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## On the clustering of mobile ad hoc networks

Raphaël Massin

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*présentée et soutenue publiquement par*

**Raphaël MASSIN**

le 8 novembre 2016

## On the Clustering of Mobile Ad Hoc Networks

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Raphaël Massin, le 9 décembre 2016

# Abstract

This thesis is about clustering of mobile ad hoc networks, which consists in building some sets of nodes, called clusters, in order to introduce hierarchy in the network and thus improve its scalability. The main goal is to design new distributed clustering algorithms suited to i) unstructured networks, where all the nodes are equal, and suited to ii) structured networks that have an inherent hierarchical structure, and in which the nodes are gathered in operational groups. In order to allow the implementation of a radio resource allocation process that is more efficient within clusters than between clusters, the proposed algorithms form clusters satisfying specific topology constraints: connectivity, maximum size and diameter. In the first part of the thesis, to compare these new solutions to the ones from the literature, independently to the medium access scheme, we introduce network cost functions which take into account the user traffic profile and the intra-cluster and inter-cluster communication costs. Then, we propose a distributed clustering algorithm suited to structured networks, and compare its performance to several clustering schemes from the literature. A salient feature of this algorithm is that it does not need to resort to the notion of cluster-head. In the last part, thanks to the coalition game theory we revisit this algorithm. This theoretical framework allows us to formalize the clustering problem in a more general context. This leads us to defining a generic algorithm suitable to any kind of ad hoc network, and enables us to acquire a better knowledge of its properties.



# Résumé

Cette thèse traite de la clusterisation des réseaux ad hoc mobiles. Ce mécanisme consiste à rassembler les nœuds du réseau en grappes appelés clusters, dans le but d'introduire de la hiérarchie dans le réseau, et ainsi de permettre son passage à l'échelle. L'objectif principal est de concevoir de nouveaux algorithmes distribués de clusterisation, adaptés aux réseaux non structurés, où tous les nœuds sont des pairs, ainsi qu'aux réseaux structurés, où il existe déjà une structure hiérarchique intrinsèque. Dans le but de permettre une allocation des ressources radio à l'intérieur des clusters plus efficace qu'entre les clusters, les algorithmes proposés forment des clusters qui satisfont certaines contraintes de topologie : la connectivité, une taille et un diamètre maximaux. Afin d'évaluer les performances de ces nouvelles solutions, comparative-ment à celles de la littérature, et de manière indépendante des mécanismes d'accès au canal radio employés, la première partie de la thèse introduit des fonctions de coût de réseau, qui incorporent le profil de trafic utilisateur ainsi que les coûts de communication intra-cluster. Ensuite, un algorithme distribué adapté aux réseaux structurés est proposé, et ses performances comparées par simulation à plusieurs autres solutions de la littérature. Une caractéristique originale de cet algorithme est qu'il ne fait pas appel à la notion de chef de cluster. Dans la dernière partie, grâce à la théorie des jeux de coalition nous revisitons l'algorithme précédemment proposé pour les réseaux structurés. Ce cadre théorique permet de formaliser le problème de la clusterisation dans un contexte plus général, et conduit à la définition d'un algorithme générique, applicable à tous types de réseaux, ainsi qu'à une meilleure connaissance théorique de ses propriétés.





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# Acronyms

1G1C	One-Group One-Cluster (clustering algorithm)
ARQ	Automatic Repeat reQuest
BFS	Breadth First Search
cdf	Cumulative Distribution Function
CDMA	Code Division Multiple Access
CD	Community Detection
CGD	Cluster Group Diversity
CH	Cluster-Head
CLQ	Clustering with Link Quality (clustering algorithm)
COG	Clustering with Operational Groups (clustering algorithm)
COMELEC	Communications et Électronique (department of Télécom ParisTech)
CSI	Channel State Indication
D2D	Device-to-Device
DCOG	Distributed Clustering with Operational Groups (clustering algorithm)
DGMA	Distributed Group Mobility Adaptive (clustering algorithm)
DSP	Dijkstra's Shortest Path
GCD	Group Cluster Diversity
GDMAC	Generalized Distributed Mobility Adaptive Clustering (clustering algorithm)
GMCA	Group Mobility-based Clustering Algorithm (clustering algorithm)
GPS	Global Positioning System
GW	Gateway
HC	Highest Connectivity (clustering algorithm)
LACA	Learning Automata-based Clustering Algorithm (clustering algorithm)
LCC	Least Cluster Change (clustering algorithm)
LID	Lowest Identifier (clustering algorithm)
LMS	Least Mean Squares
MAC	Medium Access Control
MCS	Modulation and Coding Scheme
NTU	Non Transferable Utility
OFDMA	Orthogonal Frequency Multiple Access
OLSR	Optimal Link State Routing
pdf	Probability Distribution Function
QoS	Quality of Service

QOLSR	QoS Optimal Link State Routing
RA	Resource Allocator
RPGM	Reference Point Group Mobility Model
RRA	Radio Resource Allocator
RSU	RoadSide Unit
SBS	Small cell Base Station
SECA	Signal Efficient Clustering Algorithm (clustering algorithm)
SINR	Signal Interference Noise Ratio
STR	Secteur Temps Réel (departement of Thales Communications & Security)
SU	Secondary User
TCA	Type-based Clustering Algorithm (clustering algorithm)
TDMA	Time Division Multiple Access
TU	Transferable Utility
VANET	Vehicular Network
V2V	Vehicle-to-Vehicle
VOTE	Vote-based (clustering algorithm)
WiFi	Wireless Fidelity
WSN	Wireless Sensor Networks

# Notations List

$:=$	By definition
$\alpha$	Probability that a node communicates with a member of its group
$c(\mathcal{C}_k)$	Cost of coalition $\mathcal{C}_k$
$c_i(\mathcal{C}_k)$	Cost of cluster $\mathcal{C}_k$ , calculated with cost function $c_i$
$\mathcal{C}_k$	$k^{\text{th}}$ cluster in a partition $p$ of $\mathcal{G}$ , such that $\forall k_1 \neq k_2, \mathcal{C}_{k_1} \cap \mathcal{C}_{k_2} = \emptyset$ and $\cup_{k=1}^{N_c} \mathcal{C}_k = \mathcal{V}$
$d_k$	Diameter of cluster $\mathcal{C}_k$
$d_{max}$	Maximum cluster diameter
$\mathcal{E}$	Set of edges in a graph $\mathcal{G}$
$\gamma$	Ratio of inter-cluster communication cost on intra-cluster communication cost
$\Gamma(i, j)$	The SNR measured on receiver node $j$ associated to a transmission from its neighbor node $i$
$g(\sigma_{k,\ell}(\mathcal{P}))$	Gain associated to the switch operation $\sigma_{k,\ell}(\mathcal{P})$
$g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)$	Gain associated to nodes $\{u_i\}$ leaving cluster $\mathcal{C}_k$ to join cluster $\mathcal{C}_\ell$
$\mathcal{G}$	Graph defined by its set of nodes $\mathcal{V}$ and its set of edges $\mathcal{E}$
$\mathcal{I}(\mathcal{C}_k)$	Set of the indices of the groups with at least one member in cluster $\mathcal{C}_k$
$J_0$	Network cost function based on additive metrics used to measure the quality of a partition $p \in \mathcal{P}$
$J_i$	$J_i := \sum_{k=1}^{N_c} c_i(\mathcal{C}_k)$ , with $i > 0$
$J_0^L$	Variant of $J_0$ which does not take into account the cluster structure to find the end-to-end shortest paths
$J_0^X$	Variant of $J_0$ which uses the cluster structure to find the end-to-end shortest paths
$J_0(p, i, j)$	Cost of communication from source node $i$ to destination node $j$ within a partition $p$
$m_t$	Number of members of group $\mathcal{O}_t$
$m_{t,k}$	Number of members of group $\mathcal{O}_t$ in cluster $\mathcal{C}_k$
$M$	Number of edges in a graph $\mathcal{G}$ , $M :=  \mathcal{E} $
$n_k$	Size, i.e., number of members, of cluster $\mathcal{C}_k$
$n_{max}$	Maximum cluster size
$N$	Number of vertices in a graph $\mathcal{G}$ , $N :=  \mathcal{V} $
$N_c$	Number of clusters in a partition $p$ of $\mathcal{G}$



$\mathcal{O}$	Set of groups in a structured network
$\mathcal{O}_t$	$t^{\text{th}}$ group in a set of groups $\mathcal{O}$ , such that $\forall t_1 \neq t_2, \mathcal{O}_{t_1} \cap \mathcal{O}_{t_2} = \emptyset$ and $\cup_{t=1}^T \mathcal{O}_t = \mathcal{V}$
$\pi_{j i}$	Probability of communication to node $j$ from node $i$ , with $\pi_{i i} = 0$ and $\sum_{j \in \mathcal{V}} \pi_{j i} = 1$
$p$	A partition of a graph $\mathcal{G}$
$\mathcal{P}$	Set of all partitions of a graph $\mathcal{G}$
$\sigma_{k,\ell}(\mathcal{P})$	Switch operation of nodes $\mathcal{P}$ leaving their cluster $\mathcal{C}_k$ to join a cluster $\mathcal{C}_\ell$
$T$	Number of groups in a set of groups $\mathcal{O}$
$T_0^X$	Network cost function based on the link throughput concave metric used to measure the quality of a partition $p \in \mathcal{P}$ . Uses the cluster structure to find the end-to-end shortest widest paths
$u(\mathcal{C}_k)$	Utility of coalition $\mathcal{C}_k$
$v(\mathcal{C}_k)$	Value of coalition $\mathcal{C}_k$ , $v(\mathcal{C}_k) = u(\mathcal{C}_k) - c(\mathcal{C}_k)$
$\mathcal{V}$	Set of nodes in a graph $\mathcal{G}$
$w_{i,j}$	Dimensionless weight of a link $(i, j)$

# Glossary of simulated clustering algorithms

This chapter provides a brief description of the clustering algorithms simulated in this thesis. Note that all CH-based algorithms form clusters whose members are 1-hop neighbors of the CH node.

- One-Group One-Cluster (1G1C): Centralized algorithm whose goal is to force all members of a group to be in the same cluster as soon as the cluster size and cluster diameter constraints can be satisfied. Defined in Chapter 1.
- Clustering with Link Quality (CLQ): Distributed algorithm which gathers the nodes such that the cumulated intra-cluster link capacity is as large as possible, subject to cluster size and cluster diameter constraints. No CH node. Defined in Chapter 3.
- Clustering with Operational Groups (COG): Distributed algorithm whose goal is to form clusters including entire groups, subject to cluster size and cluster diameter constraints. No CH node. Defined in Chapter 3.
- Distributed Clustering with Operational Groups (DCOG): Distributed algorithm whose goal is to form clusters including entire groups, subject to cluster size and cluster diameter constraints. No CH node. Defined in Chapter 2.
- Generalized Distributed Mobility Adaptive Clustering (GDMAC): Distributed CH-based algorithm. Within a 1-hop neighborhood, select as CH the nodes with the largest identifiers. Allows multiple neighbor CH nodes. Defined in [1].
- Least Cluster Change (LCC): Modification of the LID algorithm to improve its stability. With LCC re-clustering is only performed when two CH nodes become neighbors, or if a non-CH node is no longer in range of its CH. Defined in [2].
- Lowest Identifier (LID): Distributed CH-based algorithm. Within a 1-hop neighborhood, select as CH the node with the lowest identifier. Prevents two CH-nodes to remain CH when they become neighbor. Defined in [3].
- Signal Efficient Clustering Algorithm (SECA): Distributed CH-based algorithm. Within a 1-hop neighborhood, select as CH the node with the largest quality value, calculated as a weighted average of *i*) node average link quality (based on received signal strength), *ii*) node relative mobility, and *iii*) node degree. Defined in [4].

- Type-based Clustering Algorithm (TCA): Distributed CH-based algorithm. Within a 1-hop neighborhood, select as CH the node with the largest stability factor, calculated as a weighted average of various metrics. When calculating the stability factor of a node belonging to a given group, the contribution of its neighbors also belonging to the same group is increased. Defined in [5].
- Vote-based clustering algorithm (VOTE): Distributed CH-based algorithm. Within a 1-hop neighborhood, select as CH the node with the largest vote value, calculated as a weighted average of *i*) node degree and *ii*) battery remaining time. This algorithm builds cluster subject to a maximum cluster size constraint. Defined in [6].

# General Introduction

## Problem statement

The work presented in this PhD thesis has been produced thanks to the collaboration between the department "Communications et Électronique" (COMELEC) of Télécom ParisTech and the "Secteur Temps Réel" (STR) of Thales Communications and Security, in the framework of a "thèse en situation de travail", i.e., a PhD while working. This thesis is devoted to the study of ad hoc network clustering, which consists in building sets of nodes called clusters in order to introduce hierarchy in the network and thus improve its scalability.

An ad hoc network is an infrastructure-less multi-hop wireless network in which each node participates in routing by forwarding data for other nodes. Those networks are self-organizing and are used when usual infrastructure based networks are not available or not suitable, e.g., in wireless sensor networks (WSN), vehicular networks (VANET) or military networks. In order to implement practical large ad hoc networks, gathering nodes in clusters was proposed in the early 80s for networking purposes, and then in the 90s to sustain good Quality of Service (QoS). Notably, clustering the network has been proposed in [3] as a way to enable the rapid deployment and dynamic reconfiguration of a scalable wireless network, with support of multimedia applications combining real-time and bursty traffic. In [7], a cluster-based multichannel communications system for VANETs is detailed. In this scheme, the elected cluster-head (CH) operates as the coordinator to collect/deliver the real-time safety messages within its own cluster, and forward the consolidated safety messages to the neighboring CHs. The CH also controls channel assignments for cluster members transmitting/receiving the non-real-time traffics, to improve radio resource allocation (RRA) efficiency. In [8], the authors propose a cluster-based framework to form a wireless mesh network in the context of open spectrum sharing, where the nodes are secondary users of spectrum. When a node not yet member of any cluster finds a channel that is unused by primary users, it becomes CH and invites neighbor nodes sharing the same channel to join the new cluster. The CH node is responsible for intra-cluster channel access control and inter-cluster communications. Then, by negotiating gateway (GW) nodes between clusters, clusters are interconnected into a large network. A clustered network is proposed in [9] to implement large cognitive radio networks, in which the clustering scheme works in conjunction with a network coded cognitive control channel in order to allow cognitive radio devices to opportunistically access the unused spectrum. From a network perspective, cluster-based hierarchical routing has been proved to introduce exponential savings in the amount of information to be stored and exchanged in a large ad hoc network [10], thus allowing the routing to scale. Cellular networks have also used an approach similar to clustering to enable the deployment of

femtocells [11].

Operating a clustered ad hoc network necessitates running continuously and in parallel several processes.

- The first one, the focus of this thesis, is cluster building. During this process all nodes detect the nodes in their radio neighborhood, and exchange enough information to form the clusters.
- The second one is intra-cluster *radio resource allocation* (RRA). Within a cluster a node must be selected to be the *resource allocator* (RA) node. Similarly to what is done by base stations in cellular systems, the RA node learns about the link states between its cluster members, collects the resource requests sent from all cluster members, and locally optimizes radio resource allocation guided by appropriate goals such as maximizing the throughput, ensuring fairness, minimizing intra-cluster delay, etc.
- The third process is inter-cluster radio resource allocation. This process consists in the selection of the nodes and links used to perform inter-cluster communications, and in the associated RRA. Contrarily to intra-cluster RRA, this process involves multiple clusters and is more difficult to optimize. Following from this observation, intra-cluster RRA is more efficient than inter-cluster RRA, which explains the importance of forming the clusters cleverly.
- The fourth process is particularly important when the amount of radio resources is limited; it consists in allocating a set of radio resources to each cluster. If we consider for example TDMA, a set of radio resources could be a set of slots, associated with a carrier frequency. This step can be called graph coloring, because a set of radio resources can be associated to a color. Graph coloring must be performed such as to reduce the interference received by neighboring clusters. Achieving this goal allows each cluster to perform intra-cluster radio resource allocation without worrying about what happens in the other clusters.

One may envision a joint optimization of these four processes. Because the protocols and algorithms used to implement an ad hoc network should be fully distributed, such optimization does not seem feasible to us. Therefore, we have decided to handle separately those different issues, this thesis focusing on the cluster building one. Note that the four processes mentioned above mainly concern the lower layers of the protocol stack, i.e., physical and data link layer, and possibly the network layer depending on how the responsibilities are shared between the data link and network layers.

Let us now introduce a key assumption underlying the work presented in this document. Whatever the RRA allocation scheme, its performance is always dependent on the topology of the network, and the larger the cluster, the less efficient the RRA. To make sure that the intra-cluster RRA is efficient, we impose some basic topological *constraints* on the cluster topology. To explain these constraints, we model the network as a graph where each node is a vertex, and where an edge between two vertices means that the corresponding nodes are in radio range. We define the cluster constraints as following:

- The induced subgraph of the cluster members must be connected, meaning that only intra-cluster links are required to perform communication between two cluster members.
- The cluster size, i.e., the number of cluster members, must be limited to control the signaling overhead required by the RRA.

- The cluster diameter, i.e., the length of the longest shortest path between any pair of the vertices in the induced subgraph of the cluster, must be limited, to prevent the delay and cost of communication between cluster members from increasing too much.

Usually, in the literature, ad hoc networks are unstructured: there is no special organization of the network and all nodes share the same operational role, i.e., are equal to each other. In this thesis, beyond these usual unstructured networks, we also study the structured networks that have an inherent hierarchical structure associated with their *raison d'être*, and in which single nodes are gathered in *operational groups* (e.g., squad, section). Examples of such networks are public safety and military networks. For the sake of readability, in the sequel *group* refers to operational group. The existence of groups raises two major differences with respect to unstructured networks. First, the traffic is strongly dependent on the network hierarchical organization, being mostly concentrated within groups. Second, the nodes from the same group are very likely to move in the same direction. For these two reasons, we will show that it is beneficial to design clustering solutions that take into account group information, in order to provide more stable networks as well as better end-to-end QoS.

Most existing work about ad hoc network clustering has focused on unstructured networks. For example in [3] the authors propose the LID and HC clustering algorithms where nodes with the lowest identifier, respectively the highest degree, become CH. Non-CH nodes affiliate to their neighbor CH with the lowest identifier, respectively the highest degree. The stability of the clusters formed with LID or HC, which suffer from the *ripple effect*<sup>1</sup>, has been improved in [2] thanks to the LCC mechanism that only performs re-clustering when multiple CH nodes become neighbors. The GDMAC algorithm detailed in [1] can be seen as a generalization of LCC, allowing up to  $K$  CH nodes to be neighbors. In the VOTE proposal [6], non-CH nodes join a CH only if the number of its cluster members is below a threshold, thus limiting the cluster size. Thanks to the knowledge of node location information, DGMA [12] attempts to estimate nodes relative mobilities and capture group mobility patterns to form stable clusters. A similar approach is followed by LACA [13], which proposes to build clusters using past, current and predicted nodes' positions thanks to the help of a learning automaton, and by GMCA [14] which uses a Gauss-Markov model to calculate the velocity and direction of the nodes. The novelty of SECA [4], lies in the introduction of link qualities (based on received signal strength) combined with the nodes relative mobilities to determine if a node becomes CH. Authors in [15] use centralized genetic algorithms and particle swarm optimization to select stable CH. The above examples constitute a representative extract of a rich literature which shows that a lot of research was done, and is still being done about clustering in unstructured ad hoc networks.

In contrast, only very few papers tackle the problem of clustering in structured networks. The authors in [5] introduce the type-based clustering algorithm (TCA). This clustering scheme associates a stability factor to each node and selects as CH the nodes that have the highest stability factor in a radio neighborhood. The stability factor takes group membership (identified thanks to the IP subnet of each node) into account. A limitation of TCA lies in the fact that two CH nodes cannot be neighbors. A direct consequence in dense networks is the formation of large clusters (with a lot of members). A second example is detailed in [16] which proposes

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<sup>1</sup>The expression ripple effect describes a drawback of some clustering schemes for which a local cluster modification somewhere in the network leads to numerous cluster modifications in the whole network.

a topology management mechanism for hierarchical group oriented networks, where groups are based on geographical locations.

According to the previous state of the art we have identified the two following interesting research tracks that would need further investigation and that will be addressed in this thesis:

- **Network performance metrics.** Usually the clustering solutions are compared using metrics focused on the technical details of the clusters themselves (e.g., number of clusters, number of cluster modifications, lifetime of CH nodes, amount of signaling, etc.). These metrics provide only indirect information about the QoS provided to the user. To fill this gap, some authors have selected various metrics derived from user throughput. In that case some assumptions are made about *i*) the type of medium access control (MAC), e.g., any version of WiFi, and *ii*) the user traffic profile. As detailed above, four different processes are required to build and operate a clustered network (cluster building, allocation of a per-cluster set of radio resources, intra-cluster RRA and inter-cluster RRA). From the beginning of the thesis we wanted to be as agnostic as possible about the type of MAC used in the system. We only knew that inter-cluster communications would be implemented in a less efficient way than intra-cluster communications. Consequently, we decided to elaborate a new versatile metric which is able to take into account the clusters, the groups, the costs of intra-cluster and inter-cluster communications, and the traffic profile.
- **Distributed clustering algorithms.** In addition to the gap identified for the structured networks, the clustering algorithms proposed for unstructured networks do not fully satisfy our requirements. Indeed, none of these schemes simultaneously handle both cluster size and intra-cluster link quality. In addition, the availability of node locations and velocities is often a key underlying assumption of the most recent proposals, which cannot always be guaranteed. We thus identified the need for new distributed clustering algorithms to cover the case of structured networks, and in unstructured networks, to form clusters satisfying the topology constraints.

Additionally, the existing literature about ad hoc network clustering lacks a theoretical framework. Looking for such a framework, we found out that coalition game theory should be a relevant one. Indeed, it is a branch of game theory used to study the behavior of players when they cooperate among themselves [17]. Considering coalitions as clusters, we thus decided to use coalition game theory for our research of new clustering algorithms.

Coalition game theory has been used for many different applications. For example in [18] an algorithm is proposed for cognitive networks with primary and secondary users (SU) to form disjoint coalitions of SUs, in order to perform collaborative sensing while maximizing the utility in terms of probability of detection and accounting for a false alarm cost. In [19] the players are small cell base stations (SBS) that cooperate to share their radio resources, in order to deal with OFDMA downlink co-tier interference suffered by the small cell user devices from neighboring SBSs. The SBSs form overlapping coalitions and dedicate part of their frequency resources (OFDMA subchannels) to each coalition they belong to, in order to maximize the sum rate. Proposals suited to VANET have also been published. For example, in [20], the roadside units (RSU) receive a payoff based on the amount and class of data sent to vehicles

moving in their area. Considering that the moving vehicles can use the underlying vehicle-to-vehicle (V2V) network to exchange data received from the RSUs, the RSUs can form coalitions in order to diversify the classes of data they transmit to the vehicles. This cooperation allows them to increase their revenue. In [21], the vehicles use a coalition formation algorithm to build clusters in order to optimize the V2V exchange of context information about road conditions or driving safety. Within a cluster, the CH gathers information from all its cluster members, performs data fusion and then sends back the result. The authors of [22] propose a distributed algorithm to achieve cooperative communications in ad hoc networks (with multiple sources and one destination, similarly to a WSN), in order to increase the achievable rate. Within a coalition the communications are performed over two phases: the broadcasting and the cooperation phases. During each slot of the broadcasting phase, one coalition member performs broadcast transmission, while the other coalition members are listening. Then, during the cooperation phase, all coalition members transmit a linearly coded signal of all signals received during the previous broadcasting phase, and the destination performs multiuser detection to extract the per node signal. Device-to-device (D2D) communications are the subject of [23] which proposes a distributed resource management scheme to jointly solve the problem of resource sharing mode selection and spectrum sharing. Here sharing mode can be *i*) cellular mode, *ii*) dedicated D2D mode and *iii*) hybrid mode, where D2D reuse the resources of some cellular links. Single or pairs of cellular users form coalitions to improve the achievable sum rate of the D2D system. These examples confirm that coalitional game theory is a relevant framework to study ad hoc network clustering.

## Outline and contributions

In this section, we give the thesis outline, and we mention the most important results.

In Chapter 1 we define network cost functions to benchmark clustering solutions from a system point of view. We want these cost functions to take into account the **costs of communications along end-to-end paths**. To do that, we start by recalling that the quality of network paths depend on the routing capabilities in the network layer, and justify why a network cost function used to assess the system performance should take them into account. We then consider in detail the case of additive metrics (e.g., delay) and define a **novel network cost function  $J_0$  incorporating the traffic structure and the inter-cluster communication cost**. We then apply it to structured networks, and perform a detailed numerical analysis showing that **the clustering solutions providing the best QoS to the end-user depend on the group structure**. Then we extend  $J_0$  to better handle the constraint that cluster size should be limited. Finally we define the network cost function  $T_0$ , derived from  $J_0$ , and suited to the throughput metric.

In Chapter 2 we design a **distributed clustering algorithm suited to structured networks**. First, we identify the key parameters that should be used to build good clusters. Then, we detail the **distributed clustering with operational groups algorithm**, denoted **DCOG**. This algorithm is run at the cluster level and is designed to achieve the following properties: each cluster includes the largest possible number of members of some operational groups, each cluster



size is the closest possible to a given maximum, and the diameter of each cluster is limited to a maximum. A key characteristic of DCOG, which differentiates it from the conventional clustering schemes, is that it **does not need to resort to the notion of CH node**. After proving the **theoretical convergence of DCOG**, we compare by simulation its performance against five other clustering algorithms. The first two algorithms are GDMAC and VOTE, chosen from the literature, and the last three are extensions of GDMAC and VOTE that we designed to better handle the network group structure. Our **simulations** show that DCOG leads to better application level performance (measured thanks to  $J_0$ ) and offers a better stability to mobility.

In Chapter 3, we **revisit DCOG within the coalitional game theory framework**. After specifying how we use the common coalitional game theory notions of **coalition utility, value and cost**, we formally identify coalitions as clusters and players as nodes, and define a **generic coalition formation algorithm for clustering**. This algorithm is run at the node level, and is split in two procedures. The first one performs **switch operations**, i.e., the transfer of some nodes between two clusters, such that the sum of cluster values is always increased, thus guaranteeing **convergence to a Nash-stable partition** of the network. The second one is used when, due to mobility, a cluster no longer satisfies some topology constraints. In that case a switch operation may have to be performed to restore a correct topology. The important design parameters of this algorithm are *i*) the heuristics used to select the candidate sets of nodes involved in the switch operations, and *ii*) the utility and the associated cost functions used to calculate the switch operation gains. Then we customize this generic algorithm for the two cases of **structured and unstructured network**, leading to the **COG and CLQ algorithms** respectively. In each case, we define specific utility functions and node selection heuristics. Finally, we assess the performance of our algorithms by simulation. We first show that **COG builds stable clusters that include most members of one or several groups**. Then, in unstructured networks, we show that **CLQ performs better than LCC, SECA and VOTE**, three clustering schemes from the literature.

## Is clustering really clustering?

Clustering algorithms are also used for several other applications or contexts than the ad hoc networks considered in this thesis. Even if behind all these concepts, the idea is always to gather elements in groups called clusters, it appears that they usually differ along several lines such as the underlying graph topology hypothesis, the theoretical tools used to solve the problem, and the capability to enforce some constraints. We give here some different clustering examples (not meant to be exhaustive) and try to stress the main differences with our problem.

Clustering can be found for instance in *data analysis*. Let us quote A. K. Jain in 1999 [24]: *”Clustering is the unsupervised classification of patterns (observations, data items, or feature vectors) into groups (clusters). The clustering problem has been addressed in many contexts and by researchers in many disciplines; this reflects its broad appeal and usefulness as one of the steps in exploratory data analysis.”* With the rise of *big data*, this quote is even more true than when it was initially written. Within the large scope of data analysis domain, it is worth mentioning the field of *community detection* (CD) [25], where searching for *communities* in a graph is often referred to as clustering. This example of clustering was brought to our attention

thanks to Pierre Borgnat (ENS Lyon) and our colleague Jean-François Marcotorchino (Scientific Director at Thales Communications & Security).

Another example of clustering can be found in the signal processing community [26] where the authors use a *diffusion LMS* algorithm drawn from the field of *distributed adaptive estimation*. It is based upon the fact that nodes receiving noisy versions of a given vector of parameters can converge to a consistent estimation of the parameters by exchanging information between nodes. In this paper, they extend this principle to the case where different groups of nodes receive different vectors of parameters. They observe that the algorithm converges towards a state where information exchanged between nodes that are not receiving the same parameters vanishes (whereas within a group, the nodes still continue to estimate correctly the parameters they receive) and thus separates naturally the set of nodes in clusters.

In CD (unweighted) graphs, vertices are connected when they *share a given relation* (e.g., friends on social network). Thus researching a community can be roughly explained as identifying a group of nodes that are *more* in relationship altogether than with the rest of the remaining part of the graph. In other words, there must be more edges inside the community than edges linking vertices of the community with the rest of the graph. In wireless network (unweighted) graphs, connection between vertices (nodes) follows from the radio coverage and thus depends on the geographical nodes' distribution. In that case, the *relation* is equivalent to *the two nodes are in range*. Moreover, two nodes in range (i.e., two connected vertices) do not necessarily exchange information, which constitutes a difference with the graphs used in community detection. More important, the main difference between the two clustering contexts is that in the case of ad hoc networks, we impose constraints to the resulting cluster topology such as maximal size, maximal diameter, etc., whereas in detection community, the clusters are what they are. To illustrate this difference, let us take the extreme case (but not unrealistic in communications) of a fully connected wireless network graph (leading to a clique). Any community detection algorithm will find obviously only one community, whereas the clustering algorithm will have to find as many clusters as necessary to enforce the constraints (e.g., for a 200 node network, and a maximal cluster size of 10, the minimum number of clusters is equal to 20). Moreover, we also address the problem of clustering taking into account the fact that nodes may follow specific organizations. As a consequence, the clustering algorithms are tailored to include the maximum number of members of the same group inside the same cluster. Up to our knowledge, clustering algorithms from community detection domain do not cover such case. Lastly, it appeared to us that most of the metrics used in CD, and more generally in data analysis, require the knowledge of the whole graph, leading to *centralized* algorithms. In our case, we are looking for *non-centralized* algorithms, i.e., working with local (thus partial) information of the graph. For instance, let us consider the *modularity* measure that is a popular metric used to identify good communities [27]. Using this measure to valued graph in the context of ad hoc networks (e.g. using weights proportional to the link capacity) might work provided constraints can be handled. But computing the modularity needs to know the number of links of the whole graph. Thus, this cannot lead to non-centralized solutions since collecting this information across the network would be prohibitive. We identified recent work in [28–30], where the authors modified the *label propagation* algorithm [31] (issued from the CD domain) to perform distributed clustering in mobile ad hoc networks. However, these proposals do not take into account the cluster size and diameter constraints.

Regarding the clustering example from the signal processing paper, it appears that the context is different from ours and suffers from the main issues listed above for CD and data analysis (except the fact that it is distributed).

To conclude, since the thesis was quickly oriented towards the use of the coalition game theoretical framework, looking thoroughly to clustering algorithms from other domains as the ones mentioned above was clearly out of scope. However, our quick review shows that the other clustering algorithms cannot be applied directly to our problem and suffer from several drawbacks (not distributed, not taking constraints into account, not taking group structure into account, ...). Thus, even though a deep investigation of existing solutions along with compulsory adaptations to our context might lead to interesting algorithms, it is left for further studies (for instance, tweaking the label propagation algorithm to handle the constraints).

## Publications

The work presented in this manuscript has led to the following publications:

### Journal paper

- [32] R. Massin, C. J. Le Martret, and P. Ciblat, “A coalition formation game for distributed node clustering in mobile ad hoc networks,” submitted to *IEEE Transactions on Wireless Communications*.

### Conference paper

- [33] R. Massin, C. J. Le Martret and P. Ciblat, “A network cost function for clustered ad hoc networks: application to group based systems,” in *25th IEEE International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, Washington, D.C., September 2014.
- [34] R. Massin, C. J. Le Martret and P. Ciblat, “Distributed clustering algorithms in group-based ad hoc networks,” in *23rd European Signal Processing Conference (EUSIPCO)*, September 2015 (invited paper).
- [35] R. Massin, C. J. Le Martret and P. Ciblat, “Un Algorithme de clusterisation distribué pour les réseaux ad hoc structurés,” in *25th French GRETSI*, September 2015.
- [36] R. Massin, C. J. Le Martret and P. Ciblat, “Distributed clustering algorithm in dense group-based ad hoc networks,” in *16th IEEE Mediterranean Ad Hoc Networking Workshop (MedHocNet)*, Vilanova i la Geltru, Spain, June 2016.

### International patent

- R. Massin, and C. J. Le Martret, “Procédé pour générer des groupes au sein d’un réseau radio ad hoc,” European patent application EP16155087.6, February 2016.

# Chapter 1

## Metrics for clustered networks performance evaluation

The work presented in this chapter has been partially published in [33].

### 1.1 Introduction

Ad hoc networks clustering has attracted and is still attracting a lot of interest and a lot of algorithms have been proposed. The performance of these clustering schemes are evaluated using various metrics. A first category of metrics is dedicated to assess the performance in static networks. Examples of metrics of this kind are the number of clusters (similar to the cluster size) [1, 4–6, 16], the percentage of nodes that are not connected to the network due to clustering [16], the average path size for any pair of source and destination [16]. Another family of metrics is used to measure the performance in dynamic networks. In this category can be found the number of cluster modifications [5, 6], the lifetime of CH nodes [1, 4, 12], the duration a non CH node is affiliated to a given CH [1], the average number of cluster re-affiliations per second [12], the number of times a node becomes CH or loses its CH state, or a non CH node affiliates to another CH [1, 5]. Some metrics fit both categories, such as the one measuring the number of signaling messages sent [1, 12]. These metrics are focused on the technical details of the clustering solutions and do not give any detail about the QoS provided to the user by the network, which is the most interesting for us.

The end-to-end throughput metric from [3] is more attractive. It is defined as follows:

$$\hat{\Xi} := \sum_{i=1}^Q f_i \frac{\Xi_i}{L_i},$$

with  $Q$  the number of connected components in the graph,  $\Xi_i$  the cumulated link throughput of the  $i$ th connected component  $\mathcal{Q}_i$ .  $L_i$  the average hop length in  $\mathcal{Q}_i$  and  $f_i$  the fraction of node pairs interconnected within  $\mathcal{Q}_i$ . In a connected network this metric is written  $\hat{\Xi} = \Xi/L$  with  $\Xi$  the cumulated link throughput in the network and  $L$  the average hop length in the network. This expression neither takes into account any traffic profile nor the cluster structure of the network.

Thus the aforementioned metrics do not provide any insight on the effect of the clustering solutions on the user applications. The goal of this chapter is therefore twofold: *i*) specify

a way to benchmark clustering solutions from a system point of view, and *ii*) in the case of structured networks, prove the importance of using group membership information to build clusters. Therefore we define several network cost functions based on additive (e.g., delay) or concave (e.g., link throughput) metrics incorporating the traffic structure and the inter-cluster communication cost. Thanks to these functions we show that in the structured networks, the clustering solutions providing the best QoS to the end-user strongly depend on the group structure. Additionally, we find that to provide the best QoS to the user, routing should take the cluster structure into account.

This chapter is organized as follows. In Section 1.2 we first recall some key features of routing in ad hoc networks, and introduce the concept of network cost functions to measure the performance of clustering algorithms. We introduce the notations and the network model in Section 1.3. In Section 1.4 we define the generic network cost function  $J_0$  dedicated to additive metrics, and apply it to structured networks. In Section 1.5 we provide detailed numerical results showing that in such networks, the clustering solutions providing the best QoS should build clusters close to the groups. Then in Section 1.6 we propose an improved version of  $J_0$ , before deriving in Section 1.7 the cost function  $T_0$  dedicated to the link throughput concave metric. We conclude this chapter in Section 1.8.

## 1.2 Concept of global network cost function

### 1.2.1 Routing in ad hoc networks

Within networks, the goal of routing is to allow the end-to-end forwarding of packets from a source to one or more destinations. To find these routes across the network various procedures exist. Let us consider the example of hop by hop routing: for each node along the route from a source to a destination, hop by hop routing finds the next hop to the destination. This next hop can be found using a shortest path algorithm whose goal is to find a route whose cost is the lowest. One well known shortest path algorithm is the Dijkstra's shortest path (DSP) algorithm [37]. The information used by DSP are the nodes and links of the network. Depending on the amount and accuracy of information, the quality of the routes found by DSP is high or low. If only a binary (on/off) information is known about links, then DSP finds shortest routes in term of number of hops. Conversely, if a weight is associated with each link, a route cost can be calculated as the sum of link weights along this route. To illustrate why the latter approach leads to better results, let us take an example. Consider a network with three nodes  $\{N_1, N_2, N_3\}$  and links  $\{(N_1, N_2), (N_2, N_3), (N_1, N_3)\}$ . Looking for a shortest path from node  $N_1$  to node  $N_3$ , the direct one-hop route is shortest compared to the route through  $N_2$  that is two-hop long. Now if the cumulated weight of going through link  $(N_1, N_3)$  is larger than the one of going through links  $(N_1, N_2)$  and  $(N_2, N_3)$ , then from a cost perspective, the two-hop route is better than the direct one. Therefore, knowing the weight of links in addition to their existence allows to find better routes w.r.t. the chosen QoS criterion. Thus, depending on the QoS requirements over a network, different routing protocols should be used, each one using a various amount of input information and thus finding routes with different qualities. An example of protocol whose routes minimize the number of hops is OLSR [38]. An extension of this protocol is QOLSR [39], that finds routes with the lowest end-to-end delay.

### 1.2.2 Network cost functions to measure clustering algorithm performance

Our goal is to define network cost functions that can be used to assess the performance of the system from application layer to physical layer, in terms of QoS offered to network users. These cost functions are defined in order to take into account the costs of communications from all nodes to all nodes along the end-to-end paths calculated by the routing in the network layer. As discussed previously, the network paths found by a routing protocol depend on the available link information, which can be provided by other protocol layers, i.e., by the data link layer and potentially the physical layer. Thus, depending on the capability of the protocol stack different cost functions should be used to assess the system performance.

In the following, each cost function are identified by a superscript  $L$  or  $X$  to determine the amount of link information that can be used when calculating the routes. The first cost function defined is  $J_0$ , leading to  $J_0^L$  and  $J_0^X$ . When there is no superscript, this means that what is being discussed is applicable for both versions of the cost function. The superscript  $L$  means that only the link information (binary or link weight) can be used by the routing protocol. If binary information is available, then shortest paths minimize the hop count. This basic information can be acquired by the network layer itself (e.g., using a HELLO protocol such as the one defined in [38]) or can be provided by the data link layer. Richer link weights information can be measured by the network layer itself (e.g., in [39] the routing protocol measures link delays). It can also be provided by the data link layer which uses metrics coming from the physical layer. These metrics, such as received signal strength or signal interference noise ratio (SINR), are measured by the radio receiver during the communications. The superscript  $X$  (for cross-layer) means that in addition to link information, the cluster structure is also known, meaning that when calculating the shortest paths, the intra-cluster or inter-cluster quality of each link is taken into account. This is important because in clustered networks the efficiencies of communication over intra-cluster and inter-cluster links are different. This additional information is provided by the data link layer (in charge of building the clusters).

