			5.0	5.0	5.0	1.0	1.0	1.0	60.0		
$r_H$	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>
		12.0	5.0	5.0	5.0	1.0	1.0	1.0	80.0	60.0	<b>33.3</b>
		13.0	5.0	5.0	5.0	1.0	1.0	1.0	90.0	60.0	<b>50.0</b>
$r_L$	5.0	6.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>
		7.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>
		8.0	5.0	5.0	5.0	1.0	1.0	1.0	75.0	60.0	<b>25.0</b>
$h_H$	1.0	0.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>
$h_L$	1.0	0.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>
$c_H$	4.0	3.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>
		2.0	5.0	5.0	5.0	1.0	1.0	1.0	80.0	60.0	<b>33.3</b>
		1.0	5.0	5.0	5.0	1.0	1.0	1.0	90.0	60.0	<b>50.0</b>
$c_L$	2.0	1.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>
		0.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>

Graph II-II-I-Elite

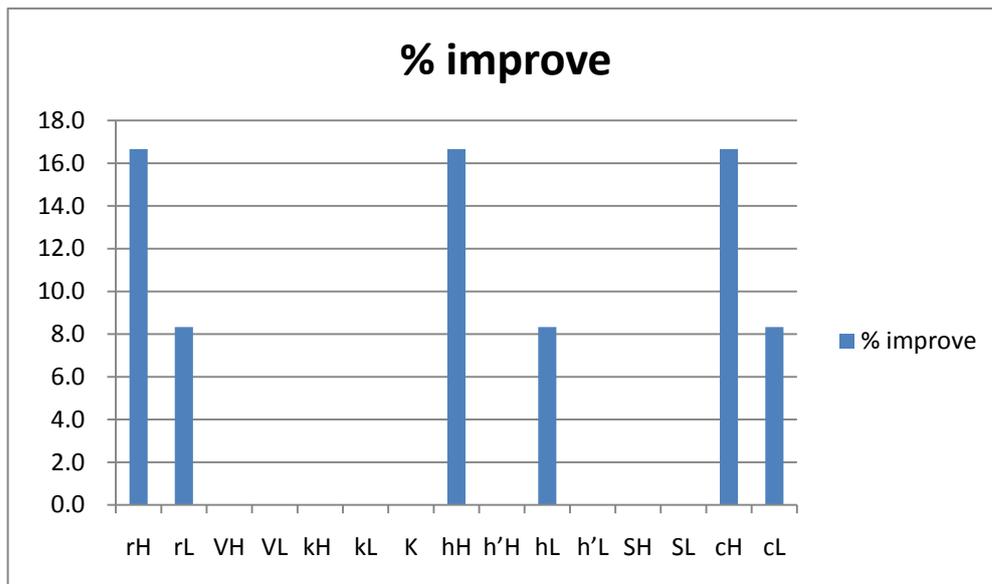


$h_H, r_H, c_H$  are the most important parameters to the retailer,  $r_L$  is the second important one.

Model 3.1, Table II-II-II Case I Sensitivity analysis with parameter settings in step II (Dimension I), step II (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,5)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			5.0	5.0	5.0	1.0	1.0	1.0	60.0		
$r_H$	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	101.7	60.0	<b>69.5</b>
$r_L$	5.0	7.1	5.0	5.0	5.0	1.0	1.0	1.0	70.4	60.0	<b>17.4</b>
$V_H$	1.0	1.4	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$V_L$	1.0	1.4	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$k^H$	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$k^L$	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
K	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$h_H$	1.0	0.6	5.0	5.0	5.0	1.0	1.0	1.0	64.2	60.0	<b>7.0</b>
$h'_H$	5.0	2.9	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$h_L$	1.0	0.6	5.0	5.0	5.0	1.0	1.0	1.0	62.1	60.0	<b>3.5</b>
$h'_L$	3.0	1.7	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$S_H$	8.0	4.7	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$S_L$	4.0	2.3	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$c_H$	4.0	2.3	5.0	5.0	5.0	1.0	1.0	1.0	76.7	60.0	<b>27.8</b>
$c_L$	2.0	1.2	5.0	5.0	5.0	1.0	1.0	1.0	64.2	60.0	<b>7.0</b>

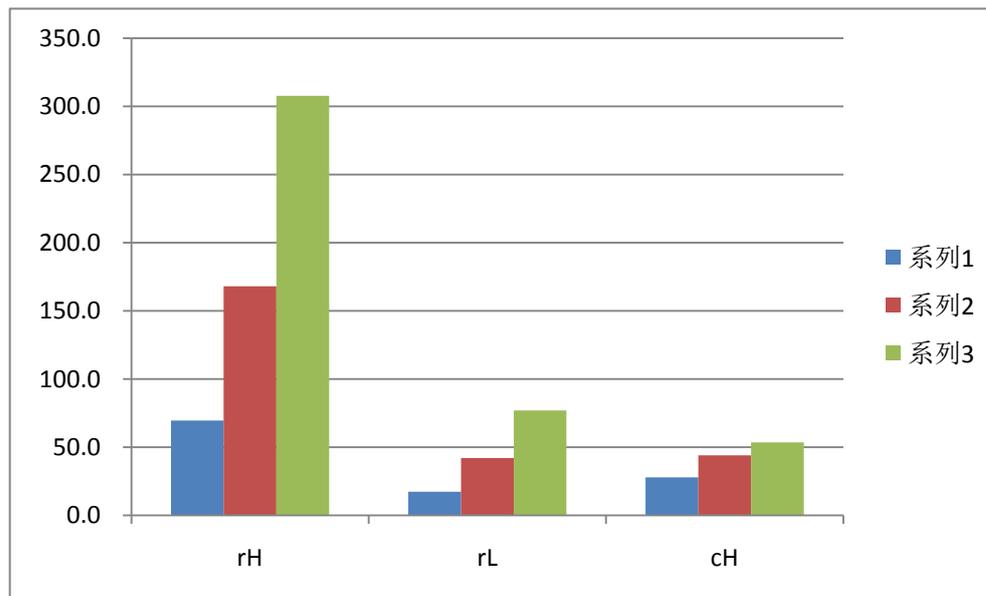
Graph II-II-II



Model 3.1, Table II-II-II-Elite Case I Sensitivity analysis with parameter settings in step II (Dimension I), step II (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,5)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			5.0	5.0	5.0	1.0	1.0	1.0	60.0		
$r_H$	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	101.7	60.0	<b>69.5</b>
		20.1	5.0	5.0	5.0	1.0	1.0	1.0	160.9	60.0	<b>168.1</b>
		28.5	5.0	5.0	5.0	1.0	1.0	1.0	244.7	60.0	<b>307.8</b>
$r_L$	5.0	7.1	5.0	5.0	5.0	1.0	1.0	1.0	70.4	60.0	<b>17.4</b>
		10.0	5.0	5.0	5.0	1.0	1.0	1.0	85.2	60.0	<b>42.0</b>
		14.2	5.0	5.0	5.0	1.0	1.0	1.0	106.2	60.0	<b>76.9</b>
$c_H$	4.0	2.3	5.0	5.0	5.0	1.0	1.0	1.0	76.7	60.0	<b>27.8</b>
		1.4	5.0	5.0	5.0	1.0	1.0	1.0	86.4	60.0	<b>44.0</b>
		0.8	5.0	5.0	5.0	1.0	1.0	1.0	92.1	60.0	<b>53.5</b>

Graph II-II-II-Elite

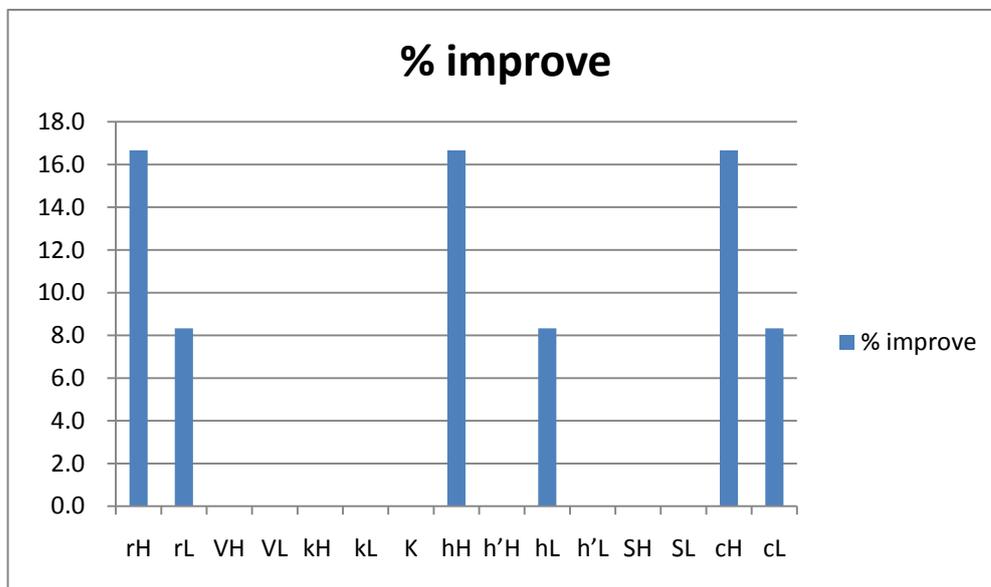


$r_H, c_H, r_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table II-III-I Case I Sensitivity analysis with parameter settings in step II (Dimension I), step III (Dimension II), step I (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,10)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			5.0	5.0	5.0	1.0	1.0	1.0	60.0			
$r_H$	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>	10.0
$r_L$	5.0	6.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>	20.0
$V_H$	1.0	2.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	100.0
$V_L$	1.0	2.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	100.0
$k^H$	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	10.0
$k^L$	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	10.0
K	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>	100.0
$h'_H$	5.0	4.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>	100.0
$h'_L$	3.0	2.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	12.5
$S_L$	4.0	3.0	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>	25.0
$c_L$	2.0	1.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>	50.0
average												41.7

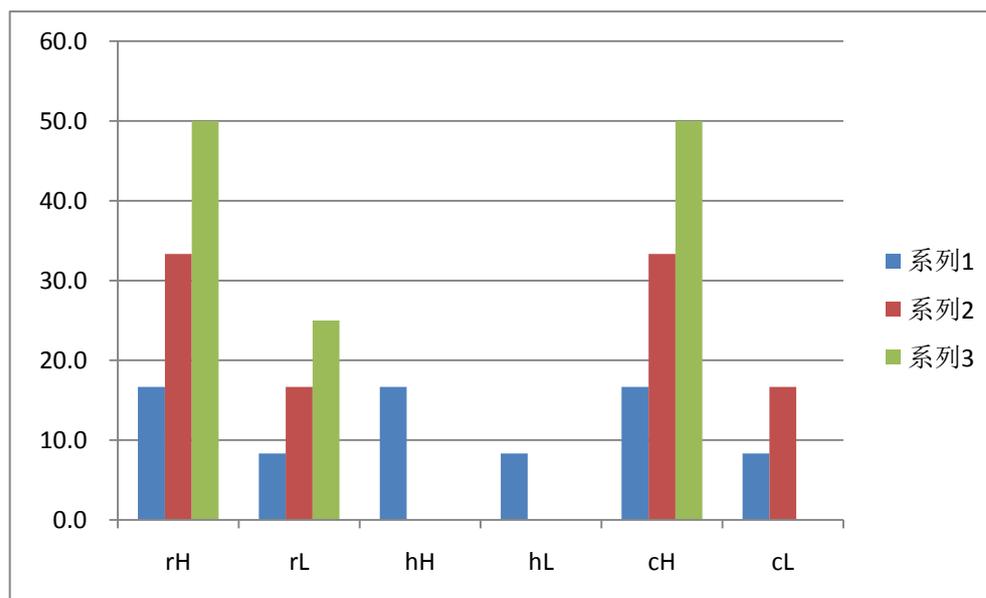
Graph II-III-I



Model 3.1, Table II-III-I-Elite Case I Sensitivity analysis with parameter settings in step II (Dimension I), step III (Dimension II), step I (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,10)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			5.0	5.0	5.0	1.0	1.0	1.0	60.0		
$r_H$	10.0	11.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>
		12.0	5.0	5.0	5.0	1.0	1.0	1.0	80.0	60.0	<b>33.3</b>
		13.0	5.0	5.0	5.0	1.0	1.0	1.0	90.0	60.0	<b>50.0</b>
$r_L$	5.0	6.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>
		7.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>
		8.0	5.0	5.0	5.0	1.0	1.0	1.0	75.0	60.0	<b>25.0</b>
$h_H$	1.0	0.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>
$h_L$	1.0	0.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>
$c_H$	4.0	3.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>
		2.0	5.0	5.0	5.0	1.0	1.0	1.0	80.0	60.0	<b>33.3</b>
		1.0	5.0	5.0	5.0	1.0	1.0	1.0	90.0	60.0	<b>50.0</b>
$c_L$	2.0	1.0	5.0	5.0	5.0	1.0	1.0	1.0	65.0	60.0	<b>8.3</b>
		0.0	5.0	5.0	5.0	1.0	1.0	1.0	70.0	60.0	<b>16.7</b>

Graph II-III-I-Elite

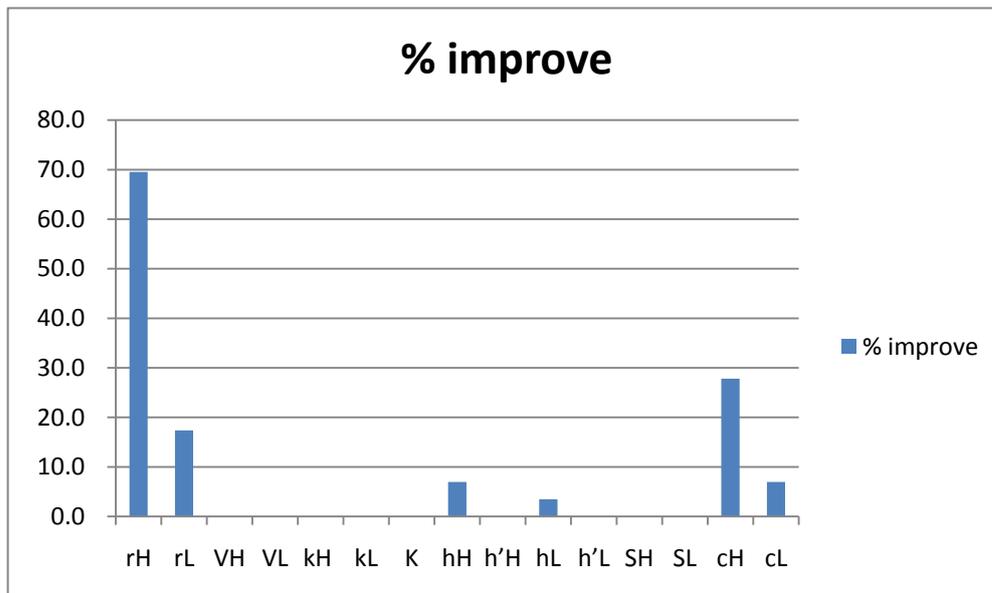


$h_H, r_H, c_H$  are the most important parameters to the retailer while  $r_L, c_L, h_L$  are the second most important parameters.

Model 3.1, Table II-III-II Case I Sensitivity analysis with parameter settings in step II (Dimension I), step III (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,10)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			5.0	5.0	5.0	1.0	1.0	1.0	60.0		
$r_H$	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	101.7	60.0	<b>69.5</b>
$r_L$	5.0	7.1	5.0	5.0	5.0	1.0	1.0	1.0	70.4	60.0	<b>17.4</b>
$V_H$	1.0	1.4	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$V_L$	1.0	1.4	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$k^H$	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$k^L$	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
K	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$h_H$	1.0	0.6	5.0	5.0	5.0	1.0	1.0	1.0	64.2	60.0	<b>7.0</b>
$h'_H$	5.0	2.9	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$h_L$	1.0	0.6	5.0	5.0	5.0	1.0	1.0	1.0	62.1	60.0	<b>3.5</b>
$h'_L$	3.0	1.7	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$S_H$	8.0	4.7	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$S_L$	4.0	2.3	5.0	5.0	5.0	1.0	1.0	1.0	60.0	60.0	<b>0.0</b>
$c_H$	4.0	2.3	5.0	5.0	5.0	1.0	1.0	1.0	76.7	60.0	<b>27.8</b>
$c_L$	2.0	1.2	5.0	5.0	5.0	1.0	1.0	1.0	64.2	60.0	<b>7.0</b>

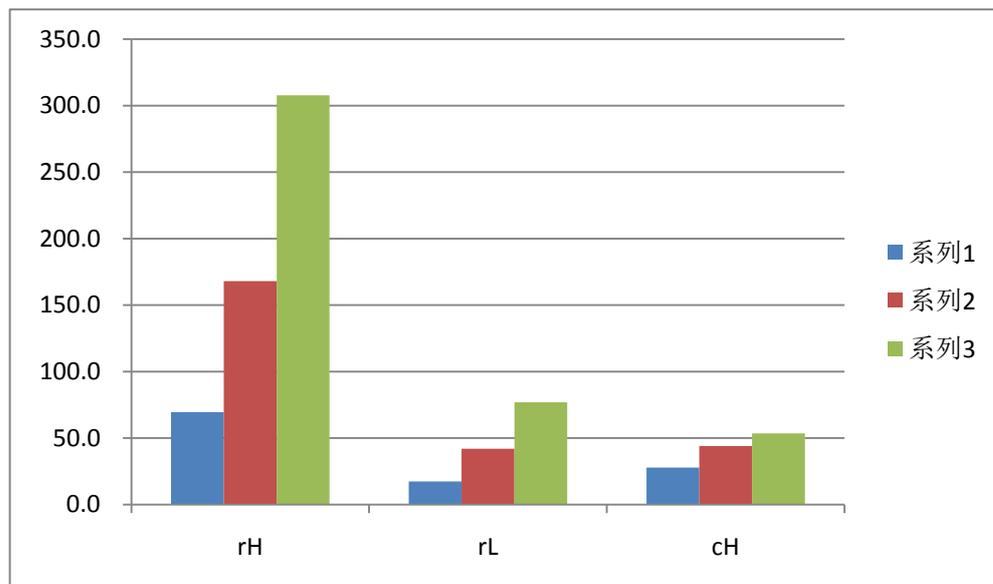
Graph II-III-II



Model 3.1, Table II-III-II-Elite Case I Sensitivity analysis with parameter settings in step II (Dimension I), step III (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(5,5,5,10)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			5.0	5.0	5.0	1.0	1.0	1.0	60.0		
$r_H$	10.0	14.2	5.0	5.0	5.0	1.0	1.0	1.0	101.7	60.0	<b>69.5</b>
		20.1	5.0	5.0	5.0	1.0	1.0	1.0	160.9	60.0	<b>168.1</b>
		28.5	5.0	5.0	5.0	1.0	1.0	1.0	244.7	60.0	<b>307.8</b>
$r_L$	5.0	7.1	5.0	5.0	5.0	1.0	1.0	1.0	70.4	60.0	<b>17.4</b>
		10.0	5.0	5.0	5.0	1.0	1.0	1.0	85.2	60.0	<b>42.0</b>
		14.2	5.0	5.0	5.0	1.0	1.0	1.0	106.2	60.0	<b>76.9</b>
$c_H$	4.0	2.3	5.0	5.0	5.0	1.0	1.0	1.0	76.7	60.0	<b>27.8</b>
		1.4	5.0	5.0	5.0	1.0	1.0	1.0	86.4	60.0	<b>44.0</b>
		0.8	5.0	5.0	5.0	1.0	1.0	1.0	92.1	60.0	<b>53.5</b>

Graph II-III-II-Elite

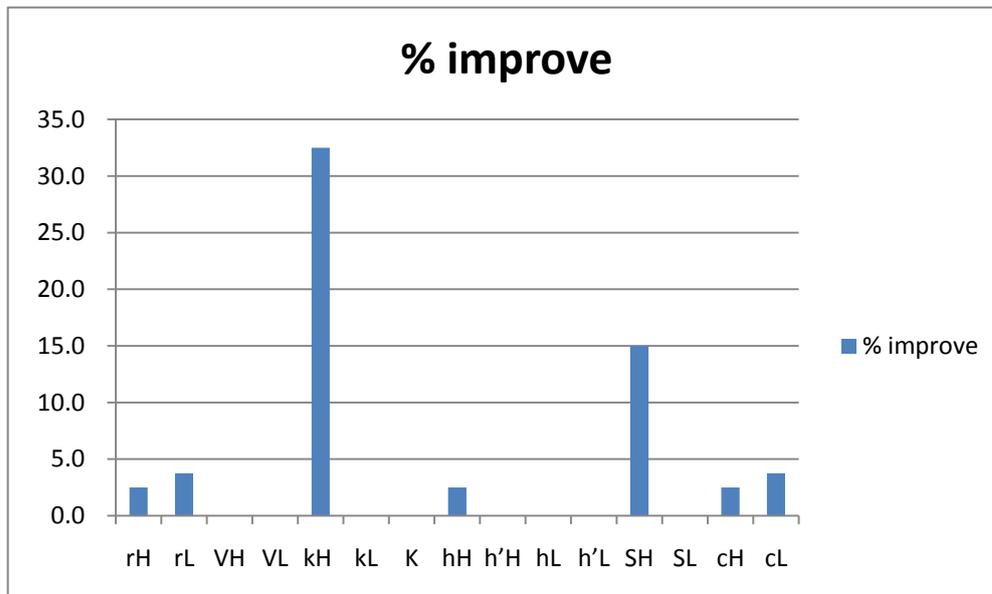


$r_H$ ,  $c_H$ ,  $r_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table III-I-I Case I Sensitivity analysis with parameter settings in step III (Dimension I), step I (Dimension II), step I (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			1.0	1.0	3.0	1.0	1.0	1.0	-80.0			
$r_H$	10.0	11.0	1.0	1.0	3.0	1.0	1.0	1.0	-78.0	-80.0	<b>2.5</b>	10.0
$r_L$	5.0	6.0	1.0	1.0	3.0	1.0	1.0	1.0	-77.0	-80.0	<b>3.8</b>	20.0
$V_H$	1.0	2.0	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>	100.0
$V_L$	1.0	2.0	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>	100.0
$k^H$	1.0	2.0	2.0	2.0	3.0	1.0	1.0	1.0	-54.0	-80.0	<b>32.5</b>	100.0
$k^L$	10.0	11.0	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>	10.0
K	10.0	11.0	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	1.0	1.0	3.0	1.0	1.0	1.0	-78.0	-80.0	<b>2.5</b>	100.0
$h'_H$	5.0	4.0	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>	100.0
$h'_L$	3.0	2.0	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	1.0	1.0	3.0	1.0	1.0	1.0	-68.0	-80.0	<b>15.0</b>	12.5
$S_L$	4.0	3.0	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	1.0	1.0	3.0	1.0	1.0	1.0	-78.0	-80.0	<b>2.5</b>	25.0
$c_L$	2.0	1.0	1.0	1.0	3.0	1.0	1.0	1.0	-77.0	-80.0	<b>3.8</b>	50.0
average												47.7

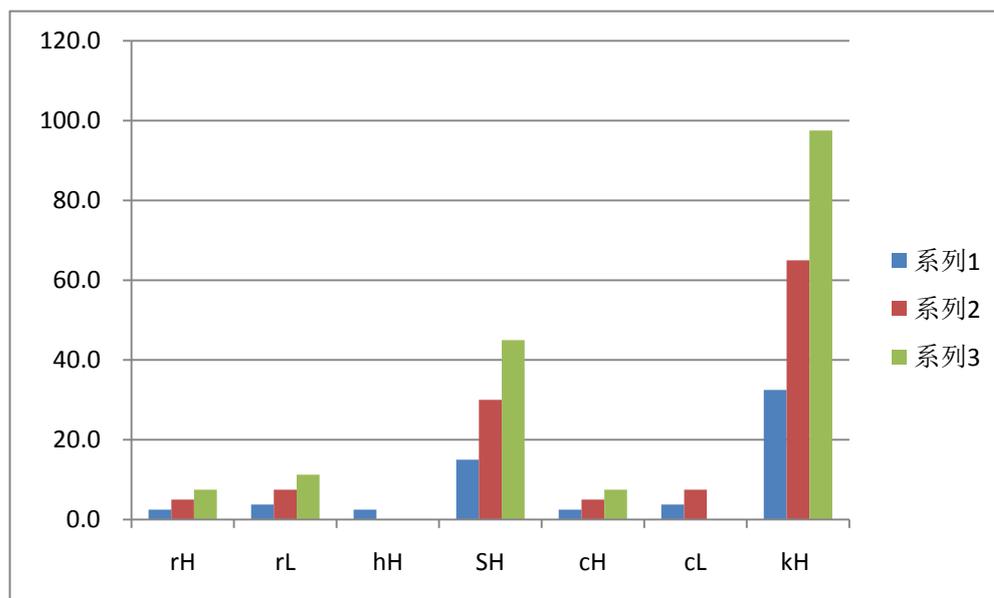
Graph III-I-I



Model 3.1, Table III-I-I-Elite Case I Sensitivity analysis with parameter settings in step III (Dimension I), step I (Dimension II), step I (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			1.0	1.0	3.0	1.0	1.0	1.0	-80.0		
$r_H$	10.0	11.0	1.0	1.0	3.0	1.0	1.0	1.0	-78.0	-80.0	<b>2.5</b>
		12.0	1.0	1.0	3.0	1.0	1.0	1.0	-76.0	-80.0	<b>5.0</b>
		13.0	1.0	1.0	3.0	1.0	1.0	1.0	-74.0	-80.0	<b>7.5</b>
$r_L$	5.0	6.0	1.0	1.0	3.0	1.0	1.0	1.0	-77.0	-80.0	<b>3.8</b>
		7.0	1.0	1.0	3.0	1.0	1.0	1.0	-74.0	-80.0	<b>7.5</b>
		8.0	1.0	1.0	3.0	1.0	1.0	1.0	-71.0	-80.0	<b>11.3</b>
$h_H$	1.0	0.0	1.0	1.0	3.0	1.0	1.0	1.0	-78.0	-80.0	<b>2.5</b>
$S_H$	8.0	7.0	1.0	1.0	3.0	1.0	1.0	1.0	-68.0	-80.0	<b>15.0</b>
		6.0	1.0	1.0	3.0	1.0	1.0	1.0	-56.0	-80.0	<b>30.0</b>
		5.0	1.0	1.0	3.0	1.0	1.0	1.0	-44.0	-80.0	<b>45.0</b>
$c_H$	4.0	3.0	1.0	1.0	3.0	1.0	1.0	1.0	-78.0	-80.0	<b>2.5</b>
		2.0	1.0	1.0	3.0	1.0	1.0	1.0	-76.0	-80.0	<b>5.0</b>
		1.0	1.0	1.0	3.0	1.0	1.0	1.0	-74.0	-80.0	<b>7.5</b>
$c_L$	2.0	1.0	1.0	1.0	3.0	1.0	1.0	1.0	-77.0	-80.0	<b>3.8</b>
		0.0	1.0	1.0	3.0	1.0	1.0	1.0	-74.0	-80.0	<b>7.5</b>
		$k^H$	1.0	2.0	1.0	1.0	3.0	1.0	1.0	1.0	-54.0
3.0	1.0	1.0		3.0	1.0	1.0	1.0	-28.0	-80.0	<b>65.0</b>	
4.0	1.0	1.0		3.0	1.0	1.0	1.0	-2.0	-80.0	<b>97.5</b>	

Graph III-I-I-Elite

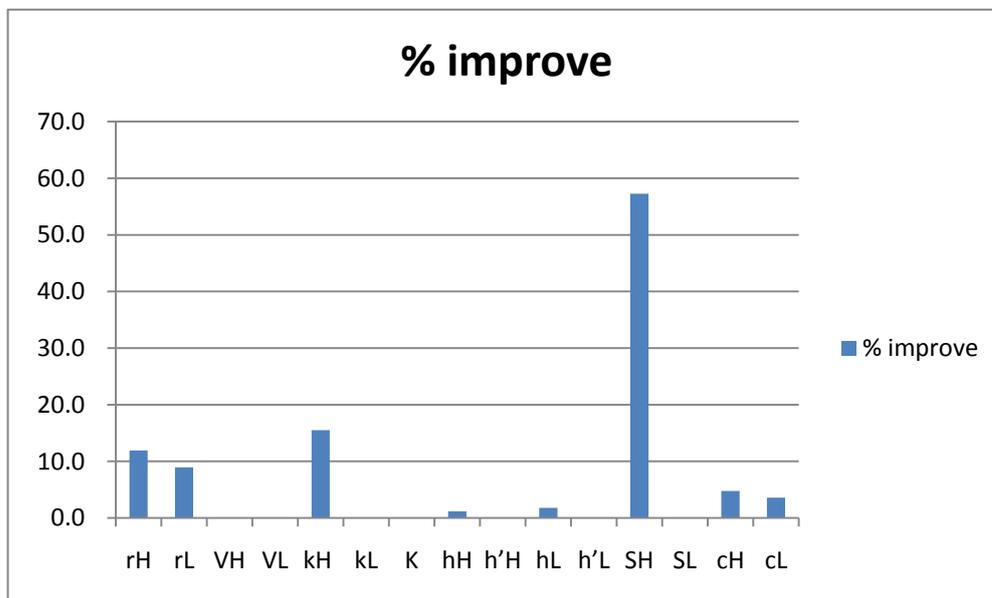


$k^H$  is the most important parameter to the retailer, followed by  $S_H, r_L$ .

