



# Operational Strategies to Foster Technology Improvement in Value Chains

Ali Shantia

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# Operations-Based Strategies to Foster Technology Improvement in the Value Chain

Thèse de doctorat de LUISS Guido Carli et de l'Université Paris-Saclay

préparée à HEC Paris

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# Operations-Based Strategies to Foster Technology Improvement in the Value Chain

by

Ali Shantia

A dissertation submitted in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy  
(Operations and Supply Chain Management)  
at LUISS Guido Carli – HEC Paris  
July 2018

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I dedicate this thesis to  
Maman & baba  
love of my life, Sara  
and my sister and brothers, Maryam, Shahram, & Amirhossein

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## PREFACE

Mon intérêt pour la recherche réside dans l’interface de la gestion de la technologie, de la finance et des opérations durables. Plus précisément, dans ma thèse, je m’efforce d’examiner les incitations des entreprises à adopter des mesures d’amélioration de la technologie (TI) qui permettent une utilisation plus efficace des intrants et affectent ainsi la structure des coûts, l’exposition au risque et la performance environnementale des entreprises. Je cherche à identifier les facteurs qui affectent — et les mécanismes par lesquels ils le font - la décision d’une entreprise d’investir dans TI: forces au sein d’une chaîne d’approvisionnement, incertitude des prix sur les marchés pour les intrants, contraintes de trésorerie, mécanismes de couverture financière, concurrence de l’industrie et stratégie de prix concurrentielle de l’entreprise.

L’investissement dans l’amélioration de la technologie réduit le cot des intrants et ajoute donc de la valeur à l’ensemble de la chaîne d’approvisionnement; Cette dynamique crée des forces qui peuvent encourager les acteurs de la chaîne d’approvisionnement à adopter ces technologies ou peuvent les dissuader d’une telle adoption. Dans mon premier chapitre, “Contrats d’amélioration de la technologie dans les chaînes d’approvisionnement sous le pouvoir de négociation asymétrique”, j’examine cette perspective pour étudier comment le pouvoir de négociation asymétrique - entre acheteurs et fournisseurs - affecte le niveau optimal d’investissement dans l’amélioration de la technologie. Les données montrent que les fournisseurs s’abstiennent d’investir dans des mesures d’amélioration de la technologie car ils craignent qu’un acheteur ayant un pouvoir de négociation plus important utilise les réductions de coûts liées à



TI pour faire baisser les prix et réduire ainsi la marge bénéficiaire du fournisseur; cette dynamique conduit à des niveaux d'investissement inefficaces dans l'informatique et conduit au soi-disant problème de rétention. L'incertitude liée aux nouvelles technologies décourage également l'investissement des fournisseurs dans TI. Ces deux questions sont étudiées via notre modèle de processus de négociation, dans une chaîne d'approvisionnement à deux niveaux, entre un fournisseur unique et un acheteur; J'analyse la manière dont l'adoption de la technologie TI par le fournisseur est influencée par le pouvoir de négociation relatif de l'acheteur et par l'incertitude technologique. Je compare les différentes dispositions contractuelles couramment utilisées dans l'industrie pour surmonter ces obstacles, y compris leurs propriétés optimales par rapport à différents critères - en particulier, le bénéfice de la chaîne d'approvisionnement et le niveau d'équilibre de l'investissement en TI. En ce qui concerne les deux critères, j'estime que les contrats d'investissement partagé donnent de meilleurs résultats que les contrats d'engagement de prix, même si ces derniers augmentent les bénéfices des fournisseurs lorsque le pouvoir de négociation de l'acheteur est relativement élevé.

La réduction de l'utilisation des intrants réduit également l'exposition de l'entreprise à l'incertitude associée au prix de ces intrants, ce qui indique que les mesures d'atténuation des risques liées à l'investissement dans des technologies durables sont des propriétés atténuantes. Dans "Gestion des risques liés aux prix des intrants: amélioration de la technologie et couverture financière", j'examine le mécanisme qui motive l'intérêt d'une entreprise pour les TI en raison de l'incertitude accrue quant au prix des intrants. Mon deuxième chapitre étudie la motivation des entreprises à investir dans des mesures de gestion des risques grâce à l'amélioration technologique: activités qui réduisent la consommation d'un intrant, entraînant moins de déchets et d'émissions, des coûts de production moindres et des opérations plus durables. Investir dans TI n'est pas une décision anodine car même si cela réduit clairement les coûts et les

risques, les entreprises peuvent tirer avantage de l'incertitude des prix des intrants, ce qui, combiné à la flexibilité de la production, crée une réticence à renoncer. J'utilise un modèle mathématique stylisé pour explorer et généraliser cette affirmation et pour préciser ses implications, dans divers scénarios, pour les décisions des entreprises d'investir dans la réduction des coûts et la gestion des risques. Je tire une expression de forme fermée qui quantifie explicitement l'attitude d'une entreprise envers le risque de prix des intrants en considérant *prime de certitude* positive ou négative (c'est-à-dire ce que l'entreprise paierait pour "verrouiller" l'entre-prix); J'établiss ensuite un lien entre cette prime et diverses caractéristiques au niveau de l'entreprise et de l'industrie. De plus, je compare les avantages de l'amélioration technologique et de la couverture financière (FH) en matière de gestion des risques et caractérise les conditions dans lesquelles ces stratégies sont complémentaires ou substitutives. Je trouve que même si l'incertitude des prix, ces entreprises peuvent toujours bénéficier d'investir dans des mesures de réduction des risques (par exemple, TI, FH), car la valeur de l'option d'incertitude pourrait ainsi augmenter. La capacité d'une entreprise à ajuster son prix en réponse à la concurrence du marché et à la variation des prix des intrants induit l'avantage des mesures de réduction des risques et affecte également la complémentarité de ces deux stratégies.

Un autre aspect important de l'amélioration technologique consiste à choisir la "capacité" d'une technologie efficace, en particulier dans un environnement dynamique où les conditions du marché peuvent changer rapidement et où l'accès de l'entreprise aux liquidités dépend de ses choix antérieurs concernant le type de technologie adopté et les dépenses sur la capacité. Dans mon troisième chapitre, intitulé "Investissement dynamique des capacités et amélioration de la technologie avec contraintes budgétaires", j'étudie le rôle des contraintes budgétaires sur le choix de la technologie et l'ajustement de la capacité correspondante. Les entreprises peuvent adopter de nouvelles technologies pour décider du portefeuille optimal de leur capacité de pro-

duction. Grâce à une gestion stratégique des capacités, ils équilibrent les avantages liés à l'acquisition de nouvelles technologies et les coûts d'investissement liés au remplacement ou à l'extension de la capacité. Les coûts d'investissement et les rendements dépendent fortement de l'état du marché. Les compagnies aériennes, par exemple, signent chaque année de nouveaux contrats pour remplacer leur ancienne flotte par de jeunes avions économes en carburant. Toutefois, la valeur des investissements dans de nouveaux avions dépend de facteurs incertains tels que la demande future et le coût du carburant. Dans le troisième chapitre, je souligne les facteurs déterminants d'une décision optimale en matière d'investissement des capacités, par exemple pour une compagnie aérienne, et caractérise la politique optimale d'investissement lorsque deux technologies de substitution sont disponibles, mais que l'entreprise est confrontée à des contraintes financières. J'utilise un cadre de programmation dynamique stochastique pour caractériser la politique optimale d'investissement dans des technologies plus efficaces lorsque la contrainte budgétaire reflète les dépenses de capacité préalables d'une entreprise et l'état de l'économie, ce qui entraîne la réalisation de la demande et donc des prix des intrants.

En collaborant avec des professeurs dans les domaines de la recherche opérationnelle, de l'économie et des finances, j'ai adopté une approche multidisciplinaire pour étudier l'adoption de technologies efficaces et durables. Outre les forces du marché économique, l'interaction entre les entreprises et la concurrence, mes recherches mettent en évidence la pertinence de l'analyse intégrant des composantes financières telles que les contraintes de flux de trésorerie et la couverture.

Cette thèse contribue également à la politique publique sur le réchauffement de la planète et le changement climatique en fournissant des lignes directrices sur la manière de stimuler l'investissement dans l'amélioration technologique afin de réduire le taux de consommation des intrants, en particulier de l'énergie. En particulier, la section un présente le choix de contrat optimal qui se traduit par un niveau d'investissement

plus élevé dans l'amélioration de la technologie. Lorsqu'un tel contrat pourrait ne pas être optimal du point de vue de l'acheteur, les décideurs ont la possibilité de fournir des incitations aux acheteurs (ex. Détaillants) pour mettre en œuvre un tel contrat avec leurs fournisseurs. De plus, la section deux caractérise les propriétés de gestion des risques de l'investissement dans TI, un facteur caché qui incite les décideurs à encourager les entreprises à faire un tel investissement.

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# ABSTRACT

Operations-Based Strategies to Foster Technology Improvement in the Value Chain

by

Ali Shantia

This thesis is in the interface of sustainable operations management, technology management, and finance. Specifically, in my thesis I strive to examine firm’s incentives to adopt ‘technology improvement’ (TI) measures that lead to the more efficient use of inputs in operations and thereby affect the cost structure, risk exposure, and environmental performance of firms. Thus I seek to identify the factors that affect—and the mechanisms by which they do so—a firm’s decision to invest in TI: forces within a supply chain, price uncertainty in the markets for inputs, cash constraints, financial hedging mechanisms, industry competition, and the firm’s competitive pricing strategy. By collaborating with professors in the fields of operations research, economics, and finance, I have embraced a multidisciplinary approach to studying the adoption of efficient and sustainable technologies.

In my first chapter, “Technology Improvement Contracting in Supply Chains under Asymmetric Bargaining Power” I examine how asymmetric bargaining power—between buyers and suppliers—affects the optimal level of investment in technology improvement. In my second chapter, “Input-price Risk Management: Technology Improvement and Financial Hedging”, I explore the mechanism driving a firm’s interest

in TI under increased uncertainty about input prices. Finally, in the third chapter, “Dynamic Capacity Investment and Technology Improvement with Financial Constraints”, I study the role of budget constraint on the choice of technology.

# Introduction

My research interest lies in the interface of technology management, finance and sustainable operations. Specifically, in my thesis I strive to examine firm's incentives to adopt 'technology improvement' (TI) measures that lead to the more efficient use of inputs in operations and thereby affect the cost structure, risk exposure, and environmental performance of firms. Thus I seek to identify the factors that affect—and the mechanisms by which they do so—a firm's decision to invest in TI: forces within a supply chain, price uncertainty in the markets for inputs, cash constraints, financial hedging mechanisms, industry competition, and the firm's competitive pricing strategy.

Investment in technology improvement reduces the cost of inputs and therefore adds value to the whole supply chain; this dynamic creates forces that may encourage supply chain players to adopt these technologies or may dissuade them from such adoption. In my first chapter, "Technology Improvement Contracting in Supply chains under Asymmetric Bargaining Power", I examine TI from this perspective to study how asymmetric bargaining power—between buyers and suppliers—affects the optimal level of investment in technology improvement. In this chapter I use a game-theoretic framework to "internalize" the interplayer bargaining process and analyze how uncertainty regarding TI investment outcomes moderates the effect of asymmetric bargaining power on the well-known hold-up problem; I also assess how well various contractual mechanisms resolve this issue.

Reduced use of inputs also reduces the firm’s exposure to uncertainty associated with the price of those inputs, which points to the risk-mitigating properties of adopting such TI measures as investing in sustainable technologies. In “Input-price Risk Management: Technology Improvement and Financial Hedging”, I explore the mechanism driving a firm’s interest in TI under increased uncertainty about input prices. In the second chapter I use a firm-level optimization framework to characterize how technology improvement—by changing the firm’s exposure to input price uncertainty—affects its risk premium (i.e., the amount a firm would pay to “lock in” the input price at its mean). I characterize the conditions under which firms invest more (or less) in TI and the effect of the subsequent investment on the risk premium. Furthermore, I show how this effect is moderated by competition and by the availability of financial or operational risk-hedging mechanisms, such as futures or the flexibility to adjust prices.

Another important aspect of technology improvement is choosing the *capacity size* of an efficient technology, especially in a dynamic setting where market conditions can change quickly and the firm’s access to cash depends on its previous choices regarding the type of technology adopted and expenditures on capacity. In my third chapter, “Dynamic Capacity Investment and Technology Improvement with Budget Constraints”, I study the role of budget constraint on the choice of technology and related capacity adjustment. By considering two different technologies—namely, a conventional (inefficient) technology and a sustainable (efficient) one—I wish to determine the optimal policy as regards replacing or expanding production capacity. In practice, the choice of technology is constrained by the available budget. I use a stochastic dynamic programming framework to characterize the optimal policy of investing in more efficient technologies when the budget constraint reflects a firm’s prior capacity expenditure and the state of the economy, which in turn drives the realization of demand and hence input prices.

By collaborating with professors in the fields of operations research, economics, and finance, I have embraced a multidisciplinary approach to studying the adoption of efficient and sustainable technologies. In addition to economic market forces, the interaction among firms, and competition, my research highlights the relevance of analysis that incorporates such financial components as cash flow constraints and hedging.

This thesis also contributes to the public policy on global warming and climate change by providing guidelines on how to foster investment in technology improvement to reduce consumption rate of input commodities, especially energy. In particular, section one introduces the optimal contract choice that results in higher investment level in technology improvement. Where such a contract might not be optimal from the buyer's perspective, there is an opportunity for policy makers to provide incentives for buyers (ex. retailers) to implement the such a contract with their suppliers. Moreover, section two characterizes the risk management properties of investment in TI, a hidden factor for policy makers to encourage firms for such an investment.

## CHAPTER I

# Technology Improvement Contracting in Supply Chains under Asymmetric Bargaining Power

Evidence shows that suppliers refrain from investing in technology improvement (TI) measures because they fear that a buyer with greater bargaining power will use TI-related cost reductions to push prices down—in the purchase bargaining process—and thereby further reduce the supplier’s profit margin; this dynamic leads to inefficient levels of investment in TI and leads to the so called holdup problem. Suppliers are also discouraged from TI investment by the uncertainty associated with new technologies. These two issues are studied via our model of the bargaining process, in a two-tier supply chain, between a single supplier and buyer; we analyze how the supplier’s TI technology adoption is affected by the buyer’s relative bargaining power and also by technology uncertainty. We compare various contracting arrangements commonly used in industry to overcome these obstacles, including price commitment by the buyer and shared investment contracts, while characterizing their optimal properties with respect to different criteria—in particular, supply chain profit and the equilibrium level of TI investment. In terms of both criteria, we find that shared investment contracts perform better than price commitment contracts, although the latter increase supplier profit when the buyer’s bargaining power is relatively high. We also show that, in a two-player model, technology uncertainty moderates how the

bargaining process affects the supplier’s investment behavior.

Keywords: supply chain coordination, renegotiation, relative bargaining power, technology uncertainty

## 1.1 Introduction

Technology improvement (TI) is one of the most effective strategies for firms to enhance their cost efficiency and increase their competitive advantage ( *Weaver et al.*, 2017). Consider, for example, the implementation of energy efficiency projects that are meant to reduce the use of energy—a costly input in many industries (cement, steel, refinery, pulp and paper, etc.)—per unit of production. According to a McKinsey report, “with an average internal rate of return (IRR) of 17 percent, [energy efficiency projects] would collectively generate annual savings ramping up to \$900 billion annually by 2020” ( *Farrell et al.*, 2008).

Yet in supply chains, a positive IRR does not guarantee the firm’s decision to invest in TI. If buyer and supplier bargaining power is strongly asymmetric, for example, then the supplier might refrain from investing in technology improvement out of fear that the buyer would renegotiate the purchasing price to capture the lion’s share of TI-related savings—a possibility that would make the investment less attractive and unfeasible.<sup>1</sup> This phenomenon, which is well known in the economics literature, is often referred to as *the holdup problem*. The case of Walmart offers an instance of such a scenario:

A supplier that invests in process improvement or capital equipment to improve energy efficiency will see [a] reduction in variable production costs. If the supplier anticipates, however, that Walmart will [re]negotiate a corresponding low purchase price which barely covers the production cost,

---

<sup>1</sup>By a “feasible” investment we mean a positive amount that creates value for the supplier net of its investment costs.



[then] the supplier is unlikely to make such investment. (*Plambeck*, 2012)

In the setting of a two-echelon supply chain that consists of a single buyer and a single supplier, we go beyond the economics literature to model explicitly their relative bargaining power as an exogenous variable and then study its effect on the supplier’s decision to invest in TI when there is technology uncertainty concerning the investment return (*Fleming*, 2001; *Aflaki et al.*, 2013). Thus we reexamine the conventional wisdom that the buyer’s bargaining power is inversely related to the supplier’s investment in TI projects. We also characterize the moderating impact of technology uncertainty on the relationship between the relative bargaining power and investment levels. Finally, we analyze different real-world contractual remedies and assess their effectiveness in increasing investment efficiency under varying degrees of bargaining power and technology uncertainty.

We shall focus on two particular arrangements: *price commitment* and *shared investment*. Price commitment (PC), or credibly forgoing the possibility of renegotiation for a reasonable time period, is a strategy that Walmart has used successfully to incentivize its suppliers’ investments in TI projects (*Cheung*, 2011; *Plambeck*, 2012). Shared investment (SI), or mutual participation in TI investment by both buyer and supplier, is also widely practiced and especially in the energy context. Buyers including Ikea, General Electric, and Ford Motor Company engage in “[o]n-site audits (fully or partially funded by the buyer) to determine a supplier’s energy performance” in addition to measures such as “financial assistance, . . . and organis[ing] assessments to identify energy saving opportunities” (*Goldberg et al.*, 2012). Note that determining and auditing such projects “can be a big driver of costs”, accounting for 5% to 20% of total TI costs.<sup>2</sup>

We develop a game-theoretic model of the bargaining process between a single supplier and a single buyer. We use this model to derive supplier-optimal TI invest-

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<sup>2</sup>See “A guide to performance contracting with escos,” Tech. rep., US Department of Energy ([http://www.pnnl.gov/main/publications/external/technical\\_reports/PNNL-20939.pdf](http://www.pnnl.gov/main/publications/external/technical_reports/PNNL-20939.pdf)).

ment as well as both the buyer’s and supplier’s equilibrium profits under various contractual scenarios. As a benchmark, we first examine the no-contract (NC) case so as to understand the supplier’s investment behavior and explicitly show a likely holdup problem. We then compare the equilibrium outcomes from this scenario to those resulting under the two relevant contract types, price commitment and shared investment. Next, for each agent in the supply chain (or “channel” for short), we derive their respective preferences regarding these different contract types. In an extension to the model, we also discuss a broader range of different contract mechanisms and describe how they neutralize any possible renegotiation process.

We find that the equilibrium investment in TI is not necessarily decreasing in the buyer’s bargaining power; in fact, the relation is characterized by an inverse U-shaped function whose maximum is at moderate levels of buyer bargaining power. When the buyer’s bargaining power is high (as in the Walmart case), a shared investment contract indeed coordinates the supply chain—and so achieves first-best (FB) investment levels—in comparison with no-contract and price commitment settings. When it comes to profits of the respective agents, we show that the supplier (resp., the buyer) should prefer price commitments (resp., shared investment) when the buyer’s bargaining power is high. Technology uncertainty plays an important role in these preferences by moderating the effect of relative bargaining power on (a) the level of TI investment and (b) the profit of each agent. So in the middle ranges of relative bargaining power, both supplier and buyer prefer SI if technology uncertainty is high whereas, if uncertainty is low, then the supplier (resp. buyer) prefers PC (resp. NC).

The rest of the paper proceeds as follows. We review the related literature on the holdup problem and supply chain coordination in Section 1.2. In Section 2.3 we develop a basic model that captures the effect of both bargaining power and technology uncertainty on the supplier’s optimal TI investment, and in Section 1.4 we study their impact on optimal TI investment as well as the buyer’s and the supplier’s

profit in more detail. Section 1.5 considers the SI and PC contracts, comparing them in terms of individual profits, channel profits, and investment levels. Extensions to our basic model are considered in Section 1.6; these include generalizing the demand function, considering alternative timing and contractual settings, and varying the extent of information asymmetry in the supply chain. Section 2.6 concludes with a discussion of our results and some closing remarks.

## 1.2 Literature Review

Incentives for investment in projects that produce surplus is the core of a stream of economics literature on the holdup problem (*Stilmant, 2015*) and is closely related to the operations management (OM) literature on supply chain coordination (*Kleindorfer et al., 2005; Linton et al., 2007*).

In the holdup literature, price renegotiation—a consequence of an initially incomplete contract, in which all possible contingencies either are not or cannot be negotiated ex ante (*Hart and Moore, 1988; Tirole, 1999*)—leads to the holdup problem when *anticipating* such renegotiation prevents efficient investment levels in any option that would create an ex post surplus. In many instances, using complete ex ante contracts is costly if not impossible; hence doing so is unlikely to prevent renegotiation and thus avoid the holdup problem (*Huberman and Kahn, 1988*).

Research in a related substream of the holdup literature studies “simple” contracts (i.e., contracts that do not fully cover the space of future outcomes), which allow the possibility of renegotiation, and then analyze their effectiveness with respect to investment efficiency (*Rogerson, 1992; Edlin and Reichelstein, 1995; Che and Hausch, 1999; Hoppe and Schmitz, 2011*). In these accounts, the timeline of events is as follows: the parties negotiate the initial contract; then the state of nature is realized; and, depending on that state, the contract’s terms are renegotiated (*Hart and Moore, 1988*). The goal is to design the initial contract in such a way that the holdup prob-

lem is minimized *Stilmant* (2015). A related substream of the literature on supply chain coordination focuses on investment in surplus-generating solutions that would benefit supply chain players but *without* the possibility of renegotiation (*Corbett and DeCroix*, 2001; *Tomlin*, 2003). The timeline of events in this literature is typically as follows: a simple contract exists; the potential for a surplus-generating action arises; and the decision to adopt that measure (which involves technology risk) is made. Our paper arises at the confluence of these two substreams by combining, into a single timeline, investment in TI and the possibility of renegotiation with a particular focus on technology uncertainty which (naturally) depends on the supplier’s decision to invest. Although we explicitly model the renegotiation process using established bargaining theory, we are able to capture important dynamics of the relationship between the parties’ relative bargaining power and the equilibrium decision to invest in TI.

As a result of considering a more complete timeline and explicitly modeling the bargaining process, some of our insights run counter to claims made in both of the source literatures. For example, some papers in the holdup literature claim that price commitment (i.e., proscribing renegotiation) resolves the holdup problem (*Edlin and Reichelstein*, 1995; *Che and Hausch*, 1999; *Hoppe and Schmitz*, 2011). Yet we find that price commitment does not fully resolve the holdup problem, as its efficiency depends on relative bargaining power. Also, *Segal and Whinston* (2002) show that no contracting (NC in our setting) can achieve higher efficiency than any noncontingent contract if the investment is either purely selfish or purely cooperative. In contrast, our results show that no contracting is superior to price commitment only under a specific range of relative bargaining power and its efficiency is always dominated by shared investment arrangement.

In the supply chain literature, *Gilbert and Cvsa* (2003) analyze the strategy of stimulating innovation in a supply chain’s downstream parties in order to reduce

manufacturing costs or increase demand. They find that one party’s cost-lowering innovations provide incentives for other parties to increase their prices opportunistically. The end result is that channel members are dissuaded from investing in innovation. *Gilbert and Cvsa* argue that price commitment agreements show promise as a mechanism for resolving this problem. Similarly, *Kim and Netessine* (2013) study how the “invisibility” of production cost and procurement contracting affect buyer–supplier collaborative efforts to reduce costs. These authors posit a contracting mechanism similar to price commitment (viz., the *expected margin* commitment) as an effective remedy for the problems due to asymmetric information in their context. The main insight from this literature is that mechanisms such as investment sharing, price commitment, and effort compensation can coordinate the channel—at least in theory. Unlike the papers cited here, however, we endogenize the pricing mechanism that emerges from a potential renegotiation process and show that outcomes depend on both relative bargaining power and investment uncertainty.

In short, our paper’s distinctive approach is to combine the holdup problem (and renegotiation) with supply chain coordination (under technology risk) to shed light on the intricate connections among the buyer’s bargaining power, the supplier’s decision to invest in TI, and these parties’ preferences as regards contracting mechanisms in the presence of technology uncertainty.

### 1.3 Modeling the Renegotiation Process and TI Investment

We model a simple supply chain consisting of a single supplier (“supplier”, for short) that sells a unit of a generic product at a wholesale price  $w$  to a single buyer (“buyer”) that in turn sells that product (or a more complete version) at ‘retail price’  $p$ . The supplier has an option to invest in technology improvement (TI) to reduce its production cost  $C$ . The parties split the total channel surplus  $p - C$  by setting the wholesale price according to their relative bargaining power, which for

buyer is denoted by  $\alpha \in [0, 1]$ . Total demand is a decreasing function  $D(p)$  of buyer's retail price  $p$ . The retail price determines the demand—and consequently the channel profit—while the wholesale price determines the respective shares captured by buyer and supplier. Appendix A spells out all the notations.

## TI Investment and Supplier's Cost Reduction

We consider a continuous technology space in which technology improvement increases with investment  $x \geq 0$ . The total supplier cost  $C$  is a stochastic function of TI investment. When there is no investment (i.e.,  $x = 0$ ) we assume, without loss of generality, that  $C = 1$ . For positive investment amounts  $x > 0$  we assume  $C = \gamma(x) + \tilde{r}$ , where  $\gamma(x)$  is a weakly decreasing and twice differentiable function in  $x \in [0, \bar{x}]$  for  $\bar{x}$  an upper bound on the feasible level of investment. The random variable  $\tilde{r} \sim \mathcal{N}(0, \sigma(x))$  is used to characterize the uncertainty in investment return. We use *investment uncertainty* and *technology uncertainty* interchangeably to refer to this uncertainty. We assume that the variance  $\sigma^2 = \sigma_n^2(x) + \sigma_x^2$ , where  $\sigma_n^2(x)$  is the uncertainty driven by investment levels and  $\sigma_x^2$  is the uncertainty that is independent of the investment level. Following the economics literature, we refer to these two components of uncertainty as (respectively) “endogenous” and “exogenous” uncertainty (see e.g. *Cukierman*, 1980; *Goel and Grossmann*, 2006; *Fragnière et al.*, 2010).

For simplicity of exposition, we let the endogenous investment uncertainty change linearly in the level of investment; that is, we assume  $\sigma_n^2 = zx$ . Note that our model encompasses cases where investment level could increase ( $z > 0$ ), decrease ( $z < 0$ ), or have no effect ( $z = 0$ ) on the level of uncertainty—depending on the nature and maturity of the focal technology.

## Sequence of Events

Figure 1.1 illustrates the sequence of events in our model.

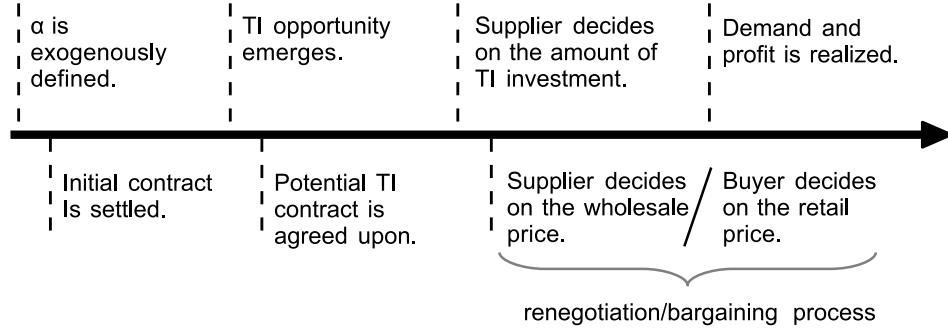


Figure 1.1: Timeline of the sequence of actions

Before realization of a TI opportunity, there exists a wholesale contract between buyer and supplier, whose respective shares of the profit are based on buyer's (exogenous) relative bargaining power  $\alpha$ . When a TI opportunity emerges, supplier has an option to choose the optimal level of TI investment and thereby maximize expected profit. Yet the subsequent realized investment return might lead to a renegotiation process that results in new wholesale (and retail) prices, which would affect each party's profit.

However, there is an opportunity—before supplier's investment decision—for the two parties to reach an agreement that will boost supplier's investment and render ex post renegotiation more efficient. Under such an agreement, buyer can commit either to (i) *not* renegotiating the wholesale price downward or to (ii) contributing some of the TI project's investment cost; alternatively (i.e., if no agreement is reached) buyer can (iii) renegotiate for the wholesale price after realization of investment return.

We focus on these three approaches to commitment—which are further elaborated in Section 1.5—because they have been extensively studied in both the holdup literature (*Che and Hausch, 1999; Schmitz, 2001*) and the OM literature (*Corbett and DeCroix, 2001; Gilbert and Cvsa, 2003; Kim and Netessine, 2013*). Following this research stream, we will refer to these three settings as price commitment (PC), shared investment (SI), and no contract (NC). It should be noted that, in our setting, the

renegotiation process occurs ex post—that is, after supplier’s investment. The case of ex ante bargaining is an extension discussed in Section 1.6.

### Ex post Renegotiation Process

We assume that supplier does not know buyer’s margin; even so, supplier’s knowledge of market demand allows it to anticipate retail prices. Assuming asymmetric information on the part of buyer and supplier is common in the supply chain literature (see e.g. *Kim and Netessine, 2013*).<sup>3</sup>

The effect of bargaining power on the wholesale price, and thus on the profit split between buyer and supplier, has been extensively studied in the bargaining literature. We use the bargaining solution proposed by *Iyer and Villas-Boas (2003)*, which models the *relative bargaining power*  $\alpha$  as a fraction of the channel profit captured by the buyer;<sup>4</sup> then  $1 - \alpha$  captures supplier’s relative bargaining power. These authors show that, given a buyer’s relative bargaining power  $\alpha$ , the outcome of negotiation is such that the wholesale price is

$$w = C + (1 - \alpha)(\hat{p} - C). \quad (1.1)$$

Here  $\hat{p}$  is the supplier’s *anticipation* of the retail price set by the buyer so as to maximize its profit  $\pi_b = (p - w)D(p)$ :

$$\hat{p} = \max_p \pi_b = (p - w)D(p). \quad (1.2)$$

The indices  $b$  and  $s$  are used (throughout) to signify “buyer” and “supplier”. It

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<sup>3</sup>This assumption is exemplified by how Toyota (the buyer) deals with its suppliers. On the one hand, Toyota asks them to share cost (not price) information and reciprocates by agreeing to help them cut costs and by forswearing all interest in the capture of any extra margins that result. On the other hand, suppliers can see the price of Toyota outputs but cannot see the margin that Toyota adds to each supplied part.

<sup>4</sup>Their solution encompasses the Nash (1950) bargaining solution as well as that of *Rubinstein (1982)*.



should be noted that buyer might incur costs other than  $w$  for preparation of the product to be sold; however, we assume that the wholesale price is the only part that is affected by supplier's TI investment. Thus focusing on buyer's profit through reduction in wholesale price  $w$ , we consider without loss of generality, that buyer does not experience any other costs.

According to the sequence of events displayed in Figure 3.1, a supplier that knows the demand structure can anticipate the equilibrium retail price  $\hat{p}$  and so can choose a wholesale price that maximizes its profit  $\pi_s = (w - C)D(\hat{p})$ .

We shall consider a linear demand function  $D = a - bp$  for market size  $a$  and retail price  $p$ , where  $b$  determines the slope of demand. Section 1.6 extends our results to general demand functions and investigates the effects of demand uncertainty and its correlation with input cost. Note our assumption that the buyer has full information regarding the supplier's cost  $C$ . Although this is widely assumed in the literature on supply chain management (see e.g. *Jeuland and Shugan, 1983; McGuire and Staelin, 1983; Moorthy, 1987*), we show in Section 1.6 that it is not crucial for the generality of our results.

The equilibrium wholesale and retail prices are derived by simultaneously solving Equations (1.1) and (1.2). Thus we derive the equilibrium profits of supplier and buyer as

$$\pi_s(\alpha, x) = \frac{\alpha(1 - \alpha)(a - b(\gamma(x) + \tilde{r}))^2}{b(1 + \alpha)^2} - x, \quad (1.3)$$

$$\pi_b(\alpha, x) = \frac{\alpha^2(a - b(\gamma(x) + \tilde{r}))^2}{b(1 + \alpha)^2}. \quad (1.4)$$

Supplier chooses a level of investment that maximizes its expected profit, which in turn determines buyer's profit. This procedure is equivalent to maximizing the TI

project's total net value subject to the nonnegative value constraint:

$$\max_{x \geq 0} \Delta \mathbb{E} \pi_s = \mathbb{E} \pi_s(\alpha, x) - \mathbb{E} \pi_s(\alpha, 0), \quad (1.5)$$

where  $\Delta g = g(x) - g(0)$  and  $\mathbb{E}$  is the expectation operator.

## 1.4 The Holdup Problem: How Bargaining Power Affects TI Investment

In this section we study how both the optimal investment in TI (i.e., the solution to Equation (1.5)) and the resulting profits for each party depend on the buyer's relative bargaining power  $\alpha$ . In order to assess the efficiency of investment, we start by characterizing the efficient investment level—also known as the first-best (FB) solution.