Several types of metrics can be used to calculate a shortest path. Within the networking community, the most common ones are the additive metrics, for which the cost of a route is the sum of its link weights. An example of this type of metric is the hop count, another one is the delay. Some other metrics cannot be cumulated. For example, the throughput of a route is not the sum but the lowest of its link throughputs. Throughput is a concave metric [40]. There are also some multiplicative metrics, such as for example the packet loss rate. In this chapter we handle in details the case of additive metrics, then propose some initial results concerning concave metrics. The case of multiplicative metrics is left for future work.

## 1.3 Network model

We consider a graph  $\mathcal{G}$  defined by its set of nodes  $\mathcal{V}$  and its set of edges  $\mathcal{E}$ . The number of nodes of  $\mathcal{G}$  is  $N := |\mathcal{V}|$ , where  $:=$  stands for *by definition*. The number of edges of  $\mathcal{G}$  is  $M := |\mathcal{E}|$ . Two nodes  $i$  and  $j$  are neighbors if  $(i, j) \in \mathcal{E}$ . Without loss of generality, the concepts developed in this chapter first concern connected<sup>1</sup> undirected graphs. A generalization to disconnected graphs is detailed in Section 1.4.6. The set of all partitions  $p$  of  $\mathcal{G}$  is called  $\mathcal{P}$ . In our model the parts of a partition are identified to the clusters. The weight of edge  $(i, j)$ ,  $\forall (i, j) \in \mathcal{E}$  is

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<sup>1</sup>In a connected graph there exists a path between any pair of vertices.

noted  $w_{i,j}$ . The weight  $w_{i,j}$  is a dimensionless quantity associated with the quality of link  $(i, j)$ . For example it can be a number of transmissions required to achieve a target packet error rate (assuming an ARQ scheme, see Section 1.4.4). The set of groups  $\mathcal{O}$  is defined as  $\{\mathcal{O}_1, \dots, \mathcal{O}_T\}$  with  $T$  the number of groups. Let us note  $m_t$  the size of group  $\mathcal{O}_t$ . Each node belongs to only one group, i.e.  $\forall t_1 \neq t_2, \mathcal{O}_{t_1} \cap \mathcal{O}_{t_2} = \emptyset$ , thus  $\sum_{t=1}^T m_t = N$ , and  $\cup_{t=1}^T \mathcal{O}_t = \mathcal{V}$ . A partition  $p$  of  $\mathcal{G}$  contains  $N_c$  clusters noted  $\mathcal{C}_k$  with  $k \in \{1, \dots, N_c\}$ . The size, or number of members, of cluster  $\mathcal{C}_k$  is  $n_k$ . Each node belongs to only one cluster, i.e.,  $\forall k \neq \ell, \mathcal{C}_k \cap \mathcal{C}_\ell = \emptyset$ , thus  $\sum_{k=1}^{N_c} n_k = N$ , and  $\cup_{k=1}^{N_c} \mathcal{C}_k = \mathcal{V}$ . The diameter<sup>2</sup> of the induced subgraph of the cluster  $\mathcal{C}_k$  is noted  $d_k$ .

## 1.4 Case of additive metrics

In this section, we define the network cost function  $J_0$  used to measure and compare the performance of clustering solutions in clustered ad hoc networks. More precisely this function measures the quality of a network partition using end-to-end path calculations with additive metrics (e.g., delay), and takes into account the fact that inter-cluster and intra-cluster communications have different costs. As a byproduct, this function is very useful for evaluating the benefit expected through the use of operational group information for obtaining the clustering solution.

### 1.4.1 Definitions

Before analyzing separately the cases of  $J_0^L$  and  $J_0^X$ , let us introduce definitions and concepts common to both functions.

In the considered network, modeled by the graph  $\mathcal{G}$ , we assume that any source node  $i \in \mathcal{V}$  may want to send some traffic to any other destination node  $j \in \mathcal{V}$ , with a probability  $\pi_{j|i}$ . By convention  $\pi_{i|i} = 0$ , and  $\sum_{j \in \mathcal{V}} \pi_{j|i} = 1, \forall i \in \mathcal{V}$ . The communication between node  $i$  and node  $j$  takes place along a network path (e.g., a shortest path), which is calculated by the routing functionality in the protocol stack, and which may depend on the partition  $p$  of  $\mathcal{G}$ . We denote by  $J_0(p, i, j)$ , the cost of transmission from node  $i$  to node  $j$  along the shortest path chosen by the routing in the partition  $p$ . Our goal is to find the partitions that minimize  $J_0$ , the average cost of communications between all pairs of nodes  $(i, j) \in \mathcal{V}^2$ , defined as:

$$J_0(p) := \frac{1}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot J_0(p, i, j), \quad (1.1)$$

where the factor  $1/N$  embodies the fact that all nodes  $i$  have equal probability to transmit.

Finding the best clustering of the network is thus equivalent to finding the set of best partitions  $\mathcal{P}'$  defined as:

$$\mathcal{P}' := \arg \min_{p \in \mathcal{P}} J_0(p). \quad (1.2)$$

### 1.4.2 Case of $J_0^L$

In this section, we consider that the routing calculates the shortest paths using the link weights information only, i.e., without taking the cluster structure into account.

---

<sup>2</sup>The diameter of a graph is the length of the longest shortest path between any pair of its vertices.

### 1.4.2.1 Definitions

Let us now explain how  $J_0^L(p, i, j)$  is elaborated. First, we assume that the routing process selects the shortest paths to establish the communications in the network. The shortest path between node  $i$  and node  $j$  is defined here as the set  $\mathcal{S}_L(i, j) = ((i, i_1), (i_1, i_2), \dots, (i_{K-1}, i_K), (i_K, j))$  for which the cumulated weights along this path,  $h_L(i, j)$ , defined as:

$$h_L(i, j) := \sum_{(i', j') \in \mathcal{S}_L(i, j)} w_{i', j'}, \quad (1.3)$$

is minimum. Note that  $\mathcal{S}_L(i, j)$  is independent of the partition  $p$  (and thus on the clusters). Fig. 1.1 provides an example of shortest path  $\mathcal{S}_{1,4}$  between source node 1 and destination node 4.

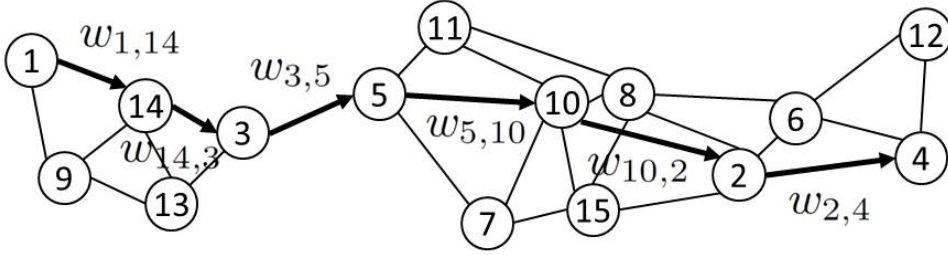


Figure 1.1: Example of multi-hop shortest path.

In clustered networks, inter-cluster communications can be implemented using a MAC different from the one used for intra-cluster communications. Within a cluster a RA node optimizes locally the RRA on behalf of the cluster members. Conversely, inter-cluster RRA is done in a more distributed way (e.g., among the RAs of neighbor clusters) and is thus more difficult to optimize. Therefore, we reasonably assume that the cost of intra-cluster communications is lower than the one of inter-cluster communications. Consequently, we split the path  $\mathcal{S}_L(i, j)$  into the two subsets  $\hat{\mathcal{S}}_L(p, i, j)$  of intra-cluster links and  $\tilde{\mathcal{S}}_L(p, i, j)$  of inter-cluster links, leading respectively to the cumulative weights  $\hat{h}_L(p, i, j)$  and  $\tilde{h}_L(p, i, j)$ :

$$\hat{h}_L(p, i, j) := \sum_{(i', j') \in \hat{\mathcal{S}}_L(p, i, j)} w_{i', j'}, \quad (1.4)$$

$$\tilde{h}_L(p, i, j) := \sum_{(i', j') \in \tilde{\mathcal{S}}_L(p, i, j)} w_{i', j'}. \quad (1.5)$$

Following (1.3)-(1.4)-(1.5), we have:

$$h_L(i, j) = \hat{h}_L(p, i, j) + \tilde{h}_L(p, i, j).$$

For example in Fig. 1.2,  $\hat{h}_L(p, 1, 4) = w_{1,14} + w_{14,3} + w_{5,10} + w_{2,4}$  and  $\tilde{h}_L(p, 1, 4) = w_{3,5} + w_{10,2}$ , with  $p = \{\{1, 3, 9, 13, 14\}, \{5, 7, 8, 10, 11, 15\}, \{2, 4, 6, 12\}\}$ .

In order to account for the difference between intra and inter-cluster communications, we define the cost from node  $i$  to node  $j$  as the weighted sum of  $\hat{h}_L(p, i, j)$  and  $\tilde{h}_L(p, i, j)$ :

$$J_0^L(p, i, j) := \hat{\gamma} \cdot \hat{h}_L(p, i, j) + \tilde{\gamma} \cdot \tilde{h}_L(p, i, j), \quad (1.6)$$

where  $\hat{\gamma} > 0$  is the cost associated with the intra-cluster communications, and  $\tilde{\gamma} \geq \hat{\gamma}$  the cost associated with the inter-cluster communications. Note that (1.4)-(1.5)-(1.6) clearly show that  $J_0^L$  is applicable to additive metrics.



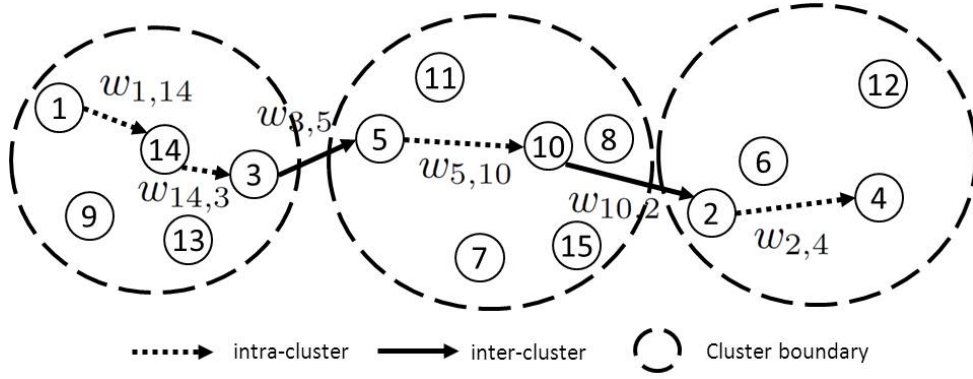


Figure 1.2: Multi-hop shortest path with cluster boundaries.

From (1.1), noting  $\gamma := \tilde{\gamma}/\hat{\gamma}$  and summing (1.6) over all pairs of nodes we get:

$$J_0^L(p) = \frac{\hat{\gamma}}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot (\hat{h}_L(p, i, j) + \gamma \cdot \tilde{h}_L(p, i, j)). \quad (1.7)$$

One can remark that the factor  $\hat{\gamma}/N$  in (1.7) plays no role in the optimization problem of (1.2).

Because  $h_L(i, j)$  is independent of the way the network is clustered, it is useful to make it appear in (1.7) leading to:

$$J_0^L(p) = A + (\gamma - 1) \cdot B(p), \quad (1.8)$$

with

$$A := \frac{\hat{\gamma}}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot h_L(i, j),$$

$$B(p) := \frac{\hat{\gamma}}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot \tilde{h}_L(p, i, j).$$

So far, the expressions are functions of probabilities  $\pi_{j|i}$  which makes the model very general and applicable to several contexts. Particular values for  $\pi_{j|i}$  are considered in Section 1.4.5 in order to take into account the hierarchical organization of some ad hoc networks.

#### 1.4.2.2 Theoretical results and additional constraints

Thanks to (1.8) we can prove the following result.

**Result 1.1** *Considering  $J_0^L$ , the solutions of the problem in (1.2) are:*

1. If  $\gamma = 1$ ,  $\mathcal{P}' = \mathcal{P}$ , and  $\forall p \in \mathcal{P}$ ,  $J_0^L(p) = A$ .
2. If  $\gamma > 1$ ,  $\mathcal{P}' = \{\mathcal{V}\}$ .

The first part of Result 1.1 means that when  $\gamma = 1$ , the way clusters are built is not important. This case is not interesting in practice since there is always a difference of cost between intra and inter-cluster communications. When  $\gamma > 1$ , the best and trivial solution is to build one cluster corresponding to the whole network. This solution is not acceptable because of the constraints related to the size of the clusters. If the cluster is too large, then the RRA becomes complex and the goal of seeking simple intra-cluster RRA cannot be fulfilled.

Consequently, we add constraints on the clusters and define  $\mathcal{P}^c$  the subset of valid partitions as follows:

$$\mathcal{P}^c = \{p \in \mathcal{P} \text{ s.t. } p \text{ satisfies } C_1, C_2, C_3\}, \quad (1.9)$$

with

$$\begin{aligned} C_1 &: \mathcal{C}_k \text{ is connected } \forall k \in \{1, 2, \dots, N_c\}, \\ C_2 &: n_{min} \leq n_k \leq n_{max} \forall k \in \{1, 2, \dots, N_c\}, \\ C_3 &: d_k \leq d_{max} \forall k \in \{1, 2, \dots, N_c\}. \end{aligned}$$

First,  $C_1$  ensures that each cluster is a connected subgraph of  $\mathcal{G}$ , allowing intra-cluster communication between all the cluster members. Then  $C_2$  forces the number of cluster members to be neither too small nor too large, which makes sense from a RRA point of view. Finally  $C_3$  prevents nodes from the same cluster from being too far (in number of hops) from the RA (in charge of RRA). The values of the parameters  $n_{min}$ ,  $n_{max}$  and  $d_{max}$  depend on the RRA process.

Now, our goal is to find the set of partitions  $\mathcal{P}_L^*$  solving the following problem:

$$\mathcal{P}_L^* = \arg \min_{p \in \mathcal{P}^c} J_0^L(p). \quad (1.10)$$

The two following results detail some properties of  $\mathcal{P}_L^*$ .

**Result 1.2** *The partial ordering between  $J_0^L$  values of different partitions does not depend on  $\gamma$ .*

**Proof** In (1.8),  $A$  does not depend on the partition whereas  $B$  does. Consequently (1.8) can be written:

$$J_0^L(\gamma, p) = A + (\gamma - 1)B(p). \quad (1.11)$$

Let  $p_1$  and  $p_2$  two different partitions of  $\mathcal{P}$  such that:

$$\begin{aligned} J_0^L(\gamma, p_1) &< J_0^L(\gamma, p_2), \\ \Leftrightarrow A + (\gamma - 1)B(p_1) &< A + (\gamma - 1)B(p_2), \\ \Leftrightarrow B(p_1) &< B(p_2). \end{aligned}$$

The last equation does not involve  $\gamma$ , which proves the result. ■

Result 1.2 induces the following result:

**Result 1.3**  *$\mathcal{P}_L^*$  is independent of  $\gamma$ .*

**Proof** Application of Result 1.2 for optimal partitions. ■

### 1.4.3 Case of $J_0^X$

In this section we consider that both link weights and cluster structure are available to calculate the shortest paths. We proceed like in Section 1.4.2 to study the case of  $J_0^X$ .

### 1.4.3.1 Definitions

Let us define graph  $\mathcal{G}(p)$  as the set of nodes  $\mathcal{V}$  connected by edges in  $\mathcal{E}$  whose weights are  $\hat{\gamma} \cdot w_{i,j}$  if  $(i,j)$  is an intra-cluster link, and  $\tilde{\gamma} \cdot w_{i,j}$  if  $(i,j)$  is an inter-cluster link. The shortest path between nodes  $i$  and  $j$  in  $\mathcal{G}(p)$  is denoted by  $\mathcal{S}_X(p, i, j)$ , with cost  $h_X(p, i, j)$ . This shortest path can be split into its intra-cluster part  $\hat{\mathcal{S}}_X(p, i, j)$  and its inter-cluster part  $\tilde{\mathcal{S}}_X(p, i, j)$  with respective costs  $\hat{h}_X(p, i, j)$  and  $\tilde{h}_X(p, i, j)$ :

$$h_X(p, i, j) := \hat{h}_X(p, i, j) + \tilde{h}_X(p, i, j), \quad (1.12)$$

with:

$$\hat{h}_X(p, i, j) := \sum_{(i',j') \in \hat{\mathcal{S}}_X(p, i, j)} \hat{\gamma} \cdot w_{i',j'} \quad (1.13)$$

$$\tilde{h}_X(p, i, j) := \sum_{(i',j') \in \tilde{\mathcal{S}}_X(p, i, j)} \tilde{\gamma} \cdot w_{i',j'}. \quad (1.14)$$

Similarly to what we did for (1.6) we define  $J_0^X(p, i, j)$  as follows:

$$J_0^X(p, i, j) := h_X(p, i, j). \quad (1.15)$$

Here  $\hat{\gamma}$  and  $\tilde{\gamma}$  are already included in  $\hat{h}_X(p, i, j)$  and  $\tilde{h}_X(p, i, j)$  respectively, which justifies that  $J_0^X(p, i, j)$  is equal to  $h_X(p, i, j)$ .

From (1.1), and summing (1.15) over all pairs of nodes we get:

$$J_0^X(p) := \frac{1}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot J_0^X(p, i, j). \quad (1.16)$$

### 1.4.3.2 Theoretical results

Let us now prove that Result 1.1 applicable to  $J_0^L$  is also valid for  $J_0^X$ . First we need the following two intermediate results.

**Result 1.4**  $\forall p \in \mathcal{P}$  with  $p \neq \{\mathcal{V}\}$ , when  $\gamma > 1$  there is at least one pair of nodes  $(i, j) \in \mathcal{V}^2$  such that  $h_X(p, i, j) > h_X(\{\mathcal{V}\}, i, j)$ .

**Proof** Because  $p \neq \{\mathcal{V}\}$ , partition  $p$  has at least two clusters. Let us choose clusters  $\mathcal{C}_1$  and  $\mathcal{C}_2$  and two 1-hop neighbor nodes  $i$  and  $j$  such that  $i \in \mathcal{C}_1$  and  $j \in \mathcal{C}_2$ , and the shortest path  $\mathcal{S}_X(\{\mathcal{V}\}, i, j)$  from  $i$  to  $j$  is link  $(i, j)$ :  $h_X(\{\mathcal{V}\}, i, j) = \hat{\gamma} \cdot w_{i,j}$ . Consider the shortest path  $\mathcal{S}_X(p, i, j)$  from  $i$  to  $j$  in  $\mathcal{G}(p)$ .

**Case 1:** If  $\mathcal{S}_X(p, i, j) = (i, j)$  then  $h_X(p, i, j) = \tilde{\gamma} \cdot w_{i,j}$  and because  $\tilde{\gamma} > \hat{\gamma}$ , we have  $h_X(p, i, j) > h_X(\{\mathcal{V}\}, i, j)$ .

**Case 2:** If  $\mathcal{S}_X(p, i, j) \neq (i, j)$  then let nodes  $\{i_1, i_2, \dots, i_{K-1}, i_K\} \subset \mathcal{V}$ , with  $i_1 = i$  and  $i_K = j$  such that  $\mathcal{S}_X(p, i, j) = (i_1, \dots, i_K)$ .  $\exists \alpha \in \{i_1, \dots, i_{K-1}\}$  such that nodes  $i_\alpha$  and  $i_{\alpha+1}$  belong to two different clusters. Because link  $(i_\alpha, i_{\alpha+1})$  belongs to a shortest path in  $\mathcal{G}(p)$ , it is also a shortest path between  $i_\alpha$  and  $i_{\alpha+1}$ , and  $h_X(p, i_\alpha, i_{\alpha+1}) = \tilde{\gamma} \cdot w_{i_\alpha, i_{\alpha+1}}$ . Consequently for all network path  $\mathcal{S}(i'_1, i'_{K'}) = (i'_1, i'_2, \dots, i'_{K'-1}, i'_{K'})$  from  $i'_1 = i_\alpha$  to  $i'_{K'} = i_{\alpha+1}$ , the cost of  $\mathcal{S}(i'_1, i'_{K'})$  is written:

$$\hat{\gamma} \sum_{(u,v) \in \hat{\mathcal{S}}(i'_1, i'_{K'})} w_{u,v} + \tilde{\gamma} \sum_{(u,v) \in \tilde{\mathcal{S}}(i'_1, i'_{K'})} w_{u,v} \geq \tilde{\gamma} \cdot w_{i_\alpha, i_{\alpha+1}},$$

with  $\hat{\mathcal{S}}(i'_1, i'_{K'})$  and  $\tilde{\mathcal{S}}(i'_1, i'_{K'})$  the set of intra-cluster and inter-cluster links in path  $\mathcal{S}(i'_1, i'_{K'})$  respectively. Multiplying by  $\hat{\gamma}/\tilde{\gamma} < 1$  we get:

$$\begin{aligned} & \hat{\gamma}^2/\tilde{\gamma} \sum_{(u,v) \in \hat{\mathcal{S}}(i'_1, i'_{K'})} w_{u,v} + \hat{\gamma} \sum_{(u,v) \in \tilde{\mathcal{S}}(i'_1, i'_{K'})} w_{u,v} > \hat{\gamma} \cdot w_{i_\alpha, i_{\alpha+1}}, \\ \Leftrightarrow & \hat{\gamma} \left[ \sum_{(u,v) \in \hat{\mathcal{S}}(i'_1, i'_{K'})} w_{u,v} + \sum_{(u,v) \in \tilde{\mathcal{S}}(i'_1, i'_{K'})} w_{u,v} \right] > \hat{\gamma} \cdot w_{i_\alpha, i_{\alpha+1}}, \end{aligned}$$

which proves that link  $(i_\alpha, i_{\alpha+1})$  is a shortest path between nodes  $i_\alpha$  and  $i_{\alpha+1}$  in  $\mathcal{G}$ , with  $h_X(p, i_\alpha, i_{\alpha+1}) = \hat{\gamma} \cdot w_{i_\alpha, i_{\alpha+1}}$ . Because  $\tilde{\gamma} > \hat{\gamma}$ , we have  $h_X(p, i_\alpha, i_{\alpha+1}) > h_X(\{\mathcal{V}\}, i_\alpha, i_{\alpha+1})$ . ■

**Result 1.5**  $\forall p \in \mathcal{P}$  with  $p \neq \{\mathcal{V}\}$ , when  $\gamma > 1$  there is no pair of nodes  $(i, j) \in \mathcal{V}^2$  such that  $h_X(p, i, j) < h_X(\{\mathcal{V}\}, i, j)$ .

**Proof** Because  $\gamma > 1$  the weight of any link in  $\mathcal{G}(p)$  is greater or equal than its weight in  $\mathcal{G}$ . Consequently a shortest path between two nodes  $i$  and  $j$  in  $\mathcal{G}$  has a cost lower than or equal to the one of a shortest path between  $i$  and  $j$  in  $\mathcal{G}(p)$ . ■

**Result 1.6** Considering  $J_0^X$ , the solutions of the problem consisting in (1.2) are:

1. If  $\gamma = 1$ ,  $\mathcal{P}' = \mathcal{P}$ ,
2. If  $\gamma > 1$ ,  $\mathcal{P}' = \mathcal{V}$ .

**Proof**

1. Suppose  $\gamma = 1$ . Then  $\tilde{\gamma} = \hat{\gamma}$ . This means that for any pair of nodes  $(i, j) \in \mathcal{V}^2$ ,  $\mathcal{S}_X(p, i, j) = \mathcal{S}_X(\{\mathcal{V}\}, i, j)$ ,  $\forall p \in \mathcal{P}$ , which entails  $h_X(p, i, j) = h_X(\{\mathcal{V}\}, i, j)$ . Consequently  $J_0^X(p)$  does not depend on  $p$  and any partition  $p \in \mathcal{P}$  is a best partition.
2. Suppose  $\gamma > 1$ . Let us consider the partition  $\{\mathcal{V}\}$ . Let us suppose that there is a partition  $p \neq \{\mathcal{V}\}$  of  $\mathcal{G}$  such that  $J_0^X(p) \leq J_0^X(\{\mathcal{V}\})$ . Thanks to Result 1.4 there is at least one pair of nodes  $(i, j)$  such that  $h_X(p, i, j) > h_X(\{\mathcal{V}\}, i, j)$ , meaning  $J_0^X(p, i, j) > J_0^X(\{\mathcal{V}\}, i, j)$ . Thanks to Result 1.5 there is no pair of nodes  $(i', j')$  such that  $h_X(p, i', j') < h_X(\{\mathcal{V}\}, i', j')$ . This means that  $J_0^X(p, i', j') \geq J_0^X(\{\mathcal{V}\}, i', j'), \forall (i', j') \in \mathcal{V}^2$ . Consequently:

$$J_0^X(p) = J_0^X(p, i, j) + \sum_{\substack{(i', j') \in \mathcal{V}^2 \\ (i', j') \neq (i, j)}} J_0^X(p, i', j') > J_0^X(\{\mathcal{V}\}).$$

Like in Section 1.4.2.2, our goal becomes to find the set  $\mathcal{P}_X^*$  of partitions solving the following problem:

$$\mathcal{P}_X^* = \arg \min_{p \in \mathcal{P}^c} J_0^X(p), \quad (1.17)$$

with  $\mathcal{P}_c$  defined in (1.9).

Here is now a result illustrating a difference between  $J_0^X$  and  $J_0^L$ , proving that Result 1.3 does not apply to  $J_0^X$ .

**Result 1.7**  $\mathcal{P}_X^*$  is dependent on  $\gamma$ .

**Proof** Fig. 1.3 defines the partitions  $p_1 = \{\{1\}, \{2, 5, 6\}, \{3, 4, 7, 8\}\}$  and  $p_2 = \{\{1, 6\}, \{2, 3, 5\}, \{4, 7, 8\}\}$  of an 8 node network. The inner and outer colors indicate group and cluster membership respectively. In this network: *i*) the nodes  $\{1, 2, 3, 5, 6\}$  and  $\{4, 7, 8\}$  belong to group 1 and 2 respectively, and *ii*)  $w_{i,j} = 1, \forall (i, j) \in \mathcal{E}$ . Let  $n_{max} = 4, \alpha = 1.0$  and  $\hat{\gamma} = 1$ . When  $\gamma = 2$  we obtain  $p_1 \in \mathcal{P}_X^*$  and  $p_2 \notin \mathcal{P}_X^*$ , and when  $\gamma = 6$  we obtain  $p_1 \notin \mathcal{P}_X^*$  and  $p_2 \in \mathcal{P}_X^*$ . Because  $\alpha = 1.0$ , the value of  $J_0^X$  only depends on the number of intra-cluster and inter-cluster links required for intra-group communications. Within  $p_1$  and  $p_2$  the costs of intra-group communications are respectively equal to  $c(p_1) = 7\tilde{\gamma} + 14\hat{\gamma}$  and  $c(p_2) = 6\tilde{\gamma} + 18\hat{\gamma}$ . Thus  $c(p_1) \leq c(p_2) \Leftrightarrow \tilde{\gamma} \leq 4\hat{\gamma}$ . To illustrate this result we have included Table 1.1, which details the value of  $J_0^X$  for  $p_1$  and  $p_2$  with  $\gamma \in \{2, 4, 6\}$ . When  $\gamma = 4, J_0^X(p_1) = J_0^X(p_2)$ .

Partition	$\gamma = 2$	$\gamma = 4$	$\gamma = 6$
$p_1$	2.000	2.875	3.750
$p_2$	2.063	2.875	3.625

Table 1.1:  $J_0^X(p_1)$  and  $J_0^X(p_2)$  for  $\gamma \in \{2, 4, 6\}$ .

Thus, in  $\mathcal{G}(p)$ , a partition optimal for a given  $\gamma$  may become suboptimal for another value of  $\gamma$ , which concludes the proof. ■

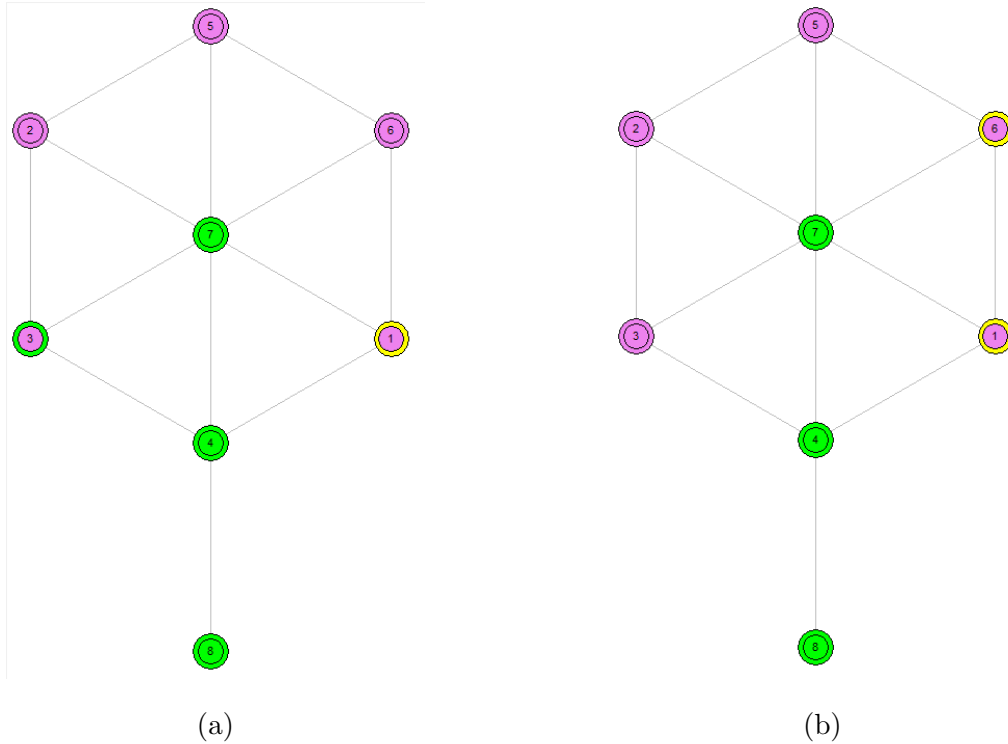


Figure 1.3: Partitions: inner and outer colors indicate group and cluster membership respectively. (a) Partition  $p_1$ . (b) Partition  $p_2$ .

#### 1.4.4 Examples for practical $J_0^X$

In this section, we discuss how to use the generic cost function  $J_0^X$  with practical QoS metrics through two examples.

First,  $J_0^X$  can be interpreted as an end-to-end delay. In that context the link weights  $w_{i,j}$  can be interpreted as the average number of transmissions required on node  $i$  to achieve a successful reception on node  $j$  (using an ARQ protocol). Using the following expression from page 52 of [41], the link weight could be written:

$$w_{i,j} := c_{max} + [1 - \Upsilon(i,j)]^{-1} + c_{max}/[\Upsilon(i,j)^{c_{max}} - 1],$$

with  $c_{max}$  the maximum number of transmissions considering that a type I hybrid ARQ is used, and  $\Upsilon(i,j)$  the packet error rate on link  $(i,j)$ , which depends on  $\Gamma(i,j)$  the SNR on the link. In this example, the parameter  $\gamma$  can be viewed as the multiplicative coefficient of the delay induced by the difference of efficiency between inter-cluster and intra-cluster RRA. The value of  $J_0^X(p, i, j)$  from (1.15) should be interpreted as the duration needed for node  $i$  to send successfully to node  $j$  one unit of traffic. Let us take the example of a TDMA based MAC in which the intra-cluster delay to send data  $\hat{\gamma}$  is equal to one 100 ms MAC frame while the inter-cluster delay  $\tilde{\gamma}$  is twice this value. Assuming a uniform traffic between all nodes, each destination node  $j$  receives an equal portion  $\pi_{j|i} = 1/14$  from node  $i$ . Considering the shortest path between nodes 1 and 4 of Fig. 1.2 and applying (1.6) with  $w_{i,j} = 1 \forall (i,j) \in \mathcal{E}$ , we get  $J_0^X(p, 1, 4) = 800$  ms. The value of  $J_0^X(p)$  in (1.8) is the average duration for a node to send successfully one unit of traffic to all nodes. Using the same example of Fig. 1.1 and Fig. 1.2 we get  $J_0^X(p) = 376$  ms.

Second,  $J_0^X$  can be used to measure the energy resource consumption, which we want to minimize. In that context the link weights  $w_{i,j}$  are associated to the energy required to perform a communication from node  $i$  to node  $j$ . This weight [42] can be written:

$$w_{i,j} := \tau(\mu P_{tx} + P_c),$$

with  $P_{tx}$  the transmit power,  $\tau$  the time in seconds required for a transmission,  $\mu$  the inverse of the efficiency of the transmit power amplifier, and  $P_c$  the power dissipated in all other circuit blocks of the transmitter and receiver to operate nodes  $i$  and  $j$ . In this example, the parameters  $\hat{\gamma}$  and  $\tilde{\gamma}$  can be viewed as the transmit power margins required to make up for the imperfect channel state indications (CSI). Compared to intra-cluster RRA, that can be centralized on RA nodes, inter-cluster RRA is a distributed process. Consequently, the CSI used by inter-cluster have less accuracy than the one used by intra-cluster RRA. Intra-cluster RRA can thus use a lower transmit power margin than inter-cluster RRA, justifying  $\gamma > 1$ .

#### 1.4.5 Application of $J_0$ to structured networks

##### 1.4.5.1 Definition

We now consider that operational groups exist and that the traffic is structured according to these groups. To capture this fact, we consider that the probability that one node communicates with a node of the same group is equal to  $\alpha \in [0, 1]$  and thus the probability that one node communicates with a node in another group is equal to  $1 - \alpha$ . Since a node of group  $\mathcal{O}_t$  can communicate to  $m_t - 1$  nodes in the same group, the probability to reach one of these nodes

is equal to  $\alpha/(m_t - 1)$ . The number of nodes of the other groups with which this node can communicate is equal to  $N - m_t$  with a corresponding probability of  $(1 - \alpha)/(N - m_t)$ . Thus, we have:

$$\pi_{j|i} := \begin{cases} \frac{\alpha}{m_t - 1} & \text{if } j \in \mathcal{O}_t, \\ \frac{1 - \alpha}{N - m_t} & \text{otherwise,} \end{cases} \quad (1.18)$$

with  $(i, j) \in \mathcal{V}^2$  and  $i \in \mathcal{O}_t$ .

Note that a group independent uniform traffic can be modeled using  $\alpha := (m_t - 1)/(N - 1)$ . In that case,  $\pi_{j|i} = 1/(N - 1)$ ,  $\forall j \in \mathcal{V}$ .

### 1.4.5.2 Examples

In order to better understand  $J_0^X$ , Fig. 1.4 shows two partitions  $p_1$  and  $p_2$ . In both figures nodes  $\{7, 8, 9\}$  belong to the same yellow group. In Fig. 1.4.a nodes  $\{7, 8, 9\}$  belong to the same yellow cluster, when in Fig. 1.4.b nodes  $\{7, 9\}$  belong to the yellow cluster, and node 8 to the green cluster. Let us consider that the main part of the traffic is exchanged within groups. In  $p_1$  all group members are included in the same clusters, but this is not the case in  $p_2$ . Consequently,  $J_0^X(p_1)$  should be lower (thus better) than  $J_0^X(p_2)$ .

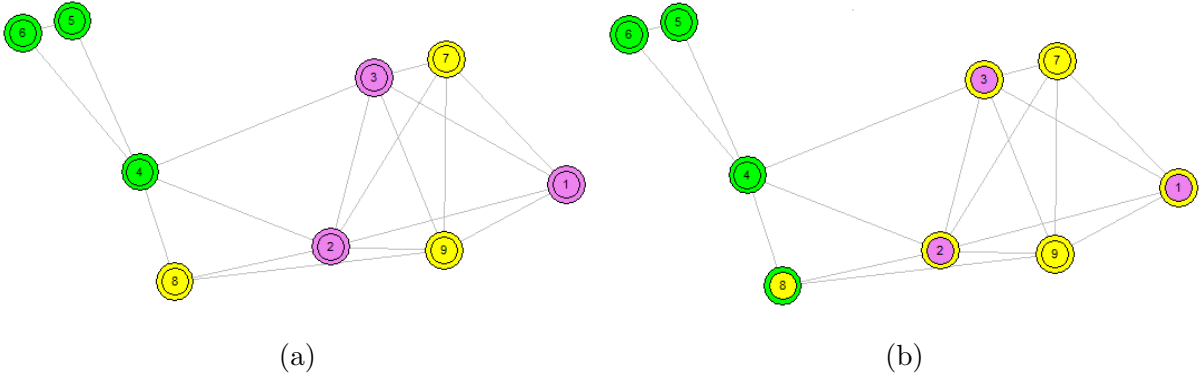


Figure 1.4: Partitions: inner and outer colors indicate group and cluster membership respectively. (a) 3 clusters partition  $p_1$ . (b) 2 clusters partition  $p_2$ .

Let  $\hat{\gamma} = 1.0$ ,  $\tilde{\gamma} = 2.0$  and  $w_{i,j} = 1.0 \forall (i, j) \in \mathcal{E}$ , Fig. 1.5 shows the values of  $J_0^X(p_1)$  and  $J_0^X(p_2)$  for  $\alpha \in \{0.5, 0.6, 0.667, 0.7, 0.8, 0.9\}$ . This figure tells us that if  $\alpha < 0.667$  then  $J_0^X(p_1) > J_0^X(p_2)$ , and if  $\alpha > 0.667$ , then  $J_0^X(p_1) < J_0^X(p_2)$ . This means that when the value of  $\alpha$  increases (i.e., the amount of intra-cluster traffic increases), then the benefit of gathering all members of the yellow group in the same cluster (partition  $p_1$ ) exceeds the one of building only two clusters (partition  $p_2$ ). This also means that for different values of  $\alpha$ , two different partitions may have their order reversed w.r.t.  $J_0^X$ .