Model 3.1, Table III-I-II Case I Sensitivity analysis with parameter settings in step III (Dimension I), step I (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			1.0	1.0	3.0	1.0	1.0	1.0	-80.0		
$r_H$	10.0	14.8	1.0	1.0	3.0	1.0	1.0	1.0	-70.5	-80.0	<b>11.9</b>
$r_L$	5.0	7.4	1.0	1.0	3.0	1.0	1.0	1.0	-72.8	-80.0	<b>8.9</b>
$V_H$	1.0	1.5	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>
$V_L$	1.0	1.5	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>
$k^H$	1.0	1.5	1.5	1.5	3.0	1.0	1.0	1.0	-67.6	-80.0	<b>15.5</b>
$k^L$	10.0	14.8	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>
K	10.0	14.8	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>
$h_H$	1.0	0.5	1.0	1.0	3.0	1.0	1.0	1.0	-79.0	-80.0	<b>1.2</b>
$h'_H$	5.0	2.6	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>
$h_L$	1.0	0.5	1.0	1.0	3.0	1.0	1.0	1.0	-78.6	-80.0	<b>1.8</b>
$h'_L$	3.0	1.6	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>
$S_H$	8.0	4.2	1.0	1.0	3.0	1.0	1.0	1.0	-34.2	-80.0	<b>57.3</b>
$S_L$	4.0	2.1	1.0	1.0	3.0	1.0	1.0	1.0	-80.0	-80.0	<b>0.0</b>
$c_H$	4.0	2.1	1.0	1.0	3.0	1.0	1.0	1.0	-76.2	-80.0	<b>4.8</b>
$c_L$	2.0	1.0	1.0	1.0	3.0	1.0	1.0	1.0	-77.1	-80.0	<b>3.6</b>

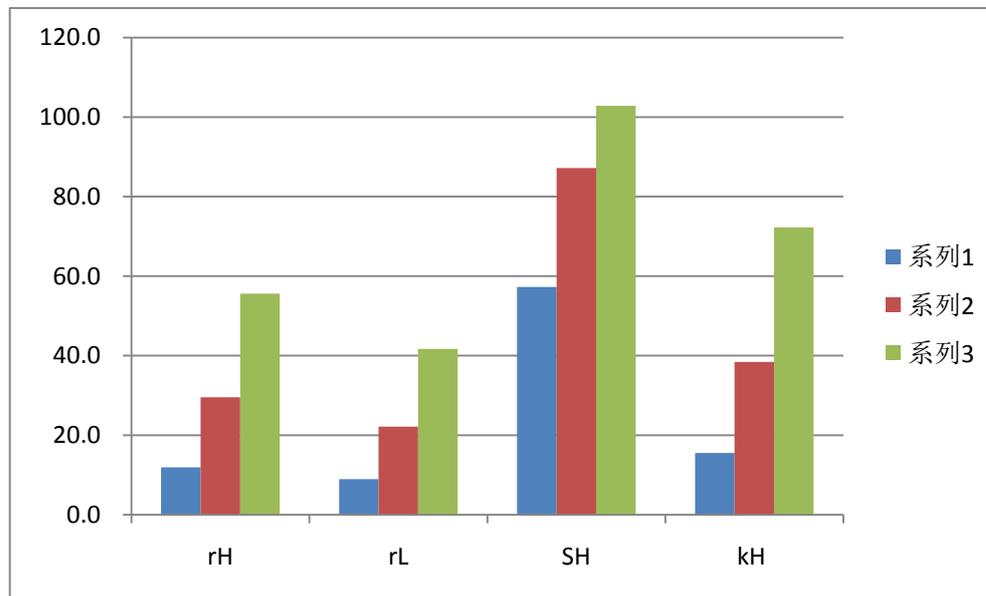
Graph III-I-II



Model 3.1, Table III-I-II-Elite Case I Sensitivity analysis with parameter settings in step III (Dimension I), step I (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			1.0	1.0	3.0	1.0	1.0	1.0	-80.0		
$r_H$	10.0	14.8	1.0	1.0	3.0	1.0	1.0	1.0	-70.5	-80.0	<b>11.9</b>
		21.8	1.0	1.0	3.0	1.0	1.0	1.0	-56.4	-80.0	<b>29.6</b>
		32.2	1.0	1.0	3.0	1.0	1.0	1.0	-35.5	-80.0	<b>55.6</b>
$r_L$	5.0	7.4	1.0	1.0	3.0	1.0	1.0	1.0	-72.8	-80.0	<b>8.9</b>
		10.9	1.0	1.0	3.0	1.0	1.0	1.0	-62.3	-80.0	<b>22.2</b>
		16.1	1.0	1.0	3.0	1.0	1.0	1.0	-46.6	-80.0	<b>41.7</b>
$S_H$	8.0	4.2	1.0	1.0	3.0	1.0	1.0	1.0	-34.2	-80.0	<b>57.3</b>
		2.2	1.0	1.0	3.0	1.0	1.0	1.0	-10.2	-80.0	<b>87.2</b>
		1.1	1.0	1.0	3.0	1.0	1.0	1.0	2.3	-80.0	<b>102.9</b>
$k^H$	1.0	1.5	1.5	1.5	3.0	1.0	1.0	1.0	-67.6	-80.0	<b>15.5</b>
		2.2	2.2	2.2	3.0	1.0	1.0	1.0	-49.3	-80.0	<b>38.4</b>
		3.2	3.2	3.2	3.0	1.0	1.0	1.0	-22.2	-80.0	<b>72.3</b>

Graph III-I-II-Elite

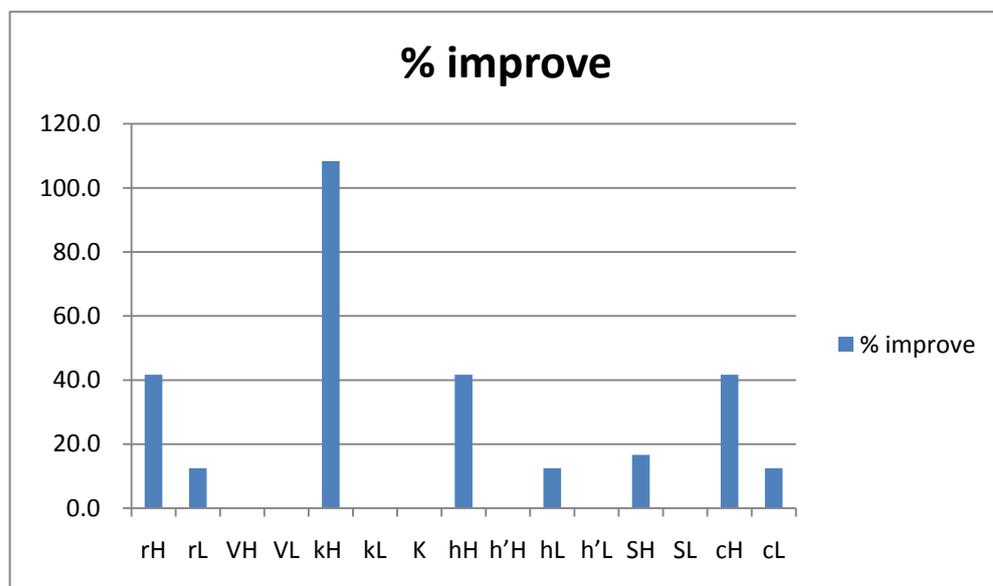


$S_H, k^H, r_H, r_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table III-II-I Case I Sensitivity analysis with parameter settings in step III (Dimension I), step II (Dimension II), step I (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,5)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			5.0	5.0	3.0	1.0	1.0	1.0	24.0			
$r_H$	10.0	11.0	5.0	5.0	3.0	1.0	1.0	1.0	34.0	24.0	<b>41.7</b>	10.0
$r_L$	5.0	6.0	5.0	5.0	3.0	1.0	1.0	1.0	27.0	24.0	<b>12.5</b>	20.0
$V_H$	1.0	2.0	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>	100.0
$V_L$	1.0	2.0	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>	100.0
$k^H$	5.0	6.0	6.0	6.0	3.0	1.0	1.0	1.0	50.0	24.0	<b>108.3</b>	20.0
$k^L$	10.0	11.0	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>	10.0
K	10.0	11.0	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	5.0	5.0	3.0	1.0	1.0	1.0	34.0	24.0	<b>41.7</b>	100.0
$h'_H$	5.0	4.0	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	5.0	5.0	3.0	1.0	1.0	1.0	27.0	24.0	<b>12.5</b>	100.0
$h'_L$	3.0	2.0	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	5.0	5.0	3.0	1.0	1.0	1.0	28.0	24.0	<b>16.7</b>	12.5
$S_L$	4.0	3.0	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	5.0	5.0	3.0	1.0	1.0	1.0	34.0	24.0	<b>41.7</b>	25.0
$c_L$	2.0	1.0	5.0	5.0	3.0	1.0	1.0	1.0	27.0	24.0	<b>12.5</b>	50.0
average												42.4

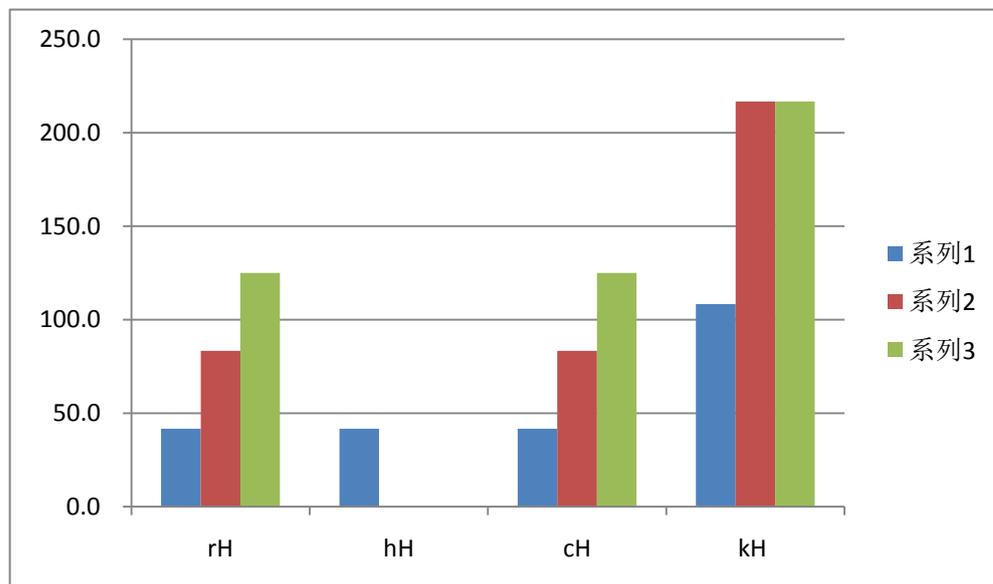
Graph III-II-I



Model 3.1, Table III-II-I-Elite Case I Sensitivity analysis with parameter settings in step III (Dimension I), step II (Dimension II), step I (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,5)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			5.0	5.0	3.0	1.0	1.0	1.0	24.0		
$r_H$	10.0	11.0	5.0	5.0	3.0	1.0	1.0	1.0	34.0	24.0	<b>41.7</b>
		12.0	5.0	5.0	3.0	1.0	1.0	1.0	44.0	24.0	<b>83.3</b>
		13.0	5.0	5.0	3.0	1.0	1.0	1.0	54.0	24.0	<b>125.0</b>
$h_H$	1.0	0.0	5.0	5.0	3.0	1.0	1.0	1.0	34.0	24.0	<b>41.7</b>
$c_H$	4.0	3.0	5.0	5.0	3.0	1.0	1.0	1.0	34.0	24.0	<b>41.7</b>
		2.0	5.0	5.0	3.0	1.0	1.0	1.0	44.0	24.0	<b>83.3</b>
		1.0	5.0	5.0	3.0	1.0	1.0	1.0	54.0	24.0	<b>125.0</b>
$k^H$	5.0	6.0	5.0	5.0	3.0	1.0	1.0	1.0	50.0	24.0	<b>108.3</b>
		7.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	24.0	<b>216.7</b>
		8.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	24.0	<b>216.7</b>

Graph III-II-I-Elite

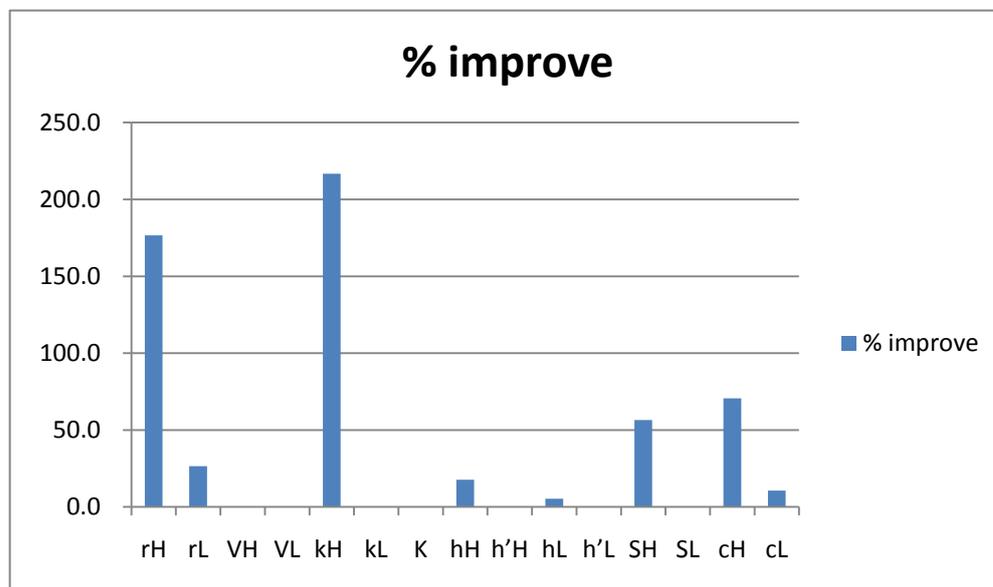


$k^H$  is the most important parameter to the retailer as long as it is less than the demand, followed by  $r_H$ ,  $c_H$ ,  $h_H$ .

Model 3.1, Table III-II-II Case I Sensitivity analysis with parameter settings in step III (Dimension I), step II (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,5)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			5.0	5.0	3.0	1.0	1.0	1.0	24.0		
$r_H$	10.0	14.2	5.0	5.0	3.0	1.0	1.0	1.0	66.4	24.0	<b>176.6</b>
$r_L$	5.0	7.1	5.0	5.0	3.0	1.0	1.0	1.0	30.4	24.0	<b>26.5</b>
$V_H$	1.0	1.4	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>
$V_L$	1.0	1.4	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>
$k^H$	5.0	7.1	7.0	7.0	3.0	1.0	1.0	1.0	76.0	24.0	<b>216.7</b>
$k^L$	10.0	14.2	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>
K	10.0	14.2	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>
$h_H$	1.0	0.6	5.0	5.0	3.0	1.0	1.0	1.0	28.2	24.0	<b>17.7</b>
$h'_H$	5.0	2.9	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>
$h_L$	1.0	0.6	5.0	5.0	3.0	1.0	1.0	1.0	25.3	24.0	<b>5.3</b>
$h'_L$	3.0	1.7	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>
$S_H$	8.0	4.6	5.0	5.0	3.0	1.0	1.0	1.0	37.6	24.0	<b>56.5</b>
$S_L$	4.0	2.3	5.0	5.0	3.0	1.0	1.0	1.0	24.0	24.0	<b>0.0</b>
$c_H$	4.0	2.3	5.0	5.0	3.0	1.0	1.0	1.0	41.0	24.0	<b>70.6</b>
$c_L$	2.0	1.2	5.0	5.0	3.0	1.0	1.0	1.0	26.5	24.0	<b>10.6</b>

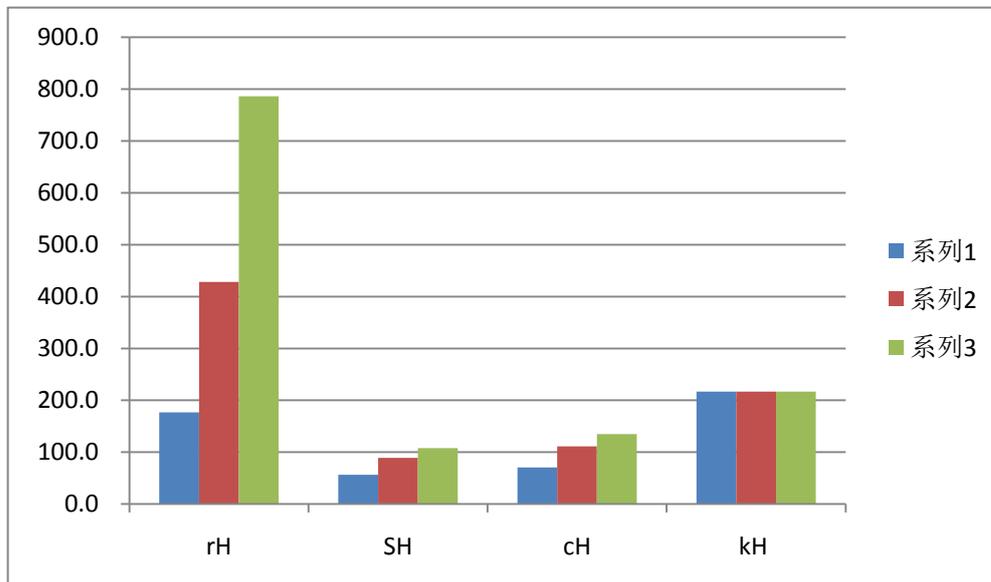
Graph III-II-II



Model 3.1, Table III-II-II-Elite Case I Sensitivity analysis with parameter settings in step III (Dimension I), step II (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,5)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			5.0	5.0	3.0	1.0	1.0	1.0	24.0		
$r_H$	10.0	14.2	5.0	5.0	3.0	1.0	1.0	1.0	66.4	24.0	<b>176.6</b>
		20.3	5.0	5.0	3.0	1.0	1.0	1.0	126.7	24.0	<b>428.1</b>
		28.9	5.0	5.0	3.0	1.0	1.0	1.0	212.7	24.0	<b>786.2</b>
$S_H$	8.0	4.6	5.0	5.0	3.0	1.0	1.0	1.0	37.6	24.0	<b>56.5</b>
		2.7	5.0	5.0	3.0	1.0	1.0	1.0	45.4	24.0	<b>89.1</b>
		1.5	5.0	5.0	3.0	1.0	1.0	1.0	49.9	24.0	<b>107.8</b>
$c_H$	4.0	2.3	5.0	5.0	3.0	1.0	1.0	1.0	41.0	24.0	<b>70.6</b>
		1.3	5.0	5.0	3.0	1.0	1.0	1.0	50.7	24.0	<b>111.3</b>
		0.8	5.0	5.0	3.0	1.0	1.0	1.0	56.4	24.0	<b>134.8</b>
$k^H$	5.0	7.1	7.0	7.0	3.0	1.0	1.0	1.0	76.0	24.0	<b>216.7</b>
		10.1	7.0	7.0	3.0	1.0	1.0	1.0	76.0	24.0	<b>216.7</b>
		14.4	7.0	7.0	3.0	1.0	1.0	1.0	76.0	24.0	<b>216.7</b>

Graph III-II-II-Elite

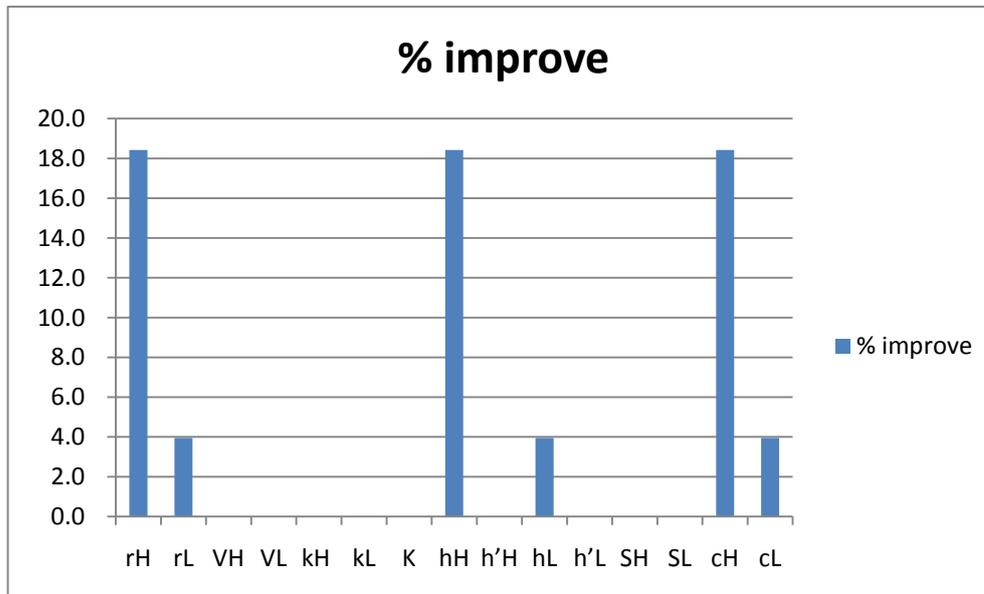


$r_L, k^H, c_H, S_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table III-III-I Case I Sensitivity analysis with parameter settings in step III (Dimension I), step III (Dimension II), step I (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,10)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			7.0	7.0	3.0	1.0	1.0	1.0	76.0			
$r_H$	10.0	11.0	7.0	7.0	3.0	1.0	1.0	1.0	90.0	76.0	<b>18.4</b>	10.0
$r_L$	5.0	6.0	7.0	7.0	3.0	1.0	1.0	1.0	79.0	76.0	<b>3.9</b>	20.0
$V_H$	1.0	2.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	100.0
$V_L$	1.0	2.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	100.0
$k^H$	10.0	11.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	10.0
$k^L$	10.0	11.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	10.0
K	10.0	11.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	10.0
$h_H$	1.0	0.0	7.0	7.0	3.0	1.0	1.0	1.0	90.0	76.0	<b>18.4</b>	100.0
$h'_H$	5.0	4.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	20.0
$h_L$	1.0	0.0	7.0	7.0	3.0	1.0	1.0	1.0	79.0	76.0	<b>3.9</b>	100.0
$h'_L$	3.0	2.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	33.3
$S_H$	8.0	7.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	12.5
$S_L$	4.0	3.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>	25.0
$c_H$	4.0	3.0	7.0	7.0	3.0	1.0	1.0	1.0	90.0	76.0	<b>18.4</b>	25.0
$c_L$	2.0	1.0	7.0	7.0	3.0	1.0	1.0	1.0	79.0	76.0	<b>3.9</b>	50.0
average												41.7

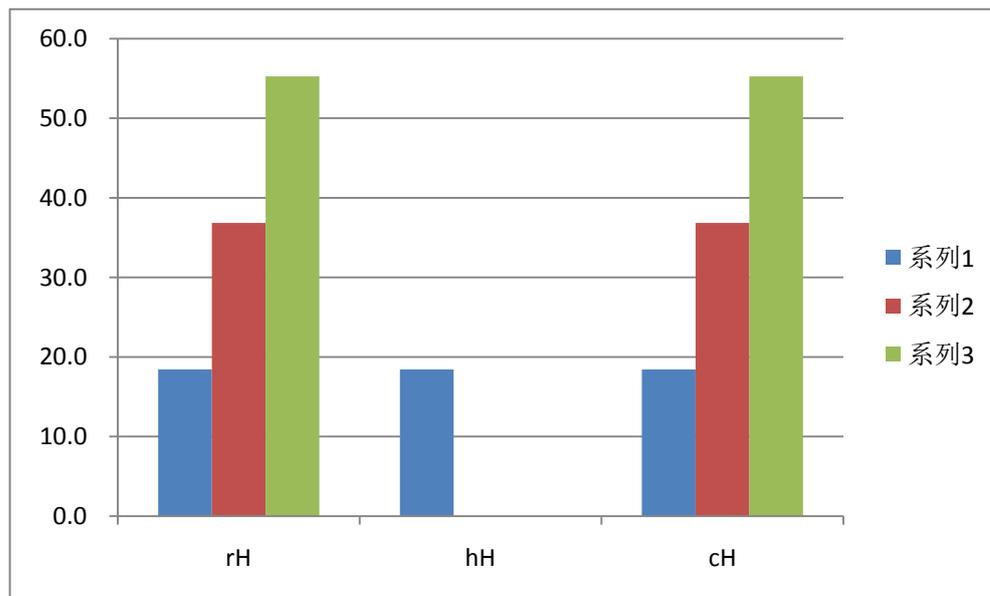
Graph III-III-I



Model 3.1, Table III-III-I-Elite Case I Sensitivity analysis with parameter settings in step III (Dimension I), step III (Dimension II), step I (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,10)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			7.0	7.0	3.0	1.0	1.0	1.0	76.0		
$r_H$	10.0	11.0	7.0	7.0	3.0	1.0	1.0	1.0	90.0	76.0	<b>18.4</b>
		12.0	7.0	7.0	3.0	1.0	1.0	1.0	104.0	76.0	<b>36.8</b>
		13.0	7.0	7.0	3.0	1.0	1.0	1.0	118.0	76.0	<b>55.3</b>
$h_H$	1.0	0.0	7.0	7.0	3.0	1.0	1.0	1.0	90.0	76.0	<b>18.4</b>
$c_H$	4.0	3.0	7.0	7.0	3.0	1.0	1.0	1.0	90.0	76.0	<b>18.4</b>
		2.0	7.0	7.0	3.0	1.0	1.0	1.0	104.0	76.0	<b>36.8</b>
		1.0	7.0	7.0	3.0	1.0	1.0	1.0	118.0	76.0	<b>55.3</b>

Graph III-III-I-Elite

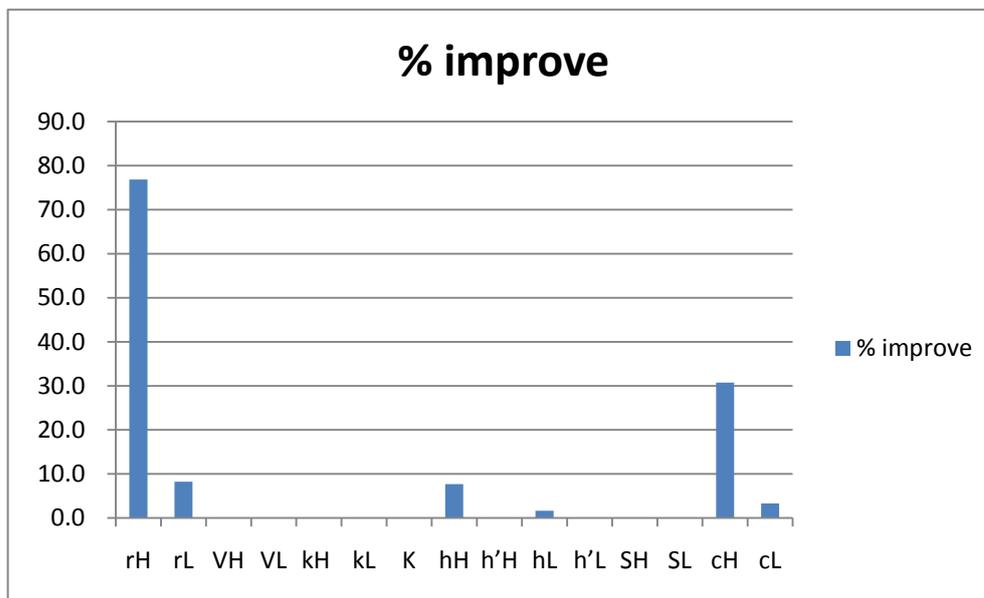


$r_H, h_H, c_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table III-III-II Case I Sensitivity analysis with parameter settings in step III (Dimension I), step III (Dimension II), step II (Dimension III) where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,10)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			7.0	7.0	3.0	1.0	1.0	1.0	76.0		
$r_H$	10.0	14.2	7.0	7.0	3.0	1.0	1.0	1.0	134.4	76.0	<b>76.9</b>
$r_L$	5.0	7.1	7.0	7.0	3.0	1.0	1.0	1.0	82.3	76.0	<b>8.2</b>
$V_H$	1.0	1.4	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
$V_L$	1.0	1.4	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
$k^H$	10.0	14.2	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
$k^L$	10.0	14.2	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
K	10.0	14.2	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
$h_H$	1.0	0.6	7.0	7.0	3.0	1.0	1.0	1.0	81.8	76.0	<b>7.7</b>
$h'_H$	5.0	2.9	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
$h_L$	1.0	0.6	7.0	7.0	3.0	1.0	1.0	1.0	77.3	76.0	<b>1.6</b>
$h'_L$	3.0	1.7	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
$S_H$	8.0	4.7	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
$S_L$	4.0	2.3	7.0	7.0	3.0	1.0	1.0	1.0	76.0	76.0	<b>0.0</b>
$c_H$	4.0	2.3	7.0	7.0	3.0	1.0	1.0	1.0	99.4	76.0	<b>30.7</b>
$c_L$	2.0	1.2	7.0	7.0	3.0	1.0	1.0	1.0	78.5	76.0	<b>3.3</b>