### Efficient Investment Level

The *efficient* investment level is the level of investment that maximizes total channel profit  $\Pi = \pi_s + \pi_b$ :

$$\max_x \Delta \mathbb{E} \left[ \Pi(x) = \frac{\alpha^2(a - b(\gamma(x) + \tilde{r}))^2}{b(1 + \alpha)^2} + \frac{\alpha(1 - \alpha)(a - b(\gamma(x) + \tilde{r}))^2}{b(1 + \alpha)^2} - x \right]. \quad (1.6)$$

Let  $x_c^*$  denote the efficient investment level that optimizes channel profit. Then the following proposition characterizes supplier's and channel's respective optimal investments,  $x_s^*$  and  $x_c^*$ , and comparing these optima reveals the potential holdup problem in this context. All proofs are given in Appendix A.

**Proposition 1.1.** (a)  $x_s^* < x_c^*$ .

(b)  $x_s^*$  is an inverse U-shaped function of  $\alpha$  that is maximized at  $\alpha_s^* = 1/3$ , and  $x_c^*$  increases with  $\alpha$ .

(c) *There exist a lower bound  $\underline{\alpha}$  and an upper bound  $\bar{\alpha}$  on relative bargaining power, where  $x_s^* \geq 0$  is feasible if and only if  $\underline{\alpha} < \alpha < \bar{\alpha}$ .*

(d)  *$x_c^* - x_s^*$  increases with  $\alpha$  and with  $\sigma^2$ .*

Part (a) of this proposition shows that, as one would expect, the holdup problem indeed occurs in that supplier makes only an inefficient investment in TI. Yet according to part (b), this inefficiency depends on  $\alpha$ , the buyer's relative bargaining power. In particular, the optimal investment  $x_s^*$  is an inverse U-shaped function of  $\alpha$ : the optimum first increases and then decreases with  $\alpha$ . The reason is that the buyer's bargaining power affects supplier profit in two opposing ways. On the one hand, if buyer's bargaining power is high then the holdup problem is severe because the renegotiation is expected to be unfavorable from supplier's perspective. On the other hand, if supplier's bargaining power is high then buyer sets a higher retail price so as to compensate for the high wholesale price. The result is lower demand overall, which eventually reduces supplier's investment return and hence the incentives for investment. These two opposing forces dictate that the optimal  $\alpha_s^* = 1/3$ , where the supplier's investment is at its maximum level. The dynamic differs for  $x_c^*$ , however:  $x_c^*$  is *always* increasing in  $\alpha$ . This follows because, when buyer's bargaining power is high, retail prices fall; the resulting higher demand increases the channel's marginal return on TI investment.

Part (c) of Proposition 1.1 follows from part (b). The inverse U-shaped relation between  $x_s^*$  and  $\alpha$  indicates that TI investment is not feasible when  $\alpha$  is either very low or very high. This result complements *Plambeck's* (2012) observation that if buyers have high bargaining power then suppliers refrain from investing in technology improvement projects; however, the same outcome prevails also when it is the supplier with high bargaining power. Finally, part (d) shows that the holdup problem is more likely to arise not only at extreme values of bargaining power but also at higher levels of TI uncertainty.

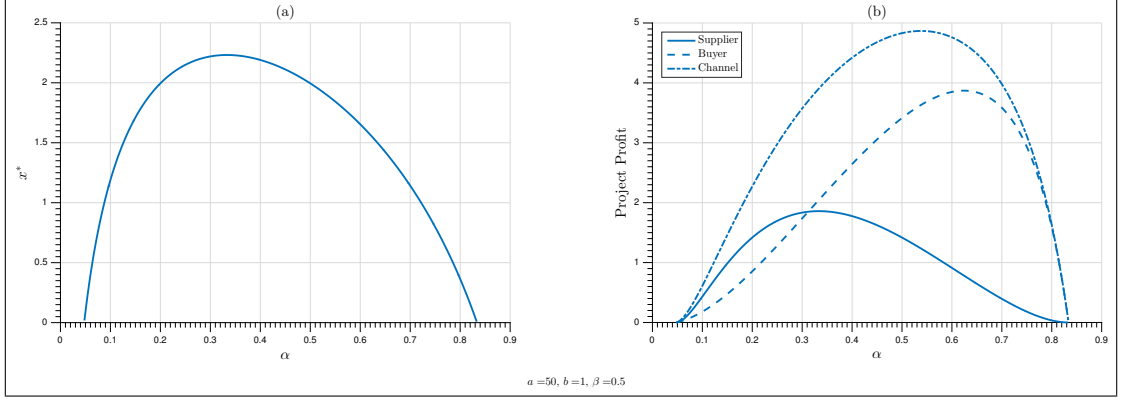


Figure 1.2: Optimal investment and equilibrium profits as a function of buyer's relative bargaining power where  $\gamma(x) = e^{-\beta x}$

We are naturally interested in understanding how the holdup problem affects supplier's and buyer's expected profit—as formalized by Equations (1.3) and (1.4)—at different levels of relative bargaining power.

**Proposition 1.2.** (a)  $\Delta \mathbb{E}\pi_s^*$  is an inverse U-shaped function of  $\alpha$  and is maximized at  $\alpha_s^* = 1/3$ .

(b)  $\Delta \mathbb{E}\pi_b^*$  is always increasing in  $x$ .

(c)  $\Delta \mathbb{E}\pi_b^*$  is an inverse U-shaped function of  $\alpha$  and is maximized at  $\alpha_b^* > \alpha_s^*$ .

(d)  $\Delta \mathbb{E}\pi_s^*$  and  $\Delta \mathbb{E}\pi_b^*$  are each increasing in  $\sigma^2$ .

Part (a) states that the optimal supplier's return from TI investment is maximized at  $\alpha_s^* = 1/3$  (where the optimal investment is maximized). Panels (a) and (b) of Figure 1.2 use a numerical example to show how the optimal TI investment and its resulting profits change with  $\alpha$ .

Part (b) of the proposition states that higher TI investment lowers supplier costs and hence the wholesale price. Thus any amount of investment—even one that is not feasible for the supplier—is profitable for the buyer. In fact, these are the circumstances that lead to the misalignment of incentives (between supplier and buyer) and are responsible for the holdup problem.

According to Proposition 1.2(c), buyer's return on a TI investment *decreases* with

an increase in its bargaining power—though only when  $\alpha$  is fairly high. This is because if  $\alpha$  is too high then supplier invests less in TI. In fact, there is an optimal amount of relative bargaining power ( $\alpha_b^*$ ) that maximizes the buyer’s profit. Moreover, this optimal value of buyer’s relative bargaining power ( $\alpha_b^*$ ) is *greater* than the optimal bargaining power  $\alpha_s^* = 1/3$  that maximizes the project’s value for the supplier. The reason is that increasing  $\alpha$  has two separate effects on the buyer’s profit. While affecting the optimal amount of TI investment and therefore the size of the “pie” to be shared, it also changes how that pie is divided up: as  $\alpha$  increases, so does the buyer’s share. So if  $\alpha$  increases beyond the value that is optimal for supplier then, notwithstanding that the channel generates fewer cost savings because of reduced supplier investment, buyer can seek to compensate for that reduction by capturing a higher share of the value associated with those savings in order to increase its profit via the renegotiation process. The implication is that buyer’s profit-maximizing level of bargaining power is higher than supplier’s profit-maximizing level.

Proposition 1.2(d) describes how supplier and buyer expected profits are affected by investment uncertainty. Because the pricing decision occurs after renegotiation, both parties’ profit functions (Equations (1.3) and (1.4)) are convex in the uncertain cost. This counterintuitive relation is a frequently cited phenomenon: when the profit function is convex in an uncertain parameter, an “option value” is created and so a risk-neutral firm’s profit increases with the level of uncertainty (Oi, 1961).

We now use Proposition 1.2 to characterize how the *channel’s* profit is affected by different levels of relative bargaining power and investment uncertainty.

**Corollary 1.1.** (a) *Channel profit evaluated at  $x_s^*$  is maximized at  $\alpha_c^*$ , where  $\alpha_s^* < \alpha_c^* < \alpha_b^*$ .*

(b)  *$\Delta \mathbb{E}\Pi(x^*)$  is increasing in  $\sigma^2$ .*

Unlike buyer expected profit, channel expected profit initially increases with TI investment and then decreases; this dynamic suggests that there is an optimal invest-

ment level  $x_c^*$  for the channel. It is no surprise that this optimal investment level is higher than the supplier's,  $x_s^*$ , since buyer profit increases with greater TI investment. A similar argument holds for the level of bargaining power  $\alpha_c^*$  that maximizes channel expected profit. According to Proposition 1.2(c), the value of this level lies (strictly) between  $\alpha_s^*$  and  $\alpha_b^*$ .

### Sensitivity of Results to Technology Characteristics

To assess the extent to which technology characteristics affect our findings, in this section we make the parametric assumption that  $\gamma(x) = e^{-\beta x}$ ; here  $\beta$  signifies technology effectiveness in reducing cost because it reflects the reduction in commodity costs that result from a single dollar being invested in technology improvement.

**Proposition 1.3.** (a)  $x_s^*$  is an inverse U-shaped function of  $\beta$ .

(b)  $\alpha_b^*$  is increasing in  $\beta$  but  $\alpha_s^*$  is independent of  $\beta$ .

Part (a) of the proposition states that a technology's effectiveness has a significant influence on the supplier's investment decision. When  $\beta$  is too high, a small investment is sufficient to reach the targeted savings in consumption rate; when  $\beta$  is too low, the investment becomes less attractive and the optimal investment will also be low. However, it is straightforward to show that the savings are in any case an increasing function of  $\beta$ .

Of greater interest, though, is the effect of  $\beta$  on the  $\alpha^*$  that maximizes expected profits. Part (b) of Proposition 1.3 states that, even though an increase in  $\beta$  has no effect on  $\alpha_s^*$ , it does increase  $\alpha_b^*$  (see Figure 1.3). This result follows from our assumption of linear demand;  $\alpha_s^*$  is independent both of market characteristics and of the supplier's cost structure, while  $\alpha_b^*$  depends on the buyer's trade-off between increasing TI investment returns and claiming a greater portion of them. A higher  $\alpha_b^*$  increases the buyer's share but reduces the overall investment. An increase in  $x_s^*$  increases the *total* return, which makes the project profitable even for higher  $\alpha_b^*$ .

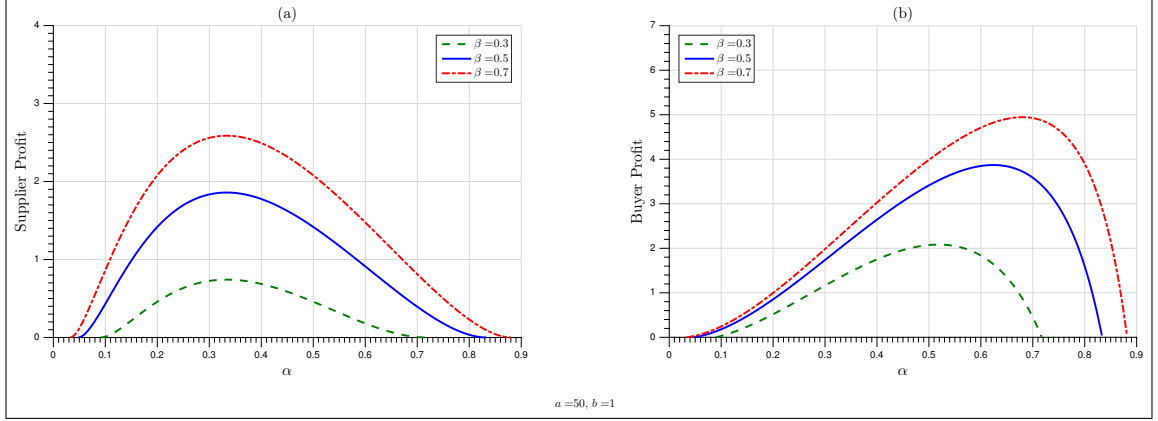


Figure 1.3: Sensitivity analysis

## 1.5 Supplier–Buyer Optimal Contracting Arrangements

In the previous section, we demonstrated the extent of inefficiency in supplier’s TI investment as a function of buyer’s relative bargaining power and investment uncertainty. However, there is an opportunity for the buyer and the supplier to make arrangements—concerning the ex post investment renegotiation—that might resolve the investment inefficiency. Several such arrangements have been used extensively in practice and in the holdup and OM literatures. For example, *Plambeck* (2012) studies the long-term commitment agreement, one of the coordination schemes that Walmart uses to encourage the adoption of TI measures by upstream suppliers, which helps alleviate the adverse effects of a buyer’s high bargaining power. *Aoki and Lennerfors* (2013) also report how Toyota follows a similar strategy in offering price commitment arrangements to its suppliers. *Edlin and Reichelstein* (1995); *Schmitz* (2001), and *Gilbert and Cvsa* (2003) all use the term *price commitment* to describe a similar arrangement that encourages cost-reducing innovation in supply chains.

The *shared investment* contract—as explored for example by *Corbett and DeCroix* (2001) and *Ida* (2012)—is another mechanism for promoting cooperative cost reduction measures in supply chains. Ikea, General Electric, and Ford motor Company promote shared investment arrangements to encourage their suppliers to engage in

TI investment (*Goldberg et al.*, 2012; *Holder*, 2016). The holdup literature refers to this arrangement as “cooperative investment”; see *Che and Hausch* (1999), who also discuss investment efficiency in the *no-contract* setting (i.e., when there is no ex ante arrangement regarding supplier investment). We discussed the NC case in Section 1.4. In this section we describe the structure of the other two arrangements and compare their efficiency under different levels of buyer’s relative bargaining power and of investment uncertainty.

### 1.5.1 Price Commitment

In a price commitment contract, buyer agrees not to bargain down the wholesale price when supplier’s profit margin increases because of cost savings realized through a TI investment. In other words, the wholesale price is fixed at its original amount before the TI investment (i.e., at  $w_0 = w|_{x=0}$ ). Although this arrangement will not increase buyer profit, a buyer might agree to it for two reasons. First, it can anticipate wholesale price reduction (via the bargaining process) once the price commitment expires; this strategy could be used also by a buyer seeking to hedge against the downside risk of future price increases.

The second and more important reason for buyer to adopt PC is that doing so yields the indirect strategic benefits of sourcing from a more sustainable supplier—for example, by leveraging the reduced carbon footprint *per unit* of its products to stimulate customer demand (*Zhu et al.*, 2007) or by reducing its exposure to regulatory risks associated with environmental legislation (see e.g. *Seuring and Müller*, 2008). Under a PC contract, the supplier’s profit function is

$$\pi_{PC}^s = (w_0 - (\gamma(x) + \tilde{r}))(a - bp_0) - x; \quad (1.7)$$

here  $w_0$  and  $p_0$  are, respectively, the wholesale and retail prices which result from



Equations (1.1) and (1.2) by setting the investment level at zero ( $x = 0$ ).

**Proposition 1.4.** *In a PC contract: (a) there exists a lower bound  $\underline{\alpha}_{PC}$  such that, for  $\alpha < \underline{\alpha}_{PC}$ , TI investment is not feasible; (b)  $x_{PC}^*$  is always increasing in  $\alpha$ ; and (c)  $x_{PC}^* < x_{FB}^*$ .*

According to part (a) of the proposition, it is only at low levels of  $\alpha$  that TI investment is not feasible for the supplier. So in contrast to the no-contract setting, there is no upper bound for feasible  $\alpha$ . Part (b) states that, under a PC contract,  $x_{PC}^*$  is increasing in  $\alpha$ . Hence contractual arrangements of this type mitigate the adverse effect of high buyer bargaining power that is observed in a NC setting. This finding may explain why buyers with high bargaining power (e.g., Walmart) propose PC agreements to encourage supplier investment in TI: without price commitments, such investing is not feasible. Because PC contracts direct that realized cost savings be fully captured by supplier, it has more incentive to invest in TI measures—especially when buyer has high bargaining power. Yet according to Proposition 1.4(c), that incentive is insufficient to justify a first-best investment.

### 1.5.2 Shared Investment

In a shared investment arrangement, the buyer finances a fraction  $\lambda$  of the whole investment; that is, buyer commits an amount equivalent to  $\lambda x$ . The parameter  $\lambda$  is the buyer's decision variable, and it is offered to the supplier on a “take it or leave it” basis. At the time of the offer, both parties anticipate a later process of bargaining on the wholesale price. It follows that the equilibrium investment level  $x_{SI}^*$  and buyer's investment share  $\lambda^*$  can be calculated by simultaneously solving each

party's problem:<sup>5</sup>

$$\max_{x \geq 0} \mathbb{E}\Delta(\pi_{SI}^s = (w - (\gamma(x) + \tilde{r}))(a - bp) - (1 - \lambda x)); \quad (1.8)$$

$$\max_{\lambda \in [0,1]} \mathbb{E}\Delta(\pi_{SI}^b = (p - w)(a - bp) - \lambda x). \quad (1.9)$$

The supplier's optimal investment clearly increases with the buyer's amount of participation in that investment. However,  $\lambda$  has two opposite effects on the buyer's profit: it increases the amount of savings (thus reducing the wholesale price); but it increases the investment cost ( $\lambda x$ ). Hence there should be an optimal  $\lambda$  that maximizes the buyer's profit—as formally stated in the following proposition.

**Proposition 1.5.** (a)  $\lambda^* = \alpha$ . (b) *A shared investment contract can coordinate the supply chain with respect to the TI investment decision:  $x_{SI}^* = x_{FB}^*$ .*

Part (a) confirms that an optimal value of the buyer's investment share exists and states that this value equals that buyer's relative bargaining power. A corollary is that the buyer's contribution increases with its relative bargaining power. Of course, higher values of  $\alpha$  result in the buyer receiving a greater portion of the return. In sum: if buyer's bargaining power is high then, by Proposition 1.1(b), supplier is less interested in TI investments; to counteract this effect, buyer must increase its supplier's incentive by financing a greater portion of the TI project.

Proposition 1.5(b) suggests that the SI contract can achieve a TI investment equal to the first-best investment level. Previous research has shown that SI contracts can achieve FB solutions in many contexts (*Corbett and DeCroix*, 2001; *Cachon and Lariviere*, 2005). Even so, we next explain how both relative bargaining power and investment uncertainty alter buyer and supplier preferences to engage in such an

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<sup>5</sup>Since both supplier and buyer contribute to a TI investment, one might question the validity of assuming that  $x^*$  is supplier's decision only. However, it is easy to show that the model where  $x^*$  is jointly determined is mathematically equivalent to the one where  $x^*$  is the supplier's decision variable. Hence we consider (1.8) and (1.9) to be the SI arrangement's governing equations.

agreement.

### 1.5.3 Comparing Optimal Contracts

In this section we compare the different contract mechanisms—and the benchmark case of no contracting—in terms of the TI investment levels (investment efficiency) and the profits accruing to buyer, supplier, and the channel. Table 1.1 summarizes the insights in this section, which are formally presented next in Propositions 1.6 and 1.7.

#### 1.5.3.1 Profits and Investment Efficiency

Higher TI investment levels result in greater savings through fewer input commodities consumed, which generally—and especially with energy efficiency investment—translates into a reduced environmental impact per unit of product.

Our next proposition relates particular contracting arrangements to maximizing the amount  $x$  of TI investment. We continue using subscripts  $PC$  and  $SI$  to reference the price commitment and shared investment settings and use  $NC$  to reference the setting with no (TI) contract. The subscripts  $np$ ,  $ns$ , and  $sp$  are used to identify values of interest in (respectively) the NC–PC, NC–SI, and SI–PC comparisons; the superscripts  $s$ ,  $b$ , and  $c$  denote supplier, buyer, and channel.

**Proposition 1.6.** (a)  $x_{SI}^* \geq \max\{x_{PC}^*, x_{NC}^*\}$ , and there exists a unique  $\alpha = \alpha_{np}$  where  $x_{NC}^* \geq x_{PC}^*$  if and only if  $\alpha \leq \alpha_{np}$ ; moreover,  $\alpha_{np}$  increases with  $\sigma_n^2$ .

(b)  $\Delta\mathbb{E}\pi_{SI}^s \geq \Delta\mathbb{E}\pi_{NC}^s$  for all  $\alpha$ , and there exists a unique  $\alpha_{sp}^s$  (resp.,  $\alpha_{np}^s$ ) such that  $\Delta\mathbb{E}\pi_{SI}^s \geq \Delta\mathbb{E}\pi_{PC}^s$  (resp.,  $\Delta\mathbb{E}\pi_{NC}^s \geq \Delta\mathbb{E}\pi_{PC}^s$ ) if and only if  $\alpha \leq \alpha_{sp}^s$  (resp.,  $\alpha \leq \alpha_{np}^s$ ). Both  $\alpha_{sp}^s$  and  $\alpha_{np}^s$  are increasing in  $\sigma_x^2$  and  $\sigma_n^2$ .

(c) There exists a unique  $\alpha = \alpha_{ns}^b$  for the buyer, where  $\Delta\mathbb{E}\pi_{NC}^b \geq \Delta\mathbb{E}\pi_{SI}^b$  if and only if  $\alpha \leq \alpha_{ns}^b$ ; furthermore,  $\alpha_{ns}^b$  decreases with increasing  $\sigma_n^2$ .

(d) The effect of uncertainty on  $\alpha_{sp}^s$ ,  $\alpha_{np}^s$ ,  $\alpha_{np}$ , and  $\alpha_{ns}^b$  is unimodal for  $\alpha \in [0, 1]$ .

Table 1.1: Comparison of Optimal Contracts

		Investment level		Supplier profit		Buyer profit	
		Low $\sigma^2$	High $\sigma^2$	Low $\sigma^2$	High $\sigma^2$	Low $\sigma^2$	High $\sigma^2$
Exogenous uncertainty: $\sigma^2 = \sigma_x^2$	Low $\alpha$	SI>NC>PC		SI>NC>PC		NC>SI>PC	
	Mid $\alpha$	SI>PC>NC		PC>SI>NC	SI>NC>PC	NC>SI>PC	
	High $\alpha$	SI>PC>NC		PC>SI>NC		SI>NC>PC	
		Investment level		Supplier profit		Buyer profit	
		Low $\sigma^2$	High $\sigma^2$	Low $\sigma^2$	High $\sigma^2$	Low $\sigma^2$	High $\sigma^2$
Endogenous uncertainty: $\sigma^2 = \sigma_x^2 + \sigma_n^2$	Low $\alpha$	SI>NC>PC		SI>NC>PC		NC>SI>PC	
	Mid $\alpha$	SI>PC>NC	SI>NC>PC	PC>SI>NC	SI>NC>PC	NC>SI>PC	SI>NC>PC
	High $\alpha$	SI>PC>NC		PC>SI>NC		SI>NC>PC	

Part (a) of this proposition shows how relative bargaining power affects the optimal choice of contract with regard to TI investment. We previously established that the optimal SI contract coordinates the channel, which explains why  $x_{SI}^*$  is greater than the equilibrium investment under other arrangements. That being said,  $\alpha$  plays an important role in the comparison of NC- versus PC-setting investment levels. Under *low*  $\alpha$  ( $\alpha < \alpha_{np}$ ), the supplier is incentivized to invest in TI because it will receive the majority of the resulting benefits; hence additional incentives from the buyer are unnecessary. When the buyer's bargaining power is *high* ( $\alpha \geq \alpha_{np}$ ), a PC contract is needed to ensure sufficient TI investment. Figure 1.4(a) illustrates how the optimal level of TI investment changes with  $\alpha$  under different contractual settings.

At the same time, investment uncertainty moderates how relative bargaining power affects investment levels under different contractual settings. The investment-dependent (endogenous) uncertainty  $\sigma_n^2 = zx$  increases the range within which NC outperforms PC in terms of investment level and supplier profit. The reason is that  $x_{PC}^*$  is independent of  $\sigma^2 = \sigma_n^2 + \sigma_x^2$  while  $x_{NC}^*$  increases with  $\sigma_n^2$  (see Panel (a) of Figure 1.4). Hence the intersection point  $\alpha_{np}$  is increasing in  $\sigma_n^2$ .

In comparing the SI and PC settings, Proposition 1.6(b) captures our intuition that supplier profit is higher under SI than PC when the buyer's bargaining power is low: a SI contract maximizes value creation, and most of that value is captured by the supplier. When the buyer's bargaining power is high, however, a supplier prefers the PC contract so as to avoid ex post bargaining in which the buyer takes most of the value. Part (b) also claims that if investment uncertainty increases then the supplier prefers either NC or SI to PC for a wider range of  $\alpha$  values. The intuition behind this result is similar to the effect of uncertainty on the optimal investment level (as described in part (a)): when it comes to supplier profit, PC removes any dependence on investment uncertainty. Yet the increase in supplier profit predicted in NC and SI settings would, in turn, increase both  $\alpha_{np}^s$  and  $\alpha_{sp}^s$  in  $\sigma^2$  (see Panel (b) of Figure 1.4).

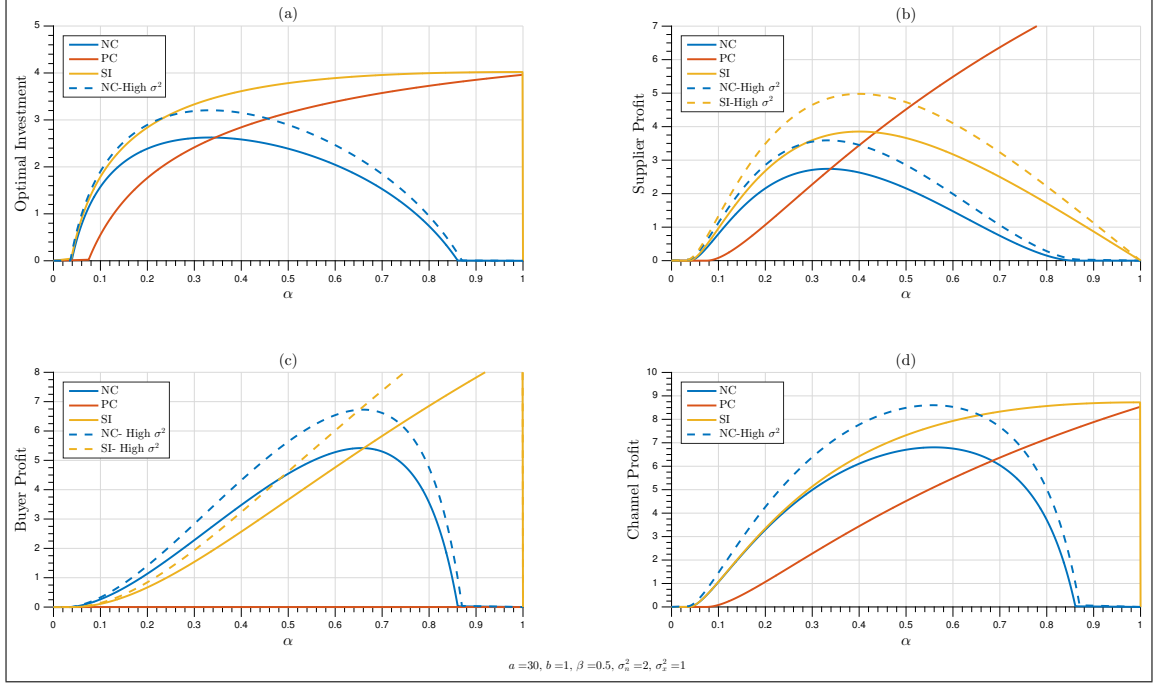


Figure 1.4: Optimal investment and profit of supplier, buyer, and channel by contract setting

Along these lines, part (c) of the proposition stipulates buyer's preferences over different contracting mechanisms. There is no direct incentive for buyer to prefer a PC contract, since then it would receive none of the value generated by a TI project. When comparing an NC versus an SI arrangement, buyer's preference is determined by  $\alpha$ : buyer profit is greater under SI, but only if its bargaining power is high enough to capture (in the ex post bargaining process) a sizable amount of the savings. Much as in the supplier's case, investment uncertainty also increases the threshold of  $\alpha_{ns}^b$ . The reason is that SI leads to a higher level of optimal investment and so the effect of  $\sigma_n^2$  on buyer profit is greater in SI than in NC settings. Hence the project-generated value for buyer is greater under SI than that under NC. Given the inverse U-shaped buyer profit under NC, this means that  $\alpha_{ns}^b$  is decreasing in  $\sigma_n^2$  (see Panel (c) of Figure 1.4).

Proposition 1.6(d) concerns the magnitude of the effect of investment uncertainty on contract preferences for various parties (i.e., how robust these preferences are to

changing levels of investment uncertainty), which is represented by the effect of  $\sigma^2$  on the threshold values discussed in parts (a)–(c). It is interesting that this effect is greatest for moderate values of  $\alpha$ —that is, when the two parties are more or less equal when it comes to bargaining power. This is because, under either high or low  $\alpha$ , the investment level approaches its boundaries: close to zero or close to its maximum (i.e., FB) level. We find that the contract preferences for each party are unchanged in these boundary cases.

### 1.5.3.2 Channel Profit and Retail Price

It is interesting also to see how channel profit (i.e, the sum of supplier and buyer profit) and retail price are related to  $\alpha$ , especially in terms of social welfare.

**Proposition 1.7.** (a)  $\Pi_{SI} > \max\{\Pi_{PC}, \Pi_{NC}\}$ .

(b) *There exists a unique  $\alpha = \alpha_{np}^c$ , where  $\Pi_{NC} \geq \Pi_{PC}^*$ , if and only if  $\alpha \leq \alpha_{np}^c$ ; also,  $\alpha_{np}^c$  increases with  $\sigma^2$ .*

(c)  $p_{SI} < \min\{p_{PC}, p_{NC}\}$ .

(d)  $p_{SI} = p_{FB}$ .

Part (a) shows that the SI contract results in higher channel profit than with the other arrangements, which is not surprising when one considers that a SI contract can coordinate the supply chain. According to part (b), if  $\alpha$  is low enough ( $\alpha \leq \alpha_{np}^c$ ) then the channel is more profitable under the NC than under the PC arrangement (see Figure 1.4(d)); this is because the PC contract makes it more difficult to reduce the market price and hence to increase demand. Yet the threshold increases with investment uncertainty, both endogenous and exogenous, for exactly the same mechanism discussed previously for the case of investment uncertainty affecting the optimal investment level.

In comparing prices under different settings, Proposition 1.7(c) states that the lowest retail price is achieved when the supply chain implements an SI contract;

Table 1.2: Equilibrium Analysis of the Stackelberg Game

First Mover	Low $\sigma^2$		High $\sigma^2$	
Always Supplier or Always Buyer	Low $\alpha$	NC		
	Mid $\alpha$	NC	SI	
	High $\alpha$	SI		
The party with higher Bargaining Power	Low $\alpha$	SI		
	Mid $\alpha$	$\alpha > 0.5$	PC	SI
		$\alpha < 0.5$	NC	SI
	High $\alpha$	SI		

according to part (d), this minimum price equals the price that results from the first-best solution.

#### 1.5.4 Equilibrium Contract Choice

The heterogeneity of preferences summarized in Table 1.1 brings about the question of which contract would prevail in equilibrium. Table 1.2 presents the equilibrium analysis of Stackelberg games in which the supplier, the buyer or the party with higher bargaining power are the first movers in offering a contract in a take-it-or-leave-it manner. Interestingly, the equilibrium outcome of such games is identical if the buyer or the supplier is always the first mover. In both cases, low bargaining power of the buyer and technology uncertainty leads to no contracting, whereas under high  $\alpha$  or  $\sigma^2$ , a shared investment contract prevails. This is not the case when the party with higher bargaining power is the first mover; here, when the relationship is highly asymmetric (both when  $\alpha$  is sufficiently low or sufficiently high) a shared investment contract prevails. This is also true when the relationship is symmetric (mid-range of  $\alpha$ ) and technology uncertainty is high. Otherwise, for a relatively established technology with low risk, either one of PC or NC arrangements could result, depending



on which party's bargaining power is slightly higher.

There are a couple of interesting takeaways from Table 1.2. First, PC contract is likely the equilibrium outcome under a symmetric relationship, with the buyer having slightly higher bargaining power, and low technology uncertainty. Secondly, we observe that SI contract (which is the preferred mechanism when it comes to the investment level and channel profit) is the equilibrium outcome under the majority of cases when the party with higher bargaining power is the first mover. It suggests that the more powerful supply chain players should be pro-active in offering the TI contract.

## 1.6 Extensions

In this section we consider some extensions of the basic model. First, we extend our analysis to the case of a general demand function as well as uncertainty in linear demand. We then study the effect of ex ante renegotiation. Next we consider the setting in which the buyer has only partial information about the supplier's cost. Finally, we study the case where the buyer has partial information regarding supplier's cost structure.

