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Inspection planning based on preventive maintenance condition in a serial multistage manufacturing system under uncertainty

Mohammad Rezaeimalek

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Mohammad REZAEIMALEK

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Inspection planning based on preventive maintenance condition in a serial multi-stage manufacturing system under uncertainty

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Chapter 1

Introduction

1.0. Chapter purpose and outline

In this chapter, first, the necessity of producing high-quality products is regarded and part quality inspection is introduced as a significant tool for this aim. (please see [Section 1.1](#)). Then, the part quality inspection planning problem in multi-stage manufacturing system is elaborated in [Section 1.2](#). [Section 1.3](#) justifies the importance of handling the part quality inspection planning problem regarding the maintenance and production issues. Finally, [Section 1.4](#) concludes this chapter and determines the position of this research in the framework of production quality paradigm.

1.1. Introduction

The paramount significance of the *Total Quality Management* has been quite understood by all the industries over the recent decades. Indeed, manufacturing companies need to reach a high level of quality in their services or products to maintain their profitability and global competitiveness. To this end, they are employing a wide range of tools to boost the quality in the manufacturing process. Some of the essential tools are Statistical Process Control (SPC), Six Sigma (6σ), process improvement and inspection ([Montgomery, 2009](#)).

Sloppy design of products, inefficient manufacturing methods, tools and machine malfunction and low quality of raw materials are some of the reasons which cause quality problems in manufacturing systems. Hence, performing an appropriate quality control program can lead system to reach a high level of quality for products in the presence of these destructive reasons. Indeed, different research directions have been followed to achieve a high level of quality for products in manufacturing systems: quality in; management, service, manufacturing, and design ([Mohammadi et al., 2015](#)). In this thesis, concentration is on a certain tool for attaining high quality in manufacturing, namely *inspection*.

Inspection is “an organized examination or formal evaluation exercise. Inspection involves the examination, measurement, testing, gauging, and comparison of materials or items. An inspection determines if the material or item is in proper quantity and condition and if it conforms to the applicable or specified requirements”

(Winchell, 1996). Inspection can be done after every production stage for product conformance assessment and based on the result of the inspection; if a defective item is detected, it can be reworked, repaired, replaced or scrapped to avoid vain processing and delivering to customer(s). The research domain of part quality inspection in a Multi-stage Manufacturing System (MMS), studies the imposed costs and obtained benefits by the inspection efforts with the purpose of assessing the manufacturing quality.

In most of the MMSs, raw materials pass a series of different manufacturing stages to be processed and changed to the final products. In these systems, the output of each manufacturing stage (except the last stage) is the input for the next one. Because of non-ideal manufacturing situations and stochastic nature of the production stages, deviations from design specifications are unavoidable and result in low-quality products (Mohammadi et al., 2017).

Part quality inspection in the MMSs is an important issue because of the multi-stage structure of the systems which causes various alternatives for *inspection*. In these systems, if an item is recognized as a defect, there are four possible actions: (1) Rework: the defective item will be sent to the previous production stage(s) for modifying, (2) Repair: the defective item will be repaired and proceeds to the next stage but in a downgraded quality form, (3) Replace: the defected item will be replaced with a conforming one, and (4) Scrap: the defective item will be scrapped. The scrap activities raise the production cost because of wasting material, time, and energy. On the other side, passing the undetected defective products through subsequent production stages raises production cost by establishing more rework and repair stages (required to restore the defect item) (Colledani et al., 2014). There are two distinct strategies, first, performing an inspection activity after each production stage to find defects quickly after corresponding operation of that stage. This would decrease the probability of passing nonconforming items across the MMS. Nevertheless, the cost of inspection after each production stage might be more elevated than the savings earned by the early recognition of defects. Second, the inspection could be employed just after the final production stage. This strategy would reduce the cost of the inspection activities, but there might be a nonconformance in the final product corresponding to the first stage. Therefore, it would be necessary to repeat all the production stages that results in an increase in

the total manufacturing cost. To minimize the total cost, in terms of production, inspection, scrap, replace, rework, and repair, it is practical to establish a balance among different costs and find an optimal inspection plan (Shetwan et al., 2011).

In the literature, the problem of finding this optimal plan is called Part Quality Inspection Planning (PQIP). The next section is dedicated to a further explanation for the PQIP problem.

1.2. PQIP problem

Figure 1.1 schematically shows the PQIP problem in an MMS in each period. This system contains N different production stages that each of them can be followed by an inspection stage m . The figure illustrates the decision problems between each two adjacent production stages. As can be seen, a typical PQIP problem consists of seven main questions as follows:

- Q1: Is Quality Characteristic (QC) k of product(s) inspected after production stage i in period t ?
- Q2: All the items are inspected or a fraction of them (i.e., sampling inspection)?
- Q3: What are the parameters of the sampling inspection?
- Q4: What is the inspection threshold or specification region?
- Q5: After the inspection performance, does the item/sample conform to the inspection threshold/specification region?
- Q6: What is the number of inspection repetitions in case of being not sure about the inspection result?
- Q7: In case of nonconformity, which waste management strategy should be performed (i.e., rework, repair, replace or scrap) for each nonconforming item?

In fact, the product may pass to the next stage or to customer(s) without at least one inspected QC, or the product may be sent to the next stage when inspection shows conformity with the design specification.

Using an inspection plan in an MMS imposes an additional cost, but in an imperfect MMS, a certain level of inspection will reduce total manufacturing cost and enhance the customer satisfaction as well. In this case, the associated cost of inspection will be covered by the benefits obtained through the detection of defects.

Noteworthy, performing an inspection after every production stage will reduce the scraping, replacing, reworking, and downgrading costs and avoid reaching nonconforming products to customer(s). On the other hand, unnecessary and often too inspections impose huge costs of equipment, staff, time, space and, interrupt the manufacturing process which results in extra Work-In-Progress (WIP) and flows. Hence, if inspections are done unnecessarily, then higher total costs will incur (Mohammadi, 2015).

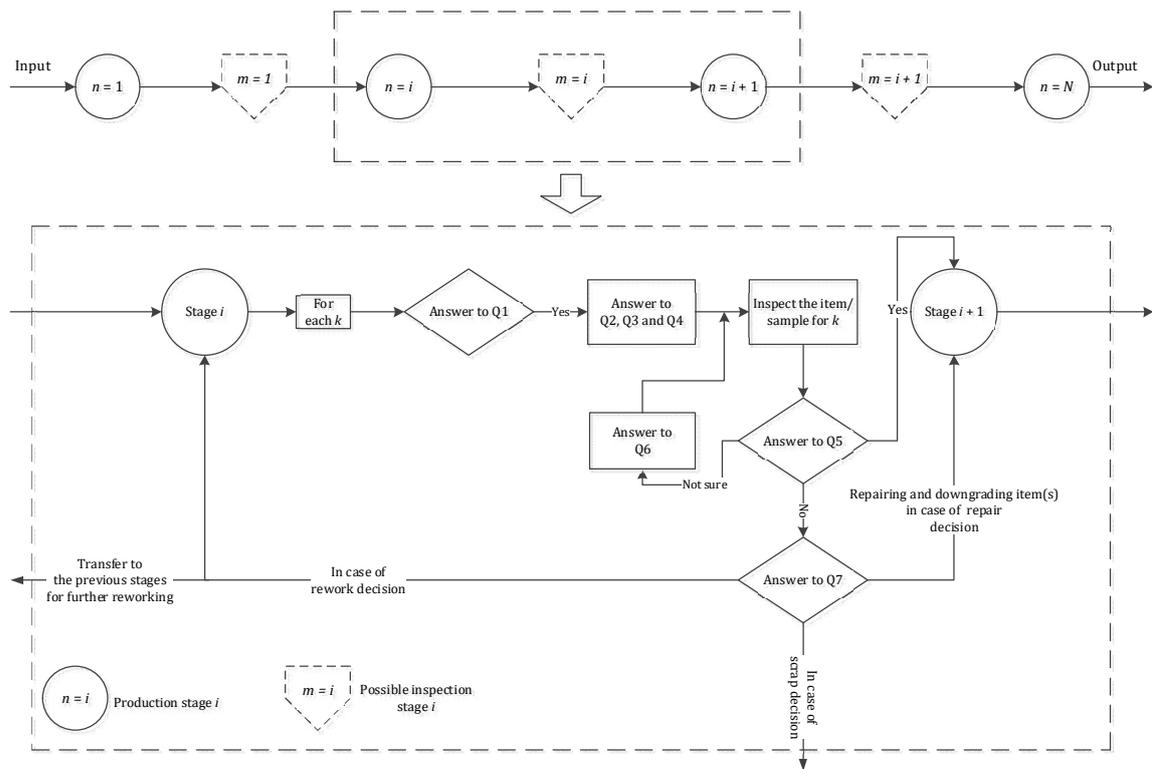


Figure 1.1. PQIP problem in an MMS in period t

In the next section, we attempt to show the impacts of the production and maintenance issues on the main decisions of PQIP problem such as location and interval of inspection activities.

1.3. The relation of PQIP with maintenance and production issues

Production, quality, and maintenance are three significant aspects of each manufacturing system and are interrelated issues (Ben-Daya and Rahim, 2001). Understanding and solving the industrial operation-related problems in manufacturing systems requires appropriate modeling of the dependencies among these aspects. In the past, production, quality, and maintenance have been mostly

considered independently. However, in the recent years, there has been a considerable interest in the models which integrate these issues (Ben-Daya and Rahim, 2001; Hadidi and Al-Turki, 2012).

Figure 1.2 depicts the relations of these aspects in manufacturing systems. The link from Production to Maintenance shows that the employed equipment in Production needs some maintenance activities to be restored to a good condition. The link from Maintenance to Production implies that maintenance activities impact on the capacity of Production. The link between Production and Quality means that if Production works on the right way, it can produce high-quality products. If the equipment is maintained in a good condition, they can operate without any fault. This impact is shown by the link between Quality and Maintenance.

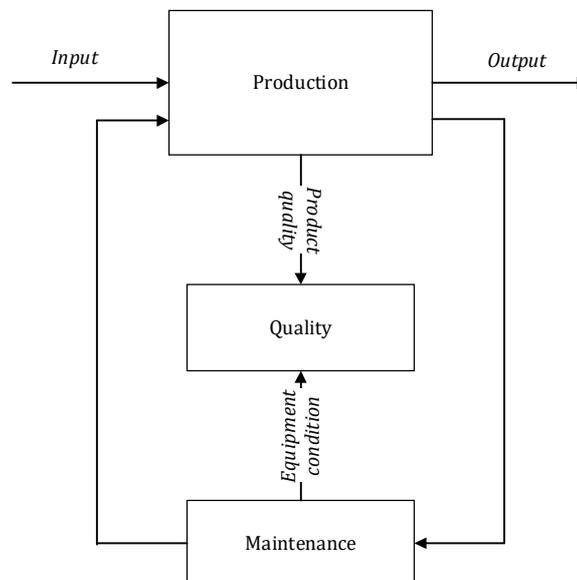


Figure1.2. Relation of production, maintenance, and quality (Adopted from Ben-Daya and Rahim, 2001)

According to the above discussions, three main aspects (i.e. production, quality, and maintenance) should be simultaneously regarded to produce high-quality products. The new developed paradigm by Colledani et al. (2014), *Production Quality*, also support this necessity. In this respect, Table 1.1 presents a planning matrix regarding the integrated view for production, quality, and maintenance. As can be seen, the PQIP is categorized as the quality related problem which should be handled in the mid-term planning horizon because as a facility planning problem, it is

Chapter 1: Introduction

not possible to change the decisions such as the location and interval of inspection activities in short time periods.

In the recent years, Industry 4.0, the fourth industrial revolution, has attracted much attention in recent research works. Cyber Physical System (CPS) is one of the main concepts of Industry 4.0. The main roles of CPS are to fulfill the agile and dynamic requirements of production, and to improve the effectiveness and efficiency of the entire industry. It will be done with the characteristics of CPS production, based on heterogeneous data and knowledge integration (Lu, 2017). Notably, for performing a successful Production quality paradigm, we need to apply CPS characteristics for integrating knowledge of production, quality and maintenance department. This thesis tries to provide an optimization framework to integrate the knowledge of these departments and propose a global plan for improving the effectiveness and efficiency of the system.

Table 1.1. Production quality paradigm; A planning matrix (Inspired from Colledani et al. (2014))

Planning horizon	Production quality paradigm parties		
	Production	Quality	Maintenance
Long term	<ul style="list-style-type: none"> Plant location Production system 	<ul style="list-style-type: none"> Quality planning 	Determination of maintenance strategies: <ul style="list-style-type: none"> Maintenance delivery Maintenance methodology Organization and work structure Support system
Mid-term	<ul style="list-style-type: none"> Master production planning Capacity planning 	<ul style="list-style-type: none"> PQIP Problem 	<ul style="list-style-type: none"> Maintenance work load Forecasting Capacity planning
Short-term	<ul style="list-style-type: none"> Lot-sizing machine Production scheduling Shop floor control 	<ul style="list-style-type: none"> Quality control and improvement 	<ul style="list-style-type: none"> Maintenance scheduling

As mentioned before, one of the factors that causes quality problems is defective/inefficient equipment. On the other side, production processes are shifting from operators to machines because of a huge improvement in automation and mechanization. Therefore, the role of equipment maintenance in controlling quantity, quality and costs is more evident and important than ever (Ben-Daya and Duffuaa, 1995).

If an equipment is not well maintained, it fails periodically and causes a low speed, a lack of precision, and consequently tends to produce defects (i.e., higher

defective production rate). On the other side, when this equipment is recently maintained, it is expected to have a lower rate in the defects production (Ben-Daya and Duffuaa, 1995). In many practical situations, part quality inspections require labors, specific test devices, and sometimes suspension of operations. So, when inspection cost is considerable, it is reasonable to inspect less frequently during the time the equipment is in healthier conditions, and, more frequently as time passes and the equipment degrades (Golmakani, 2012). On the other side, the appropriate implementation of the maintenance activities causes an increase in the production capacity and the system can meet more demands.

In conclusion, because of the maintenance impact on the defective production rate and the impact of defective production rate on the part quality inspection plan, there is a strong connection among performing maintenance activities and the required part quality inspection activities to achieve a certain production capacity for meeting the arrival demands.

1.4. Conclusion

The concentration of this thesis is on the PQIP problem, and it attempt to deal with this problem regarding the impact of maintenance activities on the defective production rate and accordingly on the production capacity. Figure 1.3 depicts the positions of traditional (gray circle) and new PQIP problem (black circle) regarding to different kinds of manufacturing system problems. The common area among Quality, Maintenance, and Production is the domain of the Production Quality paradigm. The traditional PQIP problem (gray circle) is a production-quality-based problem and it does not consider the maintenance decisions and their impacts on the production capacity. Hence, we attempt to deal with the PQIP problem with these new considerations and define the new PQIP problem as its position is shown by a black circle. As discussed in the next section, the PQIP problem have not handled regarding the effects of the maintenance activities specifically preventive type.

As there is no research work which dealt with this new PQIP problem, next chapter review the literature of the traditional PQIP problem and by discovering research gaps it justifies the importance of working on this new PQIP problem.

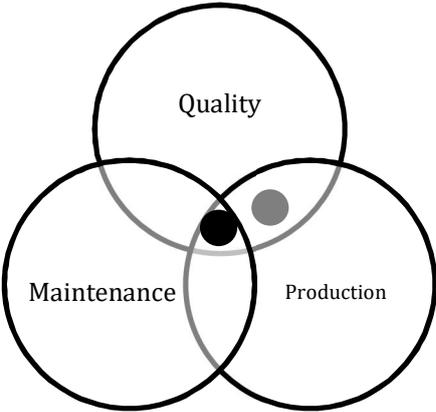


Figure 1.3. Position of traditional and new PQIP problems in the production quality paradigm

Chapter 2

Literature review

2.0. Chapter purpose and outline

Since the core of current research is the PQIP problem, this chapter aims at reviewing the PQIP literature and discovering research gaps. In this regard, [Section 2.1](#) describes the adopted framework for doing the review. Then, [Sections 2.2](#) and [2.3](#) investigate the literature regarding the characteristics of considered systems and applied methodologies, respectively. Next, [Section 2.4](#) explores the papers which handled the PQIP problem considering maintenance and production issues. Finally, the literature is analyzed in [Section 2.5](#) by providing the discovered gaps and some future research works.

2.1. Introduction

The keywords “inspection planning” in combination with “multi-stage production/manufacturing system” were used to find relevant journal articles, books, theses and conference contributions in the two most comprehensive databases; Scopus and ProQuest. 67% of the research works have been published by 11% of the journals (see [Figure 2.1](#)). Noteworthy, *International Journal of Production Research* has the highest impact (i.e., 16.7%) among all. The rest of studies (i.e. 33%) have been published by the journals with less than two research works. [Figure 2.2](#) illustrates the number of issued papers during the different periods, and as can be seen, there is an expected increasing trend for research publishing. Although the last period in [Figure 2.2](#) includes seven years (2010 to 2017), it is expected that the increasing trend of publishing in this field will be continued for the second decade of the 21st century.

The associate literature is investigated from the different viewpoints: production system characteristics, methodology characteristics and optimization framework. Although [Raz \(1986\)](#), [Mandroli et al. \(2006\)](#) and [Shetwan et al. \(2011\)](#) reviewed the literature of the PQIP problem in the different time periods, this chapter covers not only their investigated researches, but also the state-of-the-art literature up to 2017. In addition, this chapter investigates and classifies the literature based on two new and practical perspectives: Optimization under uncertainty and Integrated optimization with maintenance and

production issues. In this way, the readers of this chapter will get familiar comprehensively with the development trend and research gaps of the PQIP problem literature.

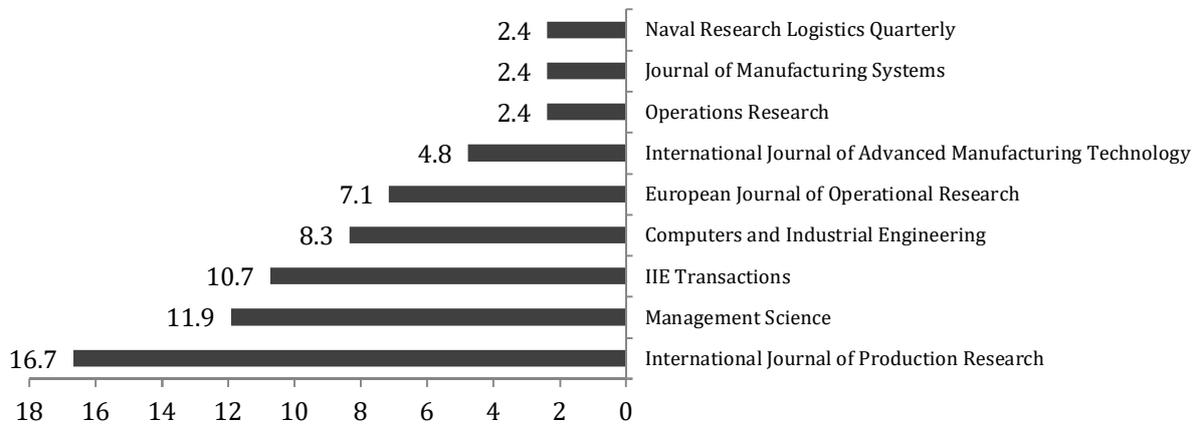


Figure 2.1 Percentage of the published research works in the most famous journals

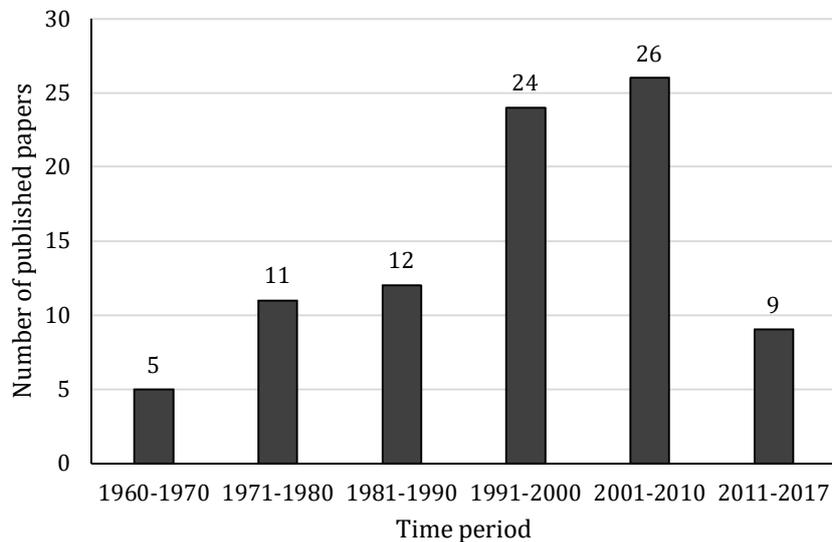


Figure 2.2. Number of published papers in different periods

2.2. MMS characteristics

Regarding the literature of the PQIP problem, the characteristics of investigated MMSs fall into seven main categories (Mohammadi, 2015); (1) manufacturing structure, (2) manufacturing/inspection flow, (3) inspection type, (4) inspection strategy, (5) inspection

errors, (6) failure type and rate, and (7) nonconforming strategy. Each of these main categories is elaborated through some sub-categories as follows. [Table 2.1](#) illustrates the features of the literature regarding these characteristics.

2.2.1. Production structure

There are three main manufacturing structures according to the product flow alongside an MMS ([Mandroli et al., 2006](#); [Mohammadi, 2015](#)):

- i. *Serial structure*: all products pass the same successive manufacturing stages sequentially.
- ii. *Convergent structure*: each item passes a certain set of successive manufacturing stages sequentially while different paths may be converged in a particular stage. On the other side, each production stage has at most one successor but many predecessor stages (e.g., assembly process).
- iii. *Nonserial structure*: each item passes distinct stages sequentially while each production stage may have several successors and predecessor stages.

In contrary to the non-serial and convergent structures, the serial structure is the most-considered in the literature (see [Figure 2.3a](#)) because the defects propagation can be easily modeled in a mathematical way. On the other hand, in the convergent structure, mathematically tracking the root defects after the convergent stage is complex.

2.2.2. Production/inspection flow

A production line in an MMS can produce either a specific product or multiple types of the same product family. In this regard, inspections of this manufacturing line can be done per item or batch/lot. Therefore, four possible disciplines for the production/inspection flow exist (see [Figure 2.3b](#)). [Figure 2.3b](#) also shows the percentage of papers which consider these different disciplines. Although the *Mixed production/batch inspection* discipline is closer to the real condition of MMSs, the modeling and solving complexity of that is greater.

2.2.3. Inspection strategy

Two different inspection strategies have been employed by the researchers in the literature: *100% or none inspection* and *Sampling inspection*. In the first strategy some of the QCs are not inspected, and if it is decided to inspect a QC, all the items are inspected. *Sampling inspection* means if it is decided to inspect a QC, a sample of items is inspected.

Figure 2.3c illustrates the percentage of papers which consider these two different strategies. Although using the *Sampling inspection* is more practical in high rate production systems, it is difficult to be considered because the other parameters of this strategy (e.g., sample size) need to be determined simultaneously in the optimization framework and this great number of decision variables increases the solution complexity of the problem.

2.2.4. Inspection errors

There are two types of errors that may happen during a part quality inspection activity:

- i. *Error type I*: occurs when a conforming item is classified wrongly as a nonconforming one.
- ii. *Error type II*: happens when a nonconforming item is classified wrongly as a conforming one.

The Error-Free assumption for the inspection activity is unrealistic, but a considerable number of works (i.e., 37%) considered the inspection activities free of any error (see Figure 2.3d).

2.2.5. Failure rate and type

A failure rate of a manufacturing stage is the proportion of defects to all the produced items. In the literature, a certain constant failure rate for each stage has been assumed in some works, whereas others have assumed either a plausible range of a failure rate or random failure under a specific distribution. In addition, two single and multiple failure types have been considered by the authors. In fact, each QC is related to a single or multiple failure modes. In a case of a single failure type for a specific QC, the QC will not be realized properly if its related failure mode is active. Similarly, for the multiple failure

types, the QC will not be realized appropriately if at least one of its related failure modes is active. In this regard, for the multiple failure types, a vector of failure rates is associated with multiple failure modes (Mandroli et al, 2006). It is notable that each manufacturing stage may contain more than one failure mode. In conclusion, as shown in Figure 2.3e, four potential combinations exist while a few papers only considered the *Multiple type/random rate* assumption.