### 1.4.5.3 Limitation of $J_0^L$ w.r.t. $J_0^X$

One may wonder about the difference between  $J_0^L$  and  $J_0^X$  w.r.t. the partition quality assessment. Let us take the example of the network of Fig. 1.6 whose partition is denoted by  $p$ , with  $w_{i,j} = 1, \forall (i, j) \in \mathcal{E}$ . Let us consider the shortest paths  $S_L^1(p, 2, 28) = (2, 14, 20, 27, 25, 10, 28)$  and  $S_L^2(p, 2, 28) = (2, 8, 20, 27, 25, 10, 28)$  between nodes 2 and 28. The number of inter-cluster links

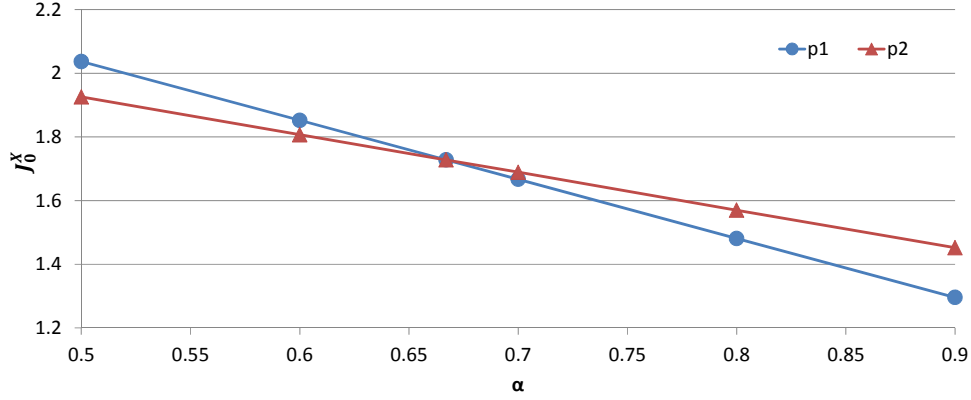


Figure 1.5:  $J_0^X(p)$  versus  $\alpha$  for 2 different partitions  $p_1$  and  $p_2$ .

in  $S_L^1(p, 2, 28)$  and  $S_L^2(p, 2, 28)$  are four - (2, 14), (14, 20), (20, 27) and (27, 25) - and two - (20, 27) and (27, 25) - respectively. Consequently, the value of  $J_0^L(p, 2, 28)$  depends on whether the selected shortest path is  $S_L^1(p, 2, 28)$  or  $S_L^2(p, 2, 28)$ . Conversely, when  $J_0^X(p, 2, 28)$  is calculated, the selected shortest path is always  $S_X(p, 2, 28) = S_L^2(p, 2, 28)$ .

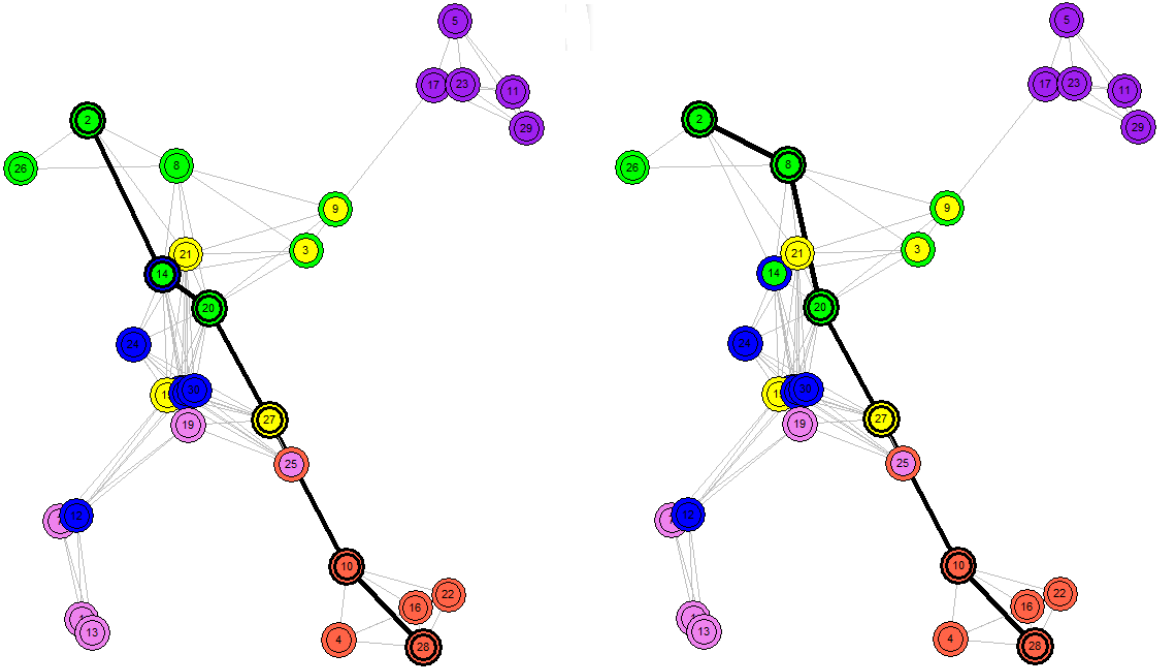


Figure 1.6: Examples of shortest paths between nodes 2 and 28 in two partitions of the same network. Inner and outer colors indicate group and cluster membership respectively.

This example shows that two different calculations of  $J_0^L$  may lead to different values because *i*) multiple shortest paths may exist between any pair of nodes, and *ii*) if more than one such path exists, because  $J_0^L$  does not take into account the cluster structure, the shortest paths may include different numbers of inter-cluster links. Thus, to get reliable results, the cluster structure must be used when calculating the cost associated with a network partition:  $J_0^X$  must be used instead of  $J_0^L$ .



To assess the error performed when using  $J_0^L$  instead of  $J_0^X$  we have used a k-shortest-path algorithm from Eppstein [43] to calculate all shortest paths between all pairs of nodes  $(i, j) \in \mathcal{V}$ . Then for each pair of nodes  $(i, j)$  we have calculated all  $J_0^L(p, i, j)$  and  $J_0^X(p, i, j)$  values. Fig. 1.7 plots  $J_0^X$  and the average, lowest and highest possible values of  $J_0^L$  for the partition of Fig. 1.6, for  $\gamma \in \{2, 5\}$  and  $\alpha \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$ .

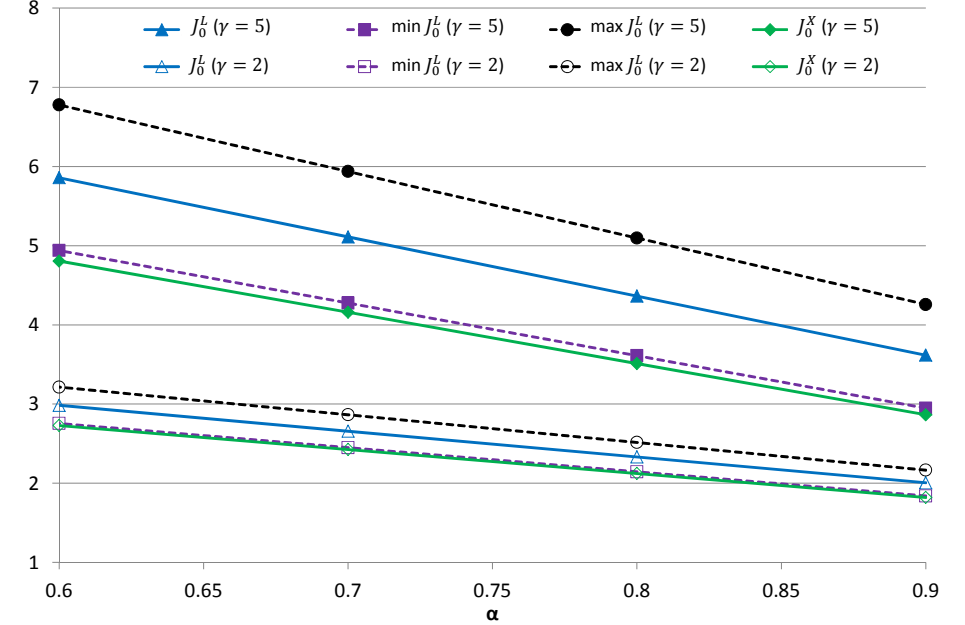


Figure 1.7:  $J_0^X(p)$  and  $J_0^L(p)$  versus  $\alpha$ .

The first conclusion about this figure is the difference between the lowest and highest values of  $J_0^L$ , which increases from about 15% to about 30% when the value of  $\gamma$  is modified from 2 to 5. Another interesting result is about  $J_0^X$  which is lower than the lowest value of  $J_0^L$ : about 1% lower if  $\gamma = 2$  and about 3% lower if  $\gamma = 5$ . To understand this surprising result, one must note that  $J_0^X$  may use paths which have a larger number of hops than  $J_0^L$ , and still have a lower cost.

#### 1.4.6 Generalization to disconnected networks

In this section the calculation of  $J_0(p)$  must be modified to take into account two aspects:

- Network nodes may be split into several connected components and,
- Some members from the same groups can be present in different connected components.

We denote by  $\mathcal{J}_0(p)$  the adaptation of  $J_0(p)$  to disconnected networks. Let us suppose that the number of connected components in the network is  $Q$ , and denote by  $\mathcal{Q}_\nu$  the  $\nu$ th connected component, with  $\nu \in \{1, \dots, Q\}$ . We denote by  $p_\nu$  the partition of  $\mathcal{Q}_\nu$ , with  $p := \cup_{\nu \in \{1, \dots, Q\}} p_\nu$ . Let us remark that no clustering scheme can enable a communication between two different connected components  $\mathcal{Q}_{\nu_1}$  and  $\mathcal{Q}_{\nu_2}$ . It is thus legitimate to ignore the absence of such communication and to separate the calculation of  $\mathcal{J}_0(p)$  into a weighted sum of values  $\mathcal{J}_0^\nu(p_\nu)$ , each associated with a connected component:

$$\mathcal{J}_0^\nu(p_\nu) := \frac{1}{|\mathcal{Q}_\nu|} \sum_{(i,j) \in \mathcal{Q}_\nu^2} \pi_{\nu,j|i} \cdot \mathcal{J}_0(p_\nu, i, j), \quad (1.19)$$

with:

$$\mathcal{J}_0(p_\nu, i, j) := \begin{cases} J_0^L(p_\nu, i, j) & \text{defined by (1.6) to calculate } \mathcal{J}_0^L(p_\nu), \\ J_0^X(p_\nu, i, j) & \text{defined by (1.15) to calculate } \mathcal{J}_0^X(p_\nu), \end{cases}$$

and

$$\pi_{\nu, j|i} := \begin{cases} \frac{\alpha}{|\mathcal{O}_{\nu, k}| - 1} & \text{if } j \in \mathcal{O}_{\nu, k}, \\ \frac{1 - \alpha}{|\mathcal{Q}_\nu| - |\mathcal{O}_{\nu, k}|} & \text{otherwise,} \end{cases}$$

with  $(i, j) \in \mathcal{Q}_\nu^2$ ,  $\mathcal{O}_{\nu, k} := \mathcal{O}_t \cap \mathcal{Q}_\nu$ , and  $i \in \mathcal{O}_{\nu, k}$ .

Then we define  $\mathcal{J}_0(p)$  as the average of  $\mathcal{J}_0^\nu(p)$  over the connected component size:

$$\mathcal{J}_0(p) := \frac{1}{N} \sum_{\nu=1}^Q |\mathcal{Q}_\nu| \cdot \mathcal{J}_0^\nu(p). \quad (1.20)$$

When the network is composed of several connected components, (1.20) must be used instead of (1.7) or (1.16) to avoid underestimating the cost of the network.

## 1.5 Numerical analysis

### 1.5.1 Assessment methodology

A first result we want to show is that the groups have an impact on the quality of the clustering solution. To do this, we find the set  $\mathcal{P}^u$  of optimal partitions when the traffic pattern does not depend on the groups, then we determine if these partitions are still good when the traffic pattern becomes dependent on the groups. A traffic independent of the groups is equivalent to  $\pi_{j|i} = \frac{1}{N-1}$ ,  $\forall (i, j) \in \mathcal{V}^2$ ,  $i \neq j$ , hence:

$$\mathcal{P}^u := \arg \min_{p \in \mathcal{P}^c} J_0^X(p) \Big|_{\pi_{j|i} = \frac{1}{N-1}, \forall (i, j) \in \mathcal{V}^2, i \neq j}.$$

To determine if the partitions  $p \in \mathcal{P}^u$  are still good when the traffic pattern becomes dependent on the groups according to (1.18), we calculate the following metric:

$$\delta_u(\alpha) := \frac{J_0^u - J_0^*}{\bar{J}_0^* - J_0^*}, \quad (1.21)$$

with  $J_0^u$  the highest value of  $J_0^X$  for all the partitions in  $\mathcal{P}^u$  when  $J_0^X$  is calculated with  $\pi_{j|i}$  defined as in (1.18),  $\bar{J}_0^* := \max_{p \in \mathcal{P}^c} J_0^X(p)$ , and  $J_0^* := \min_{p \in \mathcal{P}^c} J_0^X(p)$ . Let us call  $J_0^X$  *interval* the interval  $[J_0^*, \bar{J}_0^*]$ . The term  $\delta_u(\alpha)$  measures the performance loss obtained by not taking into account the dependence of traffic patterns to groups during cluster building. It takes its values in  $[0, 1]$  when  $J_0^u$  goes through the  $J_0^X$  interval: 0 is associated with the best partitions, and 1 with the worst partitions.

Assuming that taking the group structure into account when building clusters is of interest, new algorithms have to be designed. In order to show that it is not an easy task (which is the subject of the subsequent chapters), we propose hereafter a naive approach called one group-one cluster and denoted by 1G1C. The simplest way to follow would be to *force* all the members of a group to belong to the same cluster. However, due to the constraints it is not always possible. Therefore we propose the following procedure: *i*) for each group, find the node with the highest

degree in the subgraph of  $\mathcal{G}$  induced by the members of this group and build a cluster including this node and its 1-hop neighbors; *ii*) for each nodes not yet member of any cluster, attach it to an existing cluster while making sure that  $C_3$  is satisfied, otherwise build a new singleton cluster when  $C_3$  is not satisfied. The obtained partition is denoted by  $p_g$ . Using the same idea as (1.21), the performance of the aforementioned naive algorithm is studied through the metric:

$$\delta_g(\alpha) := \frac{J_0^X(p_g) - J_0^*}{\bar{J}_0^* - J_0^*}.$$

Note that this procedure may lead to clusters with a number of nodes less than  $n_{min}$ , thus not satisfying  $C_2$ .

## 1.5.2 Simulation setup

### 1.5.2.1 Number of partitions of a graph

Before introducing our network model, let us introduce some complexity aspects when enumerating all partitions in  $\mathcal{P}^c$ . First, the number of possible partitions in  $N_c$  clusters of a graph  $\mathcal{G}$  with  $N$  vertices is equal to the Stirling number of the second kind [25]  $S(N, N_c) := \frac{1}{N_c!} \sum_{k=0}^{N_c} (-1)^{N_c-k} \binom{N_c}{k} k^N$ . The total number of possible partitions is the  $N$ th Bell number  $B_N := \sum_{k=0}^N S(N, k)$ . This number increases very quickly with the graph size  $N$ . The first ten Bell numbers for  $N = 1$  to 10 are 6, 116, 1 928, 27 665, 364 472, 4 547 586, 54 670 463, 639 838 113, 7 338 610 159, 82 857 366 967, which means that an enumeration of all partitions  $\mathcal{P}$  of a graph is impossible unless the graph consists of very few vertices.

Fortunately, only the partitions in  $\mathcal{P}^c$  are acceptable, which significantly reduces their number. However, very quickly  $|\mathcal{P}^c|$  becomes very large, thus to cope with the limited amount of processing power and time, we have only considered networks with a limited number of nodes  $N$  and edges  $M$ .

### 1.5.2.2 Node deployment model

The networks considered in this chapter are structured networks, whose nodes are organized in groups. They are generated using the following procedure. First,  $T$  nodes are placed randomly in a  $d \times d$  square area following a uniform distribution. Each of those nodes is the first node of each group  $\mathcal{O}_t, \forall k \in \{1, 2, \dots, T\}$ , called  $V_k$ . Then all members of each group  $\mathcal{O}_t$  are placed within the disk centered on  $V_k$ , using polar coordinates  $(\rho, \theta)$ . The radius  $\rho$  is a random variable following the probability density function (pdf):

$$f_\rho(d) = \begin{cases} \frac{2}{d_1+d_2} & \text{if } 0 < d \leq d_1, \\ \frac{2(d_2-d)}{(d_1+d_2)(d_2-d_1)} & \text{if } d_1 < d \leq d_2, \\ 0 & \text{otherwise.} \end{cases} \quad (1.22)$$

The angle  $\theta$  is a random variable following a uniform pdf in  $[-\pi, +\pi)$ . When  $\theta$  leads to a node outside of the simulated area, another value is calculated.

Fig. 1.8 and 1.9 provides an example of the pdf (1.22) and cumulative distribution function (cdf) of  $\rho$  when  $d_1 = 4000$  and  $d_2 = 8000$ .

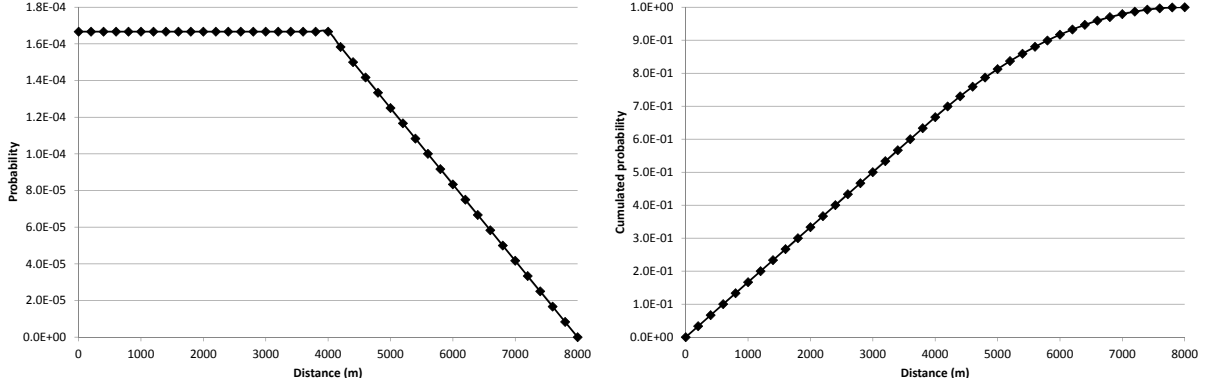


Figure 1.8: pdf of group member distance from the first group member. Figure 1.9: cdf of group member distance from the first group member.

### 1.5.2.3 Network examples and simulation parameters

Random networks were generated using parameters  $N = 30$ ,  $d_1 = 4000$ ,  $d_2 = 8000$ ,  $T = 6$ ,  $r = 5000$  and  $d = 20000$ . Fig. 1.10.a to 1.10.d displays some of these networks, ordered by increasing number of edges. In these figures the colors indicate group membership. In Fig. 1.10.a, the yellow group is composed of nodes 3, 9, 15, 21, 27. This group is not a connected component of the whole network because node 21 is not a neighbor of any other member of the group. The blue group is composed of nodes 6, 12, 18, 24, 30. It is a 2-hops diameter connected sub-graph of the whole network. The violet group is composed of nodes 1, 7, 13, 19, 25 is a 3-hops diameter connected sub-graph of the whole network. In Fig. 1.10.b, all sub-graphs induced by operational groups are connected.

Following the discussion in Section 1.5.2.1, and using the network model described in Section 1.5.2.2, we implemented an algorithm to enumerate all partitions of a graph subject to connectivity, size and diameter constraints. It is described in appendix A. This algorithm was used with the four networks of Fig. 1.10, with  $n_{min} = 4$ ,  $n_{max} = 8$  and  $d_{max} = 2$ . Table 1.2 displays the number of partitions satisfying the constraints, and the number of seconds required to calculate them (using optimized C/C++ software running on a 3.5 GHz core i5 processor). More than 9 days were required to solve the network of Fig. 1.10.d.

Fig.	Edges	Partitions	Time (s)
1.10.a	146	8 584	102
1.10.b	182	56 228	2 079
1.10.c	202	10 519 184	28 951
1.10.d	268	979 970 630	795 113

Table 1.2: Complexity for various networks.

Thus, to limit the amount of time required to enumerate partitions in  $\mathcal{P}^c$ , in this section the number of nodes has been set to  $N = 30$  and the number of edges has been limited to 230. In addition the following parameter values have been used:  $d_1 = 4000$ ,  $d_2 = 8000$ ,  $T = 6$ ,  $r = 5000$  and  $d = 20000$ . Using these parameters, the number of edges of the 100 random networks used during our simulations lies in  $[146, 222]$  with an average value of 183 edges. Their network diameter lies in  $[5, 12]$  and is equal to 7.19 on average. Among the 100 random networks, the

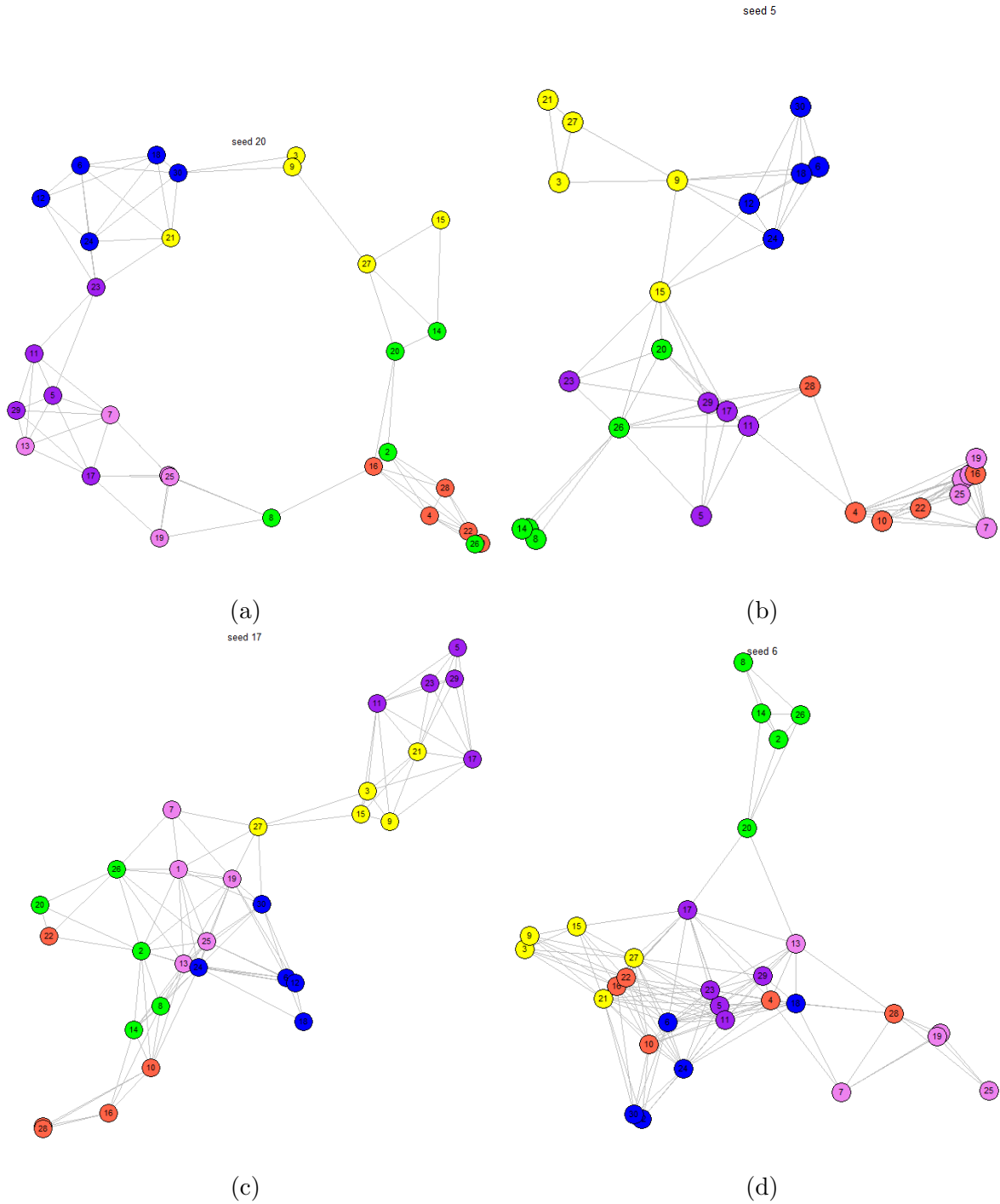


Figure 1.10: 30 nodes network. Node color indicates group membership. (a) Network with 146 edges. (b) Network with 182 edges. (c) Network with 202 edges. (d) Network with 268 edges.

minimum and maximum number of valid partitions are 672 and 49 647 650.

For the sake of simplicity, each  $w_{i,j}$  has been set constant and equal to 1,  $\forall(i, j) \in \mathcal{E}$ , and  $\gamma$  has been set to 2. Finally the constraint parameters are:  $n_{min} = 4$ ,  $n_{max} = 8$  and  $d_{max} = 2$ . All the curves of Section 1.5.3 have been obtained by averaging over 100 networks.

### 1.5.3 Results

The cdf of  $\delta_u(\alpha)$  are plotted in Fig. 1.11.

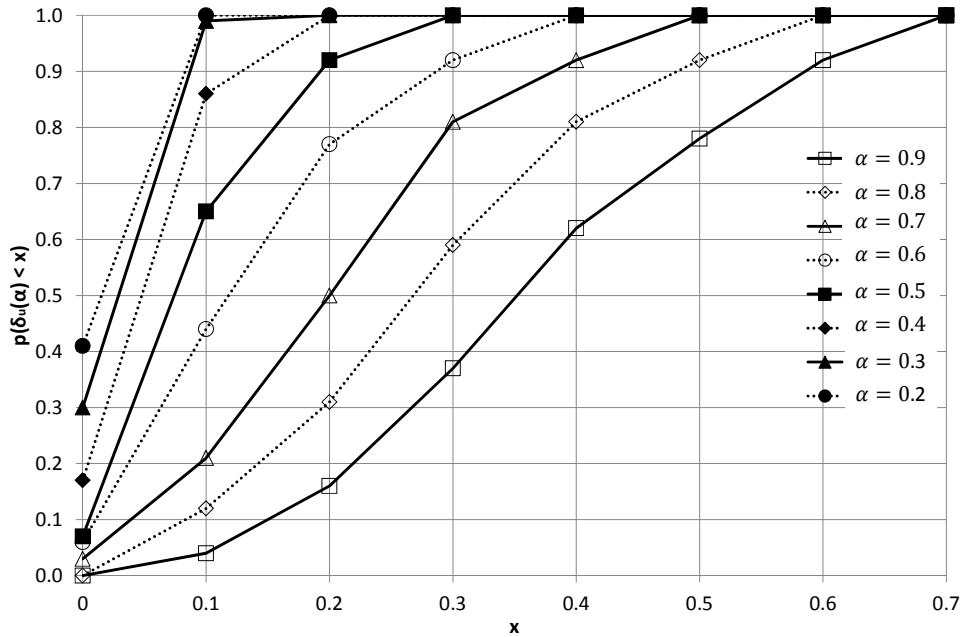


Figure 1.11: cdf of  $\delta_u(\alpha)$ .

The first information provided by these curves is that the partitions  $p \in \mathcal{P}^u$  that are the best when the traffic pattern does not depend on the groups, do not remain the best partitions when the traffic pattern depends on the group:  $\mathcal{P}^u \neq \mathcal{P}_X^*$ . For example when  $\alpha = 0.2$ , only 40% of the partitions  $p \in \mathcal{P}^u$  are also in  $\mathcal{P}_X^*$  and this proportion decreases to about 0% when  $\alpha = 0.8$  or 0.9. An additional result is that the partitions  $p \in \mathcal{P}^u$  are not only no longer the best, but also become quite bad when the value of  $\alpha$  is high enough. For example when  $\alpha = 0.7$ , only 50% of these partitions have a  $J_0^X$  value in the first 20% of the  $J_0^X$  interval. Fig. 1.11 shows that when traffic flows are concentrated within groups, the group membership should be taken into account during cluster building. This result has been achieved thanks to the introduction of the benchmark network cost function  $J_0^X$ .

Fig. 1.12 illustrates the benefit of operational clustering, using the same example of the TDMA based MAC as in Section 1.4.4. This figure plots, for one particular network, the histogram of the average delay between nodes of the same group, calculated over all groups. Thanks to  $J_0^X$ , for each value of  $\alpha$  the best partitions  $p \in \mathcal{P}_X^*$  have been found and the associated delays have been determined. Depending on  $\alpha$  the best partitions are different thus the intra-group delays are different. Fig. 1.12 shows that when  $\alpha$  increases, the average delay decreases. Additionally, the larger  $\alpha$  is, the larger is the number of pairs of nodes in the same group with a minimum delay of 100 ms. This is a practical example of the benefit provided to the end-user when the group structure is taken into account.

Another result achieved thanks to  $J_0^X$  is that the naive clustering strategy consisting in trying to build one cluster per group is not the best way to build clusters. Out of the 100 networks clustered using this heuristic, only 56 satisfied the cluster size constraints, the remaining ones included clusters with only 1 or 2 nodes. Fig. 1.13 plots the cdf of  $\delta_g(\alpha)$  for these 56 networks.

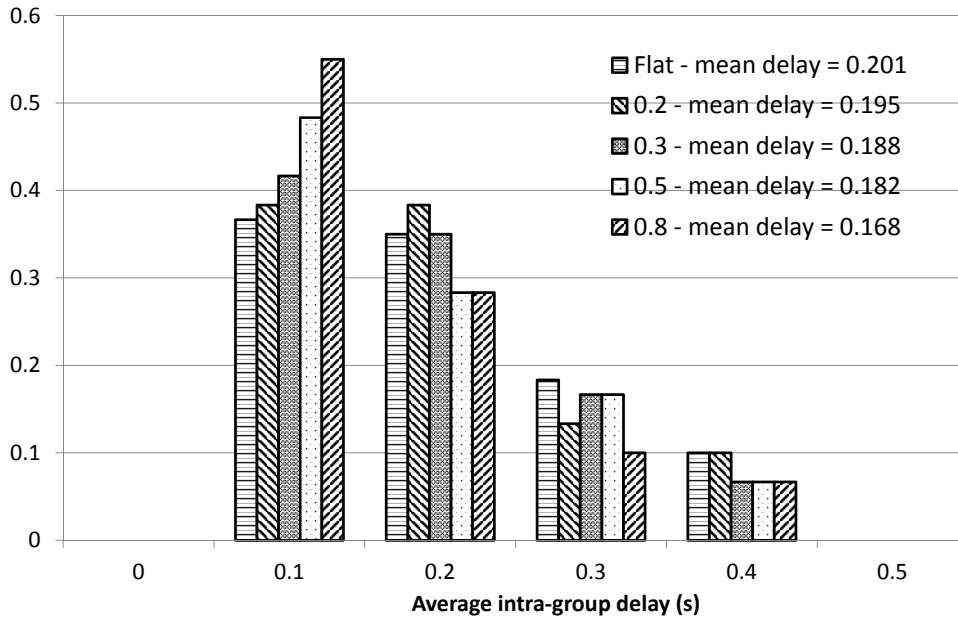


Figure 1.12: End-to-end intra-group delay histogram for several values of  $\alpha$ .

This figure shows that the need for a clustering solution more clever than the naive one increases when  $\alpha$  decreases. For example when  $\alpha = 0.9$ , about 80% of these partitions have a  $J_0^X$  value in the first 10% of the  $J_0^X$  interval. These 80% are reduced to about 55% when  $\alpha = 0.7$ , and are close to 0% when  $\alpha < 0.5$ .

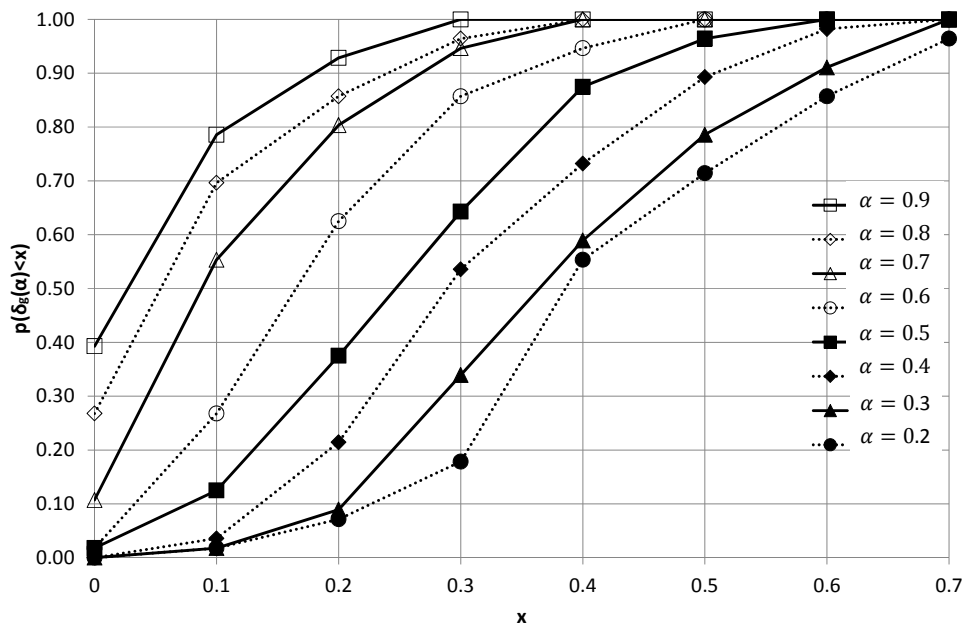


Figure 1.13: cdf of  $\delta_g(\alpha)$ .

## 1.6 Extending $J_0^X$ to take cluster size into account

A limit of  $J_0^X$  is related to the fact that its minimal (i.e. best) value is when the whole network is partitioned into a single cluster. This is a problem because a key hypothesis when defining  $J_0^X$  is  $\tilde{\gamma} \geq \hat{\gamma}$ , which is valid only if the cluster size is not too large. To handle this limit, we derive from  $J_0^X$  a new network cost function  $\bar{J}_0^X$  which takes into account the fact that the efficiency of RRA within a cluster decreases with the number of its members.

Note: in this section we extend  $J_0^X$  only, knowing that extending  $J_0^L$  could be done similarly without any difficulty.

### 1.6.1 Definition

Let us define graph  $\bar{\mathcal{G}}(p)$  as the set of nodes  $\mathcal{V}$  connected by edges in  $\mathcal{E}$  whose costs are  $\hat{\gamma} \cdot w_{i,j} \cdot \xi(i,j)$  if  $(i,j)$  is an intra-cluster link, with  $\xi(i,j) \geq 1$  a factor used to take into account cluster size, and  $\tilde{\gamma} \cdot w_{i,j}$  if  $(i,j)$  is an inter-cluster link. The shortest path between nodes  $i$  and  $j$  in  $\bar{\mathcal{G}}(p)$  is denoted by  $\bar{\mathcal{S}}_X(p,i,j)$ , with cost  $\bar{h}_X(p,i,j)$ . This shortest path can be split into its intra-cluster part  $\hat{\mathcal{S}}_X(p,i,j)$  and its inter-cluster part  $\tilde{\mathcal{S}}_X(p,i,j)$  with respective costs  $\hat{h}_X(p,i,j)$  and  $\tilde{h}_X(p,i,j)$ :

$$\hat{h}_X(p,i,j) := \sum_{(i',j') \in \hat{\mathcal{S}}_X(p,i,j)} \hat{\gamma} \cdot w_{i',j'} \cdot \xi(i',j') \quad (1.23)$$

$$\tilde{h}_X(p,i,j) := \sum_{(i',j') \in \tilde{\mathcal{S}}_X(p,i,j)} \tilde{\gamma} \cdot w_{i',j'}. \quad (1.24)$$

We define  $\bar{J}_0^X(p,i,j) := \bar{h}_X(p,i,j)$ , and define  $\bar{J}_0^X(p)$  like in (1.1):

$$\bar{J}_0^X(p) := \frac{1}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot \bar{J}_0^X(p,i,j). \quad (1.25)$$

The difference between  $\bar{J}_0^X(p,i,j)$  and  $J_0^X(p,i,j)$  lies in the factor  $\xi(i,j)$ . When  $\hat{\gamma} \cdot w_{i,j} \cdot \xi(i,j) > \tilde{\gamma} \cdot w_{i,j}$ , the cost of gathering nodes  $i$  and  $j$  in the same cluster is larger than the cost of putting them in two different clusters. The function  $\xi(i,j)$  models the decrease of RRA efficiency when the cluster size increases.

To determine the shape of  $\xi(i,j)$ , the following ideas may be considered:

- Given a slotted MAC, the number of slots per unit of time is fixed,
- The amount of slots needed per cluster member is limited. When a cluster member gets at least this number then it is considered as satisfied. Otherwise its dissatisfaction increases with the number of missing slots,

Let us consider two nodes  $i$  and  $j$  in the same cluster  $\mathcal{C}_k$ . As an example, the function  $\xi(i,j)$  can be defined as follows:

$$\xi(i,j) := \begin{cases} 1, & \text{if } n_k \leq n_{thr} \\ \frac{1}{\lambda} \left[ \frac{n_k}{n_{thr}} + \lambda - 1 \right] & \text{otherwise.} \end{cases} \quad (1.26)$$

with  $\lambda > 0$  and  $n_{thr}$  a target cluster size. The function  $\xi(i,j)$  increases linearly with the cluster size. Its value is equal to 2 when  $n_k = (\lambda + 1) \cdot n_{thr}$ .



### 1.6.2 Example

Let us consider a MAC frame including a signaling part and a data communications part, whose duration is  $d_{frame}$ . The number of time slots  $n_{data}$  in the data communication part of the MAC frame is fixed. Let us denote by  $q_{min}$  the number of slots per second needed by a cluster member to satisfy the user QoS. Let us consider a simple channel unaware equal radio resource allocation scheme in a cluster  $\mathcal{C}_k$  of size  $n_k$ . To ensure that the number of slots per second per cluster member is greater or equal than  $q_{min}$  we have:

$$\frac{n_{data}}{d_{frame} \cdot n_k} \geq q_{min} \Leftrightarrow n_k \leq \frac{n_{data}}{d_{frame} \cdot q_{min}}.$$

Let us suppose that beyond this cluster size, the QoS degrades rapidly because of the throughput decrease. We now introduce the percentage of throughput degradation  $\delta$  that can be tolerated by the application. The highest cluster size  $n_{hig}$  can be calculated as follows:

$$q_{min} \cdot (1 - \delta/100) = \frac{n_{data}}{d_{frame} \cdot n_{hig}} \Leftrightarrow n_{hig} = \frac{n_{data}}{d_{frame} \cdot q_{min} \cdot (1 - \delta/100)} \quad (1.27)$$

To calculate the  $\lambda$  parameter of (1.26), we decide that when the cluster is  $n_{hig}$ , then the intra-cluster communication cost is doubled, i.e.  $\xi(i, j) = 2$ . We thus write:

$$\frac{1}{\lambda} \left[ \frac{n_{hig}}{n_{thr}} + \lambda - 1 \right] = 2 \Leftrightarrow \lambda = \frac{n_{hig}}{n_{thr}} - 1. \quad (1.28)$$

For example let us consider a MAC frame with  $d_{frame} = 100$  ms  $n_{data} = 20$ , and the case whereby  $q_{min} = 10$ ,  $n_{thr} = 20$  and  $\delta = 20\%$ . We use (1.27) to write:

$$n_{hig} = \frac{n_{data}}{d_{frame} \cdot q_{min} \cdot (1 - \delta/100)} = \frac{20}{0.1 \cdot 10 \cdot (1 - 20/100)} = 25.$$

Using (1.28) we have  $\lambda = 25/20 - 1 = 0.25$ .