### 1.6.1 Generalized Demand

Given a general demand function, we can write supplier profit as

$$\pi_s = (w - C(x))D(p) - x.$$

The supplier's objective is to maximize its expected profit:  $\max_{x>0} \Delta E\pi_s$ . Retail and wholesale prices  $p$  and  $w$  are obtained from the following expressions, which reflect

the pricing mechanisms:

$$w = C(x) + (1 - \alpha)(\hat{p} - C(x)); \hat{p} = \arg \max_{p > C(x)} (p - w)D(p).$$

The following proposition generalizes the effect of relative bargaining power on cost-reducing investments by supply chain members.

**Proposition 1.8.** *Under a general demand function, a unique interior  $x^*$  (a) is an inverse U-shaped function of  $\alpha$  and (b) is maximized at  $\alpha^* = D'^2/(3D'^2 - DD'')$ .*

Part (a) of this proposition supports our paper's main result—namely, the existence of an inverse U-shaped relation between relative bargaining power and TI investment levels—under a general set of demand and cost functions. It may be surprising that, according to part (b), the level of  $\alpha$  at which investment is maximized depends not on the cost function's characteristics but rather on the demand function's structure. This result is unexpected in view of the cost function's direct effect on both profitability and the amount of project-related savings. It is evident that, under a linear demand structure, investment is maximized at  $\alpha = 1/3$ .

## Demand Uncertainty

We now consider the demand intercept as a random variable with normal distribution  $\tilde{a} \sim \mathcal{N}(\bar{a}, \sigma_d)$ . It is a challenge to generate analytical insights in these circumstances, but numerical analysis establishes that demand variation per se has a negligible effect on the optimal level of TI investor profits (see Figure 1.5).

### 1.6.2 Ex ante Renegotiation and Other Classes of Contracts

All the arrangements examined in this paper are based on a wholesale contract between agents in a supply chain. Such contracts allow a renegotiation process to occur ex post, or after the emergence of the TI investment possibility (Figure 3.1).

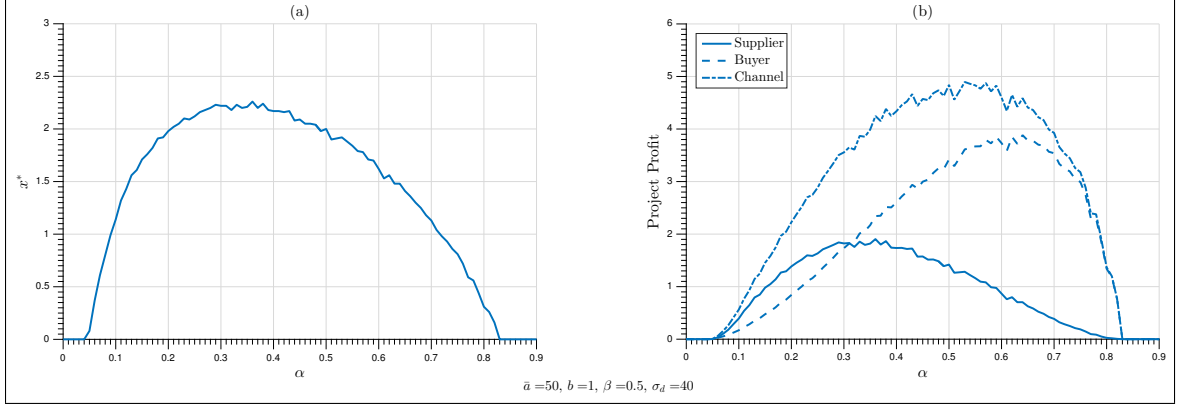


Figure 1.5: Effect of the correlation between cost and demand uncertainty on optimal TI investment and the profits of channel players

Here we briefly consider the possibility of negotiating the contract before (or in anticipation of) the emergence of a TI possibility. As mentioned in the Introduction, this setup has been partly covered by the holdup literature. In two-part tariff contracts (*Moorthy, 1987; Tirole, 1988*), for instance, the bargaining process determines how profit is shared *after* the supplier has been paid its fixed fee. *Cachon and Lariviere (2005)* discuss a revenue-sharing contract (much like a two-part tariff or price discount contract) under which the bargaining process similarly defines the profit share *ex ante* (i.e., before demand realization).

In all these settings, the objective is to maximize each agent's share of channel profit:  $\pi_s = (1-\alpha)\Pi$  and  $\pi_b = \alpha\Pi$ , where  $\Pi = (p-C)D(p)$  is the channel's profit. Thus the optimization problem is reduced to maximizing channel profit through the choice of TI investment level and retail price:  $\max_{p,x} \Pi = (p - C(x))(a - bp) - x$ . Because the optimality conditions are clearly independent of relative bargaining power, those conditions have received short shrift in this paper.

Another example is when the buyer and the supplier might renegotiate simultaneously on the wholesale and retail prices. Such a contract requires visibility on supplier's part over the retail price during the renegotiation phase (see Figure 3.1).

The optimization problem in this case can be written as

$$\begin{aligned} w &= C + (1 - \alpha)(p - C), \\ \hat{p} &= \max_p (p - w(p))D(p). \end{aligned}$$

Given this formulation, it turns out that a contract in which the wholesale and retail prices are determined simultaneously is equivalent to ex ante renegotiation case, as explained in Lemma 1.1.

**Lemma 1.1.** *The buyer's and the supplier's profit under simultaneous wholesale and retail pricing are characterized as*

$$\pi_s = (1 - \alpha)\Pi - x \quad \text{and} \quad \pi_b = \alpha\Pi.$$

According to this lemma, the profit shares are equal to those in the setting where the negotiation takes place *before* TI investment. As argued in the above, in this case relative bargaining power does not have any role in determining the optimal investment level.

### 1.6.3 Partial Information about Supplier Cost

The bargaining process discussed in Section 2.3 assumes that buyer can observe supplier's cost. Here we relax this assumption and consider a probabilistic belief, on the buyer's side, about the supplier's cost structure. Hence buyer assumes that supplier's cost is

$$C' = C + \tilde{\eta}, \tag{1.10}$$

where  $\tilde{\eta}$  is random noise distributed normally as  $\tilde{\eta} \sim \mathcal{N}(0, \sigma_\eta)$ .

**Proposition 1.9.** (a)  $\mathbb{E}_{\tilde{C}, \tilde{\eta}}[\Delta\pi_s(\tilde{C}, \tilde{\eta})] = \Delta\mathbb{E}\pi_s(\tilde{C})$ , and (b)  $\mathbb{E}_{\tilde{C}, \tilde{\eta}}[\Delta\pi_b(\tilde{C}, \tilde{\eta})] = \Delta\mathbb{E}\pi_b(\tilde{C})$ .

Proposition 1.9 shows that the  $\Delta$  operator ( $\Delta(g) = g(x) - g(0)$ ) removes the effect of uncertainty—in buyer’s belief about supplier’s cost—on investment benefits. Therefore, neither the optimal level of investment nor the TI investment outcome depend on  $\tilde{\eta}$ . In other words: the choice of optimal contract is based on comparing ex ante and ex post project profit (Section 1.5), so our main results should be robust to relaxing the assumption that buyer has full information about supplier’s cost structure.

## 1.7 Conclusion

In this study we analyze the effect of relative bargaining power, between buyer and supplier, on the latter’s adoption of TI measures under three contractual arrangements: no contract (NC), price commitment (PC), and shared investment (SI). We also examine the effect of technology uncertainty and the interaction of that effect with bargaining power.

We first develop the benchmark case in which there is no (investment) contract between buyer and supplier; in this we use a bargaining model for the pricing mechanism within a wholesale contracting framework. We show that the renegotiation resulting from an initial wholesale contract’s incomplete nature leads to the holdup problem. We also establish (in Proposition 1.1) that the level of TI investment is reduced when the buyer’s bargaining power is either extremely high or extremely low. Such investment inefficiencies can be alleviated only if certain contractual agreements are implemented. Unlike the established holdup literature, we show that the optimal choice of such agreements depends both on the level of relative bargaining power and on technology uncertainty.

In addition to analyzing a no contracting situation, we focus on two particular TI investment agreements: price commitment and shared investment. Under PC, the buyer commits to refrain from bargaining for lower wholesale prices when informed

of cost savings due to supplier's TI investments. As shown in Proposition 1.4, if the buyer's bargaining power is high then this arrangement does encourage the supplier to invest in TI, albeit not sufficiently. In such circumstances, a shared investment contract in which the buyer undertakes some the burden of TI investment leads to efficient investment levels. In this case, the optimal fraction of the investment that is covered by the buyer is equal to  $\lambda^* = \alpha$ ; the implication is that higher bargaining power warrants higher investment share on the buyer's side.

By shedding light on contract preferences from different perspectives, our results can be used as a guideline for managers (and policy makers) who seek to incentivize TI investment. According to Table 1.1, the SI contract produces the highest amount of TI investment; yet its desirability for each of the supply chain parties depends on the relative bargaining power as well as technology uncertainty. As a result, Table 1.2 shows that any of the three considered contracting mechanisms might prevail depending on different levels of these parameters.

This paper makes several contributions. First, it adds to the holdup literature by studying how relative bargaining power and technology uncertainty interact to determine the optimal contractual remedies. It also demonstrates that the improvement in technologies following an initial wholesale contract between buyer and supplier results in an inverse U-shaped investment response to relative bargaining power. Furthermore, contrary to what some of the researchers in this literature have suggested, price commitment does not resolve the holdup problem, and its efficiency depends on the relative bargaining power.

Second, this paper contributes to the OM literature on supply chain coordination choice by borrowing from an economic framework to incorporate renegotiation into the equilibrium wholesale price outcomes and thus into decisions about cost-reducing investments. The effects of incomplete contracts and their subsequent renegotiation have been overlooked in the OM and supply chain literature.

Third, this research contributes to the contracting literature by explicitly analyzing the effect of relative bargaining power on contract choice when there is misalignment of incentives within the channel. Finally, we contribute to the literature on sustainability and technology improvement by identifying the *lack* of supply chain coordination mechanisms as a current and significant barrier to the development of markets for sustainable TI. The results reported here can be used when designing supply chain incentives that will encourage channel members to adopt optimal TI arrangements, thereby reducing their respective environmental footprints by consuming lesser amounts of input commodities (e.g., energy).

The managerial implications are clear. By illustrating the strengths and weaknesses of different but frequently encountered contract types, this study can aid managers who—despite being eager to reduce their operating costs and environmental footprint—struggle to develop efficient strategies for technology improvement in their supply chains.

## CHAPTER II

### **Input-price Risk Management: Technology Improvement and Financial Hedging**

This chapter studies firms’ motivation for investing in risk management measures through technology improvement (TI): activities that reduce consumption of an input commodity, leading to fewer waste products and emissions, lower production costs, and more sustainable operations. Investing in TI is not a trivial decision because even though it clearly reduces cost and risk, firms may actually benefit from input-price uncertainty—which, when combined with production flexibility, creates an “option value” that firms are understandably reluctant to forgo. We use a stylized mathematical model to explore and generalize this claim and to specify its implications, under a variety of scenarios, for firms’ decisions to invest in cost reduction and risk management. We derive a closed-form expression that explicitly quantifies a firm’s attitude toward input-price risk by considering the firm’s positive or negative *certainty premium* (i.e., what the firm would pay to “lock in” the unit input price); we then link that premium to various firm- and industry-level characteristics. In addition, we compare the risk management advantages of technology improvement versus financial hedging (FH) and characterize conditions under which these strategies are complements or substitutes. We find that, although input-price uncertainty may be desirable even for risk-averse firms, those firms can still benefit from investing in



risk reduction measures (e.g., TI, FH) because the uncertainty’s option value could thereby increase. A firm’s ability to adjust its price in response to both market competition and input-price variation mediates the benefit of risk-reducing measures and also affects the complementarity of these two strategies.

## 2.1 Introduction

On December 19, 2014, the *Wall Street Journal* (WSJ) reported that “many airlines have raced in recent years to buy new, fuel-efficient jets to cut down on fuel bills—which typically make up about 30% of an airline’s operating costs. Amid today’s falling oil prices, there’s suddenly less urgency to do that.” One year later, the *Financial Times* wrote that “the question is whether, as some analysts have speculated, a long-term lowering of fuel prices could make airlines reluctant to invest in more fuel-efficient aircraft such as Airbus’s A320neo, Boeing’s 737 Max and A350 and 787 wide-body jets.” In the same year, WSJ quotes an industry expert responding as follows: “investment in new-technology aircraft will not stop [due to reduced fuel prices]; [but] the investment rationale will change.”

Motivated by the business case of the airline industry (and several others), this paper explores the incentives of firms to use *technology improvement* (TI)—technical changes that reduces the consumption of an input commodity—for the purpose of managing risk. Previous research has noted the use of TI as a risk management strategy: “An airline choosing to operate a newer, more fuel-efficient fleet, has less exposure to the price of jet fuel” (*Treanor et al.*, 2013). Technology improvement, such as follows from investing in fleet fuel efficiency, is thus viewed as another strategy for airlines to incorporate into their “overall risk-management program”.

Should all firms invest in technology improvement to manage input (i.e., jet fuel) price risks? To answer this question, we highlight the trade-offs involved when adopting TI initiatives. On the one hand, an extensive academic literature argues that

the firm actually benefits from uncertainty, including uncertainty about the input price, provided its profit function is convex with respect to the uncertain parameter (Oi, 1961; Mas-Colell et al., 1995; Farrell et al., 2002; Cabral, 2003). The analytical explanation for this phenomenon is that, for a convex profit function, such uncertainty creates an “option value”. In practice this means that a firm is able to adjust production levels in response to fluctuations in the price of inputs: reduce production when the price is high, increase production when it is low. Alexandrov (2015) extends this insight to the case of competition under some particular conditions. The implication is that, under those conditions, firms may benefit from deliberately exposing themselves to input-price uncertainty and so may refrain from undertaking such risk-reducing measures as adopting financial hedging instruments, including “contracts that lock in a particular price for a period of time—for example, an airline securing fuel supply for the upcoming year.” Such contracts are avoided because the uncertainty being hedged has an option value but no apparent cost. On the other hand, of course, many firms are risk-averse and so prefer to invest in risk-reducing measures irrespective of any option value that uncertainty may yield.

Our main contribution is to study the trade-off between two forces: convexity of the profit function, which results in risk-seeking preferences and higher option values; and the firm’s aversion to risky profit, which results in risk avoidance and lower option values. In this regard, we explicitly derive the (positive or negative) certainty premium that firms are willing to pay for a *constant* input price as a function of the following factors: the extent of uncertainty, the profit function’s curvature, the risk aversion parameter, and the total amount of commodity inputs that are used in the firm’s operations. The derived mechanism makes it clear why some firms prefer not to pursue TI: they prefer to continue benefitting from the uncertainty’s option value.

When it comes to input-price risk management, technology improvement is not the only option. Companies in different industries deploy an array of conventional

risk-hedging strategies—including financial hedges and long-term contracts—to shield themselves from input-price volatility. In fact, the most prominent risk management practice is financial hedging (FH) using futures and options (*Froot et al.*, 1993). Yet FH instruments do have several drawbacks,<sup>1</sup> and they are not always the best solution. According to the WSJ in 2016, “more airlines, including some of the world’s largest, are backing off [from] spending billions of dollars to hedge against rising fuel costs after getting burned by low oil prices.”

Another key difference between FH and TI is that using the latter to reduce the rate at which input commodities are consumed can affect a firm’s profit in two ways. First, TI reduces the firm’s average unit operational cost; this is the “average price” effect. However, it also changes how input-price uncertainty affects the firm’s “profit risk” by reducing the *intensity* of commodity use in both the production process and the total cost function. So in terms of risk management, the key difference between FH and TI is that FH reduces the volatility of price but not its mean, whereas TI affects both volatility and the mean.<sup>2</sup> We also identify another contrast between technology improvement and financial hedging: TI directly changes the profit function’s curvature and also the option value of uncertainty, whereas FH does not affect the certainty premium of a flexible firm.

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<sup>1</sup>There are three major limitations to the use of financial hedging. First, contingent-claim contracts (e.g., futures and options) may exist for only a limited number of commodities—as in the case of jet fuel, for which futures contracts were slow to emerge. In these cases, a firm uses the most closely related futures contract available in the market, thereby achieving only a partial cross-hedge; thus airlines have long used crude oil or heating oil futures and options to hedge jet fuel price risks (*Adams and Gerner*, 2012). The second problem with using futures and options is the uncovered exposure to quantity or production risks (*Moschini and Lapan*, 1995). Even if there is a perfect futures contract for the focal commodity, the firm must still secure a fixed number of futures positions in advance. Finally, the margin requirement for futures and the up-front payments for options necessitate that the hedging firm commit additional financial resources; hence firms with financial constraints may be unable to purchase their desired amount of hedging positions. The bankruptcy of Metallgesellschaft is a well-known example of a company that failed because it could not meet all the margin requirements of multiple hedged positions.

<sup>2</sup>This difference should not be mistaken for the subject of an extensive operations management literature that studies the benefits of adding *capacity flexibility*—a different form of technology improvement from the subject of our paper—in the face of demand uncertainty (for a review, see *Boyabatlı and Toktay*, 2011).

Technology improvement consists of implementing measures that increase the efficiency of a firm’s operations, thereby reducing the amount of input needed for the same amount of output. It follows that TI not only serves as a strategy for managing risk and reducing cost but also has direct implications for the sustainability of firms’ operations from the environmental perspective. For example, an airline that adopts a more energy-efficient engine reduces the quantity of jet fuel burned for each kilometer of flight. The oil refinery industry is another energy-intensive example. In this industry, the ratio of energy value (used to run the refining process) to value-added varies between 10% and 25%. A frequently implemented TI measure in this industry is “flare gas recovery” (FGR), which includes installing recovery compressors that reduce fuel consumption and flaring noise, operation and maintenance costs, thermal radiation, and steam consumption; FGR also reduces emissions and thus air pollution.<sup>3</sup> Our goal is therefore to develop a framework useful for characterizing instances when risk management and TI are aligned with the organization’s sustainability goals, which provides still more reasons for investing in cleaner production.

Given the literature’s previous results on the option value of uncertainty, should firms increase or rather decrease their investment in TI in response to higher input-price volatility? We respond to this question by explaining the dynamic through which the total amount of TI investment changes as a function of input-price risk; for that purpose, we use an explicitly characterized input-price certainty premium (and its sign). We show that, when investment reduces the certainty premium, firms tend to respond by increasing (resp. decreasing) TI investment in the presence of more (resp. less) uncertainty. This approach has the additional advantage of allowing us to isolate the risk management properties of TI while *ignoring* the average price effect

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<sup>3</sup>Our paper focuses on intermediary commodities (e.g., energy, water) that are not directly consumed by the end customer. Examples of production inputs that the final consumer does *not* value directly include the jet fuel for an airline, the gas for a power plant, and metals for a wind turbine producer. Customers are interested in the end product (e.g., travel services, electricity produced) rather than the input used to produce those services or goods. For that reason, in this paper we do not address technology changes that replace resources with more economical commodities.

(which is instead driven by the focal technology’s cost effectiveness).

The risk management strategies of TI and FH can affect risk as well as the profit function’s curvature and hence the option value discussed previously. However, there are various mechanisms through which a firm’s profit function is affected also by industry characteristics (e.g., product/service type or level of competition). In some industries, firms are “flexible” and can adjust their production plan or price *after* observing the realizations of uncertain input costs; the shipping industry is a prime example, with companies adjusting their freight rates in response to realized fuel costs (*Wang and Lutsey, 2013*). In other industries, the market is “committed” and so firms must decide on a production or price plan *before* observing the realization of input-price shocks. For instance, airlines offer tickets months before the flight even though the price of jet fuel changes almost daily (*Morrell and Swan, 2006*).<sup>4</sup> In this paper we delineate the circumstances under which a firm’s profit function is convex in the input price<sup>5</sup>—connecting that convexity to firms’ pricing flexibility and specifying the conditions under which a firm does (or does not) benefit from input-price uncertainty. Table 2.1 gives a summary and examples of the scenarios we examine for firms characterized by low versus high risk aversion.

As we remark in Section 2.2’s review of the literature, this research is among the first to explore firms’ responses to input-price uncertainty by considering both the risk-averse and risk-seeking motives of the firm. This study is also one of the first to examine the risk management effect of technology improvement and to compare it, from a risk perspective, with financial hedging.

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<sup>4</sup>The usual practice of hedging via contingent claims is extremely costly when the market moves in the wrong direction, as when airlines lose billions of dollars because of *low* oil prices (*Carey 2016*).

<sup>5</sup>We also show that, in a duopoly market, the profit function can actually become *concave* in the input price.

Table 2.1: Interaction between industry and firm type (RA $\uparrow$ , high risk aversion; RA $\downarrow$ , low risk aversion).

		Flexible	Committed
Noncompetitive	RA $\uparrow$	National oil and copper companies, large mining companies	Large specialized agriculture companies (e.g., monopolist coffee producers)
	RA $\downarrow$	Highly leveraged special commodity processors (e.g., Nestlé)	Nondiversified regional electricity producers
Competitive	RA $\uparrow$	Diversified freight and shipping companies	Diversified large agriculture companies producing normal crops, diversified hotel companies
	RA $\downarrow$	Refineries, solar panel producers, cement producers chemical industry, metal smelters	Airlines, small farmers, electricity companies with long-term contracts, small hotels, construction companies

In order to pursue this approach, in Section 2.3 we explicitly quantify the costs and benefits of input-price risk and connect them to a diverse range of market- and firm-level parameters, thereby shedding light on their interactions. Section 2.4 brings together operational (TI) and financial (FH) strategies for hedging risk; we address their relative effectiveness and examine whether these strategies are substitutes or complements. Section 2.5 offers an alternative strategic perspective on the problem by extending our analysis to a duopoly. We conclude in Section 2.6 with a summary and suggestions for future research.

## 2.2 Literature Review

The risk management benefits of technology improvement are widely recognized in the context of operations sustainability and management, environmental sustainability, and cost reduction. However, these benefits have received insufficient attention in the literatures on financial economics and operations finance. We contribute to the latter field by formulating and discussing the firm’s incentives to choose technology improvement not only for cost reduction and sustainability motives but also for reducing risk.

Much of the sustainable operations literature in technology management focuses on capacity investment via adopting sustainable technologies. There is a substream of papers that focus on the effect of macro-level policies and regulations (tax incentives, subsidies, emission caps, etc.) on a firm’s investment decisions (see e.g. *Drake* 2011, *Krass et al.* 2013, *Kok et al.* 2014). Another group of studies addresses the firm-level capacity investment decision, when both clean and dirty technologies are available (*Wang et al.*, 2013; *Aflaki and Netessine*, 2015; *Drake et al.*, 2015). *Drake et al.* (2015) identify the optimal investment decision when a less emission-intense but expensive technology competes with a more emission-intense but cheap technology. *Wang et al.* (2013) focus on the different investment and operating costs of these two

technologies, and *Aflaki and Netessine* (2015) discuss the intermittent drawback of green technologies as compared with conventional ones.<sup>6</sup> *Plambeck and Taylor* (2013) consider the problem of a firm facing exogenous random prices of input and output commodities; this firm must choose among improving its input efficiency, its input–output conversion efficiency, and a “flexibility option”. *Wang et al.* (2013) study the incentives to invest in a newer, and more costly, energy-efficient technology and those to invest in an older but cheaper technology that is not energy efficient. Volatile energy prices render the firm’s capacity utilization a random variable, and some scholars have derived optimal “policy bands” for the action and inaction regions of choosing a capacity portfolio. Our work differs from the sustainable operations research cited here in that we do not consider a discrete choice of technology; instead we focus on the risk management properties of technology improvement more generally while abstracting from particular policies.

Technology improvement decisions are not made independently of firms’ access to other risk-management solutions. Analyzing the respective costs and benefits reveals the interaction between TI investment and the firm’s other risk management strategies. Closely related work in the operational hedging literature studies how operations management interacts with financial hedging mechanisms. *Boyabatli and Toktay* (2004) and *Boyabatli and Toktay* (2011)<sup>7</sup> explore this interaction and discuss real (compound) options such as decentralization, production postponement, and production flexibility—types of operational strategies that can substitute or complement other firm’s risk management solutions (futures, swaps, etc.). With regard in particular to risk management and technology investment, *Goyal and Netessine* (2007) identify the optimal level of investment in two rival (product-flexible and product-dedicated) technologies in a competitive and volatile market as a function of demand for their respective products. These authors discuss how market size, correlation in

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<sup>6</sup>A more detailed review of capacity investment is provided by *Van Mieghem* 2003.

<sup>7</sup>For similar papers, see the references therein.



demand of the two products, and competition each affect the optimal investment decision vis-à-vis either of the two technologies. *Chod et al.* (2010) consider the same two technology types but also incorporate a postponement option for dealing with the risk of profit variability. Their study also shows when product flexibility (i.e., using a flexible technology) and postponement can complement or, alternatively, substitute for financial hedging as a mitigator of risk. Our work complements this literature but deviates in some important respects. First, we introduce TI investment as another operational decision that affects a firm’s risk exposure. Second, our focus is not on demand variability but rather on input-cost variability. And third, we study also the interaction between financial hedging and TI investment in a competitive environment.

In contrast to the operations literature, the finance literature has extensively studied the value of hedging and its relation to technology investment. Many researchers have sought to assess empirically whether financial hedging is beneficial in practice (see *Carter et al.* 2006 and the references therein). From the theoretical angle, *Adam et al.* (2007) develop a framework for finding the equilibrium number of firms that hedge against input-price volatility in an oligopolistic environment; *Alexandrov* (2015) describes the conditions under which hedging is *not* a profitable strategy. Both of these papers address the value of uncertainty when the profit function is convex with respect to the uncertain parameter (here, input price)—a convexity that stems from Jensen’s inequality. Our own study is similarly based on analyzing (i) the profit function’s convexity with respect to input price and (ii) the effect of TI investment on the function. We find that if hedging is not a profitable strategy, then TI investment could serve as a substitute. In addition, we extend this analysis by accounting for firms’ relative flexibility as regards a price commitment.

Finance literature also studies how capacity investment interacts with hedging and analyzes the moderating effect of a firm’s financial structure on its capital investment

decisions. For instance, *Rampini and Viswanathan* (2010, 2013) study how capital constraints affect the firm’s capital acquisition decision. In their setting, a firm can invest in capacity expansion via either leasing or buying; these papers show how a firm’s capacity acquisition decision interacts with its hedging decision—especially when the budget is tight. Our study considers neither the firm’s financial structure nor the source of TI investment funding; moreover, we suppose that the firm is not at risk of bankruptcy. We assume as well that a production technology with required capacity already exists. However, the firm can invest in improving that technology, thus reducing its input consumption rate and hence the variation in unit input price.

The behavior of firms facing uncertain input/output prices has been studied by many (e.g., *Baron* 1970, *Hartman* 1976). *Sandmo* (1971) and *Batra and Ullah* (1974) add risk aversion to the behavioral model of a firm that is producing under input price uncertainty and that must also conform to a production plan determined ex ante. *Turnovsky* (1973) and *Epstein* (1978) highlight the critical role of flexibility in adjusting a production plan before or after observing the realization of the random shock. Another group of papers (including *Moschini and Lapan* 1992 and *Viaene and Zilcha* 1998) consider the roles of production flexibility and optimal production decisions in the presence of random input/output prices and hedging instruments. The already-mentioned, more recent contribution by *Alexandrov* (2015), and the citations within, demonstrate that firms benefit from exposure to risk when they can make adjustments *after* observing the realization of a random production parameter.

The interaction between competition and technology improvement in a duopoly is a recurring subject of research in the industrial organization literature. Starting with papers such as *Brander and Spencer* (1983) and *Spencer and Brander* (1992), this line of work examines how competition affects a firm’s investment in capacity expansion and/or technology improvement. An insight from this literature is that firms may overinvest in research and development when committing to play a tough strategy.

We likewise find that, under certain conditions, firms may overinvest in technology improvement (e.g., in renewable alternatives) so as to improve their position in a duopoly game. Our results suggest that market competition may have positive effects on the sustainability of operations.

In addition to demonstrating the benefits of TI, we also discuss the shortcomings and downsides of using it for the purpose of risk management. A major issue with risk reduction through TI is the irreversible nature of investment in improved technologies. Firms that invest in improving the efficiency of commodity use in a production process may be disappointed in the results if a focal commodity subsequently becomes much cheaper. A financial hedging strategy provides more flexibility because the firm determines its optimal hedge ratio on a “rolling” basis and so does not require a perpetual commitment.

### 2.3 Modeling Input-price Uncertainty

Our basic setup includes a monopolistic, risk-averse firm that produces a homogeneous product sold in a single market at price  $p$ . We consider a linear demand function  $q(p) = a - bp$  for  $a$  the market size and  $b$  the sensitivity of demand to price. Appendix A summarizes all the notations.

The per-unit production cost  $\tilde{c} = c_0 + \tilde{\varepsilon}\gamma(x)$  consists of a constant plus a random component  $\tilde{\varepsilon}$  that is normally distributed with mean  $\mu$  and standard deviation  $\sigma_{\varepsilon}$ . Hence the variability in  $\tilde{c}$  is due to the *input-price* uncertainty of a single production element purchased on an external market over which the firm has no power. Because we aim to capture input-price uncertainty, we set (without loss of generality) the fixed component  $c_0$  of the unit price to zero. While the distribution of  $\tilde{\varepsilon}$  is exogenous and constant, firm’s investment in different financial or operational measures might differently affect the distribution of  $\tilde{c}$ . Some investments (e.g. fair financial hedging) reduce the variance but not the mean. Some may reduce the mean (e.g. a fixed

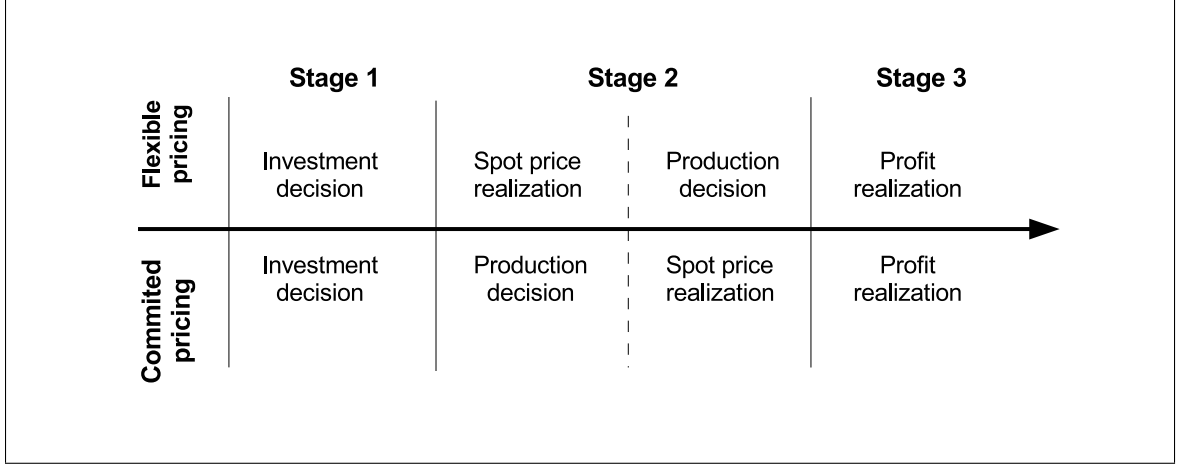


Figure 2.1: Timeline of events.

subsidy by the government) but not the variance. The firm has the option of investing in technology improvement that reduces the input cost per unit of production. This, in turn, reduces both the mean and the variance by reducing the multiplier  $0 < \gamma(x) \leq 1$ : the *unit input consumption* of the final product, which is a function of the investment  $x$  in technology improvement. This function is assumed to be both decreasing (i.e., more investment leads to less unit input consumption per product) and convex, where the latter characteristic reflects decreasing marginal returns to investment. We therefore model the product's unit cost as  $\tilde{c} = \tilde{\varepsilon}\gamma(x)$ , where  $\gamma(0) = 1$ .

### 2.3.1 The Timeline: Flexible versus Committed Firms

There are two stages of decision making. In the first stage, the firm decides on its level of investment in technology improvement. In the second stage, the firm makes its production decision—that is, it sets the output price and therefore the quantity to be produced.

As illustrated in Figure 3.1, we study two key different setups in the second stage. In the first (resp. second) setup, the pricing or quantity decision occurs after (resp. before) resolution of input-price uncertainty. The difference in these two timelines reflects the firm's power to adjust its production decisions after the realization of the

input price. In that case, the firm chooses a product price such that the input-price shock is transferred (in full or in part) to the consumer; we refer to this setup as the *flexible* case. In contrast, if the firm fixes the output price before the input price is realized (as occurs under the long-term contracts typical of many value chains), then the firm cannot adjust its production price. This setting is known as the *committed* case.<sup>8</sup> These two timelines affect firm's TI investment decision in different ways as we will describe in the next section.