2.2.6. Nonconforming strategy

When inspection recognized an item as a nonconforming product, four possible actions can be done. The item can be reworked, repaired, replaced, or scrapped. The decision about the appropriate action depends on the associated cost and knowledge of whether the nonconformity is reparable/reworkable or not. In this regard, a deterministic or probabilistic level of scrapping for nonconforming parts have been assumed by the researchers. In the deterministic level, for a given type of nonconformity, the scrapping level is given as one of the three different possibilities: all, none, or some of the nonconforming items are scrapped. On the other side, others have assumed a probabilistic level which means that a nonconforming item is scrapped with a certain probability, so some of the items may have a chance to be reworked, repaired or replaced (Mohammadi, 2015). According to the above-mentioned explanations, the nonconforming strategy is divided into four different subcategories as shown in Figure 2.3f.

Table 2.1. Literature classification based on the MMS characteristics

Author	Year	Production Structure			Product/Inspection Flow				Inspection Strategy		Inspection Errors			Failure Type and Rate				Nonconforming strategy			
		Serial	Convergent	Nonserial	Single Product/Single Inspection	Single Product/Batch Inspection	Mixed Product/Single Inspection	Mixed Product/Batch Inspection	100% or none	Sampling Inspection	Type I	Type II	Error-free	Constant Rate/Single Type	Random Rate/Single Type	Constant Rate/ Multiple Type	Random Rate/Multiple Type	No Scrap	Scrapping Some	Scrapping All	Probabilistic
Beightler and Mitten	1964	✓				✓				✓			✓		✓			✓			
Lindsay and Bishop	1964	✓				✓				✓			✓	✓						✓	
White	1966	✓				✓				✓			✓		✓			✓			
Pruzan and Jackson	1967	✓				✓				✓			✓		✓					✓	
Brown	1968	✓				✓				✓			✓	✓				✓			
Ercan	1972	✓				✓				✓			✓	✓				✓			
Garey	1972	✓			✓				✓				✓	✓						✓	
Woo and Metcalfe	1972	✓				✓			✓				✓		✓					✓	
Britney	1972			✓	✓				✓				✓		✓			✓			
Hurst	1973	✓				✓				✓	✓	✓		✓						✓	
Dietrich and Sanders	1974	✓				✓			✓				✓	✓						✓	
Eppen and Hurst	1974	✓				✓				✓	✓	✓		✓						✓	
Ercan et al.	1974	✓				✓				✓			✓	✓				✓			
Trippi	1974	✓				✓				✓			✓		✓			✓			
Enrick	1975	✓				✓				✓	✓	✓		✓					✓		
Trippi	1975	✓				✓				✓			✓		✓			✓			
Yum and McDowell	1981			✓		✓			✓		✓	✓		✓				✓			
Ballou and Pazer	1982	✓				✓				✓	✓	✓		✓						✓	
Hsu	1984	✓				✓				✓			✓	✓				✓			
Peters and Williams	1984	✓				✓				✓			✓	✓					✓		

Table 2.1. Literature classification based on the MMS characteristics (continue)

Author	Year	Production Structure			Product/Inspection Flow				Inspection Strategy	Inspection Errors			Failure Type and Rate				Nonconforming strategy					
		Serial	Convergent	Nonserial	Single Product/Single Inspection	Single Product/Batch Inspection	Mixed Product/Single Inspection	Mixed Product/Batch Inspection		100% or none	Sampling Inspection	Type I	Type II	Error-free	Constant Rate/Single Type	Random Rate/Single Type	Constant Rate/Multiple Type	Random Rate/Multiple Type	No Scrap	Scrapping Some	Scrapping All	Probabilistic
Garcia-Diaz et al.	1984		✓		✓				✓		✓		✓					✓				
Ballou and Pazer	1985	✓				✓				✓	✓		✓							✓		
Gunter and Swanson	1985		✓		✓				✓			✓	✓							✓		
Chakravarty and Shtub	1987	✓						✓		✓		✓			✓							✓
Lee and Rosenblatt	1987	✓			✓				✓		✓				✓			✓				
Peters and Williams	1987	✓				✓				✓	✓				✓					✓		
Yum and McDowell	1987	✓				✓				✓	✓	✓	✓		✓			✓				
Tayi and Ballou	1988	✓				✓				✓		✓	✓		✓			✓				
Saxena et al.	1990	✓			✓				✓				✓		✓					✓		
Barad	1990	✓				✓				✓	✓				✓							✓
Foster et al.	1990	✓			✓				✓		✓	✓			✓					✓		
Kang et al.	1990	✓				✓			✓		✓	✓			✓				✓			
Raz and Kaspi	1991	✓			✓				✓		✓	✓			✓							✓
Tang	1991	✓			✓				✓			✓		✓						✓		
Villalobos and Foster	1991	✓			✓				✓		✓	✓			✓					✓		
Villalobos et al.	1993	✓			✓				✓		✓	✓			✓					✓		
Taneja & Viswanadham	1994	✓		✓	✓				✓		✓	✓			✓					✓		
Jewkes	1995	✓			✓				✓			✓	✓		✓			✓				
Rebello et al.	1995	✓			✓				✓			✓					✓		✓			
Shin	1995	✓			✓				✓			✓			✓				✓			
Bai and Yun	1996	✓					✓			✓	✓						✓					✓

Table 2.1. Literature classification based on the MMS characteristics (continue)

Author	Year	Production Structure			Product/Inspection Flow				Inspection Strategy		Inspection Errors			Failure Type and Rate				Nonconforming strategy			
		Serial	Convergent	Nonserial	Single Product/Single Inspection	Single Product/Batch Inspection	Mixed Product/Single Inspection	Mixed Product/Batch Inspection	100% or none	Sampling Inspection	Type I	Type II	Error-free	Constant Rate / Single Type	Random Rate/Single Type	Constant Rate / Multiple Type	Random Rate/Multiple Type	No Scrap	Scraping Some	Scraping All	Probabilistic
Deliman and Feldman	1996	✓			✓				✓		✓		✓								✓
Gurnani et al.	1996	✓				✓				✓		✓	✓				✓				
Viswandham et al.	1996	✓	✓			✓			✓		✓	✓	✓						✓		
Narahari and Khan	1996			✓	✓				✓			✓	✓								✓
Chevalier and Wein	1997	✓			✓				✓		✓	✓				✓	✓				
Rabinowitz and Emmons	1997			✓	✓				✓			✓	✓						✓		
Chen et al.	1998	✓				✓			✓			✓		✓			✓				
Lee and Unnikrishnan	1998	✓					✓		✓		✓	✓			✓			✓			
Yao and Zheng	1999a	✓				✓			✓			✓		✓			✓				
Yao and Zheng	1999b	✓				✓			✓			✓			✓		✓				
Chen and Thornton	1999			✓					✓			✓			✓			✓			
Hassan and Pham	2000	✓			✓				✓		✓	✓		✓							✓
Veatch	2000	✓			✓	✓			✓			✓		✓			✓				✓
Zheng	2000		✓			✓			✓			✓			✓		✓				
Verduzco et al.	2001		✓						✓		✓	✓			✓					✓	
Zhou and Zhao	2002	✓			✓				✓			✓		✓						✓	
Shiau	2002	✓				✓			✓		✓	✓		✓					✓		
Emmons and Rabinowitz	2002			✓					✓			✓		✓						✓	
Avinadav and Raz	2003	✓				✓			✓		✓	✓		✓						✓	
Oppermann et al.	2003	✓			✓				✓		✓	✓			✓					✓	✓
Van Volsem & Van Landeghem	2003	✓			✓				✓			✓		✓						✓	

Table 2.1. Literature classification based on the MMS characteristics (continue)

Author	Year	Production Structure			Product/Inspection Flow				Inspection Strategy	Inspection Errors			Failure Type and Rate				Nonconforming strategy					
		Serial	Convergent	Nonserial	Single Product/Single Inspection	Single Product/Batch Inspection	Mixed Product/Single Inspection	Mixed Product/Batch Inspection		100% or none	Sampling Inspection	Type I	Type II	Error-free	Constant Rate/Single Type	Random Rate/Single Type	Constant Rate/Multiple Type	Random Rate/Multiple Type	No Scrap	Scrapping Some	Scrapping All	Probabilistic
Shiau	2003a	✓				✓				✓	✓			✓					✓			
Shiau	2003b	✓				✓				✓	✓			✓					✓			
Kakade et al.	2004	✓			✓				✓			✓			✓				✓			
Valenzuela et al.	2004	✓			✓				✓			✓			✓				✓			
Rau and Chu	2005	✓			✓				✓			✓			✓				✓			
Hanne and Nickel	2005	✓			✓				✓			✓	✓						✓			
Feng and Kapur	2006	✓			✓				✓		✓		✓	✓							✓	
Shiau et al.	2007	✓				✓			✓		✓		✓								✓	
Penn and Raviv	2007	✓			✓				✓			✓			✓						✓	
Van Volsem et al.	2007	✓				✓			✓			✓	✓						✓			
Penn and Raviv	2008	✓						✓				✓	✓								✓	
Vaghefi and Sarhangian	2009	✓			✓					✓	✓		✓						✓			
Ferreira et al.	2009	✓			✓					✓		✓	✓								✓	
Rau and Cho	2009	✓			✓				✓			✓	✓						✓			
Azadeh and Sangari	2010	✓			✓				✓			✓	✓								✓	
Van Volsem	2010	✓			✓				✓		✓			✓					✓			
Korytkowski	2011	✓				✓			✓			✓	✓								✓	
Rau and Cho	2011	✓			✓				✓			✓	✓								✓	
Azadeh et al.	2012	✓			✓					✓		✓	✓						✓			
Azadeh et al.	2015a	✓			✓					✓	✓		✓						✓			
Mousavi et al.	2015	✓			✓					✓	✓		✓								✓	
Mohammadi et al.	2015	✓			✓				✓		✓		✓								✓	
Mohammadi et al.	2017	✓			✓				✓		✓		✓								✓	

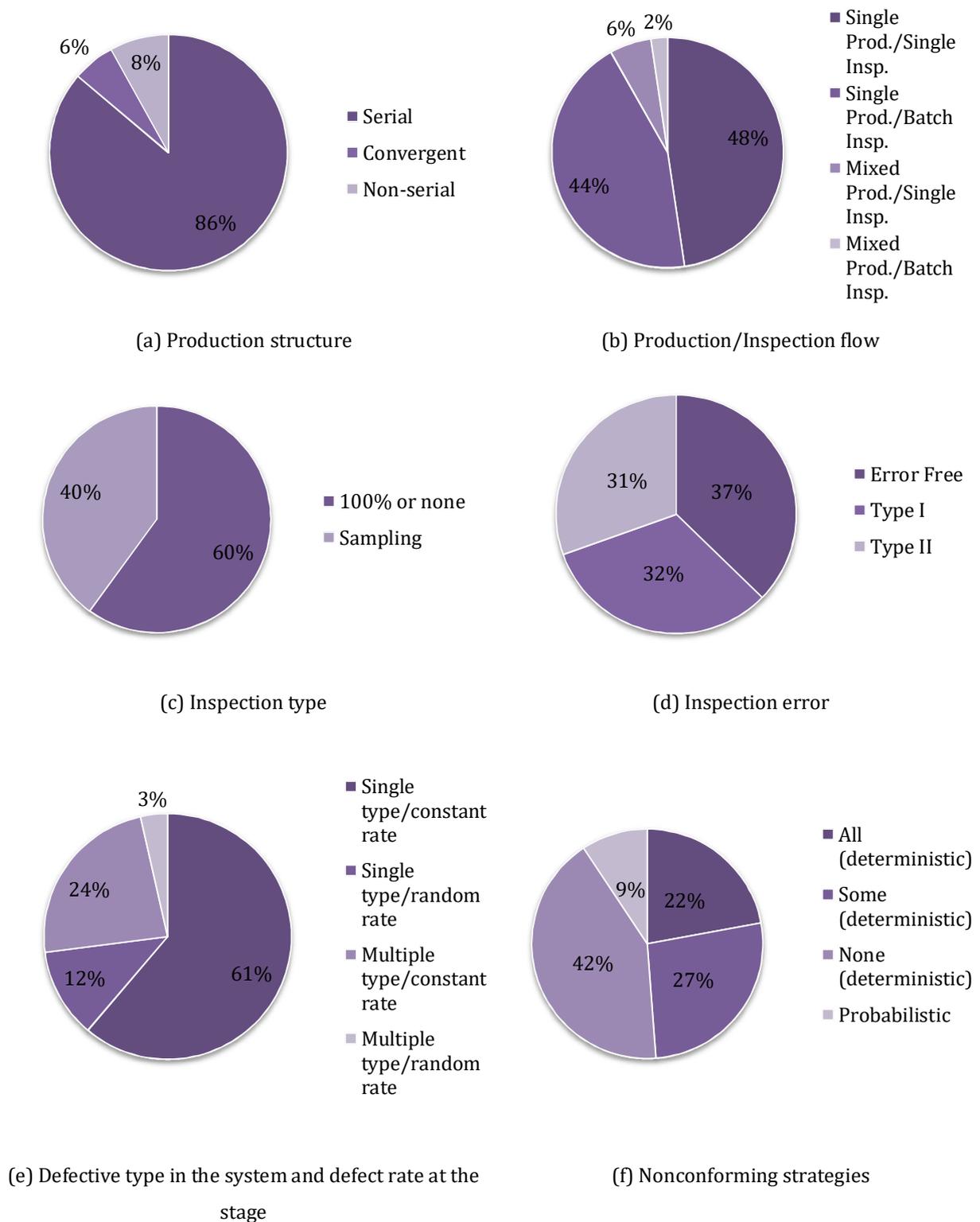


Figure 2.3. Percentage of papers which considered different assumptions

2.3. Methodology characteristics

According to the vast review of the PQIP literature, almost all the studies have dealt with the problem through an optimization formulation. In the following subsections, first, different kinds of the considered objective functions are elaborated, and then three necessary parts of an optimization formulation as (i) constraint, (ii) uncertainty approach, and (iii) solution approach are addressed. [Table 2.2](#) illustrates the features of the literature regarding the optimization characteristics.

2.3.1. Objective functions

Minimization of the total expected cost is the most common form of objective function in the literature. Total cost generally includes different cost components as production, inspection, and failure costs. The failure cost itself consists of internal and external cost. When nonconforming products are found before shipment, it imposes an internal cost to the system. This cost is specifically related to the costs of reworking, repairing, replacing, and scrapping the nonconforming product. A manufacturer undergoes the external failure costs when a defective product has been received by customer(s). These costs may be certain compensation or the lost sales and goodwill. The inspection cost contains two fixed and variable costs. The fixed inspection cost is related to a fixed amount of capital for preparing inspection tools and the variable cost directly depends on the frequency and number of inspected items. The variable inspection cost has been often assumed as a linear function, in which, the total variable inspection cost is the number of items inspected multiplied by the variable inspection cost per item ([Mohammadi, 2015](#)). There are just two works, which have treated this cost as a non-linear function (see e.g., [Britney \(1972\)](#) and [Ballou and Pazer, \(1985\)](#)). Regarding the current literature, there is no work which concurrently considers the non-linear form of variable inspection cost, fixed inspection cost and internal failure cost (i.e., scrap, rework, repair and replace). Please see [Figure 2.4a-c](#) for the details of the literature regarding the considered internal, external, and inspection cost.

Another common form of the objective function is the expected unit cost. However, there are different ways to determine the units (see [Figure 2.4d](#)). Some papers have

computed the expected unit cost as total cost divided by the number of input items (i.e. *total cost/input items*). The other versions were dividing the total cost by the number of outputs and dividing the total cost by the number of conforming outputs (i.e., *total cost/output items* or *total cost/conforming output items*).

There are only a few authors considering maximization formulations in their studies. The maximization objectives have mainly proposed in inspection scheduling problems besides to classical PQIP problem. To the best of our knowledge, no study has considered minimizing total manufacturing time. In addition, there is a lack of applying multi-objective models in the current PQIP literature. In this respect, [Mohammadi et al. \(2017\)](#) minimized total manufacturing cost and warranty cost (to capture customer satisfaction) in form of two different objective functions. However, the customer satisfaction has a non-linear behavior when he/she receives the product lot—which was neglected by [Mohammadi et al. \(2017\)](#). Indeed, the customer satisfaction and accordingly the utility of the product lot is higher when the proportion of the conforming items is greater in the delivered lot. They solved the presented model by employing a meta-heuristic algorithm, namely Differential Evolution (DE) algorithm.

For approaching to the real situation of MMSs, where they face to the conflict objectives for optimizing their systems, developing the multi-objective mathematical models which simultaneously optimize several conflict objectives is an excellent research direction for future in this field. For instance, when the quality of the system is maximized through employing more inspection stations, the production cycle time is increased. Hence, developing a new bi-objective model to establish a trade-off between these two conflict objectives seems interesting and practical.

2.3.2. Constraints

The constraints in the typical PQIP problem are mostly associated with the different types of production structure and nonconforming strategy. Moreover, other constraints have been imposed to the optimization formulation. For example, some of them are: limitations on the inspection time, number of inspection and rework stations, number of inspection repetition, the budget for production and inspection activities, required place

for an inspection station, and minimum throughput or production capacity. [Figure 2.4e](#) illustrates the percentage of papers which considered different constraints.

Other constraints in the developed optimization formulation could be the dependency between different QCs that require to be inspected. For instance, two QCs must be inspected in parallel or vice versa. In addition to QCs dependency, in some situations, there are dependent production stages, and there is no possibility for stop a particular stage to inspect a QC and you need to wait for the following operation(s) to be completed ([Mirdamadi, 2014](#); [Mohammadi, 2015](#)). There are the other applicable and realistic constraints in the domain of PQIP problem that have not considered yet such as the limited capacity of operating machines, waiting time for inspection and capable inspection tools to treat the items.

2.3.3. Optimization under uncertainty

The PQIP problem inherently contains different sources of uncertainty. Accordingly, the necessity for consideration of associate uncertain parameters and obtaining a robust solution has been implied by most of the researchers, and it should be extended more. One of the main sources of uncertainty in this problem refers to the condition of production stages for processing items in conformity with specification region. This uncertain condition results in the uncertainty of the failure production rate parameter. In this regard, many works considered the random failure rate in a probabilistic manner (see [Table 2.1](#)). They assumed a specific probability distribution (e.g., Bernoulli distribution) for this parameter. By the above-mentioned source of uncertainty, the proportion of nonconforming items which is repairable, reworkable or need to be scrapped or replaced, is uncertain. This uncertain data has been also treated as a probabilistic nonconforming strategy in the literature (See [Table 2.1](#)). Regarding the uncertainty about inspection tools and inspection operators, Error type I and Error type II have been considered probabilistically by the majority of researchers (see [Table 2.1](#)). Beside these uncertainties which are rooted in the internal reasons, there are the other uncertainties that are related to the uncertain external condition such as price fluctuations ([Azadeh et al., 2015a](#)) and demand amount. [Azadeh et al. \(2015a\)](#) described the cost components (i.e., inspection, rework and penalty of defects shipment) by applying the fuzzy numbers. They proposed a

fuzzy model for the problem and converted it into an equivalent auxiliary crisp model by employing the Jimenez 's definition of expected value. [Mohammadi et al. \(2017\)](#) considered alteration ranges for the production and inspection times, errors type I and II of the inspection activities, dispersion and misadjustment of the production processes, and developed a global robust model based on optimizing the expected value and variance of the objective function in the form of the Taguchi method. Regarding these recent works, it would be an excellent research direction to propose an approach (e.g., a robust possibilistic programming approach) to make use of the advantages of the fuzzy and robust approaches for considering uncertainties related to the external reasons which are being usually implied by experts (subjective data).

2.3.4. Solution approaches

The researchers have developed a wide variety of solution approaches for solving the PQIP problem. In the most cases, they derived a nonlinear total cost function while some of the decision variables (e.g., opening an inspection station) can only have integer values. This transforms the problem to a nonlinear and integer programming problem. For the integer programming, the discrete optimization approaches such as those using the branch-and-bound technique have been applied ([Raz and Kaspi, 1991](#)).

Because the multi-stage structure of manufacturing systems can be well considered by the stages and states of the Dynamic Programming (DP) models, 29% of the 89 reviewed papers applied DP and made this approach as the most-used one (see [Figure 2.4f](#)). However, DP becomes quite impractical when the number of production stages is high. Because possible combinations of places for performing inspection activities, increase exponentially ([Mandroli et al., 2006](#); [Shetwan et al., 2011](#)).

Indeed, a huge restriction of these approaches is their incapability of solving medium and large size problems due to the requirement of high computational time and memory. This limitation results in usage of heuristic and meta-heuristic algorithms in this domain, such as Simulated Annealing (SA) and Genetic Algorithm (GA). They obtain near-optimal solutions in considerable low computational time ([Modammadi, 2015](#)). [Mohammadi et al. \(2017\)](#) applied a Differential Evolution (DE) algorithm, which provides

promising results in comparison with SA and GA for solving a proposed mixed-integer linear programming model.

Another optimization approach is simulation; however, this method just investigates a limited number of production scenarios for optimization. Considering the current literature, no study proposes an exact solution method to large size instances. In this regard, developing decomposition methods (e.g., Benders decomposition methods) to solve large size mixed-integer programming models can be a huge contribution to this area. In addition, employing some newly-developed meta-heuristic algorithms, such as Fireworks and Cuckoo Search and comparing to the conventional SA, GA, and DE algorithms regarding computational capability can be interesting as a future research direction.