### 1.6.3 Numerical illustration

To assess numerically the difference between  $J_0^X$  and  $\bar{J}_0^X$  let us consider the network of Fig. 1.14. This network has 20 nodes with  $m_t = 4$ , and  $T = 5$ . The number of partitions of this network is very large and cannot be easily calculated. To decrease this number, we have considered only partitions including connected clusters whose size is greater or equal than two.

This number of remaining partitions is still quite large (equal to 2 477 973 765), a few days were required to calculate the values of  $J_0^X$  and  $\bar{J}_0^X$  for all these partitions. Fig. 1.15 plots the minimum values of  $J_0^X$  and  $\bar{J}_0^X$  (multiplied by the network size  $N$ ) versus the maximum cluster size. The parameters common to  $J_0^X$  and  $\bar{J}_0^X$  are  $\gamma = 2.0$  and  $\alpha = 0.9$ . To calculate  $\bar{J}_0^X$ , the value of  $n_{thr}$  has been set to five and the one of  $\lambda$  to 0.4. When the maximum cluster size is no greater than five,  $J_0^X$  and  $\bar{J}_0^X$  are equal. Beyond this value,  $\bar{J}_0^X > J_0^X$ .

Fig. 1.16 plots only  $J_0^X$ , whose minimum occurs when the whole network is a single cluster. The "wave" shape of the curve when  $n_{max} \geq 8$  is explained by the group size value  $m_t = 4$ : when the maximum cluster size is a multiple of four, then full groups can be included in the same cluster and the number of inter-cluster links used for intra-group communication is reduced to zero, thus minimizing  $J_0^X$ .

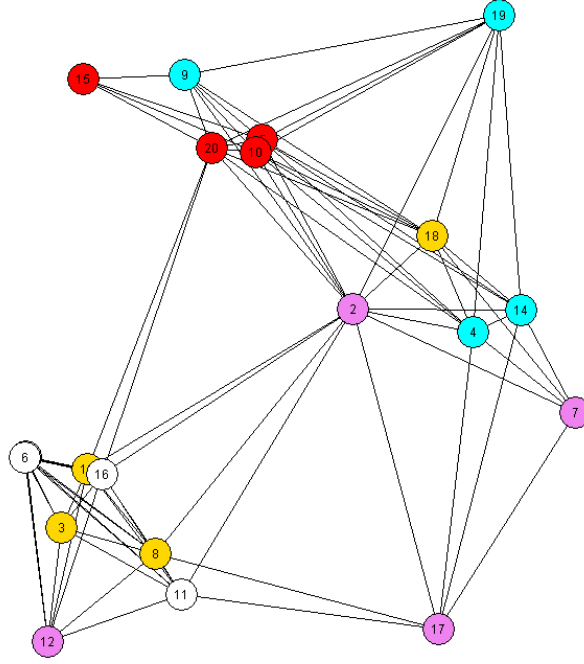


Figure 1.14: Network used to compare  $J_0^X$  and  $\bar{J}_0^X$ . Node color indicates group membership.

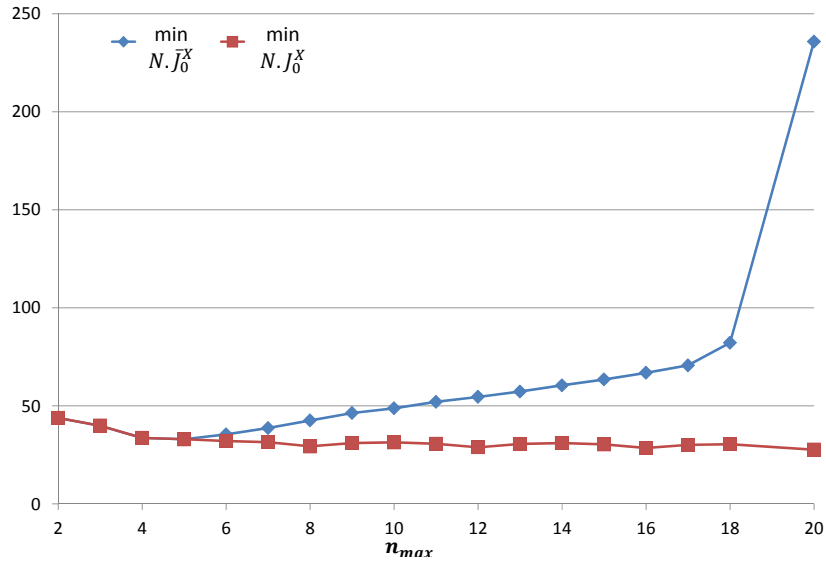


Figure 1.15: Minimum values of  $J_0^X$  and  $\bar{J}_0^X$  vs. maximum cluster size  $n_{max}$ .

## 1.7 Case of concave metrics: the throughput example

In the previous sections of this chapter, we have studied the case of additive metrics. Nevertheless, some metrics are not additive but are still very relevant to measure the performance in wireless networks. For example, the throughput metric associated with a network path is the minimum throughput value along the links of this path and not the cumulated value. The link throughput is therefore not additive. In the network community, this metric is said to be concave [40]. In this section we follow the methodology used with  $J_0$  to define  $T_0$  usable with the throughput metric. We extend  $J_0^X$  only, knowing that extending  $J_0^L$  could be done similarly.

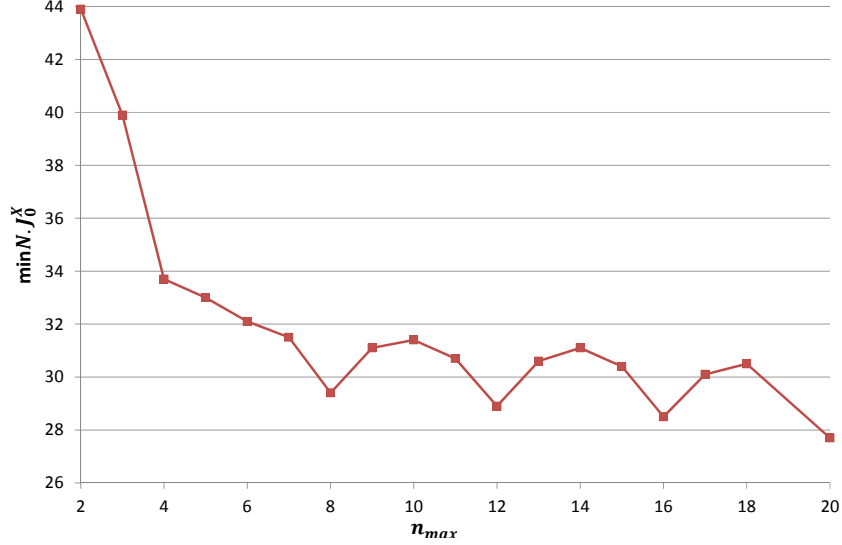


Figure 1.16: Minimum values of  $J_0^X$  vs. maximum cluster size, with group size equal to four.

### 1.7.1 Definition

When studying the case of the link throughput, the weight  $w_{i,j}$  of a link  $(i, j) \in \mathcal{E}$  can be defined as its capacity in a gaussian channel:  $w_{i,j} := \log_2[1 + \Gamma(i, j)]$ .

Let us define the graph  $\mathcal{G}'(p)$  by its vertices  $\mathcal{V}$  and its set of edges  $\mathcal{E}_T(p)$  whose weights are  $\hat{\eta} \cdot w_{i,j}$  and  $\tilde{\eta} \cdot w_{i,j}$  for intra and inter-cluster links respectively, with  $\hat{\eta}$  and  $\tilde{\eta} < \hat{\eta}$  the throughput efficiencies achieved by intra and inter-cluster RRA, and  $\eta = \tilde{\eta}/\hat{\eta} (< 1)$ . Let us denote by  $\mathcal{W}_X(p, i, j)$  the shortest widest path [44] from node  $i$  to node  $j$  in  $\mathcal{G}'(p)$ . Let us split this path  $\mathcal{W}_X(p, i, j)$  into two subsets  $\hat{\mathcal{W}}_X(p, i, j)$  and  $\tilde{\mathcal{W}}_X(p, i, j)$ , the sets of its intra and inter-cluster links respectively. Starting from (1.15), we note that link throughput is a concave metric. Therefore, to get the end-to-end path throughput  $T_0^X(p, i, j)$ , instead of a summation operator, a minimum value operator must be used on all links along a shortest path from node  $i$  to node  $j$ . The throughput  $T_0^X(p, i, j)$  that can be achieved between node  $i$  and node  $j$  is thus defined as:

$$T_0^X(p, i, j) := \min \left( \hat{\eta} \cdot \min_{(i',j') \in \hat{\mathcal{W}}_X(p,i,j)} w_{i',j'}, \tilde{\eta} \cdot \min_{(i',j') \in \tilde{\mathcal{W}}_X(p,i,j)} w_{i',j'} \right) \quad (1.29)$$

$$:= \hat{\eta} \left[ \min \left( \min_{(i',j') \in \hat{\mathcal{W}}_X(p,i,j)} w_{i',j'}, \eta \cdot \min_{(i',j') \in \tilde{\mathcal{W}}_X(p,i,j)} w_{i',j'} \right) \right] \quad (1.30)$$

Similarly to (1.1), we now introduce the metric  $T_0^X(p)$ :

$$T_0^X(p) := \frac{1}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot T_0^X(p, i, j). \quad (1.31)$$

Finding the best clustering of the network from  $T_0^X$  perspective is equivalent to finding the set of partitions  $\mathcal{P}_T^X$  defined as:

$$\mathcal{P}_T^X := \arg \max_{p \in \mathcal{P}} T_0^X(p) \quad (1.32)$$

When  $\eta < 1$  one may wonder about the quality of the single cluster network partition  $\{\mathcal{V}\}$ . Using a similar argument as for Result 1.6 page 17, it is possible to demonstrate that  $\{\mathcal{V}\}$  is the partition leading to the highest  $T_0^X$  value.

### 1.7.2 Shortest widest path

Let us recall the difference between the shortest path and the shortest widest path algorithms. In a graph  $\mathcal{G}$ , for each link  $(i, j) \in \mathcal{E}$ , let  $b_{i,j}$  and  $d_{i,j}$  the available bandwidth and the propagation delay of this link, respectively. If  $(i, j) \notin \mathcal{E}$  then  $b_{i,j} = 0$  and  $d_{i,j} = \infty$ . The length of the shortest-path  $\mathcal{S}(i, j) = (i, i_2, i_3, \dots, i_{K-1}, j)$  from node  $i$  to node  $j$  is denoted by  $D_{i,j}$  and is defined as the sum of propagation delays along this path. The shortest path algorithm from Dijkstra calculates all shortest paths from node  $i$  to all nodes  $j \neq i$ . It is defined in Table 1.3.

1	Set $L = \{i\}$ , $D_{i,i} = 0$ , and $D_{i,j} = d_{i,j}, \forall j \neq i$ .
2	<b>While</b> True:
3	Find set $K \subset \mathcal{V} \setminus L$ such that $\forall k \in K, D_{i,k} = \min_{j \notin L} D_{i,j}$ .
4	<b>If</b> $K \neq \emptyset$ <b>then</b> :
5	Choose $k \in K$ randomly.
6	Set $L = L \cup \{k\}$ .
7	<b>For all</b> $j$ such that $d_{k,j} < +\infty$ :
8	Set $D_{i,j} = \min(D_{i,j}, D_{i,k} + d_{k,j})$ .
9	<b>End For</b> .
10	<b>Else Exit</b> While loop.
11	<b>End If</b> .
12	<b>End While</b> .

Table 1.3: Dijkstra shortest path algorithm.

$B_{i,j}$  and  $D_{i,j}$  are the width and the length of the chosen shortest-widest path from node  $i$  to node  $j$ . By convention,  $B_{i,j} = \infty$  and  $D_{i,i} = 0$ . The algorithm in Table 1.4 [44] allows to find a widest path from node  $i$  to node  $j$ , and the associated bandwidth  $B_{i,j}^*$ . The widest path found by this algorithm is not the shortest widest path. To find the shortest widest path from  $i$  to  $j$  the Dijkstra algorithm must be used using the set of links  $\mathcal{E}' = \mathcal{E} \setminus \{(u, v) \in \mathcal{E} | b_{u,v} < B_{i,j}^*\}$ . Without this second step the widest path found is not a shortest one and depends on the source, as illustrated in Fig. 1.17, Fig. 1.18 and Fig. 1.19.

## 1.8 Conclusions

As first result, this chapter introduces the cost functions  $J_0^L$  and  $J_0^X$ , which can be used as a benchmark to compare different clustering solutions. These functions measure the quality of a network partition using end-to-end path calculations for additive metrics. They take into account the fact that inter-cluster and intra-cluster communications have different costs and are flexible enough to cover both cases when the traffic distribution depends on the groups or not. During the elaboration of  $J_0^X$ , it has been shown that routing should take into account the cluster structure to find good network paths and offer the best QoS to the user traffic.

A second result achieved in this chapter is that making use of group information for clustering leads to better network performance. This result was illustrated with the practical application of  $J_0^X$  to intra-group delays. Thanks to this new cost function it has also been shown that using a simple naive approach consisting of building one cluster per group does not lead to the best

1	Set $L = \{i\}$ .
2	Set $B_{i,i} = \infty$ , $D_{i,i} = 0$ , $B_{i,j} = b_{i,j}$ and $D_{i,j} = d_{i,j}, \forall j \neq i$ .
3	<b>While</b> True:
4	Find set $K \subset \mathcal{V} \setminus L$ such that $\forall k \in K, B_{i,k} = \max_{j \notin L} B_{i,j}$ .
5	<b>If</b> $K \neq \emptyset$ <b>then</b> :
6	Find $k$ such that $D_{i,k} = \min_{j \in K} D_{i,j}$ .
7	Set $L = L \cup \{k\}$ .
7	<b>For all</b> $j \notin L$ :
9	Set $B_{i,j} = \max(B_{i,j}, \min(B_{i,k}, b_{k,j}))$ .
10	<b>End For</b> .
11	<b>Else Exit</b> While loop.
12	<b>End If</b> .
13	<b>End While</b> .

Table 1.4: Modified Dijkstra widest path algorithm.

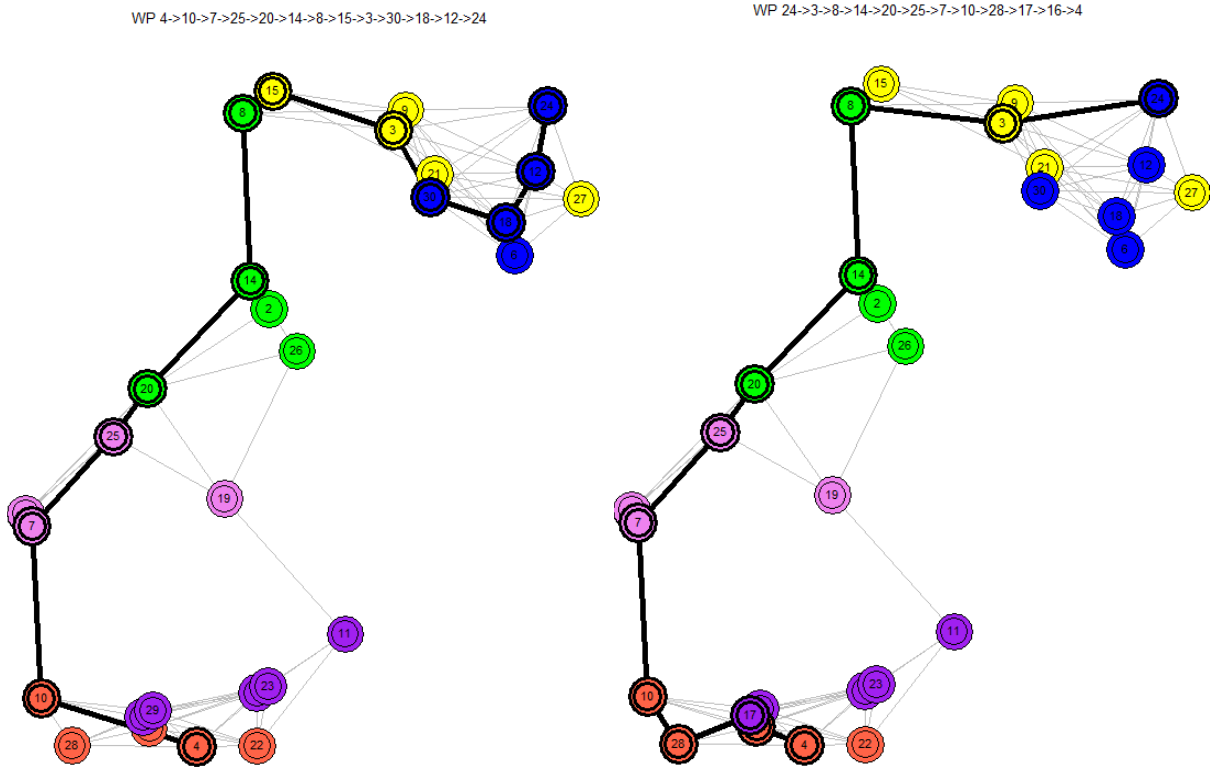


Figure 1.17: Example of a widest path from node 4 to node 24.

Figure 1.18: Example of a widest path from node 24 to node 4.

network performance. This justifies the need for more advanced clustering solutions using group information.

Finally  $J_0^X$  was extended in two ways. First, to better take cluster size into account, and avoid that  $\{\mathcal{V}\}$  be the best partition; the resulting function is denoted by  $\bar{J}_0^X$ . Second, to cope with the throughput concave metric; the resulting function is denoted by  $T_0^X$ .

The metrics introduced here are used in Chapters 2 and 3 to assess the performance of new distributed clustering algorithms.

SWP 4->10->7->25->20->14->8->3->24

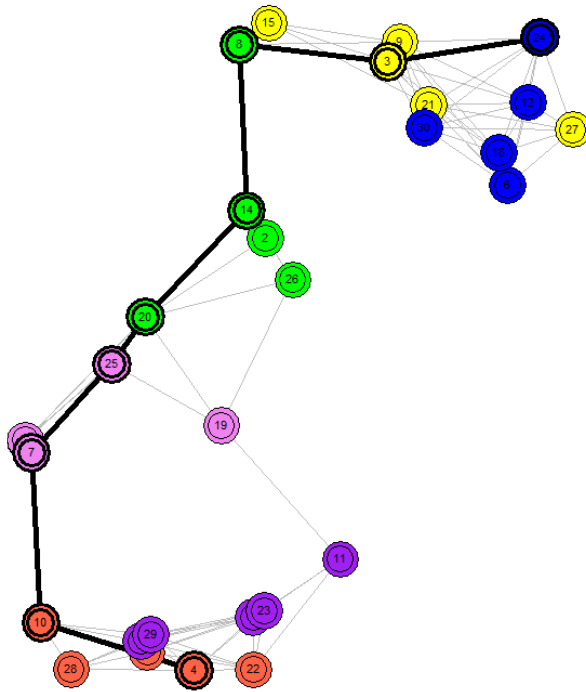


Figure 1.19: Example of a shortest widest path between nodes 4 and 24.



## Chapter 2

# Distributed clustering algorithm with operational groups

The work presented in this chapter has been partially published in [34–36].

### 2.1 Introduction

Public safety and military networks are organized into a hierarchical structure leading to the existence of *operational groups* (e.g., squad, section, etc.). In those networks, nodes exhibit group mobility behavior. Additionally, the traffic is strongly dependent on the network hierarchical organization, being mostly concentrated within operational groups. For these two reasons, the clustering solution should take into account operational group information in order to provide a better end-to-end QoS and to improve network stability. As demonstrated in Chapter 1 the trivial solution consisting in building one cluster per group is not acceptable. Thus to cope with structured networks based on operational groups, specific clustering solutions are required.

Numerous distributed clustering schemes have been proposed in the literature. These algorithmic solutions first select Cluster Head (CH) nodes, and then the other nodes affiliate to them, leading to the different clusters. A weight is commonly associated with each node and the nodes with the highest weights in a neighborhood are selected to be the CH nodes. The weights can be the node identifier, the node degree, the remaining battery power, metrics related to radio measurements etc., or a combination of them [1, 3, 6]. To get the node weight, other solutions rely on the knowledge of nodes' location and speed, obtained thanks to, for instance, a GPS device [12]. In a second step, when CH nodes have been selected, non-CH nodes affiliate to CH nodes depending on their weight or any other metric.

Only a few papers consider group information for building the clusters. The authors in [5] introduce the type-based clustering algorithm (TCA). This clustering scheme associates a stability factor to each node and selects as CH the nodes that have the highest stability factor in a radio neighborhood. The stability factor takes group membership (identified thanks to the IP subnet of each node) into account. A limitation of TCA lies in the fact that two CH nodes cannot be neighbors. A direct consequence in dense networks is the formation of clusters with a large number of members. Two distributed clustering algorithms, GDMAC [1] and VOTE [6], are well-known to handle the size of clusters but they do not take into account the group structure.

We have identified a clear gap in the ad hoc network clustering literature since there is no



clustering solution suited to structured network that properly take cluster size into account. To fill this gap we first consider extensions of existing CH-based clustering solutions. We find that the performance gains achieved thanks to these extensions are inherently limited by the algorithms from which they are derived. A challenge to overcome when designing CH-based algorithms is how to calculate the node weights used to elect the CH nodes. In structured networks, it make sense to assign the weight of a node based on its group. The drawback of this approach is that comparing the weight of nodes from different groups is very difficult. Also non-CH nodes may choose their CH based on its weight, which could depend on the current state of its cluster. For example the CH of a large cluster could have a greater weight than a CH of a small cluster. Unfortunately, this kind of strategy leads to a lot of instability in the cluster structure. We thus decided to follow a clean slate approach to design a new distributed clustering algorithm suited to structured networks, and search for algorithms that do not involve any CH. This choice allows us to form clusters whose members are peers, and thus avoids the aforementioned problem of allocating a weight to the nodes. In addition, since the CH can be a single point of failure within a cluster, our algorithm does not suffer from this problem.

Section 2.2 introduces the network model. Before going into the definition of the new distributed clustering algorithm, Section 2.3 details several new cluster cost functions and assesses their capability to be used to find good partitions with respect to the  $J_0$  metric, taken as a reference. Then Section 2.4 explains how, starting from the best cost function identified in Section 2.3, we designed our distributed clustering algorithm. The main content of this chapter is in Section 2.5 where the Distributed Clustering with Operational Groups (DCOG) algorithm is detailed. Finally, Section 2.6 is devoted to simulation results and analysis.

## 2.2 Network model

In this chapter the network model of Section 1.3 is extended introducing the following notations. The cluster group diversity  $o_k$  is the number of groups with at least one member in cluster  $\mathcal{C}_k$ . The index of the group with the highest number of members in cluster  $\mathcal{C}_k$  is  $o_k^*$ . The number of members of group  $\mathcal{O}_t$  in cluster  $\mathcal{C}_k$  is  $m_{t,k}$ . Let  $\mathcal{I}(\mathcal{C}_k) := \{t | m_{t,k} \neq 0, t \in \{1, \dots, T\}\}$  the set of the indices of the groups with at least one member in cluster  $\mathcal{C}_k$ .

To simplify notations in this chapter the cost functions  $J_0^X$  and  $T_0^X$  from Chapter 1 are denoted by  $J_0$  and  $T_0$ .

## 2.3 First step from the global network cost function to a distributed clustering solution

### 2.3.1 Definition of simple cost functions

The function  $J_0$  ( $J_0^X$  defined in Section 1.4) requires a global knowledge of the network and cannot be used during a distributed clustering process. Thus, we now define new cluster cost functions  $c_i$  that can be calculated by each cluster based on the knowledge available locally. They must take into account that:

- Nodes of the same group should belong to the same cluster, and

- The number of nodes in the clusters should not exceed a maximum value, associated with the number of nodes best handled by the radio resource allocation feature within a cluster.

Let us introduce the global network cost function  $J_i$  written as a sum of the cluster costs over all the clusters of the network:

$$J_i := \sum_{k=1}^{N_c} c_i(\mathcal{C}_k), \quad (2.1)$$

with  $c_i(\mathcal{C}_k)$  the cost of cluster  $\mathcal{C}_k$  associated with  $J_i$ .

In this section, our goal is to identify some good criteria that could be used in a distributed clustering process. We define several  $J_i$  by their corresponding  $c_i$ , and calculate their value for each partition of  $\mathcal{G}$ . We retain as best partitions the ones that minimize  $J_i$ .

The first cost function is:

$$c_1(\mathcal{C}_k) := o_k.$$

Minimizing function  $J_1$  leads to minimizing the cluster group diversity across all clusters, meaning that the clusters are more or less equal to the groups.

The second and third functions are  $c_2$  and  $c_3$ , which take into account group information in a different way:

$$c_2(\mathcal{C}_k) := -\frac{m_{o_k^*,k}}{m_{o_k^*}},$$

$$c_3(\mathcal{C}_k) := (m_{o_k^*} - m_{o_k^*,k})^2.$$

The idea of these two cost functions is to find in each cluster the group  $o_k^*$  with the highest number of members  $m_{o_k^*,k}$ , and to compare it to the total number of members in this group  $m_{o_k^*}$ . Having these numbers close to each other is another way of having clusters more or less equal to the groups.

In addition to using group related metrics, the last two functions  $c_4$  and  $c_5$  introduce a new variable that is important from a radio resource allocation point of view: the size of the clusters. They are defined as follows:

$$c_4(\mathcal{C}_k) := (n_{max} - n_k + 1) \cdot o_k,$$

$$c_5(\mathcal{C}_k) := (n_{max} - n_k + 1) \cdot (m_{o_k^*} - m_{o_k^*,k} + 1).$$

The +1 in  $c_4$  and  $c_5$  corresponds to the fact that if the size of the cluster is maximum, we still want to take group information into account.

### 2.3.2 Performance assessment

Let us define  $\mathcal{P}_i^*$ ,  $\forall i \in \{0, 1, 2, 3, 4, 5\}$ , as the set of the optimal partitions for  $J_i$ :

$$\mathcal{P}_i^* := \arg \min_{p \in \mathcal{P}^c} J_i(p).$$

A function  $J_i$ ,  $i \neq 0$ , could be considered as good as  $J_0$  if it yields the same best partitions:  $\mathcal{P}_i^* = \mathcal{P}_0^*$ . However, this never happens because none of the  $J_i$  is as good as  $J_0$ . Therefore,

our first criterion to assess if a function  $J_i$  is good or not, is whether or not this function  $J_i$  can be used to identify some of the best partitions w.r.t.  $J_0$ , i.e., if  $\mathcal{P}_i^* \cap \mathcal{P}_0^* \neq \emptyset$ . To evaluate this capability, we have simulated 100 networks using the setup (and the associated exhaustive partition enumeration procedure) described in Section 1.5.2. Firstly, let us remember that the value of  $\alpha$  is the probability that one node communicates with a node of the same group (see Section 1.4.5). For a given value of  $\alpha$ , we denote by  $n_{\alpha,i}$ ,  $\forall i \in \{1, 2, 3, 4, 5\}$ , the number of networks (out of the 100 simulated ones) for which  $\mathcal{P}_i^* \cap \mathcal{P}_0^* \neq \emptyset$ , and by  $\mathcal{L}_\alpha$  the list of  $n_{\alpha,i}$  values, sorted by ascending order. We now explain how to rank the different  $J_i$  functions. We denote by  $i_\alpha^* = \arg \max_{i \in \{1,2,3,4,5\}} n_{\alpha,i}$  and  $\bar{i}_\alpha^* = \arg \min_{i \in \{1,2,3,4,5\}} n_{\alpha,i}$  the indexes of the  $J_i$  functions which allow to find at least one best partition in the largest, respectively smallest, number of networks. Using this notation,  $J_{i_\alpha^*}$  is thus the best function, and  $J_{\bar{i}_\alpha^*}$  is the worst one. Let us define  $n_\alpha = (n_{\alpha,i_\alpha^*} + n_{\alpha,\bar{i}_\alpha^*})/2$ . For example, if for a given  $i$ ,  $n_{\alpha,i} \leq n_\alpha$ , then its  $n_{\alpha,i}$  value lies in the lower half of  $\mathcal{L}_\alpha$ . In that case, the associated  $J_i$  belongs to the 50% worst functions. In Table 2.1 we have put the values  $n_{\alpha,i}$ ,  $\forall \alpha \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ ,  $\forall i \in \{1, 2, 3, 4, 5\}$ , and have set the color of each cell to highlight the (decreasing) performance of the associated  $J_i$  function:

- Green, when  $n_{\alpha,i} = n_{\alpha,i_\alpha^*}$  (i.e., best  $J_i$  w.r.t. the associated  $\alpha$ ),
- Blue, when the  $J_i$  function is in the upper half of  $\mathcal{L}_\alpha$ ,
- Orange, when the  $J_i$  function is in the lower half of  $\mathcal{L}_\alpha$ ,
- Red, when  $n_{\alpha,i} = n_{\alpha,\bar{i}_\alpha^*}$  (i.e., worst  $J_i$ ).

The last two lines of the table are the sums of  $n_{\alpha,i}$  over several values of  $\alpha$ , for a given  $i$ .

$\alpha$	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$
0.2	1	0	4	8	4
0.3	3	0	8	15	8
0.4	5	0	13	21	14
0.5	11	2	28	26	22
0.6	15	4	32	25	25
0.7	31	17	41	23	25
0.8	63	34	56	30	31
0.9	86	55	61	25	28
$\sum_{\alpha=0.2}^{\alpha=0.9}$	215	112	243	173	157
$\sum_{\alpha=0.2}^{\alpha=0.6}$	35	6	85	95	73

Table 2.1: Number of networks with  $\mathcal{P}_i^* \cap \mathcal{P}_0^* \neq \emptyset$  (number of trials = 100).

A first remark is that all  $J_i$  achieve larger values of  $n_{\alpha,i}$  when  $\alpha$  increases. This is expected because an increasing  $\alpha$  values means that group membership has a larger impact on the user traffic load, thus taking into account groups should help in finding better partitions. The penultimate line can be used to establish a partial ordering between all  $J_i$  considering all values of  $\alpha$ :  $J_3 > J_1 > J_4 > J_5 > J_2$ . The function  $J_3$  is the best only for  $\alpha$  values from 0.5 to 0.7, and offers consistently good performance for all values of  $\alpha$ . The function  $J_2$  is consistently the worst. The case of the second best function  $J_1$  is very interesting: when groups have a low

impact on  $J_0$  values, its performance is mediocre, but improves with increasing group impact, finally becoming the best. The same kind of argument, reversed, can be made also for  $J_4$  and  $J_5$ : their performance are good for low  $\alpha$  values, but degrade as  $\alpha$  value increases ( $J_4$  even becoming the worst when  $\alpha \geq 0.8$ ). Summing the numbers over  $\alpha \in \{0.2, 0.3, 0.4, 0.5, 0.6\}$  leads to a different partial ordering between all  $J_i$ , as shown in the last line:  $J_4 > J_3 > J_5 > J_1 > J_2$ . This result hints that the  $J_i$  should be selected depending on the importance of the groups on traffic.

The fact that the proposed cost functions succeed in finding some best partitions w.r.t.  $J_0$  is a good result. However, we now prove that this criterion is not enough. Let us for example consider a cluster cost function  $c_{i_0}$  leading to a network cost function  $J_{i_0}$  such that any partition is a best partition:  $\mathcal{P}_{i_0}^* = \mathcal{P}^c$ . In that case, for any random network we have  $\mathcal{P}_{i_0}^* \cap \mathcal{P}_0^* = \mathcal{P}_0^* \neq \emptyset$ . In Table 2.1 such a function would lead to a value  $n_{\alpha,i} = 100$  and would be identified as the best. Let us define  $\bar{\mathcal{P}}_i^*$ ,  $\forall i \in \{0, 1, 2, 3, 4, 5\}$ , as the set of the worst partitions for  $J_i$ :

$$\bar{\mathcal{P}}_i^* := \arg \max_{p \in \mathcal{P}^c} J_i(p).$$

We have  $\mathcal{P}_{i_0}^* \cap \bar{\mathcal{P}}_0^* = \bar{\mathcal{P}}_0^*$ , meaning that  $J_{i_0}$  also identifies as best partitions the partitions that are the worst from the point of view of  $J_0$ . We thus need another criterion to assess the quality of a cluster cost function  $c_i$ .

We now define this second criterion, following a methodology similar to the one of Section 1.5.1. Let us write  $p_j^i$ , with  $j \in \{1, 2, \dots, |\mathcal{P}_i^*|\}$ , the members of  $\mathcal{P}_i^*$ , and define  $J_{0,i}^*$  as the highest  $J_0$  value of the partitions in  $\mathcal{P}_i^*$  (the worst according to  $J_0$ ):

$$J_{0,i}^* := \max_{p \in \mathcal{P}_i^*} J_0(p).$$

Let us write  $\bar{p}_j^i$ , with  $j \in \{1, 2, \dots, |\bar{\mathcal{P}}_i^*|\}$ , the members of  $\bar{\mathcal{P}}_i^*$ , and define  $\bar{J}_{0,i}^*$  as the lowest  $J_0$  value of the partitions in  $\bar{\mathcal{P}}_i^*$  (the best according to  $J_0$ ):

$$\bar{J}_{0,i}^* := \min_{p \in \bar{\mathcal{P}}_i^*} J_0(p).$$

Let us take the example of Fig 2.1, where  $\mathcal{P}_i^* = \{p_1^i, p_2^i, p_3^i, p_4^i, p_5^i\}$  with  $J_{0,i}^* = J_0(p_3^i)$ , and  $\bar{\mathcal{P}}_i^* = \{\bar{p}_1^i, \bar{p}_2^i, \bar{p}_3^i, \bar{p}_4^i\}$  with  $\bar{J}_{0,i}^* = J_0(\bar{p}_2^i)$ .

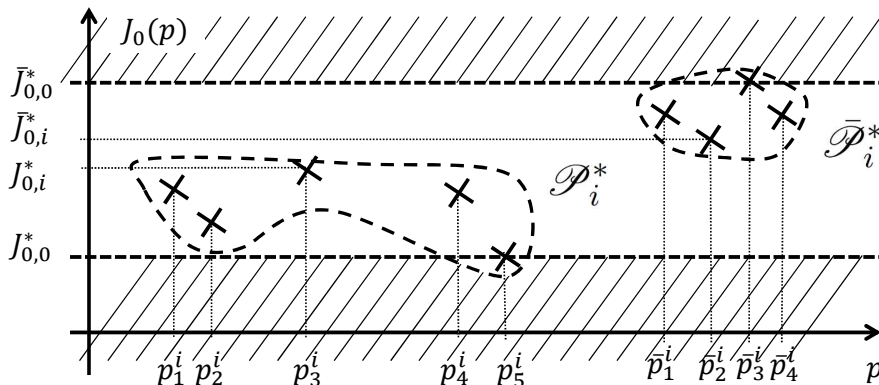


Figure 2.1: Metrics used to assess  $J_i$ : example to illustrate  $J_{0,0}^*$ ,  $J_{0,i}^*$ ,  $\bar{J}_{0,i}^*$  and  $\bar{J}_{0,0}^*$ .

To measure how good is a function  $J_i$  at finding partitions with good  $J_0$  values, we consider the quantity  $J_{0,i}^* - J_{0,0}^*$ . We also use  $\bar{J}_{0,i}^* - J_{0,0}^*$  to measure if  $J_i$  is capable to find partitions with bad  $J_0$  values. Because these values depend on the network trial,  $J_{0,i}^* - J_{0,0}^*$  and  $\bar{J}_{0,i}^* - J_{0,0}^*$  must be normalized. We now introduce the values  $\delta_i^*$  and  $\bar{\delta}_i^*$  normalized in  $[0, 1]$ :

$$\begin{cases} \delta_i^* := \frac{J_{0,i}^* - J_{0,0}^*}{\bar{J}_{0,0}^* - J_{0,0}^*}, \\ \bar{\delta}_i^* := \frac{\bar{J}_{0,i}^* - J_{0,0}^*}{\bar{J}_{0,0}^* - J_{0,0}^*}. \end{cases} \quad (2.2)$$

In order for  $J_i$  to be a good substitute to  $J_0$ ,  $\delta_i^*$  must be as close to 0 as possible. If the value of  $\bar{\delta}_i^*$  is close to 1, it means that  $J_i$  can also find very bad  $J_0$  partitions.

The cumulated distribution functions of  $\delta_i^*$  are plotted in Fig. 2.2. For example when  $\alpha = 0.2$ , the probability that  $\delta_4^*$  is less than 0.3 is 0.6, i.e., 60% of partitions considered as best by  $J_4$  have a corresponding  $J_0$  value which is in the first 30% of the  $J_0$  values interval.

These figures show that when  $\alpha$  increases from 0.2 to 0.6, the partial ordering between the  $J_i$  remains constant:  $J_5 > J_4 > J_3 > J_1 > J_2$ . This result is different from the one of Table 2.1. This means that  $J_4$  is the best if the capability of finding best partitions w.r.t.  $J_0$  is the most important, but that  $J_5$  is the best in probability: it is likely that any partition found as best using  $J_5$  will have a lower value than a partition found as best using  $J_4$ .

The function  $J_5$  remains the best for all values of  $\alpha$  no greater than 0.8. It is only when  $\alpha = 0.9$  that  $J_1$  becomes the best and  $J_5$  the second best. Also,  $J_3$  identified as best  $J_i$  in Table 2.1 is never the best in Fig. 2.2. This confirms the importance of choosing a good criterion to select the  $J_i$ . In view of its capacity to deliver greater details than our first criterion, we consider our second criterion as the most relevant.