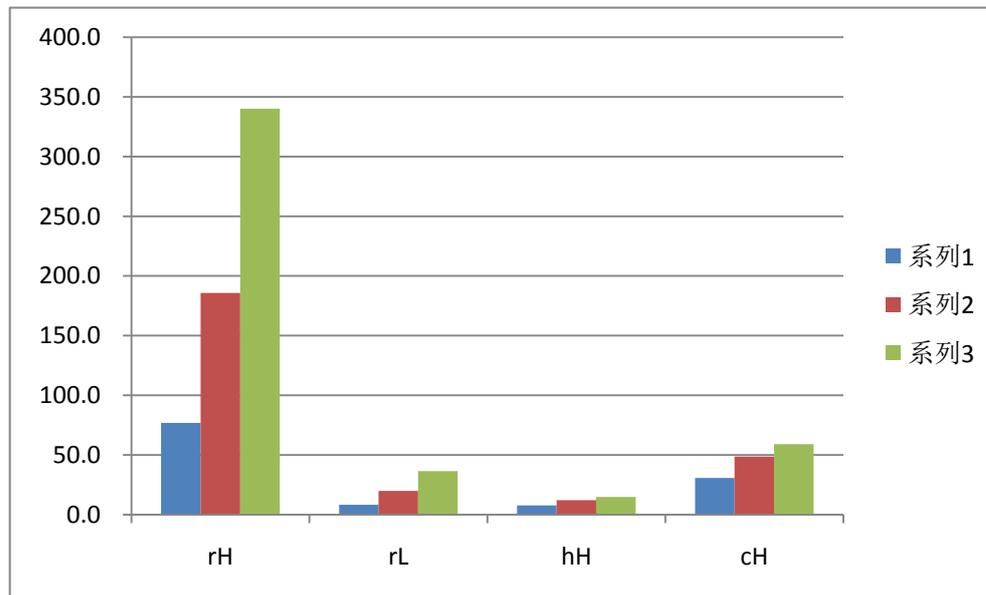
Graph III-III-II



Model 3.1, Table III-III-II-Elite Case I Sensitivity analysis with parameter settings in step III (Dimension I), step III (Dimension II), step II (Dimension III), Dimension IV where  $(D_1^H, D_2^H, D_2^L, k^H)=(7,7,3,10)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	%improve
original			7.0	7.0	3.0	1.0	1.0	1.0	76.0		
$r_H$	10.0	14.2	7.0	7.0	3.0	1.0	1.0	1.0	134.4	76.0	<b>76.9</b>
		20.1	7.0	7.0	3.0	1.0	1.0	1.0	217.2	76.0	<b>185.8</b>
		28.5	7.0	7.0	3.0	1.0	1.0	1.0	334.5	76.0	<b>340.1</b>
$r_L$	5.0	7.1	7.0	7.0	3.0	1.0	1.0	1.0	82.3	76.0	<b>8.2</b>
		10.0	7.0	7.0	3.0	1.0	1.0	1.0	91.1	76.0	<b>19.9</b>
		14.2	7.0	7.0	3.0	1.0	1.0	1.0	103.7	76.0	<b>36.4</b>
$h_H$	1.0	0.6	7.0	7.0	3.0	1.0	1.0	1.0	81.8	76.0	<b>7.7</b>
		0.3	7.0	7.0	3.0	1.0	1.0	1.0	85.2	76.0	<b>12.2</b>
		0.2	7.0	7.0	3.0	1.0	1.0	1.0	87.2	76.0	<b>14.8</b>
$c_H$	4.0	2.3	7.0	7.0	3.0	1.0	1.0	1.0	99.4	76.0	<b>30.7</b>
		1.4	7.0	7.0	3.0	1.0	1.0	1.0	113.0	76.0	<b>48.7</b>
		0.8	7.0	7.0	3.0	1.0	1.0	1.0	120.9	76.0	<b>59.1</b>

Graph III-III-II-Elite

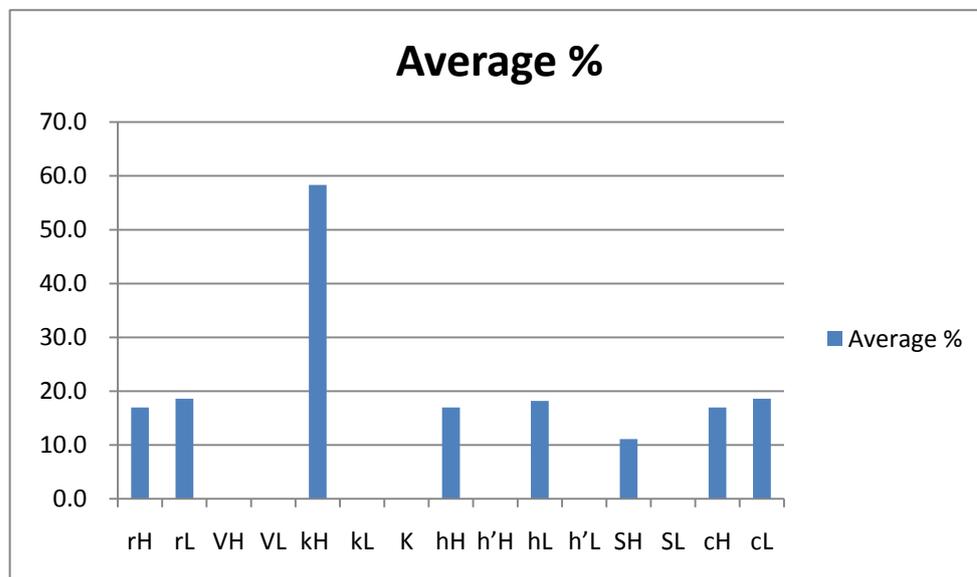


$r_H, c_H, r_L, h_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table CON-AV-I Show average profit improvement made by each parameter in Case I when they are changed 1 unit at a time

	Table I-I-I	Table I-II-I	Table I-III-I	Table II-I-I	Table II-II-I	Table II-III-I	Table III-I-I	Table III-II-I	Table III-III-I	Average %
$r_H$	25.0	13.6	13.6	4.5	16.7	16.7	2.5	41.7	18.4	<b>17.0</b>
$r_L$	87.5	15.9	15.9	11.4	8.3	8.3	3.8	12.5	3.9	<b>18.6</b>
$V_H$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$V_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h_H$	25.0	13.6	13.6	4.5	16.7	16.7	2.5	41.7	18.4	<b>17.0</b>
$h'_H$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h_L$	87.5	15.9	15.9	11.4	8.3	8.3	0.0	12.5	3.9	<b>18.2</b>
$h'_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$S_H$	50.0	0.0	0.0	18.2	0.0	0.0	15.0	16.7	0.0	<b>11.1</b>
$S_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$c_H$	25.0	13.6	13.6	4.5	16.7	16.7	2.5	41.7	18.4	<b>17.0</b>
$c_L$	87.5	15.9	15.9	11.4	8.3	8.3	3.8	12.5	3.9	<b>18.6</b>
$k^H$	325.0	0.0	0.0	59.1	0.0	0.0	32.5	108.3	0.0	<b>58.3</b>
$k^L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$K$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>

Graph CON-AV-I

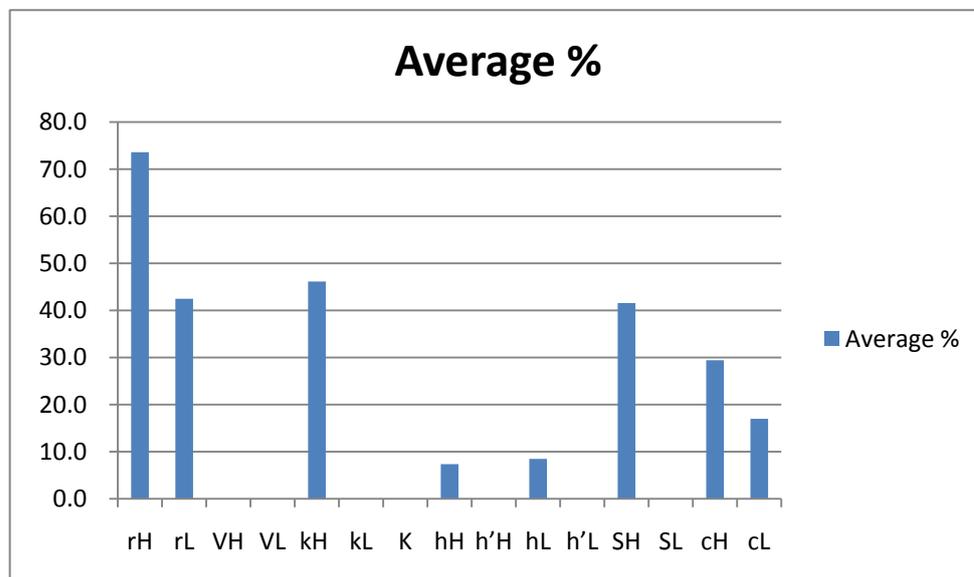


$k^H$ ,  $r_L$ ,  $c_L$ ,  $h_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table CON-AV-II Show average profit improvement made by each parameter in Case I when they are changed by the same percentage at a time

	Table I-I-II	Table I-II-II	Table I-III-II	Table II-I-II	Table II-II-II	Table II-III-II	Table III-I-II	Table III-II-II	Table III-III-II	Average %
$r_H$	119.3	59.0	57.8	21.7	69.5	69.5	11.9	176.6	76.9	<b>73.6</b>
$r_L$	208.8	34.4	33.7	27.1	17.4	17.4	8.9	26.5	8.2	<b>42.5</b>
$V_H$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$V_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$k^H$	155.1	0.0	0.0	28.2	0.0	0.0	15.5	216.7	0.0	<b>46.2</b>
$k^L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$K$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h_H$	11.9	5.9	5.8	2.2	7.0	7.0	1.2	17.7	7.7	<b>7.4</b>
$h'_H$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h_L$	41.8	6.9	6.7	5.4	3.5	3.5	1.8	5.3	1.6	<b>8.5</b>
$h'_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$S_H$	190.9	0.0	0.0	69.4	0.0	0.0	57.3	56.5	0.0	<b>41.6</b>
$S_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$c_H$	47.7	23.6	23.1	8.7	27.8	27.8	4.8	70.6	30.7	<b>29.4</b>
$c_L$	83.5	13.8	13.5	10.8	7.0	7.0	3.6	10.6	3.3	<b>17.0</b>

Graph CON-AV-II

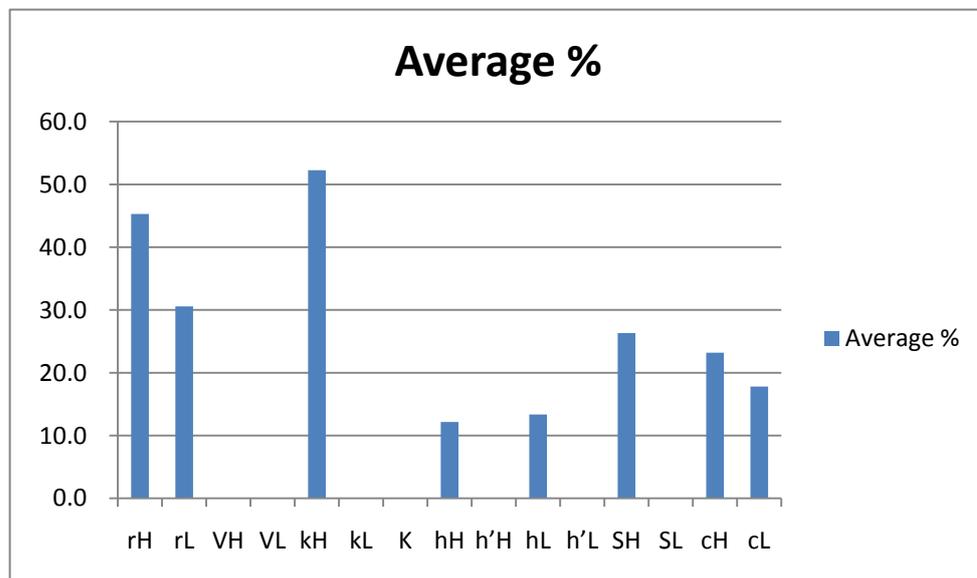


$r_H$ ,  $k^H$ ,  $r_L$ ,  $S_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table CON-AV-Overall Show average profit improvement made by each parameter in Case I (the average improvement in the above two tables).

	Table VIII-I	Table VIII-II	Average %
$r_H$	17.0	73.6	<b>45.3</b>
$r_L$	18.6	42.5	<b>30.6</b>
$V_H$	0.0	0.0	<b>0.0</b>
$V_L$	0.0	0.0	<b>0.0</b>
$k^H$	58.3	46.2	<b>52.2</b>
$k^L$	0.0	0.0	<b>0.0</b>
$K$	0.0	0.0	<b>0.0</b>
$h_H$	17.0	7.4	<b>12.2</b>
$h'_H$	0.0	0.0	<b>0.0</b>
$h_L$	18.2	8.5	<b>13.3</b>
$h'_L$	0.0	0.0	<b>0.0</b>
$S_H$	11.1	41.6	<b>26.3</b>
$S_L$	0.0	0.0	<b>0.0</b>
$c_H$	17.0	29.4	<b>23.2</b>
$c_L$	18.6	17.0	<b>17.8</b>

Graph CON-AV-Overall

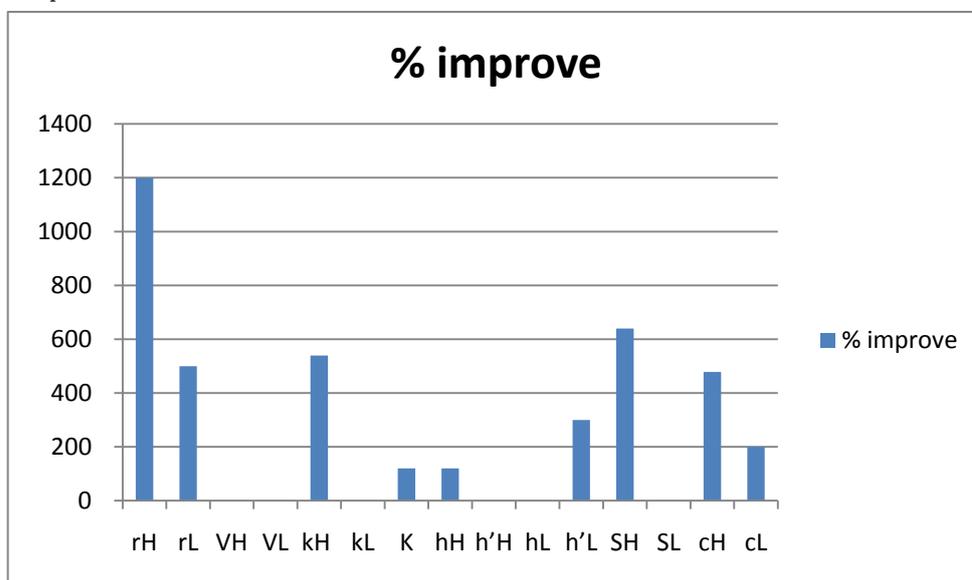


$k^H$ ,  $r_H$ ,  $S_H$ ,  $c_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table KK-I. Parameter sensitivity analysis for Case II subcase I where  $k^H$ ,  $K$  are limited:  $(k^H, k^L, K, D^H_1, D^H_2, D^L_2) = (3, 5, 3, 3, 5, 5)$  and  $(h_H - h'_H) - (h_L - h'_L) < 0$ . Parameters are changed by 40% at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			3.0	3.0	5.0	1.0	1.0	0.0	-2.0			
$r_H$	10.0	14.0	3.0	3.0	5.0	1.0	1.0	0.0	22.0	-2.0	<b>1200</b>	40.0
$r_L$	5.0	7.0	3.0	3.0	5.0	1.0	1.0	0.0	8.0	-2.0	<b>500</b>	40.0
$V_H$	1.0	1.4	3.0	3.0	5.0	1.0	1.0	0.0	-2.0	-2.0	<b>0</b>	40.0
$V_L$	1.0	1.4	3.0	3.0	5.0	1.0	1.0	0.0	-2.0	-2.0	<b>0</b>	40.0
$k^H$	3.0	4.2	4.2	3.0	5.0	0.7	1.0	0.0	8.8	-2.0	<b>539</b>	40.0
$k^L$	5.0	7.0	3.0	3.0	5.0	1.0	1.0	0.0	-2.0	-2.0	<b>0</b>	40.0
$K$	3.0	4.2	3.0	3.0	5.0	1.0	1.0	0.2	0.4	-2.0	<b>120</b>	40.0
$h_H$	1.0	0.6	3.0	3.0	5.0	1.0	1.0	0.0	0.4	-2.0	<b>120</b>	40.0
$h'_H$	5.0	3.0	3.0	3.0	5.0	1.0	1.0	0.0	-2.0	-2.0	<b>0</b>	40.0
$h_L$	1.0	0.6	3.0	3.0	5.0	1.0	1.0	0.0	-2.0	-2.0	<b>0</b>	40.0
$h'_L$	3.0	1.8	3.0	3.0	5.0	1.0	1.0	0.0	4.0	-2.0	<b>300</b>	40.0
$S_H$	8.0	4.8	3.0	3.0	5.0	1.0	1.0	0.0	10.8	-2.0	<b>640</b>	40.0
$S_L$	4.0	2.4	3.0	3.0	5.0	1.0	1.0	0.0	-2.0	-2.0	<b>0</b>	40.0
$c_H$	4.0	2.4	3.0	3.0	5.0	1.0	1.0	0.0	7.6	-2.0	<b>479</b>	40.0
$c_L$	2.0	1.2	3.0	3.0	5.0	1.0	1.0	0.0	2.0	-2.0	<b>200</b>	40.0

Graph KK-I

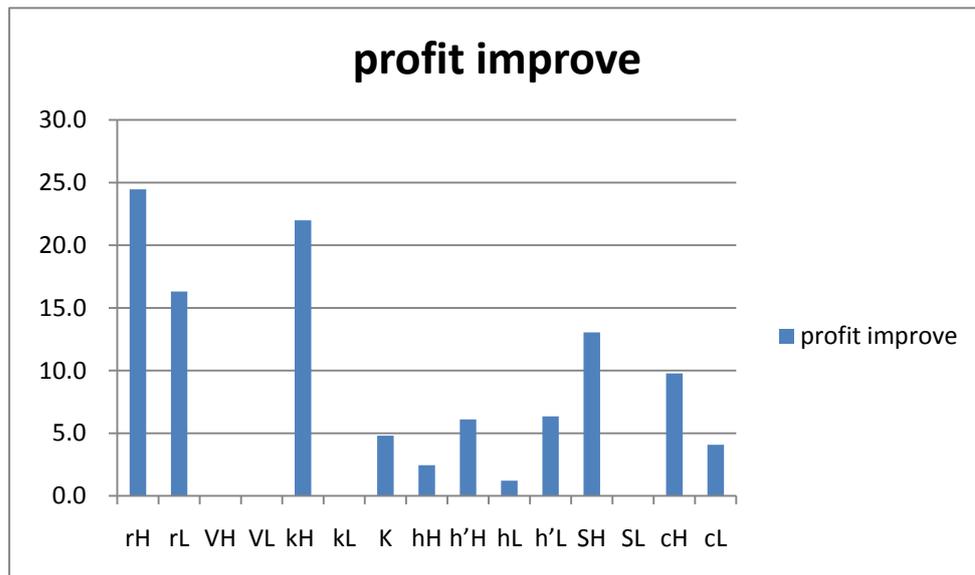


$r_H$ ,  $S_H$ ,  $k^H$ ,  $r_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table anti-KK-II Parameter sensitivity analysis for Case II subcase II where only K is limited:  $(k^H, k^L, K, D^H_1, D^H_2, D^L_2) = (3,5,3,5,5,5)$  and  $(h_H-h'_H)-(h_L-h'_L) > 0$ . Parameters are changed by 40% at a time.

parameter	original	change l	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	Profit improve
original			5.0	5.0	5.0	0.6	0.0	0.6	36.0		
$r_H$	10.0	14.1	3.0	3.0	5.0	1.0	0.0	0.6	24.5	0.0	<b>24.5</b>
$r_L$	8.0	11.3	3.0	3.0	5.0	1.0	0.0	0.6	16.3	0.0	<b>16.3</b>
$V_H$	1.0	1.4	3.0	3.0	5.0	1.0	0.0	0.6	0.0	0.0	<b>0.0</b>
$V_L$	1.0	1.4	3.0	3.0	5.0	1.0	0.0	0.6	0.0	0.0	<b>0.0</b>
$k^H$	3.0	4.2	4.2	4.2	5.0	0.7	0.0	0.6	22.0	0.0	<b>22.0</b>
$k^L$	5.0	7.0	3.0	3.0	5.0	1.0	0.0	0.6	0.0	0.0	<b>0.0</b>
K	3.0	4.2	3.0	3.0	5.0	1.0	0.0	0.8	4.8	0.0	<b>4.8</b>
$h_H$	2.0	1.2	3.0	3.0	5.0	1.0	0.0	0.6	2.4	0.0	<b>2.4</b>
$h'_H$	5.0	3.0	3.0	3.0	5.0	1.0	0.0	0.6	6.1	0.0	<b>6.1</b>
$h_L$	1.0	0.6	3.0	3.0	5.0	1.0	0.0	0.6	1.2	0.0	<b>1.2</b>
$h'_L$	5.0	3.0	3.0	3.0	5.0	1.0	1.0	0.0	6.3	0.0	<b>6.3</b>
$S_H$	8.0	4.7	3.0	3.0	5.0	1.0	0.0	0.6	13.0	0.0	<b>13.0</b>
$S_L$	4.0	2.4	3.0	3.0	5.0	1.0	0.0	0.6	0.0	0.0	<b>0.0</b>
$c_H$	4.0	2.4	3.0	3.0	5.0	1.0	0.0	0.6	9.8	0.0	<b>9.8</b>
$c_L$	2.0	1.2	3.0	3.0	5.0	1.0	0.0	0.6	4.1	0.0	<b>4.1</b>

Graph anti-KK-II

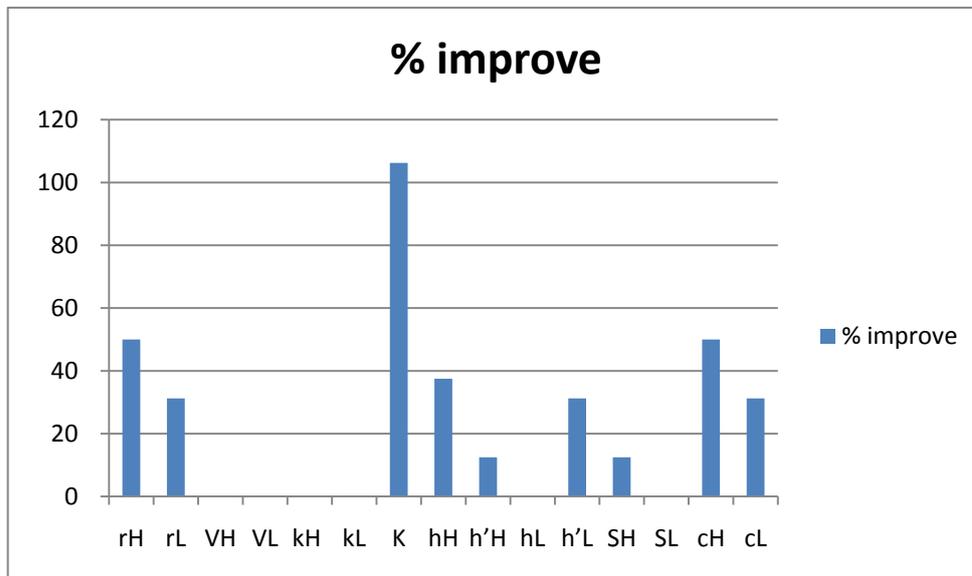


$r_H, k^H, r_L, S_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table K-I Parameter sensitivity analysis for Case III subcase I where only K is limited:  $(k^H, k^L, K, D^H_1, D^H_2, D^L_2) = (5,5,3,5,5,5)$  and  $(h_H - h'_H) - (h_L - h'_L) < 0$ . Parameters are changed one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			5	3	5	1	1	0	16			
$r_H$	10	11	5	3	5	1	1	0	24	16	50	10
$r_L$	5	6	5	3	5	1	1	0	21	16	31	20
$V_H$	1	2	5	3	5	1	1	0	16	16	0	100
$V_L$	1	2	5	3	5	1	1	0	16	16	0	100
$k^H$	5	6	5	3	5	1	1	0	16	16	0	20
$k^L$	5	6	5	3	5	1	1	0	16	16	0	20
K	3	4	5	4	5	1	1	0	33	16	106	33
$h_H$	1	0	5	3	5	1	1	0	22	16	38	100
$h'_H$	5	4	5	3	5	1	1	0	18	16	13	20
$h_L$	1	0	5	3	5	1	1	0	16	16	0	100
$h'_L$	3	2	5	3	5	1	1	0	21	16	31	33
$S_H$	8	7	5	3	5	1	1	0	18	16	13	13
$S_L$	4	3	5	3	5	1	1	0	16	16	0	25
$c_H$	4	3	5	3	5	1	1	0	24	16	50	25
$c_L$	2	1	5	3	5	1	1	0	21	16	31	50
average												45

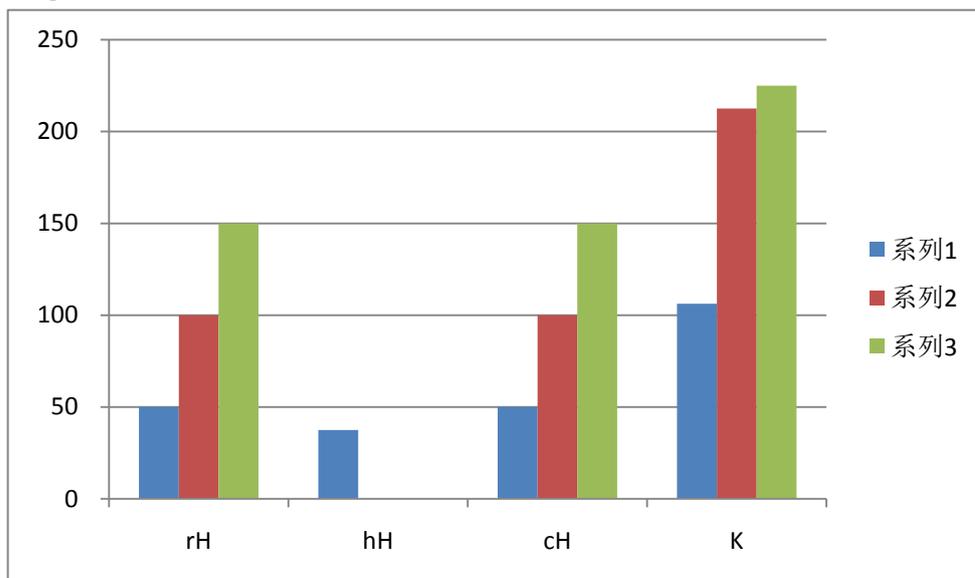
Graph K-I



Model 3.1, Table K-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

parameter	original	change l	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			5.0	3.0	5.0	0.6	1.0	0.0	16.0		
$r_H$	10	11	5	3	5	1	1	0	24	16	50
		12	5	3	5	1	1	0	32	16	100
		13	5	3	5	1	1	0	40	16	150
$h_H$	1	0	5	3	5	1	1	0	22	16	38
$c_H$	4	3	5	3	5	1	1	0	24	16	50
		2	5	3	5	1	1	0	32	16	100
		1	5	3	5	1	1	0	40	16	150
K	3	4	5	4	5	1	1	0	33	16	106
		5	5	5	5	1	1	0	50	16	213
		6	5	5	5	1	1	0	52	16	225

Graph K-I-Elite

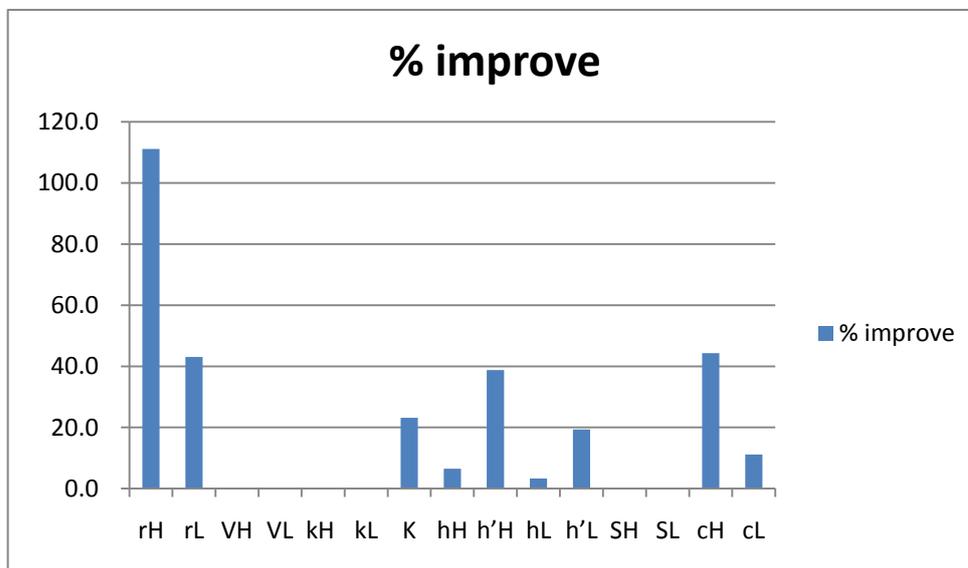


$K$ ,  $r_H$ ,  $c_H$ ,  $h_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table anti-K-II Parameter sensitivity analysis for Case III subcase II where only K is limited:  $(k^H, k^L, K, D_1^H, D_2^H, D_2^L) = (5, 5, 3, 5, 5, 5)$  and  $(h_H - h'_H) - (h_L - h'_L) > 0$ . Parameters are changed by 40% at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			5.0	5.0	5.0	0.6	0.0	0.6	36.0		
$r_H$	10.0	14.0	5.0	5.0	5.0	0.6	0.0	0.6	76.0	36.0	<b>111.1</b>
$r_L$	8.0	11.2	5.0	5.0	5.0	0.6	0.0	0.6	51.5	36.0	<b>43.1</b>
$V_H$	1.0	1.4	5.0	5.0	5.0	0.6	0.0	0.6	36.0	36.0	<b>0.0</b>
$V_L$	1.0	1.4	5.0	5.0	5.0	0.6	0.0	0.6	36.0	36.0	<b>0.0</b>
$k^H$	5.0	7.0	5.0	5.0	5.0	0.6	0.0	0.6	36.0	36.0	<b>0.0</b>
$k^L$	5.0	7.0	5.0	5.0	5.0	0.6	0.0	0.6	36.0	36.0	<b>0.0</b>
K	3.0	4.2	5.0	5.0	5.0	0.8	0.0	0.8	44.3	36.0	<b>23.2</b>
$h_H$	2.0	1.2	5.0	5.0	5.0	0.6	0.0	0.6	38.3	36.0	<b>6.5</b>
$h'_H$	5.0	3.0	5.0	5.0	5.0	0.6	0.0	0.6	50.0	36.0	<b>38.8</b>
$h_L$	1.0	0.6	5.0	5.0	5.0	0.6	0.0	0.6	37.2	36.0	<b>3.3</b>
$h'_L$	5.0	3.0	5.0	5.0	5.0	0.6	0.6	0.0	43.0	36.0	<b>19.4</b>
$S_H$	8.0	4.8	5.0	5.0	5.0	0.6	0.0	0.6	36.0	36.0	<b>0.0</b>
$S_L$	4.0	2.4	5.0	5.0	5.0	0.6	0.0	0.6	36.0	36.0	<b>0.0</b>
$c_H$	4.0	2.4	5.0	5.0	5.0	0.6	0.0	0.6	52.0	36.0	<b>44.3</b>
$c_L$	2.0	1.2	5.0	5.0	5.0	0.6	0.0	0.6	40.0	36.0	<b>11.1</b>