Table 2.2. Literature classification based on the methodology

Author	Year	Cost component							Objective Function			Constraint			Solution Approach						
		Internal Failure			External Failure		Inspection Cost		Production Cost	Total/Input	Total/Output	Total/Conforming Output	Inspection Time	No. of Inspection Station	No. of Repeated Inspection	Budget	Dynamic Programming	Integer Programming	Nonlinear Programming	Heuristics & Metaheuristics	Simulation
		Rework	Replace or repair	Scrap	Defect Dependent	Defect Independent	Fixed	Variable													
Beightler and Mitten	1964					✓		✓	✓							✓					
Lindsay and Bishop	1964			✓				✓	✓							✓					
White	1966	✓	✓	✓		✓	✓	✓	✓				✓			✓					
Pruzan and Jackson	1967					✓	✓	✓	✓							✓					
Brown	1968			✓		✓		✓	✓							✓					
Britney	1972	✓				✓			✓								✓				
Ercan	1972	✓		✓		✓		✓	✓								✓	✓			
Garey	1972							✓	✓							✓					
Woo and Metcalfe	1972			✓		✓		✓			✓					✓					
Hurst	1973			✓				✓	✓												
Dietrich and Sanders	1974					✓		✓	✓											✓	
Eppen and Hurst Jr.	1974			✓		✓		✓	✓							✓					
Ercan et al.	1974	✓	✓	✓		✓		✓	✓												
Trippi	1974	✓		✓	✓		✓	✓	✓				✓					✓			
Enrick	1975	✓		✓		✓		✓	✓							✓					
Trippi	1975		✓		✓		✓	✓	✓								✓				
Yum and McDowell	1981	✓	✓			✓		✓	✓								✓				
Ballou and Pazer	1982			✓		✓		✓			✓							✓			
Garcia-Diaz et al.	1984	✓				✓		✓	✓							✓					
Hsu	1984			✓		✓		✓	✓							✓					

Table 2.2. Literature classification based on the methodology (continue)

Author	Year	Cost component								Objective Function			Constraint				Solution Approach					
		Internal Failure			External Failure		Inspection Cost			Production Cost	Total/Input	Total/Output	Total/Conforming Output	Inspection Time	No. of Inspection Station	No. of Repeated Inspection	Budget	Dynamic Programming	Integer Programming	Nonlinear Programming	Heuristics & Metaheuristics	Simulation
		Rework	Replace or repair	Scrap	Defect Dependent	Defect Independent	Fixed	Variable														
							Linear	Nonlinear														
Peters and Williams	1984	✓		✓	✓		✓	✓		✓	✓							✓				
Ballou and Pazer	1985			✓		✓		✓		✓	✓							✓				
Gunter and Swanson	1985			✓		✓	✓				✓					✓						
Chakravarty and Shtub	1987	✓		✓	✓		✓	✓		✓	✓					✓		✓				
Lee and Rosenblatt	1987					✓	✓			✓	✓							✓				
Peters and Williams	1987			✓		✓	✓			✓		✓				✓						
Yum and McDowell	1987	✓	✓	✓		✓	✓			✓							✓					
Tayi and Ballou	1988	✓		✓		✓	✓			✓	✓							✓				
Saxena et al.	1990			✓			✓			✓	✓								✓			
Barad	1990	✓	✓	✓		✓	✓			✓	✓								✓			
Foster et al.	1990			✓		✓	✓			✓			✓						✓			
Kang et al.	1990	✓		✓		✓	✓			✓	✓									✓		
Raz and Kaspi	1991	✓		✓		✓	✓			✓	✓						✓					
Tang	1991			✓		✓	✓			✓	✓					✓	✓					
Villalobos and Foster	1991			✓		✓	✓			✓						✓						
Villalobos et al.	1993			✓		✓	✓			✓			✓				✓					
Taneja & Viswanadham	1994			✓		✓	✓			✓	✓			✓	✓				✓			
Jewkes	1995	✓				✓	✓			✓	✓							✓				
Rebello et al.	1995	✓		✓	✓		✓			✓		✓				✓			✓			
Shin	1995	✓					✓			✓												
Bai and Yun	1996			✓		✓	✓			✓				✓			✓					

Table 2.2. Literature classification based on the methodology (continue)

Author	Year	Cost component							Objective Function			Constraint			Solution Approach						
		Internal Failure			External Failure		Inspection Cost		Production Cost	Total/Input	Total/Output	Total/Conforming Output	Inspection Time	No. of Inspection Station	No. of Repeated Inspection	Budget	Dynamic Programming	Integer Programming	Nonlinear Programming	Heuristics & Metaheuristics	Simulation
		Rework	Replace or repair	Scrap	Defect Dependent	Defect Independent	Fixed	Variable													
Deliman and Feldman	1996	✓		✓		✓		✓	✓									✓			
Gurnani et al.	1996	✓						✓	✓						✓						
Narahari and Khan	1996	✓	✓						✓									✓			
Viswandham et al.	1996	✓		✓		✓		✓	✓				✓						✓		
Chevalier and Wein	1997	✓				✓		✓	✓									✓			
Rabinowitz and Emmons	1997							✓	✓								✓				
Chen et al.	1998	✓				✓		✓	✓						✓						
Lee and Unnikrishnan	1998	✓			✓			✓	✓			✓						✓			
Chen and Thornton	1999			✓			✓		✓										✓		
Yao and Zheng	1999a	✓				✓		✓	✓						✓						
Yao and Zheng	1999b	✓			✓			✓	✓						✓						
Hassan and Pham	2000	✓		✓		✓		✓		✓									✓		
Veatch	2000			✓	✓			✓		✓									✓		
Zheng	2000	✓			✓			✓	✓						✓						
Zhou and Zhao	2002			✓	✓			✓			✓				✓				✓		
Emmons and Rabinowitz	2002							✓	✓										✓		
Shiau	2002	✓	✓	✓		✓		✓	✓				✓						✓		
Oppermann et al.	2003	✓		✓		✓			✓						✓						
Avinadav and Raz	2003				✓		✓	✓		✓									✓		
Van Volsem & Van Landeghem	2003	✓				✓	✓	✓		✓										✓	
Shiau	2003a	✓	✓	✓		✓		✓	✓				✓						✓		

Table 2.2. Literature classification based on the methodology (continue)

Author	Year	Cost component								Objective Function			Constraint				Solution Approach					
		Internal Failure			External Failure		Insp. Cost			Production Cost	Total/Input	Total/Output	Total/Conforming Output	Inspection Time	No. of Inspection Station	No. of Repeated Inspection	Budget	Dynamic Programming	Integer Programming	Nonlinear Programming	Heuristics & Metaheuristics	Simulation
		Rework	Replace or repair	Scrap	Defect Dependent	Defect Independent	Fixed	Variable														
							Linear	Nonlinear														
Shiau	2003b	✓	✓	✓		✓	✓			✓				✓						✓		
Kakade et al.	2004	✓			✓		✓					✓								✓		
Valenzuela et al.	2004					✓						✓								✓		
Rau and Chu	2005	✓		✓	✓		✓	✓	✓	✓										✓		
Hanne and Nickel	2005	✓		✓			✓	✓	✓	✓										✓		
Feng and Kapur	2006			✓			✓	✓	✓	✓												
Shiau et al.	2007	✓		✓			✓	✓		✓				✓						✓		
Penn and Raviv	2007						✓	✓	✓	✓						✓						
Van Volsem et al.	2007	✓			✓		✓	✓		✓										✓		
Penn and Raviv	2008			✓			✓	✓				✓								✓		
Vaghefi and Sarhangian	2009	✓		✓		✓	✓	✓	✓	✓											✓	
Ferreira et al.	2009	✓		✓		✓	✓	✓		✓										✓		
Rau and Cho	2009	✓	✓	✓				✓		✓										✓		
Azadeh and Sangari	2010						✓	✓		✓										✓		
Van Volsem	2010	✓		✓			✓	✓		✓										✓	✓	
Korytkowski	2011						✓	✓		✓			✓							✓		
Rau and Cho	2011	✓		✓			✓	✓		✓										✓		
Azadeh et al.	2012	✓		✓		✓	✓	✓		✓										✓		
Azadeh et al.	2015a	✓		✓		✓	✓	✓		✓										✓	✓	
Mousavi et al.	2015			✓		✓		✓		✓										✓		
Mohammadi et al.	2015			✓	✓		✓	✓		✓			✓	✓					✓	✓	✓	
Mohammadi et al.	2017			✓	✓		✓	✓		✓			✓	✓					✓	✓	✓	

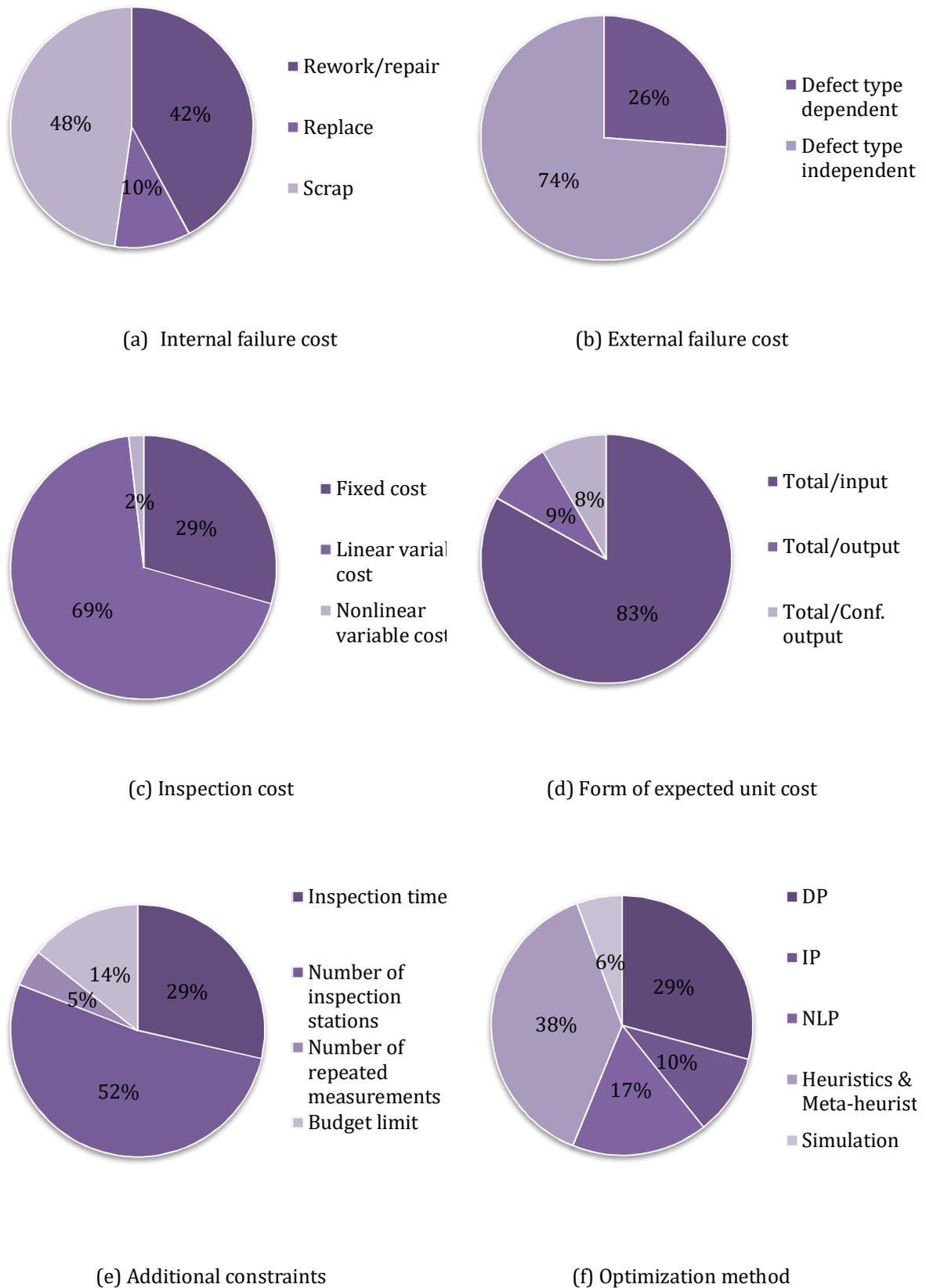


Figure 2.4. Percentage of papers considered different methodology characteristics

2.4. Integrated optimization

Production, quality, and maintenance are three significant aspects of each manufacturing system and are interrelated problems (Ben-Daya and Rahim, 2001). The link between these issues is explained in Section 1.2. Noteworthy, the aim of this section is not to review the papers, which proposed integrated planning for quality, production, and maintenance. Since the main problem of this thesis is the PQIP problem, the papers that are reviewed here are those whose principal problem is the PQIP and have tried to incorporate the production and maintenance issues into it. Indeed, there are a few works in the literature incorporated production logistic decisions, such as optimum lot size (Tayi and Ballou, 1988), production rate (Gunter and Swanson, 1985; Penn and Raviv, 2007; Penn and Raviv, 2008), sequence of operations (Raz and Kaspi, 1991), inspection scheduling (Emmons and Rabinowitz, 2002), inspection resource assignment (Shiau, 2002), assignment of operations to operators (Hanne and Nickel, 2005), and process scheduling (Shiau et al., 2007) into the PQIP problem.

Maintenance as an important tool for the quality-assurance program, has a strong effect on the optimum PQIP. The British Standard Institution defines maintenance as “a combination of all technical and associated administrative activities needed to maintain equipment, installations and other physical assets in the appropriate operating condition or bring them back to this condition” (Muchiri et al, 2014). Maintenance activities in manufacturing systems are categorized according to the time they are activated in. Corrective maintenance is a kind of the maintenance activity, which is only activated after a failure has been realized. However, Preventive Maintenance (PM) is performed to avoid a possible failure. Some of the PM activities are performed in specific points of system lifetime, called time/aged-based maintenance, and the others are performed after two or more monitored indicators showing that the system is going to be failed or its performance are deteriorating (i.e., condition-based maintenance). In the condition-based maintenance strategy, the time for performing a maintenance activity is determined based on the real condition of the system, but in the time/aged-based maintenance, it is based on the historical data of the system (Liu et al., 2017; Nicolai and Dekker, 2008). Indeed, different

maintenance strategies attempt to preserve the efficiency of degrading resources over time by employing pro-active and predictive capabilities (Iung et al., 2009).

The system degradation not only increases chance of a failure to happen, but also causes defective product output (Kuo, 2006). Indeed, the previous studies within the PQIP literature just considered a non-deteriorating system with constant defect production probability of different production stages. Although one of the conventional solutions is to conduct PM activities to reduce the probability of defect production (Colledani et al., 2014), the PQIP literature lacks a study, which uses the advantage of PM performance (decreasing the defect production rate) to plan the part quality inspection activities accordingly. Regarding this lack of research, defining and solving an integrated planning problem which simultaneously determines the optimum plan of part quality inspection and preventive maintenance is an interesting research direction which conforms with the Industry 4.0 concept.

2.5. Conclusion

In the framework of a quality assurance program, the PQIP is a significant decision problem, which all the MMSs are dealing with. Indeed, a proper part quality inspection plan not only avoids delivering nonconforming products to customer(s), but also increases the efficiency of the MMS by decreasing waste of time and material.

In this chapter, the existing research works on the PQIP optimization problem have been reviewed from the viewpoint of the MMS characteristics as well as the modeling and solution methodologies. In addition, the literature has been investigated regarding the integrated optimization of the part quality inspection with the maintenance and production planning.

Based on the conducted review, some research gaps were mentioned in the previous sections. Accordingly, this thesis considers a deteriorating serial multi-stage manufacturing system with minimum required conforming items, and attempts to contribute to the literature by:

- Developing a novel integrated optimization approach for to simultaneously plan the part quality inspection and PM activities.

Chapter 2: Literature review

- Introducing a non-linear measure for system productivity and considering it as a new objective function besides traditional total cost.
- Considering uncertainty about cost components (i.e., production, inspection, maintenance, repair, scrap, and penalty cost for the delivered nonconforming items) and demand amount (i.e., minimum required conforming items).

To perform these new contributions, a bi-objective mixed-integer non-linear mathematical model is presented. Then, a piecewise linear approximation technique is used to linearize the model. Next, a robust possibilistic approach is used to handle the uncertain parameters and guarantee the robustness of the obtained solution. Finally, the augmented ε -constraint method is employed to deal with the bi-objective model and obtain the Pareto frontier.

Chapter 3

Mathematical Formulation

3.0. Chapter purpose and outline

In this chapter, first, the main problem of this thesis is elaborated in [Section 3.1](#). Then, a mixed-integer linear programming is proposed in [Section 3.1.5](#) to mathematically model the problem. Next, an extended version of the problem and its newly added features are described in [Section 3.2](#) and corresponding bi-objective mixed-integer non-linear programming model is developed in [Section 3.2.5](#). Finally, [Section 3.3](#) provides a summary of this chapter.

3.1. Main problem description

Consider a Serial Multi-Stage Production Line (SMSPL). Material enters the SMSPL and passes all the stages and after the last one, it is transformed to a final product which is ready for selling to customer(s). Each production stage is responsible for a certain QC. The production stages are technologically incapable for processing items in a perfect quality, so there is a possibility to do an inspection activity after each stage. Each inspection activity is capable to detect the defective items associated to the preceding stage and then they can be scrapped or be repaired in a known cost which depends on the processing stage number. Two types of errors (i.e., Type-I and Type-II) can be happen during each inspection activity.

In the considered SMSPL, each stage is deteriorating in time. To be exact, when a stage starts operating, it starts deteriorating after a while (increasing defective production rate) and finally it fails. For instance, consider a drill machine, the drill bit becomes blunt after a while and does not have the precise and capability of the beginning ([Colledani et al., 2014](#)). During the deterioration period, probability of a conforming item acquires a defect at a stage, ε , is increasing in time and it is equal to 1 at the end (i.e., the stage fails) ([Muchiri et al., 2014](#)). Furthermore, in the beginning of each period, it is possible to do a PM activity in a known cost to completely restore the stage. [Figure 3.1](#) shows an example for the ε behavior of a stage and the impact of the PM activities on it.

Regarding the above-described SMSPL, the problem is to concurrently plan the part quality inspection and PM activities during a planning horizon while minimizing the total costs including production, inspection, repair, PM and the

penalty of shipped defective items. The planning horizon consists of equal time periods and it is needed to determine in which time periods, inspection and PM activities require to be done. Figure 3.2 illustrates the above-mentioned problem schematically. Below, first, the required notations are defined, and then the equivalent model of our mixed-integer linear programming is developed.

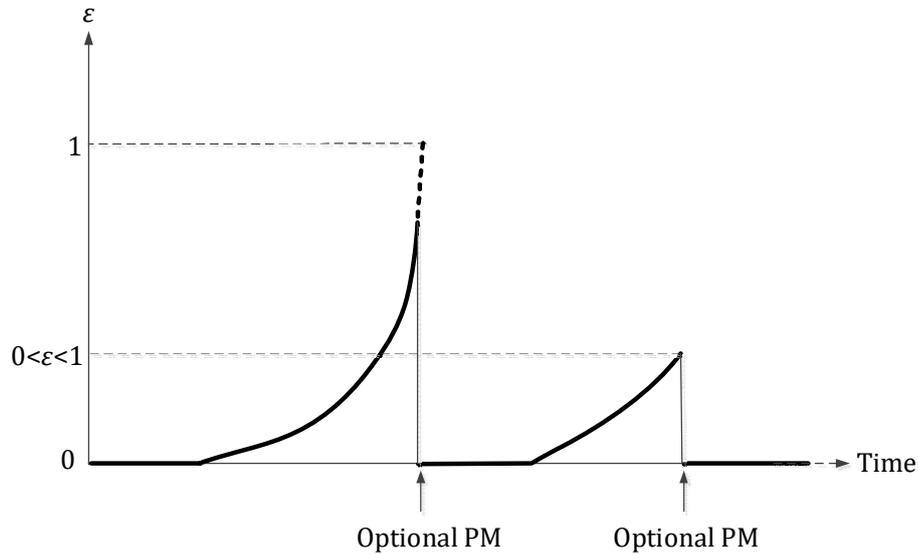


Figure 3.1. Impact of the PM on the value of ε

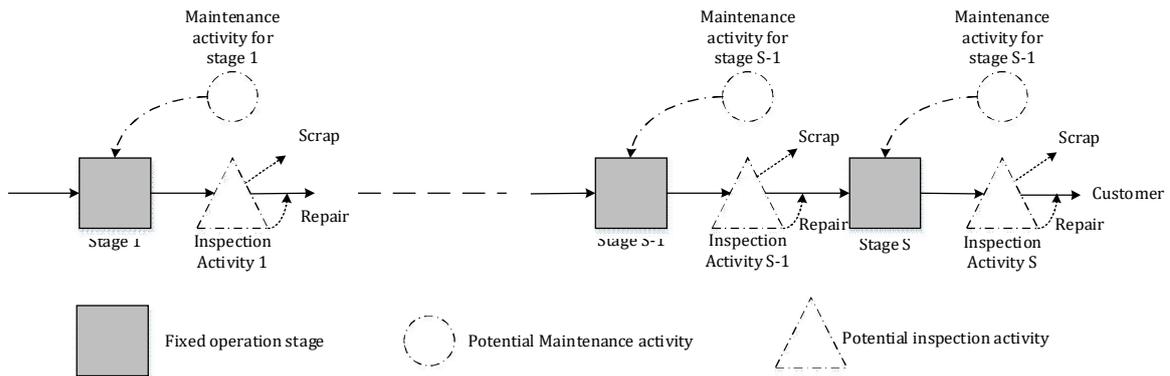


Figure 3.2. Schematic plan of the considered problem in the sample period t

3.1.1. Assumptions

The main assumptions of the considered main problem are as follows:

- Part defects are created only at the manufacturing stages.
- Each stage is deteriorating in time and the probability of defect production is increasing.

Chapter 3: Mathematical Formulation

- The values of Type-I and Type-II errors are certain constant quantities during the planning horizon.
- The repair function is done perfectly and transforms the rejected items to the conforming ones.

3.1.2. Indices

- j index of production stages ($j = 1, \dots, n$)
 t index of time periods ($t = 1, \dots, T$)
 i index of passed periods since the last performed PM ($i = 1, \dots, T$)

3.1.3. Parameters

- n number of production stages in the SMSPL
 de demand amount for conforming products in each period
 $w_{t,0,1}$ number of unit material that enters the SMSPL in period t
 ε_{t0} non-conforming fraction of material that enters the SMSPL in period t
 ε_{ij} probability of an workpiece acquires a defect during doing an operation in stage j where i period passed since the last performed PM
 α_j probability of Type-I error for the inspection activity j
 β_j probability of Type-II error for the inspection activity j
 f_{j1} fraction of the rejected items repaired after the inspection activity j
 f_{j2} fraction of the rejected items scrapped after the inspection activity j
 p_j unit production cost in stage j
 ic_j unit inspection cost of the inspection activity j
 rc_{j1} unit repair cost of a conforming item rejected by the inspection activity j
 rc_{j2} unit repair cost of a non-conforming item rejected by the inspection activity j
 sc_j scrap cost per rejected unit in stage j
 mc_j PM cost for the stage j
 pc penalty cost of delivering a non-conforming item to customer(s)
 y_i auxiliary parameter that its value is equal to the number of its subscript (i.e., i)

3.1.4. Variables

- d_{tj} 1, if the inspection activity associated to the stage j is performed in period t and 0, otherwise

Chapter 3: Mathematical Formulation

m_{tj}	1, if a PM activity corresponding to the stage j is done in the beginning of the period t and 0, otherwise
ω_{tj}	number of time periods have been passed since the last PM activity on the manufacturing stage j in period t
w_{tj1}	expected conforming items entering the stage j in period t
w_{tj2}	expected non-conforming items entering the stage j in period t
u_{tj1}	$= \alpha_j \left(1 - \sum_{i=\omega^{LO}}^{\omega^{UP}} x_{itj} \varepsilon_{ij} \right) w_{tj1} d_{tj}$; expected conforming items rejected by the j -th inspection activity opportunity in period t
u_{tj2}	$= (1 - \beta_j) \left(\sum_{i=1}^I x_{itj} \varepsilon_{ij} w_{tj1} + w_{tj2} \right) d_{tj}$; expected non-conforming items rejected by the j -th inspection activity opportunity in period t
x_{itj}	auxiliary binary variable to activate appropriate ε_{ij} parameter in the model
P	total production cost of the SMSPL
IC	total inspection cost of the SMSPL
RC	total repair cost of the SMSPL
MC	total PM cost of the SMSPL
SC	total scrap cost of the SMSPL
PC	total penalty cost of the SMSPL for delivering non-conforming items to customer(s)

3.1.5. Mixed-integer linear mathematical programming model

Based on the above-mentioned definitions, mixed-integer linear mathematical formulation of the problem is provided as follows:

Model 1:

$$\text{Min } z = RC + SC + P + IC + MC + PC \quad (3.1)$$

s.t.