Note that, for flexible and committed timelines both, the decision on TI investment occurs *before* realization of the input price; this timing means that the firm's expected profit/utility is maximized in stage 3. Now taking into account the TI investment expenditure,  $x$ , we can write the firm's profit as

$$\pi(\tilde{\varepsilon}, x) = q(p)(p - \tilde{\varepsilon}\gamma(x)) - x = (a - bp)(p - \tilde{\varepsilon}\gamma(x)) - x. \quad (2.1)$$

### Price Optimization Problem: The Flexible Firm.

In the flexible setting, the firm chooses—for a fixed level of TI investment—a market price  $\hat{p}$  in stage 2. Because input-price uncertainty is resolved before this stage, the pricing decision can be based strictly on maximizing profit. In the second stage, then, the optimal price is obtained by solving

$$\hat{p} = \arg \max_p \{(a - bp)(p - \tilde{\varepsilon}\gamma(x)) - x\}. \quad (2.2)$$

Note that the accent “^” refers to optimal values for a flexible firm.

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<sup>8</sup>The airline, construction, and agriculture production industries all feature committed production plans. Hence firms in these sectors must decide on a production plan prior to observing actual input costs (e.g. labor and, respectively, jet fuel, cement, and fertilizer) at the time of production. In contrast, refineries, restaurants, power plants, and food processors are examples of industries that observe their input costs (within a reasonable time frame) and then choose the optimal production quantity.

### Price Optimization Problem: The Committed Firm.

In the committed setting, pricing decisions must be made before uncertainty is resolved. The second-stage optimization problem is therefore

$$p^* = \arg \max_p \{\mathbb{E}[U(\pi(\tilde{\varepsilon}, x))]\}; \quad (2.3)$$

where  $U(\cdot)$  is the firm's utility function, which characterizes its attitude toward uncertainty. Here the superscript “ \* ” refers to the optimal values for a committed firm. We solve this problem by following much of economics and finance literature (*Levy and Markowitz, 1979; Pulley, 1983*) in representing the firm's attitude toward risk via the well-known mean–variance preferences

$$\mathbb{E}[U(\pi)] = \mathbb{E}[\pi] - \frac{1}{2}\lambda \text{Var}(\pi), \quad (2.4)$$

where  $U(\pi)$  is the firm's concave utility function, coming from the market incompleteness assumption. The term  $\lambda = -U''(\cdot)/U'(\cdot)$  is the Arrow–Pratt measure, which captures the firm's level of constant absolute risk aversion (CARA),<sup>9</sup> and the derivatives are with respect to the variant input price  $\tilde{\varepsilon}$ .

#### 2.3.2 Determinants of Firms' Attitudes toward Input-price Risk

Input-price risk is not always undesirable, even for a risk averse firm. To understand this, consider a firm purchasing an input commodity at price  $\tilde{\varepsilon} = \mu + \tilde{r}$ , where  $\mu$  is the mean (base) price of the input commodity and  $\tilde{r} \sim \mathcal{N}(0, \sigma_\epsilon)$  represents unexpected shocks to the base price. Positive input cost dictates the constraint  $\tilde{r} \in [-\mu, a/b - \mu]$ . The certainty premium  $\omega$  is then the amount that makes the firm indifferent between purchasing the risky input and purchasing the commodity at the

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<sup>9</sup>The mean–variance preferences are exact representations if the firm's risk preferences are characterized by a CARA utility function *and* the uncertain parameter is normally distributed. Otherwise, these representations are considered to be approximate (*Gollier, 2004*).

pre-set price of  $\mu + \omega$ :

$$U(\pi|\tilde{\epsilon} = \mu + \omega) = \mathbb{E}[U(\pi|\tilde{\epsilon} = \mu + \tilde{r})]. \quad (2.5)$$

The following lemma will be instrumental in the rest of our results. All the proofs are provided in Appendix C.

**Lemma 2.1.** (a) *Assume that the firm is risk neutral. For a flexible (resp. committed) firm,  $\hat{\pi} = \pi(\tilde{\epsilon}, x)$  is a decreasing convex (resp. linear) function of  $\tilde{\epsilon}$ .*

(b) *The certainty premium  $\omega$  can be approximated as*

$$\omega = \underbrace{\sigma_{\epsilon}^2}_{\text{market risk}} \times \underbrace{\zeta}_{\text{risk exposure}}, \quad \text{where } \zeta = \underbrace{\frac{\pi''|_{\tilde{\epsilon}=\mu}}{\pi'|_{\tilde{\epsilon}=\mu}}}_{\text{profit structure effect}} + \underbrace{\lambda\gamma(x)q(\mathbf{p})}_{\text{risk attitude effect}}, \quad (2.6)$$

where  $q(\mathbf{p} = \hat{p})$  is the total quantity for the flexible setting and  $q(\mathbf{p} = p^*)$  denotes that for the committed setting.

Given Jensen's inequality,<sup>10</sup> an important implication of Lemma 2.1(a) is that a risk-neutral flexible firm does in fact benefit from the randomness inherent in the input cost. As mentioned in the Introduction, this result, although seemingly counterintuitive, is well known in the literature (Oi, 1961). The idea is that the profit function's convexity in the input price delivers the option to expand production during favorable times (i.e., when input is cheap) and to contract that output in unfavorable times (when input is expensive). Part (b) of the lemma helps us understand the factors that drive a firm's attitude on input-price risk, which we now discuss in turn.

### Risk Aversion.

Lemma 2.1(b) extends the result in part (a) to explain why it is not only risk-neutral firms that desire input price uncertainty. A risk-averse firm also might have a

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<sup>10</sup>Jensen's inequality is that  $\mathbb{E}[f(\tilde{X})] \geq f(\mathbb{E}[\tilde{X}])$  for a convex function  $f(\cdot)$  and a random variable  $\tilde{X}$ .

positive attitude toward input-price risk if it is not “too” risk averse. In other words,  $\omega < 0$  when  $\lambda$  is sufficiently small.

### **Market Risk.**

The effect of  $\sigma_\epsilon^2$  on  $\omega$  is multiplicative; thus it simply magnifies the effect of a firm’s risk exposure without affecting its sign. Hence market risk does not influence the firm’s attitude. In other words: if the firm prefers to accept input-price risk—that is, if  $\omega < 0$ —then increasing (resp. decreasing) uncertainty will amplify (resp. attenuate) that preference.

### **Profit Structure.**

The profit function’s structure figures largely in the firm’s attitude toward input-price uncertainty. One important factor is whether the firm is flexible or committed in its pricing decision. In particular, if one considers part (a) of Lemma 2.1 then  $\pi''(\mu) = 0$  for a committed firm; this means that, unlike a flexible firm, a (risk-averse) committed firm does *not* benefit from input-price uncertainty. We shall establish that competition, technology improvement, and the availability of hedging mechanisms all have important implications for the curvature of a firm’s profit function.

### **Quantity of Input Used.**

Recall that, for a fixed technology level  $x$ , we use  $\gamma(x)q(\mathbf{p})$  to denote the total amount of the input commodity used in the production process, where  $\mathbf{p}$  represents the optimal price. When this term is high, in which case the input commodity is used in relatively large quantities, uncertainty about the input price becomes less attractive; it follows that firms prefer input-price uncertainty about commodities that the production process consumes in relatively small quantities. Technology improvement



changes both the curvature of the profit function and the quantity of input used.<sup>11</sup> The latter is the main difference between financial risk management (e.g., via hedging) and TI measures: FH never affects the quantity of input used and so cannot affect the firm's certainty premium directly (i.e., by changing input quantities) *except*, as we will show, in Section 2.5, in a competitive setting.

## 2.4 Options for Managing Input-price Risk

As discussed in the previous section, technology improvement can significantly affect the value (for the firm) of input-price uncertainty. Furthermore, the availability of financial risk management mechanisms should affect a firm's risk preferences and thus their view on input-price uncertainty. In this section we explore these effects for the two strategies and compare them from the perspective of risk.

### 2.4.1 Investment in Technology Improvement

Technology improvement affects the total amount of input commodity used and also the profit function's curvature. Here we will consider the amount of TI investment as a decision variable and examine its relation to input-price uncertainty.

The firm's stage-1 optimization problem with respect to TI investment in the flexible setting is

$$\max_x \mathbb{E}[U(\pi(\tilde{\epsilon}, x))]. \quad (2.7)$$

**Proposition 2.1.** *For a flexible firm: (a)  $\omega$  increases in  $x$  if and only if (iff)  $\lambda < \lambda_f$ ; and (b) the optimal investment level  $\hat{x}$  is decreasing in  $\sigma_\epsilon^2$  iff  $\lambda < \lambda_f$ .*

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<sup>11</sup>How uncertainty affects the expected return also defines the riskiness of firms's revenue process. The weighted average cost of capital (WACC) is a function of a firm's market Beta (i.e. co-movement of firm's stock return and the return of the aggregate market index) and more risky firms need to pay a higher premium to the equity market to raise capital. TI investment might also affect the riskiness of the firm by changing the exposure to macroeconomic factors (inherent in the price of input commodities such as crude oil), making it cheaper to raise capital. If TI reduces the firm riskiness, it reduces the cost of capital for future units of investment and introduces additional incentive to cut risk. This paper, however, does not include this effect in the analysis.

For a committed firm: (c)  $\omega$  decreases in  $x$  iff  $\lambda < \lambda_c$ ; and (d)  $x^*$  is increasing in  $\sigma_\epsilon^2$  iff  $\lambda < \lambda_c$ .

To explain the intuition behind part (a) of the proposition, we point out that investment in TI has two notable effects on the firm's profit function. On the one hand, it reduces the curvature of that function, thereby diminishing (for a flexible firm) the option value of input-price uncertainty. On the other hand, such investment also reduces total input use—which increases firm profit. From (2.6) (in Lemma 2.1(b)) it is clear how these two opposing forces determine the firm's certainty premium  $\omega$ . If  $\lambda$  is low, then the option value is more important to the firm, i.e. the firm increasingly prefers to *seek* for input-price uncertainty. Thus where investment reduces the uncertainty, the certainty premium always increases with such an investment. Yet for firms that are sufficiently risk averse, the input-quantity effect dominates; and given that  $\gamma(x)q(\hat{p})$  is decreasing in  $x$  for large  $\lambda$ , the certainty premium decreases (i.e., exposure to uncertainty becomes less expensive for the firm).

Part (b) of Proposition 2.1 is a direct consequence of part (a). Technology improvement reduces the mean *and* the variance of input used. Greater variance in the input price does not change the expected use, but it does increase the risk associated with input price. With reference to part (a), we can see that if  $\lambda$  is low (resp. high) then the firm seeks to increase (resp. decrease) input-price uncertainty; so as variance increases, the firm invests less (resp. more) in TI. See panel (a) of Figure 2.2. Thus a risk-neutral flexible firm reduces its TI investment in response to increased uncertainty. At the limit, if input-price uncertainty is extremely high then TI investment may be deemed unprofitable. The thresholds for  $\lambda$  in part (a) and (b) differ only because of the approximate nature of mean–variance preferences. Numerical results indicate that those thresholds are fairly close to each other:  $|\lambda_{f'} - \lambda_f|/\lambda_{f'} < 10^{-2}$ .

The main difference in the case of a committed firm is that its profit function is linear with respect to (w.r.t.) the input price (cf. Lemma 2.1); as a result, the

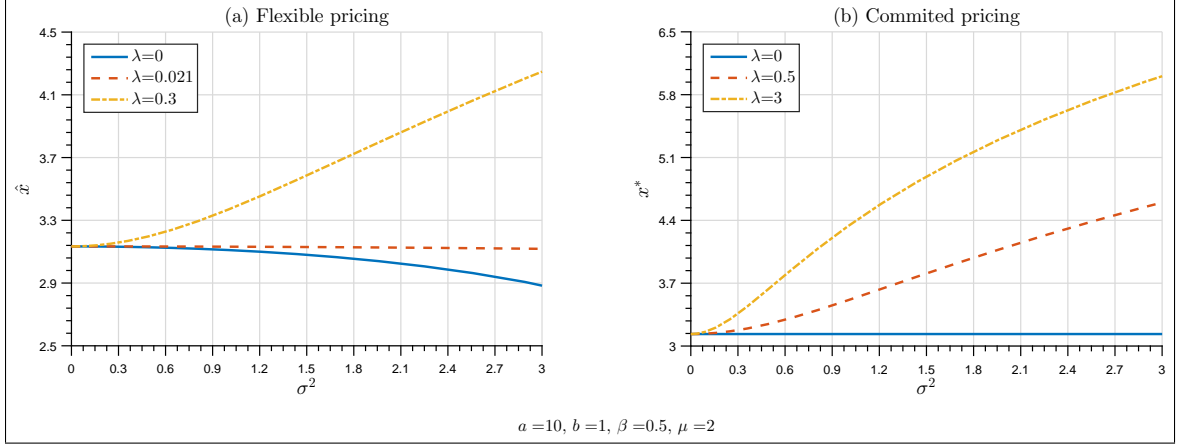


Figure 2.2: Investment in technology improvement. For the numerical illustration, we assume  $\gamma(x) = e^{-\beta x}$ .

first term in (2.6) is eliminated. In other words, input-price risk no longer has any option value. When  $\gamma(x)q(p^*)$  decreases (resp. increases) with  $x$  for  $\lambda < \lambda_c$  (resp.  $\lambda > \lambda_c$ ), there is a subsequent decrease (resp. increase) in  $\omega$ . Technology improvement affects  $\gamma(x)q(p^*)$  in two ways: because it lowers the output price (per Equation (2.3)), TI increases output quantity  $q(p^*)$  and lowers the consumption rate  $\gamma(x)$ . Part (c) of Proposition 2.1 states that this dynamic affects how  $\omega$  behaves at different levels of TI investment. The total input quantity used  $\gamma(x)q(p^*)$  decreases only if  $\lambda < \lambda_c$ . The same mechanism relating  $\omega$  and the effect of  $\sigma_\epsilon^2$  on the optimal TI investment holds here as well, and it leads to a similar result. When  $\lambda = 0$  (the case of a risk-neutral committed firm), input-price uncertainty has *no* effect on TI investment because the certainty premium  $\omega$  is zero in this setting (c.f. Equation (2.6)).

These results hint at an important feature of TI investment: risk-averse firms can use it as a tool for managing risk. Indeed, cutting a random variable by a fraction reduces both its mean and its variance. If the firm prefers input-price variance because of the associated option value, then TI loses its value. But if the firm is risk averse and would prefer to avoid input-price risk, then TI is a good means to manage that risk and also to reduce the average price.

### 2.4.2 Financial Hedging

Financial hedging is a conventional option for managing input-price uncertainty. The availability of FH mechanisms (futures contracts or options) to manage input risk has a direct influence on the firm's pricing decision, its attitude toward input-price variation (which we represent by the certainty premium  $\omega$ ), and its utility. So in this section we consider a setting where, in stage 1, the firm can adopt both strategies simultaneously.

When hedging is available, the net profit of a firm in stage 3 will be the sum of returns from operational activity and from hedging:  $\pi = \pi_o + \pi_h$ , where

$$\pi_o = (a - bp)(p - \gamma(x)\tilde{\varepsilon}) - x, \quad (2.8)$$

$$\pi_h = h(\tilde{\varepsilon} - f). \quad (2.9)$$

In the latter equality,  $h$  is the firm's hedging position and  $f$  is the futures price. To keep the analysis tractable, we consider linear hedging instruments, i.e, forward contracts.<sup>12</sup> It should also be noted that since the shocks are i.i.d, the optimal level of hedging remains constant in time and no dynamic hedging is required.

In a "fair" hedging contract,  $f = \mathbb{E}[\tilde{\varepsilon}]$ ;<sup>13</sup> here a risk-averse firm always chooses a hedging position  $\bar{h} = q(\mathbf{p}_h)$ , where  $\mathbf{p}_h = \hat{p}_h$  ( $\mathbf{p}_h = p_h^*$ ) is the optimal price in flexible (committed) setting provided that hedging is available. It is clear that if  $f > \mathbb{E}[\tilde{\varepsilon}]$  then  $h < \bar{h}$ , from which it follows that  $\bar{h} = q(\mathbf{p}_h)$  is an upper limit for the firm's

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<sup>12</sup>Options contracts are an alternative instrument, which allow for non-linear risk exposure. For the call options  $\pi(\tilde{\varepsilon}) = (\tilde{\varepsilon} - s)^+$  where  $s$  is the strike price. However, options needs an upfront premium payment; which is in contrast with forward contracts that do not require a payment beyond the margin account.

<sup>13</sup>We assume forward market is a fair market in a sense that forward prices are equal to the expected value of the realized spot prices. There is also no basis risk. This is possible when there are no major frictions and the market is risk-neutral at the "aggregate" level. The aggregate risk-neutrality does not imply that each agent to be risk-neutral; it just requires that the total risk-aversion of long and short sides of the market are equal. We assume the hedging pressures of two sides neutralize each other and the future contract are unbiased one even under the physical measure.

hedging position.

**Proposition 2.2.** *The optimal hedging positions and the optimal prices (for both the flexible and committed settings) are as follows.*

$$(a) \quad \hat{h} = h^* = \frac{1}{2}\gamma(x)(a - b\gamma(x)\mu) = \gamma(x)q(\mathbf{p}_h|\tilde{\epsilon} = \mu),$$

(b) *In the flexible setting,*

$$\mathbf{p}_h = \hat{p}_h = \frac{a + b\tilde{\epsilon}\gamma(x)}{2b}; \quad (2.10)$$

*in the committed setting,*

$$\mathbf{p}_h = p_h^* = \frac{ab\lambda\sigma_\epsilon^2\gamma(x)^2 + a + b\gamma(x)(\mu - \lambda\sigma_\epsilon^2h)}{b^2\gamma(x)^2\lambda\sigma_\epsilon^2 + 2b}. \quad (2.11)$$

(c) *Finally,  $p_h^*(h^*) = \mathbb{E}[\hat{p}_h]$ .*

According to part (a) of this proposition, it is optimal for a risk-averse firm—irrespective of its pricing flexibility—to devise a hedge against all of the input commodity used (i.e.,  $\gamma(x)q(\mathbf{p})$ ).<sup>14</sup> When it comes to pricing, in the flexible setting of Proposition 2.2(b) we can see that the pricing decision is independent of the hedging position (i.e.,  $h$  is eliminated in the derivative of  $\pi$  w.r.t.  $p$ ; see (3.15)). Yet given that firms in the committed setting maximize their expected utility, the hedging term will affect the optimal price. Indeed, it is clear from (2.11) that the optimal price decreases with the firm’s hedging position. Note, however, that we cannot thereby conclude that prices are lower in the committed setting because—by part (c) of Proposition 2.2—the expected price in the flexible setting is equal to to the optimal price in the committed setting.

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<sup>14</sup>In practice, firms often hedge less than the optimal quantity. One reason is that hedging contracts are not always fair. Another is that firms adopt cross-hedging strategies—that is, taking the opposite position in another commodity with the same or similar price changes (see for example the airlines’ hedging positions described by *Carter et al.*, 2006, which cover 15% on average of the next year’s (expected) required fuel).

### ***Certainty Premium in the Presence of Financial Hedging.***

The following lemma extends the results of Section 2.3.2 to the case where financial hedging is a viable option.

**Lemma 2.2.**

$$\omega_h = \sigma_\epsilon^2 \left( \frac{\pi_o''}{\pi_o'} + \lambda(\gamma(x)q(\mathbf{p}_h) - h) \right). \quad (2.12)$$

The possibility of setting up a hedging position does not affect the option benefit due to the profit function's convexity. However, FH availability does affect any factor that includes the risk aversion parameter  $\lambda$ —that is, when the firm is risk averse. A risk-neutral firm is neither better- nor worse-off after making a fair hedge. This finding augments *Alexandrov* (2015) by including risk-averse firms—though with a low level of risk aversion—with risk-neutral firms in terms of preferences for exposure to risk. The main difference here is that FH tends to increase risk-seeking behavior by making  $\omega$  even more negative.

### **2.4.3 Hedging via Financial Instruments versus Investing in Technology Improvement**

#### **Risk Management Perspective.**

An immediate consequence of Lemma 2.2 is that financial hedging reduces the certainty premium. It allows firms to enjoy greater benefits from uncertainty in the input price if the level of risk aversion is low. This is because FH reduces the volatility of total cash-flow without affecting the beneficial convexity of the profit function. This result contrasts with the effect of technology improvement, which can either increase or decrease  $\omega$  (see Proposition 2.1).

Quantifying the certainty premium value for firms that engage in TI versus FH allows us to compare the two strategies strictly from the risk management perspective—that is, without considering the average input-price effect mentioned in the introduc-

tion. Our next proposition will prove useful in comparisons, from this risk management perspective, between the case of hedging without TI investment ( $h = \mathbf{h}$  and  $x = 0$ , where  $\mathbf{h}$  is the optimal hedging position) and that of TI investment without hedging ( $h = 0$  and  $x = \mathbf{x}$ , where  $\mathbf{x}$  is the optimal level of investment).

**Proposition 2.3.** *In the flexible setting,  $\omega_h(h = \hat{h}, x = 0) < \omega_h(h = 0, x)$ ; in the committed setting,  $\omega_h(h = h^*) = 0 < \omega_h(h = 0, x)$  for all  $x > 0$ .*

For both flexible and committed settings, this optimal hedging position is chosen so as to minimize the firm’s certainty premium, and indeed the certainty premium in optimal hedging is lower than that of optimal TI investment. In the flexible setting, the optimal hedging position removes the effect of input quantity used; hence there remains only the negative effect of profit structure, which reduces TI. In the committed setting, the effect of profit structure is zero ( $\pi'' = 0$ ) and, since  $h^*$  removes all of the “quantity used” effect, we have  $\omega_h(h^*) = 0$ . Yet even with a positive input quantity used under TI, the resulting certainty premium in hedging is still lower than its counterpart in TI.

### **Firm’s Utility Perspective.**

A comparison of the firm’s utility under a fair hedging position with its utility under a non-budget-neutral TI investment (which also leads to an average cost reduction) would disadvantage financial hedging and therefore not be meaningful. That is why we use the firm’s certainty premium to compare the risk management properties of these two solutions. However, we remark that numerical experiments reveal that hedging’s certainty premium advantage might also outweigh the cost reductions due to TI. Figure 2.3 illustrates that, at low levels of uncertainty, financial hedging yields less for the firm—in both flexible and committed settings—owing to the cost reduction benefits of TI. Yet the gap between the risk management benefits of FH

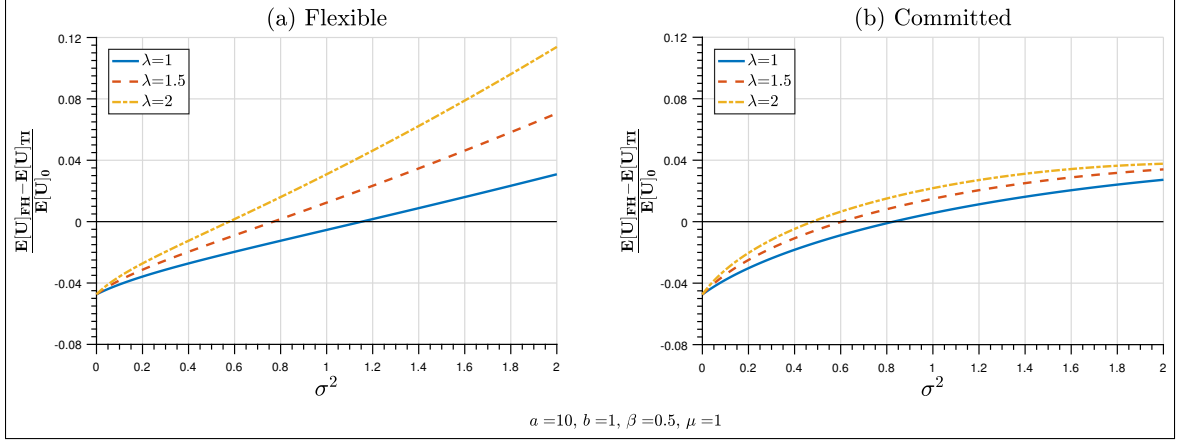


Figure 2.3: Normalized difference of expected utilities in TI and FH in flexible and committed settings. The terms  $\mathbb{E}[U]_{\text{TI}}$  and  $\mathbb{E}[U]_{\text{FH}}$  represent the optimal expected utility after adopting, respectively technology improvement and financial hedging measures;  $\mathbb{E}[U]_0$  is the baseline expected utility—that is, before investment or hedging.

and TI increases with uncertainty, so at some point the value of hedging exceeds the cost reduction benefits of TI.

#### 2.4.4 Technology Improvement and Financial Hedging: Substitutes or Complements?

With respect to the value that each strategy might add to a firm's expected utility, the following proposition addresses whether they are substitutes for or complements of each other.

**Proposition 2.4.** (a) *In the flexible setting, TI and FH are always substitutes.*

(b) *In the committed setting, there exists a  $\lambda_s$  such that TI and FH are substitutes iff  $\lambda < \lambda_s$ .*

This proposition considers both the cost reduction and certainty premium effects of TI. It suggests that the effect of hedging on a firm's expected utility can always be replicated by substituting TI; in the committed setting, however, if the firm is sufficiently risk averse then the two strategies are complements. Proposition 2.2 helps to understand this effect. It suggests that, in both flexible and committed settings,



the optimal  $h$  falls as  $x$  rises. Since the optimal quantity in the flexible setting is independent of  $h$ , it follows that the effect of  $h$  on expected utility is moderated by the decreasing effect of  $x$  on  $h$ ; hence we conclude that these two strategies are substitutes. Yet the optimal price is a function of  $h$  in the committed setting (see Proposition 2.2(b)) and so the effect of  $h$  on the firm's expected utility could also be moderated by the optimal price, resulting in a more complicated dynamic.

## 2.5 Duopoly

Market competition affects the power of an individual firm in setting the output price. In many situations, the competition mitigates the value of existing options because the actions of rival firms reduces one firm's ability to harness the option. We examine this general observations in the case of TI investment by comparing the incentives of firms operating in monopoly and duopoly markets. We begin by noting that a monopolist firm's profit margin is critically dependent on its own decisions; whereas, in the highly competitive industries the margin is determined by aggregate market forces and can be much less volatile than margins of a monopolist firm.<sup>15</sup> Therefore, it is reasonable to form a prior that market competition would have implications for the effectiveness of operational as well as financial hedging in reducing the firms' risk exposure (*Shaffer*, 1982): an interaction that has been argued to be relatively complex (*Caldentey and Haugh*, 2009).

Competition affects the firm's costs and benefits associated with input-price uncertainty by (i) changing the profit function's curvature and (ii) changing the optimal output quantity and hence the total input commodity to be used. In this section we consider competition between two firms; both of them have access to financial

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<sup>15</sup>Prices in a competitive industry are typically more volatile than a monopolist. Competitive markets pass the entire demand or supply shocks to consumers; whereas, a monopolist absorbs part of demand and supply shocks in its profit margin. Thus, while prices are more volatile in competitive markets, the profit margin of a single firm is more stable.

hedging, but only one invests in technology improvement. Our goal is to identify the competitive (dis)advantages of TI investment in a strategic environment.

Our model assumes a duopoly Cournot competition between two risk-averse firms. The timeline of decisions is illustrated by Figure 3.1 (in Section 2.3). Both the TI and FH decisions occur in stage 1, and the production decisions are made in stage 2. Profits are realized in stage 3. Formally, we have

$$\pi_i = q_i(p - \tilde{\varepsilon}\gamma(x)) + h_i(\tilde{\varepsilon} - f) - x, \quad (2.13)$$

$$\pi_n = q_n(p - \tilde{\varepsilon}) + h_n(\tilde{\varepsilon} - f); \quad (2.14)$$

here  $i$  is used to index the TI-investing firm and  $n$  the noninvesting firm.

### 2.5.1 Flexible Firms

Denote the profit of each firm by  $\pi_j = \pi_{jo} + \pi_{jh}$  the sum of operational profit and hedging return where  $j \in \{i, n\}$ . Anticipating the optimal second-stage production decision and hedging (for any given investment level), the investing firm solves the following optimization problem in stage 1:

$$\max_x \mathbb{E}[U(\pi_{io}(\tilde{\varepsilon}, x) + \pi_{ih}(\hat{h}_i))]. \quad (2.15)$$

In order to compare the risk management implications of FH and TI, we once again calculate the certainty premium of investing firm in the (flexible) competitive setting.

**Lemma 2.3.**

$$\omega_i = \sigma_\epsilon^2 \left( \frac{\pi''}{\pi'} + \lambda \left( \frac{2}{3} (2\gamma(x) - 1) \hat{q}_i - h_i \right) \right); \quad (2.16)$$

$$\frac{\pi''}{\pi'} = \frac{b(1 - 2\gamma(x))}{a + b\mu - 2b\mu\gamma(x)}. \quad (2.17)$$

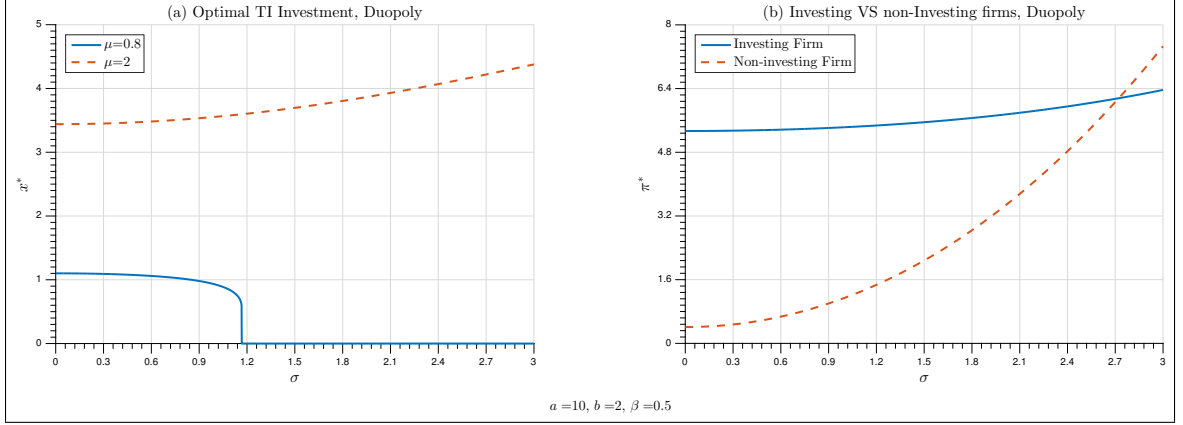


Figure 2.4: Duopoly setting: TI investment with flexible pricing.

Recall from (2.6) that, in the monopoly setting, the first term related to the profit curvature is always negative. Hence were it not for the effect of risk aversion (introduced in the second term), the firm would always benefit from input-price uncertainty. However, this generalization does not hold in the competitive setting. Lemma 2.3 shows in particular how, under duopoly, that first term can be either positive or negative—a dynamic that clarifies the effect of technology improvement.

The following proposition—whose claims are illustrated by the graphs in Figure 2.4—characterizes optimal hedging positions and expresses formally how competition changes the firm’s attitude toward input-price uncertainty.

**Proposition 2.5.** (a) *Technology improvement is a substitute for financial hedging.*

(b)  $\hat{h}_i = \frac{2}{9}(2\gamma(x) - 1)(a + b\mu - 2b\mu\gamma(x)) = \frac{2}{3}(2\gamma(x) - 1)\hat{q}_i$  and

$\hat{h}_n = \frac{2}{9}(2 - \gamma(x))(a - b\mu + 2b\mu\gamma(x)) = \frac{2}{3}(2\gamma(x) - 1)\hat{q}_n$ . (c) *There exists a  $\mu_d$  such that  $\omega(\hat{h}, \hat{x}) < 0$  iff  $\mu < \mu_d$ .* (d)  $\hat{x}$  is decreasing in  $\sigma_\epsilon^2$  iff  $\mu < \mu_d$ .

Part (a) of this proposition suggests that, similarly to the monopolistic case, TI and FH are substitute strategies for managing input-price risk. As before, this follows because the pricing decision is independent of the hedging position, which is a decreasing function of  $x$ . Thus more investment results in  $h$  having less of an effect on expected utility.

There is a contrast with part (b), however. Unlike the monopoly setting, in which optimal hedging is equivalent to optimal quantity at any level of  $x$ , it is never optimal for the investing firm to hedge against *all* of its input commodity use. In fact, even without investment ( $\gamma(x = 0) = 1$ ) the firm optimally hedges just two thirds of the input quantity used, since part of the expected profit is due to its rival's quantity and hedging decisions. At the optimal hedging position, the effect of input quantity used disappears and so financial hedging further increases the firm's benefit from input-price uncertainty; that is, hedging eliminates uncertainty's "negative" effects (resulting from risk aversion) but retains the "positive" effects.

Part (c) of Proposition 2.5 suggests that  $\gamma(x) < 1/2$  iff  $\mu > \mu_d$ , which results in a positive certainty premium. Thus the firm's attitude toward risk in the optimal TI investment depends on the expected price of inputs. In fact,  $\mu_d$  (which depends on the structure of  $\gamma(\cdot)$  and the level of  $\sigma_\epsilon^2$ ) determines whether the optimal TI investment results in  $\gamma(x)$  above or below 0.5 (see the proof of Proposition 2.5 in Appendix C). Market risk magnifies the effect of optimal TI investment in the direction of  $\mu - \mu_d$ , which accounts for parts (c) and (d) of the proposition. For  $\mu < \mu_d$ , the firm is risk seeking and so prefers more input cost variation; yet such variation is a decreasing function of TI (because the latter reduces the commodity consumption rate), and the result is less investment in TI. When  $\mu > \mu_d$ , however, the firm becomes risk averse and so would prefer less price variation; here the result is more investment in TI.