Graph anti-K-II

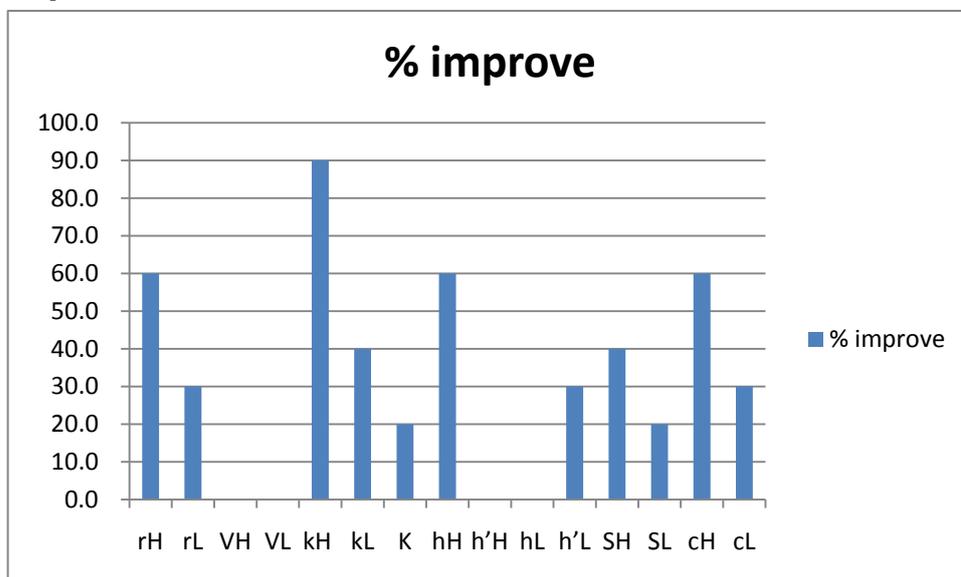


$r_H, r_L, c_H, h'_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table KKK-I Parameter sensitivity analysis for Case IV subcase I where  $k^H, k^L, K$  are limited:  $(k^H, k^L, K, D^H_1, D^H_2, D^L_2) = (3,3,3,5,5,5)$ . and  $(h_H-h'_H)-(h_L-h'_L) < 0$ . Parameters are changed one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3.0	3.0	3.0	1.0	1.0	0.0	-10.0		
$r_H$	10.0	11.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
$r_L$	5.0	6.0	3.0	3.0	3.0	1.0	1.0	0.0	-7.0	-10.0	<b>30.0</b>
$V_H$	1.0	2.0	3.0	3.0	3.0	1.0	1.0	0.0	-10.0	-10.0	<b>0.0</b>
$V_L$	1.0	2.0	3.0	3.0	3.0	1.0	1.0	0.0	-10.0	-10.0	<b>0.0</b>
$k^H$	3.0	4.0	4.0	3.0	3.0	0.8	1.0	0.0	-1.0	-10.0	<b>90.0</b>
$k^L$	3.0	4.0	3.0	3.0	4.0	1.0	1.0	0.0	-6.0	-10.0	<b>40.0</b>
$K$	3.0	4.0	3.0	3.0	3.0	1.0	1.0	0.3	-8.0	-10.0	<b>20.0</b>
$h_H$	1.0	0.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
$h'_H$	5.0	4.0	3.0	3.0	3.0	1.0	1.0	0.0	-10.0	-10.0	<b>0.0</b>
$h_L$	1.0	0.0	3.0	3.0	3.0	1.0	1.0	0.0	-10.0	-10.0	<b>0.0</b>
$h'_L$	3.0	2.0	3.0	3.0	3.0	1.0	1.0	0.0	-7.0	-10.0	<b>30.0</b>
$S_H$	8.0	7.0	3.0	3.0	3.0	1.0	1.0	0.0	-6.0	-10.0	<b>40.0</b>
$S_L$	4.0	3.0	3.0	3.0	3.0	1.0	1.0	0.0	-8.0	-10.0	<b>20.0</b>
$c_H$	4.0	3.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
$c_L$	2.0	1.0	3.0	3.0	3.0	1.0	1.0	0.0	-7.0	-10.0	<b>30.0</b>

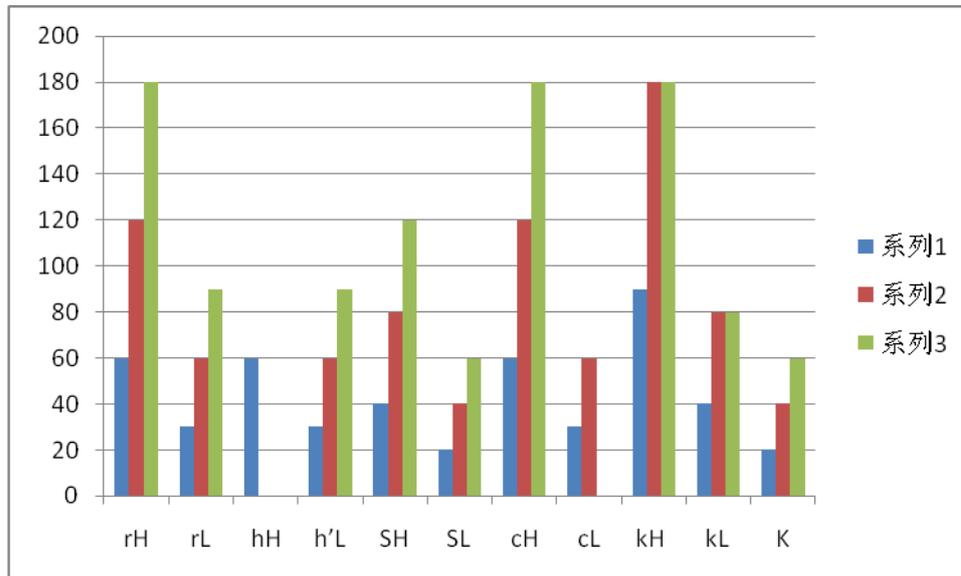
Graph KKK-I



Model 3.1, Table KKK-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3.0	3.0	3.0	1.0	1.0	0.0	-10.0		
$r_H$	10.0	11.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
		12.0	3.0	3.0	3.0	1.0	1.0	0.0	2.0	-10.0	<b>120.0</b>
		13.0	3.0	3.0	3.0	1.0	1.0	0.0	8.0	-10.0	<b>180.0</b>
$r_L$	5.0	6.0	3.0	3.0	3.0	1.0	1.0	0.0	-7.0	-10.0	<b>30.0</b>
		7.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
		8.0	3.0	3.0	3.0	1.0	1.0	0.0	-1.0	-10.0	<b>90.0</b>
$h_H$	1.0	0.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
$h'_L$	3.0	2.0	3.0	3.0	3.0	1.0	1.0	0.0	-7.0	-10.0	<b>30.0</b>
		1.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
		0.0	3.0	3.0	3.0	1.0	1.0	0.0	-1.0	-10.0	<b>90.0</b>
$S_H$	8.0	7.0	3.0	3.0	3.0	1.0	1.0	0.0	-6.0	-10.0	<b>40.0</b>
		6.0	3.0	3.0	3.0	1.0	1.0	0.0	-2.0	-10.0	<b>80.0</b>
		5.0	3.0	3.0	3.0	1.0	1.0	0.0	2.0	-10.0	<b>120.0</b>
$S_L$	4.0	3.0	3.0	3.0	3.0	1.0	1.0	0.0	-8.0	-10.0	<b>20.0</b>
		2.0	3.0	3.0	3.0	1.0	1.0	0.0	-6.0	-10.0	<b>40.0</b>
		1.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
$c_H$	4.0	3.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
		2.0	3.0	3.0	3.0	1.0	1.0	0.0	2.0	-10.0	<b>120.0</b>
		1.0	3.0	3.0	3.0	1.0	1.0	0.0	8.0	-10.0	<b>180.0</b>
$c_L$	2.0	1.0	3.0	3.0	3.0	1.0	1.0	0.0	-7.0	-10.0	<b>30.0</b>
		0.0	3.0	3.0	3.0	1.0	1.0	0.0	-4.0	-10.0	<b>60.0</b>
$k^H$	3.0	4.0	3.0	3.0	3.0	1.0	1.0	0.0	-1.0	-10.0	<b>90.0</b>
		5.0	5.0	3.0	3.0	1.0	1.0	0.0	8.0	-10.0	<b>180.0</b>
		6.0	5.0	3.0	3.0	0.6	1.0	0.0	8.0	-10.0	<b>180.0</b>
$k^L$	3.0	4.0	3.0	3.0	3.0	1.0	1.0	0.0	-6.0	-10.0	<b>40.0</b>
		5.0	3.0	3.0	5.0	1.0	1.0	0.0	-2.0	-10.0	<b>80.0</b>
		6.0	3.0	3.0	5.0	1.0	1.0	0.0	-2.0	-10.0	<b>80.0</b>
$K$	3.0	4.0	3.0	3.0	3.0	1.0	1.0	0.3	-8.0	-10.0	<b>20.0</b>
		5.0	3.0	3.0	3.0	1.0	1.0	0.7	-6.0	-10.0	<b>40.0</b>
		6.0	3.0	3.0	3.0	1.0	1.0	1.0	-4.0	-10.0	<b>60.0</b>

Graph KKK-I-Elite

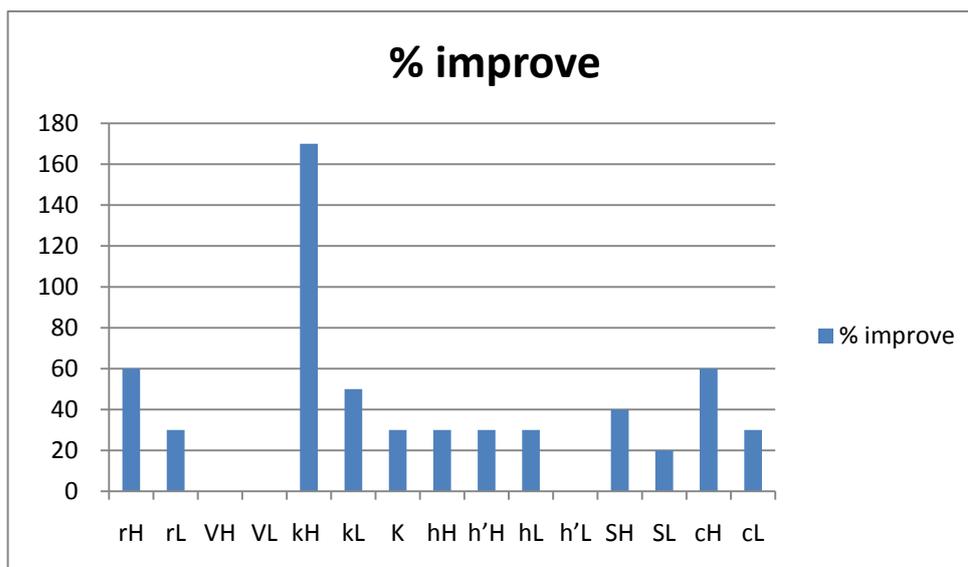


$k^H$  the most important parameter to the retailer (when it is limited) and  $r_H, c_H, h_H$  are the second most important parameters to the retailer.

Model 3.1, Table anti-KKK-I Parameter sensitivity analysis for Case IV subcase II where  $k^H, k^L, K$  are limited:  $(k^H, k^L, K, D^H_1, D^H_2, D^L_2) = (3,3,3,5,5,5)$ . and  $(h_H - h'_H) - (h_L - h'_L) > 0$ . Parameters are changed one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve	% change
original			3	3	3	1	0	1	-10			
$r_H$	10	11	3	3	3	1	0	1	-4	-10	<b>60</b>	0
$r_L$	8	9	3	3	3	1	0	1	-7	-10	<b>30</b>	0
$V_H$	1	2	3	3	3	1	0	1	-10	-10	<b>0</b>	1
$V_L$	1	2	3	3	3	1	0	1	-10	-10	<b>0</b>	1
$k^H$	3	4	4	3	3	1	0	1	7	-10	<b>170</b>	0
$k^L$	3	4	3	3	4	1	0	1	-5	-10	<b>50</b>	0
$K$	3	4	3	3	3	1	0	1	-7	-10	<b>30</b>	0
$h_H$	2	1	3	3	3	1	0	1	-7	-10	<b>30</b>	1
$h'_H$	5	4	3	3	3	1	0	1	-7	-10	<b>30</b>	0
$h_L$	1	0	3	3	3	1	0	1	-7	-10	<b>30</b>	1
$h'_L$	5	4	3	3	3	1	0	1	-10	-10	<b>0</b>	0
$S_H$	8	7	3	3	3	1	0	1	-6	-10	<b>40</b>	0
$S_L$	4	3	3	3	3	1	0	1	-8	-10	<b>20</b>	0
$c_H$	4	3	3	3	3	1	0	1	-4	-10	<b>60</b>	0
$c_L$	2	1	3	3	3	1	0	1	-7	-10	<b>30</b>	1
average												0

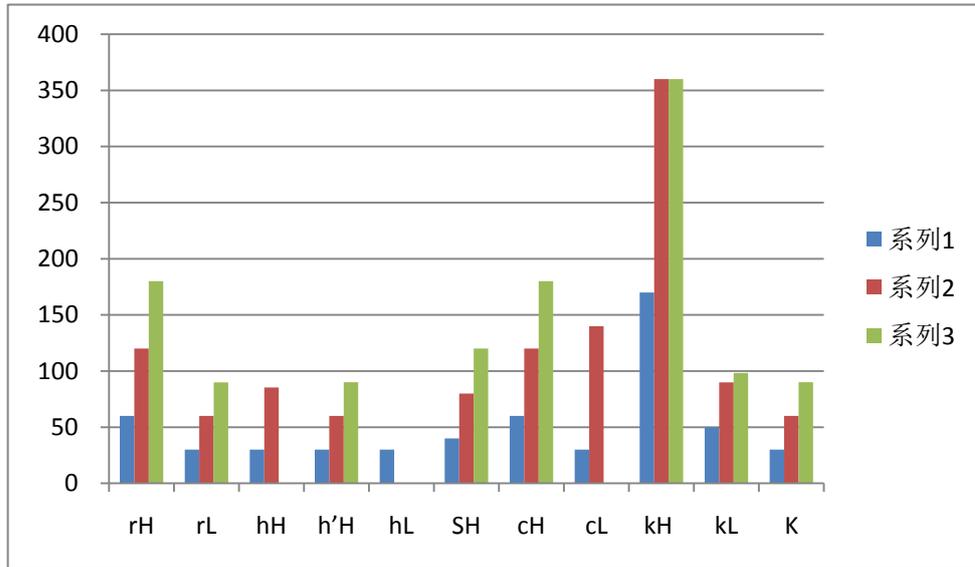
Graph anti-KKK-I



Model 3.1, Table anti-KKK-I-Elite Parameter sensitivity analysis for Case IV subcase II where  $k^H, k^L, K$  are limited:  $(k^H, k^L, K, D_1^H, D_2^H, D_2^L) = (3, 3, 3, 5, 5, 5)$ . and  $(h_H - h'_H) - (h_L - h'_L) > 0$ . Parameters are changed one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3	3	3	1	0	1	-10		
$r_H$	10	11	3	3	3	1	0	1	-4	-10	<b>60</b>
		12	3	3	3	1	0	1	2	-10	<b>120</b>
		13	3	3	3	1	0	1	8	-10	<b>180</b>
$r_L$	8	9	3	3	3	1	0	1	-7	-10	<b>30</b>
		10	3	3	3	1	0	1	-4	-10	<b>60</b>
		11	3	3	3	1	0	1	-1	-10	<b>90</b>
$h_H$	2	1	3	3	3	1	0	1	-7	-10	<b>30</b>
		0	3	3	3	1	1	0	-1	-10	<b>85</b>
$h'_H$	5	4	3	3	3	1	0	1	-7	-10	<b>30</b>
		3	3	3	3	1	0	1	-4	-10	<b>60</b>
		2	3	3	3	0	0	1	-1	-10	<b>90</b>
$h_L$	1	0	3	3	3	1	0	1	-7	-10	<b>30</b>
$S_H$	8	7	3	3	3	1	0	1	-6	-10	<b>40</b>
		6	3	3	3	1	0	1	-2	-10	<b>80</b>
		5	3	3	3	1	0	1	2	-10	<b>120</b>
$c_H$	4	3	3	3	3	1	0	1	-4	-10	<b>60</b>
		2	3	3	3	1	0	1	2	-10	<b>120</b>
		1	3	3	3	1	0	1	8	-10	<b>180</b>
$c_L$	2	1	3	3	3	1	0	1	-7	-10	<b>30</b>
		0	3	3	3	1	0	1	4	-10	<b>140</b>
$k^H$	3	4	3	3	3	1	0	1	7	-10	<b>170</b>
		5	5	5	3	1	0	1	26	-10	<b>360</b>
		6	5	5	3	1	0	1	26	-10	<b>360</b>
$k^L$	3	4	3	3	3	1	0	1	-5	-10	<b>50</b>
		5	3	3	5	1	0	0	-1	-10	<b>90</b>
		6	3	3	5	1	0	1	0	-10	<b>98</b>
$K$	3	4	3	3	3	1	0	1	-7	-10	<b>30</b>
		5	3	3	3	1	1	1	-4	-10	<b>60</b>
		6	3	3	3	1	1	1	-1	-10	<b>90</b>

Graph anti-KKK-I-Elite

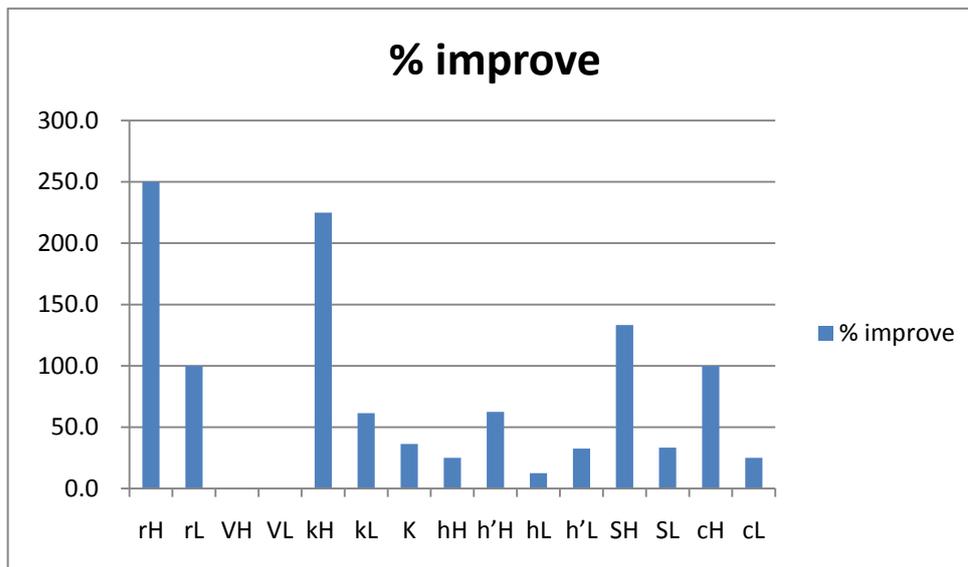


$k^H$  the most important parameter to the retailer (when it is limited) and  $r_H, c_H$  are the second most important parameters to the retailer.

Model 3.1, Table anti-KKK-II Parameter sensitivity analysis for Case IV subcase II where  $k^H, k^L, K$  are limited:  $(k^H, k^L, K, D^H_1, D^H_2, D^L_2) = (3,3,3,5,5,5)$ . and  $(h_H-h'_H)-(h_L-h'_L) > 0$ . Parameters are changed by 41.67% at a time.

parameter	original	change l	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3	3	3	1	0	1	-10		
$r_H$	10	14	3	3	3	1	0	1	15	-10	250
$r_L$	8	11	3	3	3	1	0	1	0	-10	100
$V_H$	1	1	3	3	3	1	0	1	-10	-10	0
$V_L$	1	1	3	3	3	1	0	1	-10	-10	0
$k^H$	3	4	4	4	3	1	0	1	12	-10	225
$k^L$	3	4	3	3	4	1	0	1	-4	-10	61
$K$	3	4	3	3	3	1	0	1	-6	-10	36
$h_H$	2	1	3	3	3	1	0	1	-8	-10	25
$h'_H$	5	3	3	3	3	1	0	1	-4	-10	63
$h_L$	1	1	3	3	3	1	0	1	-9	-10	12
$h'_L$	5	3	3	3	3	1	1	0	-7	-10	32
$S_H$	8	5	3	3	3	1	0	1	3	-10	133
$S_L$	4	2	3	3	3	1	0	1	-7	-10	33
$c_H$	4	2	3	3	3	1	0	1	0	-10	100
$c_L$	2	1	3	3	3	1	0	1	-8	-10	25

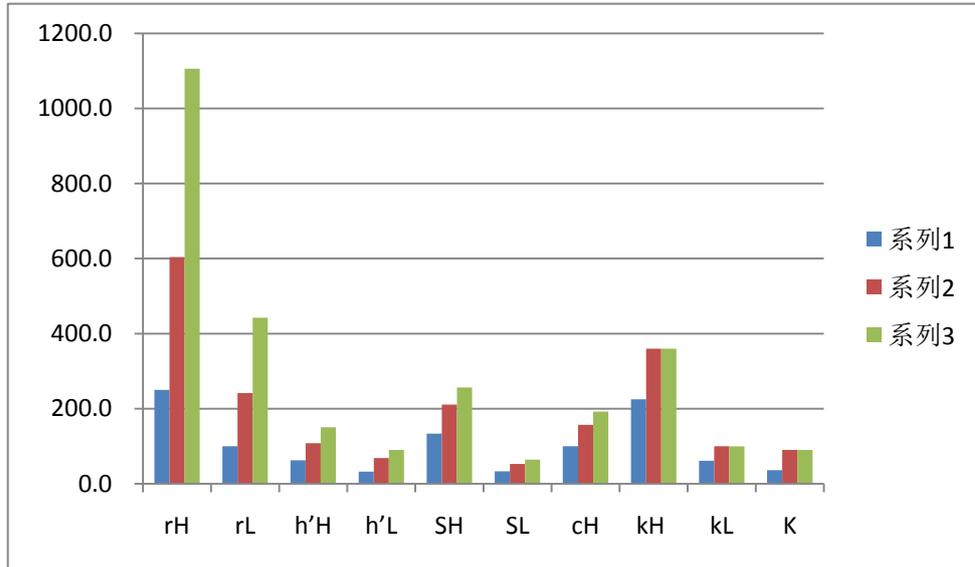
Graph anti-KKK-II



Model 3.1, Table anti-KKK-II-Elite Continue to relax the most important parameters in the above table by changing them by the same percentage at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			3	3	3	1	0	1	-10		
$r_H$	10	14	3	3	3	1	0	1	15	-10	<b>250</b>
		20	3	3	3	1	0	1	50	-10	<b>604</b>
		28	3	3	3	1	0	1	101	-10	<b>1106</b>
$r_L$	8	11	3	3	3	1	0	1	0	-10	<b>100</b>
		16	3	3	3	1	0	1	14	-10	<b>241</b>
		23	3	3	3	1	0	1	34	-10	<b>442</b>
$h'_H$	5	3	3	3	3	1	0	1	-4	-10	<b>63</b>
		2	3	3	3	0	0	1	1	-10	<b>108</b>
		1	3	3	3	0	0	1	5	-10	<b>150</b>
$h'_L$	5	3	3	3	3	1	0	1	-7	-10	<b>32</b>
		2	3	3	3	1	1	0	-3	-10	<b>69</b>
		1	3	3	3	1	1	0	-1	-10	<b>90</b>
$S_H$	8	5	3	3	3	1	0	1	3	-10	<b>133</b>
		3	3	3	3	1	0	1	11	-10	<b>211</b>
		2	3	3	3	1	0	1	16	-10	<b>256</b>
$S_L$	4	2	3	3	3	1	0	1	-7	-10	<b>33</b>
		1	3	3	3	1	0	1	-5	-10	<b>53</b>
		1	3	3	3	1	0	1	-4	-10	<b>64</b>
$c_H$	4	2	3	3	3	1	0	1	0	-10	<b>100</b>
		1	3	3	3	1	0	1	6	-10	<b>157</b>
		1	3	3	3	1	0	1	9	-10	<b>192</b>
$k^H$	3	4	4	4	3	1	0	1	12	-10	<b>225</b>
		6	5	5	3	1	0	1	26	-10	<b>360</b>
		9	5	5	3	1	0	1	26	-10	<b>360</b>
$k^L$	3	4	3	3	4	1	0	1	-4	-10	<b>61</b>
		6	3	3	5	1	0	1	0	-10	<b>100</b>
		9	3	3	5	1	0	1	0	-10	<b>100</b>
$K$	3	4	3	3	3	1	0	1	-6	-10	<b>36</b>
		6	3	3	3	1	1	1	-1	-10	<b>90</b>
		9	3	3	3	1	1	1	-1	-10	<b>90</b>

Graph anti-KKK-II-Elite

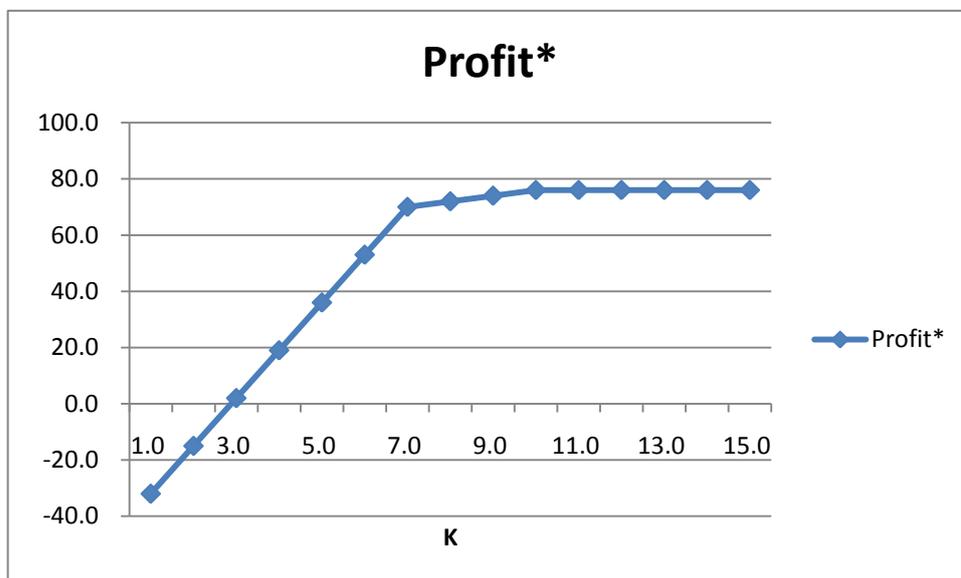


$r_H, k^H, S_H, r_L$  are the most important parameters to the retailer (from the most important to the least important).

Model 3.1, Table 3.3 Total profit changes with K where initially  $(k^H, k^L, K, D_1^H, D_2^H, D_2^L) = (1, 10, 10, 7, 7, 3)$ .

K	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*
1.0	7.0	1.0	3.0	0.1	1.0	0.0	-32.0
2.0	7.0	2.0	3.0	0.3	1.0	0.0	-15.0
3.0	7.0	3.0	3.0	0.4	1.0	0.0	2.0
4.0	7.0	4.0	3.0	0.6	1.0	0.0	19.0
5.0	7.0	5.0	3.0	0.7	1.0	0.0	36.0
6.0	7.0	6.0	3.0	0.9	1.0	0.0	53.0
7.0	7.0	7.0	3.0	1.0	1.0	0.0	70.0
8.0	7.0	7.0	3.0	1.0	1.0	0.3	72.0
9.0	7.0	7.0	3.0	1.0	1.0	0.7	74.0
10.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0
11.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0
12.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0
13.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0
14.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0
15.0	7.0	7.0	3.0	1.0	1.0	1.0	76.0

Graph 3.3 Total profit changes with K where initially  $(k^H, k^L, K, D_1^H, D_2^H, D_2^L) = (1, 10, 10, 7, 7, 3)$ .



the retailer can concavely increase the total profit by increasing K.

Model 3.1, Table C-1 Show average cost percentage

	C	H	R	S	V	H/R
I-I-I	-22	-9	55	-32	0	<b>0.16</b>
I-II-I	-38	-13	95	0	0	<b>0.14</b>
I-III-I	-38	-13	95	0	0	<b>0.14</b>
II-I-I	-18	-7	45	-64	0	<b>0.16</b>
II-II-I	-50	-15	125	0	0	<b>0.12</b>
II-III-I	-50	-15	125	0	0	<b>0.12</b>
III-I-I	-14	-5	35	-96	0	<b>0.14</b>
III-II-I	-46	-13	115	-32	0	<b>0.11</b>
III-III-I	-62	-17	155	0	0	<b>0.11</b>
Average						<b>0.13</b>

Average cost percentage in Case I is 13.3%

## Model 4.1

Model 4.1, Table I

Show profit changes with  $k^H$  where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2) = (3, 3, 7, 6, 6, 0.7)$ .