$$RC = \sum_{t=1}^T \sum_{j=1}^n f_{j1} rc_{j1} u_{tj1} + \sum_{t=1}^T \sum_{j=1}^n f_{j1} rc_{j2} u_{tj2} \quad (3.2)$$

$$SC = \sum_{t=1}^T \sum_{j=1}^n f_{j2} sc_j u_{tj1} + \sum_{t=1}^T \sum_{j=1}^n f_{j2} sc_j u_{tj2} \quad (3.3)$$

$$P = \sum_{t=1}^T \sum_{j=1}^n p_j (w_{tj1} + w_{tj2}) \quad (3.4)$$

$$IC = \sum_{t=1}^T \sum_{j=1}^n ic_j(w_{tj1} + w_{tj2})d_{tj} \quad (3.5)$$

$$MC = \sum_{t=1}^T \sum_{j=1}^n mc_j \times md_{tj} \quad (3.6)$$

$$PC = \sum_{t=1}^T pc \times w_{t,n+1,2} \quad (3.7)$$

$$w_{t,1,1} = (1 - \varepsilon_{t0})w_{t,0,1} \quad \forall t \quad (3.8)$$

$$w_{t,1,2} = \varepsilon_{t0}w_{t01} \quad \forall t \quad (3.9)$$

$$\begin{aligned} w_{t,j+1,1} = & \left(1 - \sum_{i=1}^I x_{itj}\varepsilon_{ij}\right)w_{tj1} \\ & + \left(f_{j1} \left(\alpha_j \left(1 - \sum_{i=1}^I x_{itj}\varepsilon_{ij}\right) + (1 - \beta_j) \left(\sum_{i=1}^I x_{itj}\varepsilon_{ij}\right)\right) \right. \\ & \left. - \alpha_j \left(1 - \left(\sum_{i=1}^I x_{itj}\varepsilon_{ij}\right)\right)\right)w_{t,j,1}d_{tj} + f_{j1}(1 - \beta_j)w_{tj2}d_{tj} \quad \forall t, j \\ & \neq n \end{aligned} \quad (3.10)$$

$$\begin{aligned} w_{t,n+1,1} = & \left(1 - \left(\sum_{i=1}^I x_{itn}\varepsilon_{in}\right)\right)w_{tn1} \\ & + \left(f_{n1} \left(\alpha_n \left(1 - \left(\sum_{i=1}^I x_{itn}\varepsilon_{in}\right)\right) + (1 - \beta_n) \left(\sum_{i=1}^I x_{itn}\varepsilon_{in}\right)\right) \right. \\ & \left. - \alpha_n \left(1 - \left(\sum_{i=1}^I x_{itn}\varepsilon_{in}\right)\right)\right)w_{tn1}d_{tn} + f_{n1}(1 - \beta_n)w_{tn2}d_{tn} \quad \forall t \end{aligned} \quad (3.11)$$

$$\begin{aligned} w_{t,j+1,2} = & w_{tj2} + \left(\sum_{i=1}^I x_{itj}\varepsilon_{ij}\right)w_{tj1} - (1 - \beta_j) \left(\sum_{i=1}^I x_{itj}\varepsilon_{ij}\right)w_{tj1}d_{tj} \\ & - (1 - \beta_j)w_{tj2}d_{tj} \quad \forall t, j \end{aligned} \quad (3.12)$$

$$\omega_{tj} = (\omega_{t-1,j} + 1)(1 - md_{tj}) \quad \forall j, t \neq 1 \quad (3.13)$$

$$\omega_{1,j} = 0 \quad \forall j \quad (3.14)$$

$$md_{1j} = 1 \quad \forall j \quad (3.15)$$

$$\omega_{tj} = \sum_{i=1}^T x_{tji} y_i \quad \forall t, j \quad (3.16)$$

$$\sum_{i=1}^T x_{tji} = 1 \quad \forall t, j \quad (3.17)$$

$$w_{t,n+1,1} \geq de \quad \forall t \quad (3.18)$$

$$w_{tj1}, w_{tj2}, u_{tj1}, u_{tj2}, RC, IC, P, MC, SC, PC \geq 0 \quad \forall t, j \quad (3.19)$$

$$md_{tj}, d_{tj} : \text{binary decisions} \quad \forall t, j \quad x_{itj} : \text{Binary} \quad \forall t, j, i \quad \omega_{tj} : \text{Positive Integer} \quad \forall t, j \quad (3.20)$$

Objective function 3.1 minimizes the total cost including repair, scrap, production, inspection, PM, and penalty cost. Equations 3.2–3.7 calculate the different cost components. Equations 3.8–3.12 obtain the expected number of conforming and non-conforming parts entering the different manufacturing stages in each period. Equation 3.13 obtains the number of passed time periods after the last implemented PM activity. It is assumed that a PM activity is performed for all the manufacturing stages in the first period (please see Equations 3.14 and 3.15). Equations 3.16 and 3.17 activate the corresponding auxiliary binary variable x_{itj} to ε_{ij} and accordingly the term $(\sum_{i=1}^T x_{itj} \varepsilon_{ij})$ in each period t take the value of corresponding ε_{ij} regarding the number of passed period since the last performed PM. Constraint 3.18 imposes the SMSPL to satisfy the demand of customer(s) in each period. Equation 3.19 is a non-negativity restriction, and Equations 3.20 shows that performing an inspection activity and doing a PM are binary decisions and the x_{itj} are auxiliary binary variables and the ω_{tj} are integer positive variables.

Model 1 is a single objective mixed-integer non-linear programming model. We attempt to linearize **Model 1** through some conventional operations research techniques. To this aim, new variables and constraints are added to **Model 1** for the linearization purpose (see Table 3.1).

By doing the afore-mentioned linearization operation, **Model 1** is transformed to the linear **Model 2** as follows:

Model 2:

$$\text{Min } z_1 = RC + SC + P + IC + MC + PC \quad (3.21)$$

s.t.

$$\begin{aligned}
 RC = & \sum_{t=1}^T \sum_{j=1}^n f_{j1}rc_{j1}\alpha_j wd_{tj1} - f_{j1}rc_{j1}\alpha_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) \\
 & + \sum_{t=1}^T \sum_{j=1}^n f_{j1}rc_{j2} \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) + f_{j1}rc_{j2}wd_{tj2} \\
 & - f_{j1}rc_{j2}\beta_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) - f_{j1}rc_{j2}\beta_j wd_{tj2}
 \end{aligned} \tag{3.22}$$

$$\begin{aligned}
 SC = & \sum_{t=1}^T \sum_{j=1}^n f_{j2}sc_j\alpha_j wd_{tj1} - f_{j2}sc_j\alpha_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) \\
 & + \sum_{t=1}^T \sum_{j=1}^n f_{j2}sc_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) + f_{j2}sc_j wd_{tj2} \\
 & - f_{j2}sc_j\beta_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) - f_{j2}sc_j\beta_j wd_{tj2}
 \end{aligned} \tag{3.23}$$

$$PC = \sum_{t=1}^T \sum_{j=1}^n p_j(w_{tj1} + w_{tj2}) \tag{3.24}$$

$$IC = \sum_{t=1}^T \sum_{j=1}^n ic_j(wd_{tj1} + wd_{tj2}) \tag{3.25}$$

$$MC = \sum_{t=1}^T \sum_{j=1}^n mc_j \times md_{tj} \tag{3.26}$$

$$PC = \sum_{t=1}^T pc \times w_{t,n+1,2} \tag{3.27}$$

$$w_{t,1,1} = (1 - \varepsilon_{t0})w_{t01} \quad \forall t \tag{3.28}$$

$$\begin{aligned}
 w_{t,j+1,1} = & w_{tj1} - \left(\sum_{i=1}^T \varepsilon_{ij}xw_{tji1} \right) + f_{j1}\alpha_j wd_{tj1} - f_{j1}\alpha_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) \\
 & + f_{j1} \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) - f_{j1}\beta_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) - \alpha_j wd_{tj1} \\
 & - \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) + f_{j1}wd_{tj2} - f_{j1}\beta_j wd_{tj2} \quad \forall t, j \neq n
 \end{aligned} \tag{3.29}$$

$$\begin{aligned}
 w_{t,n+1,1} = & w_{tn1} - \left(\sum_{i=1}^T \varepsilon_{in}xw_{tni1} \right) + f_{n1}\alpha_n wd_{tn1} - f_{n1}\alpha_n \left(\sum_{i=1}^T \varepsilon_{in}xwd_{tni1} \right) \\
 & + f_{n1} \left(\sum_{i=1}^T \varepsilon_{in}xwd_{tni1} \right) - f_{n1}\beta_n \left(\sum_{i=1}^T \varepsilon_{in}xwd_{tni1} \right) - \alpha_n wd_{tn1} \\
 & - \alpha_n \left(\sum_{i=1}^T \varepsilon_{in}xwd_{tni1} \right) + f_{n1}wd_{tn2} - f_{n1}\beta_n wd_{tn2} \quad \forall t
 \end{aligned} \tag{3.30}$$

$$\begin{aligned}
 w_{t,j+1,2} = & w_{tj2} + \left(\sum_{i=1}^T \varepsilon_{in}xw_{tji1} \right) - \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) + \beta_j \left(\sum_{i=1}^T \varepsilon_{ij}xwd_{tji1} \right) \\
 & - wd_{tj2} + \beta_j wd_{tj2} \quad \forall t, j
 \end{aligned} \tag{3.31}$$

$$\omega_{tj} = \omega_{t-1,j} + 1 - \omega md_{t-1,t,j} + md_{tj} \quad \forall j, t \neq 1 \tag{3.32}$$

$$\omega_{1,j} = 0 \quad \forall j \tag{3.33}$$

$$md_{1j} = 1 \quad \forall j \tag{3.34}$$

$$\omega_{tj} = \sum_{i=1}^T x_{tji} y_i \quad \forall t, j \quad (3.35)$$

$$\sum_{i=1}^T x_{tji} = 1 \quad \forall t, j \quad (3.36)$$

$$w_{t,n+1,1} \geq de \quad \forall t \quad (3.37)$$

$$wd_{tj1} \leq M \times d_{tj} \quad \forall t, j \quad (3.38)$$

$$wd_{tj1} \leq w_{tj1} \quad \forall t, j \quad (3.39)$$

$$wd_{tj1} \geq w_{tj1} - (1 - d_{tj}) \times M \quad \forall t, j \quad (3.40)$$

$$wd_{tj2} \leq M \times d_{tj} \quad \forall t, j \quad (3.41)$$

$$wd_{tj2} \leq w_{tj2} \quad \forall t, j \quad (3.42)$$

$$wd_{tj2} \geq w_{tj2} - (1 - d_{tj}) \times M \quad \forall t, j \quad (3.43)$$

$$\omega_{tj}^{LO} \times md_{tj} \leq \omega md_{t-1,t,j} \leq \omega_{tj}^{UP} \times md_{tj} \quad \forall j, t \neq 1 \quad (3.44)$$

$$\omega_{t-1,j} - \omega_{tj}^{UP} \times (1 - md_{tj}) \leq \omega md_{t-1,t,j} \leq \omega_{t-1,j} - \omega_{tj}^{LO} \times (1 - md_{tj}) \quad \forall j, t \neq 1 \quad (3.45)$$

$$wd_{itj1} \leq M \times x_{itj} \quad \forall t, j, i \quad (3.46)$$

$$xwd_{itj1} \leq wd_{tj1} \quad \forall t, j, i \quad (3.47)$$

$$xwd_{itj1} \geq wd_{tj1} - (1 - x_{itj}) \times M \quad \forall t, j, i \quad (3.48)$$

$$xwd_{itj2} \leq M \times x_{itj} \quad \forall t, j, i \quad (3.49)$$

$$xwd_{itj2} \leq wd_{tj2} \quad \forall t, j, i \quad (3.50)$$

$$xwd_{itj2} \geq wd_{tj2} - (1 - x_{itj}) \times M \quad \forall t, j, i \quad (3.51)$$

$$xw_{itj1} \leq M \times x_{itj} \quad \forall t, j, i \quad (3.52)$$

$$xw_{itj1} \leq w_{tj1} \quad \forall t, j, i \quad (3.53)$$

$$xw_{itj1} \geq w_{tj1} - (1 - x_{itj}) \times M \quad \forall t, j, i \quad (3.54)$$

$$w_{tj1}, w_{tj2}, u_{tj1}, u_{tj2}, RC, IC, P, MC, SC, PC, wd_{tj1}, wd_{tj2}, xwd_{itj1}, xwd_{itj2}, xw_{itj1} \geq 0 \quad \forall t, j, i \quad \omega md_{t-1,t,j} \geq 0 \quad \forall j, t \neq 1 \quad (3.55)$$

$$md_{tj}, d_{tj} : \text{binary decisions } \forall t, j \quad x_{itj} : \text{Binary } \forall t, j, i \quad \omega_{tj} : \text{Positive Integer } \forall t, j \quad (3.56)$$

Model 2 is a single objective mixed-integer linear programming model. However, most of the real-world problems have a multi-objective nature. In this regard, for the real applications of the SMSPLs, companies seek not only a cost-efficient line, but also a productive one. On the other side, the cost and demand fluctuations in the today's market cause many difficulties for companies because of their impacts on the expected costs and accordingly on the optimum planning. Hence, obtaining a robust planning that is immunized against the cost and demand fluctuations seems necessary in this dynamic environment. Next section extends the

Chapter 3: Mathematical Formulation

main problem regarding multi-objectivity nature and uncertainty about cost components and demand amount.

Table 3.1. Introduction of new variables and constraints for the linearization purpose

		New variables	
		$wd_{tj1} = w_{tj1} \times d_{tj}$	$wd_{tj2} = w_{tj2} \times d_{tj}$
New constraints	$wd_{tj1} \leq M \times d_{tj}$		$wd_{tj2} \leq M \times d_{tj}$
	$wd_{tj1} \leq w_{tj1}$		$wd_{tj2} \leq w_{tj2}$
	$wd_{tj1} \geq w_{tj1} - (1 - d_{tj}) \times M$		$wd_{tj2} \geq w_{tj2} - (1 - d_{tj}) \times M$
	$wd_{tj1} \geq 0$		$wd_{tj2} \geq 0$
		New variables	
		$xw_{itj1} = x_{itj} \times w_{tj1}$	$xwd_{itj1} = x_{itj} \times wd_{tj1}$
New constraints	$xw_{itj1} \leq M \times x_{itj}$		$xwd_{itj1} \leq M \times x_{itj}$
	$xw_{itj1} \leq w_{tj1}$		$xwd_{itj1} \leq wd_{tj1}$
	$xw_{itj1} \geq w_{tj1} - (1 - x_{itj}) \times M$		$xwd_{itj1} \geq wd_{tj1} - (1 - x_{itj}) \times M$
	$xw_{itj1} \geq 0$		$xwd_{itj1} \geq 0$
		New variables	
		$xwd_{itj2} = x_{itj} \times wd_{tj2}$	$\omega md_{t-1,t,j} = \omega_{t-1,j} \times md_{tj}$
New constraints	$xwd_{itj2} \leq M \times x_{itj}$		$\omega_{tj}^{LO} \times md_{tj} \leq \omega md_{t-1,t,j}$
	$xwd_{itj2} \leq wd_{tj2}$		$\leq \omega_{tj}^{UP} \times md_{tj}$
	$xwd_{itj2} \geq wd_{tj2} - (1 - x_{itj}) \times M$		$\omega_{t-1,j} - \omega_{tj}^{UP} \times (1 - md_{tj})$
	$xwd_{itj2} \geq 0$		$\leq \omega md_{t-1,t,j}$
			$\leq \omega_{t-1,j} - \omega_{tj}^{LO}$
			$\times (1 - md_{tj})$
		$\omega md_{t-1,t,j} \geq 0$	

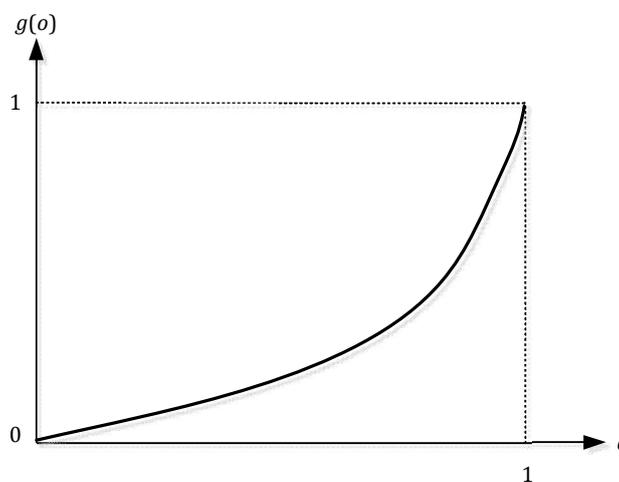
3.2. Extended problem description

Consider a Serial n -Stage Manufacturing System (SnSMS) which produces a single type product. Raw material enters this discrete manufacturing process and each stage realizes a specific QC on the product. However, the stages are not technologically capable to process all the workpieces in a perfect quality. Hence, an item incurs a defect after each stage with a certain probability. This probability is increasing in time (i.e., the manufacturing stages are deteriorating) and its value depends on the number of periods which has passed since the last performed PM. Therefore, in the beginning of each period, there is an opportunity to do a PM in an *ambiguous cost*. On the other side, after processing from each stage, to prevent this defects propagation and defects delivery to end-customer(s), the system can perform

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an inspection activity in an *imprecise cost*. Each inspection is capable to detect the defect incurred in the preceding manufacturing stage. In this way, the defects propagation and accordingly material and energy wastage would be decreased. It should be mentioned that Error types I and II exist for the inspection activities and a certain fraction of the rejected items can be repaired, and the rest should be scrapped while imposing *ambiguous expenses* to the system.

The *SnSMS* needs a simultaneous plan determining the right time and place for the part quality inspection and PM activities to produce conforming items during a planning horizon (consisting of equal time periods) while minimizing total cost including production, inspection, repair, scrap, PM, penalty of defect delivery and maximizing system productivity. The cost components are not precisely known and are represented by fuzzy rather than crisp values. To measure system productivity, this thesis considers a utility function of the fraction of the produced conforming products, $g(o)$, where o is the proportion of the produced conforming products to the input workpieces. This function reflects that the system productivity and accordingly customer satisfaction is greater when it delivers more conforming products (i.e., the o proportion is closer to 1) (see [Figure 3.3](#)). This behavior is implied by the sales experts in the considered case study in [Chapter 5](#) and we believe that this kind of behavior is common in the most of manufacturing industries as this non-linear reaction seems rational when customer receives a lot of products.



[Figure 3.3](#). Shape of the utility function for the fraction of the produced conforming products

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Below, the assumptions of the extended problem are provided. Then, the required notations are defined. Finally, a bi-objective mixed-integer non-Linear programming model is developed for the above-mentioned extended problem.

3.2.1. Assumptions

The principle assumptions of the considered extended problem are provided as follows:

- System productivity is a nonlinear function of produced conforming items.
- The cost components and demand amount of the considered system are uncertain parameters.
- Part defects are created only at the manufacturing stages.
- The values of Error type-I and Error type-II errors are known constant quantities during the planning horizon.
- The repair operation is performed perfectly and transforms the rejected workpieces to the conforming ones.

3.2.2. Indices

- j index of production stages ($j = 1, \dots, n$)
 t index of time periods in the planning horizon ($t = 1, \dots, T$)
 i index of passed periods since the last performed PM ($i = 1, \dots, T$)

3.2.3. Parameters

- n number of manufacturing stages in the $SnSMS$
 $w_{t,0,1}$ number of unit initial workpieces that enter the $SnSMS$ in the period t
 ε_{ij} probability of an workpiece acquires a defect during doing an operation in the stage j where i period passed since the last performed PM
 ε_{t0} non-conforming fraction of initial workpieces that enter the $SnSMS$ in the period t
 α_j probability of the Error type-I of the inspection activity j
 β_j probability of the Error type-II error of the inspection activity j
 f_{j1} fraction of the rejected workpieces repaired at the inspection activity j
 f_{j2} fraction of the rejected workpieces scrapped at the inspection activity j
 \tilde{p}_j unit production cost in the stage j
 \tilde{ic}_j unit inspection cost of the inspection activity j

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$\widetilde{r}c_{j1}$	unit repair cost of a conforming workpiece rejected by the inspection activity j
$\widetilde{r}c_{j2}$	unit repair cost of a non-conforming workpiece rejected by the inspection activity j
\widetilde{sc}_j	scrap cost per rejected unit in the stage j
\widetilde{mc}_j	PM cost for the stage j
\widetilde{pc}	penalty cost of delivering a non-conforming product to customer(s)
y_i	auxiliary parameter that its value is equal to the number of its subscript (i.e., i)

3.2.4. Variables

d_{tj}	1; if inspection activity associated to the stage j is performed in the period t , 0; otherwise
m_{tj}	1; if a PM activity corresponding to the stage j is done in the beginning of the period t , 0; otherwise
ω_{tj}	number of time periods have been passed since the last PM activity on the manufacturing stage j in the time period t
w_{tj1}	expected conforming items entering the stage j in the period t
w_{tj2}	expected non-conforming items entering the stage j in the period t
u_{tj1}	$= \alpha_j \left(1 - \sum_{i=\omega}^{\omega^{UP}} x_{itj} \varepsilon_{ij} \right) w_{tj1} d_{tj}$; expected conforming items rejected by the j -th inspection activity opportunity in the period t
u_{tj2}	$= (1 - \beta_j) \left(\sum_{i=1}^I x_{itj} \varepsilon_{ij} w_{tj1} + w_{tj2} \right) d_{tj}$; expected non-conforming items rejected by the j -th inspection activity opportunity in the period t
x_{itj}	auxiliary binary variable to activate appropriate ε_{ij} parameter in the model
\widetilde{P}	total production cost of the SnSMS
\widetilde{IC}	total inspection cost of the SnSMS
\widetilde{RC}	total repair cost of the SnSMS
\widetilde{MC}	total PM cost of the SnSMS
\widetilde{SC}	total scrap cost of the SnSMS
\widetilde{PC}	total penalty cost of the SnSMS for delivering non-conforming items to customer(s)

3.2.5. Bi-objective mixed-integer non-linear mathematical model

Regarding the above-mentioned annotation, a bi-objective, mixed-integer, and non-

linear mathematical formulation of the problem is as follows:

Model 3:

$$\text{Max } z_1 = \sum_{t=1}^T f \left(w_{t,n+1,1} / w_{t,0,1} \right) \quad (3.57)$$

$$\text{Min } z_2 = \widetilde{RC} + \widetilde{SC} + \widetilde{P} + \widetilde{IC} + \widetilde{MC} + \widetilde{PC} \quad (3.58)$$

s.t.

$$\begin{aligned} \widetilde{RC} = & \sum_{t=1}^T \sum_{j=1}^n f_{j1} r \widetilde{c}_{j1} \alpha_j w_{tj1} - f_{j1} r \widetilde{c}_{j1} \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) \\ & + \sum_{t=1}^T \sum_{j=1}^n f_{j1} r \widetilde{c}_{j2} \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) + f_{j1} r \widetilde{c}_{j2} w_{tj2} \\ & - f_{j1} r \widetilde{c}_{j2} \beta_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) - f_{j1} r \widetilde{c}_{j2} \beta_j w_{tj2} \end{aligned} \quad (3.59)$$

$$\begin{aligned} \widetilde{SC} = & \sum_{t=1}^T \sum_{j=1}^n f_{j2} \widetilde{s} c_j \alpha_j w_{tj1} - f_{j2} \widetilde{s} c_j \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) \\ & + \sum_{t=1}^T \sum_{j=1}^n f_{j2} \widetilde{s} c_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) + f_{j2} \widetilde{s} c_j w_{tj2} \\ & - f_{j2} \widetilde{s} c_j \beta_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) - f_{j2} \widetilde{s} c_j \beta_j w_{tj2} \end{aligned} \quad (3.60)$$

$$\widetilde{PC} = \sum_{t=1}^T \sum_{j=1}^n \widetilde{p}_j (w_{tj1} + w_{tj2}) \quad (3.61)$$

$$\widetilde{IC} = \sum_{t=1}^T \sum_{j=1}^n \widetilde{i} c_j (w_{tj1} + w_{tj2}) \quad (3.62)$$

$$\widetilde{MC} = \sum_{t=1}^T \sum_{j=1}^n \widetilde{m} c_j \times m d_{tj} \quad (3.63)$$

$$\widetilde{PC} = \sum_{t=1}^T \widetilde{p} c \times w_{t,n+1,2} \quad (3.64)$$

$$w_{t,1,1} = (1 - \varepsilon_{t0}) w_{t,0,1} \quad \forall t \quad (3.65)$$

$$\begin{aligned} w_{t,j+1,1} = & w_{tj1} - \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) + f_{j1} \alpha_j w_{tj1} - f_{j1} \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) \\ & + f_{j1} \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) - f_{j1} \beta_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) - \alpha_j w_{tj1} \\ & - \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) + f_{j1} w_{tj2} - f_{j1} \beta_j w_{tj2} \quad \forall t, j \neq n \end{aligned} \quad (3.66)$$

$$\begin{aligned} w_{t,n+1,1} = & w_{tn1} - \left(\sum_{i=1}^T \varepsilon_{in} x w_{tni1} \right) + f_{n1} \alpha_n w_{tn1} - f_{n1} \alpha_n \left(\sum_{i=1}^T \varepsilon_{in} x w_{tni1} \right) \\ & + f_{n1} \left(\sum_{i=1}^T \varepsilon_{in} x w_{tni1} \right) - f_{n1} \beta_n \left(\sum_{i=1}^T \varepsilon_{in} x w_{tni1} \right) - \alpha_n w_{tn1} \\ & - \alpha_n \left(\sum_{i=1}^T \varepsilon_{in} x w_{tni1} \right) + f_{n1} w_{tn2} - f_{n1} \beta_n w_{tn2} \quad \forall t \end{aligned} \quad (3.67)$$