### 2.5.2 Committed Firms

Given the optimal production quantity in stage 2 and the hedging position that maximizes the firm's expected utility, TI investment is the solution to the following optimization problem:

$$\max_x \mathbb{E}[U(\pi_{io}(\tilde{\epsilon}, x) + \pi_{ih}(h_i^*))]. \quad (2.18)$$

Our next lemma characterizes the behavior of a firm's certainty premium in committed settings.

**Lemma 2.4.**

$$\omega_i = \sigma_\epsilon^2 \left( \frac{\pi''}{\pi'} + \lambda(q_i^* - h_i) \right); \quad (2.19)$$

$$\frac{\pi''}{\pi'} = 0. \quad (2.20)$$

This lemma indicates that the profit curve's convexity (recall (2.6)) has no effect on the certainty premium in this setting and, furthermore, that the firm's attitude toward risk is determined solely by the input quantity used.

**Proposition 2.6.** (a)  $h_i^* = q_i^* \alpha(x)$  and  $\alpha(x) \in [0, \frac{(2+b\lambda\sigma_\epsilon^2)^2}{(2+b\lambda\sigma_\epsilon^2)^2-1}]$ ;

(b)  $\alpha(x)$  is decreasing in  $x$ .

In light of Lemma 2.4, Proposition 2.6 suggests that—in contrast to the flexible setting, where  $\hat{h}_i \leq \hat{q}_i$ —in the committed setting we find that  $h_i^*$  can be either less or greater than  $q_i^*$ ; thus the *sign* of a firm's certainty premium depends on the level of its TI investment. According to parts (a) and (b), if there is no TI investment then the optimal hedging position exceeds the optimal quantity because  $\max\{\alpha(x)\} > 1$ , which results in  $\omega(x) < 0$ . The  $\alpha$  decreases with  $x$  and approaches zero as  $x$  approaches infinity; hence some levels of TI investment should yield  $\omega = 0$ , in which case the firm becomes risk neutral for most practical purposes. It is also worth noticing that if  $\lambda$  is too large, the upper bound of for  $\alpha(x)$  becomes one.

## 2.6 Conclusion

This paper's main objectives are to enrich our understanding of firms' attitudes toward input-price uncertainty—and to assess the relative effectiveness of technology improvement (TI) and financial hedging (FH) as risk management solutions—in a

context where firms could benefit from increased volatility in the price of production inputs. We define and quantify a firm's preferences vis-à-vis input-price uncertainty as the premium the firm is willing to pay (or receive) to replace the uncertain price with its expected value. For that purpose, we decompose the forces affecting the certainty premium into two main elements: the profit function's *structure* (i.e., how convex or concave it is) and the multiplicative *product* of the firm's risk aversion and the quantity of inputs used. These two components constitute the mechanism behind changes in the firm's certainty premium as a function of its investment in TI. The availability of financial hedging adds an additional term (viz., hedging return) to the firm's total profit function. We establish that a firm's hedging profit directly affects the certainty premium by changing the effect of input quantities used. However, our analysis of the case that incorporates competition suggests that hedging affects the firm's certainty premium through the profit structure as well. For both monopoly and competition cases, we show that the firm's input certainty premium is affected by TI directly and also indirectly (i.e., through the optimal hedging position). These effects can either increase or reduce the value derived from investing in technology improvement, which explains why the optimal level of TI investment should vary in response to changes in input-price uncertainty. Table 2.2 summarizes the aforementioned results.

Technology improvement reduces the quantity of inputs needed and, by extension, the variance associated with their cost; this property makes TI comparable to financial hedging. Our analysis reveals that—contrary to what is assumed in the economics and finance literature—it is not only risk-neutral firms with convex profit functions that benefit from input-price uncertainty (and so would prefer not to reduce that uncertainty). In particular, a risk-averse firm (with a convex profit function) might also exhibit a negative certainty premium for input prices if the extent of its risk aversion is not too great. In the absence of financial hedging, if risk aversion is low then the firm reduces its TI investing in response to an increase in price uncertainty;

the reason is that investing in technology improvement reduces the option value associated with being exposed to input-price uncertainty. At the same time, this setting results in higher exposure to risk if firms do engage in financial hedging. Comparing TI and FH from the risk management perspective, we demonstrate that hedging always results in a comparatively lower certainty premium; that is, the option value resulting from exposure to input-price uncertainty is higher with FH than with TI. This difference in the risk management value of these strategies increases with the level of uncertainty. It follows that, even though the cost reduction benefits of TI might result in higher expected utility under low uncertainty, the risk value benefits of FH under high uncertainty might exceed those due to TI.

Our analysis also answers the question of whether a firm is better-off adopting both strategies. From the risk management perspective we observe that TI, in isolation, can *lower* the firm's certainty premium by reducing the input quantity used. Because optimal hedging can *eliminate* the effect of input quantity used, FH can always substitute for TI. From the standpoint of total expected utility, our analysis delivers a more nuanced result: the two strategies are substitutes in flexible settings, but they are substitutes also in committed settings only when risk aversion is low. So for a firm that has pricing flexibility yet is extremely averse to risk, it would be preferable to adopt *both* strategies.

Duopoly competition adds an instructive dimension to our analysis of the certainty premium. In a duopoly, the flexibly pricing firm's profit is not always convex with respect to the input price. We show that there is a level of TI investment at which the profit could be either convex or concave, resulting in a (respectively) negative or positive effect on the certainty premium. The threshold at which TI affects the certainty premium depends on the expected input costs—a relation that explains why TI investment decreases (resp. increases) with uncertainty when the mean of input cost is low (resp. high).

Table 2.2: Interaction between industry and firm type (RA  $\uparrow$ : high risk aversion, RA  $\downarrow$ : low risk aversion).  $\omega(x) > 0$  ( $\omega(x) < 0$ ) means the effect of the option value of the uncertainty through profit structure is lower (greater) than the effect of firm's risk attitude. (c.f. Equation (2.6)).  $\omega' > 0$  ( $\omega' < 0$ ) means the effect of TI on firm's risk attitude is more (less) salient than its effect of option value of the uncertainty.

		Flexible		Committed	
		without hedging	with hedging	without hedging	with hedging
Noncompetitive	RA $\uparrow$	$\omega(x) > 0$ $\omega' < 0$	$\left\{ \begin{array}{l} \omega(x) < 0 \\ \omega' > 0 \end{array} \right\}$	$\omega(x) > 0$ $\omega' > 0$	$\left\{ \begin{array}{l} \omega(x) = 0 \\ \omega' = 0 \end{array} \right\}$
	RA $\downarrow$	$\omega(x) < 0$ $\omega' > 0$		$\omega(x) \approx 0$ $\omega' < 0$	
		with hedging		with hedging	
Competitive	RA $\uparrow$	$\left\{ \begin{array}{ll} \mu < \mu_d & \omega(x) < 0 \\ & \omega' > 0 \end{array} \right\}$		$\omega(x) > 0$ $\omega' > 0$	$\omega(x) \leq 0 \text{ or } \omega(x) \geq 0$ $\omega' \leq 0 \text{ or } \omega' \geq 0$
	RA $\downarrow$				



In the flexible setting we also observe that TI and FH are substitutes. Yet in the (committed) duopoly setting, although profit curve convexity has no effect on the certainty premium ( $\pi'' = 0$ ), the effect of input quantity used could result in risk-seeking behavior (i.e., a negative certainty premium). The interaction between TI and FH plays an important role in this dynamic. The level of an optimal hedging position might exceed the input quantity used, resulting in a negative certainty premium. That level falls when TI investment rises; when the firm is hedging less than the optimal quantity, the certainty premium becomes positive.

Although absent from our exposition so far, we should also note that TI investments often have direct implications for the “sustainability” of firms’ operations. The higher the TI, the lower the consumption rate of variable inputs (e.g. fossil fuels) will be, which is in line with core promises of sustainable supply chain. Thus, our proposed mechanism also shows when firms risk management motives is aligned with their activities toward a cleaner production strategy. Interestingly, our results reveal another mechanism why not all firms have a motive to invest in sustainable technologies because it reduces their (potentially desirable) exposure to input price risk. As such, our model predicts that, *ceteris paribus*, investment in sustainability (e.g. energy efficiency initiatives) is higher for: 1) industries with less flexibility in pricing; 2) private firms (which tend to be more risk-averse); 3) firms in industries with missing markets for forward contracts of input commodity; 4) industries with lower heterogeneity of production efficiency across different plants (resulting in lower convexity of the profit function). In other word, if firms of the industry use similar technologies, they will be more willing to invest on sustainability; 5) industries with higher price elasticity of demand (higher sensitivity of consumers purchase to prices). It is important to note, however, that the net environmental and sustainability effect of a firm’s TI crucially depends on the relative footprint of the input commodity (e.g. jet fuel) and the technology used to improve the efficiency (e.g. rare materials for an

efficient jet engine). A full life-cycle analysis is required to quantify the overall effect of risk-reduction motives on the environmental footprint of the firm.

Our study offers insights on the real value of investment in TI and of financial hedging as well as on how their interaction affects the option value of a firm’s exposure to input-price uncertainty. This vein of research could be extended in several directions. First, we assume that the investment in TI occurs only once, in the first period (stage 1 of the model). In reality, of course, firms can engage in multiple rounds of TI investment; that option is being explored and formalized in another study by the authors. Second, the connection between FH and TI is more salient when the risk of bankruptcy is taken into account. In particular, a firm that uses external funding to invest in TI might need to hedge against high input prices in order to guarantee the return of external funding and to avoid bankruptcy. Although this paper’s model can be considered to incorporate bankruptcy risk into the firm’s level of risk aversion, one could—by explicitly incorporating external funding and the risk of bankruptcy—clarify even further the mechanisms that explain how input price uncertainty affects TI investment. Third, this study considers the two extreme cases of full flexibility or full commitment. However, some industries (e.g., shipping) exhibit *partial* flexibility in adjusting their output prices to reflect input costs. This intermediate variation is an approach that merits further analysis. Finally, another direction is to consider the effect of TI investment on price of input commodity. If the investing firm has a large market share, then its investment on TI significantly affects the demand and consequently the price of input commodity. This not only affects the investment decision of such firms, it also might result in cases where non-investing firms free-ride the effort of investing firms—a phenomenon coined as “green paradox” in the literature.

## CHAPTER III

# Dynamic Capacity Investment and Technology Improvement with Financial Constraints

### 3.1 Introduction

Firms can adopt new technologies when deciding on the optimal portfolio of their production capacity. With an strategic capacity management, they balance the benefits of acquiring new technologies and investment costs of replacing or expanding capacity. Investment cost and returns depends heavily on the state of the market. Airlines, for example, sign new contracts every year to replace their old fleet with young fuel-efficient planes, however, the value of investing on new planes depends significantly on uncertain factors like future demand and fuel cost. Through this chapter we highlight the driving factors of an optimal capacity investment decision, for example for an airline, and characterize the optimal capacity investment policy when two substitute technologies are available, yet firm faces financial constraints.

In airline industry, fuel costs account for about 30% of airlines' operating costs and advent of new design of engines and fuselage could considerably reduce fuel consumption rate: "New aircraft are 70% more fuel efficient than 40 years ago and 20% better than 10 years ago" (*IATA*, 2015). While it can be expected that the rise and fall of fuel cost directly affect airlines' capacity replacement decision, airplane

manufacturer order books state differently:

Falling fuel prices are encouraging airlines to keep less efficient aircraft in service for longer[...]. The value of in-service aircraft is rising as 80% to 90% of airlines renting aircraft have extended their use [...however] representatives for the plane makers have argued that airlines make aircraft investment decisions based on long-term business assumptions, not on short-term oil price fluctuations.

—*Robert Wall, Wall Street Journal, January 2015*

There is clearly an interaction between short-term and long-term decisions, and the link is firm's financial position. Short term decisions, driven by fuel price shocks or seasonal demand change directly affect airline financial status which consequently affects airline's future financial commitments to purchase new airplanes and replace the old fleet:

Airlines also compete for market share by increasing or decreasing their capacity. [...] The airline industry is highly cyclical, and the level of demand for air travel is correlated to the strength of the U.S. and global economies. [...] Aircraft fuel is critical to the Company's operations and [...] has historically been the Company's most volatile operating expense due to the highly unpredictable nature of market prices for fuel. [...] To protect against increases in the market prices of fuel, the Company may hedge a portion of its future fuel requirements.

—*United Airlines Annual Report on Form 10-k, February 2017*

The way airlines manage their capacity, i.e. to expand, to shrink, and to renew their old fleet, and how they confront volatility of demand and fuel cost can guarantee the sustainability of their business in long-run.

We study a firm's long-run strategic decision making in expanding or replacing its capacity when two types of technologies are available: a low-efficiency technology with cheaper investment cost but expensive running cost and a high-efficiency technology

with high upfront investment cost and low running cost. The running cost comes from different consumption rate of input commodity, e.x. fuel. The firm's long-run decision is modeled through a dynamic stochastic program and we aim to find the optimal policy regarding how much capacity to acquire from either of the available technologies. Motivated by the challenges in airline industry, we study how firm's budget constraint affect the optimal policy. We consider input price and demand uncertainty the main drivers of short-term profit volatility.

### 3.2 Literature Review

Strategic capacity management involves a couple of main elements which are broadly discussed in operations management (OM) and finance literature. While OM literature considers strategic capacity management as a tool to maximize expected profit, increase operational flexibility, and reduce environmental impact of production, finance literature focuses on how capacity investment decisions interacts with capital structure, budget constraint and financial risk management where firm maximizes its value or expected future dividends.

Most studies in OM literature consider capacity decisions with only one technology available. The earliest works had the basic concern of how to meet the growing demand with the trade off of using economies-of-scale large size expansion versus opportunity cost of acquiring capacity right before it is needed (*Manne*, 1961). This stream of works is developed considering probabilistic demand, capacity deterioration, and multi-period horizons (see *Van Mieghem*, 2003, for comprehensive literature review). We continue this stream considering two types of technologies that influence input commodity consumption rate when in addition to demand, the input cost is also uncertain. The input price uncertainty is also a link to extend the analysis and consider the interaction of financial hedging and capacity investment as a new flavor in this stream of research.

Technology choice, when more than one technology is available, depends on how technologies are different from each other. OM literature extensively study technology choice when one technology can be used to produce two different product (flexible), and the other can be used to produce only one product (dedicated). *Goyal and Netessine* (2007); *Boyabatlı and Toktay* (2011); *Boyabatlı et al.* (2015) study the technology choice (flexible or dedicated) and capacity investment decision in competition, budget constraint, and multi-agent settings respectively. These works are considered the early stage of introducing operational flexibility as a risk hedging mechanism, although they did not study the interaction of financial hedging in their analysis. The technologies we consider also differs from theirs in terms of providing more efficiency in the production line versus flexibility. Sustainable OM, on the other hand, considers technologies which differs in terms of  $CO_2$  emission. *Drake* (2011); *Drake et al.* (2015), for instance, study the effect of carbon tariff and regulations on sustainable technology adoption when emission allowance price is uncertain. Closer to our setting, *Kleindorfer et al.* (2012) consider two technologies with different input commodity (electricity and gas) in a fleet renewal problem, however, in a deterministic setting where the whole capacity will be utilized. This assumption results in a single policy of using only one type of technology. A recent work by (*Wang et al.*, 2013) is the closest one to our study. They consider two types of technologies, a conventional and an environment-friendly one, which use different type of input commodities. They consider a dynamic, multi-horizon, and stochastic setting and develop a policy for the optimal level of investment on capacity from either of the technologies in different realization of demand and input cost and initial capacity in hand. We extend their work by considering budget constraint for the investing firm.

Finance literature approaches the problem of capacity management focusing on optimal way to finance the investment (capital structure). Investing in capacity is the link between capital structure and firm value. Firms utilize either available cash or

external funding to acquire capacity and make an uncertain profit in future periods. The profits in future periods are used to manage production capacity and return debts, thus it is crucial that the uncertainty of the profit is managed accordingly. Maximizing total firm value or expected dividends, a firm decides on the level of investment in acquiring capacity and the capital structure as well as using financial instruments, e.x. futures options, to hedge against profit volatility. *Rampini and Viswanathan* (2013); *Rampini et al.* (2014) study capacity investment of a unique technology when firm is budget constrained but has access to financial markets to get external funding. The firm has also a volatile cash flow, originated from uncertainty of input commodity cost or demand. Firm has also the option to adopt financial hedging strategies to reduce volatility of future cash flow. The former study also considers leasing as an option to acquire capacity with low upfront investment cost. Our work differs mainly from these works in term of introducing two technologies that might differently be valued when input cost could be low or high.

### 3.3 General Model Characterization

We develop a dynamic capacity replacement/expansion model for a firm which maximizes its total expected discounted profit  $V_t$  along  $T$  periods of production where demand and input price are non-stationary random variables. We consider two types of technology  $(l, h)$ :  $l$  has lower upfront investment cost but higher input consumption rate and  $h$  is more expensive to acquire but more efficient, resulting in lower consumption rate. At each period, firm decides to replace its current capacity  $\mathbf{k} = (k_l, k_h)$  with  $\bar{\mathbf{k}} = (\bar{k}_l, \bar{k}_h) \in \mathcal{R}^{2+}$ . Through out the text, **bold** formatted variables denote vector of variables for the two technologies.

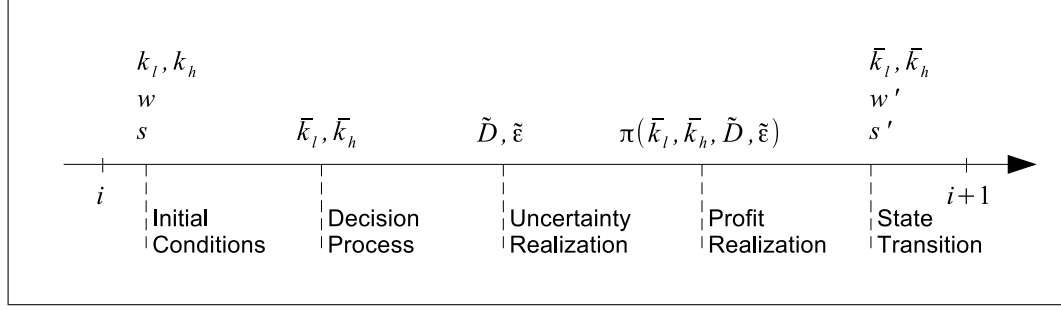


Figure 3.1: Timeline of events at period  $i$

### 3.3.1 Environment

Demand<sup>1</sup>  $\tilde{D}(s)$  and input cost  $\tilde{\epsilon}(s)$  are non-stationary random variables whose distribution parameters are realized in state  $s$ , which we assume to follow a Markov process  $S(t) \equiv \{s_0, s_1, \dots, s_N\}$ .  $\tilde{D}_t(s)$  follows a Poisson distribution with mean  $\lambda_t(s) = \lambda s$  and  $\tilde{\epsilon}(s)$  follows a Normal distribution  $\mathcal{N}(\mu_t(s), \sigma_t)$  where  $\mu_t(s) = \mu(\Lambda - s)$  and  $\Lambda > s_N$ . We assume the variation of  $\epsilon$ , given  $\mu(s)$ , has a stationary standard deviation of  $\sigma_t = \sigma$ . States  $s_0$  and  $s_N$  represent the worst and the best global economy respectively. Let  $s(t) = s_i$ , then  $s(t+1) = s_{i\pm 1}$  with probability  $\rho$  and  $s(t+1) = s_i$  with probability  $(1 - 2\rho)$  unless for  $i = 0, N$  where probability of  $s(t+1) = s_i$  is  $(1 - \rho)$ . Note that due to dependency of  $\tilde{D}$  and  $\tilde{\epsilon}$  on state  $s$ , demand and input cost are correlated through the whole horizon; however, at each period after realization of  $s$ , the distributions of demand and input cost are independent.

### 3.3.2 Timeline

The timeline of events is as follows: at the beginning of each period, the state is  $s$ , firm has capacity  $\mathbf{k}$  and available cash is  $w$ . Before realization of uncertainties, firm decides on the new production capacity available in the same period,  $\bar{\mathbf{k}}$ .

After realization of demand  $D$  and input cost  $\epsilon$ , firm uses  $\bar{\mathbf{k}}$  to satisfy demand and make profit  $\pi(\bar{\mathbf{k}}, D, \epsilon)$ . Next period starts with state variables  $\mathbf{k}' = \bar{\mathbf{k}}$ ,  $w'$ , and  $s'$ , as

<sup>1</sup>Throughout the paper we denote random variables with accent  $\tilde{\cdot}$ ; we do not use any accent for the realization of random variables.



shown in Figure 3.1.

### 3.3.3 Firm's problem

With state variables  $\mathbf{k}$ ,  $w$ , and  $s$ , and the decisions  $\bar{\mathbf{k}}$ , we define  $V_t(\mathbf{k})$  the expected profit from periods  $t$  to  $T + 1$ ,

$$V_t(\mathbf{k}, w, s) = \max_{\bar{\mathbf{k}} \in \mathcal{A}(\mathbf{k}, w)} \{-RC(\mathbf{k}, \bar{\mathbf{k}}) + g_t(\bar{\mathbf{k}}, w, s)\} \quad (3.1)$$

$$g_t(\bar{\mathbf{k}}, w, s) = \mathbb{E}[\pi(\bar{\mathbf{k}}, \tilde{D}, \tilde{\epsilon})] + \alpha \mathbb{E}[V_{t+1}(\bar{\mathbf{k}}, w', s')] \quad (3.2)$$

$$s.t. \quad w' = w - RC(\mathbf{k}, \bar{\mathbf{k}}) + \pi(\bar{\mathbf{k}}, D, \epsilon), \quad (3.3)$$

where  $w'$  and  $s'$  are the available cash and the new state in the beginning of the next period, and  $\mathcal{A}(\mathbf{k}, w) = \{\bar{\mathbf{k}} | RC(\bar{\mathbf{k}}, \mathbf{k}) \leq w\}$ ,  $\alpha$  is discount factor,  $r$  is one period interest rate and  $RC(\cdot)$  is defined by Equation (3.6).

The terminal value,  $V_{T+1}$ , equals to the salvage price of capacity  $\mathbf{k}$  at the beginning of period  $T + 1$ :

$$V_{T+1}(\mathbf{k}) = \mathbf{s}\mathbf{k}. \quad (3.4)$$

The output price  $p$  is exogenous and the production profit after realization of demand and cost is as follows:

$$\pi(\bar{\mathbf{k}}, D, \epsilon) = \sum_{i \in \{l, h\}} \{\theta_i(D, k_h, k_l)(p - \epsilon\gamma_i)\}, \quad (3.5)$$

where  $\theta_i(\cdot)$  denotes the usage of capacity  $i$  to meet the demand,  $\gamma_i$  represents the consumption rate of input for technology  $i$ , where  $\gamma_l > \gamma_h$ , and  $\mu$  is the expected input cost at each period. We assume  $p \geq \epsilon\gamma_i$  so the firm always has incentive to produce. Note that costs other than input cost can be incorporated into  $p$ . It is easy to decide the amount of each type of capacity to use. If  $D \geq \bar{k}_l + \bar{k}_h$ , then all capacity will be used. Otherwise, with  $\gamma_l > \gamma_h$ , the less efficient capacity is more costly and

the firm first exhausts the more efficient capacity,  $\bar{k}_h$ , and then uses the less efficient capacity,  $\bar{k}_l$ . It results in

$$\begin{aligned}\theta_h &= \min(D, k_h) \\ \theta_l &= \min((D - k_h)^+, k_l).\end{aligned}$$

Function  $RC(\cdot)$  determines the cost of changing capacity in the beginning of period,  $\mathbf{k}$ , to the new capacity,  $\bar{\mathbf{k}}$ , and can be formulated as

$$RC(\mathbf{k}, \bar{\mathbf{k}}) = \sum_{i \in \{h, l\}} b_i(\bar{k}_i - k_i)^+ - s_i(k_i - \bar{k}_i)^+. \quad (3.6)$$

### 3.4 The Basic Model: Capacity investment with no budget constraint

The base model includes capacity replacement decisions with no budget constraint. The firm's problem then becomes

$$V_t(\mathbf{k}, s) = \max_{\bar{\mathbf{k}}} \{-RC(\mathbf{k}, \bar{\mathbf{k}}) + g_t(\bar{\mathbf{k}}, s)\} \quad (3.7)$$

$$g_t(\bar{\mathbf{k}}, s) = \mathbb{E}[\pi(\bar{\mathbf{k}}, \tilde{D}, \tilde{\epsilon})] + \mathbb{E}[V_{t+1}(\bar{\mathbf{k}}, s')]. \quad (3.8)$$

It should be noted that dependency of  $\tilde{D}$  and  $\tilde{\epsilon}$  on  $s$  is suppressed in the above formulation. We provide some of the properties of optimal solution to Problem (3.7).

**Lemma 3.1.** (a)  $\mathbb{E}[\pi(\bar{\mathbf{k}}, \tilde{D}, \tilde{\epsilon})]$  is concave and weakly increasing in  $\bar{\mathbf{k}}$ .

(b)  $\mathbb{E}[\pi(\bar{\mathbf{k}}, \tilde{D}, \tilde{\epsilon})]$  is submodular in  $\bar{\mathbf{k}}$ .

*Proof:*

For (a), we use a simple property of concave functions:

(i): Let  $g(x) = \max_{y \in \mathcal{A}(x)} f(x, y)$ . If  $f(x, y)$  is jointly concave and  $\cup_x \mathcal{A}(x)$  is a convex set, then  $g(x)$  is concave in  $x$ .

Defining  $h(\bar{\mathbf{k}}, \theta) = \sum_{i \in \{l, h\}} \theta_i (p - (1 + \epsilon) \gamma_i)$ , Equation (3.5) (with no hedging) can be written as

$$\pi(\bar{\mathbf{k}}, D, \epsilon) = \max_{(\theta_l, \theta_h) \in \mathcal{A}(\bar{\mathbf{k}})} h(\bar{\mathbf{k}}, \theta_l, \theta_h)$$

where  $\mathcal{A}(\bar{\mathbf{k}}) = \{(\theta_l, \theta_h) \in (\mathcal{R}^+)^2 | \theta_l \leq \bar{k}_l, \theta_h \leq \bar{k}_h, \theta_l + \theta_h \leq D\}$ . Clearly  $h(\bar{\mathbf{k}}, \theta)$  is linear in  $\bar{\mathbf{k}}$  and  $\theta$  and, thus, jointly concave. Convexity of  $\cup_{\bar{\mathbf{k}}} \mathcal{A}(\bar{\mathbf{k}})$  in  $(\theta, \bar{\mathbf{k}})$  implies in concavity of  $\pi(\bar{\mathbf{k}}, D, \epsilon)$ . Since expectation preserves concavity,  $\mathbb{E}[\pi(\bar{\mathbf{k}}, D, \epsilon)]$  is concave.

With definition of  $\mathcal{A}(\bar{\mathbf{k}})$  we have  $\mathcal{A}(\bar{\mathbf{k}}_1) \subseteq \mathcal{A}(\bar{\mathbf{k}}_2)$  if  $\bar{\mathbf{k}}_1 \leq \bar{\mathbf{k}}_2$ , therefore,  $h(\bar{\mathbf{k}}, \theta)$  is weakly increasing in  $\bar{\mathbf{k}}$ .

For part (b), we consider the capacities  $\bar{k}_{l1} < \bar{k}_{l2}$  and  $\bar{k}_{h1} < \bar{k}_{h2}$ . With the definition of submodularity we show  $\Delta = \pi(\bar{k}_{l1}, \bar{k}_{h1}) + \pi(\bar{k}_{l2}, \bar{k}_{h2}) - \pi(\bar{k}_{l1}, \bar{k}_{h2}) - \pi(\bar{k}_{l2}, \bar{k}_{h1}) \leq 0$  for a realized  $\tilde{D}$  and  $\tilde{\epsilon}$ . There are several combinations with respect to  $\bar{k}_i$  and realized demand  $D$ .

Case (i): if  $\bar{k}_{h1} \geq D$ , then clearly  $\Delta = 0$ .

Case (ii): if  $\bar{k}_{h2} \geq D > \bar{k}_{h1}$ , then  $\pi(\bar{k}_{l2}, \bar{k}_{h2}) = \pi(\bar{k}_{l1}, \bar{k}_{h2})$  and  $\pi(\bar{k}_{l1}, \bar{k}_{h1}) \leq \pi(\bar{k}_{l2}, \bar{k}_{h1})$ , thus  $\Delta \leq 0$ .

Case (iii): if  $D \geq \bar{k}_{h2} > \bar{k}_{h1}$ , the capacities  $\bar{k}_{h1}$  and  $\bar{k}_{h2}$  will be fully used and thus will be canceled out in calculating  $\Delta$ . The remaining parts are the usage of capacities  $\bar{k}_{l1}$  and  $\bar{k}_{l2}$ . With  $D - \bar{k}_{h1} > D - \bar{k}_{h2}$  and  $\bar{k}_{l2} > \bar{k}_{l1}$ , due to the supermodularity of  $\min(x, y)$  we can write  $\min(D - \bar{k}_{h1}, \bar{k}_{l1}) - \min(D - \bar{k}_{h2}, \bar{k}_{l1}) \leq \min(D - \bar{k}_{h1}, \bar{k}_{l2}) - \min(D - \bar{k}_{h2}, \bar{k}_{l2})$ , resulting in  $\Delta \leq 0$ .

Since expectation preserves inequality in linear operations, for all possible cases  $\Delta \leq 0$  and proof is completed. ■

**Lemma 3.2.** For all  $t < T + 1$ ,

$$(a) \mathbf{k}_1 \neq \mathbf{k}_2, \text{ then } V_t(\mathbf{k}_1) - V_t(\mathbf{k}_2) \geq \mathbf{s}(\mathbf{k}_1 - \mathbf{k}_2)^+ - \mathbf{b}(\mathbf{k}_2 - \mathbf{k}_1)^+$$

(b)  $\bar{\mathbf{k}}_t \geq \mathbf{k}_t$

*Proof:*

(a)  $V_t(\mathbf{k}_i)$  is the maximum profit-to-go when the initial capacity is  $\mathbf{k}_i$ . Therefore, when  $\mathbf{k}_1 \neq \mathbf{k}_2$ , the profit-to-go of  $\mathbf{k}_1$  should be weekly greater than any other possible solutions including adjusting the capacity from  $\mathbf{k}_1$  to  $\mathbf{k}_2$  first and then receiving the profit-to-go of  $\mathbf{k}_2$ , i.e.  $V(\mathbf{k}_1) \geq V(\mathbf{k}_2) - RC(\mathbf{k}_1, \mathbf{k}_2)$ . Writing it component-wise yields

$$V_t(\mathbf{k}_1) - V_t(\mathbf{k}_2) \geq \sum_{i \in \{l, h\}} s_i(k_{i1} - k_{i2})^+ - b_i(k_{i2} - k_{i1})^+$$

(b) Let assume  $t_o < T$  is the latest period when it is optimal for the firm to salvage at least one type of capacity, e.g.  $k_h$  by  $\delta$ . Let  $\vec{\mathbf{k}}_t = \bar{\mathbf{k}} + \delta(0, 1)$  for  $t = t_o, \dots, T-1$ . Note that  $\pi_t(\vec{\mathbf{k}}) \geq \pi_t(\bar{\mathbf{k}})$  and that  $V_{T+1}(\vec{\mathbf{k}}) = V_{T+1}(\bar{\mathbf{k}}) + s_h\delta$ .

Thus,  $V_t(\vec{\mathbf{k}}) \geq V_t(\bar{\mathbf{k}})$ , and it is not optimal for the firm to salvage capacities in periods  $t < T+1$ . ■

**Lemma 3.3.**  $V_t(\mathbf{k})$  is concave in  $\mathbf{k}$  for  $t < T+1$

*Proof:*

With Lemma 3.2, we have  $\bar{\mathbf{k}}_t \geq \mathbf{k}_t$  for  $t < T+1$ . Thus (3.7) results in

$$V_t(\mathbf{k}) = \max_{\bar{\mathbf{k}} \geq \mathbf{k}} \{-\mathbf{b}(\bar{\mathbf{k}} - \mathbf{k}) + g_t(\bar{\mathbf{k}})\}. \quad (3.9)$$

If  $V_{t+1}(\bar{\mathbf{k}})$  concave in  $\bar{\mathbf{k}}$ , since all the terms in the objective function are jointly concave (c.f. Lemma 3.1) and  $\cup_{\mathbf{k}} \bar{\mathbf{k}} \geq \mathbf{k}$  is convex in  $\mathbf{k}$  and  $\bar{\mathbf{k}}$ , then  $V_t(\mathbf{k})$  is concave in  $\mathbf{k}$ . ■

Similar to *Eberly and Van Mieghem (1997)* but without dis-investment (salvaging) action, we define I-S policy as below:

**Definition 3.1.** I-S Policy.

The I-S policy is characterized by two non-negative functions  $\hat{k}_h(k_l)$  and  $\hat{k}_l(k_h)$  with a unique intersection at  $(k_l^*, k_h^*)$ :

1. If  $k_l \leq k_l^*$  and  $k_h \leq k_h^*$ :  $\bar{k}_l = k_l^*$  and  $\bar{k}_h = k_h^*$ .
2. If  $k_l > k_l^*$  and  $k_h \leq \hat{k}_h(k_l)$ :  $\bar{k}_l = k_l$  and  $\bar{k}_h = \hat{k}_h(k_l)$ .
3. If  $k_h > k_h^*$  and  $k_l \leq \hat{k}_l(k_h)$ :  $\bar{k}_l = \hat{k}_l(k_h)$  and  $\bar{k}_h = k_h$ .
4. Otherwise:  $\bar{k}_l = k_l$  and  $\bar{k}_h = k_h$ .