The frequency diagrams of  $\bar{\delta}_i^* - \delta_i^*$  have been drawn in Fig. 2.3 and Fig. 2.4 for  $J_5$ , identified as the globally best  $J_i$  (except for  $\alpha = 0.9$ ). The  $x$  values on the x-axis, indicate values of  $\bar{\delta}_i^* - \delta_i^*$  in the  $[x - 0.2, x]$  interval. When strictly positive these values quantify, the larger the better, the fact that  $J_i$  finds as its best partitions ( $p_i^* \in \mathcal{P}_i^*$ ), partitions that have better  $J_0$  values than the partitions that  $J_i$  finds as its worst partitions ( $\bar{p}_i^* \in \bar{\mathcal{P}}_i^*$ ). Negative values are associated with the inverse undesired behavior. The y value indicates the number of networks. For example in the frequency diagram associated with  $\alpha = 0.2$  of Fig. 2.3, the y value of the bar at  $x = 0.2$  is 34. This means that for 34 of the 100 random networks, all partitions identified as the best by  $J_5$  have better  $J_0$  values, by a factor in  $[0, 0.2]$ , than all partitions identified as the worst by  $J_5$ .

Fig. 2.3 and Fig. 2.4 show the capability of  $J_5$  to discriminate between good and bad partitions w.r.t.  $J_0$ . It is lower with low value of  $\alpha$  and improves when  $\alpha$  increases. As soon as  $\alpha \geq 0.5$ , it is always the case.

The results obtained so far allow to determine that if group membership is essential w.r.t. the traffic load (i.e. high values of  $\alpha$ ),  $J_1$  is the best function. However,  $J_5$  is always good  $\forall \alpha$ , indicating that cluster size is also important. This makes sense because the number of inter-cluster communications also depend on the number of clusters, i.e., on cluster size. Thus, in the following we use the cost function  $c_5$  to design a distributed clustering algorithm.

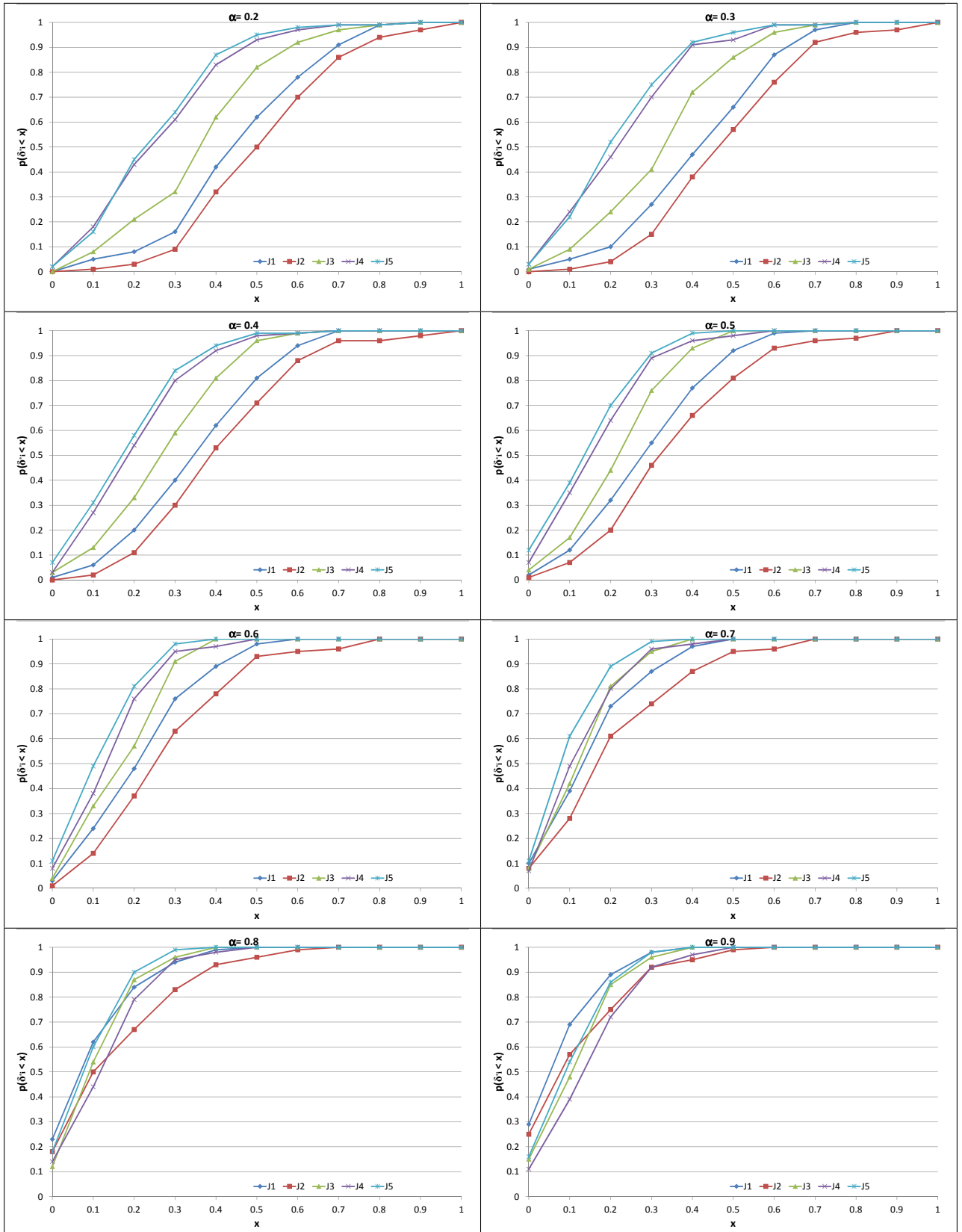


Figure 2.2: cdf of  $\delta_i^*$  vs.  $\alpha$ .

## 2.4 Towards a distributed clustering solution

This section replicates in chronological order the steps that led to the definition of the cost function used in the distributed clustering scheme proposed in this chapter, as well as the way

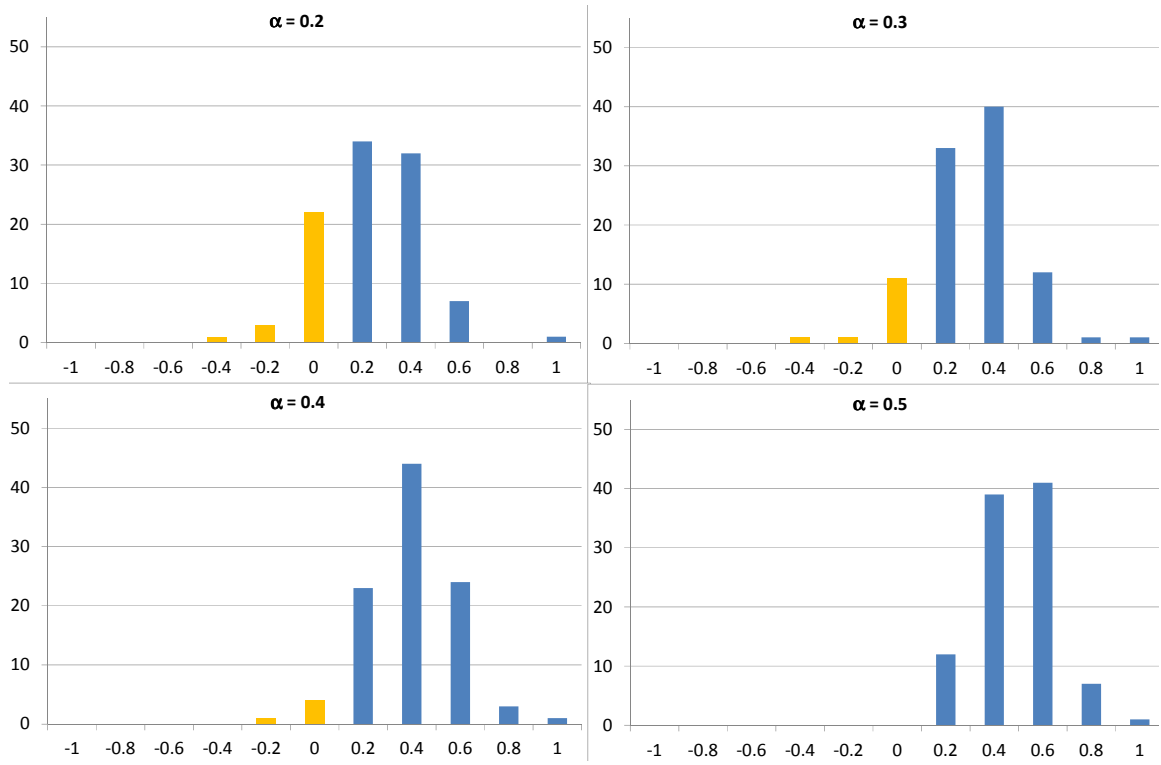


Figure 2.3:  $\bar{\delta}_5^* - \delta_5^*$  frequency diagrams for  $\alpha \in \{0.2, 0.3, 0.4, 0.5\}$ .

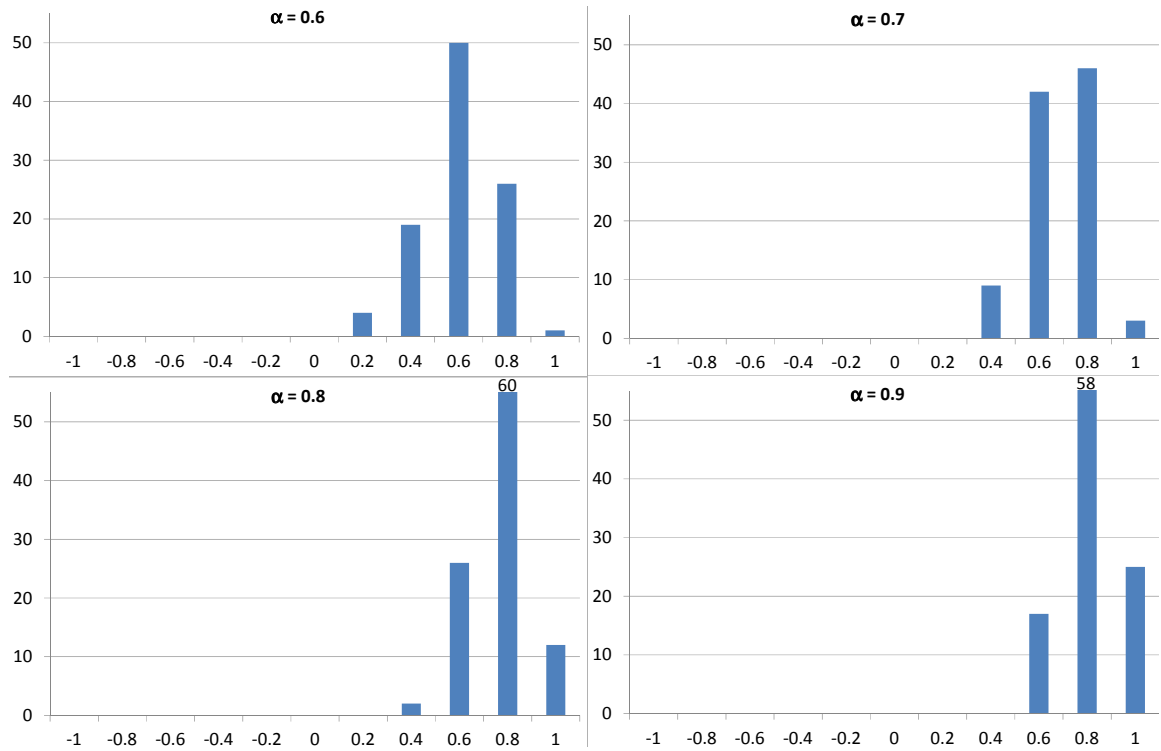


Figure 2.4:  $\bar{\delta}_5^* - \delta_5^*$  frequency diagrams for  $\alpha \in \{0.6, 0.7, 0.8, 0.9\}$ .

to handle it. Accordingly with the analysis performed in Section 2.3, we chose  $c_5$  as initial cost function. Concerning the algorithm describing how the clusters are modified, we initialize it

with each node being a singleton cluster. Let us now detail some of the steps that led to the definition of the distributed clustering solution of Section 2.5.

### 2.4.1 First intermediate solution

The principle of the first algorithm was to split time in rounds, and to divide each round in two steps: the decision making and its application. We found in Section 2.3 that the global cost function  $J_5$  allowed to select good partitions. Consequently, to perform the first step of the algorithm we first chose the cluster cost function  $c_5$  derived from  $J_5$ :

$$c_5(\mathcal{C}_k) := (n_{max} - n_k + 1) \cdot (m_{o_k^*} - m_{o_k^*,k} + 1).$$

#### 2.4.1.1 Step 1: decision making

During step 1 in each cluster  $\mathcal{C}_k$  each node  $i$  checks if it is valuable to move from its current cluster to each of its neighboring cluster  $\mathcal{C}_\ell$ . To check this, the loss induced by node  $i$  leaving its cluster is calculated:

$$\text{loss}(i, \mathcal{C}_k) := c(\mathcal{C}_k \setminus \{i\}) - c(\mathcal{C}_k). \quad (2.3)$$

Then the gain induced by the arrival of node  $i$  in  $\mathcal{C}_\ell$  is calculated:

$$\text{gain}_1(i, \mathcal{C}_k, \mathcal{C}_\ell) := c(\mathcal{C}_\ell \cup \{i\}) - c(\mathcal{C}_k). \quad (2.4)$$

with  $c$  the chosen cluster cost function, i.e.  $c_5$ . We decided that a node could leave its cluster  $\mathcal{C}_k$  to join another cluster  $\mathcal{C}_\ell$  if:

$$\text{gain}_1(i, \mathcal{C}_k, \mathcal{C}_\ell) > -\text{loss}(i, \mathcal{C}_k). \quad (2.5)$$

If more than one neighbor cluster  $\mathcal{C}_\ell$  exist, and that, at the same time satisfy (2.5) and maximize the  $\text{gain}_1(i, \mathcal{C}_k, \mathcal{C}_\ell)$ , then one is randomly chosen among them and node  $i$  is selected to join  $\mathcal{C}_\ell$ . Otherwise node  $i$  does not leave its cluster  $\mathcal{C}_k$ .

In addition to this condition, to allow a node to join a neighbor cluster, the connectivity, size and diameter constraints must also be satisfied.

#### 2.4.1.2 Step 2: decision application

At the end of step 1, it has been decided that some nodes should leave their current cluster to join a neighboring one. We call this an inter-cluster nodes swap, or more shortly a swap. During step 2, the selected swap is performed.

#### 2.4.1.3 Behavior of first intermediate solution

Several problems were identified with this solution:

- When multiple groups have the same number of members in a cluster  $\mathcal{C}_k$ , if this number is also the highest among all groups in  $\mathcal{C}_k$ , then  $o_k^*$  has not a unique value. If in addition the groups do not have the same size (multiple values for  $m_{o_k^*,k}$ ), then the value of  $c_5(\mathcal{C}_k)$  is implementation dependent.
- Notwithstanding this problem, when multiple groups have at least one member in a cluster, only the one with the highest number of members in the cluster is taken into account by  $J_5$ . The problem is similar for all  $J_i$  from Section 2.3.



## 2.4.2 Second intermediate solution

### 2.4.2.1 Modification of the cost function

To solve both problems identified in the previous solution, we introduced within the cost function a sum over all groups with at least one member in the cluster. We also wanted to achieve the following three goals:

1. For each group, we wanted that the cost be a decreasing function of the number of members of this group in the cluster. For each group, we thus introduced the function  $1 - f_1(m_{t,k})$ , with  $f_1(m_{t,k}) := m_{t,k}/K$  a linear function of  $m_{t,k}$ , and  $K$  a constant.
2. Also, we decided to favor the formation of clusters including the groups that have the largest size. Accordingly, we chose to set  $K := m_{min}$ , with  $m_{min} := \arg \min_{t \in \{1, \dots, N_g\}} m_t$ , the size of the smallest group.
3. Finally, to enforce the cluster size constraint, we decided to limit the number of groups within a cluster, adding a penalty to the clusters including some members from too many groups.

We thus defined the new following cost function:

$$c'_5(\mathcal{C}_k) := \sum_{t \in \mathcal{I}(\mathcal{C}_k)} \left[ 1 - \frac{m_{t,k}}{m_{min}} \right] + 1_{|\mathcal{I}(\mathcal{C}_k)| \leq \Omega} \left[ \Omega - |\mathcal{I}(\mathcal{C}_k)| \right], \quad (2.6)$$

with  $\Omega$  a constant.

### 2.4.2.2 Behavior of second intermediate solution

The loss induced by a node  $i$  leaving a cluster  $\mathcal{C}_k$  is the same as the gain induced by the arrival of node  $i$  in any cluster  $\mathcal{C}_\ell$ . Consequently, the algorithm remains in its initial state.

## 2.4.3 Third intermediate solution

### 2.4.3.1 Modification of the cost function

To solve the aforementioned problem, we modified  $c'_5$ . We replaced the linear function  $f_1$  used in (2.6) by function  $f_2$  defined as:

$$f_2(m_{t,k}) := \frac{m_{t,k}(m_{t,k} + 1)}{m_{min}(m_{min} + 1)}. \quad (2.7)$$

Thanks to (2.7) a gain equal to  $n + 1$  is achieved when in a cluster the number of members of a group increases from  $n$  to  $n + 1$ . Let us notice that the following property holds:

$$f_2(m_{t,k} + 1) - f_2(m_{t,k}) < f_2(m_{t,k} + 2) - f_2(m_{t,k} + 1).$$

Thanks to this property, it is always better from a cost perspective to add members of a group to a cluster already including the largest number of members of this group.

We thus defined the following cost function:

$$c''_5(\mathcal{C}_k) := \sum_{t \in \mathcal{I}(\mathcal{C}_k)} \left[ 1 - \frac{m_{t,k}(m_{t,k} + 1)}{m_{min}(m_{min} + 1)} \right] + 1_{|\mathcal{I}(\mathcal{C}_k)| \leq \Omega} \left[ \Omega - |\mathcal{I}(\mathcal{C}_k)| \right]. \quad (2.8)$$

In Chapter 3, we introduce a class of functions which includes (2.7) as a specific case.

### 2.4.3.2 Modification of the decision making step

Because  $c(\mathcal{C}_k)$  appears in both  $\text{gain}_1(i, \mathcal{C}_k, \mathcal{C}_\ell)$  and  $\text{loss}(i, \mathcal{C}_k)$ , a node in group  $\mathcal{O}_t$  may leave its cluster  $\mathcal{C}_k$  to join cluster  $\mathcal{C}_\ell$ , even if there are more members of  $\mathcal{O}_t$  in  $\mathcal{C}_k$  than in  $\mathcal{C}_\ell$ . To fix this issue, we defined the new gain  $\text{gain}_2(i, \mathcal{C}_\ell)$  induced by the arrival of node  $i$  in cluster  $\mathcal{C}_\ell$ :

$$\text{gain}_2(i, \mathcal{C}_\ell) := c(\mathcal{C}_\ell \cup \{i\}) - c(\mathcal{C}_\ell). \quad (2.9)$$

Also, when using the algorithm of Section 2.4.1.1, the loss induced by a node leaving its singleton cluster is equal to the gain induced by this node joining a cluster not yet including any node of its group. To allow a cluster to include members of several groups the loss associated with the vanishing of a singleton cluster was set to zero.

### 2.4.3.3 Behavior of third intermediate solution

Some clustering now happens: provided that the constraints allow it, the nodes of the same groups join together into the same cluster, forming one cluster per group. In addition to this, we would like that if the maximum cluster size allows it, more than one group can be included in a single cluster.

## 2.4.4 Fourth intermediate solution

### 2.4.4.1 Modification of the cost function

We found out that in (2.8) the indicator function was unnecessary and thus we removed it. We also modified the cost function to make it normalized in  $[0, 1]$ . The next cost function was defined as:

$$c_5'''(\mathcal{C}_k) := 1 - \frac{1}{n_{max}} \sum_{t \in \mathcal{I}(\mathcal{C}_k)} m_{t,k} \cdot \frac{m_{t,k}(m_{t,k} + 1)}{m_t(m_t + 1)}. \quad (2.10)$$

In this equation, multiplying the fraction  $[m_{t,k}(m_{t,k} + 1)]/[m_t(m_t + 1)]$  by  $m_{t,k}$  favors the completion of the biggest groups.

### 2.4.4.2 Modification of the decision making step

Instead of allowing single nodes to be swapped between clusters, we now allow sets of nodes to change cluster together. More precisely, within a cluster all members of the same group may leave their current cluster to join a neighboring cluster. This modification allows to gather members from multiple groups in the same cluster. Also, to allow a set of nodes to move to a neighboring cluster, the connectivity, size and diameter constraints must be satisfied in both the source and the destination cluster.

### 2.4.4.3 Behavior of fourth intermediate solution

This solution seemed to succeed in building adequate clusters for a variety of networks. Yet a last improvement to simplify the decision making step could be done.

### 2.4.5 Final solution

In Section 2.4.3, the decision making step was modified to set to zero the loss associated with the vanishing of a singleton cluster. This policy had also to be applied when a non-singleton cluster vanishes because all its members from the same group move to another cluster. We modified (2.10) to avoid handling these cases in a specific way. We introduced an additional term  $f_3(n_k)$  depending on the cluster size, such that the gain (2.9) would always be strictly greater than the loss (2.3), when forming a larger cluster. We defined this term along the same line as (2.7) in Section 2.4.3:

$$f_3(n_k) := \frac{n_k(n_k + 1)}{n_{max}(n_{max} + 1)}. \quad (2.11)$$

The new cost function became:

$$c_6(\mathcal{C}_k) := 1 - \left[ \frac{n_k(n_k + 1)}{n_{max}(n_{max} + 1)} \cdot \epsilon + \frac{1}{n_{max}} \sum_{t \in \mathcal{I}(\mathcal{C}_k)} m_{t,k} \cdot \frac{m_{t,k}(m_{t,k} + 1)}{m_t(m_t + 1)} \cdot (1 - \epsilon) \right], \quad (2.12)$$

with  $\epsilon$  chosen in order to either favor ( $\epsilon$  close to 0) the formation of clusters including complete groups, or to only favor ( $\epsilon$  close to 1) large clusters.

In Chapter 3 we detail the properties that must be fulfilled by cluster cost functions usable to perform distributed clustering, (2.12) being only a specific example.

### 2.4.6 Simulations

The final cost function  $c_6$  defined by (2.12) has been designed to be used by a distributed clustering algorithm. Before evaluating the performance of this algorithm, we now assess the capability of  $c_6$  to find the best partitions, using the same methodology as described in Section 2.3, based on a centralized partition enumeration procedure. We set  $\alpha = 0.9$  and  $\epsilon = 0$  (in order to only take into account group membership). Simulations have been performed for the 100 random networks from Section 1.5.2. For each network the best partition w.r.t.  $J_6$  has been found and compared to the partitions with the lowest and highest  $J_0$  values. Fig. 2.5 details the simulation results, including the ones for the functions from Section 2.3:  $J_1$ ,  $J_2$ ,  $J_3$ ,  $J_4$  and  $J_5$  (which are only usable with the centralized partition enumeration procedure of Section 1.5.2).

The partitions obtained thanks to  $J_6$  are nearly as good as the one obtained thanks to  $J_1$ , which is the best for  $\alpha = 0.9$ . This achievement foretells good results for our distributed algorithm using  $c_6$ . Now that the methodology that we used to define our distributed clustering algorithm has been detailed, we can introduce the algorithm itself.

## 2.5 Distributed Clustering with Operational Group

In this section, we start by introducing the principles of DCOG, starting from a generic cluster cost function  $c$ . Then, we detail the DCOG algorithm based on the results of Section 2.4, i.e., a way to compare node swaps between clusters, and a cluster cost function requiring only local cluster information. Finally we explain how DCOG is adapted to node mobility.

### 2.5.1 Principles of DCOG

The DCOG algorithm is a distributed clustering algorithm that is run continuously by all network nodes in order to build clusters satisfying the two following properties: any cluster should

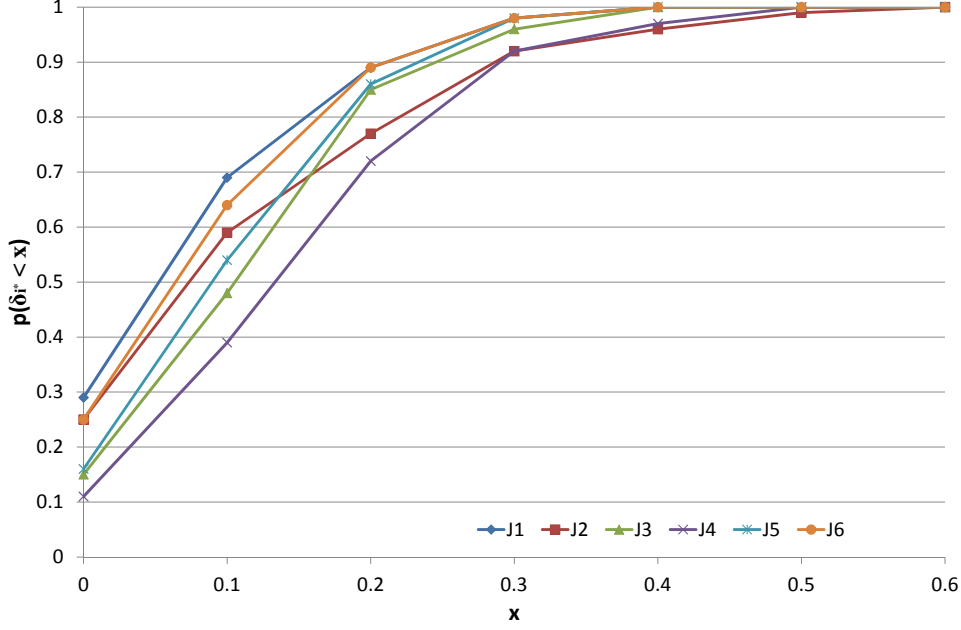


Figure 2.5: cdf of  $\delta_i^*$  for  $\alpha = 0.9$ .

both *i*) include nodes of the same operational groups (as much as possible), and *ii*) include the largest number of nodes, while satisfying the following constraints: it must be connected, its size must be less or equal to  $n_{max}$ , and its diameter must be less or equal to  $d_{max}$ . Note that for (*i*), all the nodes of a group may not be able to be gathered into one single cluster due to the constraints.

Conversely to most of the clustering algorithms that can be found in the literature, DCOG algorithm does not need to resort to the notion of CH node. Instead, DCOG manages a trading process between the clusters, each cluster evaluating periodically the opportunity to give away some of its nodes to a neighbor cluster. Once an opportunity is found, the swap is implemented until the next opportunity.

In order to evaluate if a swap of nodes is possible between two clusters, we introduce two different metric functions: a cluster one (noted  $c$ ) that gives a score depending on the nodes inside the cluster, and a gain one (noted  $g$ ) that is a function of  $c$  for both clusters involved (details are given in the subsequent sections). The decision to let the swap to happen is based upon the enforcement of the two following conditions: *i*)  $g > 0$ , and *ii*) the constraints are met.

## 2.5.2 Functions related to DCOG

In this section, we denote by  $\mathcal{C}$  the set of all possible clusters.

The cluster cost function used in DCOG is an application that associates a real value in  $[0, 1]$  to a cluster  $\mathcal{C}_k$ , defined as:

**Definition 2.1** *The DCOG cluster cost function  $c$  is defined as:*

$$c: \mathcal{C} \mapsto [0, 1]$$

$$\mathcal{C}_k \mapsto c(\mathcal{C}_k),$$

such that  $c(\emptyset) = 1$ , and  $c(\mathcal{C}_k) = 0$  when  $\mathcal{C}_k$  fulfills the targeted properties.

Function  $c$  is used to define the gain  $g$ , and specific implementation is given in Section 2.5.5.

As described previously, DCOG uses a function  $g$  to evaluate the benefit of swapping a set of nodes  $\{u_i\}$  from one cluster  $\mathcal{C}_k$  to a cluster  $\mathcal{C}_\ell$ , with  $k \neq \ell$ . A gain function is defined as:

**Definition 2.2** *The DCOG cluster gain function  $g$  is defined,  $\forall k, \forall \ell$ , with  $k \neq \ell$  as:*

$$\begin{aligned} g : \quad \mathcal{V} \times \mathcal{C} \times \mathcal{C} &\mapsto \mathbb{R} \\ (\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) &\mapsto g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) \end{aligned}$$

with:

$$g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) := [c(\mathcal{C}_\ell) - c(\mathcal{C}_\ell \cup \{u_i\})] - [c(\mathcal{C}_k \setminus \{u_i\}) - c(\mathcal{C}_k)]. \quad (2.13)$$

The gain  $g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)$  results from the difference of: *i*) the gain associated with the arrival of nodes  $\{u_i\}$  in  $\mathcal{C}_\ell$  previously introduced in Section 2.4.3.2 and equal to  $c(\mathcal{C}_\ell) - c(\mathcal{C}_\ell \cup \{u_i\})$ , and *ii*) the loss associated with the departure of nodes  $\{u_i\}$  from  $\mathcal{C}_k$  and already defined in Section 2.4.1.1, equal to  $c(\mathcal{C}_k \setminus \{u_i\}) - c(\mathcal{C}_k)$ .

To make sure that any inter-cluster nodes swap decided by DCOG is beneficial, we impose the following property:

**Property 2.1** *With DCOG, an inter-cluster nodes swap is allowed only if the associated gain is strictly positive.*

The action of the DCOG clustering algorithm is to only allow swaps that induce a strictly positive gain. To understand the consequence of this property of DCOG, we now introduce the concept of network cost function as the sum of the costs of all clusters in the network. The evolution of the network cost during DCOG execution is instrumental to prove its convergence to a stable partition.

Let us denote by  $\mathcal{C}_k(t)$  and  $p(t)$  the cluster  $\mathcal{C}_k$  and partition  $p$  at time  $t$ , respectively. Omitting the subscript  $i$ , let us consider the cost function  $c$  associated to the global cost function  $J$  defined in (2.1).

**Result 2.1** *If between time  $t_n$  and  $t_{n+1}$ , the only swap that happened is the departure of nodes  $\{u_i\}$  from cluster  $\mathcal{C}_k$  towards cluster  $\mathcal{C}_\ell$ , then the following equality holds:*

$$J(p(t_{n+1})) = J(p(t_n)) - g(\{u_i\}, \mathcal{C}_k(t_n), \mathcal{C}_\ell(t_n)). \quad (2.14)$$

**Proof** Before nodes  $\{u_i\}$  move from cluster  $\mathcal{C}_k$  to cluster  $\mathcal{C}_\ell$  we have:  $J(p(t_n)) = c(\mathcal{C}_k(t_n)) + c(\mathcal{C}_\ell(t_n)) + \sum_{j \neq k, j \neq \ell} c(\mathcal{C}_j(t_n))$ . At time  $t_{n+1}$  we have:

$$\begin{aligned} J(p(t_{n+1})) &= c(\mathcal{C}_k(t_{n+1})) + c(\mathcal{C}_\ell(t_{n+1})) + \sum_{j \neq k, j \neq \ell} c(\mathcal{C}_j(t_{n+1})) \\ &= c(\mathcal{C}_k(t_n) \setminus \{u_i\}) + c(\mathcal{C}_\ell(t_n) \cup \{u_i\}) + \sum_{j \neq k, j \neq \ell} c(\mathcal{C}_j(t_n)) \\ &= c(\mathcal{C}_k(t_n) \setminus \{u_i\}) + c(\mathcal{C}_\ell(t_n) \cup \{u_i\}) + J(p(t_n)) - c(\mathcal{C}_k(t_n)) - c(\mathcal{C}_\ell(t_n)) \\ &= J(p(t_n)) - \{[c(\mathcal{C}_\ell(t_n)) - c(\mathcal{C}_\ell(t_n) \cup \{u_i\})] - [c(\mathcal{C}_k(t_n) \setminus \{u_i\}) - c(\mathcal{C}_k(t_n))]\} \\ &= J(p(t_n)) - g(\{u_i\}, \mathcal{C}_k(t_n), \mathcal{C}_\ell(t_n)). \end{aligned}$$

■

**Result 2.2** *If between time  $t_n$  and  $t_{n+1}$ , the only swap that happened is the departure of nodes  $\{u_i\}$  from cluster  $\mathcal{C}_k$  towards cluster  $\mathcal{C}_\ell$ , then the following inequality holds:*

$$J(p(t_{n+1})) < J(p(t_n)).$$

**Proof** Immediate consequence of Property 2.1 and Result 2.1. ■

### 2.5.3 Detailed DCOG description

The DCOG algorithm which builds the clusters is described in Table 2.2.

It is split in two steps after which existing clusters satisfy the connectivity, size, and diameter constraints. The first one (lines 1-22) lets each cluster determine if some of its members should leave the cluster and join another neighboring one. During step 2 (lines 22-35), nodes are swapped between clusters as determined in previous step. Step 1 involves nodes of a single cluster, and step 2 involves nodes from a cluster and some of its neighboring clusters. In addition, the clustering algorithm is performed independently by all clusters. This means that the decisions made during step 1 by some nodes  $\{u_i\}$  of a cluster  $\mathcal{C}_k$  to join a cluster  $\mathcal{C}_\ell$  may no longer make sense because for example step 2 of cluster  $\mathcal{C}_\ell$  has been executed before step 2 of cluster  $\mathcal{C}_k$ , and  $\mathcal{C}_\ell$  has been modified. Consequently, in step 2, before modifying clusters it is verified if the decided swaps still make sense. These two steps are repeated until there is no more any possible cluster modification.

During step 1, lines 3-4 ensure that a cluster  $\mathcal{C}_k$  will still satisfy connectivity and diameter constraints after some nodes  $\{u_i\}$  have joined a neighboring cluster. Lines 6-8 check that if nodes  $\{u_i\}$  were to join cluster  $\mathcal{C}_\ell$ , then size, connectivity, and diameter constraints would still be satisfied. Lines 9-16 are used to find the set of neighboring clusters for which the gain achieved when nodes  $\{u_i\}$  leave cluster  $\mathcal{C}_k$  is maximal. Lines 17-21 check if there are some nodes  $\{u_i\}$  that could join a neighboring cluster. If this is the case then one neighboring cluster is selected randomly among the neighboring clusters inducing the maximum gain.

During step 2, only at most one set of nodes  $\{u_i\}$  may leave cluster  $\mathcal{C}_k$ , preferably the one inducing the highest gain. Lines 24-29 check that a swap decided in step 1 is still authorized. The first condition is that the cluster source of the leaving nodes must remain connected. Also the size and diameter constraints must still be satisfied if the leaving nodes are included in the destination cluster. Finally the gain induced by the swap must still be positive. Line 30 checks that the destination cluster  $\mathcal{C}_\ell$  is ready to accept new nodes, i.e., it is not currently running the clustering algorithm (i.e., running step 1, or step 2 with another cluster). If all those conditions are satisfied, then the source and destination clusters are modified accordingly and line 33 ensures that only one swap will be performed with cluster  $\mathcal{C}_k$  as source. Otherwise, clusters  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  are left unchanged.

Note: convenient initial conditions are when all nodes form their own singleton clusters.

Fig. 2.6.a illustrates a swap search by DCOG within a 16 node network organized into four clusters, in which all nodes are in range. The group of each node is identified by its shape. Clusters boundaries are indicated by solid lines. During the execution of DCOG, the cluster  $\mathcal{C}_1$  evaluates the swap gain induced by the 1 and 2 from the circle group joining each of the neighboring clusters. Among these three possible swaps, it is the one inducing the maximal strictly positive gain which is selected, i.e., the one where the two nodes join  $\mathcal{C}_2$ . The cluster structure resulting from this swap is depicted in Fig. 2.6.b.

<p><b>Step 1</b></p> <p>1 Set <math>\mathcal{M} = \emptyset</math>, <math>\mathcal{L} = \emptyset</math> and <math>g = 0</math></p> <p>2 <b>For each</b> set of nodes <math>\{u_i\}</math> members of the same group in <math>\mathcal{C}_k</math> <b>do</b>:</p> <p>3   <b>If</b> <math>\mathcal{C}_k \setminus \{u_i\}</math> is not connected, <b>go to</b> line 2. <b>End If.</b></p> <p>4   <b>If</b> <math>d(\mathcal{C}_k \setminus \{u_i\}) &gt; d_{max}</math>, <b>go to</b> line 2. <b>End If.</b></p> <p>5   <b>For each</b> cluster <math>\mathcal{C}_\ell</math> neighbor of cluster <math>\mathcal{C}_k</math> <b>do</b>:</p> <p>6       <b>If</b> <math> \mathcal{C}_\ell \cup \{u_i\}  &gt; n_{max}</math>, <b>go to</b> line 5. <b>End If.</b></p> <p>7       <b>If</b> <math>\mathcal{C}_\ell \cup \{u_i\}</math> is disconnected, <b>go to</b> line 5. <b>End If.</b></p> <p>8       <b>If</b> <math>d(\mathcal{C}_\ell \cup \{u_i\}) &gt; d_{max}</math>, <b>go to</b> line 5. <b>End If.</b></p> <p>9       Calculate <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)</math> thanks to (2.13).</p> <p>10       <b>If</b> <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) &gt; g</math> <b>then</b>:</p> <p>11           Set <math>g = g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)</math>.</p> <p>12           Set <math>\mathcal{L} = \{\mathcal{C}_\ell\}</math>.</p> <p>13       <b>Else if</b> <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) = g</math> <b>then</b>:</p> <p>14           Set <math>\mathcal{L} = \mathcal{L} \cup \mathcal{C}_\ell</math>.</p> <p>15       <b>End If.</b></p> <p>16   <b>End For.</b></p> <p>17   <b>If</b> <math>g = 0</math> <b>then</b> nodes <math>\{u_i\}</math> remain in cluster <math>\mathcal{C}_k</math>.</p> <p>18   <b>Else then</b></p> <p>19       Choose randomly <math>\mathcal{C}_\ell \in \mathcal{L}</math>.</p> <p>20       Set <math>\mathcal{M} = \mathcal{M} \cup (\{u_i\}, \mathcal{C}_\ell, g)</math>.</p> <p>21   <b>End If.</b></p> <p>22 <b>End For.</b></p>
<p><b>Step 2</b></p> <p>23 <b>For each</b> <math>(\{u_i\}, \mathcal{C}_\ell, g) \in \mathcal{M}</math> considered in decreasing <math>g</math> values</p> <p>24   <b>If</b> <math>\mathcal{C}_k \setminus \{u_i\}</math> is not connected, <b>go to</b> line 23. <b>End If.</b></p> <p>25   <b>If</b> <math>\mathcal{C}_\ell = \emptyset</math>, <b>go to</b> line 23. <b>End If.</b></p> <p>26   <b>If</b> <math> \mathcal{C}_\ell \cup \{u_i\}  &gt; n_{max}</math>, <b>go to</b> line 23. <b>End If.</b></p> <p>27   <b>If</b> <math>d(\mathcal{C}_\ell \cup \{u_i\}) &gt; d_{max}</math>, <b>go to</b> line 23. <b>End If.</b></p> <p>28   Calculate <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)</math> thanks to (2.13).</p> <p>29   <b>If</b> <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) \leq 0</math>, <b>go to</b> line 23. <b>End If.</b></p> <p>30   <b>If</b> <math>\mathcal{C}_\ell</math> is available <b>then</b>:</p> <p>31       Set <math>\mathcal{C}_\ell = \mathcal{C}_\ell \cup \{u_i\}</math>.</p> <p>32       Set <math>\mathcal{C}_k = \mathcal{C}_k \setminus \{u_i\}</math>.</p> <p>33       <b>Exit For loop.</b></p> <p>34   <b>End If.</b></p> <p>35 <b>End For.</b></p>

Table 2.2: Dynamic clustering algorithm applied to cluster  $\mathcal{C}_k$ .