Dimension II	$X_{0^H}^*$	$X_{1^H}^*$	$X_{0^L}^*$	$a_{0^H}^*$	$a_{1^H}^*$	$a_{1^L}^*$	Profit*
$K^H=1$	0.1	0.1	9.9	1.0	1.0	1.0	<b>-36.7</b>
$K^H=5$	5.0	1.5	7.1	1.0	1.0	0.5	<b>10.6</b>
$K^H=10$	6.0	0.9	6.8	1.0	1.0	0.5	<b>14.0</b>
$K^H=15$	5.9	0.9	6.9	1.0	1.0	0.5	<b>12.8</b>

Expected profit decreases as  $k^H$  increases, which is incorrect. One possible reason is that the objective function may have many local optimal solutions. Due to the Monte Carlo method, the total error is relatively too big comparing to the value of the optimal solution itself, which is another possible reason.

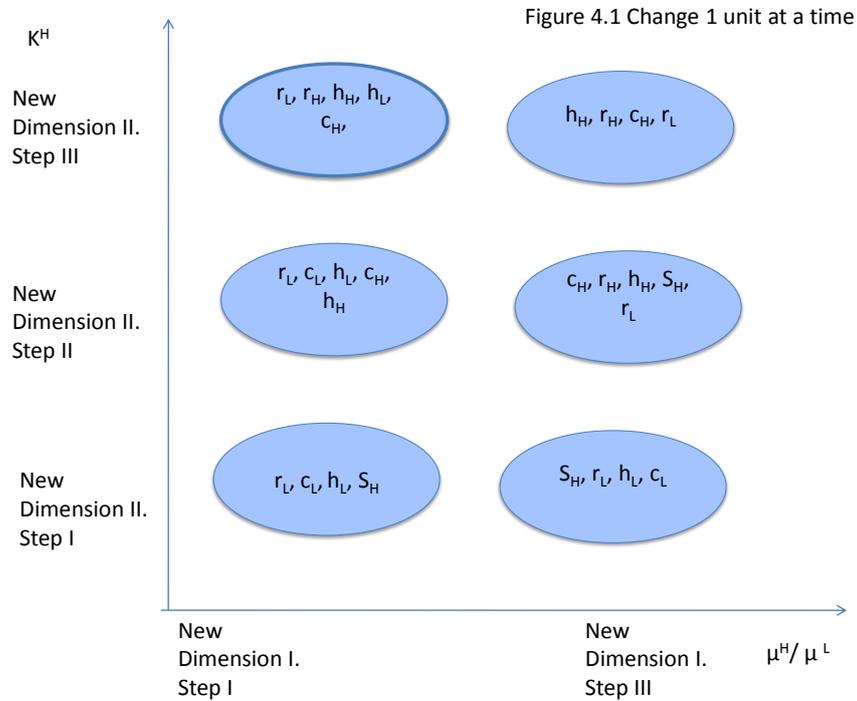
Model 4.1, Table 4.1 Parameter settings

Dimension I \ Dimension II	Step I $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2)$ =(300, 300, 700, 600, 600, 70)	Step II $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2)$ =(500, 500, 500, 1000, 1000, 50)	Step III $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2)$ =(700, 700, 300, 1400, 1400, 30)
Step I	$k^H=1$	$k^H=1$	$k^H=1$
Step II	$k^H=200$	$k^H=500$	$k^H=500$
Step III	$k^H=400$	$k^H=1000$	$k^H=3000$

Settings of the steps in New Dimension I and New Dimension II.

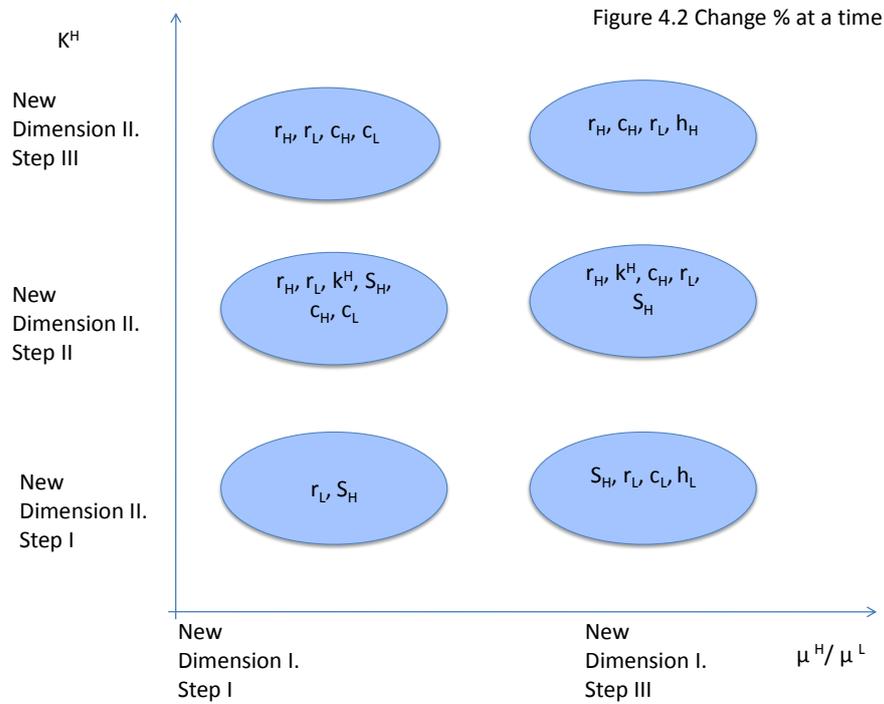
Model 4.1, Figure 4.1

Show the most important parameters (when changed one unit at a time) in each combination of the steps in Dimension I (except step II) and the steps in Dimension II



No parameters can always be the most important one. We indicate the most important parameters (changed 1 unit at a time) in each combination of the steps in New Dimension I (except step II) and the steps in New Dimension II.

Model 4.1, Figure 4.2. Show the most important parameters (when changed by the same percentage at a time) in each combination of the steps in Dimension I (except step II) and the steps in Dimension II



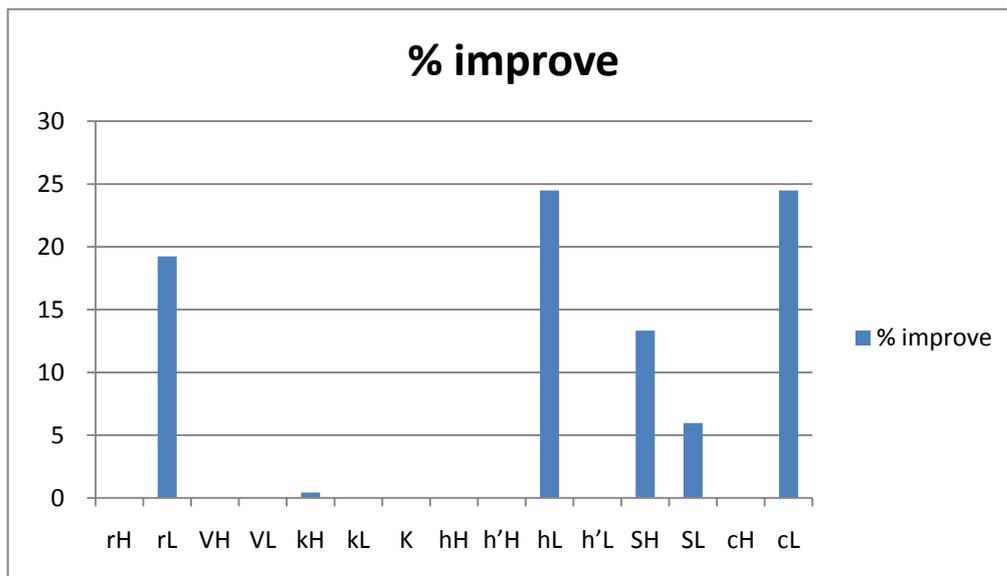
No parameters can always be the most important one. We indicate the most important parameters (changed by the same percentage at a time) in each combination of the steps in New Dimension I (except step II) and the steps in New Dimension II.

Model 4.1, Table I-I-I

Sensitivity analysis with parameter settings in step I (New Dimension I), step I (New Dimension II), step I (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 1)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve	% change
original			1	0	694	1	0	1	-2833			
$r_H$	10	11	1	0	694	1	0	1	-2832	-2833	<b>0</b>	10
$r_L$	5	6	1	0	638	1	0	1	-2288	-2833	<b>19</b>	20
$V_H$	1	2	1	0	694	1	0	1	-2833	-2833	<b>0</b>	100
$V_L$	1	2	1	0	694	1	0	1	-2832	-2833	<b>0</b>	100
$k^H$	1	2	2	0	694	1	0	1	-2820	-2833	<b>0</b>	100
$k^L$	1000	1001	1	0	694	1	0	1	-2833	-2833	<b>0</b>	0
K	1000	1001	1	0	694	1	0	1	-2833	-2833	<b>0</b>	0
$h_H$	1	0	1	0	694	1	0	1	-2832	-2833	<b>0</b>	100
$h'_H$	5	4	1	0	694	1	0	1	-2833	-2833	<b>0</b>	20
$h_L$	1	0	1	0	694	1	0	1	-2139	-2833	<b>24</b>	100
$h'_L$	3	2	1	0	694	1	0	1	-2833	-2833	<b>0</b>	33
$S_H$	8	7	1	0	638	1	0	1	-2455	-2833	<b>13</b>	13
$S_L$	4	3	1	0	566	1	0	1	-2664	-2833	<b>6</b>	25
$c_H$	4	3	1	0	694	1	0	1	-2832	-2833	<b>0</b>	25
$c_L$	2	1	1	0	694	1	0	1	-2139	-2833	<b>24</b>	50
average												46

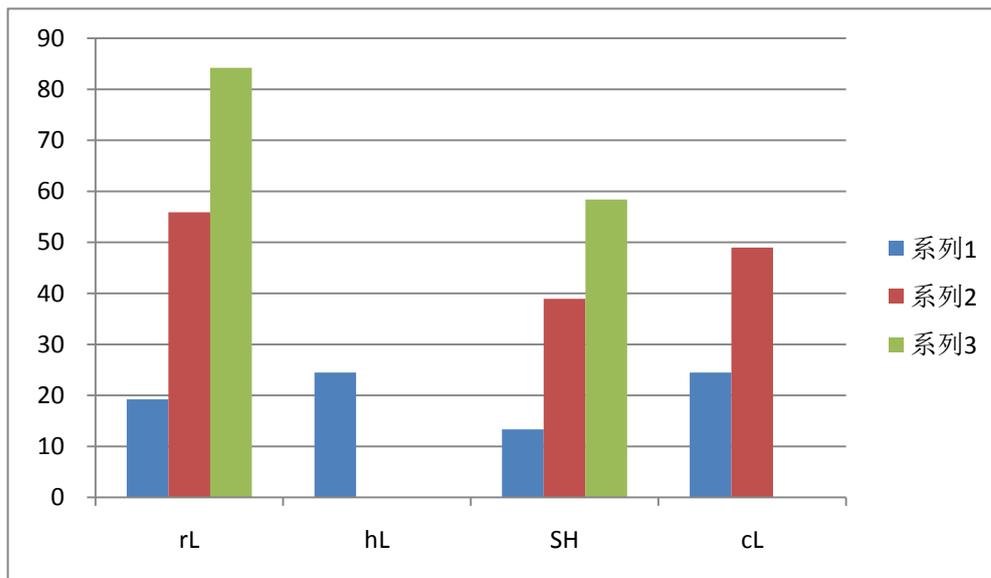
Graph I-I-I



Model 4.1, Table I-I-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$\hat{a}_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			1	0	694	1	0	1	-2833		
$r_L$	5	6	1	0	638	1	0	1	-2288	-2833	<b>19</b>
		7	1	0	566	1	0	1	-1249	-2833	<b>56</b>
		8	1	0	566	1	0	1	-447	-2833	<b>84</b>
$h_L$	1	0	1	0	694	1	0	1	-2139	-2833	<b>24</b>
$S_H$	8	7	1	0	638	1	0	1	-2455	-2833	<b>13</b>
		6	1	0	694	1	0	1	-1730	-2833	<b>39</b>
		5	1	0	694	1	0	1	-1179	-2833	<b>58</b>
$c_L$	2	1	1	0	694	1	0	1	-2139	-2833	<b>24</b>
		0	1	0	694	1	0	1	-1446	-2833	<b>49</b>

Graph I-I-I-Elite



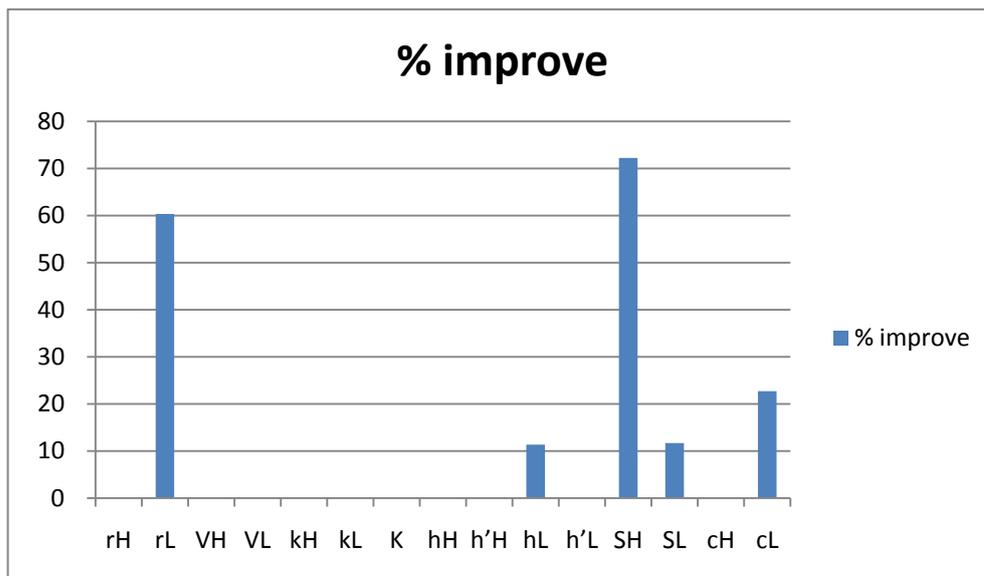
$h_L, c_L, r_L, S_H$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed one unit at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 1)$ .

Model 4.1, Table I-I-II

Sensitivity analysis with parameter settings in step I (New Dimension I), step I (New Dimension II), step II (Dimension III) where  $(\mu^{H_1}, \mu^{H_2}, \mu^{L_2}, \sigma^{H_1}, \sigma^{H_2}, \sigma^{L_2}, k^H) = (300, 300, 700, 600, 600, 70, 1)$ .

parameter	original	change 40%	$X_0^{H^*}$	$X_1^{H^*}$	$X_0^{L^*}$	$q_1^H$	$a_1^{H^*}$	$a_1^{L^*}$	Profit*	original	% improve
original			1	0	694	1	0	1	-2833		
$r_H$	10	15	1	0	694	1	0	1	-2829	-2833	<b>0</b>
$r_L$	5	7	1	0	694	1	0	1	-1123	-2833	<b>60</b>
$V_H$	1	1	1	0	694	1	0	1	-2833	-2833	<b>0</b>
$V_L$	1	1	1	0	694	1	0	1	-2833	-2833	<b>0</b>
$k^H$	1	1	1	0	694	1	0	1	-2827	-2833	<b>0</b>
$k^L$	1000	1464	1	0	694	1	0	1	-2833	-2833	<b>0</b>
K	1000	1464	1	0	694	1	0	1	-2833	-2833	<b>0</b>
$h_H$	1	1	1	0	694	1	0	1	-2833	-2833	<b>0</b>
$h'_H$	5	3	1	0	694	1	0	1	-2833	-2833	<b>0</b>
$h_L$	1	1	1	0	694	1	0	1	-2511	-2833	<b>11</b>
$h'_L$	3	2	1	0	694	1	0	1	-2833	-2833	<b>0</b>
$S_H$	8	4	1	0	694	1	0	1	-786	-2833	<b>72</b>
$S_L$	4	2	1	0	566	1	0	1	-2501	-2833	<b>12</b>
$c_H$	4	2	1	0	694	1	0	1	-2831	-2833	<b>0</b>
$c_L$	2	1	1	0	694	1	0	1	-2189	-2833	<b>23</b>

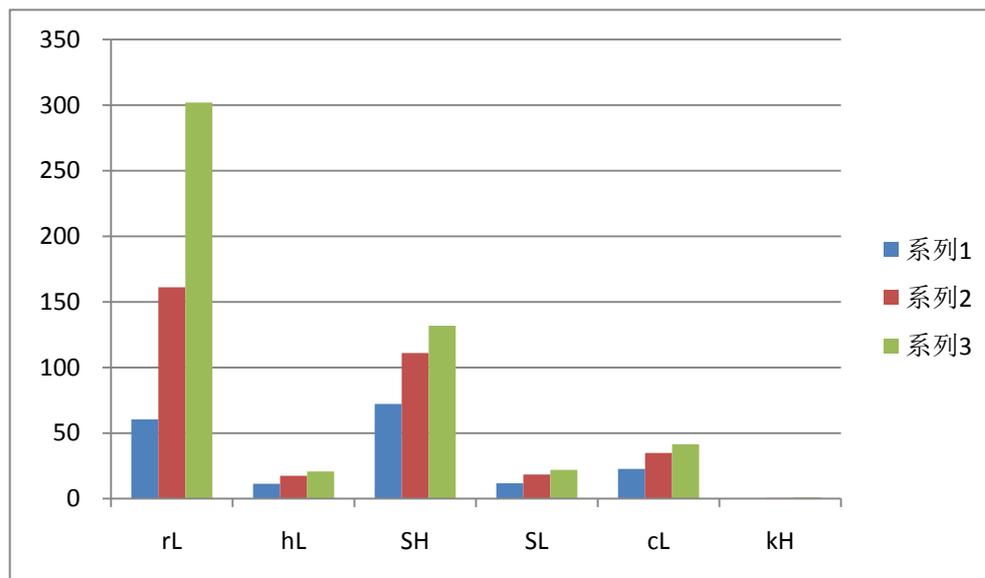
Graph I-I-II



Model 4.1, Table I-I-II-Elite Continue to relax the most important parameters in the above table by changing them by the same percentage at a time.

parameter	original	change 40%	$X_0^{H^*}$	$X_1^{H^*}$	$X_0^{L^*}$	$\sigma_1^H$	$\sigma_1^{H^*}$	$\sigma_1^{L^*}$	Profit*	original	% improve
original			1	0	694	1	0	1	-2833		
			1	0		1	0	1		-2833	
$r_L$	5	7	1	0	694	1	0	1	-1123	-2833	<b>60</b>
		11	1	0	566	1	0	1	1732	-2833	<b>161</b>
		16	1	0	566	1	0	1	5722	-2833	<b>302</b>
$h_L$	1	1	1	0	694	1	0	1	-2511	-2833	<b>11</b>
		0	1	0	694	1	0	1	-2339	-2833	<b>17</b>
		0	1	0	694	1	0	1	-2246	-2833	<b>21</b>
$S_H$	8	4	1	0	694	1	0	1	-786	-2833	<b>72</b>
		2	1	1	694	1	0	1	312	-2833	<b>111</b>
		1	1	0	694	1	0	1	901	-2833	<b>132</b>
$S_L$	4	2	1	0	566	1	0	1	-2501	-2833	<b>12</b>
		1	1	0	566	1	0	1	-2312	-2833	<b>18</b>
		1	1	0	566	1	0	1	-2211	-2833	<b>22</b>
$c_L$	2	1	1	0	694	1	0	1	-2189	-2833	<b>23</b>
		1	1	0	694	1	0	1	-1844	-2833	<b>35</b>
		0	1	0	694	1	0	1	-1659	-2833	<b>41</b>
$k_H$	1	1	1	0	694	1	0	1	-2827	-2833	<b>0</b>
		2	2	0	694	1	0	1	-2819	-2833	<b>1</b>
		3	3	0	694	1	0	1	-2806	-2833	<b>1</b>

Graph I-I-II-Elite



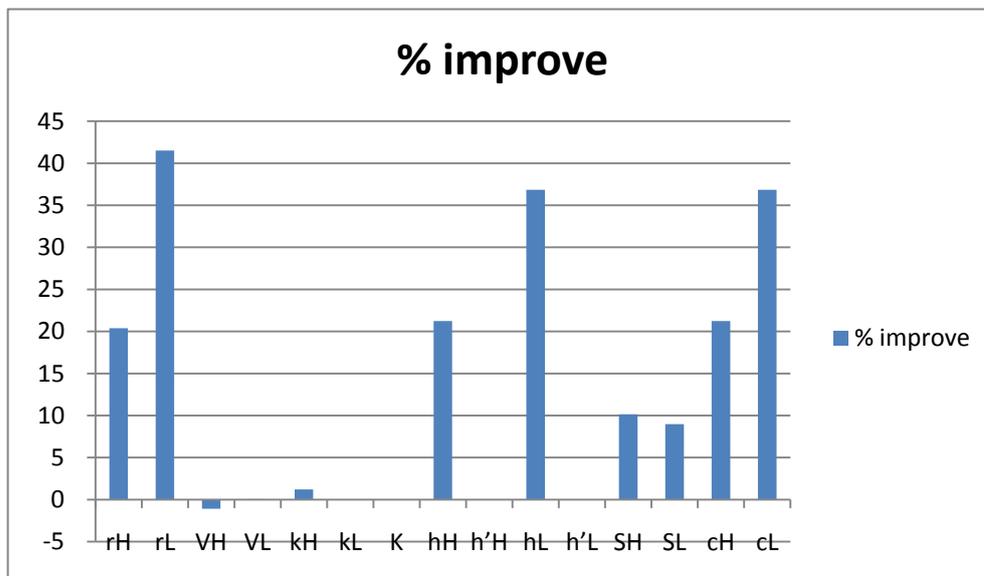
$r_L, S_H, c_L, h_L, S_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed by the same percentage at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 1)$ .

Model 4.1, Table I-II-I

Sensitivity analysis with parameter settings in step I (New Dimension I), step II (New Dimension II), step I (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 200)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit <sup>1</sup>	original	% improve	Pa % change
original			200	200	694	1	1	1	1883			
$r_H$	10	11	200	200	694	1	1	1	2267	1883	<b>20</b>	10
$r_L$	5	6	200	200	566	1	1	1	2664	1883	<b>42</b>	20
$V_H$	1	2	200	200	566	1	1	1	1862	1883	<b>-1</b>	100
$V_L$	1	2	200	200	694	1	1	1	1884	1883	<b>0</b>	100
$k^H$	200	201	201	201	694	1	1	1	1906	1883	<b>1</b>	1
$k^L$	1000	1001	200	200	694	1	1	1	1883	1883	<b>0</b>	0
K	1000	1001	200	200	694	1	1	1	1883	1883	<b>0</b>	0
$h_H$	1	0	200	200	694	1	1	1	2283	1883	<b>21</b>	100
$h'_H$	5	4	200	200	694	1	1	1	1883	1883	<b>0</b>	20
$h_L$	1	0	200	200	694	1	1	1	2577	1883	<b>37</b>	100
$h'_L$	3	2	200	200	694	1	1	1	1883	1883	<b>0</b>	33
$S_H$	8	7	200	200	694	1	1	1	2074	1883	<b>10</b>	13
$S_L$	4	3	200	200	566	1	1	1	2052	1883	<b>9</b>	25
$c_H$	4	3	200	200	694	1	1	1	2283	1883	<b>21</b>	25
$c_L$	2	1	200	200	694	1	1	1	2577	1883	<b>37</b>	50
											Average %	40

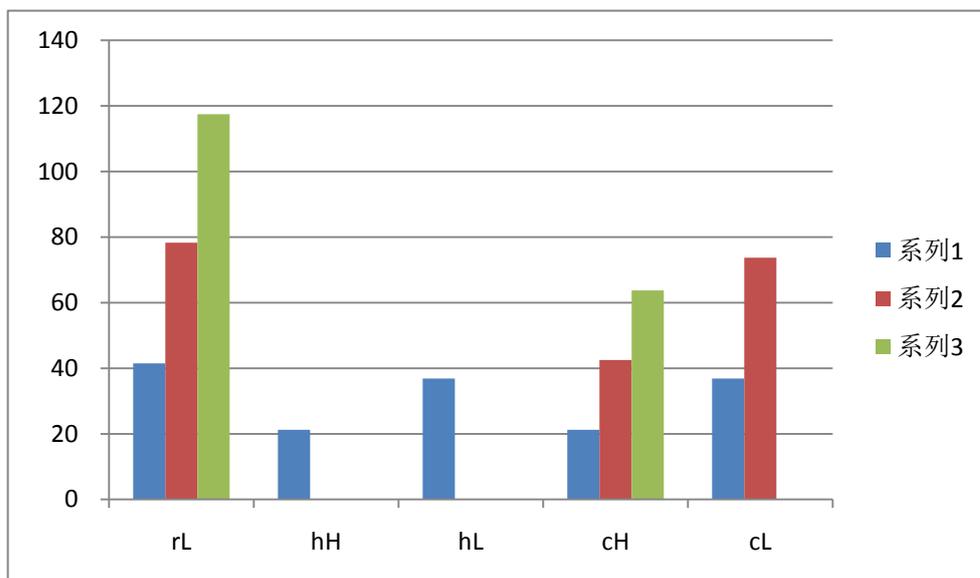
Graph I-II-I



Model 4.1, Table I-II-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			200	200	694	1	1	1	1883		
$r_L$	5	6	200	200	566	1	1	1	2664	1883	<b>42</b>
		7	200	200	694	1	1	1	3357	1883	<b>78</b>
		8	200	200	694	1	1	1	4094	1883	<b>117</b>
$h_H$	1	0	200	200	694	1	1	1	2283	1883	<b>21</b>
$h_L$	1	0	200	200	694	1	1	1	2577	1883	<b>37</b>
$c_H$	4	3	200	200	694	1	1	1	2283	1883	<b>21</b>
		2	200	200	694	1	1	1	2683	1883	<b>42</b>
		1	200	200	694	1	1	1	3083	1883	<b>64</b>
$c_L$	2	1	200	200	694	1	1	1	2577	1883	<b>37</b>
		0	200	200	694	1	1	1	3270	1883	<b>74</b>

Graph I-II-I-Elite

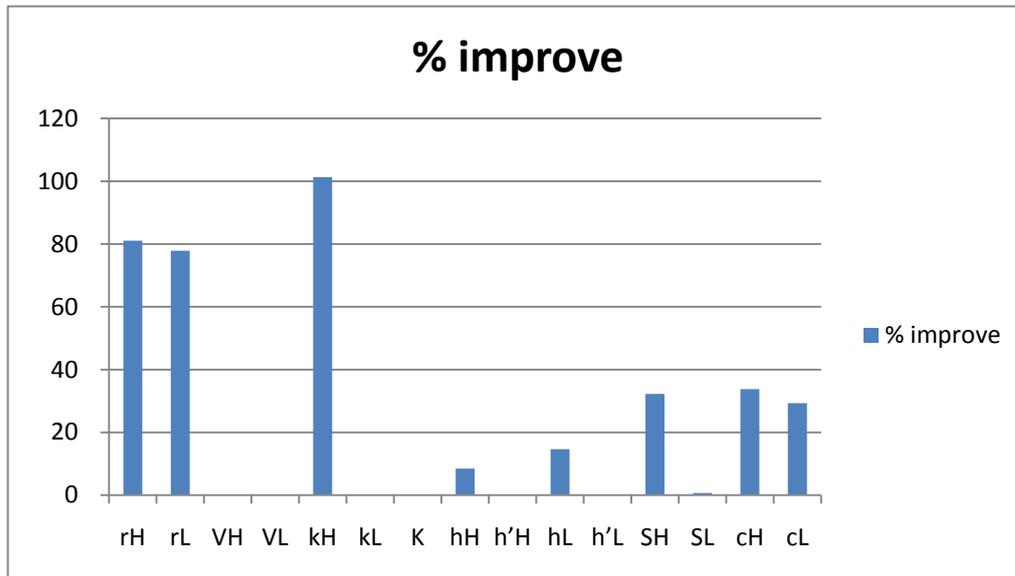


$r_L, c_L, h_L, c_H, h_H$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed one unit at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 200)$ .