The above definition forms an inaction region (4) within which no investment takes place. We discuss the boundaries of this region in Property 3.1.

**Theorem 3.1.** *For a finite-horizon problem with no budget constraint, no discounting, and no hedging, the optimal policy is I-S policy.*

*Proof:*

Problem (3.9) is separable in  $\bar{k}_i$  and  $k_i$  for  $i \in \{l, h\}$ . With concavity of  $V_t(\mathbf{k})$  in  $\mathbf{k}$  (Lemma 3.3), and hence, concavity of  $g_t(\bar{\mathbf{k}})$  in  $\bar{\mathbf{k}}$ , there exist  $\bar{\mathbf{k}}^*$ , a global maximizer of  $g_t(\bar{\mathbf{k}}) - \mathbf{b}\bar{\mathbf{k}}$ . We define  $\hat{k}_j(k_i)$  the global maximizer of  $g_t(\bar{\mathbf{k}}) - \mathbf{b}\bar{\mathbf{k}}$  when  $\bar{k}_i = k_i$ . If  $\mathbf{k} \leq \mathbf{k}^*$ , firm increases its initial capacity  $k_i$  to  $k_i^*$ , thus  $\bar{\mathbf{k}} = \mathbf{k}^*$  (region 1). Let  $k_i > k_i^*$  and  $k_j \leq \hat{k}_j(k_i)$ . Consider a situation  $\bar{k}_i \geq k_i$ . Since  $g(\bar{\mathbf{k}}) - \mathbf{b}\bar{\mathbf{k}}$  is concave and  $\mathbf{k}^*$ , the global maximizer, is outside the convex region  $\bar{k}_i \geq k_i$ , we must have  $\bar{k}_i = k_i$  and the maximum subject to  $\bar{k}_i = k_i$  is  $(k_i, \hat{k}_j(k_i))$ . Thus, for  $k_j \leq \hat{k}_j(k_i)$  the maximizing capacity is feasible and we have the optimal solution  $\bar{k}_j = \hat{k}_j(k_i)$  (regions 2 and 3). Consider  $k_i > \hat{k}_i(k_j)$  and  $k_j > \hat{k}_j(k_i)$ . For all  $\bar{k}_i \geq k_i$  and  $\bar{k}_j \geq k_j$ ,  $g(\bar{\mathbf{k}}) - \mathbf{b}\bar{\mathbf{k}}$  is concave and the maximizer  $(k_i^*, k_j^*)$  is outside of the feasible region. Thus, the optimal solution must be on the “boundary” of the convex set, i.e. either  $\bar{k}_i = k_i$  or  $\bar{k}_j = k_j$ .

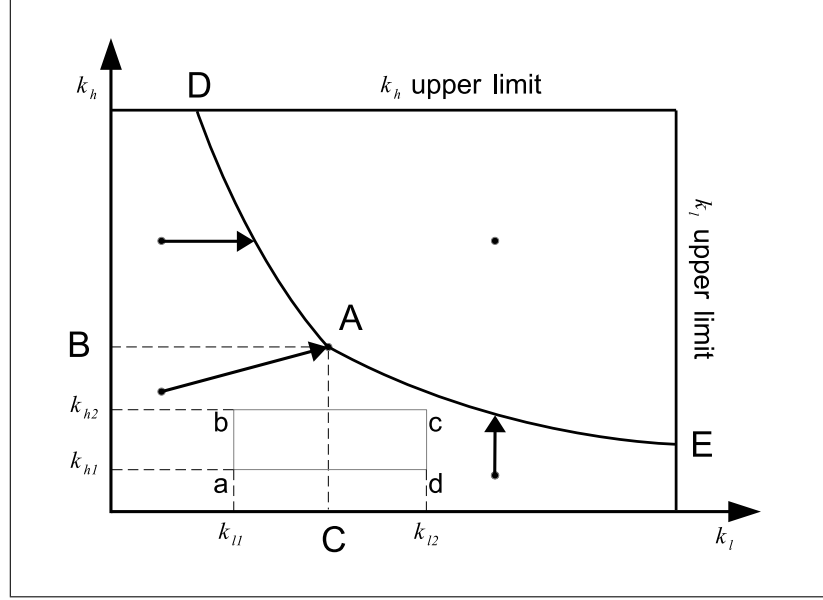


Figure 3.2: Unconstraint I-S policy

Assume  $\bar{k}_i = k_i$  and  $\bar{k}_j \geq k_j$ . Note that  $g(k_i, \bar{k}_j) - \mathbf{b}(k_i, \bar{k}_j)$  is concave and the maximum is at  $\hat{k}_j(k_i) < k_j$ . This implies that  $\bar{k}_j = k_j$  (region 4). ■

**Property 3.1.** (a)  $V_t(\mathbf{k})$  is submodular in  $\mathbf{k}$ .

(b)  $g_t(\bar{\mathbf{k}})$  is submodular in  $\bar{\mathbf{k}}$

(c)  $d\bar{k}_i/dk_j \leq 0, i \neq j \in \{l, h\}$

*Proof:*

(a) With  $k_{l1} < k_{l2}$  and  $k_{h1} < k_{h2}$ , we show  $\Delta = V(k_{l1}, k_{h1}) + V(k_{l2}, k_{h2}) - V(k_{l1}, k_{h2}) - V(k_{l2}, k_{h1}) \leq 0$ . Let assume  $k_{h1}$  is below arc  $BAE$  in Figure 3.2. Therefore,  $V(k_{l1}, k_{h1}) - V(k_{l1}, k_{h2}) = b(k_{h1} - k_{h2})$  since both pairs  $(k_{l1}, k_{h1})$  and  $(k_{l1}, k_{h2})$  results in a unique optimal solution. However, since  $k_{h2}$  could be below or above the arc  $BAE$  at  $k_{l2}$ , we use Lemma 3.2,  $V(k_{l2}, k_{h1}) - V(k_{l2}, k_{h2}) \geq -b_h(k_{h2} - k_{h1})$ , resulting in  $\Delta \leq 0$ . Similar analogy is used if the pairs of initial capacities are located on the left side of arc  $DAC$ .

(b) With (a) and Lemma 3.1,  $g_t(\bar{\mathbf{k}})$ , a sum of two submodular functions is submodular.

(c) The arcs  $DAC$  and  $BAE$  represent  $\hat{k}_l(k_h)$  and  $\hat{k}_h(k_l)$ , the global maximizer of  $g(k_i, \bar{k}_j) - \mathbf{b}(k_i, \bar{k}_j)$  in  $\bar{k}_j$  for  $i \in \{l, h\}$  and  $j \neq i$ . For given  $k_i$  and  $\bar{k}_j = \hat{k}_j(k_i)$ , we have  $\partial g / \partial \bar{k}_j - b_j = 0$ . Thus, implicit function theorem yields

$$\frac{d\bar{k}_j}{dk_i} = \frac{-\frac{\partial^2 g}{\partial \bar{k}_j \partial k_i}}{\frac{\partial^2 g}{\partial \bar{k}_j^2}}. \quad (3.10)$$

With  $k_i = \bar{k}_i$  and (b), we have  $\partial^2 g / \partial \bar{k}_i \partial \bar{k}_j \leq 0$  (submodularity of  $g(\cdot)$ ). Concavity of  $\pi(\bar{\mathbf{k}})$  and  $V_{t+1}(\bar{\mathbf{k}})$  also results in concavity of  $g(\bar{\mathbf{k}})$ , therefore,  $\partial^2 g / \partial \bar{k}_i^2 \leq 0$  which results in  $d\bar{k}_i / d\bar{k}_j \leq 0$  that completes the proof. ■

**Property 3.2.**  $d\bar{\mathbf{k}}/dt \geq 0$  if  $t < T + 1$

*Proof:* Lemma 3.2 suggests no salvaging is optimal, thus the capacities never reduce in time unless in the last period. ■

### 3.5 Capacity investment with budget constraint

If a firm is financially constrained, the capacity acquisition decision is bounded by the budget. Denoting the discount factor as  $\alpha$ , the firm's objective becomes

$$V_t(\mathbf{k}, w, s) = \max_{\bar{\mathbf{k}} \in \mathcal{A}(\mathbf{k}, w)} \{-RC(\mathbf{k}, \bar{\mathbf{k}}) + g_t(\bar{\mathbf{k}}, w', s)\} \quad (3.11)$$

$$g_t(\bar{\mathbf{k}}, w', s) = \mathbb{E}[\pi(\bar{\mathbf{k}}, \tilde{D}, \tilde{\epsilon})] + \alpha \mathbb{E}[V_{t+1}(\bar{\mathbf{k}}, w', s')] \quad (3.12)$$

$$s.t. \quad w' = w - RC(\mathbf{k}, \bar{\mathbf{k}}) + \pi(\bar{\mathbf{k}}, D, \epsilon), \quad (3.13)$$

where  $\mathcal{A}(\mathbf{k}, w) = \{\bar{\mathbf{k}} | RC(\bar{\mathbf{k}}, \mathbf{k}) \leq w\}$  and  $RC(\cdot)$  is defined by Equation (3.6).

#### Approximate Separation of $g_t(\bar{\mathbf{k}}, w')$

If Equation (3.11) is separable in  $w$  and  $\bar{\mathbf{k}}$ , budget constraint will only affect the feasible optimal solution at that very period, resulting in a simple tractable optimal

policy. In this subsection, we intend to show the negligibly of the error of separability assumption.

Consider a setting, in a two-period problem, where budget constraint in period one ( $w_1$ ) and the corresponding optimal capacities  $\bar{\mathbf{k}}_1$  result in  $w_2$ , the initial budget at period two. Assume  $w_2$  also constrains the optimal solutions in period two. We calculate  $\bar{\mathbf{k}}_1$ , using two different solutions: one calculates  $g_1$  based on  $V_2(\bar{\mathbf{k}}_1, w_2)$  (coupled), the other calculates  $g_1$  based on  $V_2(\bar{\mathbf{k}}_1)$  (separated), i.e. value to go in period two only depends on initial capacities at period two and not  $w_2$ .

A budget and capacity discretized algorithm used for both the solutions shows a small gap between the results of the two solutions, and the error of separated solution decreases when the size of discretization reduces. To understand whether the source of the error is the separation of the value function in the two solutions and not the discretization of the space, we implemented a forward two-period dynamic program with no discretization of the capacity and budget. Setting the convergence tolerance of  $1e - 12$  in numerical evaluations of the result, the forward algorithm shows the error is in the order of  $1e - 06$ . Figure 3.3 demonstrates the numerical example of the error of the separate solution. Whenever it is obvious we omit some of the variables, e.g.  $g_t(\bar{\mathbf{k}}, w')$  represents  $g_t(\bar{\mathbf{k}}, w', s')$ . We conclude that separation assumption w.r.t. initial budget at each period results in a small-order error, supporting the following assumption for the budget constraint setting:

**Assumption 3.1.**  $g_t(\bar{\mathbf{k}}, w')$  is separable in  $\bar{\mathbf{k}}$  and  $w'$ .

This assumption applies throughout the remainder of this paper.

**Lemma 3.4.**  $\forall w : \cup_{\mathbf{k}} \mathcal{A}(\mathbf{k}, w)$  is a convex set in  $(\mathbf{k}, \bar{\mathbf{k}})$ .

Proof:



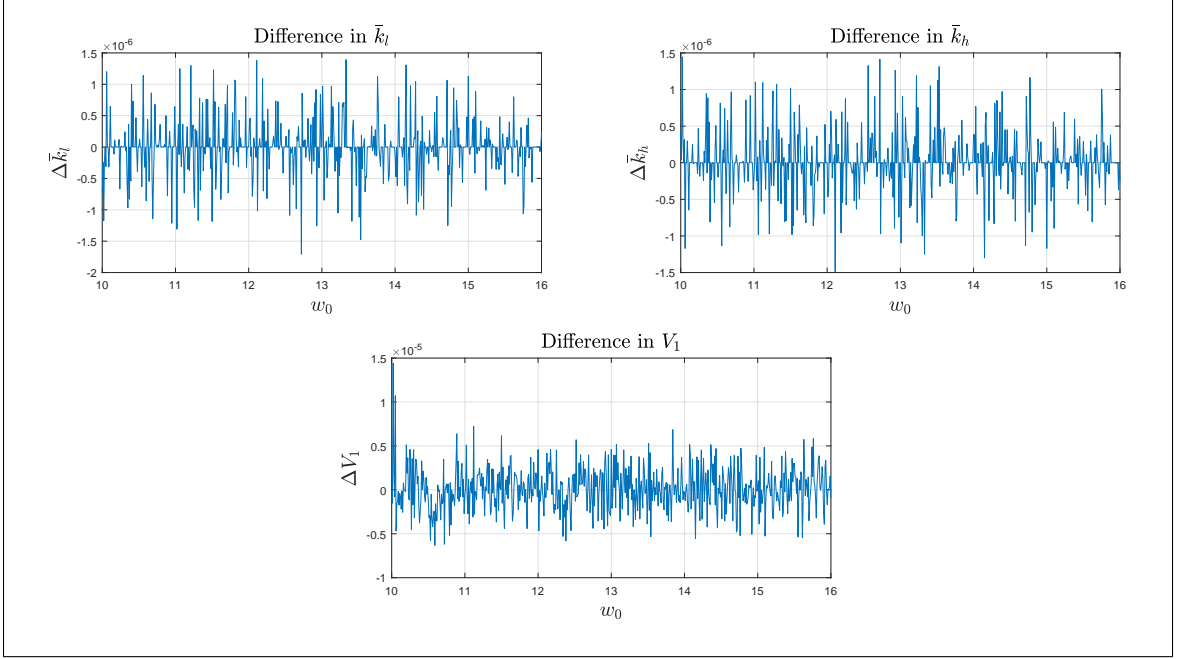


Figure 3.3: Error magnitude in coupled and separate solution.  $w_0$  is the available budget in the beginning of period one.  $\Delta \bar{\mathbf{k}}$  and  $\Delta V$  are the difference of the optimal capacity and value to go from the coupled and separated solutions respectively.

Denote  $\bar{\mathcal{A}}(w) = \cup_{\mathbf{k}} \mathcal{A}(\mathbf{k}, w)$ .  $RC(\mathbf{k}, \bar{\mathbf{k}})$  can be equivalently defined as

$$RC(\mathbf{k}, \bar{\mathbf{k}}) = \min_{p_i, m_i} \sum_{i \in \{l, h\}} b_i p_i - s_i m_i, \quad (3.14)$$

where  $\bar{k}_i = k_i + p_i - m_i$ , and  $p_i, m_i \geq 0$ . Since  $\sum_{i \in \{l, h\}} b_i p_i - s_i m_i \leq w$  is linear, the feasible set in extended space  $(\mathbf{k}, \bar{\mathbf{k}}, w, p, m)$  is convex. Projection of a convex set is convex. Thus  $\bar{\mathcal{A}}$  is convex in  $\mathbf{k}$ , and  $\bar{\mathbf{k}}$ . ■

**Lemma 3.5.** (a)  $g_t(\bar{\mathbf{k}}, w, s)$  is concave in  $\bar{\mathbf{k}}$ .

(b)  $V_t(\mathbf{k}, w, s)$  is concave in  $\mathbf{k}$ .

*Proof:*

The proof is by induction. Since  $V_{T+1}(\mathbf{k}, w, s) = \mathbf{s}\mathbf{k}$  in  $\mathbf{k}$ . (b) holds for  $T+1$ .

Assume that (b) holds for  $t+1$ .

(a) Since the expectation over  $s'$  preserves concavity,  $\alpha \mathbb{E}_{s'}[V_{t+1}(\bar{\mathbf{k}}, w', s')|s]$



- $f_l^R(k_h)$  and  $f_h^R(k_l)$  are decreasing functions defined for  $k_h \in [0, k_h^{ID}]$  and  $k_l \in [0, k_l^{DI}]$  respectively, where  $f_h^R(k_l^{ID}) = f_h^D(k_l^{ID})$  and  $f_l^R(k_h^{DI}) = f_l^D(k_h^{DI})$ .
- $f_h^L(k_l)$  is defined on  $[0, K_l^{II}]$  and increasing, where  $f_h^L(0) = 0$  and  $f_h^L(k_l^{II}) = k_h^{II}$ .

The IRSD policy is defined as follows:

- Region 1a:  $k_l < f_l^I(k_h)$  &  $k_h^{II} < k_h < k_h^{ID}$ . The policy is to invest in  $k_l$  upto  $\min(w/b_l, f_l^I(k_h))$  and not to change  $k_h$ . Similarly, Region 1b:  $k_h < f_h^I(k_l)$  &  $k_l^{II} < k_l < k_l^{DI}$ : the policy is to invest in  $k_h$  upto  $\min(w/b_h, f_h^I(k_l))$  and not to change  $k_l$ .
- Region 2a:  $k_l > f_l^D(k_h)$  &  $k_h^{DI} \leq k_h \leq k_h^{DD}$ . The policy is to disinvest  $k_l$  to  $f_l^D(k_h)$  and not to change  $k_h$ . Similarly, Region 2b:  $k_h > f_h^D(k_l)$  &  $k_l^{ID} \leq k_l \leq k_l^{DD}$ : the policy is to disinvest  $k_h$  to  $f_h^D(k_l)$  and not to change  $k_l$ .
- Region 3:  $k_l > k_l^{DD}$  &  $k_h > k_h^{DD}$ . The policy is to disinvest both capacities to  $\mathbf{K}^{DD}$ .
- Region 4a:  $k_l < k_l^{ID}$  &  $k_h > k_h^{ID}$ . The policy is to invest in  $k_l$  upto  $\min(k_l + w/b_l, k_l^{ID})$ . If  $k_l + w/b_l \geq k_l^{ID}$ ,  $k_h$  is salvaged down to  $\bar{\mathbf{k}} = (k_l^{ID}, k_h^{ID})$ . If  $w/b_l + k_l < k_l^{ID}$ ,  $k_h$  is salvaged down to  $f_h^R(k_l + w/b_l)$ , then more of  $k_h$  is replaced with  $k_l$  along the function  $f_h^R(\cdot)$  to  $\bar{\mathbf{k}} = (\bar{k}_l, f_h^R(\bar{k}_l))$  where  $b_l(\bar{k}_l - k_l) - s_h(k_h - f_h^R(\bar{k}_l)) = w$ . Region 4b:  $k_h < k_h^{DI}$  &  $k_l > k_l^{DI}$ . The policy is to invest in  $k_h$  upto  $\min(k_h + w/b_h, k_h^{DI})$ . If  $w/b_h + k_h \geq k_h^{DI}$ ,  $k_l$  is salvaged down to  $\bar{\mathbf{k}} = (k_l^{DI}, k_h^{DI})$ . If  $w/b_h + k_h < k_h^{DI}$ ,  $k_l$  is salvaged down to  $f_l^R(k_h + w/b_h)$ , then more of  $k_l$  is replaced with  $k_h$  along the function  $f_l^R(\cdot)$  to  $\bar{\mathbf{k}} = (\bar{k}_l, f_l^R(\bar{k}_l))$ .
- Region 5a:  $k_l < k_l^{II}$ ,  $k_h < k_h^{II}$ , and  $k_h > f_h^L(k_l)$ . The policy is to invest in  $k_l$  upto  $\min(k_l + w/b_l, (f_h^L)^{-1}(k_h))$  and if  $w/b_l > (f_h^L)^{-1}(k_h) - k_l$  the investment continues in both capacities along the function  $f_h^L(\cdot)$  upto  $(\bar{k}_l, f_h^L(\bar{k}_l))$  where  $b_l(\bar{k}_l - k_l) + b_h(f_h^L(\bar{k}_l) - k_h) = w$ . Region 5b:  $k_l < k_l^{II}$ ,  $k_h < k_h^{II}$ , and  $k_h <$

$f_h^L(k_l)$ . The policy is to invest first in  $k_h$  upto  $\min(k_h + w/b_h, f_h^L(k_l))$ . Then, if  $w/b_h > f_h^L(k_l) - k_h$ , the additional investment continues in both capacities along the function  $f_h^L(\cdot)$  upto  $(\bar{k}_l, f_h^L(\bar{k}_l))$ .

- Region 6: If  $f_l^I(k_h) \leq k_l \leq f_l^D(k_h)$  &  $f_h^I(k_l) \leq k_h \leq f_h^D(k_l)$ , the optimal policy is to change neither of the capacities.

We use strict concavity of  $g(\bar{\mathbf{k}})$  to define  $f_i^I(\cdot)$ ,  $f_i^R(\cdot)$ ,  $f_i^D(\cdot)$ , and  $f_l^h(\cdot)$ , ( $i \neq j \in \{l, h\}$ ).

**Definition 3.3.** The function  $f_i^I(\cdot)$ ,  $f_i^D(\cdot)$ ,  $f_i^R(\cdot)$ , and  $f_h^L(\cdot)$  are defined as follows

1.  $f_i^I(k_j)$  is  $k_i^I$  such that  $\frac{\partial g(k_j, \cdot)}{\partial k_i} \big|_{k_i=k_i^I} = b_i$  if such  $k_i^I$  exists, and otherwise, it is the point in  $[0, \infty)$  that has derivative closest to  $b_i$ .
2.  $f_i^D(k_j)$  is  $k_i^D$  such that  $\frac{\partial g(k_j, \cdot)}{\partial k_i} \big|_{k_i=k_i^D} = s_i$  if such  $k_i^D$  exists, and otherwise, it is the point in  $[0, \infty)$  that has derivative closest to  $s_i$ .
3.  $f_i^R(k_j)$  is  $k_i^R$  such that  $\frac{\partial g(k_j, \cdot)}{\partial k_i} \big|_{k_i=k_i^R} - s_i = -\frac{s_i}{b_j} \left( \frac{\partial g(k_i^R, \cdot)}{\partial k_j} - b_j \right)$  if such  $k_i^R$  exists, and otherwise, it is the point in  $[f_i^D(k_j), \infty)$  that  $\frac{\partial g(k_j, \cdot)}{\partial k_i} \big|_{k_i=k_i^R}$  is closest to  $-\frac{s_i}{b_j} \frac{\partial g(k_i^R, \cdot)}{\partial k_j}$ .
4.  $f_h^L(k_l)$  is  $k_h^L$  such that  $\frac{\partial g(k_l, \cdot)}{\partial k_h} \big|_{k_h=k_h^L} - b_h = \frac{b_h}{b_l} \left( \frac{\partial g(k_h^L, \cdot)}{\partial k_l} - b_l \right)$  if such  $k_h^L$  exists, and otherwise, it is the point in  $[0, k_h^{II}]$  that  $\frac{\partial g(k_l, \cdot)}{\partial k_h} \big|_{k_h=k_h^L}$  is closest to  $\frac{b_h}{b_l} \frac{\partial g(k_h^L, \cdot)}{\partial k_l}$ .

A corollary of Lemma 3.5 proves the existence of  $f_i^I(k_j)$  and  $f_i^D(k_j)$ .

**Corollary 3.1.**  $\forall k_j \geq 0$ , functions  $f_i^I(k_j)$  and  $f_i^D(k_j)$  are uniquely defined, where  $f_i^I(k_j) < f_i^D(k_j)$ .

*Proof:* Deciding on  $k_i$ , if its initial value  $k_{i0} = 0$ , firm maximizes the concave function of  $g(k_i, k_j) - b_i k_i$ . Thus, for any given  $k_j$ , there exists  $k_i^I = f_i^I(k_j)$  where the first derivative  $\frac{\partial g(k_j, \cdot)}{\partial k_i} \big|_{k_i=k_i^I} - b_i$  becomes the closest to zero. The concavity of  $g(\cdot)$  also suggests that the result remains valid for all  $k_i \in [0, k_i^I]$ .

If  $k_{i0} = \infty$ , firm's maximization problem becomes  $g(k_i, k_j) - s_i k_i$ . Similarity, concavity of  $g(k_j, \cdot)$  proves that there exists  $k_i^D = f_i^D(k_j)$  where the first derivative  $\frac{\partial g(k_j, \cdot)}{\partial k_i} \big|_{k_i=k_i^I} - s_i$  becomes the closest to zero. The result also holds for  $k_i \in [k_i^D, \infty)$ . Additionally,  $b_i > s_i$  results in  $f_i^I(k_j) < f_i^D(k_j)$ . ■

**Lemma 3.6.** (a)  $\forall k_j < f_j^I(k_i)$ , there exists a unique function  $f_i^R(k_j)$  where  $f_i^R(k_j) > f_i^D(k_j)$ ,

(b)  $\forall k_l < k_l^{II}$ , there exists a unique function  $f_h^L(k_l)$  where  $f_h^L(k_l) < f_h^D(k_l)$ .

*Proof:*

(a) Assume  $j = l$ . Corollary 3.1 suggests that  $\forall k_l < k_l^{ID}, \exists k_h^D = f_h^D(k_l)$  where  $x = -\frac{\partial g(k_l, \cdot)}{\partial k_h} \big|_{k_h=k_h^D} + s_i > 0$  has the closest value to zero. In addition,  $k_l < k_l^{ID}$  suggests that  $y = \frac{\partial g(k_h^D, \cdot)}{\partial k_l} - b_l > 0$ . Given  $k_l$ , we explore how  $x/y$  changes in  $k_h \geq k_h^D$ . Concavity of  $g(k_l, \cdot)$  results in  $\partial(x)/\partial k_h > 0$ . In addition, submodularity of  $g(\cdot)$  (c.f. Property 3.1) results in  $\partial(y)/\partial k_h < 0$ . Therefore,  $z = x/y$  will be an increasing function of  $k_h$  with a minimum at  $(k_l, k_h^D)$ . If  $z(k_l, k_h^D) < s_h/b_l$ , there exists  $k_h^R = f_h^R(k_l) > k_h^D$  where  $z(k_l, k_h^R) = s_h/b_l$ . If  $z(k_l, k_h^D) \geq s_h/b_l$ ,  $k_h^R = f_h^R(k_l) = k_h^D$  has the closest  $z$  to  $s_h/b_l$ .

(b) Consider a direct line between  $(0,0)$  and  $K^{II}$  and an arbitrary point  $A(k_l) = (k_l, k_l \frac{k_l^{II}}{k_l^{II}})$  on the line where  $k_l \in [0, k_l^{II}]$ . Denote line  $L$  that passes through  $A(k_l)$  with slope  $-b_l/b_h$ . The joint concavity of firm's maximization problem  $J(k_l, k_h) = g(k_l, k_h) - b_l k_l - b_h k_h$  along line  $L$  suggests that there exists a unique point  $K^L(k_l)$  on  $L$  that maximizes  $J(k_l, k_h)$ . Connecting  $K^L(k_l)$  on all parallel lines  $L$ —passing through points  $A(k_l)$ —forms a curve  $f_h^L(k_l)$  for  $k_l \in [0, k_l^{II}]$ . With respect to the properties of  $f_h^L(k_l)$ , notice that each arbitrary line  $L$  represents a fixed capacity replacement cost  $RC(k_l, k_h, k_{l0}, k_{h0})$ , i.e.  $b_l(k_l - k_{l0}) + b_h(k_h - k_{h0}) = C$  where  $k_{i0}$  is the initial capacity of type  $i$ . This results in  $k_l = \frac{-b_h}{b_l} k_h + C'$ , where  $C' = (C - b_l k_{l0} - b_h k_{h0})/b_l$ . At the maximizing point  $k_h = k_h^L(k_l)$ ,  $\partial J(\cdot)/\partial k_h$  is closest to 0 where  $\frac{\partial J(k_l, k_h)}{\partial k_h} =$

$\frac{\partial g(k_l, k_h)}{\partial k_h} - b_h - \frac{b_h}{b_l} \left( \frac{\partial g(k_l, k_h)}{\partial k_l} - b_l \right)$ . Equivalently, for all  $k_l < k_l^{II}$ , there exists  $k_h^L = f_h^L(k_l)$  so that  $\frac{\partial g(k_l, \cdot)}{\partial k_h} |_{k_h=k_h^L} - b_h$  is closest to  $\frac{b_h}{b_l} \left( \frac{\partial g(\cdot, k_h^L)}{\partial k_l} - b_l \right)$ . ■

**Theorem 3.2.** *For a finite-horizon problem with budget constraint and discounting, and without borrowing and hedging, the optimal policy is the IRSD policy.*

*Proof:* Denote  $\mathbf{K}^{II}$ ,  $\mathbf{K}^{ID}$ ,  $\mathbf{K}^{DI}$ , and  $\mathbf{K}^{DD}$  where  $\mathbf{K}^{mn} = (k_l^{mn}, k_h^{mn})$  is the intersection of  $f_l^m(k_h)$  and  $f_h^n(k_l)$  and  $\mathcal{A}(\mathbf{k}, w) = \{\bar{\mathbf{k}} | RC(\bar{\mathbf{k}}, \mathbf{k}) \leq w\}$  the feasible region for  $\bar{\mathbf{k}}$ . The proof starts with the replacement policy.

(a) Region 4a,  $k_l < k_l^{ID}$  &  $k_h > k_h^{ID}$ . In this region, Corollary 3.1 suggests that if the constraint does not bind,  $\bar{\mathbf{k}} = \mathbf{K}^{ID}$ . If the constraint binds, the extend to which  $k_h$  should be replaced by  $k_l$  depends on  $f_h^R(\cdot)$ ; yet before salvaging  $k_h$ , firm can invest in  $k_l$  upto  $k_l + w/b_l$ . According to Lemma 3.6-a, the value of replacing  $k_h$  with  $k_l$ , i.e.  $-\left(\frac{\partial g(k_l, \cdot)}{\partial k_h} - b_h\right) + \frac{s_h}{b_l} \left(\frac{\partial g(k_h, \cdot)}{\partial k_l} - b_l\right)$  is positive if and only if (iff)  $k_h > k_h^R = f_h^R(k_l)$ . Thus if  $k_h \leq f_h^R(k_l + w/b_l)$  it is not optimal to salvage  $k_h$  and the optimal solution is  $\bar{\mathbf{k}} = (k_l + w/b_l, k_h)$ . If  $k_h = k_h > f_h^R(k_l + w/b_l)$ , replacement is optimal, thus firms salvages  $k_h$  down to  $(\bar{k}_l, \bar{k}_h^R)$  where  $\bar{k}_h^R = f_h^R(\bar{k}_l)$ . Similar analogy holds for Region 4b,  $k_h < k_h^{DI}$  &  $k_l > k_l^{DI}$ . If  $\bar{k}_l < f_l^R(k_h)$ , salvaging  $k_l$  to purchase  $k_h$  has lower value than salvaging  $k_h$  to buy  $k_l$ .

(b) Region 5a and 5b,  $k_l < k_l^{II}$ ,  $k_h < k_h^{II}$ . If constraint does not bind,  $k_l < f_l^I(k_h)$  and  $k_h < f_h^I(k_l)$  results in  $\bar{\mathbf{k}} = \mathbf{K}^{II}$ . If budget constraint binds,  $RC(\mathbf{k}, \bar{\mathbf{k}}) = w$  limits the investment on both of the capacities to reside on the segment  $AB$  where  $A = (k_l, k_h + w/b_h)$  and  $B = (k_l + w/b_l, k_h)$ . Lemma 3.6-b suggests the value replacing  $k_l$  with  $k_h$ ,  $\frac{\partial g(k_l, \cdot)}{\partial k_h} - b_h - \frac{b_h}{b_l} \left( \frac{\partial g(k_h, \cdot)}{\partial k_l} - b_l \right)$  is positive iff  $k_h < k_h^L = f_h^L(k_l)$ . Thus, we increase  $k_l$  upto  $k_l + w/b_l$ , without changing  $k_h$ . If  $k_h < f_h^L(k_l + w/b_l)$ , it is optimal to replace  $k_l$  with of  $k_h$ , i.e. moving along line  $BA$  upto the intersection of  $BA$  and  $f_h^L(\bar{k}_l)$  at  $\bar{\mathbf{k}} = (\bar{k}_l, f_h^L(\bar{k}_l))$  or  $\bar{\mathbf{k}} = A$ , whichever comes first. If  $k_h > f_h^L(k_l + w/b_l)$ ,  $\bar{\mathbf{k}} = B$ .