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$$w_{t,j+1,2} = w_{tj2} + \left(\sum_{i=1}^T \varepsilon_{in} x w_{tji1} \right) - \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) + \beta_j \left(\sum_{i=1}^T \varepsilon_{ij} x w_{tji1} \right) - w_{tj2} + \beta_j w_{tj2} \quad \forall t, j \quad (3.68)$$

$$\omega_{tj} = \omega_{t-1,j} + 1 - \omega m d_{t-1,t,j} + m d_{tj} \quad \forall j, t \neq 1 \quad (3.69)$$

$$\omega_{1,j} = 0 \quad \forall j \quad (3.70)$$

$$m d_{1j} = 1 \quad \forall j \quad (3.71)$$

$$\omega_{tj} = \sum_{i=1}^T x_{tji} y_i \quad \forall t, j \quad (3.72)$$

$$\sum_{i=1}^T x_{tji} = 1 \quad \forall t, j \quad (3.73)$$

$$w_{t,n+1,1} \geq \tilde{d}e \quad \forall t \quad (3.74)$$

$$w_{tj1} \leq M \times d_{tj} \quad \forall t, j \quad (3.75)$$

$$w_{tj1} \leq w_{tj1} \quad \forall t, j \quad (3.76)$$

$$w_{tj1} \geq w_{tj1} - (1 - d_{tj}) \times M \quad \forall t, j \quad (3.77)$$

$$w_{tj2} \leq M \times d_{tj} \quad \forall t, j \quad (3.78)$$

$$w_{tj2} \leq w_{tj2} \quad \forall t, j \quad (3.79)$$

$$w_{tj2} \geq w_{tj2} - (1 - d_{tj}) \times M \quad \forall t, j \quad (3.80)$$

$$\omega_{tj}^{LO} \times m d_{tj} \leq \omega m d_{t-1,t,j} \leq \omega_{tj}^{UP} \times m d_{tj} \quad \forall j, t \neq 1 \quad (3.81)$$

$$\omega_{t-1,j} - \omega_{tj}^{UP} \times (1 - m d_{tj}) \leq \omega m d_{t-1,t,j} \leq \omega_{t-1,j} - \omega_{tj}^{LO} \times (1 - m d_{tj}) \quad \forall j, t \neq 1 \quad (3.82)$$

$$w_{ditj1} \leq M \times x_{itj} \quad \forall t, j, i \quad (3.83)$$

$$x w_{ditj1} \leq w_{ditj1} \quad \forall t, j, i \quad (3.84)$$

$$x w_{ditj1} \geq w_{ditj1} - (1 - x_{itj}) \times M \quad \forall t, j, i \quad (3.85)$$

$$x w_{ditj2} \leq M \times x_{itj} \quad \forall t, j, i \quad (3.86)$$

$$x w_{ditj2} \leq w_{ditj2} \quad \forall t, j, i \quad (3.87)$$

$$x w_{ditj2} \geq w_{ditj2} - (1 - x_{itj}) \times M \quad \forall t, j, i \quad (3.88)$$

$$x w_{itj1} \leq M \times x_{itj} \quad \forall t, j, i \quad (3.89)$$

$$x w_{itj1} \leq w_{itj1} \quad \forall t, j, i \quad (3.90)$$

$$x w_{itj1} \geq w_{itj1} - (1 - x_{itj}) \times M \quad \forall t, j, i \quad (3.91)$$

$$w_{tj1}, w_{tj2}, u_{tj1}, u_{tj2}, RC, IC, P, MC, SC, PC, w_{ditj1}, w_{ditj2}, x w_{ditj1}, x w_{ditj2}, x w_{itj1} \geq 0 \quad \forall t, j, i \quad \omega m d_{t-1,t,j} \geq 0 \quad \forall j, t \neq 1 \quad (3.92)$$

$$m d_{tj}, d_{tj} : \text{binary decisions } \forall t, j \quad x_{itj} : \text{Binary } \forall t, j, i \quad \omega_{tj} : \text{Positive Integer } \forall t, j \quad (3.93)$$

Objective function 3.57 maximizes the system productivity through maximizing the utility function of the produced conforming products to the input

workpieces (see [Figure 3.3](#)). Hereafter this proportion is called as o_t variable. The rest of equations have been described in the [Model 1](#) and [Model 2](#).

3.3. Conclusion

In this chapter, first, a mixed-integer linear programming model (i.e., [Model 2](#)) was proposed for the main problem of integrated planning of inspection and PM activities. Then, the main problem was extended regarding uncertain and multi-objective nature of the integrated planning. In this regard, a bi-objective mixed-integer non-linear programming model (i.e., [Model 3](#)) was developed to incorporate the system productivity objective and uncertain cost components and demand amount into the mathematical model. As can be seen [Model 3](#) is a non-linear programming because of considering a non-linear function for the system productivity (i.e. Objective function [3.57](#)). Furthermore, the uncertain cost components and demand amount are not handled by an uncertain optimization approach. Therefore, a single objective counterpart of the developed bi-objective model is required that also handles the uncertainties. Hence, the next chapter goes through these issues and applies appropriate techniques to deal with them.

Chapter 4

Methodology

4.0. Chapter purpose and outline

As can be seen in Chapter 3, the proposed Model 3 is a bi-objective mixed-integer non-linear programming model including some uncertain parameters. Figure 4.1 depicts the proposed 3-stage approach to deal with the non-linearity, uncertainty, and multi-objectivity of the presented Model 3. In the first stage, the non-linearity of the Model 3, which is because of considering a non-linear function for the system productivity, is handled through a piecewise linear approximation technique (please see Section 4.1). Section 4.2 describes the second stage which adopts the robust possibilistic programming for dealing with the uncertain parameters. Section 4.3 is dedicated to the third stage explaining the ϵ -constraint method for transforming the bi-objective model to a single objective counterpart. Finally, the concluding remarks of this chapter are collected in Section 4.4.

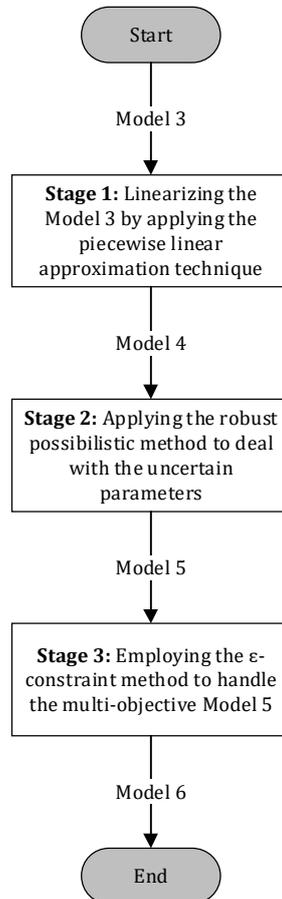


Figure 4.1. Flowchart of the proposed 3-stage approach

4.1. Stage 1: Piecewise linear approximation

After the performance of conventional linearization procedure in the Chapter III, Model 3 is still non-linear because of the first objective function. Generally, Non-linear models are not easy to solve and in the most cases are time-consuming. In the operations research problems, generally, researchers apply metaheuristic algorithms (e.g., GA) to obtain near optimal solution; however, in our case, this non-linearity can be handled through employing a piecewise linear approximation to the non-linear function which is shown for a four-segment approximation in Figure 4.2. Applying this method decreases the solution complexity and commercial software like GAMS can be used easily, which is an advantage for practitioners. For our case the non-linear behavior (Figure 3.3) is divided into four straight line portions according to the experts' opinion and it is formulated as Equation 4.1:

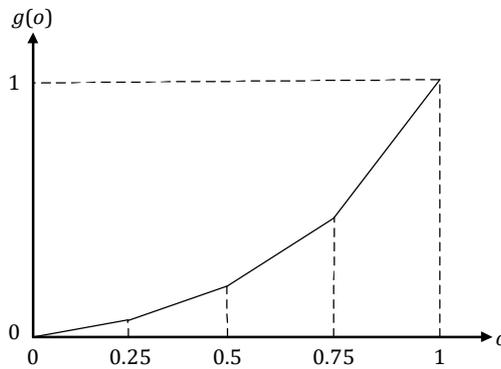


Figure 4.2. Piecewise linear approximation of $f(o)$

$$g = g(o) = \begin{cases} \frac{4o}{13} & o < 0.25 \\ \frac{8o - 1}{13} & 0.25 \leq o < 0.5 \\ \frac{16o - 5}{13} & 0.5 \leq o < 0.75 \\ \frac{24o - 11}{13} & o \geq 0.75 \end{cases} \quad (4.1)$$

The aim is to eliminate the non-linear term $f(o_t)$ from the Model 3. This can be performed by replacing it through the single linear term ϑ_t . Now, it is possible to relate ϑ_t to o_t by the following relationships.

$$o_t - 0 \times \lambda_1 + \frac{1}{4} \times \lambda_2 + \frac{1}{2} \times \lambda_3 + \frac{3}{4} \times \lambda_4 + 1 \times \lambda_5 = 0 \quad \forall t \quad (4.2)$$

$$\vartheta_t - 0 \times \lambda_1 + \frac{1}{13} \times \lambda_2 + \frac{3}{13} \times \lambda_3 + \frac{7}{13} \times \lambda_4 + 1 \times \lambda_5 = 0 \quad \forall t \quad (4.3)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 1 \quad \forall t \quad (4.4)$$

λv are new variables which are added into the [Model 3](#). They can be implied as ‘weights’ to be attached to the vertices of the curve in [Figure 4.2](#). In addition, it is necessary to consider another stipulation regarding λv .

$$\text{At most two adjacent } \lambda v \text{ can be non-zero} \quad (4.5)$$

Stipulation [4.5](#) can be done by introducing λv under SOS2 condition. At most two variables within the SOS2 can obtain on non-zero values. The two non-zero values must be for adjacent variables in that set. Hence, this type of variable definition is used in our case and coded in General Algebraic Modeling System (GAMS) software ([Rezaei-Malek et al., 2016](#)). Stipulation [4.5](#) guarantees that corresponding values of ϑ_t and o_t lie on one of the straight-line segments. For example, if $\lambda_2 = 0.5$ and $\lambda_3 = 0.5$ (i.e., other λv are zero), we can get $o_t = 0.375$ and $\vartheta_t = 0.154$. Apparently, ignoring Stipulation [4.5](#) will incorrectly allow the possibility of values o_t and ϑ_t off the piecewise straight lines. For further information about the piecewise approximation approach, the enthusiastic readers can refer to ([Williams, 2013; p. 177–182](#)).

By doing the afore-mentioned linearization operation, [Model 3](#) is transformed to the linear [Model 4](#) as follows:

Model 4:

$$\text{Max } z_1 = \sum_{t=1}^T \vartheta_t \quad (4.6)$$

s.t.

$$\lambda v: \text{ Positive and SOS2} \quad v \in \{1, \dots, 5\} \quad (4.7)$$

With Equations [\(3.58\)](#), [\(3.59\)–\(3.93\)](#) and [\(4.2\)–\(4.4\)](#).

4.2. Stage 2: Robust possibilistic programming model

As mentioned in the [Chapter II](#), the uncertainty of PQIP problem inherently refers to the internal and external sources. The uncertain external condition such as cost and demand fluctuations are regarded in the current thesis. Indeed, the cost components and demand amount in the PQIP problem tainted by high degree of uncertainty in the real-life situation because of dynamic nature of market and the tactical horizon of planning decisions. Therefore, neglecting the uncertainty in the planning of manufacturing system may impose high risks to the company. The risks such as considerable amount of back order and misestimated manufacturing cost.

To deal with these uncertain parameters which are based on the subjective opinion of the decision-maker and to obtain a robust solution, this thesis employs the Robust Possibilistic Approach (RPP), which has been developed by [Pishvae et al. \(2012\)](#).

Fuzzy mathematical programming can be classified into two major categories ([Zahiri et al., 2014](#)): possibilistic programming and flexible programming. Possibilistic programming handle ambiguous coefficients of objective functions and constraints that are generally modeled considering available objective data and subjective knowledge of the decision-maker. However, the latter (i.e., flexible programming) is applied to handle flexible target value of goals and limitations (i.e., fuzziness in the form of vague/unsharp boundaries) ([Zahiri et al., 2014](#)). With the above-mentioned definitions, [Model 4](#) belongs to the possibilistic programming category.

[Pishvae et al. \(2012\)](#) extended the theory of robust programming into the possibilistic programming framework to be benefited from the advantages and capabilities of both possibilistic programming and the concept of robustness. In comparison with the basic possibilistic chance constrained programming, the RPP approach avoids subjective judgment about the best value of chance constraints' confidence levels (i.e., θ) as well as ensuring to find the global optimum value for them. This advantage is more valuable when the number of chance constraints increases and there is no need to apply complex and time-consuming processes such as simulation experiments to find the optimum value for confidence levels.

Generally, the RPP model seeks for a reasonable trade-off between: (1) average performance, (2) optimality robustness and (3) feasibility robustness.

Chapter 4: Methodology

Several versions of the RPP approach was introduced by Pishvae et al. (2012). Because in our application it is desirable for decision-makers to obtain a lower total cost when compared to the expected optimal value in any realization. So, in these cases the Robust Possibilistic Programming-II (RPP-II) version is applied. For more information about the other version of the RPP programming approach, the enthusiastic readers can refer to Pishvae et al. (2012). Here, according to the available data type in the considered case study in the Chapter 5, we adopt trapezoidal possibility distributions (see Figure 4.3) for modeling imprecise parameters that can be defined by their four prominent points, e.g., $\tilde{\xi} = (\xi^{(1)}, \xi^{(2)}, \xi^{(3)}, \xi^{(4)})$.

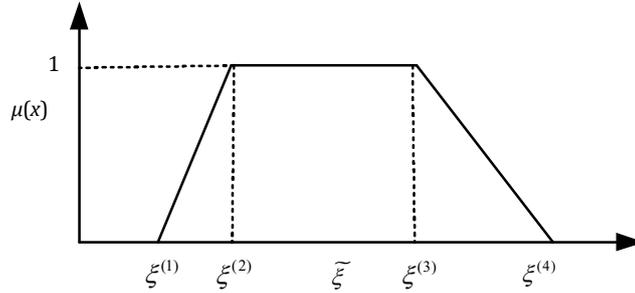


Figure 4.3. Trapezoidal possibility distribution of fuzzy parameter $\tilde{\xi}$

Model 5:

This model is completely same as Model 4 except the second objective function (i.e., Equation 3.58) and Constraint 3.74 which are changed based on the RPP-II as follows:

$$\text{Min } E[z_1] + \gamma(z_1^{\max} - E[z_1]) + \delta[de^{(4)} - (1 - \theta)de^{(3)} - \theta de^{(4)}] \quad (4.8)$$

s.t.

$$w_{t,n+1,1} \geq (1 - \theta)de^{(3)} + \theta de^{(4)} \quad \forall t \quad (4.9)$$

$$\text{Where :} \quad (4.10)$$

$$E[z_2] = E[RC] + E[SC] + E[P] + E[IC] + E[MC] + E[PC] \quad (4.11)$$

$$z_2^{\max} = RC^{\max} + SC^{\max} + P^{\max} + IC^{\max} + MC^{\max} + PC^{\max} \quad (4.12)$$

$$\begin{aligned}
 E[RC] = & \sum_{t=1}^T \sum_{j=1}^n f_{j1} \left(\frac{rc_{j1}^{(1)} + rc_{j1}^{(2)} + rc_{j1}^{(3)} + rc_{j1}^{(4)}}{4} \right) \alpha_j w d_{tj1} \\
 & - f_{j1} \left(\frac{rc_{j1}^{(1)} + rc_{j1}^{(2)} + rc_{j1}^{(3)} + rc_{j1}^{(4)}}{4} \right) \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & + \sum_{t=1}^T \sum_{j=1}^n f_{j1} \left(\frac{rc_{j2}^{(1)} + rc_{j2}^{(2)} + rc_{j2}^{(3)} + rc_{j2}^{(4)}}{4} \right) \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & + f_{j1} \left(\frac{rc_{j2}^{(1)} + rc_{j2}^{(2)} + rc_{j2}^{(3)} + rc_{j2}^{(4)}}{4} \right) w d_{tj2} \\
 & - f_{j1} \left(\frac{rc_{j2}^{(1)} + rc_{j2}^{(2)} + rc_{j2}^{(3)} + rc_{j2}^{(4)}}{4} \right) \beta_j \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & - f_{j1} \left(\frac{rc_{j2}^{(1)} + rc_{j2}^{(2)} + rc_{j2}^{(3)} + rc_{j2}^{(4)}}{4} \right) \beta_j w d_{tj2}
 \end{aligned} \tag{4.13}$$

$$\begin{aligned}
 RC^{max} = & \sum_{t=1}^T \sum_{j=1}^n f_{j1} rc_{j1}^{(4)} \alpha_j w d_{tj1} - f_{j1} rc_{j1}^{(4)} \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & + \sum_{t=1}^T \sum_{j=1}^n f_{j1} rc_{j2}^{(4)} \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) + f_{j1} rc_{j2}^{(4)} w d_{tj2} \\
 & - f_{j1} rc_{j2}^{(4)} \beta_j \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) - f_{j1} rc_{j2}^{(4)} \beta_j w d_{tj2}
 \end{aligned} \tag{4.13}$$

$$\begin{aligned}
 E[SC] = & \sum_{t=1}^T \sum_{j=1}^n f_{j2} \left(\frac{sc_j^{(1)} + sc_j^{(2)} + sc_j^{(3)} + sc_j^{(4)}}{4} \right) \alpha_j w d_{tj1} \\
 & - f_{j2} \left(\frac{sc_j^{(1)} + sc_j^{(2)} + sc_j^{(3)} + sc_j^{(4)}}{4} \right) \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & + \sum_{t=1}^T \sum_{j=1}^n f_{j2} \left(\frac{sc_j^{(1)} + sc_j^{(2)} + sc_j^{(3)} + sc_j^{(4)}}{4} \right) \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & + f_{j2} \left(\frac{sc_j^{(1)} + sc_j^{(2)} + sc_j^{(3)} + sc_j^{(4)}}{4} \right) w d_{tj2} \\
 & - f_{j2} \left(\frac{sc_j^{(1)} + sc_j^{(2)} + sc_j^{(3)} + sc_j^{(4)}}{4} \right) \beta_j \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & - f_{j2} \left(\frac{sc_j^{(1)} + sc_j^{(2)} + sc_j^{(3)} + sc_j^{(4)}}{4} \right) \beta_j w d_{tj2}
 \end{aligned} \tag{4.14}$$

$$\begin{aligned}
 SC^{max} = & \sum_{t=1}^T \sum_{j=1}^n f_{j2} sc_j^{(4)} \alpha_j w d_{tj1} - f_{j2} sc_j^{(4)} \alpha_j \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & + \sum_{t=1}^T \sum_{j=1}^n f_{j2} sc_j^{(4)} \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & + f_{j2} \left(\frac{sc_j^{(1)} + sc_j^{(2)} + sc_j^{(3)} + sc_j^{(4)}}{4} \right) w d_{tj2} - f_{j2} sc_j^{(4)} \beta_j \left(\sum_{i=1}^T \varepsilon_{ij} x w d_{tji1} \right) \\
 & - f_{j2} sc_j^{(4)} \beta_j w d_{tj2}
 \end{aligned} \tag{4.15}$$

$$E[P] = \sum_{t=1}^T \sum_{j=1}^n \left(\frac{p_j^{(1)} + p_j^{(2)} + p_j^{(3)} + p_j^{(4)}}{4} \right) (w_{tj1} + w_{tj2}) \quad (4.16)$$

$$P^{max} = \sum_{t=1}^T \sum_{j=1}^n p_j^{(4)} (w_{tj1} + w_{tj2}) \quad (4.17)$$

$$E[IC] = \sum_{t=1}^T \sum_{j=1}^n \left(\frac{ic_j^{(1)} + ic_j^{(2)} + ic_j^{(3)} + ic_j^{(4)}}{4} \right) (w_{tj1} + w_{tj2}) d_{tj} \quad (4.18)$$

$$IC^{max} = \sum_{t=1}^T \sum_{j=1}^n ic_j^{(4)} (w_{tj1} + w_{tj2}) d_{tj} \quad (4.19)$$

$$E[MC] = \sum_{t=1}^T \sum_{j=1}^n \left(\frac{mc_j^{(1)} + mc_j^{(2)} + mc_j^{(3)} + mc_j^{(4)}}{4} \right) \times md_{tj} \quad (4.20)$$

$$MC^{max} = \sum_{t=1}^T \sum_{j=1}^n mc_j^{(4)} \times md_{tj} \quad (4.21)$$

$$E[PC] = \sum_{t=1}^T \left(\frac{pc_j^{(1)} + pc_j^{(2)} + pc_j^{(3)} + pc_j^{(4)}}{4} \right) \times w_{t,n+1,2} \quad (4.22)$$

$$PC^{max} = \sum_{t=1}^T pc_j^{(4)} \times w_{t,n+1,2} \quad (4.23)$$

γ represents the weight/importance of the second term against the other terms in objective function and this term controls optimality robustness. The third term determines the confidence level of each chance constraint in which δ is the penalty unit of possible violation of each constraint including imprecise parameter(s) and $[de^{(4)} - (1 - \theta)de^{(3)} - \theta de^{(4)}]$ indicates the difference between the worst case value of imprecise demand parameter and the value that is used in chance constraints. Indeed, this term controls the feasibility robustness of the solution vector. Noteworthy, that is not just a theoretical and meaningless parameter, rather the value of penalty can be determined based on application context properly (Pishvae et al., 2012).

4.3. Stage 3: Single objective counterpart model

Several methods have been developed to tackle the multi-objective mathematical models, such as the weighted-sum, Tchebycheff-based methods, and the fuzzy programming. In this thesis the ε -constraint method, which is the best-known technique to solve multicriteria optimization problems, is applied to dandle the bi-objective Model 5. This method works for general problems and no convexity assumption is needed. In the ε -constraint method, the most significant objective

function (i.e., the first objective in this thesis that is chosen regarding the opinion of the decision-maker) is optimized while the other objective (i.e., the second function) are transformed into constraints as follows.

Model 6:

$$\text{Max } z_1 = \sum_{t=1}^T \vartheta_t \quad (4.24)$$

s.t.

$$\text{Equation 4.8} \leq \varepsilon_2 \quad (4.25)$$

With Equations 3.65–3.73, 3.75–3.93, 4.2–4.4, 4.7, and 4.9.

Then, the efficient solutions of the Model 6 are obtained by parametrical variation in the right-hand side (i.e. ε_2) of the constrained objective functions (Mavrotas, 2009; Rezaei-Malek et al., 2016). The range of ε_2 can be obtained by optimizing the constrained second objective functions separately considering the constraints and constructing the pay-off table (Rezaei-Malek et al., 2016). Next, different values for ε_2 can be obtained by dividing the range of constrained second objective (i.e. r_2) to q equal intervals as follows.

$$r_2 = z_2^{max} - z_2^{min}; \quad \varepsilon_2^l = z_2^{max} - \frac{r_2}{q} \times l \quad l = 0, \dots, q-1 \quad (4.26)$$

4.4. Conclusion

In this chapter, a 3-stage approach has been proposed to deal with the bi-objective mixed-integer non-linear programming Model 3. First, the non-linear system productivity function has been transformed to some linear segments through the piecewise linear approximation (see Model 4). Then, the RPP-II has been employed to obtain a robust solution for the problem in presence of the uncertainty about cost components and demand amount (see Model 5). Finally, the ε -constraint method, as the most well-known method of multi-objective optimization, has been used to develop a single objective counterpart model (see Model 6).