Model 4.1, Table I-II-II Sensitivity analysis with parameter settings in step I (New Dimension I), step II (New Dimension II), step II (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 200)$ .

parameter	original	change 39.76889%	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			200	200	694	1	1	1	1883		
$r_H$	10	14	200	200	694	1	1	1	3409	1883	<b>81</b>
$r_L$	5	7	200	200	694	1	1	1	3349	1883	<b>78</b>
$V_H$	1	1	200	200	694	1	1	1	1883	1883	<b>0</b>
$V_L$	1	1	200	200	694	1	1	1	1883	1883	<b>0</b>
$k^H$	200	280	280	280	566	1	1	1	3790	1883	<b>101</b>
$k^L$	1000	1398	200	200	694	1	1	1	1883	1883	<b>0</b>
K	1000	1398	200	200	694	1	1	1	1883	1883	<b>0</b>
$h_H$	1	1	200	200	694	1	1	1	2042	1883	<b>8</b>
$h'_H$	5	3	200	200	694	1	1	1	1883	1883	<b>0</b>
$h_L$	1	1	200	200	694	1	1	1	2159	1883	<b>15</b>
$h'_L$	3	2	200	200	694	1	1	1	1883	1883	<b>0</b>
$S_H$	8	5	200	200	694	1	1	1	2491	1883	<b>32</b>
$S_L$	4	2	200	200	693	1	1	1	1896	1883	<b>1</b>
$c_H$	4	2	200	200	694	1	1	1	2519	1883	<b>34</b>
$c_L$	2	1	200	200	694	1	1	1	2435	1883	<b>29</b>

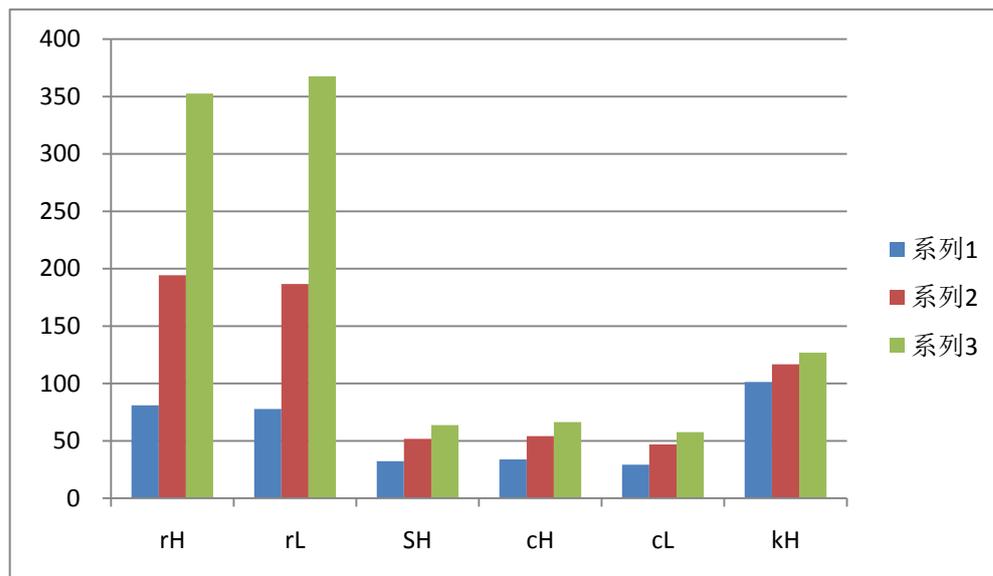
Graph I-II-II



Model 4.1, Table I-II-II-Elite Continue to relax the most important parameters in the above table by changing them by the same percentage at a time.

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			200	200	694	1	1	1	1883		
$r_H$	10	14	200	200	694	1	1	1	3409	1883	<b>81</b>
		20	200	200	694	1	1	1	5541	1883	<b>194</b>
		27	200	200	694	1	1	1	8522	1883	<b>353</b>
$r_L$	5	7	200	200	694	1	1	1	3349	1883	<b>78</b>
		10	200	200	694	1	1	1	5397	1883	<b>187</b>
		14	200	200	566	1	1	1	8803	1883	<b>368</b>
$S_H$	8	5	200	200	694	1	1	1	2491	1883	<b>32</b>
		3	198	200	694	1	1	1	2858	1883	<b>52</b>
		2	195	200	694	1	1	1	3084	1883	<b>64</b>
$c_H$	4	2	200	200	694	1	1	1	2519	1883	<b>34</b>
		1	200	200	694	1	1	1	2902	1883	<b>54</b>
		1	200	200	694	1	1	1	3133	1883	<b>66</b>
$c_L$	2	1	200	200	694	1	1	1	2435	1883	<b>29</b>
		1	200	200	694	1	1	1	2767	1883	<b>47</b>
		0	200	200	694	1	1	1	2967	1883	<b>58</b>
$k^H$	200	280	280	280	566	1	1	1	3790	1883	<b>101</b>
		391	391	201	639	1	1	1	4081	1883	<b>117</b>
		546	492	181	566	1	1	1	4272	1883	<b>127</b>

Graph I-II-II-Elite

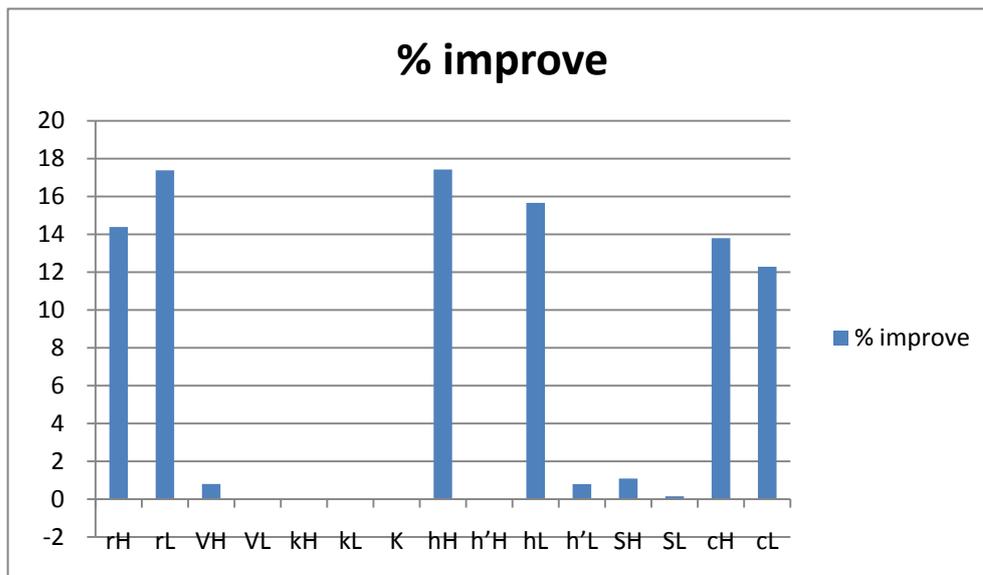


$r_L, r_H, k^H$  are the most important parameters (the first two are more sensitive but  $k^H$  better improves the profit at the beginning) to the retailer (from the most important to the least important) when parameters are changed by the same percentage at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 200)$ .

Model 4.1, Table I-III-I Sensitivity analysis with parameter settings in step I (New Dimension I), step III (New Dimension II), step I (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 400)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^L$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve	
original		387	195	693	1	1	1	4227				
$r_H$	10	11	395	191	693	1	1	1	4836	4227	14	<b>10</b>
$r_L$	5	6	387	195	693	1	1	1	4963	4227	17	<b>20</b>
$V_H$	1	2	382	289	566	1	1	1	4261	4227	1	<b>100</b>
$V_L$	1	2	387	195	693	1	1	1	4226	4227	0	<b>100</b>
$k^H$	400	401	387	195	693	1	1	1	4227	4227	0	<b>0</b>
$k^L$	1000	1001	387	195	693	1	1	1	4227	4227	0	<b>0</b>
K	1000	1001	387	195	693	1	1	1	4227	4227	0	<b>0</b>
$h_H$	1	0	397	278	566	1	1	1	4964	4227	17	<b>100</b>
$h'_H$	5	4	387	195	693	1	1	1	4227	4227	0	<b>20</b>
$h_L$	1	0	380	197	693	1	1	1	4890	4227	16	<b>100</b>
$h'_L$	3	2	396	192	693	1	1	1	4261	4227	1	<b>33</b>
$S_H$	8	7	388	193	693	1	1	1	4273	4227	1	<b>13</b>
$S_L$	4	3	387	195	693	1	1	1	4234	4227	0	<b>25</b>
$c_H$	4	3	397	192	693	1	1	1	4811	4227	14	<b>25</b>
$c_L$	2	1	400	197	639	1	1	1	4747	4227	12	<b>50</b>
average												40

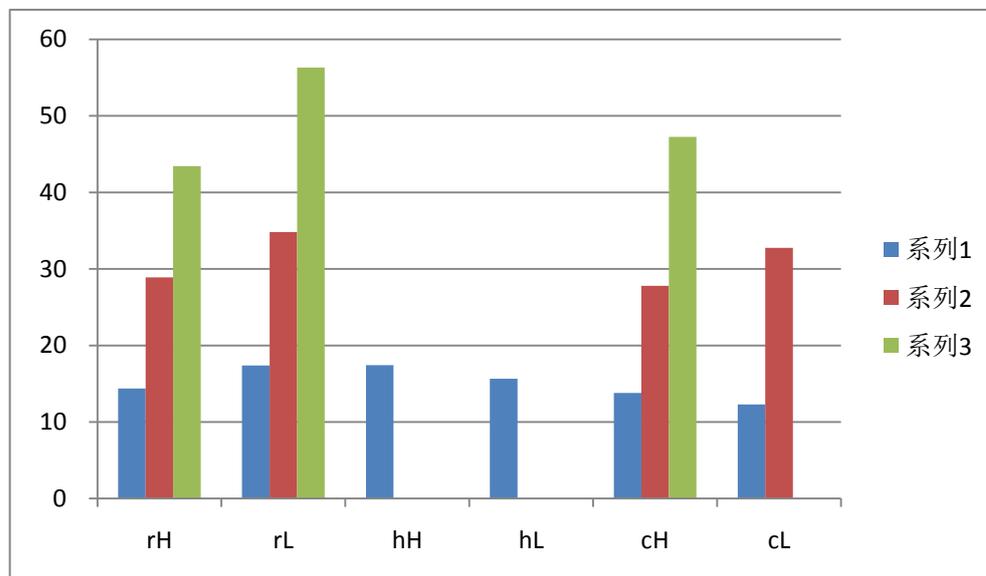
Graph I-III-I



Model 4.1, Table I-III-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Orininal	% improve
original			387	195	693	1	1	1	4227		
$r_H$	10	11	395	191	693	1	1	1	4836	4227	<b>14</b>
		12	400	189	693	1	1	1	5450	4227	<b>29</b>
		13	400	190	693	1	1	1	6064	4227	<b>43</b>
$r_L$	5	6	387	195	693	1	1	1	4963	4227	<b>17</b>
		7	387	195	693	1	1	1	5700	4227	<b>35</b>
		8	385	286	566	1	1	1	6608	4227	<b>56</b>
$h_H$	1	0	397	278	566	1	1	1	4964	4227	<b>17</b>
$h_L$	1	0	380	197	693	1	1	1	4890	4227	<b>16</b>
$c_H$	4	3	397	192	693	1	1	1	4811	4227	<b>14</b>
		2	400	193	693	1	1	1	5403	4227	<b>28</b>
		1	400	286	566	1	1	1	6225	4227	<b>47</b>
$c_L$	2	1	400	197	639	1	1	1	4747	4227	<b>12</b>
		0	387	195	694	1	1	1	5612	4227	<b>33</b>

Graph I-III-I Elite

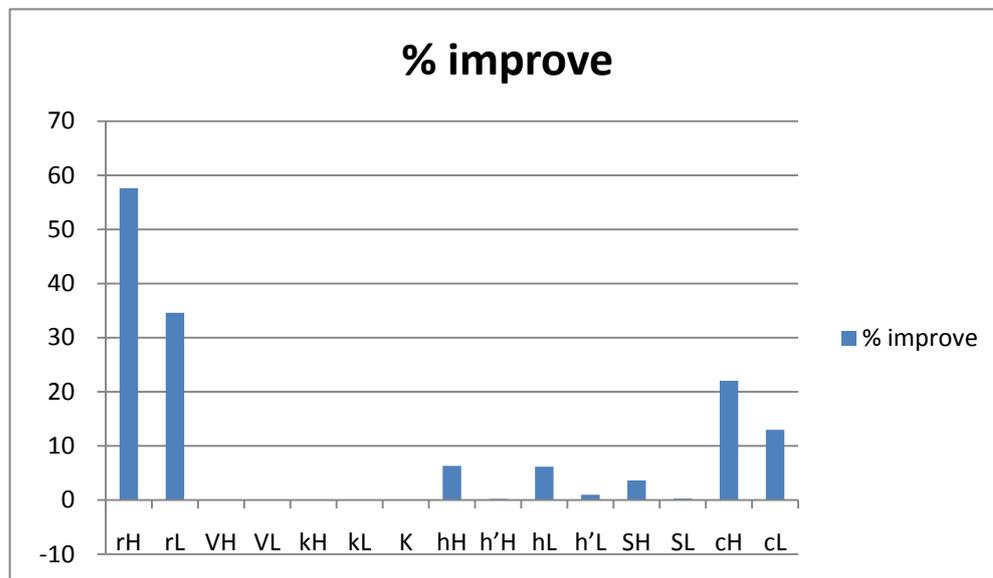


$r_L, h_H, r_H, h_L, c_H,$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed one unit at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 400)$ .

Model 4.1, Table I-III-II Sensitivity analysis with parameter settings in step I (New Dimension I), step III (New Dimension II), step II (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 400)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Orininal	% improve
original			387	195	693	1	1	1	4227		
$r_H$	10	14	400	190	693	1	1	1	6663	4227	<b>58</b>
$r_L$	5	7	387	195	693	1	1	1	5690	4227	<b>35</b>
$V_H$	1	1	388	194	693	1	1	1	4228	4227	<b>0</b>
$V_L$	1	1	387	195	693	1	1	1	4226	4227	<b>0</b>
$k^H$	400	559	387	195	693	1	1	1	4225	4227	<b>0</b>
$k^L$	1000	1398	387	195	693	1	1	1	4225	4227	<b>0</b>
K	1000	1398	387	195	693	1	1	1	4225	4227	<b>0</b>
$h_H$	1	1	398	191	693	1	1	1	4494	4227	<b>6</b>
$h'_H$	5	3	389	194	693	1	1	1	4237	4227	<b>0</b>
$h_L$	1	1	384	195	693	1	1	1	4489	4227	<b>6</b>
$h'_L$	3	2	398	191	693	1	1	1	4269	4227	<b>1</b>
$S_H$	8	5	389	190	693	1	1	1	4381	4227	<b>4</b>
$S_L$	4	2	387	195	693	1	1	1	4238	4227	<b>0</b>
$c_H$	4	2	400	192	693	1	1	1	5160	4227	<b>22</b>
$c_L$	2	1	387	194	693	1	1	1	4777	4227	<b>13</b>

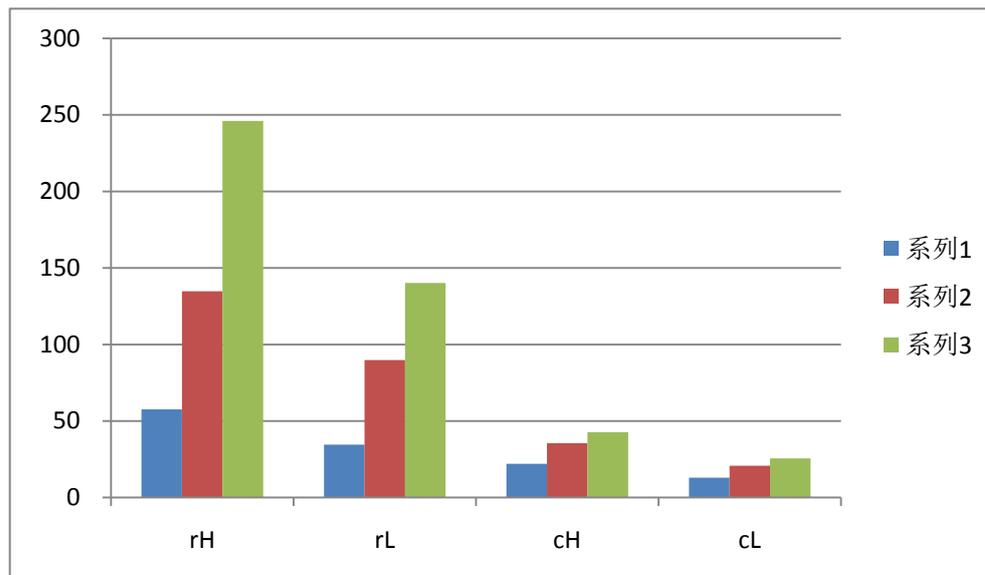
Graph I-III-II



Model 4.1, Table I-III-II-Elite Continue to relax the most important parameters in the above table by changing them by the same percentage at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Orininal	% improve
original			387	195	693	1	1	1	4227		
$r_H$	10	14	400	190	693	1	1	1	6663	4227	<b>58</b>
		20	400	197	640	1	1	1	9925	4227	<b>135</b>
		27	400	263	692	1	1	1	14631	4227	<b>246</b>
$r_L$	5	7	387	195	693	1	1	1	5690	4227	<b>35</b>
		10	385	286	566	1	1	1	8024	4227	<b>90</b>
		14	352	264	693	1	1	1	10157	4227	<b>140</b>
$c_H$	4	2	400	192	693	1	1	1	5160	4227	<b>22</b>
		1	400	194	693	1	1	1	5728	4227	<b>35</b>
		1	400	265	692	1	1	1	6031	4227	<b>43</b>
$c_L$	2	1	387	195	693	1	1	1	4777	4227	<b>13</b>
		1	387	195	693	1	1	1	5109	4227	<b>21</b>
		0	387	195	694	1	1	1	5309	4227	<b>26</b>

Graph I-III-II Elite

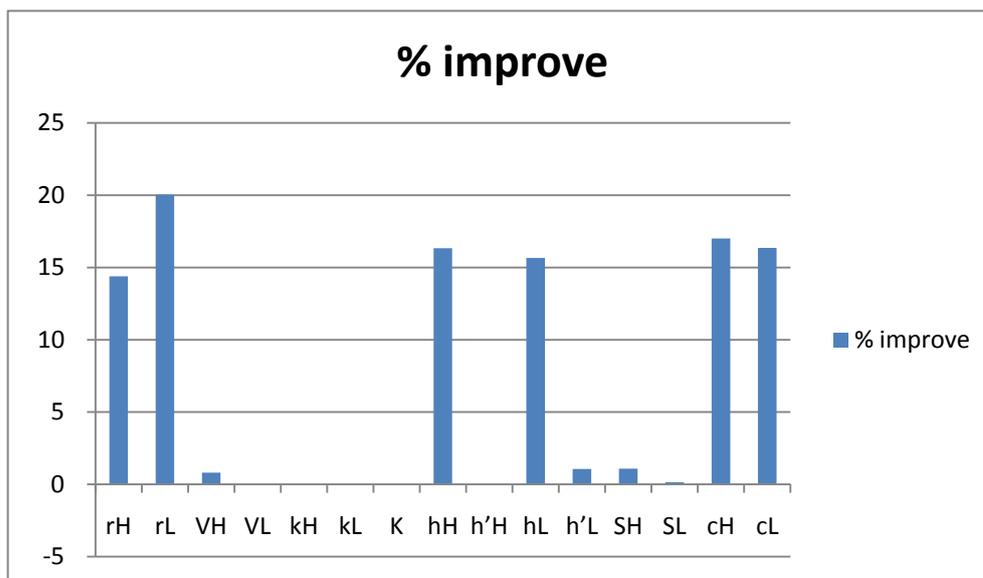


$r_H, r_L, c_H, c_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed by the same percentage at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 400)$ .

Model 4.1, Table I-IV-I Sensitivity analysis with parameter settings in step I (New Dimension I), step IV (New Dimension II), step I (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 500)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve	% change
original			387	195	693	1	1	1	4227			
$r_H$	10	11	395	192	693	1	1	1	4836	4227	<b>14</b>	10
$r_L$	5	6	492	181	566	1	1	1	5074	4227	<b>20</b>	20
$V_H$	1	2	382	289	566	1	1	1	4261	4227	<b>1</b>	100
$V_L$	1	2	386	195	693	1	1	1	4226	4227	<b>0</b>	100
$k^H$	500	501	387	195	693	1	1	1	4227	4227	<b>0</b>	0
$k^L$	1000	1001	387	195	693	1	1	1	4227	4227	<b>0</b>	0
K	1000	1001	387	195	693	1	1	1	4227	4227	<b>0</b>	0
$h_H$	1	0	421	184	693	1	1	1	4918	4227	<b>16</b>	100
$h'_H$	5	4	387	195	693	1	1	1	4227	4227	<b>0</b>	20
$h_L$	1	0	380	197	693	1	1	1	4890	4227	<b>16</b>	100
$h'_L$	3	2	492	181	566	1	1	1	4272	4227	<b>1</b>	33
$S_H$	8	7	388	193	693	1	1	1	4273	4227	<b>1</b>	13
$S_L$	4	3	387	195	693	1	1	1	4234	4227	<b>0</b>	25
$c_H$	4	3	495	181	566	1	1	1	4947	4227	<b>17</b>	25
$c_L$	2	1	387	195	693	1	1	1	4919	4227	<b>16</b>	50
average												40

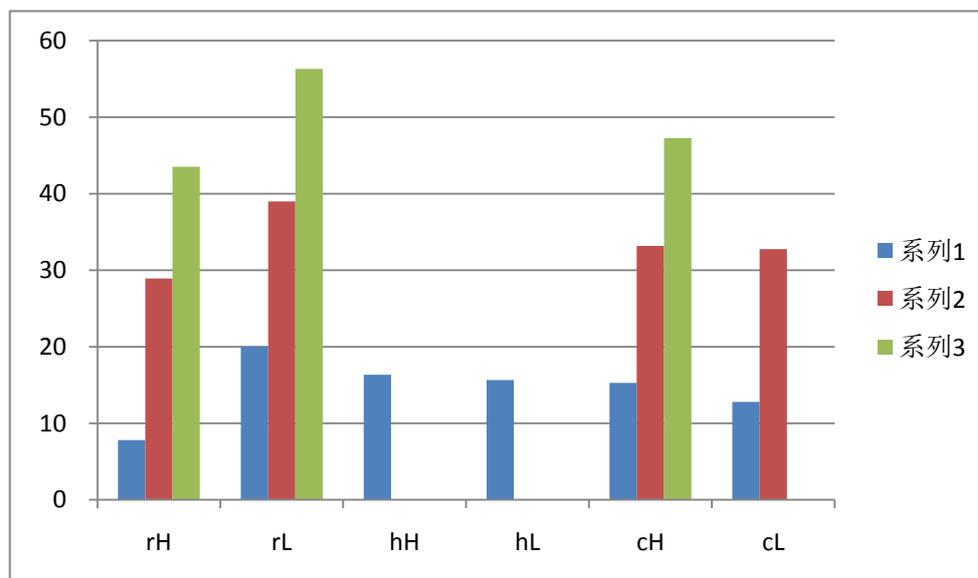
Graph I-IV-I



Model 4.1, Table I-IV-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			387	195	693	1	1	1	4227		
$r_H$	10	11	355	281	637	1	1	1	4557	4227	<b>8</b>
		12	403	188	693	1	1	1	5450	4227	<b>29</b>
		13	410	186	693	1	1	1	6067	4227	<b>44</b>
$r_L$	5	6	492	181	566	1	1	1	5074	4227	<b>20</b>
		7	492	181	566	1	1	1	5876	4227	<b>39</b>
		8	385	286	566	1	1	1	6608	4227	<b>56</b>
$h_H$	1	0	421	184	693	1	1	1	4918	4227	<b>16</b>
$h_L$	1	0	378	197	693	1	1	1	4889	4227	<b>16</b>
$c_H$	4	3	387	286	566	1	1	1	4873	4227	<b>15</b>
		2	500	185	566	1	1	1	5630	4227	<b>33</b>
		1	417	283	566	1	1	1	6226	4227	<b>47</b>
$c_L$	2	1	417	187	638	1	1	1	4769	4227	<b>13</b>
		0	387	194	694	1	1	1	5612	4227	<b>33</b>

Graph I-IV-I-Elite



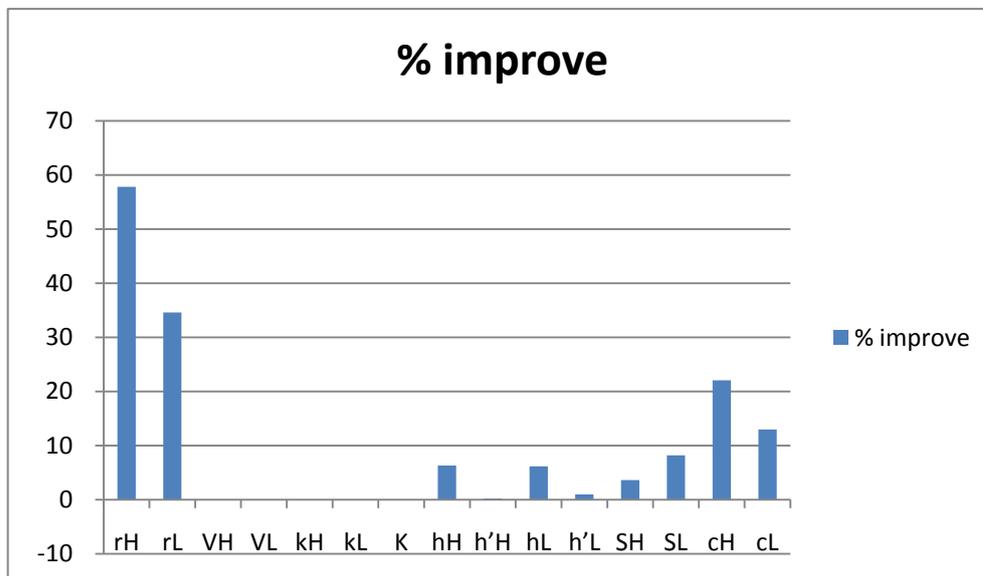
$r_L$  is the most important.  $h_H$ ,  $h_L$ ,  $c_L$ ,  $c_H$ , are the second most important parameters to the retailer (from the most important to the least important) when parameters are changed one unit at a time and

$$(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 500).$$

Model 4.1, Table I-IV-II Sensitivity analysis with parameter settings in step I (New Dimension I), step IV (New Dimension II), step II (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 500)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve	% change
original			387	195	693	1	1	1	4227			
$r_H$	10	14	417	184	693	1	1	1	6672	4227	<b>58</b>	10
$r_L$	5	7	387	195	693	1	1	1	5690	4227	<b>35</b>	20
$V_H$	1	1	388	194	693	1	1	1	4228	4227	<b>0</b>	100
$V_L$	1	1	387	195	693	1	1	1	4226	4227	<b>0</b>	100
$k^H$	500	699	387	195	693	1	1	1	4227	4227	<b>0</b>	0
$k^L$	1000	1397	387	195	693	1	1	1	4227	4227	<b>0</b>	0
K	1000	1397	387	195	693	1	1	1	4227	4227	<b>0</b>	0
$h_H$	1	1	398	191	693	1	1	1	4494	4227	<b>6</b>	100
$h'_H$	5	3	389	194	693	1	1	1	4237	4227	<b>0</b>	20
$h_L$	1	1	384	195	693	1	1	1	4489	4227	<b>6</b>	100
$h'_L$	3	2	398	191	693	1	1	1	4269	4227	<b>1</b>	33
$S_H$	8	5	389	190	693	1	1	1	4381	4227	<b>4</b>	13
$S_L$	4	2	493	181	566	1	1	1	4574	4227	<b>8</b>	25
$c_H$	4	2	405	190	693	1	1	1	5160	4227	<b>22</b>	25
$c_L$	2	1	387	195	693	1	1	1	4777	4227	<b>13</b>	50
average												40

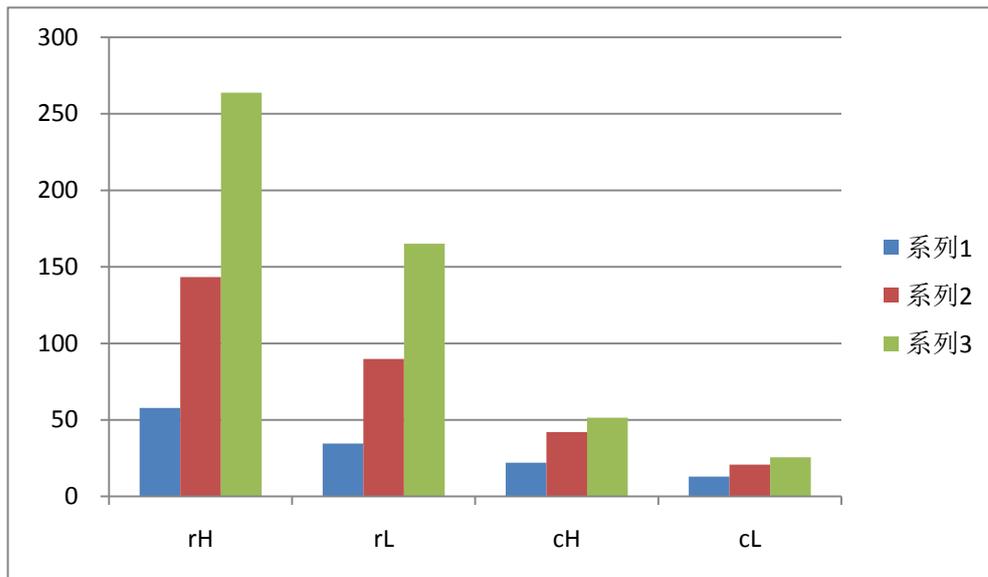
Graph I-IV-II



Model 4.1, Table I-IV-II-Elite Continue to relax the most important parameters in the above table by changing them by the same percentage at a time.

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			387	195	693	1	1	1	4227		
$r_H$	10	14	417	184	693	1	1	1	6672	4227	<b>58</b>
		20	500	181	693	1	1	1	10286	4227	<b>143</b>
		27	500	183	693	1	1	1	15381	4227	<b>264</b>
$r_L$	5	7	387	195	693	1	1	1	5690	4227	<b>35</b>
		10	385	286	566	1	1	1	8024	4227	<b>90</b>
		14	493	181	566	1	1	1	11208	4227	<b>165</b>
$c_H$	4	2	405	190	693	1	1	1	5160	4227	<b>22</b>
		1	500	187	566	1	1	1	6006	4227	<b>42</b>
		1	500	189	566	1	1	1	6403	4227	<b>51</b>
$c_L$	2	1	387	195	693	1	1	1	4777	4227	<b>13</b>
		1	387	195	693	1	1	1	5109	4227	<b>21</b>
		0	387	195	694	1	1	1	5309	4227	<b>26</b>

Graph I-IV-II-Elite

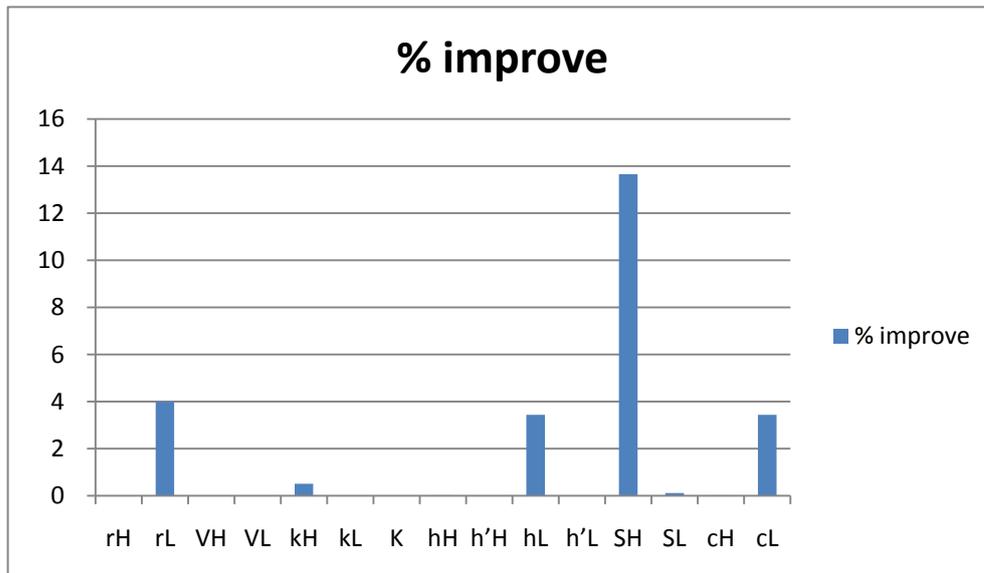


$r_H, r_L, c_H, c_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed by the same percentage at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (300, 300, 700, 600, 600, 70, 500)$ .