(c) Region 1a,  $k_l < f_l^I(k_h)$   $k_h^{II} < k_h < k_h^{ID}$ . Corollary 3.1 suggests  $\bar{k}_h = k_h$  for  $f_l^I(k_h) < k_l < f_l^D(k_h)$ . Thus the optimal solution, given  $k_l < f_l^I(k_h)$ , is to invest to  $\min(f_l^I(k_h), k_l + w/b_l)$ . If constraint binds, increase in  $k_l$  is only feasible by replacing it with  $k_h$ . Yet, since  $k_h < f_h^D(k_l) < f_h^R(k_l)$ , according to part (a), it is not optimal to replace  $k_h$  with  $k_l$ . In addition, since  $k_l < f_l^I(k_h) < f_l^D(k_h)$ , and thus  $k_l < f_l^R(k_h)$ , it is not optimal to replace  $k_l$  with  $k_h$ . Therefore, the optimal solution becomes  $\bar{\mathbf{k}} = (\min(f_l^I(k_h), k_l + w/b_l), k_h)$ . The same analogy holds for Region 1b,  $k_h < f_h^I(k_l)$  and  $k_l^{II} < k_l < k_l^{DI}$ .

(d) Region 2a,  $k_l > f_l^D(k_h)$   $k_h^{DI} < k_h < k_h^{DD}$ . Corollary 3.1 suggests  $\bar{k}_h = k_h$  for  $f_l^I(k_h) < k_l < f_l^D(k_h)$ . Thus the optimal solution, given  $k_l > f_l^D(k_h)$ , is to dis-invest to  $f_l^D(k_h)$ . In this setting the constraint does not bind and  $\bar{\mathbf{k}} = (f_l^D(k_h), k_h)$ . The same analogy holds for Region 2b,  $k_h > f_h^D(k_l)$  and  $k_l^{ID} < k_l < k_l^{DD}$  where  $\bar{\mathbf{k}} = (k_l, k_h^D(k_l))$ .

(e) Region 3,  $k_l > k_l^{DD}$  and  $k_h > k_h^{DD}$ . Corollary 3.1 suggests that salvaging both capacities down to  $f_l^D(k_h)$  and  $f_h^D(k_l)$  is optimal. The budget constraint does not bind and the optimal solution is  $\bar{\mathbf{k}} = \mathbf{K}^{DD}$ .

(f) According to Corollary 3.1, if  $f_l^I(k_h) < k_l < f_l^D(k_h)$  and  $f_h^I(k_l) < k_h < f_h^D(k_l)$ , neither investment nor salvaging in either of the capacities adds value, and thus, when the constraint does not bind,  $\bar{\mathbf{k}} = \mathbf{k}$ . ■

### 3.6 Conclusion and Potential Extensions

In this chapter we define and characterize the optimal policy of investing in new capacity or replacing the current capacity when two technologies are available: efficient but expensive, and inefficient but cheap. This study contributes to the intersection of OM and finance literature, where OM mostly fails to address the effect of budget constraint on capacity decision and finance literature overlooks the availability of competing production technologies. This study introduces the IRSD (i.e. Invest,

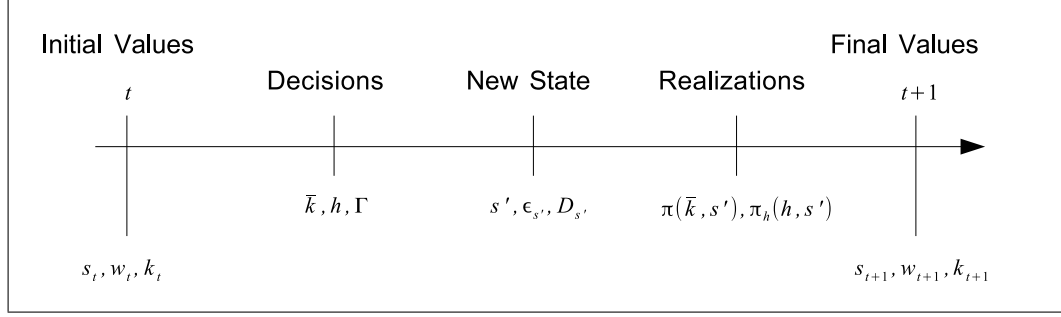


Figure 3.5: Timeline of events at period  $t$

Replace, Stay-put, and Dis-invest) policy that characterizes firm's optimal decisions with respect the choice of technology and the size of capacity based on the initial capacity, budget constraint, and the economic state of the market.

Finance literature highlights an important link between capacity decisions, budget constraint, and financial risk management (*Rampini and Viswanathan, 2013; Rampini et al., 2014*). The rationale is that financial risk management instruments, although reduces the variance of cash flow/revenue, it might result in negative return if the spot price becomes lower than futures price and firm's cash flow should be able to compensate such a loss. Otherwise, there will be a risk of bankruptcy as discussed in the famous case of Metallgesellschaft (*Hawkins and Weyns, 1994*).

Our model is capable of extending the problem to the situation where firm has access to external funds and financial risk management market. Similar to section two we consider financial hedging as the instrument firm use to shield itself against the variability of input price. Thus firm decides on the optimal technology with capacity size  $\bar{k}$ , the amount of external funding  $\Gamma$ , and the hedging position  $h$ . The production profit  $\pi_o$  and the hedging return  $\pi_h$  occur after realization of the new state  $s'$ . The budget at the end of the period should be large enough to repay the external funding  $\Gamma$  with the interest rate  $r$ . We slightly modify the the timeline of events, provided in Figure 3.5, to better formulate the problem with external borrowing and financial hedging.



The net profit of a firm  $\pi$  will be the sum of returns from operational activities and hedging,  $\pi = \pi_o + \pi_h$ , where

$$\pi_o = \sum_{i \in \{l, h\}} \theta_i (p - (1 + \epsilon) \gamma_i), \quad (3.15)$$

$$\pi_h = x(\tilde{\varepsilon} - f), \quad (3.16)$$

where  $f$  is the futures price, and  $\theta_i$  is defined by Equation (3.6). We assume forward market is a fair market in a sense that forward prices are equal to the expected value of the realized spot prices, i.e.  $f = \mathbb{E}[\tilde{\varepsilon}] = \mu$ .

In period  $T + 1$ , the cash residual and the salvage value of the acquired capacity will be the amount the firm is worth, and thus the firm objective is to maximize the net worth in this period. Defining the value to go  $V_t(\mathbf{k}, w, s)$  as the expected discounted net worth in period  $T + 1$ , the firm's objective becomes

$$V_t(\mathbf{k}, w, s) = \max_{\bar{\mathbf{k}}, h, \Gamma \in \mathcal{A}(\mathbf{k}, w, s)} \{g_t(\bar{\mathbf{k}}, w', s')\}, \quad (3.17)$$

$$g_t(\bar{\mathbf{k}}, w', s') = \alpha \mathbb{E}[V_{t+1}(\bar{\mathbf{k}}, w', s')], \quad (3.18)$$

$$s.t. \quad w' = w - RC(\mathbf{k}, \bar{\mathbf{k}}) + \pi(\bar{\mathbf{k}}, h, s') - r\Gamma, \quad (3.19)$$

where  $\mathcal{A}(\mathbf{k}, w, s) = \{\bar{\mathbf{k}} | RC(\bar{\mathbf{k}}, \mathbf{k}) \leq w, w' \geq 0\}$  and  $RC(\cdot)$  is defined by Equation (3.6). The terminal value is  $V_{T+1}(\mathbf{k}, w, \cdot) = \mathbf{s}\mathbf{k} + w$ .

If the budget constraint binds in period  $t + 1$ , the constrained capacity decision  $\bar{\mathbf{k}}_{t+1}$  affects the optimal policy in period  $t$ . With no financial hedging and external borrowing, we showed in Section 3.5 that such an effect is small enough to consider the assumption of the separability of value to in  $\mathbf{k}$  and  $w$  (c.f. Assumption 3.1). In the current setting, although the firm can borrow money to loosen the budget constraint, the borrowing might be capped by the firm's ability to repay the money plus the interest. Thus the budget constraint will not be necessarily removed by the

use of external money and we still need to have this assumption for characterizing the optimal policy.

The separability assumption makes the prospective optimal policy independent of the available cash. However, with the presence of external funding and financial hedging, and the constraints introduced in  $\mathcal{A}(\mathbf{k}, w, s)$  (c.f. Equation (3.17)), the available cash is updated to  $w_b = w + \Gamma$ , but it remains for the further steps to determine how much of the updated cash will be used to finance the capacity decision and how much will be used for risk management by allowing higher levels of hedging position.

## APPENDICES

## APPENDIX A

### List of Symbols and Variables

Table A.1: Description of Symbols and Variables

Chapter	Variable	Description
Common variables	$a$	Potential market size; linear demand intercept
	$b$	Slope of the demand
	$C$	Supplier's cost
	$D$	Market demand
	$p$	Retail price
	$\pi$	Profit
	$\sim$	Indicator of random variables

Continued on next page

Table A.1 – continued from previous page

Chapter	Variable	Description
	$\hat{\gamma}/*$	Indicator of optimal values
Chapter 1	$\bar{a}$	Mean market size
	$w$	Wholesale price
	$x$	Level of investment in technology improvement
	$z$	Coefficient of endogenous uncertainty
	$\alpha$	Buyer's relative bargaining power
	$\beta$	Effectiveness of technology improvement
	$\gamma(x)$	Normalized input consumption rate
	$\sigma^2$	Total Technology uncertainty
	$\sigma_n^2$	Endogenous technology uncertainty
	$\sigma_x^2$	Exogenous technology uncertainty
	$\sigma_d^2$	Demand intercept uncertainty
	$\pi_s$	Supplier profit
	$\pi_b$	Buyer profit
	$\Pi$	Channel profit
Chapter 2	$h$	Hedging position
	$q$	Production quantity
	$\sigma_\epsilon$	Variance of input price
Continued on next page		

Table A.1 – continued from previous page

Chapter	Variable	Description
	$\mu$	Expected value of input cost
	$\lambda$	Arrow–Pratt measure of risk aversion
	$\varepsilon$	Input price
	$\zeta$	Risk exposure
Chapter 3	$h$	Hedging position
	$\mathbf{k}$	Current capacity vector
	$\bar{\mathbf{k}}$	Capacity vector next period
	$w$	Available budget
	$w'$	Budget next period
	$\Gamma$	External funding
	$s$	Economic state of the nature
	$\mathbf{b}$	Capacity purchase price vector
	$\mathbf{s}$	Capacity salvage price vector
	$\gamma$	Consumption rate vector

## APPENDIX B

### Proofs of Chapter 1

#### Proof of Proposition 1.1

We take the expected value of supplier and buyer profit (Equations (1.3) and (1.4)) to be as follows:

$$\Delta\mathbb{E}\pi_s = \frac{\alpha(1-\alpha)(b(zx + \sigma_x^2 + 1)\gamma(x)^2 + (2b - 2a)\gamma(x) + 2a - 3b)}{(1+\alpha)^2} - x; \quad (\text{B1})$$

$$\Delta\mathbb{E}\pi_b = \frac{(b(zx + \sigma_x^2 + 1)\gamma(x)^2 + (2b - 2a)\gamma(x) + 2a - 3b)\alpha^2}{(1+\alpha)^2}. \quad (\text{B2})$$

Then, according to (1.6), the channel profit will be

$$\Delta\mathbb{E}\Pi = \frac{\alpha(b(zx + \sigma_x^2 + 1)\gamma(x)^2 + (2b - 2a)\gamma(x) + 2a - 3b)}{(1+\alpha)^2}. \quad (\text{B3})$$

Regarding part (a), we calculate the first-order conditions (FOCs) of (B1) and (B3).

This results in

$$\frac{\partial\Delta\mathbb{E}\Pi}{\partial x} - \frac{\partial\Delta\mathbb{E}\pi_s}{\partial x} = -\frac{\alpha^2((2a - 2b\gamma(x))\gamma'(x) + bz)}{(1+\alpha)^2}; \quad (\text{B4})$$

that is, the FOC of channel profit is always greater than the FOC of supplier profit because  $\gamma'(x) < 0$  and  $a$  is relatively large. Our calculation also shows that this difference is increasing in  $z$ ; that is,  $x_c^* - x_s^*$  increases with exogenous uncertainty (part (d)).

For part (b), the monotone comparative statics of the FOC of (B1) with respect to (w.r.t.)  $x$  suggests that

$$\text{sign}\left\{\frac{\partial x_s^*}{\partial \alpha}\right\} = \text{sign}\left\{\frac{2(3\alpha - 1)}{(1 + \alpha)^3}(a - b\gamma(x))\gamma'(x) - bz\right\}. \quad (\text{B5})$$

The “sign” function indicates that  $x_s^*$  increases (resp., decreases) in  $\alpha$  if  $\alpha < 1/3$  (resp.,  $\alpha > 1/3$ ) and is maximized at  $\alpha = 1/3$ .

Regarding part (c), supplier profit is concave w.r.t.  $x$  and so we calculate its FOC at  $x = 0$ ; hence

$$\left.\frac{\partial \Delta \mathbb{E} \pi_s}{\partial x}\right|_{x=0} = \frac{-2\alpha(1 - \alpha)(a - b)\gamma'(0) - \alpha^2(bz + 1) + bz\alpha - 2\alpha - 1}{(1 + \alpha)^2}. \quad (\text{B6})$$

Given  $\gamma'(0) < 0$ , it is straightforward to show that Equation (B6) is concave in  $\alpha$ . This expression is equal to  $-1$  at  $\alpha = 0$  and also at  $\alpha = 1$ , so a necessary condition for the feasibility of  $x_s^*$  is that (B6) be positive in the range  $\alpha \in [\alpha_l, \alpha_h]$ , where  $0 < \alpha_l < \alpha_h < 1$ .

## Proof of Proposition 1.2

To show part (a), we note that

$$\frac{\partial \Delta \mathbb{E} \pi_s}{\partial \alpha} = \frac{\partial \Delta \mathbb{E} \pi_s}{\partial x} \frac{\partial x}{\partial \alpha} + \frac{d \Delta \mathbb{E} \pi_s}{d \alpha}. \quad (\text{B7})$$



At  $x = x_s^*$ , we have  $\partial \Delta \mathbb{E} \pi_s / \partial x = 0$ . Hence

$$\frac{\partial \Delta \mathbb{E} \pi_s}{\partial \alpha} \Big|_{x=x_s^*} = \frac{d \Delta \mathbb{E} \pi_s}{d \alpha} = -\frac{(3\alpha - 1)}{(1 + \alpha)^3} (2a - 2a\gamma(x) + b(zx + \sigma_x^2 - 1) + b\gamma(x)^2); \quad (\text{B8})$$

that is,  $\Delta \mathbb{E} \pi_s$  is increasing (resp., decreasing) in  $\alpha$  for  $\alpha < 1/3$  (resp.,  $\alpha > 1/3$ ) and is maximized at  $\alpha = 1/3$ .

Regarding part (b), a simple derivative of  $\Delta \mathbb{E} \pi_b$  w.r.t.  $x$  results in

$$\frac{\partial \Delta \mathbb{E} \pi_b}{\partial x} = -\frac{2\alpha^2}{(1 + \alpha)^2} ((a - b\gamma(x))\gamma'(x) + bz); \quad (\text{B9})$$

that is, the buyer's profit is always increasing in  $x$ .

For part (c), Propositions 1.1(b) and 1.2(b) together suggest that  $\Delta \mathbb{E} \pi_b$  first increases with  $\alpha$  but then decreases. At  $\alpha = 1/3$ , where  $\partial x_s^* / \partial \alpha = 0$ , we have

$$\frac{\partial \Delta \mathbb{E} \pi_b^*}{\partial \alpha} = \frac{9}{32} (1 - \gamma(x))(2a - b\gamma(x) - b) > 0; \quad (\text{B10})$$

that is,  $\alpha_b^* > \alpha_s^*$ .

For part (d), with  $\sigma^2 = zx + \sigma_x^2$  we take the derivatives of  $\Delta \mathbb{E} \pi_s(x(\sigma^2), \sigma^2)$  and  $\Delta \mathbb{E} \pi_b(x(\sigma^2), \sigma^2)$  w.r.t.  $z$  and  $\sigma_x^2$ . Then

$$\frac{\partial \Delta \mathbb{E}(\pi_s)}{\partial z} = \frac{\alpha(1 - \alpha)}{(1 + \alpha)^2} (2b\gamma(x)x'(z)\gamma'(x) - 2ax'(z)\gamma'(x) + bx). \quad (\text{B11})$$

The monotone comparative statistics of the FOC of (B1) w.r.t.  $z$  suggests that  $\text{sign}\{x'(z)\} = \text{sign}\{b\alpha(1 - \alpha)/(1 + \alpha^2)\}$ ; that is,  $x'(z) > 0$ . Therefore,  $\partial \Delta \mathbb{E}(\pi_s^*) / \partial z > 0$ . It is also straightforward to show that  $\partial \Delta \mathbb{E}(\pi_s^*) / \partial (\sigma_x^2) = b\alpha(1 - \alpha)/(1 + \alpha^2) > 0$ . Therefore, since  $\Delta \mathbb{E} \pi_s^*$  increases with both  $z$  and  $\sigma_x^2$ , it is also increasing in  $\sigma^2 = zx_s^* + \sigma_x^2$ . An analogous argument for the case of buyer profit completes the proof of part (d).

### Proof of Proposition 1.3

For part (a), we replace  $\gamma(x) = e^{-\beta x}$  in (B1). The monotone comparative statics of its FOC w.r.t.  $\beta$  results in

$$\frac{4\alpha e^{-\beta x}(1-\alpha)}{(1+\alpha)^2} (b(2\beta x - 1)e^{-\beta x} - a(\beta x - 1)). \quad (\text{B12})$$

Therefore, how  $x_s^*$  changes in  $\beta$  is determined by the sign of  $\mathcal{A} = b(2\beta x - 1)e^{-\beta x} - a(\beta x - 1)$ . The second derivative of  $\mathcal{A}$  shows that its derivative is decreasing (resp. increasing) in low (resp. high)  $\beta$ ; hence  $\mathcal{A}$  can have at most two roots. It is easy to show that  $\mathcal{A}$  is positive in  $\beta = 0$  and is negative as  $\beta \rightarrow \infty$ . Thus  $\mathcal{A}$  has only one root,  $\beta^*$ , where  $\mathcal{A} > 0$  (resp.,  $\mathcal{A} < 0$ ) if  $\beta < \beta^*$  (resp.,  $\beta > \beta^*$ ) and where the sign of  $\partial x_s^*/\partial \beta$  matches that of  $\mathcal{A}$ .

Regarding part (b), we seek to characterize

$$\left. \frac{\partial^2 \Delta \mathbb{E} \pi_b(x, \alpha, \beta)}{\partial \beta \partial \alpha} \right|_{x=x_s^*},$$

where  $x = x(\alpha, \beta)$ . Let  $J = \partial x^*/\partial \alpha$ ,  $H = \partial x^*/\partial \beta$ , and  $L = \partial^2 x^*/\partial \beta \partial \alpha$ . Then we can calculate the term just displayed as

$$\begin{aligned} \left. \frac{\partial^2 \pi_b(x, \alpha, \beta)}{\partial \beta \partial \alpha} \right|_{x^*} &= \frac{2\alpha e^{-\beta x}}{(1+\alpha)^3} \left( (-\beta\alpha(1+\alpha)L + (2H\beta^2 + 2\beta x - 1)(1+\alpha)\alpha J - 2x \right. \\ &\quad \left. - 2\beta H)be^{-\beta x} - ((HJ\beta^2 + (Jx - L)\beta - J)\alpha^2 + (HJ\beta^2 + (Jx \right. \\ &\quad \left. - L)\beta - J)\alpha - 2\beta H - 2x)a \right) \Big|_{x=x^*}. \end{aligned}$$

Given the FOC of (B1), we calculate  $H$  and  $L$  using the implicit function theorem:

$$H = \frac{a - \beta a x + 2be^{-\beta x}\beta x - be^{-\beta x}}{a - \beta^2(2be^{-\beta x})}, \quad (\text{B13})$$

$$L = \frac{-J(a^2 - 3bae^{-\beta x} + 4b^2e^{-2\beta x})}{(a - 2b\beta e^{-\beta x})^2}. \quad (\text{B14})$$

Substituting  $H$  and  $L$  into the preceding expression now yields

$$\begin{aligned} \frac{\partial^2 \pi_b(x, \alpha, \beta)}{\partial \beta \partial \alpha} \Big|_{x=x^*} &= -\frac{2\alpha(a - be^{-\beta x})e^{-\beta x}}{(2be^{-\beta x} - a)^2(1 + \alpha)^3} \\ &\quad \times ((J\alpha^2\beta + J\alpha\beta)(a^2 - 3bae^{-\beta x} + 4b^2e^{-2\beta x}) - (2a^2 - 6bae^{-\beta x} \\ &\quad - 4b^2e^{-2\beta x})). \end{aligned}$$

Since  $\alpha_b^* > \alpha_s^* = 1/3$  and since  $J < 0$  for  $\alpha > 1/3$  (by Proposition 1.1(b)), it follows that the displayed expression is positive in  $\beta$  and hence that  $\alpha_b^*$  is increasing in  $\beta$ . Additionally,  $\alpha_s^* = 1/3$  and it is independent of  $\beta$ .

#### Proof of Proposition 1.4

Under a PC contract, the supplier's problem (Equation (1.7)) becomes

$$\max_x \left\{ \mathbb{E}\Delta_{PC}^s = \frac{-\alpha(a - b)\gamma(x) + \alpha(a - b)}{1 + \alpha} - x \right\}; \quad (\text{B15})$$

as a result,

$$\frac{\partial \mathbb{E}\Delta_{PC}^s}{\partial x} = \frac{-\alpha(a - b)\gamma'(x)}{1 + \alpha} - 1. \quad (\text{B16})$$

Part (a) is a consequence of the necessary condition for  $x_s^* > 0$  to be feasible:  $\partial \mathbb{E}\Delta_{PC}^s / \partial x|_{x=0} > 0$ . This condition suggests that  $\alpha > \underline{\alpha}$ , where

$$\underline{\alpha} = \frac{-1}{\gamma'(0)(a - b) + 1}; \quad (\text{B17})$$

the displayed expression is positive for sufficiently large  $a$  when  $\gamma'(x) < 0$  for all  $x$ .

For part (b), a comparative statics analysis on (B16) suggests that

$$\text{sign} \left\{ \frac{\partial x_s^*}{\partial \alpha} \right\} = \text{sign} \left\{ \frac{-(a - b)\gamma'(x)}{(1 + \alpha)^2} \right\}; \quad (\text{B18})$$

that is,  $\partial x_s^* / \partial \alpha > 0$ .

As for part (c), comparing the FOC of the channel (see (B6)) with the FOC of  $\mathbb{E}\Delta_{PC}^s$  yields

$$\frac{\partial \Delta \Pi}{\partial x} - \frac{\partial \mathbb{E} \Delta \pi_{PC}^s}{\partial x} = -\frac{\alpha}{(1+\alpha)^2} (\gamma'(x)((1-\alpha)(a-b) - zb - 2b\gamma(x))) > 0; \quad (\text{B19})$$

that is,  $x_{PC}^* < x_{FB}^*$ .

### Proof of Proposition 1.5

If we calculate the expected values of (1.8) and (1.9) then supplier and buyer profit in the SI setting become, respectively,

$$\begin{aligned} \Delta \mathbb{E} \pi_{SI}^s = \frac{1}{(1+\alpha^2)} & (b\alpha(1-\alpha)\gamma(x)^2 - 2\alpha(1-\alpha)\gamma(x)a + \alpha^2(b(1-zx - \sigma_x^2) - x(1-\lambda) \\ & - 2a) + \alpha(b(zx + \sigma_x^2 - 1) - 2x(1-\lambda) + 2a) - x(1-\lambda)) \end{aligned} \quad (\text{B20})$$

$$\begin{aligned} \Delta \mathbb{E} \pi_{SI}^b = \frac{1}{(1+\alpha^2)} & (b\alpha^2\gamma(x)^2 - 2\alpha^2\gamma(x)a + (b(zx + \sigma_x^2 - 1) - x\lambda + 2a)\alpha^2 - 2x\lambda\alpha - x\lambda). \end{aligned} \quad (\text{B21})$$

It is straightforward to show that, at  $\lambda^* = \alpha$ ,

$$\frac{\partial \Delta \mathbb{E} \pi_{SI}^s}{\partial x} = \frac{\mathbb{E} \pi_{SI}^b}{\partial x} \quad \forall x; \quad (\text{B22})$$

that is, the optimal investment is the same for both supplier and buyer.

For part (b), we compare the FOCs of  $\Delta \mathbb{E} \Pi$  and  $\Delta \mathbb{E} \pi_{SI}^s$ . Thus

$$\frac{\partial \Delta \mathbb{E} \pi_{SI}^s}{\partial x} = (1-\alpha) \frac{\partial \Delta \mathbb{E} \Pi}{\partial x}, \quad (\text{B23})$$

where at  $x_c^*$  we have  $\partial \Delta \mathbb{E} \pi_{SI}^s / \partial x|_{x_c^*} = \partial \Delta \mathbb{E} \Pi / \partial x|_{x_c^*} = 0$ .

## Proof of Proposition 1.6

(a) As discussed in the proof of Proposition 1.5,  $x_{SI}^*$  is equal to  $x_{FB}^*$ , which is greater than both  $x_{NC}^*$  and  $x_{PC}^*$  (per Propositions 1.1 and 1.4, respectively). To compare  $x_{NC}^*$  with  $x_{PC}^*$ , we calculate the difference between the FOCs of supplier profit in the two settings:

$$\frac{\partial \Delta \mathbb{E} \pi_{NC}}{\partial x} - \frac{\partial \Delta \mathbb{E} \pi_{PC}}{\partial x} = \frac{\alpha}{(1+\alpha)^2} (2b(1-\alpha)\gamma(x) + \alpha(3a-b) - a-b)\gamma'(x) + bz(1-\alpha). \quad (\text{B24})$$

Solving Equation (B24) for  $\alpha$  results in

$$\alpha_{np} = \frac{(2b\gamma(x) - a - b)\gamma'(x) + bz}{(2b\gamma(x) + b - 3a)\gamma'(x) + bz}. \quad (\text{B25})$$

It is easy to show that  $1/3 < \alpha_{np} < 1$ . Moreover, the equalities  $\partial \alpha_{np} / \partial z > 0$  (c.f. Equation (B25)) and  $\partial \alpha_{np} / \partial (\sigma_x^2) = 0$  also suggest that  $\alpha_{np}$  increases with uncertainty when  $z \neq 0$ .

(b) For all  $x$ ,

$$\Delta \mathbb{E} \pi_{SI}^s - \Delta \mathbb{E} \pi_{NC}^s = \alpha x; \quad (\text{B26})$$

therefore,  $\Delta \mathbb{E} \pi_{SI}^s(x_{SI}^*) - \Delta \mathbb{E} \pi_{NC}^s(x_{NC}^*) > 0$ .

Comparing the optimal supplier profit in NC and SI versus PC settings, we find that  $\Delta \mathbb{E} \pi_{NC}$  and  $\Delta \mathbb{E} \pi_{SI}$  are strictly concave functions of  $\alpha$  whereas  $\Delta \mathbb{E} \pi_{PC}$  strictly increases with  $\alpha$ . In particular:

$$\frac{\partial^2 \Delta \mathbb{E} \pi_{NC}}{\partial \alpha^2} = \frac{\partial^2 \Delta \mathbb{E} \pi_{SI}}{\partial \alpha^2} = -\frac{6(1-\alpha)}{(1+\alpha)^4} (2a(1-\gamma(x)) + b(zx + \sigma_x^2 - 1) + b\gamma(x)^2) < 0, \quad (\text{B27})$$

which shows that the functions are strictly concave, while

$$\frac{\partial \Delta \mathbb{E} \pi_{PC}}{\partial \alpha} = \frac{(a-b)(1-\gamma(x))}{(1+\alpha)^2} > 0 \quad (\text{B28})$$

indicates that supplier profit increases with  $\alpha$ . It is straightforward to show that, at  $\alpha = 1/3$  and for all  $x$ , we have  $\Delta\mathbb{E}\pi_{NC} - \Delta\mathbb{E}\pi_{PC} = 1/(8b)((1 - \gamma(x))^2 + zx + \sigma_x^2 > 0)$  and  $\Delta\mathbb{E}\pi_{SI} - \Delta\mathbb{E}\pi_{PC} = 1/(8b)((1 - \gamma(x))^2 + 3(zx + \sigma_x^2) > 0)$ . That is, both NC and SI result in *higher* optimal profit (than does PC) at  $\alpha = 1/3$ . At  $\alpha = 1$ , however,  $\Delta\mathbb{E}\pi_{NC} - \Delta\mathbb{E}\pi_{PC} = -1/2(1 - \gamma(x))(a - b)$  and  $\Delta\mathbb{E}\pi_{SI} - \Delta\mathbb{E}\pi_{PC} = -1/2(1 - \gamma(x))(a - b) + x$ ; that is, both NC and SI result in *lower* supplier profit (than does PC) at  $\alpha = 1$ . Because supplier profit is concave in NC and SI, we conclude that there exists an  $\alpha_{sp}^s$  (resp., an  $\alpha_{np}^s$ ) for which  $\Delta\mathbb{E}\pi_{SI}^* > \Delta\mathbb{E}\pi_{PC}^*$  (resp.,  $\Delta\mathbb{E}\pi_{NC}^* > \Delta\mathbb{E}\pi_{PC}^*$ ) only if  $\alpha < \alpha_{sp}^s$  (resp.,  $\alpha < \alpha_{np}^s$ ).

Equation (B15) shows that  $\Delta_{PC}^s$  is independent of  $z$  and  $\sigma_x^2$ , while (B1) and (B20) show that both  $\Delta_{NC}^s$  and  $\Delta_{SI}^s$  are increasing in  $z$  and  $\sigma_x^2$ . Therefore, if  $\Delta_{NC}^s$  (resp.,  $\Delta_{SI}^s$ ) is equal to  $\Delta_{PC}^s$  at  $\alpha_{np}$  (resp., at  $\alpha_{sp}$ ), then the thresholds will increase with uncertainty.

(c) Proposition 1.2(c) states that, in an NC setting, buyer profit is an inverse U-shaped function of  $\alpha$ . We show buyer profit in an SI setting to be an increasing function of  $\alpha$ . First consider that

$$\frac{\partial \Delta\mathbb{E}\pi_{SI}^b}{\partial x} = -\frac{\alpha}{(1 + \alpha)^2}((a - b\gamma(x))\gamma'(x) + \alpha bz) - 1, \quad (\text{B29})$$

which is positive for sufficiently large  $a$ . Proposition 1.4(b) also suggests that  $\partial x^*/\partial \alpha > 0$  in the SI setting. Therefore,  $\partial \Delta\mathbb{E}\pi_{SI}^b/\partial \alpha = \partial \Delta\mathbb{E}\pi_{SI}^b/\partial x^* \partial x^*/\partial \alpha > 0$ ; that is,  $\Delta\mathbb{E}\pi_{SI}^b(x_{SI}^*)$  always increases with  $\alpha$ . Hence there exists an  $\alpha_{ns}^b$  for which NC yields higher buyer profit when  $\alpha < \alpha_{ns}^b$ .

It is straightforward to show that  $\Delta\mathbb{E}\pi_{SI}^b(x_{SI}^*) - \Delta\mathbb{E}\pi_{NC}^b(x_{NC}^*)$  is an increasing function of  $z$  and hence that  $\alpha_{ns}^b$  increases with uncertainty.

(d) Differentiating Equation (B24) w.r.t.  $z$  results in

$$\frac{\partial}{\partial z} \left( \frac{\partial \Delta \mathbb{E} \pi_{NC}}{\partial x} - \frac{\partial \Delta \mathbb{E} \pi_{PC}}{\partial x} \right) = \frac{b\alpha(1-\alpha)}{(1+\alpha)^2}; \quad (\text{B30})$$

that is, the difference between  $x_{NC}^*$  and  $x_{PC}^*$  is unimodal. In other words, the threshold  $\alpha_{np}$  is a unimodal function of  $\sigma^2$ . We can similarly use Equation (B1) to show that  $\partial \Delta \mathbb{E} \pi_{NC}^s(x_{NC}^*)/\partial z = b\alpha x_{NC}^*(1-\alpha)/(\sigma_x^2 + \alpha)^2$  and that  $\partial \Delta \mathbb{E} \pi_{NC}^s(x_{NC}^*)/\partial(\sigma_x^2) = b\alpha(1-\alpha)/(\sigma_x^2 + \alpha)^2$ , from which it follows that  $\alpha_{sp}^s$  and  $\alpha_{np}^s$  are each unimodal functions in  $\sigma^2$ . An analogous argument proves the corresponding statement for  $\alpha_{ns}^b$ .