#### 2.5.4 DCOG convergence

**Result 2.3** *For a fixed topology, the DCOG algorithm defined in Table 2.2 converges in a finite number of iterations to a stable clustering structure, i.e., when there is no more any possible*

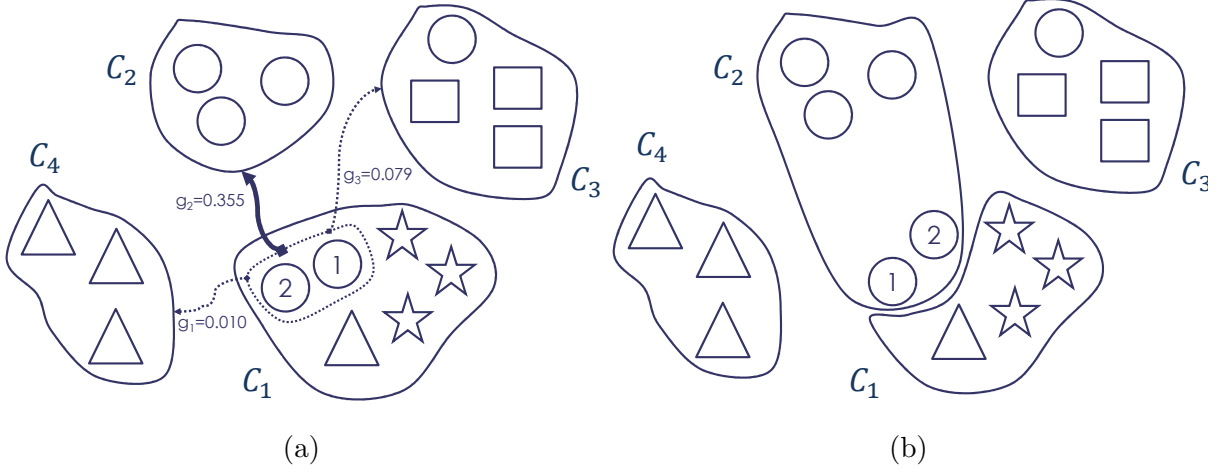


Figure 2.6: (a) Potential swaps and associated gains during DCOG operation in cluster  $\mathcal{C}_1$  for its members  $\{1, 2\}$  from the circle group. (b) Swap execution: formation of a cluster with five members from the circle group.

*swap with strictly positive gain.*

**Proof** Any swap selected in step 1 and executed in step 2 implies a decreasing of the network cost function  $J(p(t))$  thanks to Result 2.2. Consequently DCOG cannot choose a partition that has already been selected, which prevents loops. Furthermore since there is a finite number of partitions, the algorithm converges to a stable partition after a finite number of iterations. ■

### 2.5.5 DCOG cluster cost function

A cost is associated with each cluster. The goal of the clustering algorithm is to build clusters whose costs are minimum. The work of Section 2.3 has allowed to identify that both the number of groups with at least one member in a cluster, and the cluster size, should be used to build good clusters. Therefore, a cost function associated with a cluster must fulfill the following goals (also used in Chapter 3):

**Goal 1 ( $\mathbf{G}_1$ )** *Build clusters whose size is maximal, i.e., equal to  $n_{max}$ .*

**Goal 2 ( $\mathbf{G}_2$ )** *Build clusters including the highest number of members from the same group.*

The work of Section 2.4 has identified (2.12) as an example of cost function achieving these two goals. This cost function can be written:

$$c(\mathcal{C}_k) := c_6(\mathcal{C}_k) = 1 - \left[ f_{n_{max}}(n_k) \cdot \epsilon + \frac{1 - \epsilon}{n_{max}} \sum_{t \in \mathcal{I}(\mathcal{C}_k)} m_{t,k} \cdot f_{m_t}(m_{t,k}) \right], \quad (2.15)$$

with  $\epsilon \in [0, 1]$  selected to favor either  $\mathbf{G}_1$  or  $\mathbf{G}_2$ , and function  $f_n(m)$  defined as:

$$f_n(m) := \frac{1}{n(n+1)} \cdot m(m+1). \quad (2.16)$$

In Chapter 3 we generalize the expression of (2.16) and prove how  $\epsilon$  should be set to favor  $\mathbf{G}_2$  over  $\mathbf{G}_1$ . Here we can only state that to favor  $\mathbf{G}_2$  over  $\mathbf{G}_1$ , a small value of  $\epsilon$  must be selected.

Note that if the size  $n_k$  of cluster  $\mathcal{C}_k$  is equal to  $n_{max}$ , and cluster  $\mathcal{C}_k$  is composed of full groups only (i.e.,  $\forall t \in \mathcal{I}(\mathcal{C}_k), m_{t,k} = m_t$ ), then  $c(\mathcal{C}_k) = 0$ .



## 2.5.6 Adaptation to mobility

Because of mobility, a node may move away from the other nodes of the same cluster  $\mathcal{C}_k$ , leading to an invalid cluster, which does not fulfill the diameter or connectivity constraint. In this case, in order to restore the constraints, we have decided that this cluster  $\mathcal{C}_k$  should be split in several clusters, each of them verifying the constraints. Remembering that in structured networks, the members of a given group usually remain close to each other, we have chosen to split an invalid cluster  $\mathcal{C}_k$  in at least as many clusters as the number of groups with at least one member in  $\mathcal{C}_k$ , each one gathering some members of a single group. This procedure, performing adaptation to mobility, is detailed in Table 2.3. For each group  $\mathcal{O}_t$  with at least one member in  $\mathcal{C}_k$ , the procedure first finds (lines 4-7) the connected components gathering the cluster  $\mathcal{C}_k$  members that also belong to  $\mathcal{O}_t$ . Then (lines 9-15), if required, it splits those components which do not satisfy the diameter constraint. Finally (lines 16-18), it creates the new clusters, which satisfy the constraints by construction.

1	<b>If</b> $\mathcal{C}_k$ is not connected or $d(\mathcal{C}_k) > d_{max}$ <b>then:</b>
2	Let $\mathcal{M} = \emptyset$ .
3	<b>For each</b> set of nodes $\{u_i\}$ members of the same group $\mathcal{O}_t$ in $\mathcal{C}_k$ <b>do:</b>
4	Let $\mathcal{C} = \{\{u_i\}\}$ .
5	<b>If</b> $\{u_i\}$ is not connected <b>then:</b>
6	Split $\{u_i\}$ into its $m$ connected components $\{\{u_i^1\}, \dots, \{u_i^m\}\}$ .
7	Set $\mathcal{C} = \{\{u_i^1\}, \dots, \{u_i^m\}\}$ .
8	<b>End If.</b>
9	<b>For each</b> set of nodes $\{u_i^j\}$ in $\mathcal{C}$ <b>do:</b>
10	<b>If</b> $d(\{u_i^j\}) > d_{max}$ <b>then:</b>
11	Split $\{u_i^j\}$ into its $m'$ subcomponents $\{\{v_i^1\}, \dots, \{v_i^{m'}\}\}$ each one satisfying the diameter constraint.
12	Set $\mathcal{M} = \mathcal{M} \cup \{\{v_i^1\}, \dots, \{v_i^{m'}\}\}$ .
13	<b>Else</b> set $\mathcal{M} = \mathcal{M} \cup \{u_i^j\}$ .
14	<b>End If.</b>
15	<b>End For.</b>
16	<b>For each</b> set of nodes $\{v_i\}$ in $\mathcal{M}$ <b>do:</b>
17	Create a new cluster with $\{v_i\}$ .
18	<b>End For.</b>
19	<b>End If.</b>

Table 2.3: Adaptation to node mobility for cluster  $\mathcal{C}_k$ .

Let us now consider the particular case when  $d_{max} = 2$ . In that case, the following heuristic can be used in line 11, to split a set of nodes  $\{u_i^j\}$  into subsets all satisfying the diameter constraint: *i*) form a first subset including the node with the highest degree within the induced subgraph of  $\{u_i^j\}$ , along with all its 1-hop neighbors in this subgraph, and *ii*) build connected components with the remaining nodes of  $\{u_i^j\}$ .

## 2.6 Numerical results

### 2.6.1 Performance metrics

To assess DCOG performance, the following metrics have been selected:

- The time needed to reach a stable clustered network, measured in number of time units.
- The cluster size. The target cluster size is  $n_{max}$ .
- The cluster group diversity (CGD), i.e., the average number of groups per cluster with at least one member within the cluster. Ideally all members of a group should be in the same cluster and this number should not be larger than the cluster size divided by the group size.
- The group cluster diversity (GCD), i.e., the average number of clusters per group including at least one member of the group. This metric should have a low value, meaning that the members of a group tend to be in the same cluster.
- The application level performance measured using  $J_0^X$  as defined in Section 1.4.3. Here its values are comparable to the average end-to-end delays from all nodes to all nodes. This means that the lower the  $J_0^X$  value, the better. Two salient features of this metric are that *i*) it uses two different costs for inter-cluster ( $\tilde{\gamma}$ ) and intra-cluster ( $\hat{\gamma}$ ) communications; and *ii*) it can be used to concentrate traffic within groups thanks to its  $\alpha$  parameter. The  $J_0^X$  parameter values are:  $\hat{\gamma} = 1$ ,  $\tilde{\gamma} = 2$  and  $\alpha = 0.9$ . This means that the inter-cluster communication cost is twice the one of intra-cluster communication, and that 90% of the traffic is exchanged between members of the same groups.
- The application level performance measured using  $T_0^X$  as defined in Section 1.7. Its values are comparable to average bandwidth on the widest path from all nodes to all nodes. Thus the larger the  $T_0^X$  value, the better. The  $T_0^X$  parameters are:  $\hat{\eta} = 1$ ,  $\tilde{\eta} = 0.5$  and  $\alpha = 0.9$ .

Except in Section 2.6.7.2, all simulation results provided in this section are average values over 100 different random networks.

### 2.6.2 Reference clustering algorithms

We have selected as reference the two distributed clustering protocols GDMAC [1] and VOTE [6] because they allow to adapt the number of clusters to the network density. We have also defined extensions of these protocols in order to take into account the group structure of the network.

#### 2.6.2.1 GDMAC

The goal of the distributed clustering protocol GDMAC is to build stable clusters in presence of node mobility. In this protocol CH nodes are elected first, and then non-CH nodes affiliate to a neighbor CH, leading to the clusters. To do that, a weight (depending on the context) is associated with each node. Within a radio neighborhood, nodes with the highest weights are chosen as CH nodes. To increase stability in presence of mobility GDMAC introduces the  $K$  parameter, whose value is equal to the number of CH nodes that are allowed to be neighbors of a  $(K + 1)$ th CH node. A non-CH node affiliates to the CH node within its neighborhood whose weight is the highest. In order to obtain stable clusters, a non-CH node remains affiliated to its current CH unless there is a CH node in its neighborhood whose weight exceeds the one of the

current CH node by at least a positive lower-bound denoted by  $H$ . The GDMAC algorithm is defined in Table 2.4. Note that after execution of lines 1-17, some nodes may have decided to join the cluster for a node which itself has made the same kind of decision. Consequently the former nodes will believe to be in a cluster that does not exist in reality. This is handled by these nodes the next time they perform the GDMAC algorithm.

1	<b>If</b> local node is CH <b>then:</b>
2	Find list of neighbor CH with larger weight than node $i$ .
3	<b>If</b> the number of such neighbors is strictly greater than $K$ <b>then:</b>
4	Local node chooses one of the $K$ neighbor CH with the highest weights as its new CH.
5	<b>Else</b> local node remains CH.
6	<b>End If.</b>
7	<b>Else:</b>
8	<b>If</b> local node CH is no longer a neighbor or is no longer a CH <b>then:</b>
9	Find the list of local node's neighbor CH.
10	<b>Else:</b>
11	Find the list of local node's neighbor CH whose weight exceeds by at least $H$ the weight of local node CH.
12	<b>End If.</b>
13	<b>If</b> this list is not empty <b>then:</b>
14	Local node chooses randomly as CH among the $K+1$ ones with the highest weight.
15	<b>Else:</b>
16	<b>If</b> current CH is no longer a neighbor CH <b>then:</b>
17	Local node becomes CH.
18	<b>End If.</b>
19	<b>If</b> current CH is still a neighbor CH <b>then:</b>
20	Local node keeps its current CH as CH.
21	<b>End If.</b>
22	<b>End If.</b>
23	<b>End If.</b>

Table 2.4: GDMAC algorithm.

In the GDMAC paper [1], the weight is allocated randomly. In this work, we prefer to use the node identifier as node weight, like in [3]. We refer to this approach as GDMAC-std. To extend GDMAC so as to take group membership into account, we propose to calculate the node weights in a different way, and also to modify the way non-CH nodes affiliate to their CH. This leads to two new versions of the GDMAC, denoted by GDMAC-new1 and GDMAC-new2.

**GDMAC-new1:** the weight used is the stability factor defined in [5]. In that original paper the stability factor has been introduced for clustering structured networks but is associated with a very simple algorithm. Here we thus propose to associate this stability factor with GDMAC.

The stability factor of a node is a linear combination of the average relative speed with its neighbors, the average distance with its neighbors, the average number of neighbors, and its remaining energy. The three mentioned averages are weighted averages. In order to take into account the group structure, lower weights in the average are used for neighbors that are members of the same group as the one of the current node. The TCA protocol from [5] can be seen as a particular case of GDMAC-new1 by setting the GDMAC parameter  $K$  to zero. We call it TCA-std.

**GDMAC-new2:** it is an extension of GDMAC-new1 where we modify the non-CH node affiliation strategy. Indeed, a non-CH node affiliates, if possible, to a CH node that is also member of its group instead of choosing it with respect to its weight.

According to the proposed modifications, GDMAC-new1 and GDMAC-new2 are expected to be better suited to the context of structured networks.

### 2.6.2.2 VOTE

Similarly to GDMAC, VOTE [6] selects some CH nodes based on their weight, and then non-CH nodes affiliate to a neighbor CH node. In VOTE, the weight of each node is called its *vote* and is a linear combination of the normalized degree and the battery remaining time. The main difference between GDMAC and VOTE does not lay in the weight definition but in the way each CH node manages its cluster size: VOTE limits the cluster size to  $n_{max}$  and GDMAC does not handle it. When the number of nodes affiliated to a CH node is equal to  $n_{max} - 1$ , non-CH nodes refrain to affiliate to this CH. Nevertheless, in the case of simultaneous affiliations a cluster may include more than  $n_{max}$  members. Then, the concerned CH randomly rejects as many members as required to satisfy the cluster size constraint. The VOTE algorithm is defined in Table 2.5.

In order to take into account the group structure we propose here to apply the algorithm VOTE (denoted by VOTE-new) by using the stability factor as the weight. Initial VOTE defined in [6] is hereafter denoted as VOTE-std.

Note: like for GDMAC-new2, we defined VOTE-new2 which was intended to improve VOTE-new such as a non CH node affiliates preferentially to a CH that is also member of its group. However, the convergence of VOTE-new2 was not ensured because of bad interactions between this affiliation scheme and the cluster size maintenance process. Thus we dismissed VOTE-new2.

## 2.6.3 Simulation setup

In this section, we describe the general simulation setup.

### 2.6.3.1 Network setup

The simulated network has  $N$  nodes. The group size is constant:  $\forall t, m_t = 10$ . The communication range of a node is  $d_{ref} = 250$  distance units. The nodes are deployed in a square area whose side is 1500 distance units long, following a modified RPGM model [45], different from the one in Section 1.5.2.2 because it does not handle mobility. At the beginning of the simulation a randomly located virtual center  $A_t(0)$  is associated with each group  $t$ , and all group  $t$  members are deployed randomly in a disk of radius  $d_{ref}$  centered at  $A_t(0)$ . In mobile networks, time is split in intervals of fixed duration  $\Lambda$  seconds. At time  $k \cdot \Lambda$  (i.e., the beginning of interval number  $k \in \mathbb{N}$ ), the virtual center  $A_t(k + 1)$  of group  $t$  is randomly located, and the coordinates

```

1 If local node is CH then:
2   Find list of neighbor CH with larger weight than node  $i$ ,
   whose cluster size is lesser than the maximum size.
3   If this list is non empty then:
4     Local node chooses as its CH its neighbor CH with the
     highest weight.
5   Else local node remains CH.
6   End If.
7 Else:
8   If local node CH is no longer a neighbor, or is no longer a
   CH, or local node has been excluded from the cluster of its
   previous CH then:
9     Find the list of local node's neighbor CH whose cluster
     size is lesser than the maximum size.
10  Else:
11  Find the list of local node's neighbor CH whose weight is
     strictly greater than the one of local node CH, and whose
     cluster size is lesser than the maximum size.
12  End If.
13  If this list is not empty then:
14    Local node chooses as its CH its neighbor CH with the
     highest weight.
15  Else:
16    If current CH is no longer a neighbor CH then:
17      Local node becomes CH.
18    End If.
19    If current CH is still a neighbor CH then:
20      Local node keeps its current CH as CH.
21    End If.
22  End If.
23 End If.

```

Table 2.5: VOTE cluster-head selection algorithm.

of group  $t$  members at time  $(k+1) \cdot \Lambda$  are randomly chosen in the disk of radius  $d_{ref}$  centered at  $A_t(k+1)$ . Then during time interval  $[k \cdot \Lambda, (k+1) \cdot \Lambda]$ , each node  $i$  follows a uniform rectilinear motion with a speed limited to a maximum  $v_{max}$  between its coordinates at time  $k \cdot \Lambda$  and time  $(k+1) \cdot \Lambda$ . Note: when new coordinates are selected for a virtual center  $A_t(k)$ , if the distance between  $A_t(k)$  is smaller than  $d_{ref}$  from the deployment area boundary, then a new location is drawn to make sure that no group member is placed outside of the deployment area.

Fig. 2.7 depicts an example of random network with  $N = 100$  nodes, such that nodes in the same group share the same color.

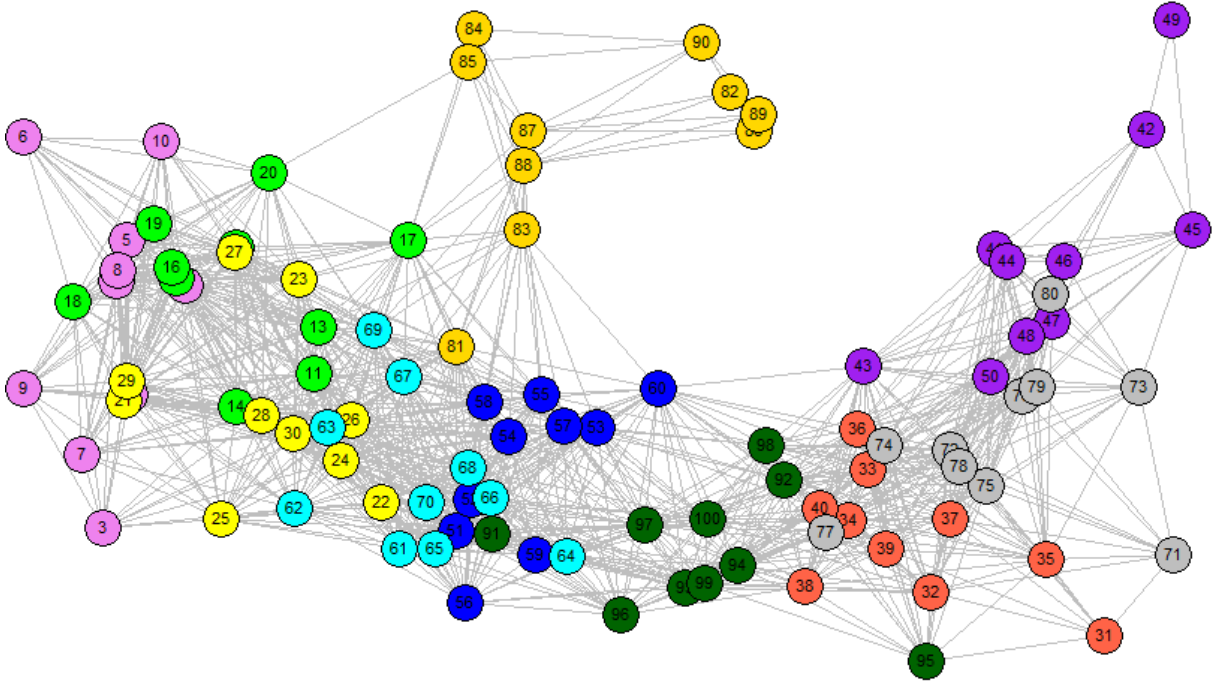


Figure 2.7: Example of random structured network with  $N = 100$  nodes. Node color indicates group membership.

### 2.6.3.2 Clustering schemes setup

The clustering algorithms parameters common to all simulations are now detailed. We consider  $H = 10$  for GDMAC-std, and  $H = 30$  for GDMAC-new1 and GDMAC-new2. Due to the difficulty to set its value, the values of parameter  $K$  are selected in Section 2.6.3.3. The stability factor of a node is calculated using weighted linear combination of the average relative speed and distance between this node and its neighbors (with equal weights 0.5), averaged on five samples. In these average values, group membership is taken into account by allocating a weight (denoted by  $\lambda$  in [5]) equal to 0.5 to nodes in the same group as the local node, and equal to 1 to other nodes. We set  $n_{max} = 20$ , and  $d_{max} = 2$ . Finally, the value of DCOG utility function (2.15)  $\epsilon$  parameter is set to a value small enough to make sure that whatever the cluster size, increasing the number of members of a group in a cluster always takes precedence over increasing its size.

Fig. 2.8 shows an example of the network from Fig. 2.7 clustered with DCOG. In this figure the nodes in the same cluster share the same outer circle color.

### 2.6.3.3 GDMAC $K$ parameter values

Whereas DCOG and VOTE have a maximum cluster size parameter, GDMAC does not and leads to clusters with large size differences: in the same geographical area very small clusters coexist with very large clusters. Therefore to ensure a meaningful comparison with VOTE and DCOG,  $K$  is set to reduce cluster size variability. For each GDMAC extension and each network size, we have determined through simulations the best value of  $K$  as indicated in Table 2.6. The selected value is the lowest one leading to an average highest cluster size no greater than  $n_{max}$ . GDMAC-std succeeds in filling this criterion only for 100 and 200 nodes. Consequently static networks GDMAC-std simulation results are not provided in Section 2.6.6.

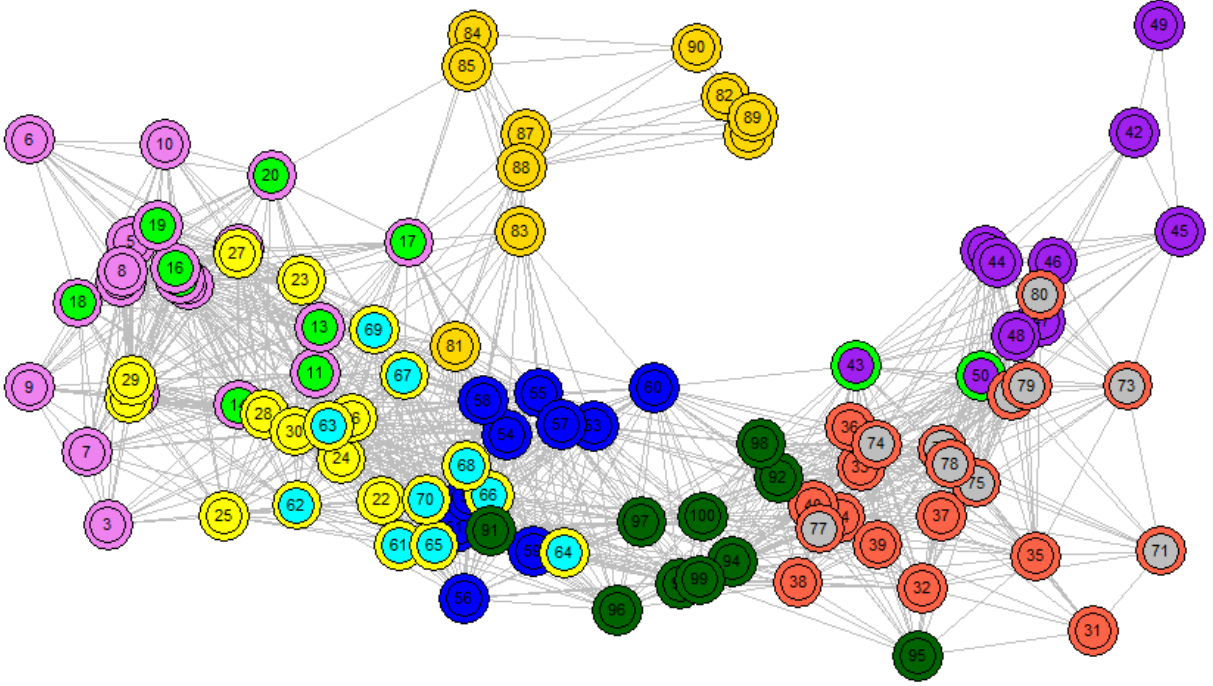


Figure 2.8: Example of DCOG action on a random structured network with  $N = 100$  nodes. Inner and outer colors indicate group and cluster membership respectively.

Nodes	100	200	300	400	500
GDMAC-std	2	5	-	-	-
GDMAC-new1	2	3	5	7	9
GDMAC-new2	1	2	4	6	8
Nodes	600	700	800	900	1000
GDMAC-new1	11	13	17	20	22
GDMAC-new2	11	13	17	19	23

Table 2.6: Values of  $K$  parameter.

## 2.6.4 Asynchronous and synchronous modeling for DCOG

To simulate DCOG we defined two models: *i*) a fully distributed model with no synchronization between the clusters, and *ii*) a simpler distributed synchronous model. Concerning metrics unrelated to time, we show that both models lead to the same performance. This allows us to perform subsequent simulations using the synchronous model.

### 2.6.4.1 Distributed asynchronous model

Let us first define our fully distributed asynchronous model. In this model a cluster can be in one of the following six states:

- *Deciding*: when step 1 of DCOG is being performed (see Table 2.2) and a decision is made to select of set of nodes that will leave the cluster to join a neighboring one. The duration of this state is  $\delta_d$ .
- *Leaving*: when step 2 of DCOG is being performed and the set of nodes selected in *Deciding*

state potentially leave their cluster to join the chosen neighboring one. The duration of this state is  $\delta_l$ .

- *Checking*: when the cluster checks if connectivity, diameter and size constraints are still satisfied. The duration of this state is  $\delta_c$ .
- *Adapting*: when, because of node mobility the constraints are no longer satisfied and adaptation to mobility must be performed. The duration of this state is  $\delta_a$ .
- *Waiting*: when the cluster is idle, ready to accept new nodes. The duration of this state is  $\delta_w$ , exponentially distributed with parameter  $\lambda$ .
- *Joining*: when some nodes are joining the cluster. The duration of this state is  $\delta_l$ .

The asynchronous DCOG state machine is described in Fig. 2.9. In this figure, lines illustrate the state transitions, and the text associated with each transition details the duration required for this transition. In this model, all the clusters are always supposed to know the internal information about their neighbor clusters.

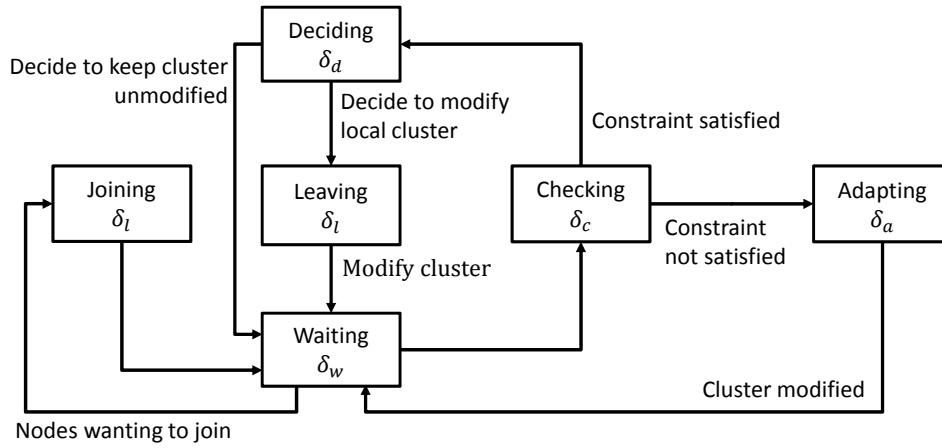


Figure 2.9: Asynchronous DCOG state machine.

To better understand this state diagram, the Fig. 2.10 to 2.12 detail the different state transitions using time diagrams. The Fig. 2.10 details the transition from *Deciding* to *Waiting* state.

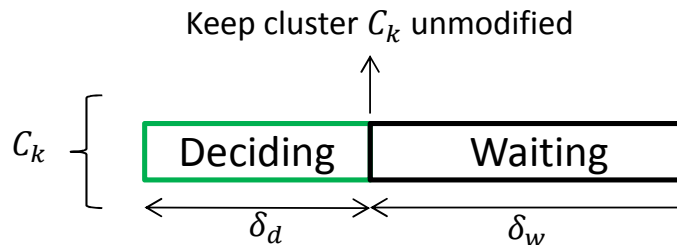


Figure 2.10: Asynchronous DCOG: transition from *Deciding* to *Waiting* state.

The Fig. 2.11 details the two cases when a cluster  $C_k$  leaves the *Deciding* state and some of its nodes want to join a neighboring cluster  $C_\ell$ . Firstly (① in Fig. 2.11), if cluster  $C_\ell$  state is *Waiting* when cluster  $C_k$  enters the *Leaving* state, then cluster  $C_\ell$  state becomes *Joining* and the selected



nodes leave  $C_k$  and join  $C_\ell$ . Note that in that case, both clusters  $C_k$  and  $C_\ell$  remain in their respective state *Leaving* and *Joining* during the same duration  $\delta_l$ . Secondly (② in Fig. 2.11), if cluster  $C_\ell$  state is not *Waiting*, then cluster  $C_k$  state becomes *Leaving* but the selected nodes remain in  $C_k$ .

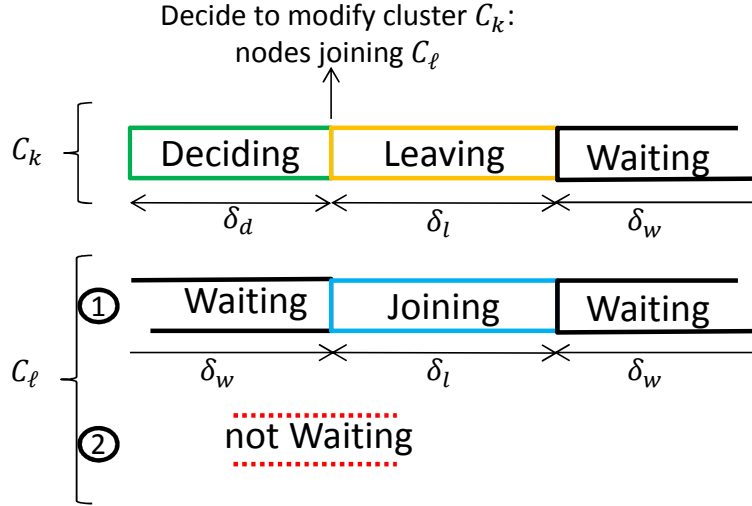


Figure 2.11: Asynchronous DCOG: cases when a cluster leaves *Deciding* state and some of its nodes want to join a neighboring cluster.

Fig. 2.12 details what happens each time a cluster leaves the *Waiting* state to check if the constraints are still satisfied. In the positive case (① in Fig. 2.12), then the cluster state becomes *Deciding*, otherwise (② in Fig. 2.12), *Adapting*.

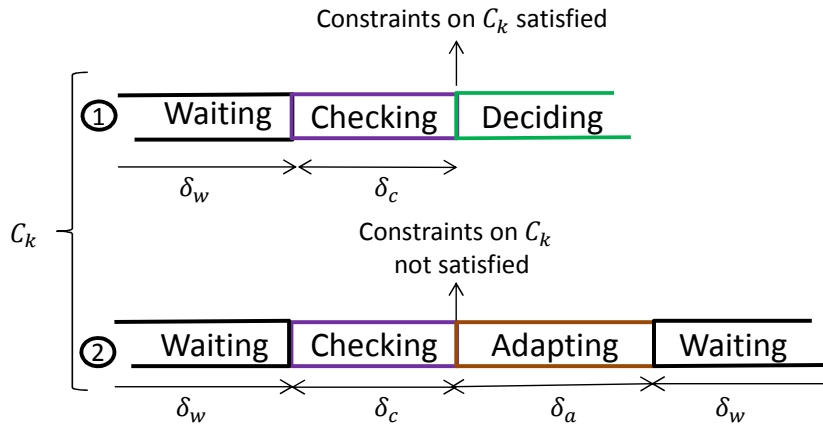


Figure 2.12: Asynchronous DCOG: cases when the cluster constraints are checked.

This asynchronous model is used to verify the convergence of DCOG through simulation in a fully distributed setting. This model requires several parameters ( $\delta_d$ ,  $\delta_l$ ,  $\delta_c$ ,  $\delta_a$  and  $\lambda$ ). Consequently, as soon as DCOG convergence has been verified then we use a simpler and faster (from the point of view of simulation time) synchronous model to measure its performance.

### 2.6.4.2 Distributed synchronous model

In the synchronous model, time is split in rounds. At the beginning of a round, all clusters are supposed to know the internal information about their neighbor clusters. During a round all clusters perform the following actions:

- Apply sequentially to all clusters the algorithm of Table 2.3 to adapt to mobility.
- Apply sequentially step 1 of Table 2.2 to all clusters not modified in this round due to mobility.
- Apply sequentially step 2 of Table 2.2 to all clusters not modified in this round due to mobility.

### 2.6.5 Asynchronous DCOG simulation results in static networks

Simulations of asynchronous DCOG have been performed with  $N = 100$  static nodes. Due to the lack of mobility, when a node enters the *Checking* state, the constraints are always satisfied, leading to the *Deciding* state.

First, the values of asynchronous DCOG parameters  $\delta_d$ ,  $\delta_l$  and  $\delta_a$  have been set to 2 time units, and  $\delta_c$  to 1 time unit. Table 2.7 provides the results of the simulation of 100 random networks for each value of the  $\lambda$  parameter selected in  $\{0.5, 1, 2, 5, 10, 20, 30, 40, 50\}$ . The last line whose cell with  $\lambda$  value equal to "sync" provides the results when DCOG is simulated using its synchronous model.

$\lambda$	Cluster size			CGD	GCD	$J_0^X$	$T_0^X$
	avg	std dev	max				
0.5	11.42	4.00	18.73	1.27	1.12	1.61	5.49
1	11.34	4.12	18.92	1.26	1.12	1.61	5.50
2	11.27	4.14	18.84	1.25	1.12	1.61	5.48
5	11.28	4.12	18.88	1.25	1.11	1.61	5.49
10	11.34	4.18	19.04	1.26	1.12	1.61	5.50
20	11.38	4.16	19.04	1.26	1.12	1.61	5.51
30	11.37	4.13	18.81	1.26	1.12	1.61	5.50
40	11.32	4.12	18.97	1.26	1.12	1.61	5.49
50	11.35	4.15	18.85	1.28	1.13	1.61	5.49
sync	11.04	4.12	18.87	1.24	1.12	1.61	5.49

Table 2.7: Asynchronous DCOG simulation results, with  $\delta_d = \delta_l = \delta_a = 2$  and  $\delta_c = 1$ .

The main result from Table 2.7 is that all metrics are nearly independent of the value of  $\lambda$ . Moreover, the values from asynchronous and synchronous models are similar. The largest difference concerns the cluster size which is always smaller than 3.5%. Concerning the application layer metrics  $J_0^X$  and  $T_0^X$ , they are within 0.3% of the values obtained with the synchronous model.

With the asynchronous model, the clusters are considered stable when at least 100 time units have elapsed since at least one cluster entered the *Leaving* state. The time required to converge to a stable cluster structure is plotted in Fig. 2.13. For example, in this figure, when  $\lambda = 0.5$  the average convergence time is 300.84 time units, meaning that at event time greater or equal than 400.84, no cluster had entered state *Leaving* for 100 time units. With the synchronous model,

the convergence time is in rounds, the clusters are considered stable when no modification has been performed during two consecutive rounds.

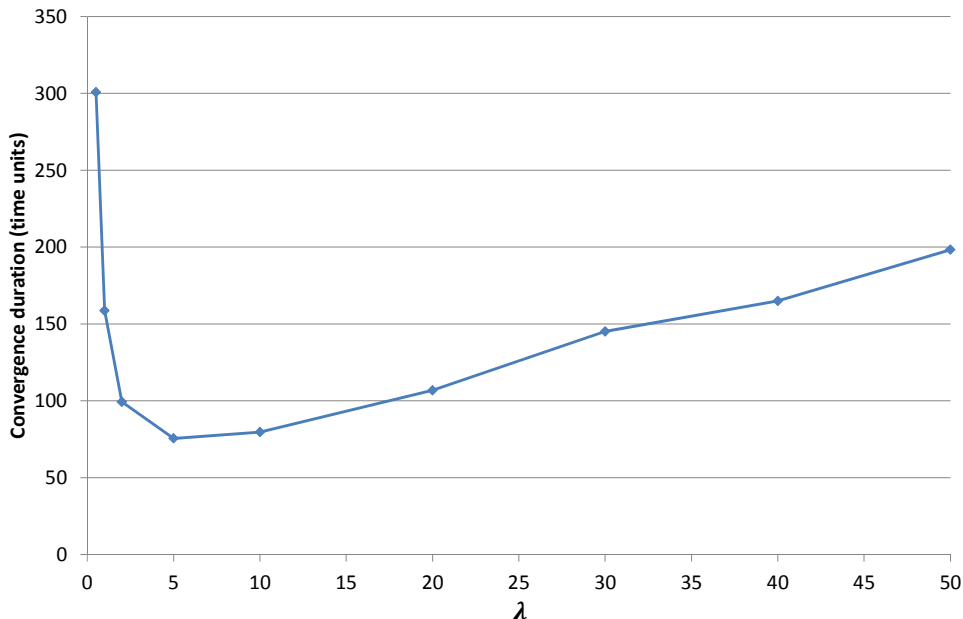


Figure 2.13: Asynchronous DCOG convergence time vs.  $\lambda$ , with  $\delta_d = \delta_l = \delta_a = 2$  and  $\delta_c = 1$ .