Although considering the linear segments in comparison with the original

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non-linear system productivity function reduces the accuracy of the model, it decreases the complexity and accordingly the obtained linear model can be optimally solved by commercial solvers like CPLEX. It should be mentioned that the RPP is the best-suited approach to obtain a robust solution for the problem of this thesis because of two reasons: (1) the fuzzy nature of the available data of our considered case study, which is elaborated in the next section, and (2) its better performance over the basic possibilistic chance constrained programming. Finally, the advantages of the ε -constraint method over the traditional techniques like weighted sum and the simplicity of the usage are of the main reasons for applying the ε -constraint.

Chapter 5

Experimental Results

5.0. Chapter purpose and outline

According to the previous two chapters, [Model 2](#) and [Model 5](#) are the final proposed models for the main problem of this thesis (see [Section 3.1](#)) and its extended version (see [Section 3.2](#)), respectively. Hereafter, [Model 2](#) is called [Main Model](#) and [Model 5](#) is named [Extended Model](#). To display that [Main Model](#) and [Extended Model](#) and the proposed solution approach comply with the integrated planning specifications and meet the needs of SMSPLs, two main sections are structured in this chapter. [Section 5.1](#) aims at verifying the proposed approach through exploring a test problem. Then, [Section 5.2](#) investigates a real case study and hereby provides a validation study and some managerial insights. Finally, [Section 5.3](#) concludes this chapter.

5.1. Numerical example

To verify the proposed models and solution approach, a numerical example is investigated. This example and its specifications are inspired from a part of a real SMSPL, which is an assembly line for “PC250” air piston compressors. It contains a three-stage system, in which the duration of the planning horizon is 12 periods. The deterioration processes of all the three stages are the same because the system includes identical machines and tools for doing different operations. Regarding the historical data, the ε_{ij} behavior of each stage as a function of i is estimated $\varepsilon_{ij} = 0.05 \times i$. Similarly, the PM cost is considered US\$ 50. The number of unit material entering the SMSPL is 100 units per period and 5% of them are non-conforming, i.e., $\varepsilon_{t0} = 0.05$. The cost for replacing a delivered non-conforming item is estimated US\$ 20 and the amount of demand is 50 units per period, i.e. $de = 50$. The rest of input data is provided in [Table 5.1](#).

[Main Model](#) and [Extended Model](#) coded in the GAMS software (ver. 24.1.2) and solved by the CPLEX solver (ver. 12.5.1.0), which is capable to solve MILP models through the branch-and-cut algorithm, on the data of the numerical example and the case study using a laptop with Intel® Core™ i5-6300U CPU, 2.50 GHz, 8 GB of RAM. The next subsections present the obtained results.

Table 5.1. Production parameters of the 3-stage assembly line

Parameter	Stage		
	1	2	3
α	0.02	0.05	0.01
β	0.01	0.1	0.02
ic	0.5	0.5	0.5
rc_1	5	0	0
rc_2	20	0	0
p	10	20	5
sc	0	-2	40
f_1	0.2	0	0.5
f_2	0.8	1	0.5
mc	50	50	50

5.1.1. Main model

In comparison with the existing literature, specifically those presented mathematical models for the conventional PQIP problem (e.g., see [Yum and McDowellj \(1987\)](#), [Mohammadi et al. \(2015\)](#) and [\(2017\)](#)), the most important achievement of this research is that the proposed technology of integrated planning of PM and part quality inspection activities results in a decrease in the total manufacturing cost of the SMSPL as this was predictable because of the inherent interconnectivity of these two types of operation. In the investigated example, this saving is 26.94%. If we define two approaches: *Without PM* (refers to the conventional PQIP) and *With PM Possibility* (refers to the proposed integrated planning), the cost objective function value for the *Without PM* approach is 51587 and by adding the implementation possibility of PM activities (i.e., the *With PM Possibility* approach), it decreases to 37689 (see [Figure 5.1](#)), in which this will be a huge saving for a manufacturing company. [Figure 5.1](#) also illustrates the different cost components when we applied the *With PM Possibility* and *Without PM* approaches for the SMSPL. As can be seen, the extreme cost differences refer to the scrap and production costs because when the *Without PM* approach is applied, most of the time the production stages are in the downgraded state and produce and proceed much more defective items in comparison to the *With PM Possibility* approach. Although the *With PM Possibility* approach imposes US\$ 1500 PM cost to the system, this cost acts as an investment and reduces the scrap and production costs in a way that the total manufacturing cost is decreased.

[Tables 5.2](#) and [5.3](#) show the places and periods in which the PM and part

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quality inspection activities are done when the *Without PM* and *With PM Possibility* approaches are employed, respectively. Table 5.2 indicates that, for the *Without PM approach*, the most of inspection activities are performed for the last stage because of its high scrap cost. However, when *With PM Possibility* approach is adopted (see Table 5.3), the model decides to establish inspection stations when and where the PM activities are not taken to be done.

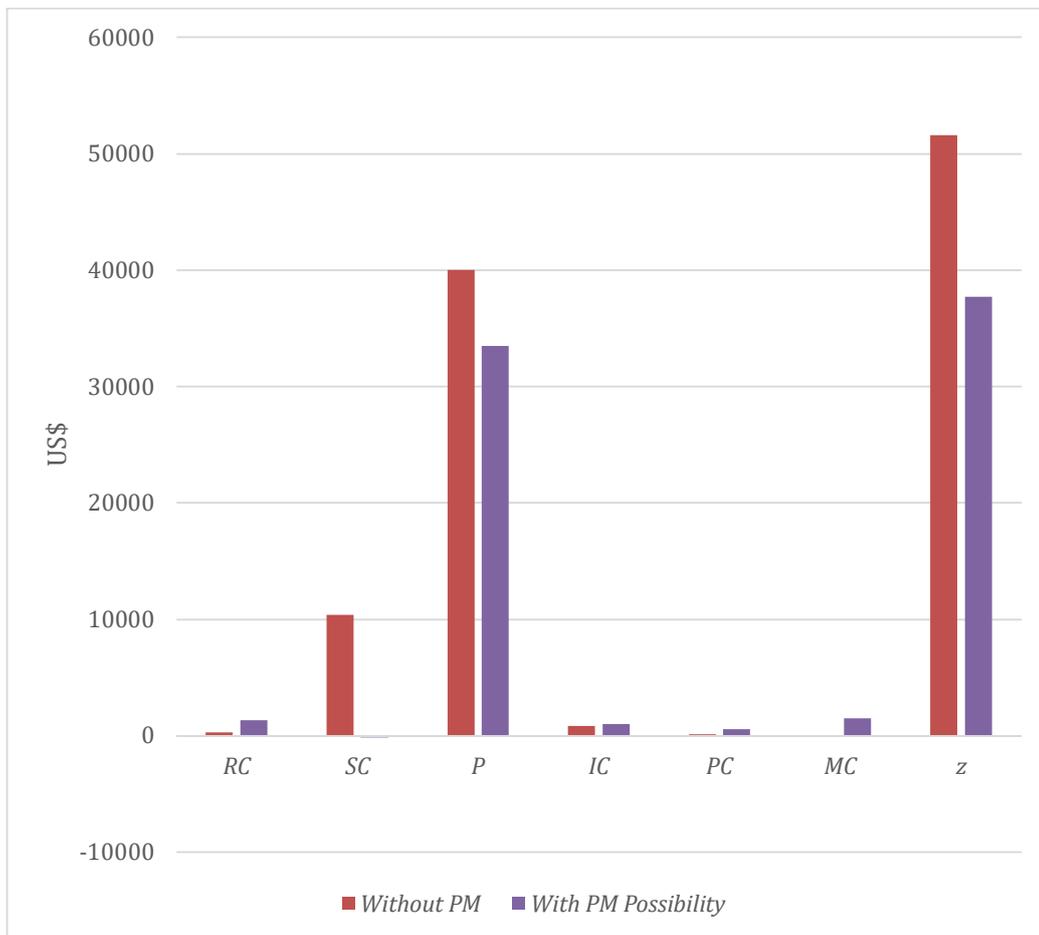


Figure 5.1. Cost components of the adopted approaches

The reason for the PM activities done mostly for the last stage is that the scrap cost for the last stage is too high in comparison with the other stages and in this way, the model avoided a considerable waste of money.

The obtained solution of the presented model and accordingly the associated saving cost highly depend on the PM cost (i.e., mc). As shown in Figure 5.2, by increasing the cost of PM, the cost saving is being decreased as there is not any monetary benefit of applying the proposed integrated approach when the cost of a PM is higher than 3000. However, “3000” unit cost for a PM seems unrealistic

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because it is 20 times more than “150” unit cost which we have inspired from the real case.

Table 5.2. Inspection (I) activities done in the SMSPL while applying the *Without PM approach*

Period	Stage		
	1	2	3
1	I		
2	I	I	
3	I	I	
4	I	I	
5	I	I	
6	I		I
7	I		I
8			I
9			I
10			I
11			I
12			I

Table 5.3. Maintenance (M) and inspection (I) activities done in the SMSPL while applying the *With PM Possibility approach*

Period	Stage		
	1	2	3
1	M,I	M	M
2	I	I	M
3	I	I	M
4	I	I	M
5	I	I	M
6	I	I	M,I
7	I	M	M
8	I	I	M
9	I	I	M
10	I		M,I
11	I	M,I	M
12	I		M,I

The other critical parameter for the SMSPL system is the unit inspection cost. **Figure 5.3** depicts the objective function value when the unit cost of inspection changes. As can be seen, the *With PM Possibility* approach is the cost-effective approach even when there is no cost for inspection. In addition, the cost saving of the *With PM Possibility* approach increases when the unit inspection cost is being raised, so the *With PM Possibility* approach is more cost-effective whenever the unit cost of inspection is higher. Please note that after “10” unit inspection cost, the value of the objective function under the *With PM Possibility* approach does not change because

Main Model decides not to do any inspection activity.

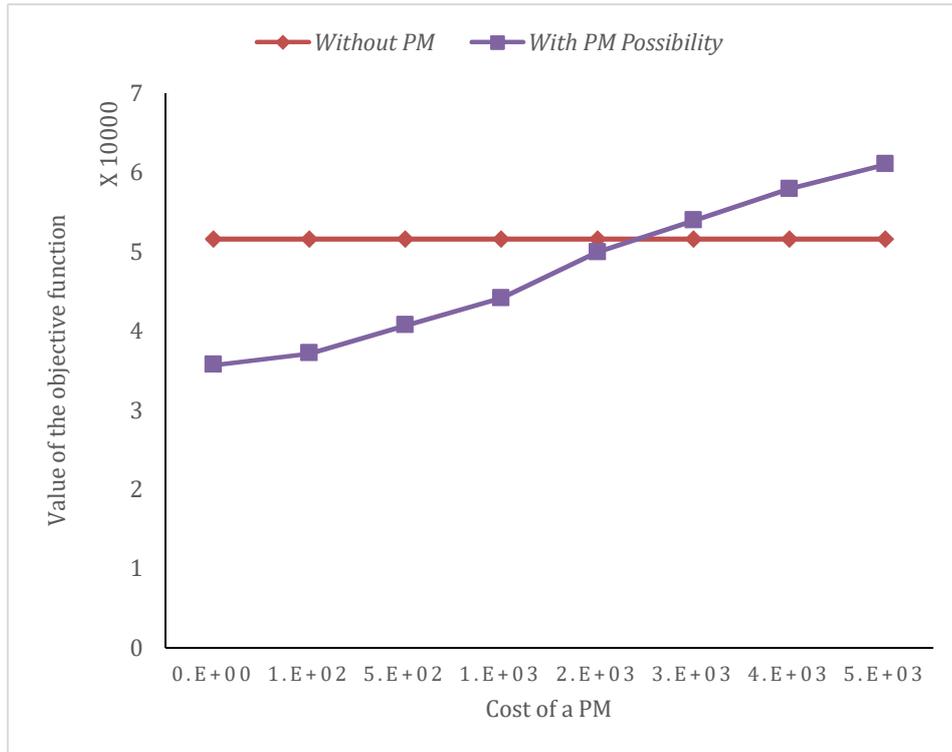


Figure 5.2. Objective function value by considering different values for the cost of a PM

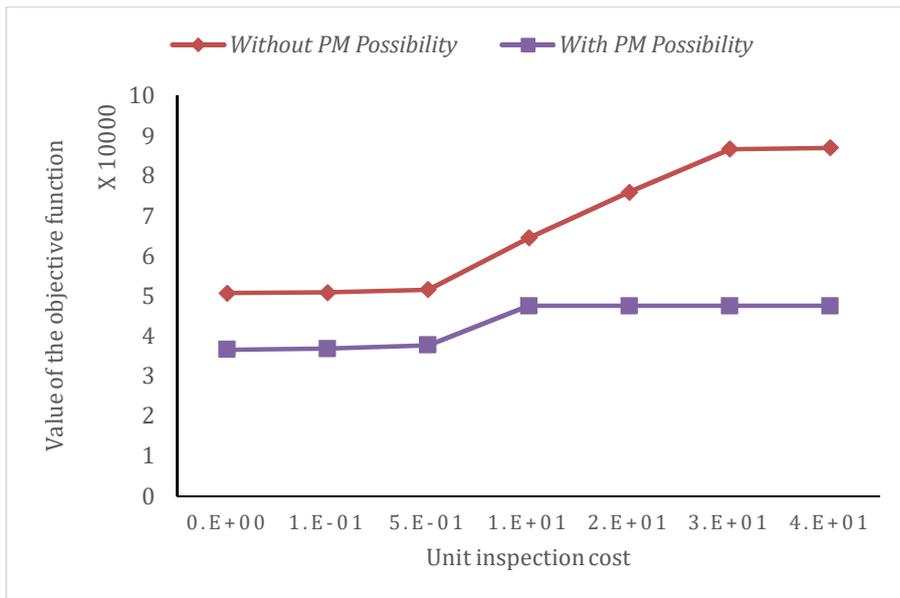


Figure 5.3. Objective function values by considering different values for the unit inspection cost

5.1.2. Extended model

To verify the influence of considering two different objective functions on the integrated planning, the generated solutions using different objective functions of the proposed **Extended Model** are investigated. **Extended Model** is solved for the

extended version of the above-mentioned numerical example. Uniform distribution is used to randomly generate prominent values of each trapezoidal fuzzy number while the ranges are estimated according to the available data and experts' knowledge of the assembly line for "PC250" (see [Table 5.4](#)). Similarly, the value of γ and δ are considered 0.2 and 100, respectively.

[Table 5.5](#) depicts the generated solutions using different objective functions of the proposed [Model 5](#). Since both the performed PM and inspection activities in each solution are different, it can be implied that the results from employing different objective functions are not necessarily consistent, and so objective functions should be considered separately. In addition, [Table 5.3](#) in comparison with the right part of [Table 5.5](#) proves the influence of the RPP-II method on the optimum obtained solution.

[Figure 5.4](#) depicts the values of the objective functions of the Pareto-optimal solutions obtained by solving [Extended Model](#). This observation also proves the abovementioned conclusion as an increase of system productivity (the second objective function) leads to an increase in the cost objective and vice versa. Notably, the cost objective function has a tendency towards production of less final conforming items via minimizing usage of PM activities (i.e., achieving the cost-efficiency) and on the other hand the first objective function has a tendency towards production of more conforming items to maximize the system productivity. For instance, consider the results of the single objective counterpart of the [Extended Model](#) (i.e., [Model 6](#)). In this case, when $z_2 = 11.335$ and $z_1 = 74126.635$ (the last Pareto-optimal solution), the number of produced conforming items is 1164 and the system deliver no non-conforming item, whereas when $z_2 = 9.143$ and $z_1 = 67471.280$ (the first Pareto-optimal solution), the number of produced conforming items is 1045 and the system deliver 10 non-conforming items.

Table 5.4. Data of the test problem

Uncertain parameter	Distribution
$\tilde{p}_j = (p_j^{(1)}, p_j^{(2)}, p_j^{(3)}, p_j^{(4)})$	$\sim U(1,50); p_j^{(i+1)} > p_j^{(i)}$
$\tilde{ic}_j = (ic_j^{(1)}, ic_j^{(2)}, ic_j^{(3)}, ic_j^{(4)})$	$\sim U(0,2); ic_j^{(i+1)} > ic_j^{(i)}$
$\tilde{rc}_{j1} = (rc_{j1}^{(1)}, rc_{j1}^{(2)}, rc_{j1}^{(3)}, rc_{j1}^{(4)})$	$\sim U(1,10); rc_{j1}^{(i+1)} > rc_{j1}^{(i)}$
$\tilde{rc}_{j2} = (rc_{j2}^{(1)}, rc_{j2}^{(2)}, rc_{j2}^{(3)}, rc_{j2}^{(4)})$	$\sim U(1,50); rc_{j2}^{(i+1)} > rc_{j2}^{(i)}$
$\tilde{sc}_j = (sc_j^{(1)}, sc_j^{(2)}, sc_j^{(3)}, sc_j^{(4)})$	$\sim U(-5,55); sc_j^{(i+1)} > sc_j^{(i)}$
$\tilde{mc}_j = (mc_j^{(1)}, mc_j^{(2)}, mc_j^{(3)}, mc_j^{(4)})$	$\sim U(50,170); mc_j^{(i+1)} > mc_j^{(i)}$
$\tilde{pc} = (pc^{(1)}, pc^{(2)}, pc^{(3)}, pc^{(4)})$	$\sim U(15,25); pc^{(i+1)} > pc^i$
$\tilde{de} = (de^{(1)}, de^{(2)}, de^{(3)}, de^{(4)})$	$\sim U(40,60); de^{(i+1)} > de^i$

Table 5.5. The PM (M) and inspection (I) activities done in the S3SMS considering different objectives

Objective 2 Period	Stage			Objective 1 Period	Stage		
	1	2	3		1	2	3
1	I,M	M	M	1	I,M	M	M
2	I,M	M	M	2	I		
3	I,M	M	M	3	I	I	M
4	I,M	M	M	4	I	I	
5	I,M	M	M	5	I	I	M
6	I,M	M	M	6	I	I	
7	I,M	M	M	7	I	I	M
8	I,M	M	M	8	I	I	
9	I,M	M	M	9	I	M	M
10	I,M	M	M	10	I		
11	I,M	M	M	11	I	I	
12	I,M	M	M	12	I	I	M

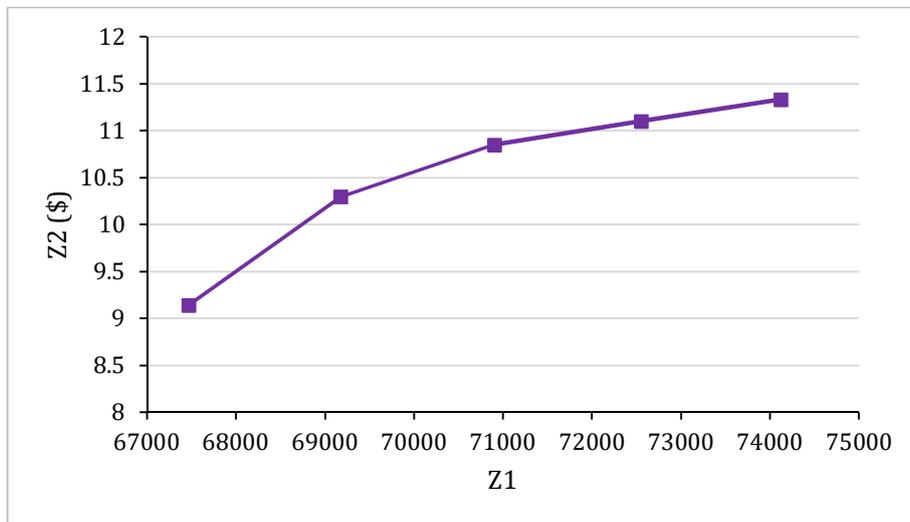


Figure 5.4. Pareto curve

To assess the usefulness of the applied RPP-II approach, first, 10 random realizations of the extended numerical example are generated. For example, if $\xi = (\xi^{(1)}, \xi^{(2)}, \xi^{(3)}, \xi^{(4)})$ is an imprecise parameter with trapezoidal possibility distribution function, the realization is produced by generating a random number uniformly between the two extreme points of the corresponding possibility distribution function. Then, the obtained optimal solutions by [Main Model](#) and [Extended Model](#) under deterministic (x^*, y^*) and nominal data (x^{**}, y^{**}) will be replaced respectively in the linear programming models (considering realizations) that their compact forms are as follows.

$$\text{Min } f_{real}y^* + c_{real}x^*$$

s.t.

$$Ax^* \geq de_{real}$$

$$Bx^* = 0$$

(5.1)

$$\text{Min } f_{real}y^{**} + c_{real}x^{**} + \delta R^d$$

s.t.

$$Ax^{**} + R^d \geq de_{real}$$

$$Bx^{**} = 0$$

$$R^d \geq 0$$

(5.2)

In the linear programming model [\(5.2\)](#), R^d is the only decision variables that specifies the violation of chance constraints under random realization. The average and standard deviation of objective function values under random realizations are employed as the performance measures to assess the developed models. The results of these experiments are shown in [Table 5.6](#). As can be seen the obtained solution by the [Extended Model](#) provides lower total costs for different realizations. On average, it results in 1.1% reduction in the total costs. In addition, according to the standard deviation, applying the [Extended Model](#) reduces 0.3% the risk of additional imposed costs to the system.

The first objective function contains the piecewise linear segments to model the non-linear behavior of the system productivity regarding the produced conforming items (see [Section 4.1](#)). By increasing the linear segments, we can achieve more accurate estimation for the non-linear behavior. [Figure 5.5](#) depicts the obtained pareto carve while applying different numbers of linear segments. The 3-segment approximation has the lowest accuracy and employing more than three segments almost causes the same result/accuracy (as it is shown for the 4- and 5-segment case). [Table 5.7](#) provides the obtained Pareto optimal solutions for the

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different numbers of segments and shows that the average obtained values of the first and second objective function for the 4- and 5-segment approach are almost the same and accordingly they provide similar accuracy for the system.

Table 5.6. Performance of the proposed models under realizations

No. of realization	Main Model	Extended Model
1	66407.115	65705.241
2	74281.224	73569.489
3	56613.398	55881.472
4	49663.407	48963.740
5	53234.151	52687.414
6	66000.509	65553.609
7	43519.911	43095.735
8	67653.938	67039.298
9	58425.099	57735.315
10	53832.192	53067.514
Average	58963.09	58329.88
Standard deviation	8972.447	8946.283

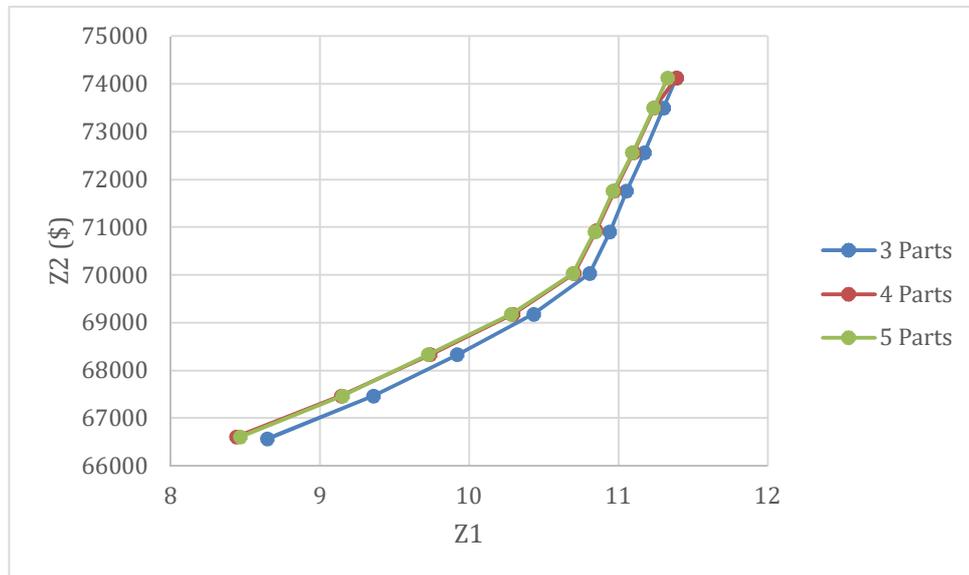


Figure 5.5. Pareto curve while applying different numbers of segments

5.2. Case study

A real case study within the automotive industry in France is investigated in this section to validate the developed [Main Model](#) and [Extended Model](#) and to show the effectiveness and significance of the contributions of this thesis which are proposing the integrated planning of part quality inspection and PM activities,

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developing of the system productivity objective function, and integrating the uncertainty of the cost components and demand amount into the problem.