Model 4.1, Table III-I-I Sensitivity analysis with parameter settings in step III (New Dimension I), step I (New Dimension II), step I (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 1)$ .

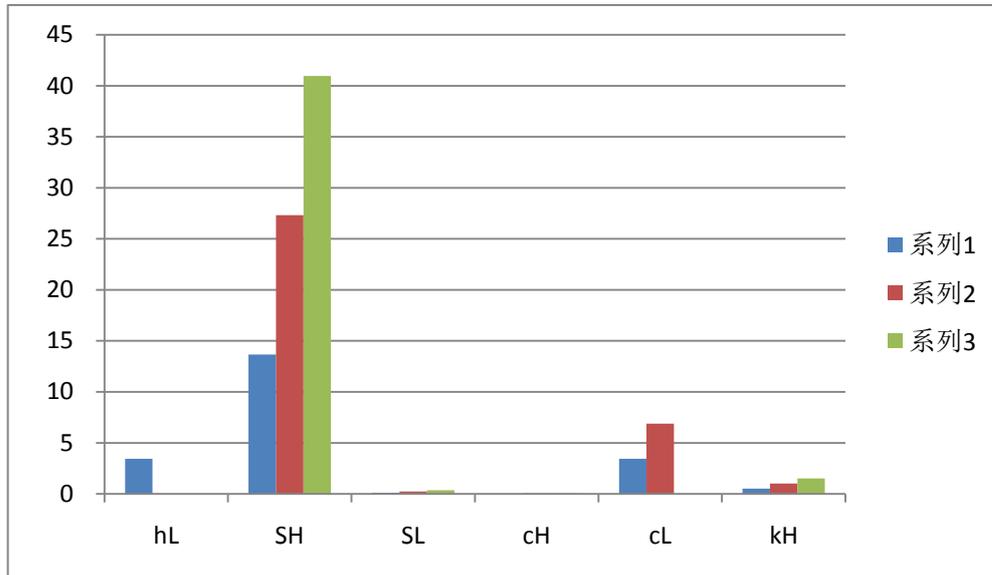
parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit *	Orininal	% improve	% para
original			1	1	292	1	1	1	-8476			
$r_H$	10	11	1	1	292	1	1	1	-8475	-8476	0	10
$r_L$	5	6	1	1	292	1	1	1	-8138	-8476	4	20
$V_H$	1	2	1	1	292	1	1	1	-8476	-8476	0	100
$V_L$	1	2	1	1	292	1	1	1	-8476	-8476	0	100
$k^H$	1	2	2	2	292	1	1	1	-8433	-8476	1	100
$k^L$	1000	1001	1	1	292	1	1	1	-8476	-8476	0	0
K	1000	1001	1	1	292	1	1	1	-8476	-8476	0	0
$h_H$	1	0	1	1	292	1	1	1	-8474	-8476	0	100
$h'_H$	5	4	1	1	292	1	1	1	-8476	-8476	0	20
$h_L$	1	0	1	1	292	1	1	1	-8185	-8476	3	100
$h'_L$	3	2	1	1	292	1	1	1	-8476	-8476	0	33
$S_H$	8	7	1	1	292	1	1	1	-7319	-8476	14	13
$S_L$	4	3	1	1	292	1	1	1	-8466	-8476	0	25
$c_H$	4	3	1	1	292	1	1	1	-8474	-8476	0	25
$c_L$	2	1	1	1	292	1	1	1	-8185	-8476	3	50
average												46

Graph III-I-I



Model 4.1, Table III-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

Graph III-I-I-Elite

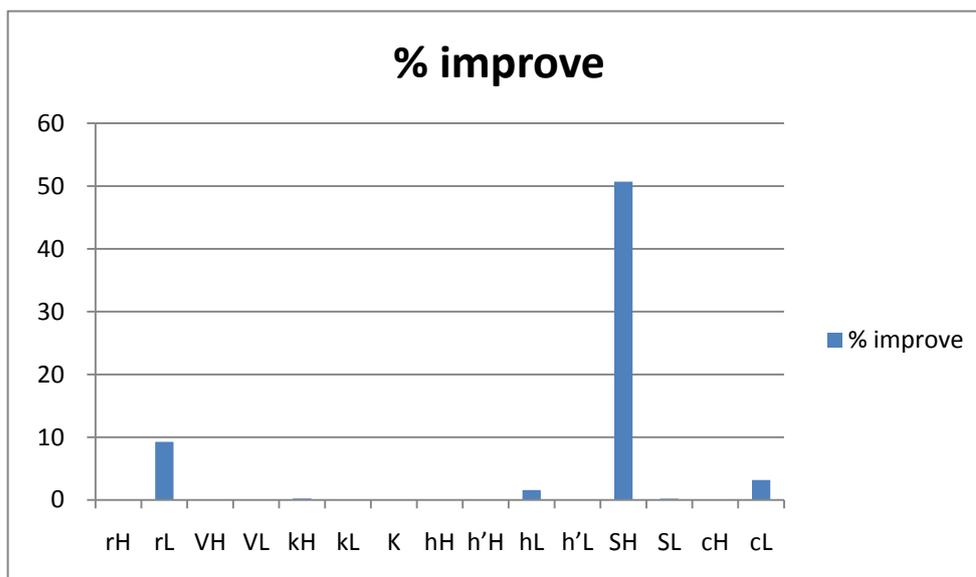


$S_H, c_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed one unit at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 1)$ .

Model 4.1, Table III-I-II Sensitivity analysis with parameter settings in step III (New Dimension I), step I (New Dimension II), step II Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 1)$ .

parameter	original	change 1	$X_0^{H^*}$	$X_1^{H^*}$	$X_0^{L^*}$	$a_0^{H^*}$	$a_1^{H^*}$	$a_1^{L^*}$	Profit*	Original	% improve
original			1	1	292	1	1	1	-8476		
$r_H$	10	15	1	1	292	1	1	1	-8469	-8476	0
$r_L$	5	7	1	1	292	1	1	1	-7691	-8476	9
$V_H$	1	1	1	1	292	1	1	1	-8476	-8476	0
$V_L$	1	1	1	1	292	1	1	1	-8476	-8476	0
$k^H$	1	1	1	1	292	1	1	1	-8456	-8476	0
$k^L$	1000	1464	1	1	292	1	1	1	-8476	-8476	0
K	1000	1464	1	1	292	1	1	1	-8476	-8476	0
$h_H$	1	1	1	1	292	1	1	1	-8475	-8476	0
$h'_H$	5	3	1	1	292	1	1	1	-8476	-8476	0
$h_L$	1	1	1	1	292	1	1	1	-8341	-8476	2
$h'_L$	3	2	1	1	292	1	1	1	-8476	-8476	0
$S_H$	8	4	1	1	292	1	1	1	-4179	-8476	51
$S_L$	4	2	1	1	292	1	1	1	-8458	-8476	0
$c_H$	4	2	1	1	292	1	1	1	-8473	-8476	0
$c_L$	2	1	1	1	292	1	1	1	-8205	-8476	3

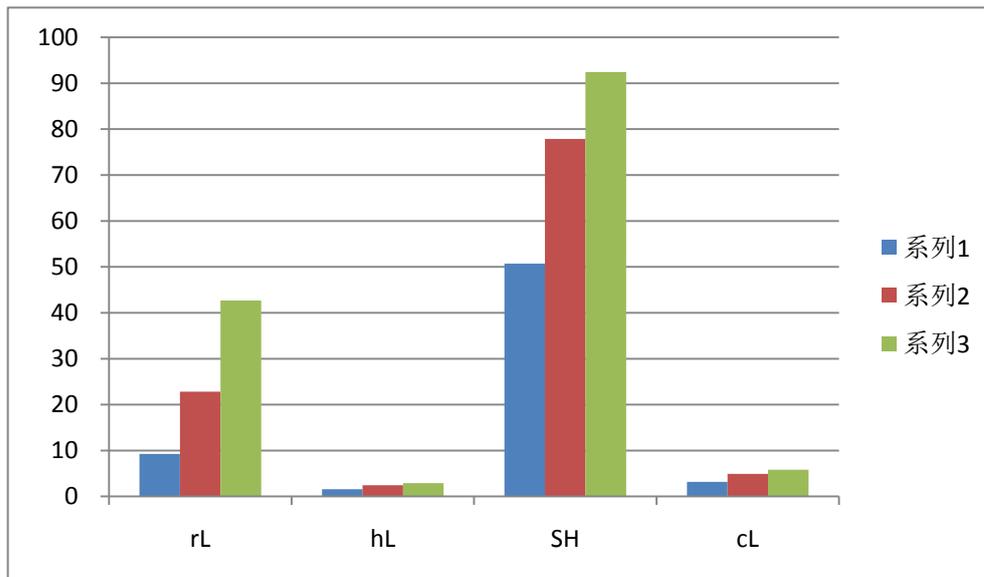
Graph III-I-II



Model 4.1, Table III-I-II-Elite Continue to relax the most important parameters in the above table by changing them by the same percentage at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	Original	% improve
original			1	1	292	1	1	1	-8476		
$r_L$	5	7	1	1	292	1	1	1	-7691	-8476	<b>9</b>
		11	1	1	292	1	1	1	-6541	-8476	<b>23</b>
		16	1	1	292	1	1	1	-4858	-8476	<b>43</b>
$h_L$	1	1	1	1	292	1	1	1	-8341	-8476	<b>2</b>
		0	1	1	292	1	1	1	-8268	-8476	<b>2</b>
		0	1	1	292	1	1	1	-8229	-8476	<b>3</b>
$S_H$	8	4	1	1	292	1	1	1	-4179	-8476	<b>51</b>
		2	1	1	292	1	1	1	-1876	-8476	<b>78</b>
		1	1	1	292	1	1	1	-641	-8476	<b>92</b>
$c_L$	2	1	1	1	292	1	1	1	-8205	-8476	<b>3</b>
		1	1	1	292	1	1	1	-8060	-8476	<b>5</b>
		0	1	1	292	1	1	1	-7983	-8476	<b>6</b>

Graph III-I-II-Elite

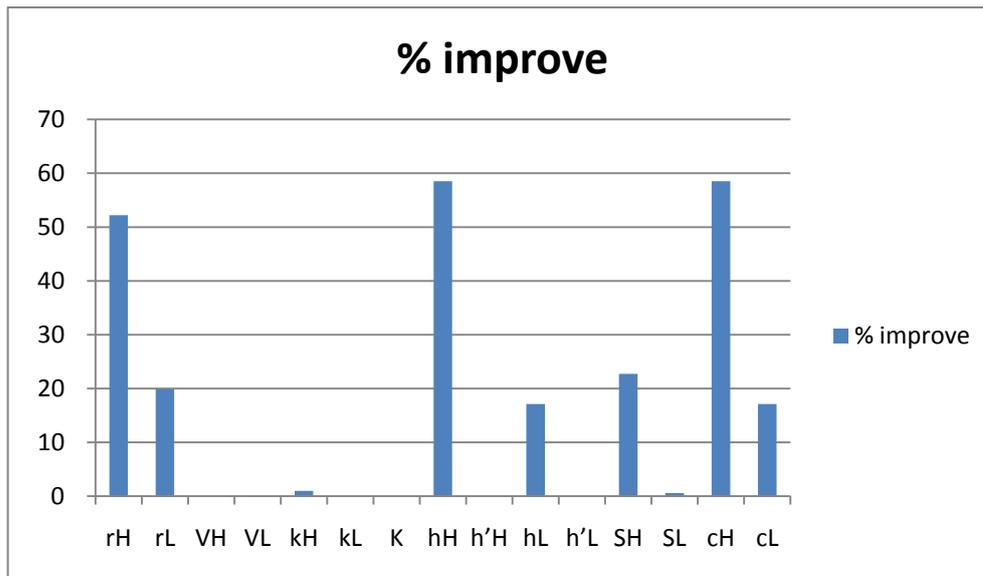


$S_H, r_L, c_L, h_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed by the same percentage at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 1)$ .

Model 4.1, Table III-II-I Sensitivity analysis with parameter settings in step III (New Dimension I), step II (New Dimension II), step I (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 500)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve	% change
original			500	485	292	1	1	1	1704			
$r_H$	10	11	500	479	292	1	1	1	2594	1704	<b>52</b>	10
$r_L$	5	6	500	485	292	1	1	1	2043	1704	<b>20</b>	20
$V_H$	1	2	500	485	292	1	1	1	1704	1704	<b>0</b>	100
$V_L$	1	2	500	485	292	1	1	1	1704	1704	<b>0</b>	100
$k^H$	500	501	501	486	292	1	1	1	1721	1704	<b>1</b>	0
$k^L$	1000	1001	500	485	292	1	1	1	1704	1704	<b>0</b>	0
K	1000	1001	500	485	292	1	1	1	1704	1704	<b>0</b>	0
$h_H$	1	0	500	500	292	1	1	1	2702	1704	<b>59</b>	100
$h'_H$	5	4	500	485	292	1	1	1	1704	1704	<b>0</b>	20
$h_L$	1	0	500	485	292	1	1	1	1996	1704	<b>17</b>	100
$h'_L$	3	2	500	485	292	1	1	1	1704	1704	<b>0</b>	33
$S_H$	8	7	500	474	292	1	1	1	2092	1704	<b>23</b>	13
$S_L$	4	3	500	485	292	1	1	1	1714	1704	<b>1</b>	25
$c_H$	4	3	500	500	292	1	1	1	2702	1704	<b>59</b>	25
$c_L$	2	1	500	485	292	1	1	1	1996	1704	<b>17</b>	50
average												40

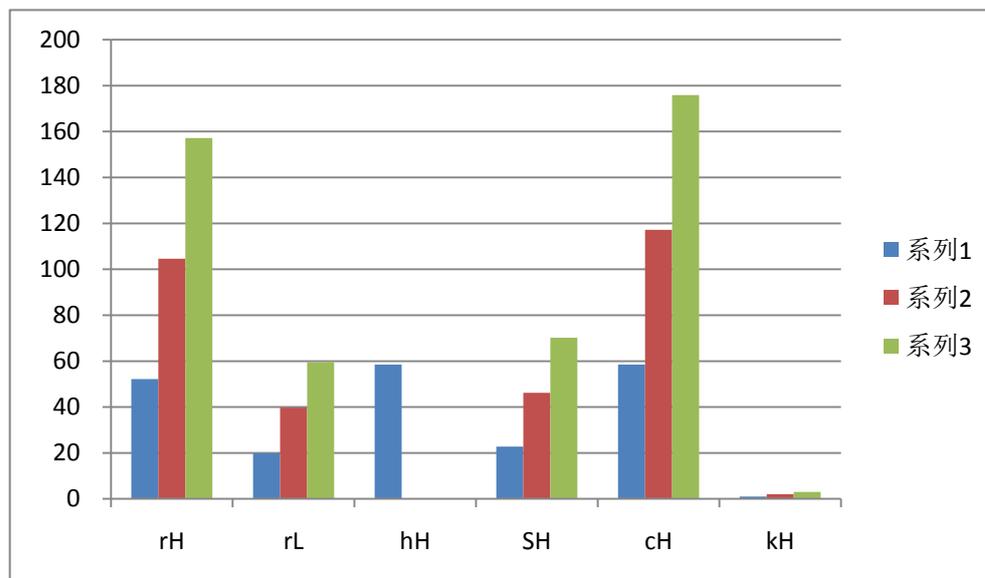
Graph III-II-I



Model 4.1, Table III-II-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			500	485	292	1	1	1	1704		
$r_H$	10	11	500	479	292	1	1	1	2594	1704	<b>52</b>
		12	500	476	292	1	1	1	3487	1704	<b>105</b>
		13	500	474	292	1	1	1	4382	1704	<b>157</b>
$r_L$	5	6	500	485	292	1	1	1	2043	1704	<b>20</b>
		7	500	485	292	1	1	1	2381	1704	<b>40</b>
		8	500	485	292	1	1	1	2720	1704	<b>60</b>
$h_H$	1	0	500	500	292	1	1	1	2702	1704	<b>59</b>
$S_H$	8	7	500	474	292	1	1	1	2092	1704	<b>23</b>
		6	500	468	292	1	1	1	2492	1704	<b>46</b>
		5	500	464	292	1	1	1	2900	1704	<b>70</b>
$c_H$	4	3	500	500	292	1	1	1	2702	1704	<b>59</b>
		2	500	500	292	1	1	1	3702	1704	<b>117</b>
		1	500	500	292	1	1	1	4702	1704	<b>176</b>
$k_H$	500	501	501	486	292	1	1	1	1721	1704	<b>1</b>
		502	502	486	292	1	1	1	1739	1704	<b>2</b>
		503	503	486	292	1	1	1	1756	1704	<b>3</b>

Graph III-II-I-Elite

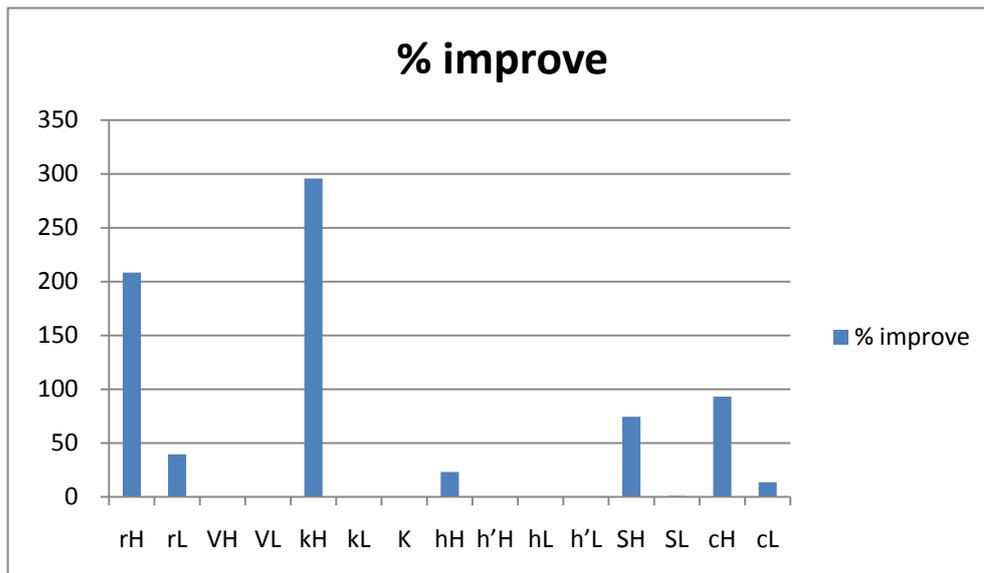


$c_H, h_H, r_H, S_H, r_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed one unit at a time and  $(\mu_1^H, \mu_2^H, \mu_2^L, \sigma_1^H, \sigma_2^H, \sigma_2^L, k^H) = (700, 700, 300, 1400, 1400, 30, 500)$ .

Model 4.1, Table III-II-II Sensitivity analysis with parameter settings in step III (New Dimension I), step II (New Dimension II), step II Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 500)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			500	485	292	1	1	1	1704		
$r_H$	10	14	500	472	292	1	1	1	5256	1704	<b>208</b>
$r_L$	5	7	500	485	292	1	1	1	2377	1704	<b>39</b>
$V_H$	1	1	500	485	292	1	1	1	1704	1704	<b>0</b>
$V_L$	1	1	500	485	292	1	1	1	1704	1704	<b>0</b>
$k^H$	500	699	699	655	292	1	1	1	6745	1704	<b>296</b>
$k^L$	1000	1397	500	485	292	1	1	1	1704	1704	<b>0</b>
K	1000	1397	500	485	292	1	1	1	1704	1704	<b>0</b>
$h_H$	1	1	500	500	292	1	1	1	2099	1704	<b>23</b>
$h'_H$	5	3	500	485	292	1	1	1	1704	1704	<b>0</b>
$h_L$	1	1	500	485	292	1	1	1	1820	1704	<b>0</b>
$h'_L$	3	2	500	485	292	1	1	1	1704	1704	<b>0</b>
$S_H$	8	5	500	463	292	1	1	1	2974	1704	<b>75</b>
$S_L$	4	2	500	485	292	1	1	1	1720	1704	<b>1</b>
$c_H$	4	2	500	500	292	1	1	1	3292	1704	<b>93</b>
$c_L$	2	1	500	485	292	1	1	1	1936	1704	<b>14</b>

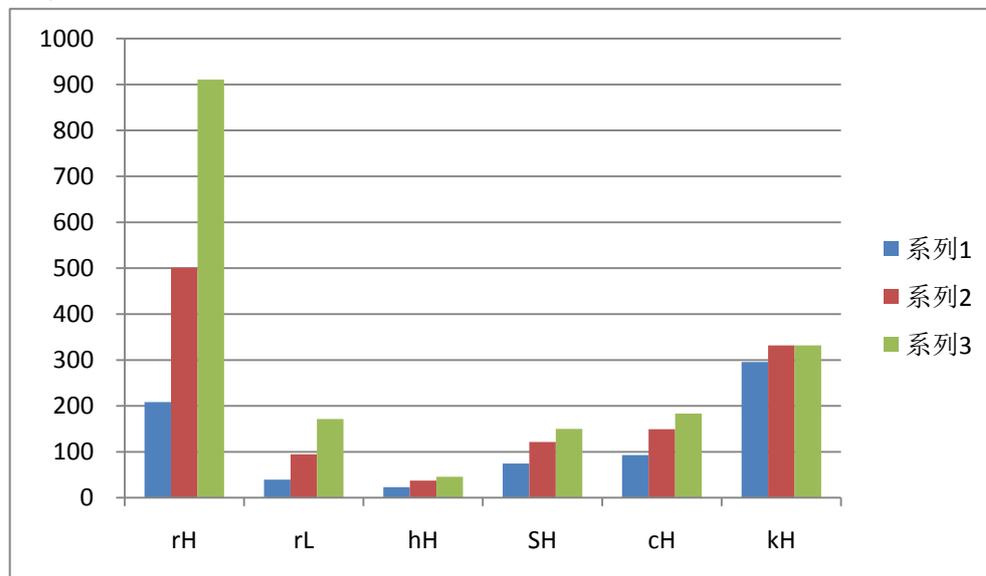
Graph III-II-II



Model 4.1, Table III-II-II-Elite Continue to relax the most important parameters in the above table by changing them by the same percentage at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			500	485	292	1	1	1	1704		
$r_H$	10	14	500	472	292	1	1	1	5256	1704	<b>208</b>
		20	500	467	292	1	1	1	10245	1704	<b>501</b>
		27	500	464	292	1	1	1	17227	1704	<b>911</b>
$r_L$	5	7	500	485	292	1	1	1	2377	1704	<b>39</b>
		10	500	485	292	1	1	1	3317	1704	<b>95</b>
		14	500	485	292	1	1	1	4631	1704	<b>172</b>
$h_H$	1	1	500	500	292	1	1	1	2099	1704	<b>23</b>
		0	500	500	292	1	1	1	2339	1704	<b>37</b>
		0	500	500	292	1	1	1	2483	1704	<b>46</b>
$S_H$	8	5	500	463	292	1	1	1	2974	1704	<b>75</b>
		3	500	457	292	1	1	1	3773	1704	<b>121</b>
		2	500	454	292	1	1	1	4260	1704	<b>150</b>
$c_H$	4	2	500	500	292	1	1	1	3292	1704	<b>93</b>
		1	500	500	292	1	1	1	4250	1704	<b>149</b>
		1	500	500	292	1	1	1	4827	1704	<b>183</b>
$k^H$	500	699	699	655	292	1	1	1	6745	1704	<b>296</b>
		976	793	575	292	1	1	1	7360	1704	<b>332</b>
		1365	793	575	292	1	1	1	7360	1704	<b>332</b>

Graph III-II-II-Elite

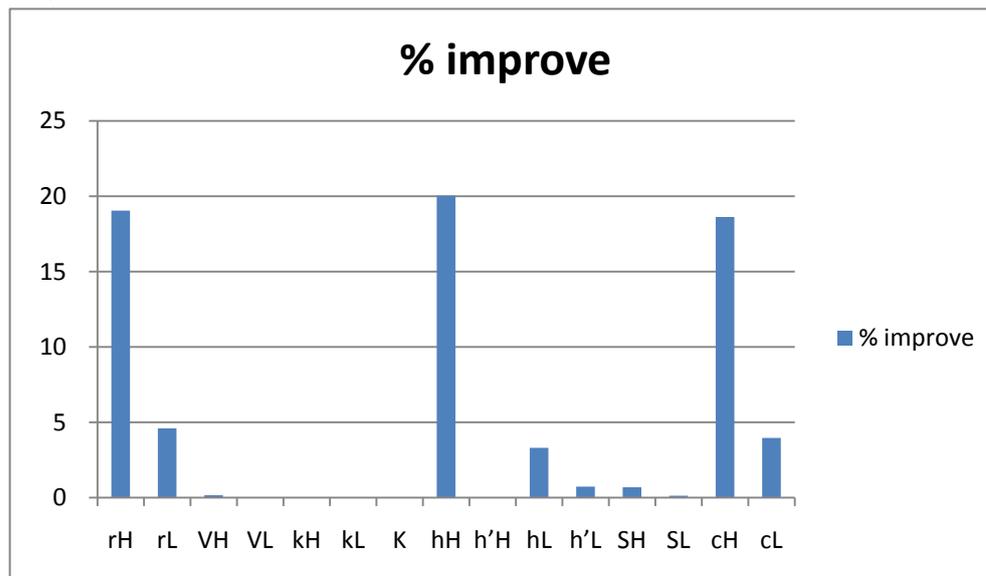


$r_H, k^H, c_H, S_H, r_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed by the same percentage at a time and  $(\mu_1^H, \mu_2^H, \mu_2^L, \sigma_1^H, \sigma_2^H, \sigma_2^L, k^H) = (700, 700, 300, 1400, 1400, 30, 500)$ .