The slope of the curve in Fig. 2.13 can be explained as follows:

- When  $\lambda < 2$ , the *Waiting* state duration is short. Therefore, when some nodes from a cluster  $\mathcal{C}_k$  want to join a cluster  $\mathcal{C}_\ell$ , it is usually in a state different from *Waiting*, thus canceling the inter-cluster swap. This explains why the convergence duration decreases when  $\lambda$  increases.
- Conversely, when  $\lambda \geq 2$ , the clusters are usually in the *Waiting* state, thus allowing inter-cluster swaps. As  $\lambda$  value increases, the time in the *Waiting* state also increases and clusters need more time to enter the *Deciding* state to build the clusters, thus increasing the time required to reach stable clusters.

Note: in ad hoc networks, we expect the medium access delay to be at least 100 ms. Setting the time unit to this duration, the time needed by DCOG to converge is lower than 10 s, provided that the value of  $\lambda$  lies in the  $[2, 20)$  time units interval.

To assess the influence of the simulation parameters, simulations have been performed with all parameters values multiplied by 3:  $\delta_d = \delta_l = \delta_a = 6$ ,  $\delta_c = 3$  and  $\lambda \in \{1.5, 3, 6, 15, 30, 60, 90, 120, 150\}$ . Simulation results are provided in Table 2.8. Again, all metrics are nearly independent of the value of  $\lambda$ . The maximum difference with the value achieved using synchronous model is 3.4% for average cluster size, 3.3% for CGD and 4.5% for GCD. The differences for application level metrics  $J_0^X$  and  $T_0^X$  are less than 0.5%.

Convergence time is plotted in Fig. 2.14. Comparing Fig. 2.14 and Fig. 2.13 reveals that both curves have similar shape, which was expected. It can be noted that there is an optimal value of  $\lambda$  for which the convergence time is the shortest. This optimal value could be used to design a radio access scheme ensuring the fastest convergence of the clustering algorithm.

The proof of DCOG convergence is independent of the considered model (asynchronous or synchronous). Additionally, the results of this section show that there are only small differences

$\lambda$	Cluster size			CGD	GCD	$J_0^X$	$T_0^X$
	avg	std dev	max				
1.5	11.42	4.00	18.73	1.27	1.12	1.61	5.49
3	11.34	4.12	18.92	1.26	1.12	1.61	5.50
6	11.27	4.14	18.84	1.25	1.12	1.61	5.48
15	11.28	4.12	18.88	1.25	1.11	1.61	5.49
30	11.34	4.20	19.04	1.26	1.12	1.61	5.50
60	11.34	4.17	19.01	1.27	1.13	1.61	5.50
90	11.31	4.13	18.79	1.29	1.14	1.61	5.49
120	11.18	4.14	18.80	1.28	1.15	1.61	5.47
150	11.10	4.13	18.70	1.28	1.17	1.62	5.47
sync	11.04	4.12	18.87	1.24	1.12	1.61	5.49

Table 2.8: Asynchronous DCOG simulation results, with  $\delta_d = \delta_l = \delta_a = 6$  and  $\delta_c = 3$ .

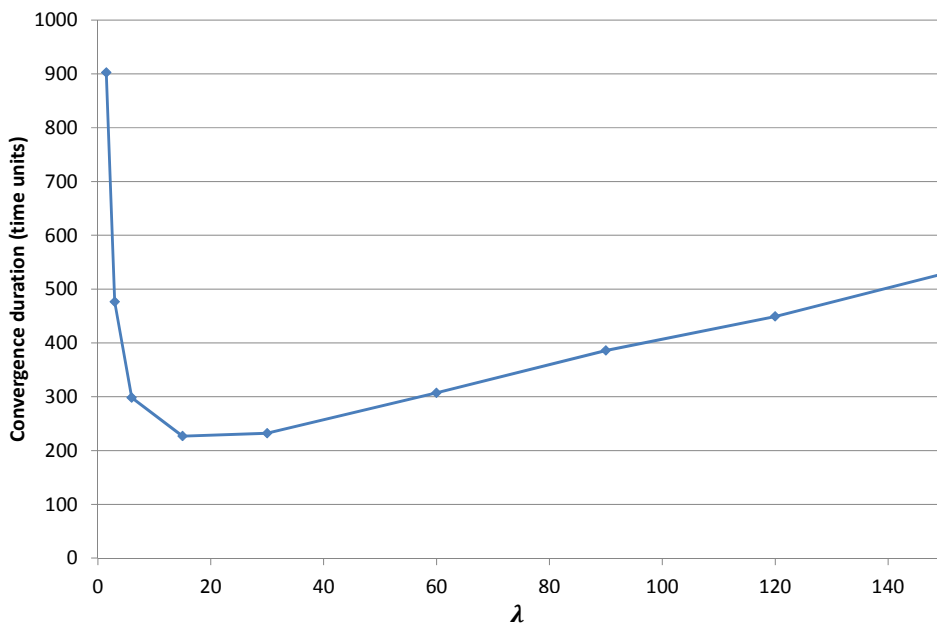


Figure 2.14: Asynchronous DCOG convergence time vs.  $\lambda$ , with  $\delta_d = \delta_l = \delta_a = 6$  and  $\delta_c = 3$ .

between the results of asynchronous DCOG and synchronous DCOG simulations. Consequently in the following, in order to speed them up, the simulations are performed using the synchronous DCOG model described Section 2.6.4.2.

Let us now describe the synchronous model used to simulate the reference clustering algorithms of Section 2.6.2. Initially all nodes are CH and nodes learn about their neighbors. Then every round, the following actions are performed: *i*) update each node state (CH or non-CH, selected CH) using the appropriate algorithm, and *ii*) for each node update the knowledge of its neighbors' state.

## 2.6.6 Performance in static networks

This section is devoted to the analysis of the static networks simulation results.

### 2.6.6.1 Node average degree

Simulations have been performed for an increasing number of nodes. The size of the deployment area is constant, therefore the average node degree increases with  $N$ . The Fig. 2.15 shows the node average degree as well as the 5th and 95th percentiles when  $N$  increases from 100 to 1000. The average number of 1-hop neighbors increases linearly from a moderate density of about 19 to a high density of about 145.

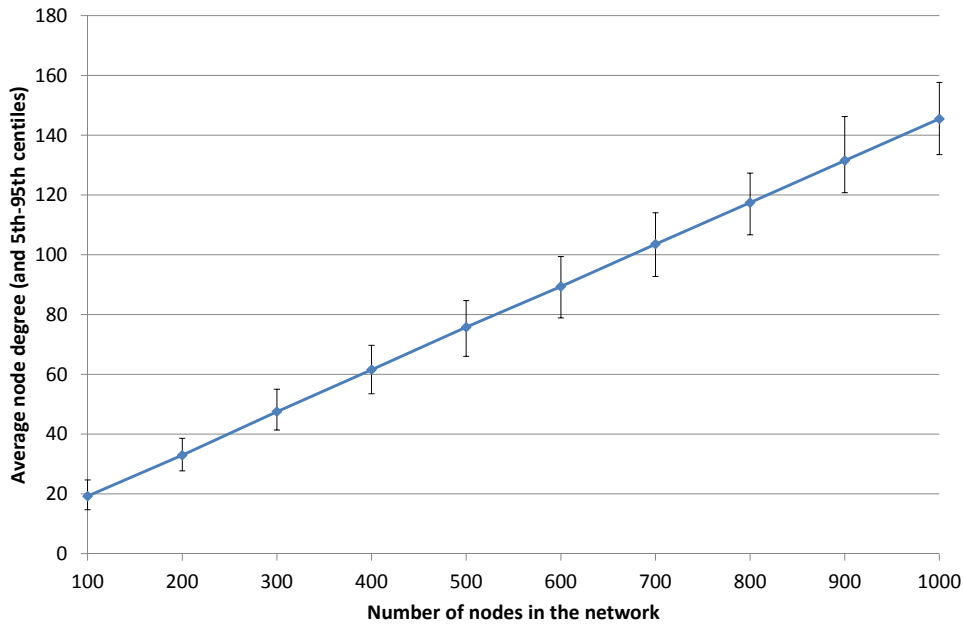


Figure 2.15: Average degree and 5th-95th percentiles in static networks vs.  $N$ .

### 2.6.6.2 Convergence

The number of rounds needed to reach a stable cluster structure depending on the number of nodes in the network is illustrated in Fig. 2.16.

The three algorithms GDMAC-new1, GDMAC-new2 and DCOG converge much faster than VOTE-std and VOTE-new. The two VOTE-based algorithms need an increasing amount of time when the number of nodes increases. This happens because when too many nodes are members of a cluster, the CH rejects as many nodes (chosen randomly) as required to reduce this number to the limit  $n_{max}$ . Those nodes then simultaneously join neighboring clusters, which usually cause these clusters to exceed their maximum size, causing additional nodes being excluded from these clusters (the highest duration before convergence required by VOTE-std in a 100 node network is 222 rounds).

Fig. 2.17 zooms on GDMAC-new1, GDMAC-new2 and DCOG. GDMAC-new1 needs about 3 rounds to converge, when GDMAC-new2 needs 4 or 5 rounds. The additional time needed by GDMAC-new2 is due to its affiliation procedure slightly more complex than the one of

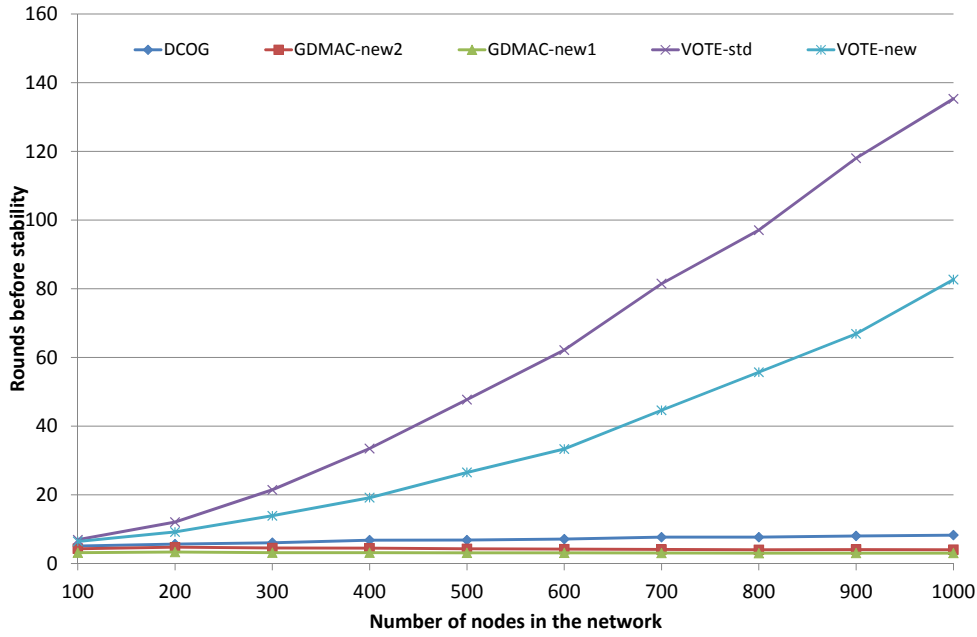


Figure 2.16: Convergence duration in static networks vs. the number of nodes.

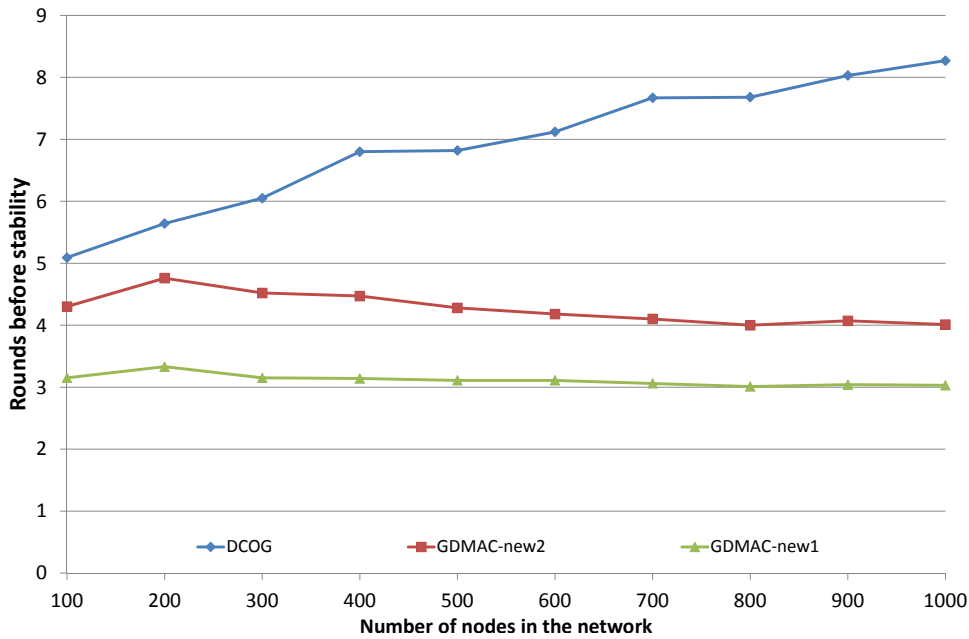


Figure 2.17: Convergence duration (DCOG, GDMAC-new1 and GDMAC-new2) in static networks with 100 to 1000 nodes.

GDMAC-new. Compared to VOTE-std and VOTE-new the GDMAC variants do not manage the cluster size and do not suffer from the problem associated with the exclusion of nodes from too large clusters. The time needed by DCOG to converge is slightly larger than for the GDMAC variants, and increases with the number of nodes from about 5 to 8 rounds. This is justified by the greater complexity of DCOG, which is mitigated by its capacity to implement node cluster swaps involving multiple nodes.

### 2.6.6.3 Cluster size

Fig. 2.18 plots the average number of members in a cluster with respect to the number of nodes  $N$  in the network.

Because TCA-std only allows one CH node in a neighborhood, the cluster size is expected to increase with the number of nodes. This is exactly what can be seen in Fig. 2.18. This behavior is undesired because in clusters whose maximum size is controlled, it is not possible to ensure efficient radio resource allocation. Consequently, even if TCA-std is the only reference [5] from the state of the art taking into account group membership<sup>1</sup>, we no longer consider it in the remainder of our work. Thanks to the appropriate choice of GDMAC  $K$  parameter, GDMAC and VOTE-based algorithms as well as DCOG yield clusters whose maximum size is almost the same. However, their average cluster size are substantially different.

Both GDMAC-new1 and GDMAC-new2 limit the number of neighbor CH to  $K + 1$ , therefore their average cluster sizes are similar. However, their average clusters size are quasi-independent of the network size, and are much smaller than  $n_{max}$ . This means that these protocols create too many clusters. This is the drawback of using the parameter  $K$  to limit the cluster size, which has not been introduced by the authors of GDMAC for that purpose. A more detailed analysis shows that GDMAC-new2 builds more balanced clusters than GDMAC-new1. This is due to their different affiliation procedure. With GDMAC-new1, non-CH nodes tend to affiliate to their neighbor CH with the highest weight leading to large clusters. Conversely, only a few nodes affiliate to the CH with lower weights, leading to small clusters. GDMAC-new2 ensures that non-CH nodes affiliate to a neighbor CH of the same group, if any. Thanks to this rule, if only one CH of a given group exists in a radio neighborhood, this CH has at least as many cluster members as the number of its neighbors belonging to its group.

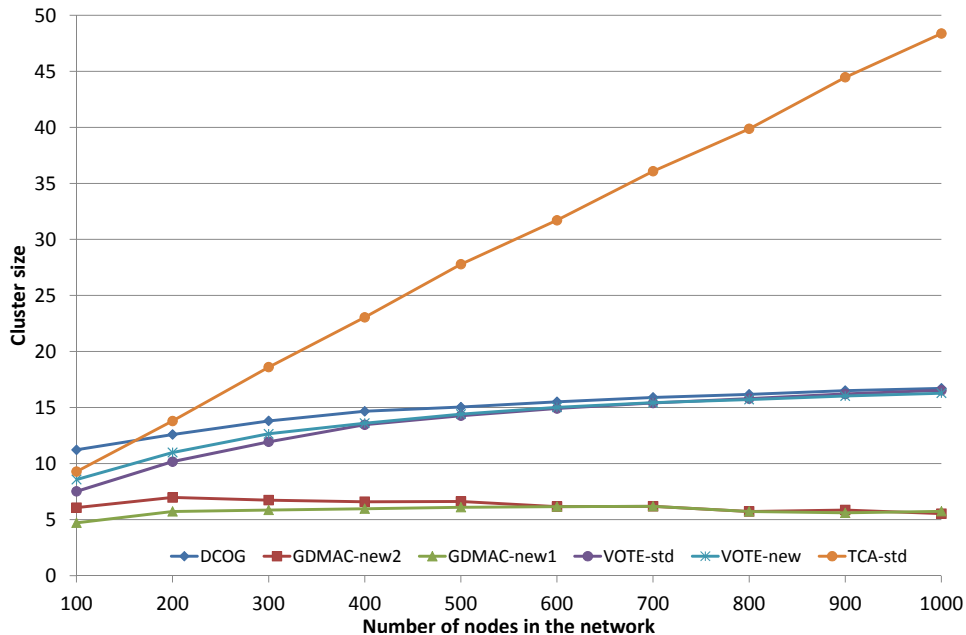


Figure 2.18: Average cluster size in static networks vs. the number of nodes.

VOTE-std and VOTE-new determine the number of CH dynamically, and build clusters

<sup>1</sup>We did not succeed in implementing [16].

whose size is limited by  $n_{max}$ . This leads to clusters of similar size, increasing with the number of nodes in the network. Nevertheless the number of small clusters is still large. Even if VOTE-std and VOTE-new build clusters with larger average size than GDMAC-new1 and GDMAC-new2, the cluster size variability is still large.

Finally concerning DCOG, the cluster size starts at 11.2 for 100 nodes, and increases with the number of nodes in the network to reach 16.7 for 1000 nodes. A more detailed analysis shows that DCOG builds clusters whose size are usually around 10 or around 20. Also, the number of maximum size clusters increases with the number of nodes. To understand this behavior, the CGD and GCD must be analyzed, which is done in the next section of this document.

#### 2.6.6.4 Cluster group and group cluster diversities

Following the definitions in Section 2.6.1, the CGD and GCD have been evaluated and plotted vs. the number of nodes in Fig. 2.19 and Fig. 2.20 respectively.

First let us analyze the results for the three algorithms GDMAC-new1, VOTE-std and VOTE-new which lead to the highest (i.e. worst) values. None of these algorithms take the group membership into account to choose the CH they affiliate to, leading to a large numbers of groups per cluster. As expected, the values also increase with the number of nodes in the network.

- GDMAC-new1: the CGD increases from 2 to 5 and the GCD increases from 4 to nearly 8.5. It is interesting to note that the larger the number of nodes, the closer the CGD is to the cluster size itself. This means that on average, each cluster includes only one member of each operational group. Also, when the number of nodes increases, the number of clusters also increases which raises the probability that the members of each group are split among different clusters. This is confirmed by larger GCD values.
- VOTE-std and VOTE-new: a similar analysis as the one performed for GDMAC-new1 can be done. The difference is that they both build larger clusters. Consequently they lead to a larger average CGD value, but a smaller GCD one. This is confirmed by Fig. 2.19 and Fig. 2.20.

Let us now study the GDMAC-new2 and DCOG metrics, which are clearly better.

- Thanks to its affiliation procedure GDMAC-new2 succeeds in attaining lower values for both metrics: the CGD increases from 1.5 to only 2.5, and the GCD increases from 2.5 to nearly 4.5. These results must be put in perspective with the fact that GDMAC-new2 builds small clusters, as GDMAC-new1.
- DCOG achieves the lowest CGD, which lies between 1 and 2. This means that the clusters built by DCOG usually contain members from only one or two operational groups. This is confirmed by GCD which is always close to 1, meaning that DCOG succeeds in collecting nearly all the members of one or more groups in the same cluster. This conclusion is true independently of the number of nodes. In addition to its cluster size management feature, this property is one of the main strengths of DCOG. Let us now interpret in a different way the reason why the DCOG cluster size metric increases with the number of nodes. When  $N$  increases, the average node degree becomes larger, thus the probability that two entire groups are included in a single cluster increases, which logically raises the average cluster size.



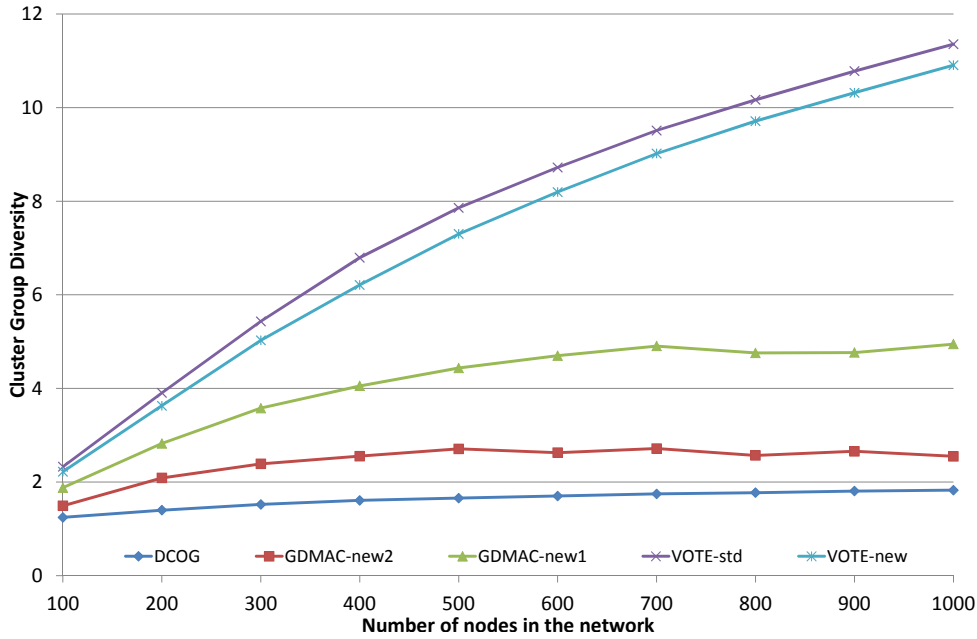


Figure 2.19: Cluster group diversity in static networks vs. the number of nodes.

### 2.6.6.5 Additional insights on DCOG performance

Different group size  $m_t$  have been simulated, keeping constant the maximal cluster size  $n_{max} = 20$  and leading to different  $m_t/n_{max}$  ratios. To ensure that all groups are complete, the number of nodes in the network has been slightly adjusted, leading to the  $(N, m_t, m_t/n_{max})$  3-tuples listed in Table 2.9.

$N$	105	100	104	102
$m_t$	15	10	8	6
$m_t/n_{max}$	1.3	2	2.5	3.3

Table 2.9:  $N$ ,  $m_t$  values for various  $m_t/n_{max}$  ratios.

The distribution of cluster size for all ratios  $m_t/n_{max}$  is plotted in Fig. 2.21. When the ratio is 1.3, 81% of the clusters formed by DCOG are as large as a group, and 16% are smaller. When the ratio is equal to 2 or 2.5 then DCOG mainly builds cluster whose size is either equal to the group size or equal to twice the group size. This behavior is justified by  $\mathbf{G}_2$ , which focuses on forming clusters with the highest possible number of members from the same groups. For the same reason, when the ratio is equal to 3.3, DCOG mainly creates cluster whose size is once, twice or thrice the group size. When the ratio is greater than two, the probability to build clusters including a whole group is larger than building clusters including two (or three) entire groups. To understand this, one should remember that the nodes are deployed randomly in groups, and that there is no guarantee that the members of different groups can be gathered in a cluster satisfying the connectivity, size and diameter constraints. We explain in the same way the fact that DCOG also sometimes builds clusters whose size is not a multiple of the group size.

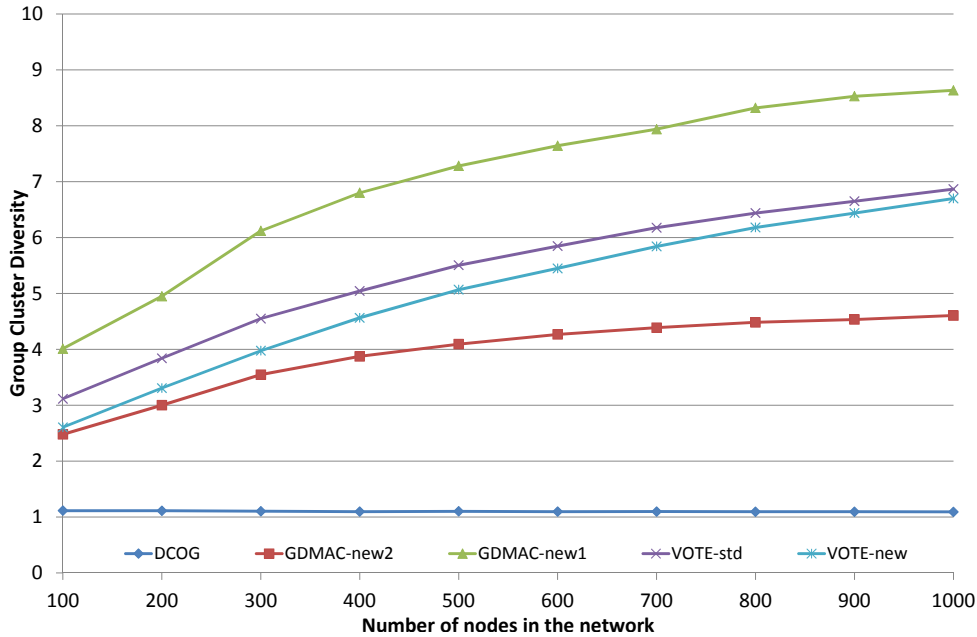


Figure 2.20: Group cluster diversity in static networks vs. the number of nodes.

#### 2.6.6.6 Application level performance: average end-to-end delay

The values of  $J_0^X$  as defined in Section 1.4.3 are plotted in Fig. 2.22 for GDMAC-new1, GDMAC-new2, VOTE-std and VOTE-new. A box plot is drawn to detail the statistical results associated with DCOG. In this box plot the lowest bar is the lowest value, the second lowest is the 5th percentile, then the box boundaries provide the first quartile, median and third quartile. The second highest bar is the 95th percentile, and the highest bar is the highest value. The average value is identified by a disk.

First, except for VOTE-std and VOTE-new which have similar  $J_0^X$  values, the difference between the different clustering solutions are significant. To understand why, let us recall that the metric  $J_0^X$  is related to the average end-to-end delay from all nodes to all nodes. To calculate  $J_0^X$  we have set the link weight  $w_{i,j}$  using the following expression from page 52 of [41]:

$$w_{i,j} := c_{max} + [1 - \Upsilon(i,j)]^{-1} + c_{max}/[\Upsilon(i,j)^{c_{max}} - 1],$$

with  $c_{max} = 4$  the maximum number of transmissions considering that a type I hybrid ARQ is used, and  $\Upsilon(i,j)$  the packet error rate on link  $(i,j)$ , which depends on  $\Gamma(i,j)$  the signal noise ratio (SNR) on the link. In our simulations a link  $(i,j)$  exists only if  $\Gamma(i,j) > 0$  dB, for which  $\Upsilon(i,j) = 0.13$ . The consequence is that  $w_{i,j} \in [1, 1.15]$ , meaning that the delay is never significantly greater than 1. In fact it is slightly greater than 1 only for a small number of links which tend to be avoided during the shortest path calculations performed to get the  $J_0^X$  value. Therefore the end-to-end delay strongly depends on the number of inter-cluster links, thus on the clustering solution.

Second, the DCOG  $J_0^X$  values are significantly lower than the ones of the other clustering schemes. Let us define  $\triangleleft_J$  a comparison operator between two clustering schemes  $clu_1$  and  $clu_2$  such as  $clu_1 \triangleleft_J clu_2$  if the  $J_0^X$  value achieved by  $clu_1$  is greater than the one of  $clu_2$ . Comparing

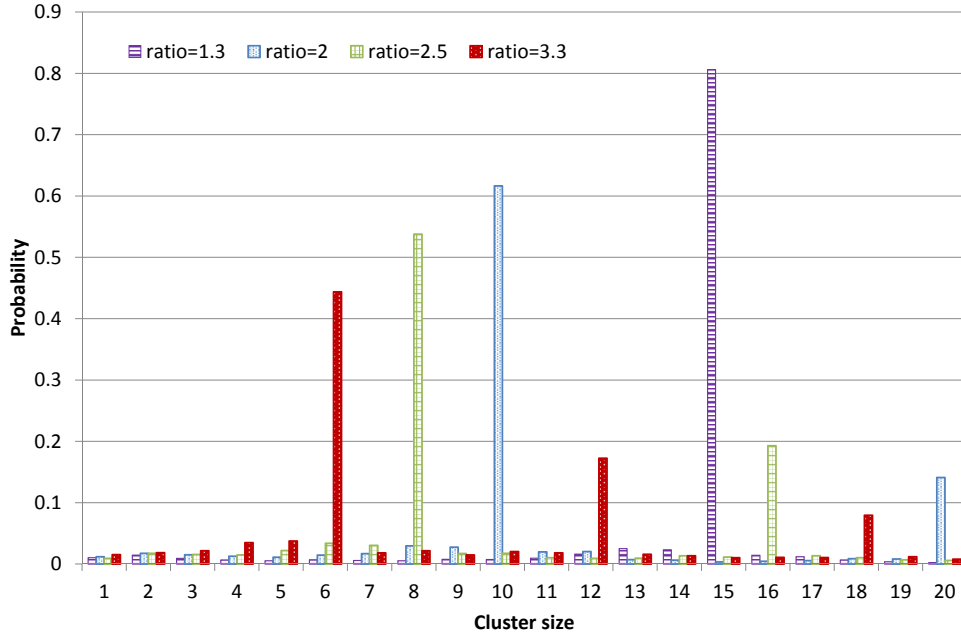


Figure 2.21: Cluster size distribution vs.  $m_t/n_{max}$  ratio.

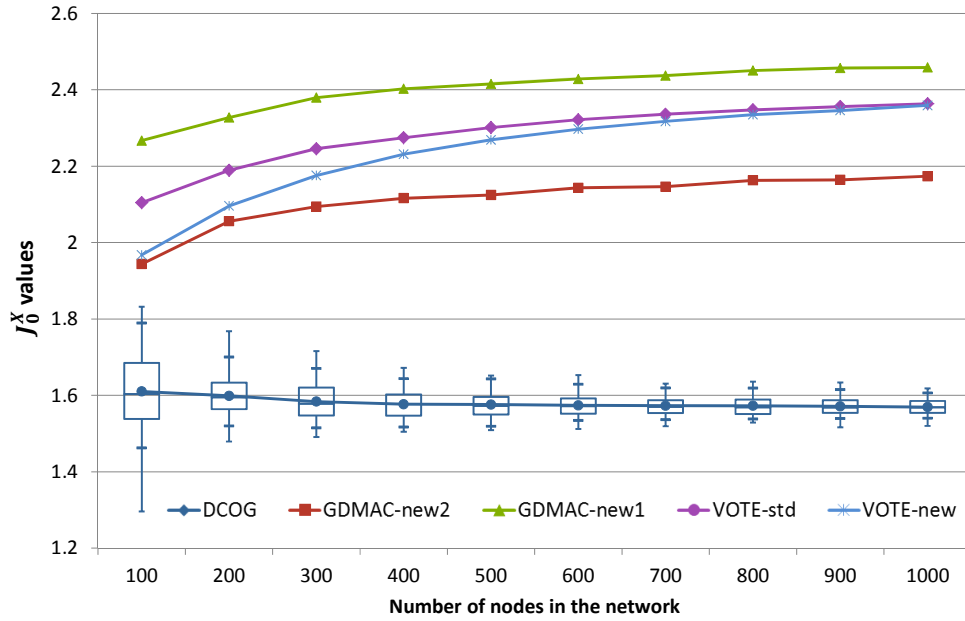


Figure 2.22:  $J_0^X$  values in static networks vs. the number of nodes.

the five clustering solutions we have:

$$\text{GDMAC-new1} \triangleleft_J \text{VOTE-std} \triangleleft_J \text{VOTE-new} \triangleleft_J \text{GDMAC-new2} \triangleleft_J \text{DCOG}.$$

DCOG achieves a gain in [29%, 36%] vs. GDMAC-new1, [23%, 34%] vs. VOTE-std, [18%, 33%] vs. VOTE-new, and [17%, 28%] vs. GDMAC-new2.  $J_0^X$  decreases when the amount of inter-cluster traffic decreases, which is the case also when the CGD and GCD decrease. This is the reason why DCOG, whose CGD and GCD are the lowest, achieves the best performance here.

The same justification holds to justify why GDMAC-new2 is the second best. It is interesting to note that the two VOTE variants, that manage the cluster size, have lower  $J_0^X$  performance than GDMAC-new2. This allows to conclude that it is more important to gather in the same clusters the members of the same operational groups than to handle cluster size. But both are important, and this justifies the good performance of DCOG.

To better estimate the quality of DCOG, let us remark that no partition  $p$  leads to a highest  $J_0^X$  value than the partition consisting in as many singleton clusters as nodes in the network. In this case any link is an inter-cluster link, and its contribution to  $J_0^X$  is maximum (cf. Section 1.4.3). Similarly, even if the cluster  $\mathcal{V}$  is likely to violate the constraints, a convenient lower bound for  $J_0^X$  is associated to the network partition  $\{\mathcal{V}\}$ . During our simulations we calculated both bounds and found that whatever the number of nodes in the network,  $J_0^X(p) \in [1.4, 2.8]$ ,  $\forall p \in \mathcal{P}^c$ . The  $J_0^X$  values achieved thanks to DCOG are close to 1.6, and are thus not far from the theoretical lower bound equal to 1.4 (knowing that the lowest  $J_0^X$  value achieved by a partition satisfying the constraints is somewhere in the interval  $[1.4, 1.6]$ ).

Finally, the box plot in Fig. 2.22 shows that the distribution of  $J_0^X$  becomes more and more narrow as the number of nodes increases from 100 to 600-700 and then remains stable. This is explained thanks to the result of Section 2.6.6.1, showing that the network diameter remains quasi-unchanged when the  $N$  increases, leading to similar route lengths in the network, and thus similar  $J_0^X$  values independently of the number of nodes.

### 2.6.6.7 Application level performance: average end-to-end bandwidth

In a similar way as for the values of  $J_0^X$ , the values of  $T_0^X$  defined in Section 1.7 are plotted in Fig. 2.23.

A first remark is that, even if DCOG achieves the best result, the performance of all solutions are much closer than when measured with  $J_0^X$ . To calculate  $T_0^X$ , related to the average throughput from all nodes to all nodes, we have set the link weight  $w_{i,j}$  equal to the link capacity, defined as follows:

$$w_{i,j} := \log_2[1 + \Gamma(i, j)].$$

Contrarily to the link delay used with  $J_0^X$ , the link capacity may vary a lot depending on the SNR. For example if  $\Gamma(i, j) = 0$  dB then the capacity is equal to 1.0, but if  $\Gamma(i, j) = 10$  dB (which is not a large value) then the capacity is increased to 3.3. Nevertheless, the main difference between  $T_0^X$  and  $J_0^X$  is the use of a  $\min()$  operation instead of a sum to calculate  $T_0^X(p, i, j)$ . Due to this  $\min()$  operation, the contribution of the source-destination pair  $(i, j)$  to  $T_0^X$  is given by the link with the lowest bandwidth. A single bad link between source  $i$  and destination  $j$  hurts  $T_0^X$  performance a lot more than  $J_0^X$  performance. Because DCOG takes into account the group membership and not the link quality, it is expected that its advantage over the other algorithms from a  $T_0^X$  perspective would be smaller than from a  $J_0^X$  perspective.

Let us define  $\triangleleft_T$  a comparison operator between two clustering schemes  $\text{clu}_1$  and  $\text{clu}_2$  such as  $\text{clu}_1 \triangleleft_T \text{clu}_2$  if the  $T_0^X$  value achieved by  $\text{clu}_1$  is lower than the one of  $\text{clu}_2$ . A second remark is that the partial ordering between the clustering algorithms is modified w.r.t. the one of the  $J_0^X$  metric:

$$\text{GDMAC-new1} \triangleleft_T \text{GDMAC-new2} \triangleleft_T \text{VOTE-std} \triangleleft_T \text{VOTE-new} \triangleleft_T \text{DCOG}.$$

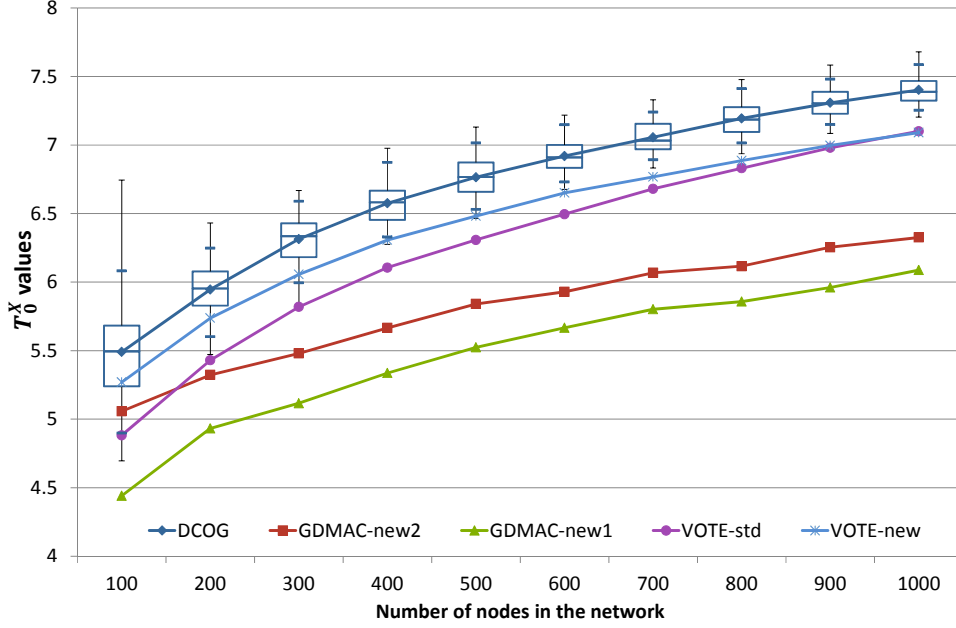


Figure 2.23:  $T_0^X$  values in static networks vs. the number of nodes.

DCOG still achieves the best performance but GDMAC-new2 is now the second worst solution instead of being the second best. Concerning capacity, it is more important to build bigger clusters which reduces the number of inter-cluster links than collecting together in the same cluster members of the same groups. This is better achieved by the VOTE variants than by the GDMAC variants. Because DCOG takes into account both cluster size and group membership, it succeeds in reducing the most the number of inter-cluster links, thus achieving the best performance w.r.t. the  $T_0^X$  metric.