Figure 5.6 illustrates the solid frame of the considered product and its 15 QCs are indicated in Figure 5.7. This product is manufactured by a plant in the supply chain of “Renault Groupe”. The 15 QCs are processed along the 15 different manufacturing stages.

Table 5.7. Obtained Pareto solution while considering different numbers of segments

	3-Segment		4-Segment		5-Segment	
	Z1	Z2	Z1	Z2	Z1	Z2
Pareto solutions	11.388	74126.64	11.388	74126.64	11.33	74126.64
	11.302	73498.81	11.241	73498.81	11.236	73498.81
	11.173	72557.07	11.101	72557.07	11.094	72557.07
	11.055	71766.05	10.973	71766.06	10.965	71766.05
	10.942	70905.91	10.85	70925.18	10.841	70905.91
	10.808	70033.3	10.704	70033.3	10.695	70033.3
	10.433	69179.63	10.297	69179.63	10.284	69179.63
	9.921	68336.2	9.74	68336.2	9.726	68336.2
	9.36	67471.28	9.143	67471.28	9.153	67471.28
	8.65	66566.81	8.443	66610.86	8.468	66611.74
Mean	10.875	70469.6	10.777	<i>70479.24</i>	10.768	<i>70469.6</i>

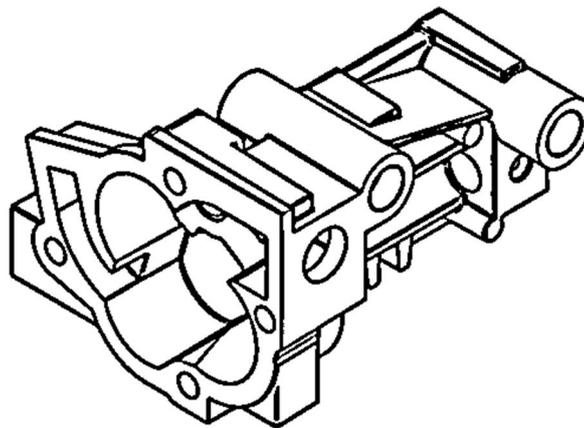


Figure 5.6. Solid frame of the oil pump housing

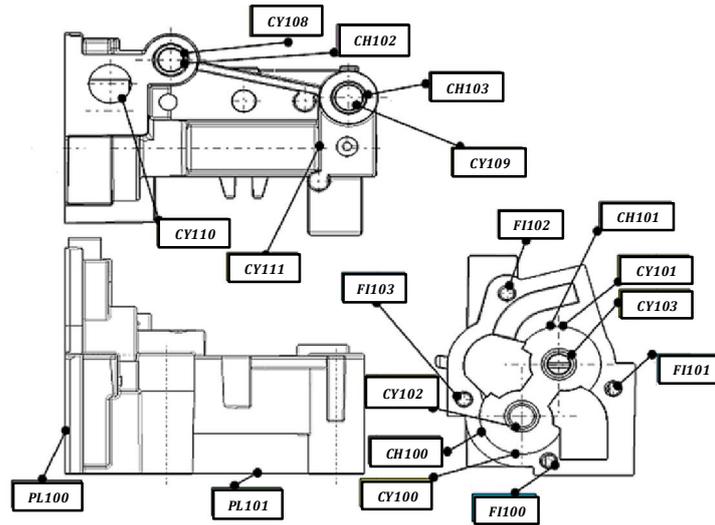


Figure 5.7. QCs of the oil pump housing

5.2.1. Main model

For the deterministic situation, regarding the proposed [Main Model](#), the data of case study are presented in [Tables 5.8](#). In addition, unit inspection cost is estimated US\$ 0.018, the demand amount is 600 units per period, non-conforming fraction of material that enters the SMSPL in each period is 0.05, penalty cost of delivering a non-conforming item to customer is US\$ 20, and the planning horizon for this SMSPL is 12 periods (i.e., 12 months). [Table 5.9](#) shows the defective production rate for the different stages regarding the number of periods has been passed since the last performed PM. This recent data has been obtained considering the historical behavior of the system. In the considered plant, the maintenance department was doing a PM activity for each stage in the first period (*separate planning*). Now, the system benefits from the integrated planning of the part quality inspection and PM activities, which is provided by applying the proposed [Main Model](#) (*integrated planning*).

In this section, we aim at comparing the performance of this SMSPL before and after applying the proposed *integrated planning*. For this purpose, some key performance indicators and cost components are regarded. [Figure 5.8](#) shows the comparison of the different cost components when the system used the *separate* and *integrated planning* approach. As can be seen, all the cost components except PM cost are improved after applying the *integrated planning* approach. It is notable that both approaches delivered no non-conforming items to customers. [Tables 5.10](#) and [5.11](#)

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show the places and periods, in which the part quality inspection and PM activities are performed considering the *separate* and *integrated planning* approaches, respectively. *Integrated planning* reduced the number of inspection and increased the PM activities, but it succeeds to decrease the total cost (4.82%) while satisfying the minimum quantity of demand for the conforming items, 600 units. Noteworthy, the number of delivered conforming item to customer(s) is increased 7.16% by applying the *Integrated planning*

Table 5.8. Production parameters of the oil pump housing

No.	Operation name	Details				
		α	β	p	sc	mc
1	Rough milling PL100	0.0027	0.00005	0.308333	1	2.50
2	Rough milling PL100	0.0027	0.00005	0.345833	1	2.50
3	Rough milling PL101	0.0027	0.00005	0.277083	1	2.50
4	Boring CY110	0.0027	0.00005	0.320833	1	3.00
5	Rough drilling CY108 & CY109	0.0027	0.00005	0.1875	1	3.40
6	Chamfering CY108 & CY109	0.0027	0.00005	0.520833	1	2.90
7	Chamfering CY100 & CY101	0.0027	0.00005	0.535417	1	2.90
8	Boring CY100	0.0027	0.00005	0.535417	1	3.00
9	Boring CY101	0.0027	0.00005	0.254167	1	3.00
10	Rough drilling CY102 & CY103	0.0027	0.00005	0.227083	1	3.40
11	Rough drilling CY111	0.0027	0.00005	0.279167	1	3.40
12	Boring CY108 & CY109	0.0027	0.00005	0.254167	1	3.00
13	Boring CY102 & CY103	0.0027	0.00005	0.254167	1	3.00
14	Boring CY111	0.0027	0.00005	0.24375	1	3.00
15	Finish milling PL100	0.0027	0.00005	0.26875	1	3.40

No.	Operation name	Details				
		ic	f_1	f_2	rc_1	rc_2
1	Rough milling PL100	0.018	0.97	0.03	0.030833	0.061667
2	Rough milling PL100	0.018	0.75	0.25	0.034583	0.069167
3	Rough milling PL101	0.018	0.33	0.67	0.027708	0.055417
4	Boring CY110	0.018	0.08	0.92	0.032083	0.064167
5	Rough drilling CY108 & CY109	0.018	0.87	0.13	0.01875	0.0375
6	Chamfering CY108 & CY109	0.018	0.42	0.58	0.052083	0.104167
7	Chamfering CY100 & CY101	0.018	0.69	0.31	0.053542	0.107083
8	Boring CY100	0.018	0.88	0.12	0.053542	0.107083
9	Boring CY101	0.018	0.05	0.95	0.025417	0.050833
10	Rough drilling CY102 & CY103	0.018	0.90	0.10	0.022708	0.045417
11	Rough drilling CY111	0.018	0.91	0.09	0.027917	0.055833
12	Boring CY108 & CY109	0.018	0.44	0.56	0.025417	0.050833
13	Boring CY102 & CY103	0.018	0.41	0.59	0.025417	0.050833
14	Boring CY111	0.018	0.02	0.98	0.024375	0.04875
15	Finish milling PL100	0.018	0.30	0.70	0.026875	0.05375

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Table 5.9. Defective production rate regarding the number of passed periods since the last performed PM

Stage	Number of passed periods since the last PM												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0071	0.0126	0.0245	0.0348	0.0443	0.0471	0.0484	0.0639	0.0662	0.0750	0.0812	0.0879	0.0889
2	0.0065	0.0068	0.0080	0.0088	0.0169	0.0169	0.0307	0.0476	0.0480	0.0516	0.0734	0.0877	0.0965
3	0.0162	0.0171	0.0188	0.0319	0.0498	0.0710	0.0777	0.0780	0.0783	0.0783	0.0902	0.0903	0.0938
4	0.0008	0.0196	0.0289	0.0303	0.0382	0.0414	0.0473	0.0595	0.0675	0.0712	0.0743	0.0911	0.0984
5	0.0041	0.0170	0.0270	0.0330	0.0427	0.0463	0.0472	0.0504	0.0551	0.0608	0.0629	0.0753	0.0870
6	0.0070	0.0115	0.0133	0.0153	0.0322	0.0331	0.0568	0.0645	0.0659	0.0761	0.0822	0.0886	0.0909
7	0.0024	0.0049	0.0063	0.0103	0.0129	0.0371	0.0413	0.0551	0.0612	0.0756	0.0926	0.0991	0.0998
8	0.0056	0.0065	0.0115	0.0118	0.0182	0.0192	0.0254	0.0524	0.0553	0.0609	0.0610	0.0751	0.0973
9	0.0099	0.0176	0.0279	0.0402	0.0411	0.0431	0.0461	0.0483	0.0520	0.0560	0.0584	0.0719	0.0951
10	0.0002	0.0239	0.0304	0.0379	0.0517	0.0605	0.0723	0.0775	0.0860	0.0875	0.0891	0.0900	0.1000
11	0.0012	0.0086	0.0143	0.0442	0.0452	0.0499	0.0534	0.0545	0.0560	0.0612	0.0659	0.0913	0.0919
12	0.0118	0.0118	0.0197	0.0210	0.0297	0.0319	0.0483	0.0654	0.0672	0.0832	0.0896	0.0971	0.0999
13	0.0047	0.0121	0.0138	0.0181	0.0329	0.0379	0.0408	0.0499	0.0536	0.0587	0.0694	0.0769	0.0917
14	0.0026	0.0155	0.0171	0.0206	0.0235	0.0372	0.0377	0.0430	0.0581	0.0642	0.0886	0.0891	0.0992
15	0.0130	0.0160	0.0214	0.0233	0.0290	0.0309	0.0600	0.0614	0.0657	0.0732	0.0821	0.0827	0.0955

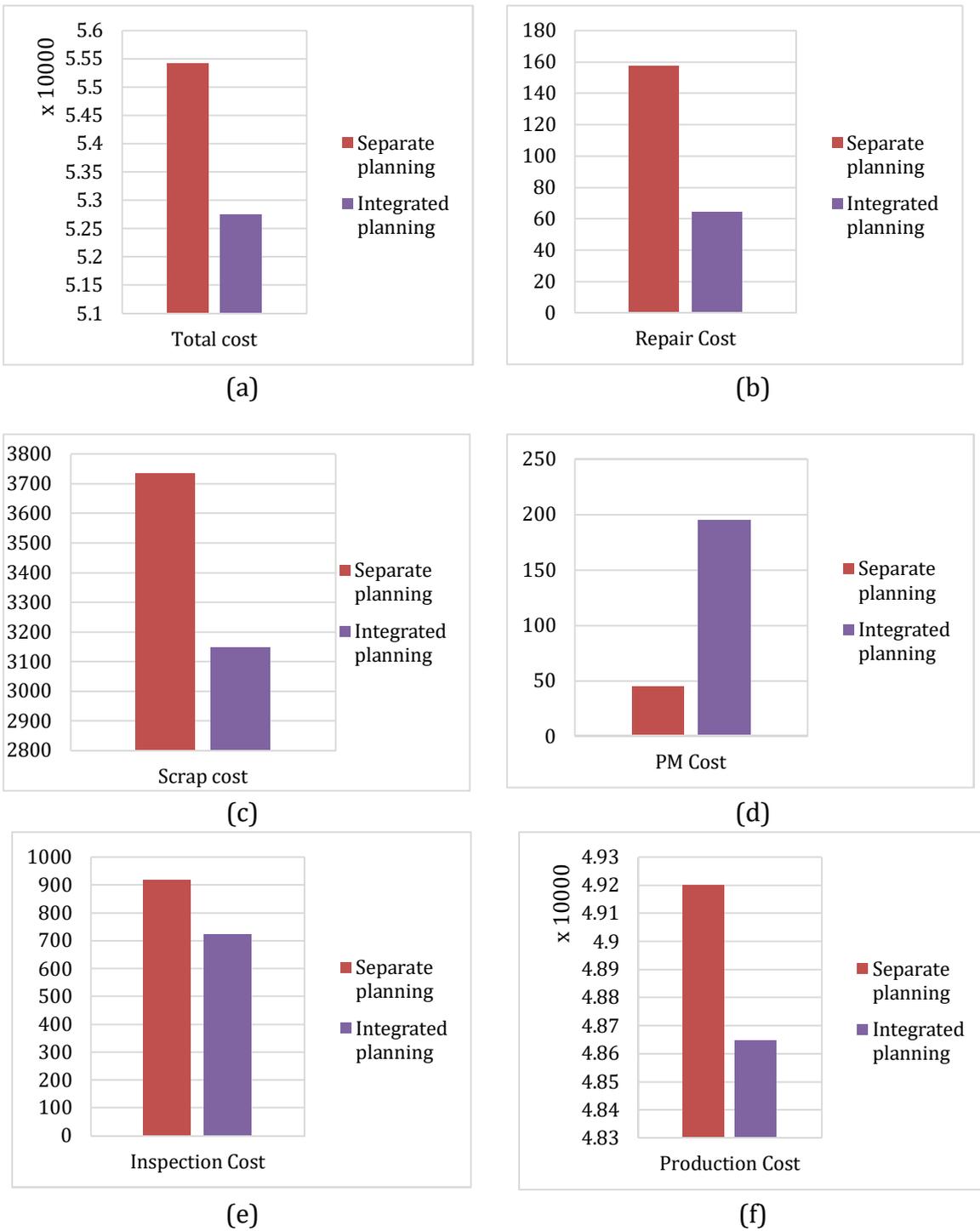


Figure 5.8. Cost components of the case study

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Table 5.10. Places and periods in which inspection (I) and preventive maintenance (M) activities are performed by applying the *Separate Planning* approach

		Stage														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Period	1	M	M	M	M,I	M	M	M	M	M	M	M	M	M	M	M
	2				I					I			I			I
	3				I		I			I			I			I
	4				I		I			I			I			I
	5				I		I			I			I			I
	6				I		I			I			I			I
	7				I		I			I		I		I		I
	8				I		I	I				I		I		I
	9				I	I	I	I				I		I		I
	10			I	I		I					I		I		I
	11			I	I	I						I		I		I
	12		I		I	I						I		I		I

Table 5.11. Places and periods in which inspection (I) and preventive maintenance (M) activities are performed by applying the *Integrated Planning* approach (*Main Model*)

		Stage														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Period	1	M	M	M	M,I	M	M	M	M	M	M	M	M	M	M	M
	2				I	M				I	M	M	M	M	M	M
	3				I		M			I	M	M	M	M	M	M
	4				I		I			I	M	M	M	M	M	M
	5				I		I			I	M	I	M	M	M	M
	6				I		I			I			M	I	M	M
	7				I		I			M,I	M	I			M,I	M
	8				I		I			I			I	M,I		M,I
	9				I		I	I		I		I		M	I	M
	10				I		M,I	I		I	M	I				M,I
	11				I		I	I		M,I		I	M	M	M,I	M
	12				I		I	I		M,I	I	M	M	M	M	M

5.2.2. Extended model

According to the proposed **Extended Model**, the imprecise parameters of case study are presented in **Tables 5.12**. To estimate the possibility distribution of imprecise parameters, a focus group of field experts and firm's managers has been formed to specify the four prominent values of each trapezoidal fuzzy number according to the available data and their knowledge. Similarly, the value of γ and δ are considered 0.2 and 100, respectively. In addition, the planning horizon for this SMSPL is 12 periods (i.e., 12 months) and the demand amount is estimated $\tilde{d}e = (550, 600, 630, 680)$.

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Table 5.12. Imprecise parameters of the case study

Stage	Details	
	$\tilde{p}_j = (p_j^{(1)}, p_j^{(2)}, p_j^{(3)}, p_j^{(4)})$	$\tilde{ic}_j = (ic_j^{(1)}, ic_j^{(2)}, ic_j^{(3)}, ic_j^{(4)})$
1	(0.26, 0.31, 0.36, 0.41)	(0.000999, 0.003999, 0.006999, 0.009999)
2	(0.30, 0.35, 0.40, 0.45)	(0.001936, 0.004936, 0.007936, 0.010936)
3	(0.23, 0.28, 0.33, 0.38)	(0.002561, 0.005561, 0.008561, 0.011561)
4	(0.27, 0.32, 0.37, 0.42)	(0.000817, 0.003817, 0.006817, 0.009817)
5	(0.14, 0.19, 0.24, 0.29)	(0.003380, 0.006380, 0.009380, 0.012380)
6	(0.47, 0.52, 0.57, 0.62)	(0.005665, 0.008665, 0.011665, 0.014665)
7	(0.49, 0.54, 0.59, 0.64)	(0.002364, 0.005364, 0.008364, 0.011364)
8	(0.49, 0.54, 0.59, 0.64)	(0.004962, 0.007962, 0.010962, 0.013962)
9	(0.20, 0.25, 0.30, 0.35)	(0.003381, 0.006381, 0.009381, 0.012381)
10	(0.18, 0.23, 0.28, 0.33)	(0.001140, 0.001857, 0.004857, 0.007857)
11	(0.23, 0.28, 0.33, 0.38)	(0.004292, 0.007292, 0.010292, 0.013292)
12	(0.20, 0.25, 0.30, 0.35)	(0.001139, 0.004139, 0.007139, 0.010139)
13	(0.20, 0.25, 0.30, 0.35)	(0.002847, 0.005847, 0.008847, 0.011847)
14	(0.19, 0.24, 0.29, 0.34)	(0.004841, 0.007841, 0.010841, 0.013841)
15	(0.22, 0.27, 0.32, 0.37)	(0.000916, 0.003916, 0.006916, 0.009916)

Stage	Details	
	$\tilde{rc}_{j1} = (rc_{j1}^{(1)}, rc_{j1}^{(2)}, rc_{j1}^{(3)}, rc_{j1}^{(4)})$	$\tilde{rc}_{j2} = (rc_{j2}^{(1)}, rc_{j2}^{(2)}, rc_{j2}^{(3)}, rc_{j2}^{(4)})$
1	(0.020833, 0.030833, 0.040833, 0.050833)	(0.0517, 0.0617, 0.0717, 0.0817)
2	(0.024583, 0.034583, 0.044583, 0.054583)	(0.0592, 0.0692, 0.0792, 0.0892)
3	(0.017708, 0.027708, 0.037708, 0.047708)	(0.0454, 0.0554, 0.0654, 0.0754)
4	(0.022083, 0.032083, 0.042083, 0.052083)	(0.0542, 0.0642, 0.0742, 0.0842)
5	(0.008750, 0.018750, 0.028750, 0.038750)	(0.0275, 0.0375, 0.0475, 0.0575)
6	(0.042083, 0.052083, 0.062083, 0.072083)	(0.0942, 0.1042, 0.1142, 0.1242)
7	(0.043542, 0.053542, 0.063542, 0.073542)	(0.0971, 0.1071, 0.1171, 0.1271)
8	(0.043542, 0.053542, 0.063542, 0.073542)	(0.0971, 0.1071, 0.1171, 0.1271)
9	(0.015417, 0.025417, 0.035417, 0.045417)	(0.0408, 0.0508, 0.0608, 0.0708)
10	(0.012708, 0.022708, 0.032708, 0.042708)	(0.0354, 0.0454, 0.0554, 0.0654)
11	(0.017917, 0.027917, 0.037917, 0.047917)	(0.0458, 0.0558, 0.0658, 0.0758)
12	(0.015417, 0.025417, 0.035417, 0.045417)	(0.0408, 0.0508, 0.0608, 0.0708)
13	(0.015417, 0.025417, 0.035417, 0.045417)	(0.0408, 0.0508, 0.0608, 0.0708)
14	(0.014375, 0.024375, 0.034375, 0.044375)	(0.0388, 0.0488, 0.0588, 0.0688)
15	(0.016875, 0.026875, 0.036875, 0.046875)	(0.0438, 0.0538, 0.0638, 0.0738)

Stage	Details	
	$\tilde{sc}_j = (sc_j^{(1)}, sc_j^{(2)}, sc_j^{(3)}, sc_j^{(4)})$	$\tilde{mc}_j = (mc_j^{(1)}, mc_j^{(2)}, mc_j^{(3)}, mc_j^{(4)})$
1	(0.4, 0.6, 0.8, 1)	(2.30, 2.50, 2.70, 2.90)
2	(0.3, 0.5, 0.7, 0.9)	(2.30, 2.50, 2.70, 2.90)
3	(0.2, 0.4, 0.6, 0.8)	(2.30, 2.50, 2.70, 2.90)
4	(0.1, 0.3, 0.5, 0.7)	(2.80, 3.00, 3.20, 3.40)
5	(-0.1, 0.1, 0.3, 0.5)	(3.20, 3.40, 3.60, 3.80)
6	(0, 0.2, 0.4, 0.6)	(2.70, 2.90, 3.10, 3.30)
7	(0.1, 0.3, 0.5, 0.7)	(2.70, 2.90, 3.10, 3.30)
8	(0.5, 0.7, 0.9, 1.1)	(2.80, 3.00, 3.20, 3.40)
9	(-0.1, 0.1, 0.3, 0.5)	(2.80, 3.00, 3.20, 3.40)
10	(-0.1, 0.1, 0.3, 0.5)	(3.20, 3.40, 3.60, 3.80)
11	(0, 0.2, 0.4, 0.6)	(3.20, 3.40, 3.60, 3.80)
12	(0.1, 0.3, 0.5, 0.7)	(2.80, 3.00, 3.20, 3.40)
13	(0.3, 0.5, 0.7, 0.9)	(2.80, 3.00, 3.20, 3.40)
14	(0.2, 0.4, 0.6, 0.8)	(2.80, 3.00, 3.20, 3.40)
15	(0.1, 0.3, 0.5, 0.7)	(2.50, 2.70, 2.90, 3.10)

In the previous subsection, the integrated planning problem of this case study was optimized while minimizing total cost under deterministic cost components and

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demand amount. Now, the RPP-II approach is employed to handle the above-mentioned uncertainties. In this respect, to show the advantage of the obtained solution by the RPP-II technique, a similar experiment to which was done in [Section 5.1.2](#) is performed here (see [Table 5.13](#)). According to [Table 5.13](#), the obtained solution for [Main Model](#) is feasible just for 50% of the realizations. Hence, it can be concluded that applying [Main Model](#), imposes a huge feasibility risk to the system.

[Tables 5.11](#) and [5.14](#) depict the obtained optimal solution by [Main Model](#) and [Extended Model](#), respectively. As can be seen, the [Extended Model](#)'s optimum solution costs 13.46% more than [Main Model](#)'s, but as mentioned before, it can respond to all the realizations and decreases the feasibility risk around 50%.