Model 4.1, Table III-III-I Sensitivity analysis with parameter settings in step III (New Dimension I), step III (New Dimension II), step I (Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 3000)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve	% change
original			793	575	292	1	1	1	7360			
$r_H$	10	11	797	574	292	1	1	1	8762	7360	<b>19</b>	10
$r_L$	5	6	793	575	292	1	1	1	7699	7360	<b>5</b>	20
$V_H$	1	2	794	575	292	1	1	1	7373	7360	<b>0</b>	100
$V_L$	1	2	793	575	292	1	1	1	7360	7360	<b>0</b>	100
$k^H$	3000	3001	793	575	292	1	1	1	7360	7360	<b>0</b>	0
$k^L$	1000	1001	793	575	292	1	1	1	7360	7360	<b>0</b>	0
K	1000	1001	793	575	292	1	1	1	7360	7360	<b>0</b>	0
$h_H$	1	0	812	571	292	1	1	1	8835	7360	<b>20</b>	100
$h'_H$	5	4	793	575	292	1	1	1	7360	7360	<b>0</b>	20
$h_L$	1	0	788	575	292	1	1	1	7604	7360	<b>3</b>	100
$h'_L$	3	2	798	575	292	1	1	1	7415	7360	<b>1</b>	33
$S_H$	8	7	792	573	292	1	1	1	7412	7360	<b>1</b>	13
$S_L$	4	3	793	575	291	1	1	1	7370	7360	<b>0</b>	25
$c_H$	4	3	799	575	292	1	1	1	8731	7360	<b>19</b>	25
$c_L$	2	1	793	575	292	1	1	1	7652	7360	<b>4</b>	50
average												40

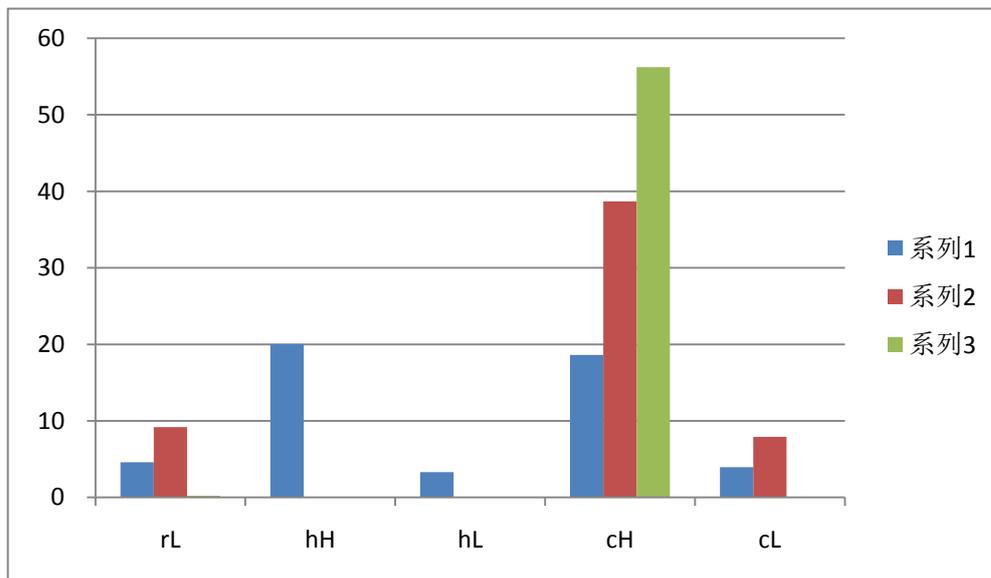
Graph III-III-I



Model 4.1, Table III-III-I-Elite Continue to relax the most important parameters in the above table by changing them one unit at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			793	575	292	1	1	1	7360		
$r_L$	5	6	793	575	292	1	1	1	7699	7360	<b>5</b>
		7	793	575	292	1	1	1	8037	7360	<b>9</b>
		8	793	575	292	1	1	1	7376	7360	<b>0</b>
$h_H$	1	0	812	571	292	1	1	1	8835	7360	<b>20</b>
$h_L$	1	0	788	575	292	1	1	1	7604	7360	<b>3</b>
$c_H$	4	3	799	575	292	1	1	1	8731	7360	<b>19</b>
		2	1000	564	292	1	1	0	10207	7360	<b>39</b>
		1	826	574	292	1	1	1	11499	7360	<b>56</b>
$c_L$	2	1	793	575	292	1	1	1	7652	7360	<b>4</b>
		0	793	575	292	1	1	1	7944	7360	<b>8</b>

Graph III-III-I-Elite

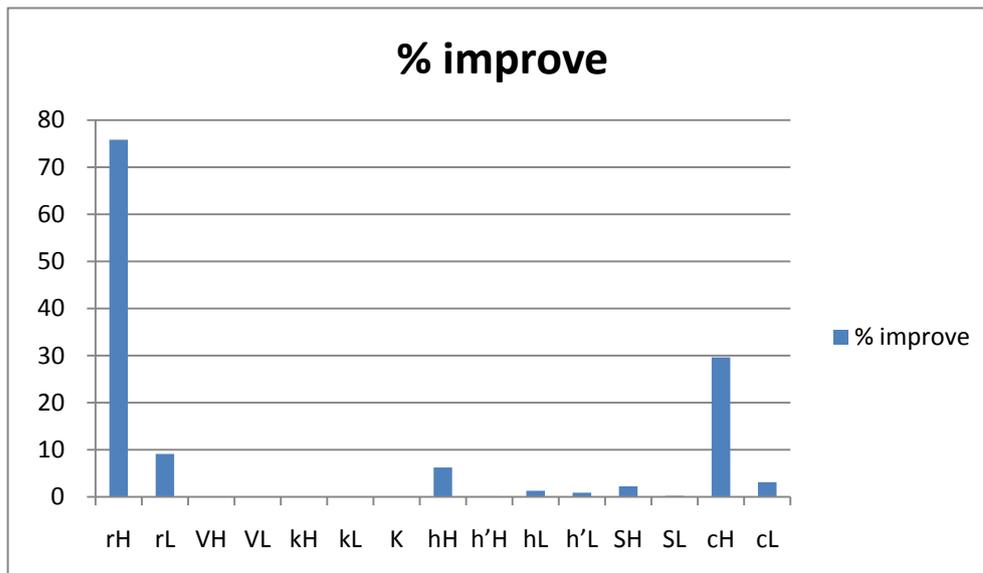


$h_H, c_H, r_L, c_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed one unit at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 3000)$ .

Model 4.1, Table III-III-II Sensitivity analysis with parameter settings in step III (New Dimension I), step III (New Dimension II), step II Dimension III) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 3000)$ .

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			793	575	292	1	1	1	7360		
$r_H$	10	14	809	570	292	1	1	1	12941	7360	<b>76</b>
$r_L$	5	7	793	575	292	1	1	1	8033	7360	<b>9</b>
$V_H$	1	1	793	575	292	1	1	1	7365	7360	<b>0</b>
$V_L$	1	1	793	575	292	1	1	1	7360	7360	<b>0</b>
$k^H$	3000	4192	793	575	292	1	1	1	7360	7360	<b>0</b>
$k^L$	1000	1397	793	575	292	1	1	1	7360	7360	<b>0</b>
K	1000	1397	793	575	292	1	1	1	7360	7360	<b>0</b>
$h_H$	1	1	1000	561	292	1	1	0	7821	7360	<b>6</b>
$h'_H$	5	3	794	575	292	1	1	1	7372	7360	<b>0</b>
$h_L$	1	1	791	575	292	1	1	1	7456	7360	<b>1</b>
$h'_L$	3	2	800	575	292	1	1	1	7426	7360	<b>1</b>
$S_H$	8	5	792	570	292	1	1	1	7527	7360	<b>2</b>
$S_L$	4	2	793	575	291	1	1	1	7376	7360	<b>0</b>
$c_H$	4	2	803	575	292	1	1	1	9542	7360	<b>30</b>
$c_L$	2	1	793	575	292	1	1	1	7592	7360	<b>3</b>

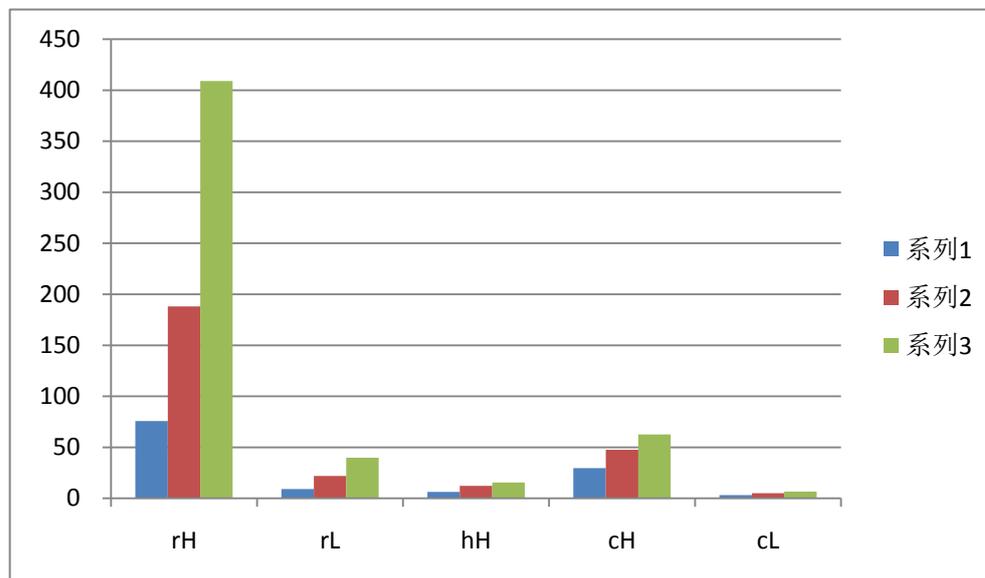
Graph III-III-II



Model 4.1, Table III-III-II-Elite Continue to relax the most important parameters in the above table by changing them by the same percentage at a time.

parameter	original	change 1	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			793	575	292	1	1	1	7360		
$r_H$	10	14	809	570	292	1	1	1	12941	7360	<b>76</b>
		20	1025	558	291	1	1	1	21208	7360	<b>188</b>
		27	2099	369	292	1	1	0	37462	7360	<b>409</b>
$r_L$	5	7	793	575	292	1	1	1	8033	7360	<b>9</b>
		10	793	575	292	1	1	1	8972	7360	<b>22</b>
		14	793	575	292	1	1	1	10285	7360	<b>40</b>
$h_H$	1	1	1000	561	292	1	1	0	7821	7360	<b>6</b>
		0	1000	561	292	1	1	0	8267	7360	<b>12</b>
		0	806	572	292	1	1	1	8509	7360	<b>16</b>
$c_H$	4	2	803	575	292	1	1	1	9542	7360	<b>30</b>
		1	815	575	292	1	1	1	10867	7360	<b>48</b>
		1	1000	566	292	1	1	0	11966	7360	<b>63</b>
$c_L$	2	1	793	575	292	1	1	1	7592	7360	<b>3</b>
		1	793	575	292	1	1	1	7732	7360	<b>5</b>
		0	793	575	292	1	1	1	7852	7360	<b>7</b>

Graph III-III-II-Elite

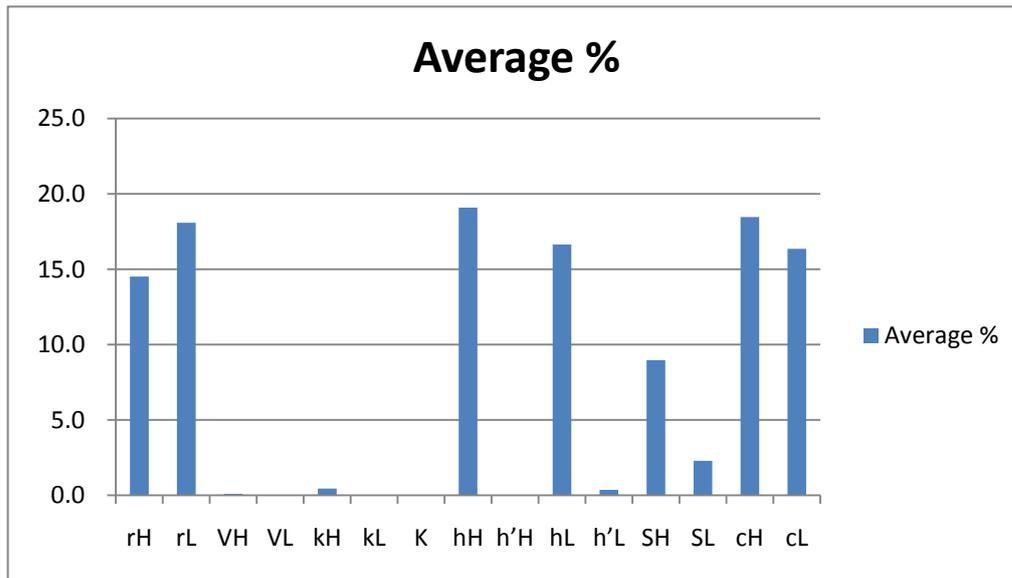


$r_H, c_H, r_L, h_H, c_L$ , are the most important parameters to the retailer (from the most important to the least important) when parameters are changed by the same percentage at a time and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H) = (700, 700, 300, 1400, 1400, 30, 3000)$ .

Model 4.1, Table IV-1 Show parameters' average improvement to the profit when changed one unit at a time.

	Table I-I-I	Table I-II-I	Table I-III-I	Table I-IV-I	Table III-I-I	Table III-II-I	Table III-III-I	Average %
$r_H$	0.0	20.4	14.4	14.4	0.0	52.2	0.2	<b>14.5</b>
$r_L$	19.2	41.5	17.4	20.0	4.0	19.9	4.6	<b>18.1</b>
$V_H$	0.0	-1.1	0.8	0.8	0.0	0.0	0.2	<b>0.1</b>
$V_L$	0.0	0.1	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h_H$	0.4	1.2	0.0	0.0	0.5	1.0	0.0	<b>0.5</b>
$h'_H$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h'_L$	0.0	21.2	17.4	16.3	0.0	58.5	20.0	<b>19.1</b>
$S_H$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$S_L$	24.5	36.8	15.7	15.7	3.4	17.1	3.3	<b>16.6</b>
$c_H$	0.0	0.0	0.8	1.1	0.0	0.0	0.7	<b>0.4</b>
$c_L$	13.3	10.1	1.1	1.1	13.7	22.7	0.7	<b>9.0</b>
$K$	6.0	9.0	0.1	0.1	0.1	0.6	0.1	<b>2.3</b>
$k_H$	0.0	21.2	13.8	17.0	0.0	58.5	18.6	<b>18.5</b>
$k_L$	24.5	36.8	12.3	16.4	3.4	17.1	4.0	<b>16.4</b>

Graph IV-I

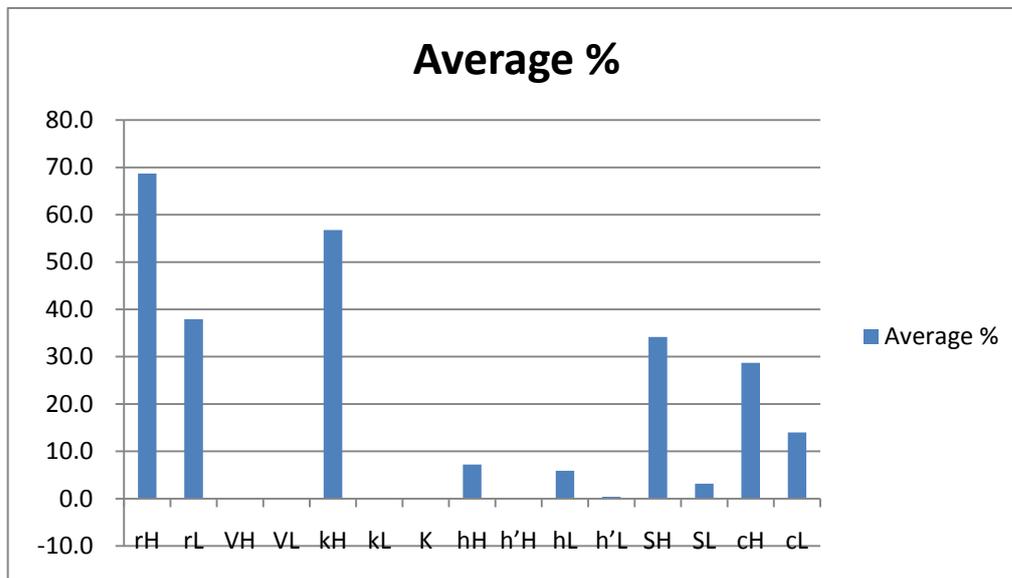


$h_H$ ,  $c_H$ ,  $r_L$ ,  $h_L$ ,  $c_L$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed one unit at a time.

Model 4.1, Table IV-1 Show parameters' average improvement to the profit when changed by the same percentage at a time.

	Table I-I-II	Table I-II-II	Table I-III-II	Table I-IV-II	Table III-I-II	Table III-II-II	Table III-III-II	Average %
$r_H$	0.2	81.0	57.6	57.8	0.1	208.4	75.8	<b>68.7</b>
$r_L$	60.4	77.8	34.6	34.6	9.3	39.5	9.1	<b>37.9</b>
$V_H$	0.0	0.0	0.0	0.0	0.0	0.0	0.1	<b>0.0</b>
$V_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h_H$	0.2	101.3	0.0	0.0	0.2	295.8	0.0	<b>56.8</b>
$h'_H$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h_L$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>
$h'_L$	0.0	8.5	6.3	6.3	0.0	23.2	6.3	<b>7.2</b>
$S_H$	0.0	0.0	0.2	0.2	0.0	0.0	0.2	<b>0.1</b>
$S_L$	11.4	14.7	6.2	6.2	1.6	0.0	1.3	<b>5.9</b>
$c_H$	0.0	0.0	1.0	1.0	0.0	0.0	0.9	<b>0.4</b>
$c_L$	72.3	32.3	3.6	3.6	50.7	74.5	2.3	<b>34.2</b>
$K$	11.7	0.7	0.3	8.2	0.2	0.9	0.2	<b>3.2</b>
$k_H$	0.1	33.8	22.1	22.1	0.0	93.2	29.6	<b>28.7</b>
$k_L$	22.7	29.3	13.0	13.0	3.2	13.6	3.1	<b>14.0</b>

Graph IV-II

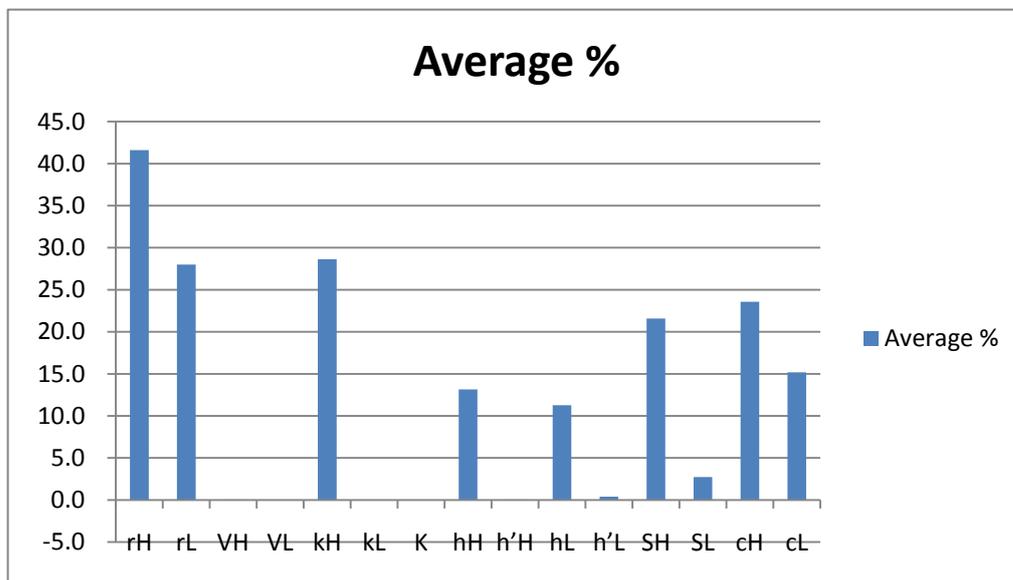


$r_H$ ,  $k^H$ ,  $r_L$ ,  $S_H$ ,  $c_H$  are the most important parameters to the retailer (from the most important to the least important) when parameters are changed by the same percentage at a time.

Model 4.1, Table IV- Overall Show parameters' average improvement to the profit.

	Table IV-I	Table IV-II	Average %
$r_H$	14.5	68.7	<b>41.6</b>
$r_L$	18.1	37.9	<b>28.0</b>
$V_H$	0.1	0.0	<b>0.1</b>
$V_L$	0.0	0.0	<b>0.0</b>
$h_H$	0.5	56.8	<b>28.6</b>
$h'_H$	0.0	0.0	<b>0.0</b>
$h_L$	0.0	0.0	<b>0.0</b>
$h'_L$	19.1	7.2	<b>13.2</b>
$S_H$	0.0	0.1	<b>0.0</b>
$S_L$	16.6	5.9	<b>11.3</b>
$c_H$	0.4	0.4	<b>0.4</b>
$c_L$	9.0	34.2	<b>21.6</b>
$K$	2.3	3.2	<b>2.7</b>
$k_H$	18.5	28.7	<b>23.6</b>
$k_L$	16.4	14.0	<b>15.2</b>

Graph IV-Overall



To conclude, for all the above cases in Model 4.1 where  $(h_H - h'_H) - (h_L - h'_L) < 0$ ,  $r_H$ ,  $k^H$ ,  $r_L$ ,  $c_H$ ,  $S_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 4.1, Table C-1 Show the average percentage of the expected holding cost of the above examples.

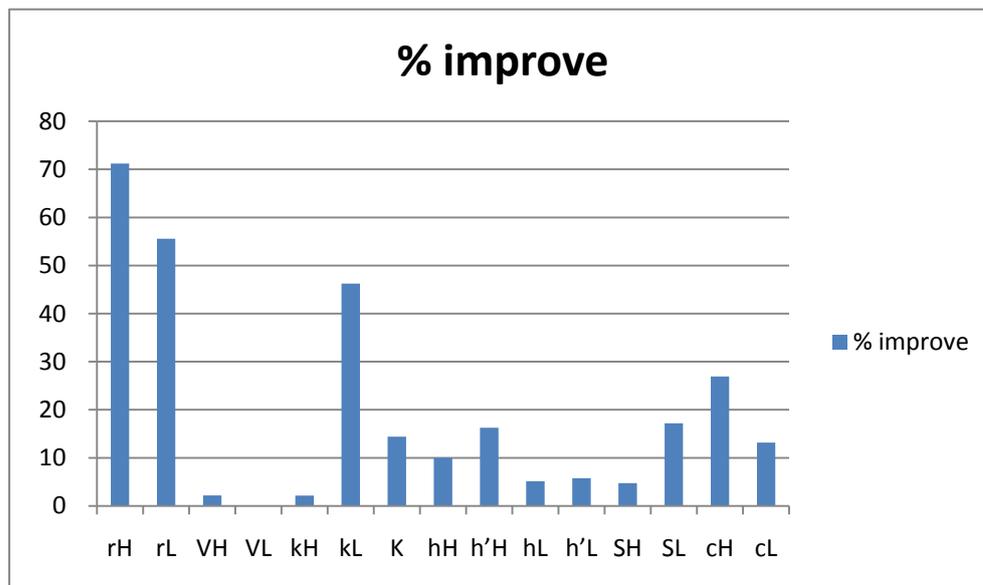
	EC	Eh	ER	ES	EV	Eh/ER
I-I-I	-1391.1	-694.6	3554.8	-4726.7	1.0	0.2
I-II-I	-2987.1	-1093.6	7390.9	-1569.6	1.0	0.1
I-III-I	-3713.6	-1423.5	9263.5	-353.9	7.2	0.2
I-IV-I	-3713.6	-1423.5	9263.5	-353.9	7.2	0.2
III-I-I	-591.4	-293.7	1528.2	-10955.0	0.1	0.2
III-II-I	-4525.2	-1277.1	11112.0	-3272.2	0.1	0.1
III-III-I	-6052.9	-1853.1	14955.0	404.2	12.2	0.1
Average						<b>0.2</b>

The average percentage of the expected holding cost of the above examples is 20%.

Model 4.1, Table anti-I. Sensitivity analysis with parameter settings in step III (New Dimension I), step III (New Dimension II), step I (Dimension III) where  $(h_H-h'_H)-(h_L-h'_L)>0$  and  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H, k^L, K) = (300, 300, 700, 600, 600, 70, 500, 500, 500)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve	% change
original			363	200	433	1	0	1	3349			
$r_H$	10	14	386	192	433	1	0	1	5734	3349	71	<b>10</b>
$r_L$	8	11	363	200	433	1	0	1	5211	3349	56	<b>13</b>
$V_H$	1	1	363	200	465	1	0	1	3423	3349	2	<b>100</b>
$V_L$	1	1	363	200	433	1	0	1	3349	3349	0	<b>100</b>
$k^H$	500	699	363	200	465	1	0	1	3421	3349	2	<b>0</b>
$k^L$	500	699	363	200	465	1	0	1	4898	3349	46	<b>0</b>
K	500	699	366	201	433	1	1	1	3832	3349	14	<b>0</b>
$h_H$	2	1	370	198	433	1	0	1	3683	3349	10	<b>50</b>
$h'_H$	5	3	372	200	465	1	0	1	3894	3349	16	<b>20</b>
$h_L$	1	1	363	200	433	1	0	1	3521	3349	5	<b>100</b>
$h'_L$	5	3	368	201	465	1	1	0	3543	3349	6	<b>20</b>
$S_H$	8	5	364	196	433	1	0	1	3508	3349	5	<b>13</b>
$S_L$	4	2	363	200	432	1	0	1	3925	3349	17	<b>25</b>
$c_H$	4	2	371	200	433	1	0	1	4251	3349	27	<b>25</b>
$c_L$	2	1	363	200	465	1	0	1	3790	3349	13	<b>50</b>

Model 4.1, Graph anti-I.

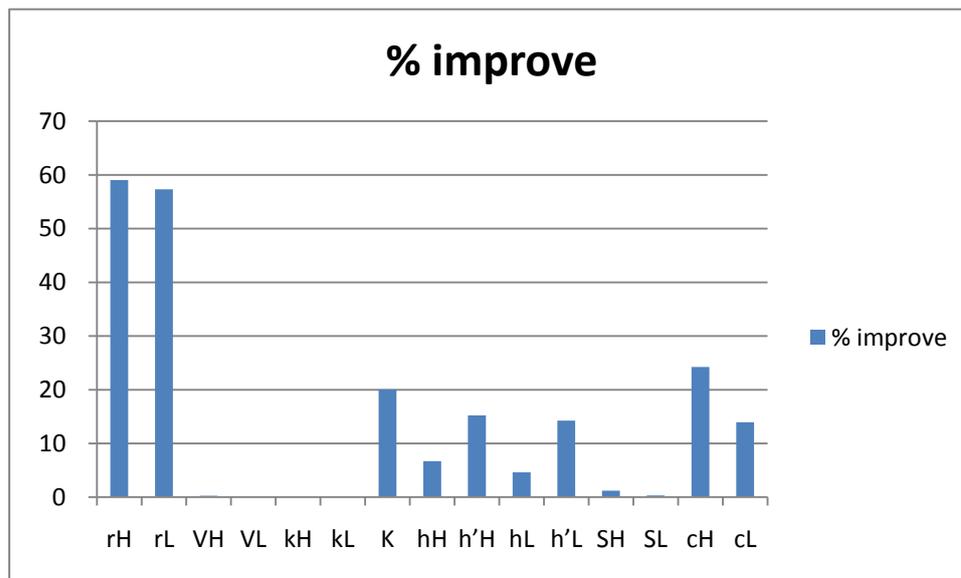


$r_H, r_L, k^L, c_H$  are the most important parameters to the retailer (from the most important to the least important).

Model 4.1, Table anti-II. Sensitivity analysis in case  $(h_H-h'_H)-(h_L-h'_L)>0$  with parameter settings in step I (New Dimension I) where  $(\mu^H_1, \mu^H_2, \mu^L_2, \sigma^H_1, \sigma^H_2, \sigma^L_2, k^H, k^L, K) = (300, 300, 700, 600, 600, 70, 500, 1000, 500)$ .

parameter	original	change %	$X_0^{H*}$	$X_1^{H*}$	$X_0^{L*}$	$a_0^{H*}$	$a_1^{H*}$	$a_1^{L*}$	Profit*	original	% improve
original			339	259	692	1	0	1	3929		
$r_H$	10	14	351	262	692	1	0	1	6249	3929	<b>59</b>
$r_L$	8	11	339	259	692	1	0	1	6182	3929	<b>57</b>
$V_H$	1	1	340	260	692	1	0	1	3939	3929	<b>0</b>
$V_L$	1	1	339	259	692	1	0	1	3931	3929	<b>0</b>
$k^H$	500	699	339	260	691	1	0	1	3931	3929	<b>0</b>
$k^L$	1000	1397	339	259	692	1	0	1	3931	3929	<b>0</b>
$K$	500	699	339	260	692	1	0	0	4717	3929	<b>20</b>
$h_H$	2	1	343	258	691	1	0	1	4192	3929	<b>7</b>
$h'_H$	5	3	344	264	691	1	0	1	4528	3929	<b>15</b>
$h_L$	1	1	339	259	691	1	0	1	4110	3929	<b>5</b>
$h'_L$	5	3	341	264	692	1	1	0	4490	3929	<b>14</b>
$S_H$	8	5	338	253	692	1	0	1	3977	3929	<b>1</b>
$S_L$	4	2	339	260	692	1	0	1	3941	3929	<b>0</b>
$c_H$	4	2	344	264	691	1	0	1	4882	3929	<b>24</b>
$c_L$	2	1	339	259	692	1	0	1	4477	3929	<b>14</b>

Model 4.1, Graph anti-II



$r_H, r_L, c_H, K, h'_H$  are the most important parameters to the retailer (from the most important to the least important).

## Résumé

### Analyse et optimisation des décisions d'approvisionnement dans une supply chain : le cas d'un distributeur et deux fournisseurs

L'objectif de cette recherche est de développer des modèles aussi bien conceptuels, analytiques et managériaux en analysant un maillon de la supply chain, à savoir la relation entre un distributeur et deux fournisseurs opérant dans un environnement incertain. Dans la première partie de la thèse, nous considérons un seul produit, plutôt haut de gamme et/ou périssable, et nous faisons l'analyse sur un horizon d'une période. Dans ce cas précis, les caractéristiques unitaires du produit sont toutes non linéaires, à savoir : le prix, le coût de production, le coût de rupture, le coût de reprise. La demande est supposée être une variable aléatoire. Dans la deuxième partie de la thèse, nous nous inspirons des pratiques de firmes internationales qui s'approvisionnent, pour une partie de leur offre, dans des pays à bas coûts. Nous développons plusieurs modèles mais dont la structure de base est similaire, à savoir : deux produits (un haut gamme acheté localement et l'autre bas de gamme acheté dans les pays à bas coûts), un horizon de trois périodes, deux fournisseurs à capacité de production limitée et un distributeur ayant des capacités de stockage limitées. Une panoplie de résultats théoriques, numériques ainsi que des insights sont présentés. Les modèles développés peuvent être utilisés comme des outils d'aide à la prise de décision dans les environnements décrits dans cette thèse.

Mots-clés: Modèles de gestion de stocks, problème 'newsboy', programmation stochastique, planification, l'affectation des ressources, gestion des approvisionnements, méthodes heuristiques.

## Abstract

### ANALYSIS AND OPTIMIZATION OF SINGLE AND DUAL SOURCING DECISIONS IN SUPPLY CHAIN

The objective of this research is to develop conceptual, analytical, and managerial models and insights by analyzing a portion of the supply chain made up of a retailer dealing with two suppliers in an uncertain environment. In the first part of this thesis, we consider a single high-end (or perishable) product, single period, variable unit price, variable unit production cost, variable unit shortage cost, variable unit salvage value, stochastic demand problem. In a second part of the thesis, we consider settings inspired by the case of large international companies sourcing some of their products from low cost countries. This structure is as follows: two products (one sourced locally and the other sourced abroad), a three-period, two-stages, two capacitated suppliers, and a single capacitated retailer. Both analytical and numerical results are provided. Important theoretical results and insights are developed for these types of settings. These models can be used as decision-making aid tools in such environments.

**Keywords:** Inventory management, newsboy problem, stochastic programming, planning, resource allocation, supply chain management, dual-sourcing, two-stage problem, lost sale, Periodic-review models, heuristic methods.