### Proof of Proposition 1.7

The FOCs associated with optimal channel profits are

$$\begin{aligned} \frac{\partial \Delta \Pi_{SI}}{\partial x} &= -\frac{2\alpha(a - b\gamma(x))\gamma'(x) - bz\alpha}{(1+\alpha)^2} - 1, \\ \frac{\partial \Delta \Pi_{PC}}{\partial x} &= -\frac{\alpha(a - b)\gamma'(x)}{1+\alpha} - 1. \end{aligned}$$

It is straightforward to show that  $\partial \Delta \Pi_{SI}/\partial x - \partial \Delta \Pi_{PC}/\partial x$  for all  $x$  is positive for  $\alpha \in [0, 1]$ . Therefore, since  $\Pi_{SI}|_{x=0} = 0$  and  $\Pi_{PC}|_{x=0} = 0$ , we conclude that  $\Pi_{SI}(x_{SI}^*) > \Pi_{PC}(x_{PC}^*)$ .

In comparing the channel profit in SI and NC settings, we easily verify that  $\Pi_{SI}(x) = \Pi_{NC}(x)$ . Furthermore, from  $x_{NC}^* \leq x_{SI}^* = x_{FB}^*$  we obtain the inequality  $\Pi_{NC}(x_{NC}^*) < \Pi_{SI}(x_{SI}^*)$ .

We now compare channel profit in the NC and PC settings. Recall from Corollary 1.1 that optimal channel profit in the NC setting is a concave function of  $\alpha$  and is maximized at  $\alpha_c^*$ . Yet Proposition 1.4 suggests that  $x_{PC}^*$  and thus also  $\pi_{PC}^*$  is increasing in  $\alpha$ . Note that, in the PC setting, buyer profit ( $\Delta \pi_{PC}^b$ ) is zero and so channel profit simply equals supplier profit, which is an increasing function of  $\alpha$ . Now

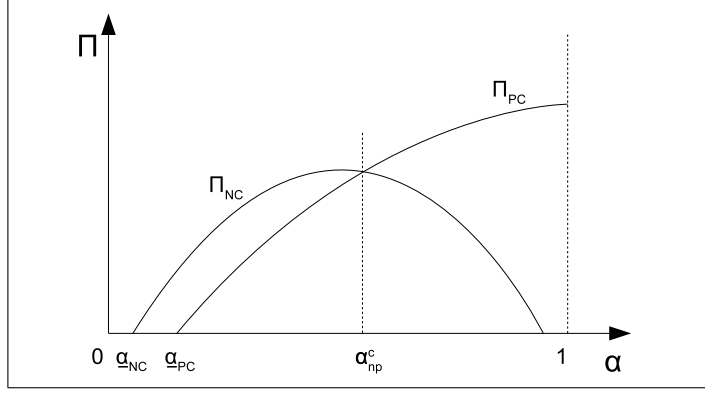


Figure B.1:  $\Pi_{NC}$  versus  $\Pi_{PC}$  as a function of  $\alpha$

we show that the minimum  $\alpha = \underline{\alpha}_{NC}$  at which  $\Pi_{NC}^* = 0$  is *lower* than the minimum  $\alpha = \underline{\alpha}_{PC}$  at which  $\Pi_{PC} = 0$ . In either case, if the optimal investment is zero then the optimal channel profit is also zero. Hence we compare  $\underline{\alpha}$  for both cases. At  $x = 0$ , we know that  $\underline{\alpha}$  guarantees a positive  $\partial\pi^s/\partial x$ . Therefore,

$$\left. \frac{\partial \Delta \mathbb{E} \pi_{NC}^s}{\partial x} \right|_{x=0} = \frac{-2\underline{\alpha}_{NC}(a-b)\gamma'(0) + bz\underline{\alpha}_{NC}}{(1 + \underline{\alpha}_{NC})^2} - 1 = 0, \quad (\text{B31})$$

$$\left. \frac{\partial \Delta \mathbb{E} \pi_{PC}^s}{\partial x} \right|_{x=0} = \frac{-2\underline{\alpha}_{PC}(a-b)\gamma'(0)}{(\sigma_x^2 + \underline{\alpha}_{PC})} - 1 = 0. \quad (\text{B32})$$

Given the above equations, one can easily verify that  $\underline{\alpha}_{NC} < \underline{\alpha}_{PC}$ . Hence there exists a unique  $\alpha_{np}^c$  such that  $\Pi_{NC}^* > \Pi_{PC}^*$  if and only if  $\alpha < \alpha_{np}^c$  (see Figure B.1).

As for the retail price, we remark that it is unchanged under a PC contract because the wholesale price remains unaltered. Yet in the SI, NC, and FB settings, the retail price is a decreasing function of TI investment:

$$p = \frac{a + b\alpha\gamma(x)}{b(1 + \alpha)}.$$

Therefore, since  $x_{NC}^* < x_{SI}^* = x_{FB}^*$  (by Propositions 1.1 and 1.5), it follows that  $p_{SI} = p_{FB} < p_{NC}$ .



### Proof of Proposition 1.8

In the general demand case, it will be helpful to focus not on the investment but rather on the resulting optimal supply cost ( $C^*$ ). For the supplier we have

$$\frac{\partial \Delta \mathbb{E} \pi_s}{\partial \alpha} = \frac{\partial \Delta \mathbb{E} \pi_s}{\partial C} \frac{\partial C}{\partial \alpha} + \frac{d \Delta \mathbb{E} \pi_s}{d \alpha}, \quad (\text{B33})$$

where at  $C^*$  we calculate the derivative in fixed  $C$ .

The bargaining process between buyer and supplier can be expressed as

$$w = C + (1 - \alpha)(p - C),$$

where supplier and buyer optimize their respective profits:

$$\pi_s = (w - C)D(p) - K; \quad \pi_b = (p - w)D(p).$$

Here  $D(p)$  denotes demand as a general function of the retail price  $p$ . Following the discussion in Section 2.3, we have the equilibrium retail price

$$p = \frac{\alpha C D' - D}{\alpha D'},$$

where  $D'$  is the first derivative of  $D$  with respect to  $p$ . Since  $D = D(p)$ , the chain rule now yields

$$\frac{\partial p}{\partial \alpha} = \frac{D' D}{\alpha(\alpha D'^2 + D'^2 - D'' D)};$$

here  $D''$  is the second derivative of demand with respect to  $p$ . We use the chain rule again to calculate  $\partial \pi_s / \partial \alpha = \partial \Delta \pi_s / \partial p \cdot \partial p / \partial \alpha$ . We also take the expectation out of the derivative, as follows:

$$\left. \frac{\partial \Delta \mathbb{E} \pi_s}{\partial \alpha} \right|_{x=x^*} = \mathbb{E} \left\{ \left. \frac{\partial \Delta \pi_s}{\partial \alpha} \right|_{x=x^*} \right\} = \mathbb{E} \left\{ \left. \frac{D^2(\alpha D'' D + (1 - 3\alpha) D'^2)}{\alpha^2 D'(D'' D - (1 + \alpha) D'^2)} \right|_{x=x^*} \right\}.$$

One can easily show that this expression is positive at  $\alpha = 0$  and negative at  $\alpha = 1$ . Hence the resulting unique  $\alpha$  that maximizes the optimal supplier's profit,

$$\alpha^* = \frac{D'^2}{3D'^2 - D''D},$$

is within the range  $[0, 1]$ .

### **Proof of Lemma 1.1**

A basic requirement for the joint determination of wholesale and retail prices is that the supplier has visibility over retail price, i.e.

$$w = C + (1 - \alpha)(p - C). \tag{B34}$$

Here  $p^* = \arg \max_p (p - w(p))(a - bp)$  results in

$$p^* = \frac{a + bC}{2b},$$

from which buyer and supplier profit follow as

$$\begin{aligned} \pi_b &= \alpha \frac{(a - bC(x))^2}{4b}, \\ \pi_s &= (1 - \alpha) \frac{(a - bC(x))^2}{4b} - x. \end{aligned}$$

These profits match those under ex ante negotiation, where  $\Pi = (a - bC(c))^2/4b$ .

### Proof of Proposition 1.9

Equation (1.10) results in new equilibrium wholesale and retail prices:

$$w = \frac{a - \alpha(a - 2b(\tilde{\eta} + C)) - 2b\tilde{\eta}}{b(1 + \alpha)},$$

$$p = \frac{a - b(\tilde{\eta} - \alpha(\tilde{\eta} + \tilde{C}))}{b(1 + \alpha)}.$$

If we substitute  $w$  and  $p$  in the supplier and buyer profit functions  $\pi_s = D(p)(w - \tilde{C}) - x$  and  $\pi_b = D(p)(p - w)$ , then it is straightforward to show that  $\Delta\mathbb{E}_{\tilde{C}, \tilde{\eta}}\pi_s$  and  $\Delta\mathbb{E}_{\tilde{C}, \tilde{\eta}}\pi_b$  are each linear in the random variable  $\tilde{\eta}$ . Therefore,  $\mathbb{E}_{\tilde{C}, \tilde{\eta}}[\Delta\pi_i(\tilde{C}, \tilde{\eta})] = \Delta\mathbb{E}_{\tilde{C}}\pi_i(\tilde{C})$  for  $i = s, b$ .

## APPENDIX C

### Proofs of Chapter 2

*Proof of Lemma 2.1* (a) The optimal prices in Equations (2.2) and (2.3) result in  $\pi'' = \frac{1}{2}b\gamma(x)$  and  $\pi'' = 0$  in the flexible and committed settings, respectively (cf. Equation (2.1)).

(b) By definition, the certainty premium is the value a firm pays ( $\omega > 0$ ) or receives ( $\omega < 0$ ) to replace an uncertain price with its expected value. Thus  $\omega$  should satisfy

$$z(\mu + \omega) = \mathbb{E}[z(\tilde{\varepsilon})], \tag{C1}$$

where  $z(\tilde{\varepsilon}) = U(\pi(\tilde{\varepsilon}))$  for  $U$  the firm's utility function. As before, we let  $\varepsilon = \mu + \tilde{r}$  denote the input price, where  $\tilde{r}$  is a normally distributed shock with the same variance as  $\tilde{\varepsilon}$ . We follow the custom in the economics and finance literature (e.g., *Pratt* 1975; *Gollier* 2004) by using the Taylor expansion  $z(\mu + \omega) \approx z(\mu) + \omega z'(\mu)$  and  $\mathbb{E}[z(\mu + \tilde{r})] \approx \mathbb{E}[z(\mu) + \tilde{r}z'(\mu) + \frac{1}{2}\tilde{r}^2 z''(\mu)]$ .<sup>1</sup> After some algebra, we obtain

$$\omega = \sigma_\epsilon^2 \zeta \quad \text{for } \zeta = z''(\mu)/z'(\mu). \tag{C2}$$

---

<sup>1</sup>Taylor expansion results in  $\mathbb{E}[z(\mu + \tilde{r})] = \mathbb{E}[z(\mu) + \tilde{r}z'(\mu) + \frac{1}{2}\tilde{r}^2 z''(\mu) + o(\sigma_\epsilon^2)]$ , and for small amounts of  $\sigma_\epsilon^2$  the approximation removes the term  $o(\sigma_\epsilon^2)$ .

Applying the chain rule in derivatives of  $z$ , we have

$$z' = \frac{\partial U(\pi(\tilde{\varepsilon}))}{\partial \tilde{\varepsilon}} = \pi' \frac{\partial U}{\partial \pi} \quad \text{and} \quad (C3)$$

$$z'' = \frac{\partial^2 U(\pi(\tilde{\varepsilon}))}{\partial \tilde{\varepsilon}^2} = (\pi')^2 \frac{\partial^2 U}{\partial \pi^2} + \pi'' \frac{\partial U}{\partial \pi}. \quad (C4)$$

Substituting into these equations and putting  $\lambda = -\frac{\partial^2 U(\pi)}{\partial \pi^2} / \frac{\partial U(\pi)}{\partial \pi}$  now yields

$$\omega = \sigma_\epsilon^2 \left( \frac{\pi''}{\pi'} - \lambda \pi' \right). \quad (C5)$$

One can use the envelope theorem and Hotelling's lemma ( $\gamma(x)\mathbf{q} = -\frac{\partial \pi}{\partial \tilde{\varepsilon}}$ ) to expand Equation (C5) by replacing  $\pi'$  with  $-\gamma(x)\mathbf{q}$ , which yields the desired result. Here  $\mathbf{q}$  is the optimal level of the production and  $\gamma(x)\mathbf{q}$  represents total input commodities used. ■

*Proof of Proposition 2.1* (a) In the flexible setting,  $\hat{p}$  and accordingly  $\hat{q} = (a - b\hat{p})$  follow from Equation (2.2). Therefore,

$$\hat{q}(\mu) = a - b\mu\gamma(x); \quad (C6)$$

$$\frac{\pi''}{\pi'} = \frac{b\gamma(x)}{a - b\mu\gamma(x)}. \quad (C7)$$

We calculate the derivative of  $\omega(x)$  in  $x$  and solve it for  $\lambda$ , which yields

$$\lambda'_f = \frac{ab}{2\gamma(x)(2a - 3b\mu\gamma(x))\hat{q}^2}. \quad (C8)$$

It is straightforward to show that  $\omega = \sigma_\epsilon^2 \left( \frac{\pi''}{\pi'} + \lambda\gamma(x)\hat{q} \right)$  is decreasing in  $x$  iff  $\lambda < \lambda'_f$ .

(b) The concavity condition of Equation (2.4), when combined with the existence

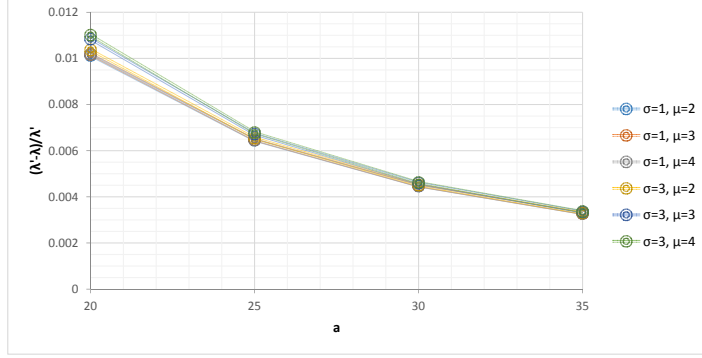


Figure C.1: A numerical illustration the error term in calculating  $\lambda_f$  and  $\lambda'_f$ , derived by  $|\lambda_{f'} - \lambda_f|/\lambda_{f'}$ .

of an interior solution for  $\hat{x}$ , requires the following conditions on  $\gamma'$  and  $\gamma''$ :

$$\gamma' < g(\gamma, \sigma_\epsilon^2, \lambda); \quad (\text{C9})$$

$$\gamma'' > f(\gamma, \gamma', \sigma_\epsilon^2, \lambda) \quad (\text{C10})$$

The first-order condition (FOC) for  $\mathbb{E}[U(\pi(\hat{p}))]$  w.r.t.  $x$  results in

$$\frac{\partial \mathbb{E}[U]}{\partial x} = -\frac{1}{4}\gamma'(\lambda\sigma_\epsilon^2\gamma(a - b\mu\gamma)^2 + (2 - b\lambda\sigma_\epsilon^2\gamma^2)(a\mu - b\gamma(\mu^2 + \sigma_\epsilon^2))) - 1. \quad (\text{C11})$$

Monotone comparative statics of (C11) w.r.t.  $\sigma_\epsilon^2$  shows that  $\hat{x}$  increases with  $\sigma_\epsilon^2$  iff  $\lambda < G(\lambda)$ , where

$$G(\lambda) = \frac{2b}{a^2 + 2b^2(\mu^2 + \sigma_\epsilon^2) - 3ab\mu}. \quad (\text{C12})$$

Since  $\gamma(x)$  is bounded in  $[0, 1]$ , it follows that  $G(\lambda)$  is bounded and has a fixed point  $\lambda_f \in \mathcal{R}^+$ .

We perform numerical analysis to estimate  $\lambda_{f'}$  and  $\lambda_f$ , after which we calculate the difference between them. We find the maximum error term to be in the order of 0.01 (see Figure C.1).

(c) In the committed setting,  $\pi'' = 0$  (cf. Lemma 2.1) and so  $\omega = \lambda\sigma_\epsilon^2 q^*$ . Solving

Equation (2.3) then yields

$$q^* = \frac{a - b\mu\gamma(x)}{2 + b\mu\sigma_\epsilon^2\gamma(x)^2}. \quad (\text{C13})$$

Calculating the derivative of  $\omega(q^*)$  w.r.t.  $x$  and solving it for  $\lambda$ , we obtain the threshold

$$\lambda_c = \frac{2(a - b\mu\gamma(x))}{ab\sigma_\epsilon^2\gamma(x)^2}; \quad (\text{C14})$$

below this threshold,  $\omega(q^*)$  is decreasing in  $x$ .

(d) The FOC for  $\mathbb{E}[U(\pi(p^*))]$  w.r.t.  $x$  results in

$$\frac{\partial \mathbb{E}[U]}{\partial x} = \frac{-\gamma'(a - b\mu\gamma)(a\lambda\sigma_\epsilon^2\gamma + 2\mu)}{(2 + b\lambda\sigma_\epsilon^2\gamma^2)} - 1. \quad (\text{C15})$$

Monotone comparative statics of (C15) w.r.t.  $\sigma_\epsilon^2$  suggests that  $x^*$  is increasing in  $\sigma_\epsilon^2$  iff  $\lambda < G$ , where

$$G = \frac{2(a - 2b\mu\gamma(x))}{ab\sigma_\epsilon^2\gamma(x)^2}. \quad (\text{C16})$$

At  $\lambda = 0$ , an interior solution  $x_0$  results in  $G_0 > 0$ . For  $\lambda \rightarrow \infty$ , we can easily show that  $\lim_{\lambda \rightarrow \infty} \mathbb{E}[U(x)] = -x$ . From that equality it follows that  $x^*|_{\lambda \rightarrow \infty} = 0$  and hence  $G|_{\lambda \rightarrow \infty} = 2(a - 2b\mu)/ab\sigma_\epsilon^2$ . The existence of a bounded  $G|_{\lambda \rightarrow \infty}$  suggests that there also exists a fixed-point solution  $\lambda_c = G$ . ■

*Proof of Proposition 2.2* (a) It is not difficult to demonstrate that  $\mathbb{E}[U(\hat{p})]$  (cf. Equation (2.10)) is strictly concave in  $h$ . Consequently, the FOC for expected utility gives us

$$\hat{h} = \frac{1}{2}\gamma(x)(a - b\mu\gamma(x)). \quad (\text{C17})$$

(b) Maximizing the firm's stage-2 profit results in  $\hat{p} = (a + b\varepsilon\gamma(x))$ ; therefore,  $\hat{q}(\mu) = (a - b\mu\gamma(x))$  and so  $\hat{h} = \gamma(x)\hat{q}$ .

For the committed setting, it is straightforward to show that the FOC for expected utility amounts to Equation (2.11). Plugging  $p^*$  into Equation (2.11) with  $q^* = a - bp^*$

now yields  $q^* = \frac{1}{2}(a - b\mu\gamma(x))$  and  $h^* = \gamma(x)q^*$ .

(c) The equality  $h^* = \gamma(x)q^*$  suggests that  $p^*(h^*) = (a + b\mu)/2b = \mathbb{E}[\hat{p}]$ . ■

*Proof of Lemma 2.2* The return  $\pi_o$  from operational activity and the return  $\pi_h$  from hedging are defined as in Equations (3.15). The first derivative of total profit  $\pi$  w.r.t.  $\tilde{\epsilon}$  yields

$$\pi' = \pi'_o + h. \quad (\text{C18})$$

Hotelling's lemma now implies that  $\pi'_o = -\gamma(x)\mathbf{q}$ , from which it follows that

$$\omega_h = \sigma_\epsilon^2 \left( \frac{\pi''_o}{\pi'_o} + \lambda(\gamma(x)\mathbf{q} - h) \right). \quad \blacksquare \quad (\text{C19})$$

*Proof of Proposition 2.3* According to Proposition 2.2, we should have  $\mathbf{h} = \gamma(x)\mathbf{q}$  in both flexible and committed settings; therefore,  $\omega_h(\mathbf{h}) = \sigma_\epsilon^2(\pi''_o)/\pi'_o$ . Given Lemma 2.1 and since firm profit is decreasing in the input cost ( $\pi'_o < 0$ ), we have  $\omega(\hat{h}) < 0$  for the flexible setting and  $\omega(h^*) = 0$  for the committed setting. ■

*Proof of Proposition 2.4* Because  $\gamma(x)' < 0$ , the second derivative of  $\mathbb{E}[U(\pi(\hat{p}))]$  w.r.t.  $x$  and  $h$  implies that

$$\frac{\partial^2 U}{\partial x \partial h} = \frac{1}{2} \lambda \sigma_\epsilon^2 \gamma(x)' \hat{q} < 0. \quad (\text{C20})$$

For the committed setting, let  $p^*$  be as defined in Proposition 2.2(b). Then the second derivative of  $\mathbb{E}[U(\pi(p^*))]$  w.r.t.  $x$  and  $h$  gives us

$$\frac{\partial^2 U}{\partial x \partial h} = (2a - ab\lambda\sigma_\epsilon^2\gamma(x)^2 + 4b\gamma(x)(h\lambda\sigma_\epsilon^2 - \mu)) \frac{\lambda\sigma_\epsilon^2\gamma(x)'}{(2 + b\lambda\sigma_\epsilon^2\gamma(x)^2)^2}. \quad (\text{C21})$$

Given any  $x$  and  $h$ , there exists a

$$\lambda_s = \max \left\{ \frac{2q^*(x)}{b\gamma(x)\sigma_\epsilon^2(a\gamma(x) - 4h)}, 0 \right\} \quad (\text{C22})$$



such that  $\frac{\partial^2 U}{\partial x \partial h} < 0$ —but if and only if  $\lambda < \lambda_s$ . ■

*Proof of Lemma 2.3* Cournot equilibrium quantities imply the following statements:

$$\hat{q}_i = \frac{1}{3}(a + b\tilde{\varepsilon}(1 - 2\gamma(x))); \quad (\text{C23})$$

$$\hat{q}_n = \frac{1}{3}(a - b\tilde{\varepsilon}(2 - \gamma(x))). \quad (\text{C24})$$

Then, by Equation (2.13), we have

$$\pi'_o(\mu) = -\frac{2}{9}(2\gamma(x) - 1)(a + b\mu(1 - 2\gamma(x))) = \frac{2}{3}(2\gamma(x) - 1)\hat{q}_i, \quad (\text{C25})$$

$$\pi''_o(\mu) = \frac{2b}{9}(2\gamma(x) - 1)^2. \quad (\text{C26})$$

Since  $\pi' = \pi'_o + h_i$ , these two equations yield in  $\omega_i$  introduced in Equation (2.16). ■

*Proof of Proposition 2.5* (a) The second derivative of  $\mathbb{E}[U(\pi(\hat{q}_i, \hat{q}_n))]$  w.r.t.  $x$  and  $h$  allows us to write

$$\frac{\partial^2 U}{\partial x \partial h} = \frac{4\lambda\sigma_\epsilon^2}{9}\gamma(x)'(a + 2b\mu(1 - 2\gamma(x))) < 0, \quad (\text{C27})$$

which implies that TI and FH are substitutes.

(b) The concavity of  $\mathbb{E}[U]$  in  $h$  is easily proved. As a consequence, the FOC for  $\mathbb{E}[U(\pi(\hat{q}_i, \hat{q}_n))]$  w.r.t.  $h$  results in the optimal hedging position

$$\hat{h} = \frac{2}{3}(2\gamma(x) - 1)\hat{q}_i. \quad (\text{C28})$$

(c) We assure concavity of  $\mathbb{E}[U]$  by considering a lower bound for  $\gamma''$  and an upper bound for  $\gamma'$ ; this approach produces an interior solution for  $\hat{x}$  that also uniquely solves the FOC for  $\mathbb{E}[U]$ . Solving the FOC for  $\mathbb{E}[U]$  at a known function  $\gamma(x) = \frac{1}{2}$

gives

$$\mu_d = \frac{-9}{4a\gamma'|_{\frac{1}{2}}}, \quad (\text{C29})$$

where  $\gamma'|_{\frac{1}{2}}$  is the derivative of  $\gamma(x)$  when  $\gamma(x) = \frac{1}{2}$ . Let  $x_{\frac{1}{2}}$  denote the solution of  $\gamma(x) = \frac{1}{2}$ . For  $\mu < \mu_d$ , the FOC at  $x_{\frac{1}{2}}$  is negative (i.e.,  $\hat{x} < x_{\frac{1}{2}}$ ) and so  $\gamma(\hat{x}) > \frac{1}{2}$ . Therefore, by Lemma 2.3,  $\omega_i(\hat{h}, \hat{x})$  is decreasing in  $x$  iff  $\mu < \mu_d$ .

(d) The FOC for  $\mathbb{E}[U(\pi(\hat{q}_i, \hat{q}_n), \hat{h}_i)]$  w.r.t.  $x$  results in

$$\begin{aligned} \frac{\partial \mathbb{E}[U(\pi(\hat{q}_i, \hat{q}_n), \hat{h}_i)]}{\partial x} &= \frac{b}{81} \left\{ -8b^2\gamma(x)\gamma(x)'(2b\lambda\sigma_\epsilon^2(4\gamma(x)^2 - 6\gamma(x) + 3) - 9(\mu^2 + \sigma_\epsilon^2)) \right. \\ &\quad \left. - \gamma(x)'(-8b^3\lambda\sigma_\epsilon^4 + 36b\mu(a + b(\mu^2 + \sigma_\epsilon^2))) - 81b \right\}. \end{aligned} \quad (\text{C30})$$

Monotone comparative statics of Equation (C30) suggests

$$\frac{\partial^2 \mathbb{E}[U(\pi(\hat{q}_i, \hat{q}_n), \hat{h}_i)]}{\partial x \partial (\sigma_\epsilon^2)} = \frac{4b}{81} \gamma(x)'(1 - 2\gamma(x))(4b\lambda\sigma_\epsilon^2(1 - 2\gamma(x))^2 - 9). \quad (\text{C31})$$

Solving Equation (C31) for  $\lambda$  results in  $\bar{\lambda}(x) = 9/(4b\sigma_\epsilon^2(1 - 2\gamma(x))^2)$ , however, we show this threshold will never be achieved. Monotone comparative statics of Equation (C30) also suggests

$$\frac{\partial^2 \mathbb{E}[U(\pi(\hat{q}_i, \hat{q}_n), \hat{h}_i)]}{\partial x \partial \lambda} = \frac{8b^2\sigma_\epsilon^4}{81} \gamma(x)'(1 - 2\gamma(x))^3. \quad (\text{C32})$$

We can easily show that if  $\gamma(x) \leq \frac{1}{2}$ ,  $\hat{x}$  reduces in  $\lambda$ ; hence  $\gamma(\hat{x}) \rightarrow \frac{1}{2}$  and  $\bar{\lambda} \rightarrow +\infty$ . If  $\gamma(x) \geq \frac{1}{2}$ ,  $\hat{x}$  increases in  $\lambda$ ; thus  $\gamma(\hat{x}) \rightarrow \frac{1}{2}$  and  $\bar{\lambda} \rightarrow +\infty$ . Therefore,  $\bar{\lambda}(x)$  is not feasible and Equation (C31) is negative iff  $\gamma(\hat{x}) > \frac{1}{2}$  or  $\mu < \mu_d$ . ■

*Proof of Lemma 2.4* Because production decisions about  $q_i$  and  $q_n$  are based on maximizing the firm's expected utility, we know that  $\pi_o$  will be linear in demand; hence  $\pi'_o = -\gamma(x)q_i^*$ . At the same time, by Lemma 2.1 we should have  $\pi'' = 0$  in the committed setting. This completes the proof. ■

*Proof of Proposition 2.6* (a) The equilibrium quantities simultaneously maximize the expected utilities of investing and noninvesting firms. We may therefore write

$$q_i^* = \frac{a - b(\mu - \lambda\sigma_\epsilon^2 h_i)(2 + b\lambda\sigma_\epsilon^2)\gamma(x) + b(\lambda\sigma_\epsilon^2(a - h_n) + \mu)}{b\lambda\sigma_\epsilon^2\gamma(x)^2(2 + b\lambda\sigma_\epsilon^2) + 2b\lambda\sigma_\epsilon^2 + 3} \quad \text{and} \quad (\text{C33})$$

$$q_n^* = \frac{a + b\gamma(x)(\lambda\sigma_\epsilon^2\gamma(x)(a - b\mu + b\lambda\sigma_\epsilon^2 h_n) + \mu - \lambda\sigma_\epsilon^2 h_i) + 2b(\lambda\sigma_\epsilon^2 h_n - \mu)}{b\lambda\sigma_\epsilon^2\gamma(x)^2(2 + b\lambda\sigma_\epsilon^2) + 2b\lambda\sigma_\epsilon^2 + 3}. \quad (\text{C34})$$

The FOC for  $\mathbb{E}[U(\pi(q_i^*, q_n^*))]$  w.r.t.  $h_i$  and  $h_n$  gives us the optimal hedging position  $h_i^*$  and  $h_n^*$  for (respectively) investing and noninvesting firms. It is straightforward to show that

$$h_i^* = \alpha(x)q_i^*(h_i^*, h_n^*) \quad \text{for} \quad (\text{C35})$$

$$\alpha(x) = \frac{\gamma(x)(2 + b\lambda\sigma_\epsilon^2\gamma(x)^2)(2 + b\lambda\sigma_\epsilon^2)}{b\lambda\sigma_\epsilon^2\gamma(x)^2(2 + b\lambda\sigma_\epsilon^2) + 2b\lambda\sigma_\epsilon^2 + 3}. \quad (\text{C36})$$

We can now easily prove that  $\alpha(x)$  is decreasing in  $x$  and that, for  $x \in [0, +\infty[$ , we have  $\alpha(x) \in [0, \frac{(2+b\lambda\sigma_\epsilon^2)^2}{(2+b\lambda\sigma_\epsilon^2)^2-1}]$ . ■

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**Titre :** Les Stratégies Opérationnelles pour Promouvoir l'Amélioration de la Technologie dans les Chaînes de Valeur

**Mots clés :** Amélioration de la technologie, Coordination des Supply Chain, Stratégies Opérationnelles

Cette recherche se situe à l'interface de la gestion des opérations durables, de la gestion de la technologie et de la finance. Plus précisément, dans mes recherches, j'essaie d'examiner les mesures incitatives des entreprises pour adopter des mesures d'amélioration technologique qui conduisent à une utilisation plus efficiente des intrants et affectent ainsi la structure des coûts, l'exposition aux risques et la performance environnementale des entreprises. Ainsi, je cherche à identifier les facteurs qui affectent --- et les mécanismes par lesquels ils le font --- la décision d'une entreprise d'investir dans TI: forces dans une chaîne d'approvisionnement, incertitude des prix sur les marchés des intrants, contraintes de trésorerie, couverture financière mécanismes, la concurrence de l'industrie et la stratégie de prix compétitive de l'entreprise. En collaborant avec des professeurs dans les domaines de la recherche opérationnelle, de l'économie et de la finance, j'ai adopté une approche multidisciplinaire

pour étudier l'adoption de technologies efficaces et durables. En particulier, dans mon premier chapitre, «L'amélioration des technologies dans les chaînes d'approvisionnement sous pouvoir de négociation asymétrique», j'examine comment le pouvoir de négociation asymétrique --- entre les acheteurs et les fournisseurs --- affecte le niveau optimal d'investissement dans l'amélioration technologique. Dans mon deuxième chapitre, «Gestion des risques liés aux prix des intrants: amélioration de la technologie et couverture financière», j'explore le mécanisme qui guide l'intérêt d'une entreprise pour TI en raison de l'incertitude accrue sur les prix des intrants. Enfin, dans le troisième chapitre, «La valeur de la gestion des risques financiers dans l'investissement dynamique de capacité et l'amélioration technologique», j'étudie le rôle de la contrainte budgétaire et de la couverture financière sur le choix de la technologie.

**Title :** Operational Strategies to Foster Technology Improvement in Value Chains

**Keywords :** Technology Improvement, Supply Chain Coordination, Operational Strategies

This thesis is in the interface of sustainable operations management, technology management, and finance. Specifically, in my thesis I strive to examine firm's incentives to adopt 'technology improvement' (TI) measures that lead to the more efficient use of inputs in operations and thereby affect the cost structure, risk exposure, and environmental performance of firms. Thus I seek to identify the factors that affect---and the mechanisms by which they do so---a firm's decision to invest in TI: forces within a supply chain, price uncertainty in the markets for inputs, cash constraints, financial hedging mechanisms, industry competition, and the firm's competitive pricing strategy. By collaborating with professors in the fields of operations research, economics, and finance, I have embraced a multidisciplinary approach to studying the adoption

of efficient and sustainable technologies. In particular, in my first chapter, "Technology Improvement Contracting in Supply Chains under Asymmetric Bargaining Power" I examine how asymmetric bargaining power---between buyers and suppliers---affects the optimal level of investment in technology improvement. In my second chapter, "Input-price Risk Management: Technology Improvement and Financial Hedging", I explore the mechanism driving a firm's interest in TI under increased uncertainty about input prices. Finally, in the third chapter, "The Value of Financial Risk Management in Dynamic Capacity Investment and Technology Improvement", I study the role of budget constraint and financial hedging on the choice of technology.