[Table 5.13](#). Performance of the proposed models under realizations

No. of realization	<i>Main Model</i>	<i>Extended Model</i>
1	54061.998	54859.966
2	Infeasible	56798.442
3	Infeasible	52771.405
4	54964.901	55742.102
5	53326.721	54014.608
6	Infeasible	53376.714
7	54229.969	54841.489
8	Infeasible	56148.684
9	53199.339	53796.880
10	Infeasible	53556.824

[Table 5.14](#). Places and periods in which inspection (I) and preventive maintenance (M) activities are performed by applying *Extended Model*

Period	Stage														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	M	M	M	M,I	M	M	M	M	M	M	M	M	M	M	M
2				I				I		M	M	M	M	M	M
3				I		I		I		M	M	M	M	M	M
4				I	M		M	I		M	M	M	M	M	M
5				I		I		I	M				I	M	M
6				I		I		I	M			I		M	M,I
7				I		I		I				I	M		M,I
8				I		I		M,I		M	M	M		I	M
9				I		I				M,I		M		M	M,I
10				I		I	M	M	M		M,I	M		M	M,I
11	I			I		I	M		M	M	M	M	I	M	M
12				I		M	I		M		M	M	M,I	M	M

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The RPP-II approach contains two technical parameters that their importance in the considered case study need to be analyzed. To this aim, a sensitivity analysis study on these two parameters are performed.

γ represents the weight/importance of the minimization of the difference between the expected total cost and the worst-case cost against the other terms in the objective function 4.8. Table 5.15 shows the value of this difference and the obtained confidence level of chance constraint (i.e., α) regarding the different applied values for γ . As can be seen, **Extended Model** has the best performance when the value of γ is considered equal to 0.5. Because the difference between the expected total cost and the worst-case cost is minimum (i.e., the least risk of imposed additional cost in the worst case) while the obtained confidence level of chance constraint is maximum (no risk). On the other side, the expected total cost and the worst-case cost have the lowest amount, which confirms cost-efficiency of this value ($\gamma = 0.5$).

Table 5.15. Impact of γ on the expected total cost, worst-case cost, and confidence level

γ	z_1^{max}	$E[z_1]$	$z_1^{max} - E[z_1]$	α
0.01	69732.01	57181.39	12550.62	1
0.1	69908.67	57340.77	12567.9	1
0.2	69532.05	57030.65	12501.4	1
0.5	69411.41	56930.76	12480.65	1
1	69986.34	57399.41	12586.93	0.996
10	69789.65	57250.43	12539.22	0.976
100	69784.04	57265.72	12518.32	0.5

The third term of the objective function 4.8, i.e., $\delta[de^{(4)} - (1 - \theta)de^{(3)} - \theta de^{(4)}]$, specifies the confidence level of the demand chance constraint in which δ is the penalty unit of possible violation of the constraint including imprecise parameter and $[de^{(4)} - (1 - \theta)de^{(3)} - \theta de^{(4)}]$ indicates the difference between the worst case value of imprecise parameter and the value that is used in the demand chance constraint. Indeed, this term controls the *feasibility robustness* of the solution vector. It is notable that δ is not just a theoretical and meaningless parameter, rather the value of penalty can be determined based on application context properly (Pishvae et al., 2012). For example, in our case study the value of δ can be considered as the penalty of non-satisfied demand or shortage that is known as a popular parameter in the context of manufacturing.

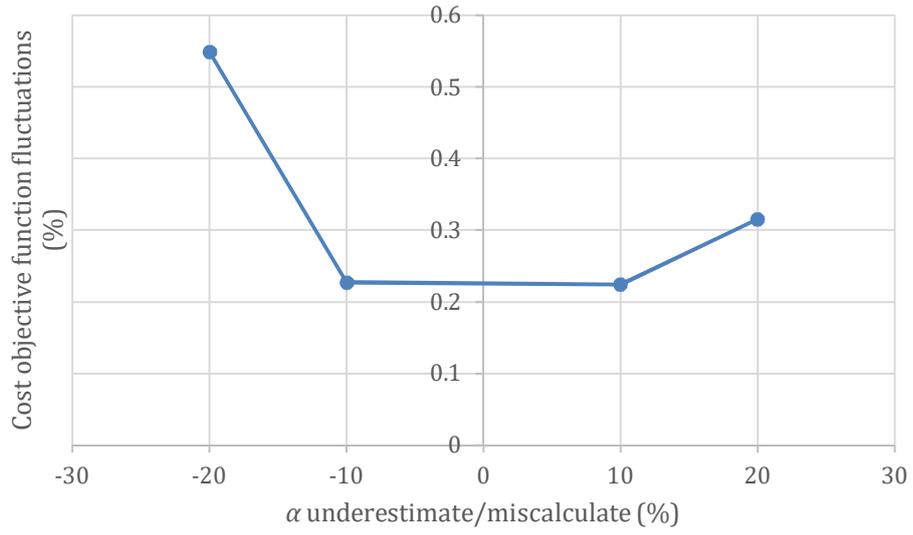
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Table 5.16 shows the performance of **Extended Model** regarding different values of δ . The best value which can be considered for δ is 100 because **Extended Model** reaches not only the highest confidence level ($\alpha = 1$), but also the least values for average and worst-case total cost. In other words, the solution obtained by considering $\delta = 100$, is the most risk-averse and cost-efficient solution.

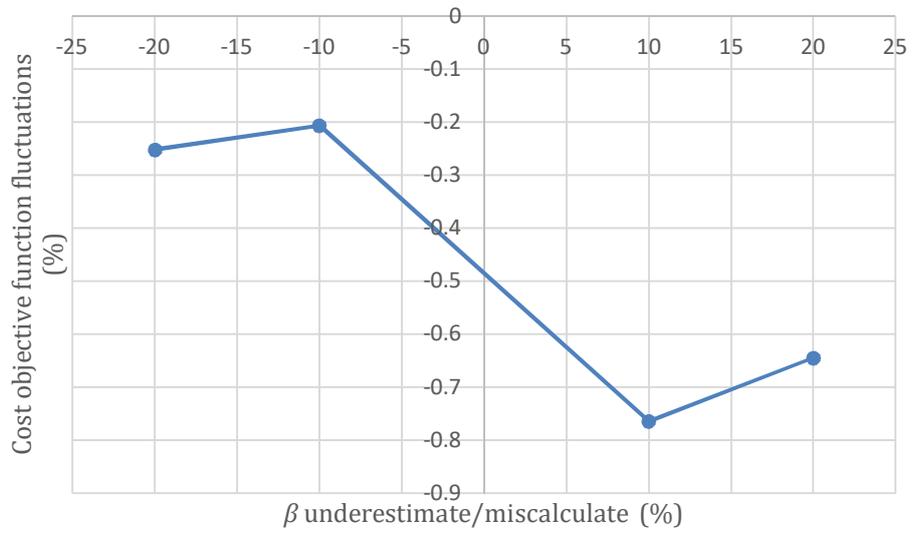
Table 5.16. Performance of Extended Model while considering different values for δ

δ	z_1^{max}	$E[z_1]$	$z_1^{max} - E[z_1]$	α
1	69684.95	57160.46	12524.49	0.526
10	69885.69	57329.88	12555.8	0.954
50	69603.77	57069.6	12534.18	0.889
100	69411.41	56930.76	12480.65	1
200	69773.81	57234.89	12538.91	1
300	69876.18	57325.14	12551.04	1
500	69974.67	57397.28	12577.38	1

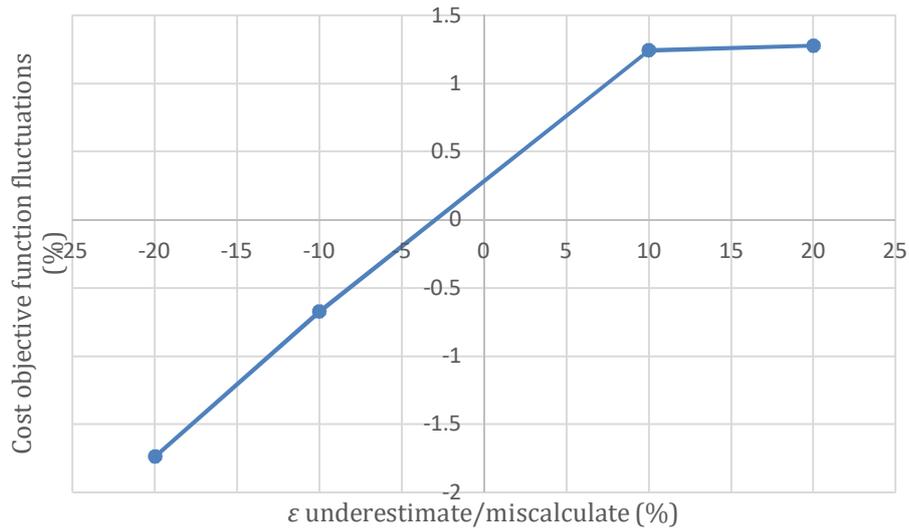
By employing the RPP-II, this thesis has tried to handle the uncertainty about cost components and demand amount (which are rooted in the external reasons) and provide a robust solution (see Section 2.3.3). However, the rest of parameters may undergo fluctuations because of the internal reasons (see Section 2.3.3). Figure 5.9 shows the impacts of these fluctuations on the total cost objective function. As can be seen, even if the system undergoes 20% fluctuations for the α , β , ε , and ε_0 parameter, the changes of the objective function are not more than 1.7%, which is negligible. Hence, it can be concluded that the obtained solution by **Extended Model** is remained robust under 20% miscalculation/underestimate of the deterministic parameters.



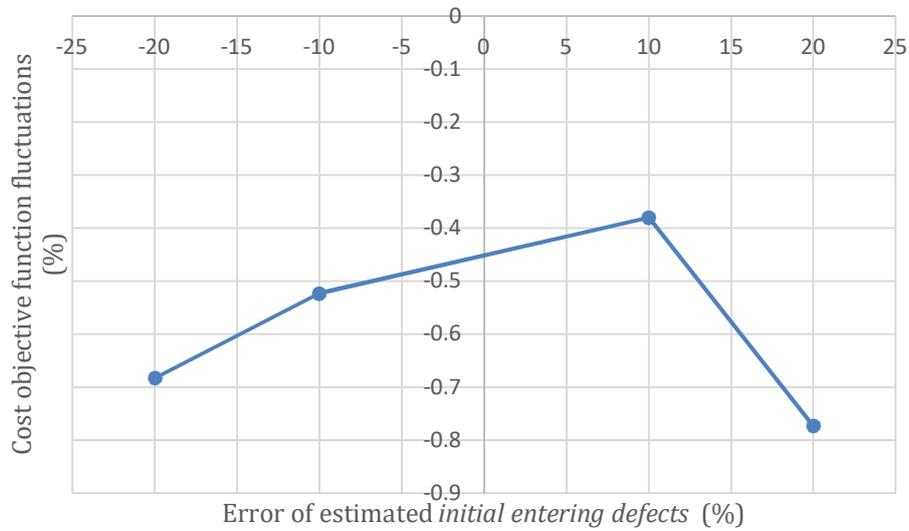
(a)



(b)



(c)



(d)

Figure 5.9. Sensitivity of the total cost objective function on the α (a), β (b), ϵ (c), and ϵ_0 (d) parameter

In this section, we utilize the 4-segment linear approximation and not go further (e.g., 6-segment and more) because of the obtained same accuracy after applying 4-segment approximation. It is notable that each run for obtaining a Pareto optimal solution takes time around 24 hours and the decision-maker is satisfied to select between these fives, but one may prefer more options to select and obtain more Pareto solutions as it is possible by consuming more time.

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Figure 5.10 and Table 5.17 show the obtained Pareto curve for the case study. As can be seen, by improving the system productivity, the total cost is getting worse. Since there is no optimal solution for the multi-objective problems, one of the Pareto optimal solutions should be selected based on the decision-maker's preference. The decision maker prefers the third Pareto solution (see Table 5.18 for the details of the solution) because the company can increase its system productivity around 15% through investing about 4% more. The fourth solution also can be a preferable solution for the decision maker, but the company is not able to invest 5% and more.



Figure 5.10. Obtained Pareto curve for the case study

Table 5.17. Pareto-optimal solutions for the case study

No	Epsilon	Total cost (\$)	System productivity
1	72330.661	63027.01	11.995
2	70554.491	63027.01	11.995
3	68778.322	62026.94	11.244
4	67002.152	60446.32	10.429
5	65225.983	58829.67	9.631

Chapter 5: Experimental Results

Table 5.18. Third Pareto-optimal solution: Places and periods in which inspection (I) and preventive maintenance (M) activities are performed

		Stage														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Period	1	M	M	M	M,I	M	M	M	M	M	M	M	M	M	M	M
	2	I	M	M	M	M	M	M	M	M	M	M	M	M	M	M
	3			I	M	M	M	M	M	M	M	I	M	M	M	M
	4	M	M	M	M	M	M	M	M	M	M	I	M	M	M	M
	5	M	M	M	M	M	M	M	M	M	M,I	M	M	M	M	M
	6	M	I	M	M	M	M	M	M	M	M	M	M	M	M	M
	7	M	M	M	M	M	M		M	M	M,I	M	M	M	M	M
	8	M	M	M	M	M	M	M	M	M	M,I	M	M	M	M	M
	9	M	M	M	M	I	M	M	M	M	M	M	M	M	M	M
	10	M	M	M	M	M	M	M	M	M	M	M,I	M	M	M	M
	11	M	M	M	M	M	M	M	M		M	I	M	M	M	M
	12	M	M	M	M	M	M	M			M,I	M	M	M	M	M

5.3. Conclusion

In this chapter, the proposed mathematical models (i.e., [Main Model](#) and [Extended Model](#)) in the previous chapter were verified and validated by investigating a numerical example and a real case study, respectively.

[Section 5.1.1](#) showed the cost-efficiency of developing an integrated plan for the part quality inspection and PM activities (i.e., [Main Model](#)) in comparison with the conventional PQIP. In addition, the significance of the two input parameters (i.e., the inspection and PM cost) was justified through a sensitivity study. The importance of the system productivity objective function and the RPP-II approach (which were incorporated into the presented [Extended Model](#)) was regarded in [Section 5.1.2](#). Furthermore, the accuracy of the piecewise linear approximation technique was tested while considering different numbers of linear segments.

[Section 5.2](#) tried to present the applicability of the presented mathematical models through a real case study in an automotive industry in France. The results confirmed the effectiveness of the developed models and specially in the presence of costs and demand uncertainties in a real manufacturing environment. The sensitivity of [Extended model](#) regarding the different values for the input parameters, was shown and the best-fitted values for the parameters of the RPP-II approach (i.e., γ and δ) were achieved. Finally, the obtained Pareto optimal solutions were presented and the pareto carve was analyzed to help the decision-maker in the selection process. The managerial insight of this investigated real case and the other concluding remarks of the presented experimental study are elaborated in the next

section.

Chapter 6

Conclusion and Future Research Direction

6.0. Chapter purpose and outline

In this chapter, [Section 6.1](#) draws the obtained conclusions from the developed [Main Model](#) (please see [Sub-section 6.1.1](#)) and [Extended Model](#) (please see [Sub-section 6.1.2](#)). Then, some future directions are recommended to work on based on the *MMS characteristics* (please see [Sub-section 6.2.1](#)) and the *Methodology characteristics* (please see [Sub-section 6.2.2](#)).

6.1. Conclusion

The obtained conclusions of this thesis are categorized into two subsections regarding the developed models as follows:

6.1.1. *Main model*

- The proposed [Main Model](#), which provides an integrated planning of part quality inspection and PM activities, results in a considerable cost saving for SMSPLs. For instance, in the considered case study, applying the proposed integrated planning decreased the costs around 4.82% in comparison with the situation that the company used the separate planning.
- The number of produced conforming final products is increased by usage of the integrated planning. In the considered case study, it is about 7.16%.
- The integrated planning imposed more PM cost to the company, but this cost acts as an investment and reduces the scrap and production costs in a way that the total manufacturing cost is decreased.
- The unit inspection and PM cost are two important input parameters of the presented model because the proportion of them impacts on the amount of cost saving. In addition, the other cost parameters almost have the same influence. Hence, it is significant to estimate these parameters properly.

6.1.2. *Extended model*

- The [Extended Model](#)'s optimum solution costs 13.46% more than the [Main Model](#)'s; however, it can respond to all the realizations and decreases the

- feasibility risk around 50% and it provides a robust solution for any realization of the cost components and demand amount.
- In the piecewise linear approximation technique, applying more segments to approximate the nonlinearity results more accuracy and run time. In the case study, the 4-segment linear approximation is used and not go further (e.g., 6-segment and more) because of the obtained same accuracy after applying 4-segment approximation.
 - If the oil housing pump manufacturer undergoes 20% fluctuations for the α , β , ε , and ε_0 parameter, the changes of the total cost are not more than 1.7%, which is negligible. Hence, it can be concluded that the obtained solution by the **Extended Model** is remained robust under 20% miscalculation/underestimate of the deterministic parameters.
 - The **Extended Model** has the best performance when the value of γ is considered equal to 0.5 for the oil housing pump manufacturer. Because the least risk of imposed additional cost in the worst case while the obtained confidence level of chance constraint is maximum. On the other side, the expected total cost and the worst-case cost have the lowest amount which confirms cost-efficiency of this value.
 - The best value which can be considered for δ is 100 because the **Extended Model** reaches not only the highest confidence level ($\alpha = 1$), but also the least values for average and worst-case total cost. In other words, the solution obtained by considering $\delta = 100$, is the most risk-averse and cost-efficient solution for the case study.
 - Since there is no optimal solution for the multi-objective problems, one of the pareto optimal solutions should be selected based on the decision-maker's preference. Generally, by improving the system productivity, the total cost is getting worse.

6.2. Future research directions

Based on the recognized shortcomings in the literature and current research work, some directions for future research are provided regarding the system and methodology characteristics:

6.2.1. *MMS characteristics*

- Most of the existing research works have focused on the PQIP optimization in serial production systems, while most of the actual production systems follow a combination of serial and non-serial structures.
- Few papers have investigated multi-product MMS with different quality characteristics, while most of the current production systems process contains different quality characteristics of multiple products.
- Incorporating the inspection tool selection problem into the classical PQIP makes the model more real and obtains more flexible inspection plan. So, manufacturers can apply inspection tools with higher precision and decrease the number of non-detected items that reach customers and consequently raise customer satisfaction.
- Considering machine selection decision at the same time as PQIP decisions causes that manufacturer can purchase machines with high capability to obtain high-quality level for essential design characteristics.

6.2.2. *Methodology characteristics*

- Developing more multi-objective PQIP models, which concurrently consider multiple conflict objectives. For example, development of a new multi-objective model to establish a trade-off among production cycle time, average outgoing quality and total cost seems so interesting and practical.
- By considering time as an objective, one significant issue that comes up is waiting time for the WIP items. Different items must wait before each processing or inspection station to receive services. These waiting times should be analyzed in the final decisions.
- Since MMSs are stochastic in nature and are impacted by different unpredictable environmental factors, machines and inspection tools are subject to disruption. Any breakdown in the production system not only increases the manufacturing cost but also significantly affects the quality of final products. Therefore, considering the reliability issue of a production system and investigating the effect of unreliable machines and inspection tools on the final PQIP could be an interesting research direction.

- Almost all the authors have ignored manufacturing constraints in their studies. Some of these constraints could be the capacity of machines and inspection tools, minimum conforming items, an upper bound for total production time, low capital for initial investment and limited places for performing inspections and so on. Considering these constraints provides more real and applicable inspection plans.
- Most of the existing research works have not taken the uncertainty of system into account. These uncertainties are of different parameters such as defective production rate and inspection times, Type I and II errors and lots of other related parameters. Taking these uncertainties into account provides a more realistic optimization of inspection plans and leads to more appropriate solutions for inspection planning systems.
- Developing efficient decomposition algorithms to achieve exact solutions is also another gap in the literature for solving large size problems.
- Recently, some researchers have tried to design manufacturing systems which are psychologically-consistent ([Azadeh et al., 2015b](#); [Rezaei-Malek et al., 2017](#); [Azadeh et al., 2017](#)). Since operators play critical roles in the inspection process in the most of manufacturing systems, it would be interesting if one can incorporate this new concept and its impact on the inspection accuracy into the PQIP problem.

List of the publications related to this thesis

- Rezaei-Malek, M., Siadat, A., Dantan, J.-Y, Tavakkoli-Moghaddam, R., 2018. A trade-off between productivity and cost for the integrated part quality inspection and preventive maintenance planning under uncertainty. *International Journal of Production Research*, <https://doi.org/10.1080/00207543.2018.1556411>.
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- Rezaei-Malek, M., Tavakkoli-Moghaddam, R., Siadat, A., Dantan, J.-Y, 2018., A novel model for the integrated planning of part quality inspection and preventive maintenance in a linear-deteriorating serial multi-stage manufacturing system. *The International Journal of Advanced Manufacturing Technology*, 96(9-12), 3633–3650.
- Rezaei-Malek, M., Siadat, A., Dantan, J.-Y, Tavakkoli-Moghaddam, R., 2018. An approximation approach for an integrated part quality inspection and preventive maintenance planning in a nonlinear deteriorating serial multi-stage manufacturing system. *IFAC-PapersOnLine*, 51(11), 270–275.
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Planification des activités d'inspection sous incertitude basée sur les conditions de maintenance préventive d'un système de production série

RESUME : La planification de l'inspection de la qualité des pièces (PQIP) est un problème important dans les systèmes de production. En raison du fort impact des activités de maintenance préventive (PM) sur le taux de non-conformité des produits (un des principaux paramètres du problème PQIP), il est nécessaire de développer une planification intégrée des activités d'inspection et les activités de maintenance afin d'obtenir une optimisation globale du système. Cette thèse présente des modèles mathématiques de programmation linéaire à nombres entiers mixtes pour le problème de planification intégrée du contrôle de la qualité et des activités de gestion de maintenance dans un système de production en série à plusieurs étapes. Les modèles déterminent simultanément le moment et l'endroit optimaux pour effectuer les activités susmentionnées en prenant en compte la détérioration des étapes de production. Ces deux décisions sont effectuées via la minimisation du coût total (y compris la production, la maintenance, l'inspection, la mise au rebut, la réparation et la pénalité des articles défectueux expédiés au client) et ainsi elles optimisent la productivité du système. Dans ce cas, la productivité du système est formulée comme une mesure non linéaire, puis linéarisée par la technique d'approximation linéaire par morceaux. En plus, l'incertitude relative à l'estimation des composantes de coût et de la quantité demandée est gérée par une approche possibiliste robuste. Un exemple numérique et une étude de cas réelle sont étudiés pour valider et vérifier les modèles proposés. Le résultat le plus important de cette recherche est que la détermination des lieux d'inspection le long d'un processus de fabrication à différentes périodes avec la prise en compte d'impact des activités de maintenance préventive sur le taux de production défectueuses se traduit par une amélioration significative de la performance du système de production.

Mots clés : Optimisation de système de production, Inspection de la qualité, Maintenance préventive, Modélisation du système de fabrication en série à plusieurs étapes, Planification intégrée, Programmation robuste, Optimisation multi-objectifs.

Inspection planning based on preventive maintenance condition in a serial multi-stage manufacturing system under uncertainty

ABSTRACT: Part Quality Inspection Planning (PQIP) is a significant problem in multi-stage manufacturing systems. Because of an existing strong impact of Preventive Maintenance (PM) activities on the defective production rate (which is the main input of the PQIP), developing an integrated planning for the part quality inspection and PM protects system from a local optimum. This thesis presents mixed-integer linear programming models for the integrated planning problem of the part quality inspection and PM activities in a serial multi-stage manufacturing system. The models concurrently determine the right time and place for performing the above-mentioned activities while the stages are deteriorating. These two decisions are made while the models are to minimize the total cost (including the production, PM, inspection, scrap, repair, and the penalty of shipped defective items) and maximize system productivity. Notably, the system productivity is formulized as a non-linear measure, and then it is linearized by the piecewise linear approximation technique. In addition, the uncertainty about the estimation of cost components and demand is handled by a robust possibilistic approach. A numerical example and a real case study are investigated to validate and verify the proposed models. The most important result of this research is that the determination of inspection locations along a manufacturing line in different periods of time regarding the impact of preventive maintenance activities on defective production probability results in a more efficient system.

Keywords : Production system optimization, Part quality inspection, Preventive maintenance, Serial multi-stage manufacturing system modeling, Integrated planning, Robust programming, Multi-objective optimization.