## 2.6.7 Performance in dynamic networks

### 2.6.7.1 Simulation setup

In this section each simulation concerns networks with  $T = 10$  and  $m_t = 10$ . The node mobility model is the aforementioned RPGM group mobility model [45], modified such as to make sure that no node ever moves with a speed greater than a maximum  $v_{max}$ . To assess performance in mobility, simulations have been run for varying maximum node speed  $v_{max} \in \{1, 3, 5, 7.5, 10, 12.5, 15, 20\}$ .

A simulation is split in two phases: the transient phase and the steady phase. When there is no mobility, the steady phase begins when nothing change any more, i.e., concerning our simulations in Section 2.6.6, after the cluster structure has become stable. In presence of node mobility, the focus is on the performance during the steady state, and the metrics measured during the transient phase (also called warmup period) must be ignored. The results of Section 2.6.6.2 show that when  $N = 100$ , the clustering algorithms converge in less than 10 rounds. Therefore we set the duration of the warmup period to 10 rounds.

Let us now justify our choice for the simulation duration. A metric that can be used to assess the amount of mobility is the average link duration. Because of the group mobility model, intra-group links duration is larger than the global average link duration. The numbers in Table 2.10

and Table 2.11 detail the average link and the intra-group links durations (both in time units), versus the maximum node speed (in distance units per time units) and the simulation duration (in time units). These numbers are plotted in Fig. 2.24.

Max. speed → Sim. duration ↓	1	3	5	7.5	10	12.5	15	20
200	158	102	73.8	55.5	45.6	39.5	35.5	30.3
400	242	127	85.2	61.6	49.9	42.7	38.2	32.1
600	290	138	90.0	64.0	52.2	44.3	39.7	33.1
800	322	145	92.4	65.9	53.6	45.4	40.6	33.7
1000	344	150	93.8	67.4	54.5	46.1	41.0	34.0
1200	359	155	95.2	68.7	-	-	-	-
1400	370	158	96.1	69.8	-	-	-	-

Table 2.10: Average link durations (in time units) vs. simulation duration (in time units) and maximum speed (in distance units per time units).

Max. speed → Sim. duration ↓	1	3	5	7.5	10	12.5	15	20
200	198	194	175	137	108	91.8	82.6	73.6
400	396	382	305	189	133	109	96.8	85.7
600	594	566	401	213	146	119	105	92.0
800	792	746	472	230	156	125	110	95.7
1000	990	922	525	245	162	129	113	98.0
1200	1190	1090	565	252	-	-	-	-
1400	1390	1260	597	261	-	-	-	-

Table 2.11: Average intra-group link durations (in time units) vs. simulation duration (in time units) and maximum speed (in distance units per time units).

A first analysis of these numbers reveals that when the maximum speed is low, the intra-group link duration strongly depends on the simulation duration. This is illustrated in Fig. 2.24.a by the different plateaus occurring when the maximum speed is 1 and 3. On the other hand, the average link duration dependence on simulation duration is weaker, as shown in Fig. 2.24.b. An important property of the simulation is that its results should not be dependent on its duration. The above analysis shows that this is not doable in a reasonable time for intra-group links when the maximum speed is very low. Table 2.12 provides the increase of link durations when the simulation duration is increased by 200. These numbers are in percentage. For example the number 53.0 in the line associated with simulation duration 400 and maximum speed 1 is calculated by dividing the number 242 which is the link duration when simulation duration is 400 (in Table 2.10) by the number 158 which is the link duration when the simulation duration is 200 (same table). Thus 53.0 means that increasing from 200 to 400 the simulation duration leads to a 53% increase in link duration. In this table, the cells have been filled in green as soon as this percentage increase is lower than 10%. This condition is satisfied for all considered maximum speed if the simulation duration is greater or equal than 1000. Consequently, we

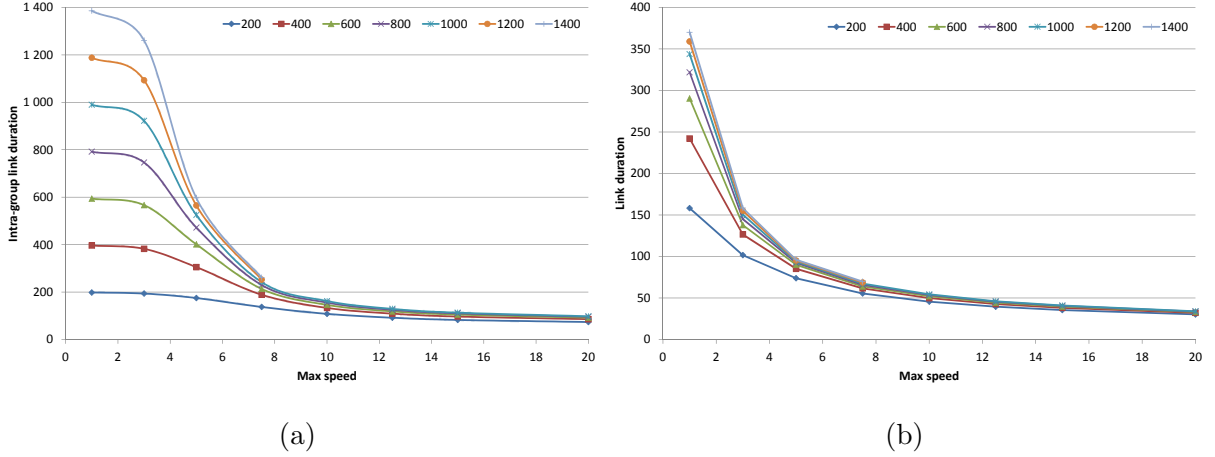


Figure 2.24: Link durations for various simulation durations increasing from 200 to 1400 time units. (a) Intra-group links. (b) Intra-group and inter-group links.

choose to set the simulation duration to 1000 rounds.

Max. speed → Sim. duration ↓	1	3	5	7.5	10	12.5	15	20
400	53.0	24.6	15.4	11.1	9.46	8.09	7.52	6.01
600	19.9	8.89	5.65	3.88	4.54	3.83	4.02	3.21
800	10.8	5.30	2.73	2.92	2.84	2.45	2.18	1.62
1000	6.82	3.68	1.50	2.37	1.52	1.46	1.01	1.10
1200	4.42	3.10	1.43	1.87	-	-	-	-
1400	3.08	2.09	0.97	1.63	-	-	-	-

Table 2.12: Average link durations difference.

### 2.6.7.2 Single simulation results

In mobility conditions, the average node degree is similar to the one during the simulations in static networks, i.e. equal to 20 as in Fig. 2.15. Nevertheless, the instantaneous node degree is different from its average value. To illustrate the instantaneous behavior of the simulation, we plot several relevant metrics vs. time in Fig. 2.25 to Fig. 2.29. These figures are the result of the simulation of a scenario with  $N = 100$  and  $v_{max} = 10.0$ .

As depicted in Fig. 2.25, the average node degree does not remain constant, showing large topological modifications along time.

Fig. 2.26 shows how many and when clustering events happened during the simulation. These events may be *i*) a swap between two clusters, or *ii*) a cluster split. A cluster that is split is destroyed and leads to a number of new clusters. Fig. 2.26 shows that during a simulation multiple such events may happen simultaneously: either no event (line NONE), one swap or one split (lines SWAP and SPLIT respectively), two swaps (line SWAP<sup>2</sup>), two splits (line SPLIT<sup>2</sup>), one swap and one split (line SWAP+SPLIT) or even three swaps (line SWAP<sup>3</sup>).

Fig. 2.27 and Fig. 2.28 depict average cluster size, CGD and GCD. The GCD is equal to 1 during 99.0% of the simulation, meaning that the groups are nearly always included in a single

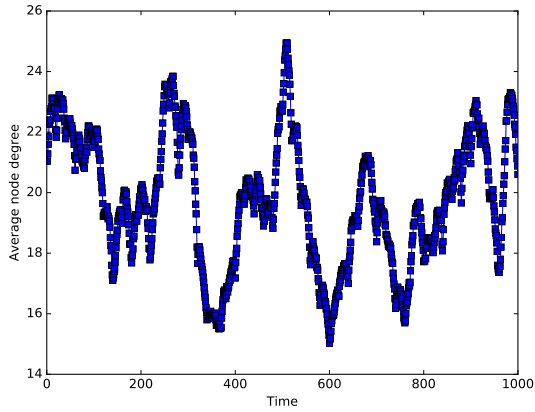


Figure 2.25: Average node degree vs. time.

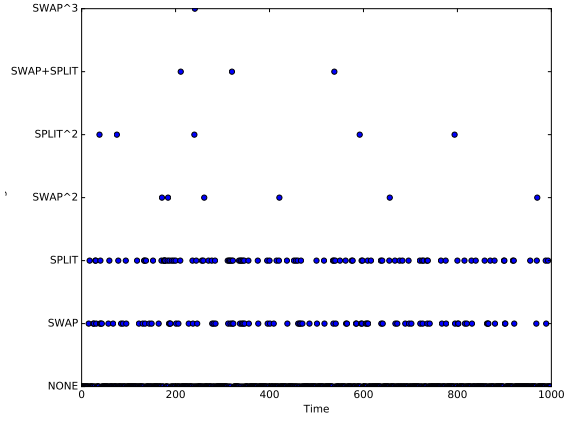


Figure 2.26: Clustering events vs. time.

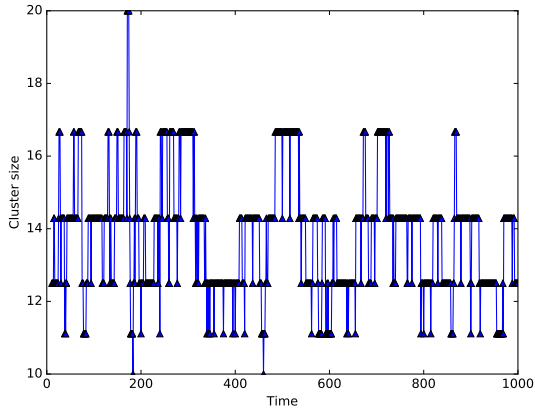


Figure 2.27: Average cluster size vs. time.

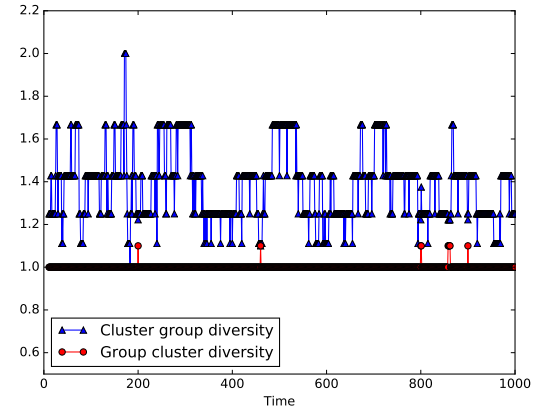


Figure 2.28: Average cluster group and group cluster diversities vs. time.

cluster. Consequently, during 99.0% of the simulation the cluster size curve is the enlarged version of the CGD curve by a factor of the group size, 10.

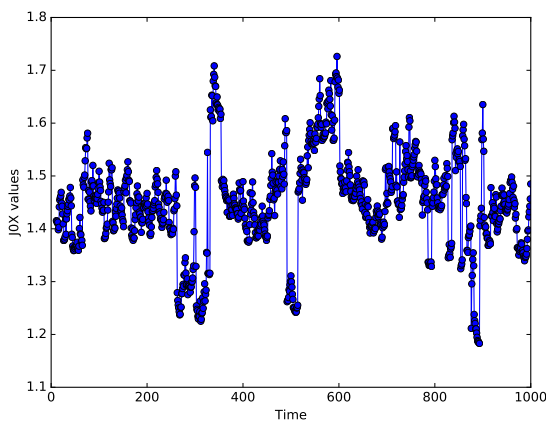


Figure 2.29:  $J_0^X$  values vs. time.

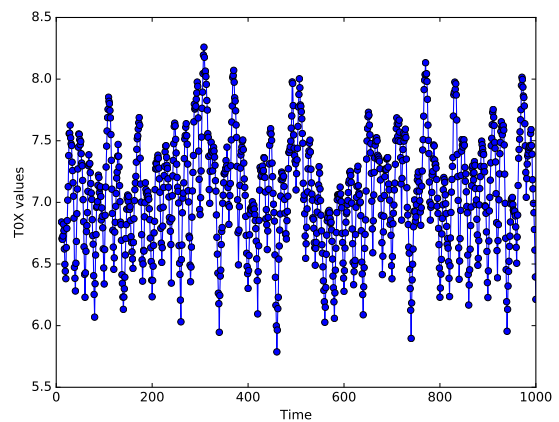


Figure 2.30:  $T_0^X$  values vs. time.

Finally Fig. 2.29 and Fig. 2.30 provide application level performance with  $J_0^X$  and  $T_0^X$  metrics. As expected (see Section 2.6.6.6 and Section 2.6.6.7) the former is less volatile than the



latter. No correlation seems to exist between any of these curves and the ones of Fig. 2.25, Fig. 2.27, or Fig. 2.28.

### 2.6.7.3 Simulation statistical results

To assess the stability of the cluster structure built by the different clustering solutions, usually metrics such as *average cluster duration*, or *average cluster head life time*, etc. are used. These metrics cannot be used to perform a fair comparison between DCOG and the reference clustering algorithms because DCOG defines a cluster through its members and does not have any CH. From the point of view of DCOG, when one or more nodes leave a cluster to join a neighboring one, then both source and destination clusters are modified. Consequently, in this section we define as stability metric the ratio of the number of simulation rounds when no cluster was modified to the simulation time. Values of this ratio close to 1 indicate stable clusters, conversely, values close to zero indicate unstable clusters.

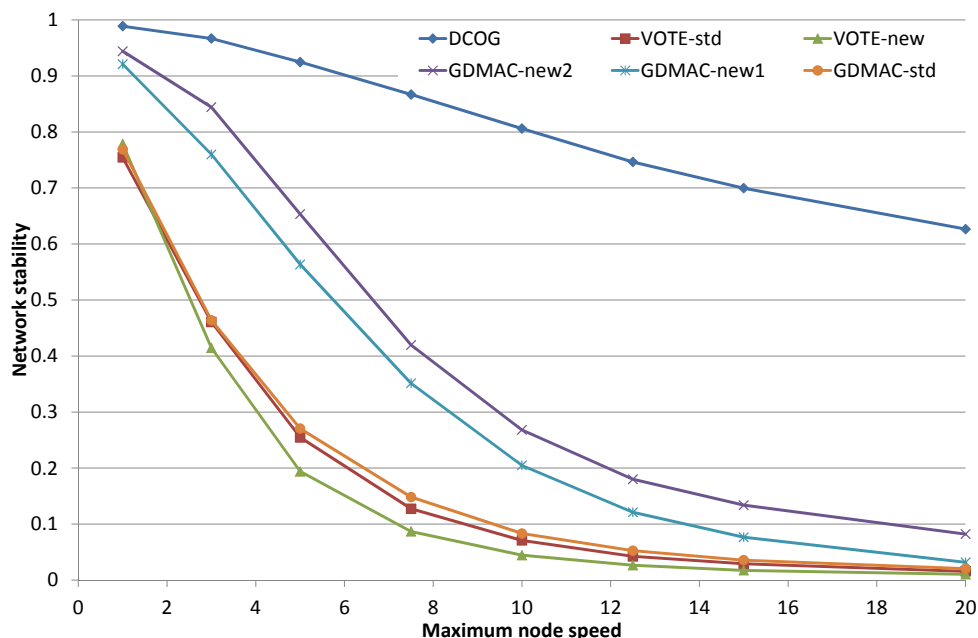


Figure 2.31: Mobile networks stability depending on maximum speed.

The plot of this metric in Fig. 2.31 shows that VOTE-std, VOTE-new and GDMAC-std are highly unstable, having less than 50% stability as soon as the node maximum speed exceeds 3 distance units per simulation round. Thanks to their small average cluster size and their use of group information GDMAC-new1 and GDMAC-new2 succeeds in keeping a stability greater than 50% up to a 5 distance units per simulation round. Finally, DCOG achieves by far the best stability, ensuring more than 60% stability even when the node maximum speed is as high as 20 distance units per simulation round. A first rationale for this good performance is the quick convergence property of DCOG, as shown in Section 2.6.6.2. This is achieved also thanks to a GCD whose values are always very close to 1, indicating that all members of a group are usually in the same cluster. Combined with the fact that nodes follow a group mobility pattern, a DCOG cluster is very stable.

## 2.7 Conclusions

In this chapter we detailed our methodology to design the novel distributed clustering algorithm DCOG suited to structured ad hoc networks such as public safety or military networks.

First, thanks to an exhaustive partition enumeration we found a cluster cost function suitable to identify good partitions, and thus good clusters. Second, we explained how from a centralized algorithm, we modified this cluster cost function and elaborated a decision making process, which led us to the distributed clustering algorithm that we called DCOG. The key feature of DCOG is its capability to build size limited clusters including entire groups, without resorting to CH nodes. Thirdly, we formalized our algorithm, and proved its theoretical convergence.

Our simulations in static and mobile conditions showed that thanks to DCOG, it is now possible to operate large scale dense networks based on groups and ensure good performance with respect to end-to-end delay and network stability. The comparison with existing solutions showed that our solution outperforms the ones from the literature. Another interesting feature of DCOG is its small number of parameters compared to other solutions, such as GDMAC, which makes it easier to use.

In Chapter 3, DCOG is revisited within the coalitional game theory framework. This enables us to characterize its final state after convergence, to suggest more general cost functions and gain insights on the design parameters. Moreover, this framework allows us to find a new algorithm for unstructured ad hoc networks that outperforms the conventional ones.



## Chapter 3

# Distributed clustering and coalitional game theory

Part of the work presented in this chapter has been submitted for future publication [32].

### 3.1 Introduction

Coalition game theory is the branch of game theory used to study the behavior of players when they cooperate among themselves [17]. Coalitional games involve a set of players who want to cooperate by forming *coalitions* in order to improve their positions in the game. In such games the coalitions are formed based on their *value*, which quantifies their worth in the game. Within a coalition the players receive a *benefit* thanks to their membership, which directly depends on the coalition value. When the value of a coalition can be divided between its members using any division rule, the game is said to be with *transferable utility* (TU). Otherwise the game is said to be with *non transferable utility* (NTU).

Numerous coalition formation algorithms have been proposed to solve many different problems. However some commonalities can be found. A first set of solutions involves the notion of *switch operation* in which a player leaves its current coalition and joins another one. Another family of proposals is based on the notion of *coalition merge and split*. In both cases, the benefit achieved thanks to the chosen algorithm must be positive to really improve the situation of the involved players. Usually the proposed algorithms own stability properties, such as for example Nash-stability [46], that guarantee the stability of the partition resulting from its execution.

Let us consider [20], where roadside units (RSU) form coalitions to diversify the classes of data they transmit to the vehicles within a VANET, in order to increase their revenue. In this paper the game is with TU, and the value of a coalition is shared between its members such that any player obtains a benefit no less than its benefit when acting non-cooperatively. In this paper, similarly to what is chosen in numerous other papers, the initial partition of the network is formed by singletons. The algorithm is synchronous: during each coalition formation iteration, the players are invoked sequentially. In the first phase each player  $i$  firsts looks for candidate switch operations, i.e., leaving its current coalition  $\mathcal{C}_k$  and joining another one  $\mathcal{C}_\ell$ , such that  $\mathcal{C}_\ell \cup \{i\} \succ_i \mathcal{C}_k$ , with  $\succ_i$  a preference relation between coalitions. In the second phase, for each player  $i$ , if one switch operation has been found in phase 1, then player  $i$  leaves its current coalition  $\mathcal{C}_k$  to join  $\mathcal{C}_\ell$ . The algorithm is repeated until a Nash-stable partition is achieved, which

is proved to always happen thanks to the fact that a graph has a finite number of partitions, and that the same partition is never visited twice by a player.

In [19] the players are small cell base stations (SBS) that cooperate to share their radio resources (OFDMA subchannels), in order to deal with the downlink co-tier interference suffered by the small cell user devices from neighboring SBSs. The SBSs form overlapping coalitions, and dedicate part of their frequency resources to each coalition they belong to. The benefit received by a coalition member is equal to the fraction of frequency resource that this members gets from its coalition multiplied by the coalition value, which is its achieved sum-rate. The game is with TU. The authors define the preference relations  $\succ_{\mathcal{C}}$  and  $\succ_{\mathcal{I}}$  to compare two coalitional structures w.r.t. the SBS benefits and the coalition values. Given the current coalition structure, thanks to these relations and for each subchannel, each SBS decides if this subchannel should be allocated to another coalition: first if possible an existing one using the preference relation  $\succ_{\mathcal{C}}$ , and otherwise a new one using the preference relation  $\succ_{\mathcal{I}}$ . When a subchannel is moved from one coalition to another, an history set is maintained in order to prevent this subchannel to be moved back later in the same coalition. This process is repeated until convergence to a stable coalition structure.

The authors in [18] detail an algorithm suited to cognitive networks with primary and secondary users (SU). Thanks to their algorithm, SUs form disjoint coalitions in order to perform collaborative sensing while maximizing utility in terms of detection probability and accounting for a false alarm cost. Here the game is with NTU, the benefit of each member of the coalition being equal to the coalition value. The algorithm is synchronous: during each coalition formation stage, coalitions are invoked sequentially to perform first merge then split operations. In the merge phase, the coalition with the highest value merges with a nearby coalition, then the merged coalition merges again, etc., until there is no more any benefit to merge the resulting coalitions. During this phase, a coalition  $\mathcal{C}_k$  may merge with a nearby coalition  $\mathcal{C}_\ell$  only if  $\{\mathcal{C}_k \cup \mathcal{C}_\ell\} \succ \{\mathcal{C}_k, \mathcal{C}_\ell\}$ , with  $\succ$  a comparison relation between collections of coalitions governed by the Pareto order. Following the merge process, the coalitions are subject to split operations. A coalition  $\{\mathcal{C}_k, \mathcal{C}_\ell\}$  may split into the two coalitions  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  only if  $\{\mathcal{C}_k, \mathcal{C}_\ell\} \succ \{\mathcal{C}_k \cup \mathcal{C}_\ell\}$ . With this relation, coalitions merge (split) only if at least one SU is able to strictly improve its benefit through this merge (split) without decreasing the benefits of other SU. The proposed algorithm is proved to converge to a  $\mathbb{D}_{hp}$ -stable partition, i.e., no player in this coalition are interested in leaving it through any merge-and-split operation to form other partitions.

Device-to-device (D2D) communications are the subject of [23] which proposes a distributed resource management scheme to jointly solve the problem of resource sharing mode selection and spectrum sharing. Here sharing mode can be *i*) cellular mode, *ii*) dedicated D2D mode and *iii*) hybrid mode where D2D reuse the resources of some cellular links. Players are either single cellular users involved in communication with the eNB, or pairs of cellular users involved in D2D communications. The game is with NTU, the benefit of a player being its achieved rate. The proposed algorithm is based on a succession of merge-and-split operations, using a preference relation  $\succ$  governed by the Pareto order to compare collection of coalitions.

The authors of [22] propose a distributed algorithm to achieve cooperative communications in ad hoc networks (with multiple sources and one destination, similarly to a WSN) in order to increase the achievable rate. Within a coalition the communications are performed over two phases: the broadcasting and the cooperation phases. During each slot of the broadcasting phase,

one coalition member performs broadcast transmission, while the other coalition members are listening. Then during the cooperation phase, all coalition members transmit a linearly coded signal of all signals received during the previous broadcasting phase, and the destination performs multiuser detection to extract the per node signal. The benefit of each coalition member is equal to its achievable rate, and the value of a coalition is the sum of its members benefits. The game is thus with NTU.

These examples prove that, when some benefit can be achieved thanks to cooperation, coalitional game theory can be used advantageously to solve numerous problems arising in the wireless communication field. In light of this, it seems natural to apply coalitional game theory to cluster building in wireless ad hoc networks. Within this chapter we thus reinterpret the DCOG algorithm from Chapter 2 in the context of this theory. This allows us to design a generic clustering algorithm which is *i*) applicable to any type of wireless mobile ad hoc network, and is *ii*) parameterized according to some utility functions and heuristics. Our analysis leads us to a specific family of utility functions and to several heuristics suitable for structured networks, and also allows us to deal with the case of unstructured networks.

Within this chapter, Section 3.2 introduces a generic clustering algorithm for mobile ad hoc networks based on coalitional game theory. It is generic in the sense that it is agnostic of the network structure. This generic algorithm is then adapted to structured and unstructured networks in Section 3.3 and Section 3.4 respectively. Finally Section 3.5 is devoted to simulation results and analysis.

## 3.2 Generic clustering algorithm based on coalition formation theory

In this section we begin by introducing the coalition formation framework that is used to derive the algorithm. We then detail the algorithm that is split into two specific procedures and prove its convergence.

### 3.2.1 Useful definitions related to coalition formation game theory

The coalition formation games in this chapter involve a set of players  $\mathcal{N}$ , also denoted by  $\mathcal{V}$  in the graph theory context of the two previous chapters. Let us now recall some definitions from the coalition game theory [17] that are instrumental in the derivation of our clustering algorithm, starting with the notions of coalition and *coalition structure*.

**Definition 3.1** A coalition structure (or coalition partition) is defined as the set  $p := \{\mathcal{C}_1, \dots, \mathcal{C}_{N_c}\}$  where  $\mathcal{C}_k \subseteq \mathcal{N}$  are disjoint coalitions verifying  $\cup_{k=1}^{N_c} \mathcal{C}_k = \mathcal{N}$ .

Within a coalition structure we associate three different quantities to each coalition  $\mathcal{C}_k$ :

- The *utility*  $u(\mathcal{C}_k) \geq 0$  quantifies the worth of the coalition, with  $u(\emptyset) = 0$ . The expression of the utility is a design parameter that depends on the goal of the coalitional game.
- The *cost*  $c(\mathcal{C}_k)$  quantifies the cost of cooperation. In this thesis, we use the cost to account for the constraints imposed to the coalitions. As a consequence, we put:  $c(\mathcal{C}_k) = 0$  if all constraints are satisfied and  $c(\mathcal{C}_k) = +\infty$  otherwise.

- The *value*  $v(\mathcal{C}_k)$  is defined as the difference between the utility achieved thanks to the cooperation and the cost of cooperation:  $v(\mathcal{C}_k) = u(\mathcal{C}_k) - c(\mathcal{C}_k)$ . Note that  $v(\mathcal{C}_k) = u(\mathcal{C}_k)$  when the constraints are satisfied,  $v(\mathcal{C}_k) = -\infty$  otherwise.

The utility and the cost of a coalition depend on the type of network (unstructured or structured). Examples of utility functions are detailed in Sections 3.3 and 3.4. In this thesis we consider coalition formation games in characteristic form, i.e., when the value of a coalition only depends on its members [17].

A transfer of nodes from one coalition to another is called a *switch operation*:

**Definition 3.2** A switch operation  $\sigma_{k,\ell}(\mathcal{P})$  is defined as the transfer of players  $\mathcal{P}$  from  $\mathcal{C}_k \in p$  to  $\mathcal{C}_\ell \in p \cup \{\emptyset\}$ ,  $\sigma_{k,\ell}(\mathcal{P}) : \mathcal{C}_k \mapsto \mathcal{C}_k \setminus \mathcal{P}$ , and  $\mathcal{C}_\ell \mapsto \mathcal{C}_\ell \cup \mathcal{P}$ .

Note 1: if  $\mathcal{C}_\ell = \emptyset$ , then  $\sigma_{k,\ell}(\mathcal{P})$  leads to the formation of a new coalition  $\mathcal{P}$ , thus increasing by one the number of coalitions. In that case, the switch operation is noted  $\sigma_{k,\emptyset}(\mathcal{P})$ .

Note 2: if  $\mathcal{P} = \mathcal{C}_k$ , then  $\sigma_{k,\ell}(\mathcal{P})$  leads to the merge of  $\mathcal{C}_k$  with  $\mathcal{C}_\ell$ , thus decreasing by one the number of coalitions.

To determine if a switch operation improves the coalition structure, we now define the *switch operation gain*.

**Definition 3.3** The switch operation gain  $g(\sigma_{k,\ell}(\mathcal{P}))$  associated with  $\sigma_{k,\ell}(\mathcal{P})$  is defined as:

$$g(\sigma_{k,\ell}(\mathcal{P})) := r_{\mathcal{P}}(\mathcal{C}_\ell \cup \mathcal{P}) - r_{\mathcal{P}}(\mathcal{C}_k), \quad (3.1)$$

with  $r_{\mathcal{P}}(\mathcal{C}_k)$  defined as:

$$r_{\mathcal{P}}(\mathcal{C}_k) := v(\mathcal{C}_k) - v(\mathcal{C}_k \setminus \mathcal{P}). \quad (3.2)$$

The  $r_{\mathcal{P}}(\mathcal{C}_k)$  quantity can be interpreted as the added value of having players  $\mathcal{P}$  in coalition  $\mathcal{C}_k$ .

Considering only valid partitions (satisfying the constraints), equations (3.1) and (3.2) prove that the gain of a switch operation only depends on the values of the two involved coalitions, and not on the way the coalition values are shared among their members. This means that the coalition formation game proposed in this chapter is with transferable utility (TU).

Note that the generic clustering algorithm we propose in Section 3.2.2 only selects switch operations with a strictly positive gain. As a consequence of this strict positivity condition, a switch operation occurs only if the nodes involved in the switch operation have a higher added value in the destination coalition than in the source coalition. In an equivalent way, this also means that a switch operation occurs only if the increase in the utility of the destination coalition is strictly greater than the decrease in the utility of the source coalition.

Let us now define the *preference relation* used by the players to compare two switch operations.

**Definition 3.4** The preference relation  $\succ$  is defined as a complete and transitive binary relation between two switch operations  $\sigma_{k,\ell}(\mathcal{P}_i)$  and  $\sigma_{k',\ell'}(\mathcal{P}_j)$  such that:

$$\sigma_{k,\ell}(\mathcal{P}_i) \succ \sigma_{k',\ell'}(\mathcal{P}_j) \Leftrightarrow g(\sigma_{k,\ell}(\mathcal{P}_i)) > g(\sigma_{k',\ell'}(\mathcal{P}_j)). \quad (3.3)$$

Similarly, we also define  $\succeq$  as:  $\sigma_{k,\ell}(\mathcal{P}_i) \succeq \sigma_{k',\ell'}(\mathcal{P}_j) \Leftrightarrow g(\sigma_{k,\ell}(\mathcal{P}_i)) \geq g(\sigma_{k',\ell'}(\mathcal{P}_j))$ .

Using our notations, the *Nash-stability* [46] of a partition can be defined as:

**Definition 3.5** A partition  $p = \{\mathcal{C}_1 \dots, \mathcal{C}_{N_c}\}$  is Nash-stable if  $\forall \mathcal{C}_k \in p$ , for any node  $i \in \mathcal{C}_k$ ,  $g(\sigma_{k,\ell}(\{i\})) \leq 0$  for all  $\mathcal{C}_\ell \in p \cup \{\emptyset\}$ .

When the partition  $p$  is Nash-stable, it means that there exists no single node switch operation with a strictly positive gain.

### 3.2.2 Generic coalition formation algorithm for clustering

In this section we propose a generic, distributed and asynchronous coalition formation algorithm to cluster the network. To do that, we consider the coalition game theory formation framework, identifying coalitions as clusters, and players as nodes. In the sequel, we preferably use the cluster/node terminology. We note  $n_k$  the size (or number of members) of cluster  $\mathcal{C}_k$ , and  $d(\mathcal{C}_k)$  the diameter of its induced subgraph.

Implementation of ad hoc networks includes protocols that enable nodes to discover their neighbors and to exchange information between them. These protocols are either specific to the communication system providers or taken from standards such as the IEEE 802.15.4 one, designed for wireless personal area networks. In our work, we assume to use the capabilities provided by such protocols to share information between nodes such as cluster membership, link qualities, etc. This allows to disseminate useful information inside clusters and between neighbor clusters to implement the clustering algorithm.

The proposed algorithm is based upon comparisons of switch operation gains. It is *distributed* since the decision to perform a switch operation is done at the node level. Moreover, the decision-making time points are assumed not to be coordinated between nodes and thus the algorithm is *asynchronous*. However, when a node is evaluating the possibility to perform a switch operation, it may consider other nodes of its current cluster. To account for the asynchronism of the decision-making instants, nodes that are involved in a switch operation are set in a *busy* status. Nodes that are not in the *busy* status are said *available*. Before implementing a switch operation, availability of the nodes involved are checked. Switch operations are done between neighbor clusters; two clusters are neighbors if at least one node in one cluster is a neighbor of at least one node in the other cluster.

When the network is static, we can prove that the algorithm converges to a stable solution where all constraints are satisfied. When nodes are mobile, the network topology changes over time. As a consequence, cluster fulfilling the constraints at one time may not satisfy them after some time. Thus, the algorithm needs to cope with this situation and to react to find a new cluster formation that respects the constraints. Our generic clustering algorithm can then be summarized as follows. As soon as a node starts a decision-making process, it first checks if the constraints of its current cluster are satisfied. If the constraints are fulfilled, it applies the procedure  $\mathbf{P}_1$  to operate the best switch operation, described in Section 3.2.2.1. If because of the mobility of its members, at least one constraint is violated, it applies another procedure  $\mathbf{P}_2$ , described in Section 3.2.2.2. Each procedure includes a selection of candidate nodes for switch operations, which is done according to common sense rules referred to as *heuristics* and noted  $\mathcal{H}_1$  for  $\mathbf{P}_1$  and  $\mathcal{H}_2$  for  $\mathbf{P}_2$ .



### 3.2.2.1 Procedure when constraints are fulfilled ( $\mathbf{P}_1$ )

When a node  $i$  starts a decision-making procedure and detects that its cluster does not satisfy the constraints, it triggers the procedure  $\mathbf{P}_1$  in order to search for a strictly positive gain switch operation and implement it. The principle of this procedure run at each node  $i$  (summarized in Table 3.1) splits into the three following successive steps:

1. **Selection of candidate switch operations (lines 1-9).** We first use the heuristic  $\mathcal{H}_1$  to find the sets of nodes  $\mathcal{P}_{1,i}(a)$  which are candidates for a switch operation, i.e., that could leave the cluster  $\mathcal{C}_k$  to join a neighbor one. We assume that this set, denoted by  $\{\mathcal{P}_{1,i}(a)\}_{a=1}^{A_1}$ , includes at least the singleton  $\{i\}$  itself, hence  $A_1 \geq 1$ . For each candidate set, the switch operation gain is evaluated against all the neighbor clusters. We keep as candidates the switch operations with strictly positive gain.
2. **Selection of one best switch operation (line 10-11).** If the set of candidate switch operations is not empty, a best switch operation is selected among those with the largest switch operation gain. Otherwise, the procedure is terminated.
3. **Implementation of the switch operation (lines 12-14).** First, we check if the identified switch operation can be performed, i.e., if all the nodes involved (actually  $\mathcal{C}_k \cup \mathcal{C}_{\ell^*}$ ) are available. Then, if the condition is met, the selected switch operation is performed, otherwise it is dropped.

```

// Selection of candidate switch operations
1 Set  $\mathcal{M} = \emptyset$ .
2 Apply heuristic  $\mathcal{H}_1$  to node  $i$  to get  $\{\mathcal{P}_{1,i}(a)\}_{a=1}^{A_1}$ , with  $\mathcal{P}_{1,i}(a) \in \mathcal{C}_k$ 
and  $A_1 \geq 1$ .
3 For each  $a \in \{1, \dots, A_1\}$  do:
4   For each  $\mathcal{C}_\ell$  neighbor of  $\mathcal{C}_k$  do:
5     If  $g(\sigma_{k,\ell}(\mathcal{P}_{1,i}(a))) > 0$  then:
6       Set  $\mathcal{M} = \mathcal{M} \cup \sigma_{k,\ell}(\mathcal{P}_{1,i}(a))$ .
7     End If.
8   End For.
9 End For.
10 If  $\mathcal{M} \neq \emptyset$  then:
// Selection of one best switch operation
11 Find  $(a^*, \ell^*)$  such that  $\sigma_{k,\ell^*}(\mathcal{P}_{1,i}(a^*)) \succeq \sigma_{k,\ell}(\mathcal{P}_{1,i}(a))$ ,
                                                                     $\forall \sigma_{k,\ell}(\mathcal{P}_{1,i}(a)) \in \mathcal{M}$ .

// Implementation of the switch operation
12 If the nodes involved in  $\sigma_{k,\ell^*}(\mathcal{P}_{1,i}(a^*))$  are all available then:
13   The nodes in  $\mathcal{P}_{1,i}(a^*)$  join  $\mathcal{C}_{\ell^*}$ .
14 End If.
15 End If //  $\mathcal{M} \neq \emptyset$ .

```

Table 3.1: Procedure  $\mathbf{P}_1$  at node  $i \in \mathcal{C}_k$  of the generic clustering algorithm when constraints are satisfied.

Given the network  $\mathcal{G}$ , the complexity of  $\mathbf{P}_1$  is governed by the number of iterations of lines 4-7 in Table 3.1, which is equal to the largest node degree  $D(\mathcal{G}) < N$ . Thus, the complexity of  $\mathbf{P}_1$  is  $O(D(\mathcal{G}))$ .

### 3.2.2.2 Procedure when constraints are not fulfilled ( $\mathbf{P}_2$ )

When a node  $i$  starts a decision-making procedure and detects that its cluster does not satisfy the constraints because of the mobility of its members, it triggers a procedure to change the cluster topology looking for a new partition matching the constraints. The procedure chooses among three different actions: *i*) do nothing, *ii*) some members of the cluster (including node  $i$ ) join another cluster, or *iii*) some members of the cluster (including node  $i$ ) form a new cluster. Action (*i*) happens when no candidate switch operation is identified. The principle of this procedure run at each node  $i$  (summarized in Table 3.2) splits into the three following successive steps:

1. **Selection of candidate switch operations (lines 1-10).** We first use the heuristic  $\mathcal{H}_2$  to find the sets of nodes  $\mathcal{P}_{2,i}(a)$  which are candidates for a switch operation. We denote this set by  $\{\mathcal{P}_{2,i}(a)\}_{a=1}^{A_2}$ . When it is not empty, we evaluate the values  $r_{\mathcal{P}_{2,i}(a)}(\mathcal{C}_\ell \cup \mathcal{P}_{2,i}(a))$  in (3.2) of the merge of all the candidate sets against all the neighbor clusters  $\mathcal{C}_\ell$  and keep those with strictly positive gains.
2. **Selection of one best switch operation (line 11).** Select one switch operation among those with the largest  $r_{\mathcal{P}}$  value.
3. **Implementation of the switch operation (lines 12-16).** First, we check if the identified switch operation can be performed, i.e., if all the nodes involved (actually  $\mathcal{P}_{2,i}(a^*) \cup \mathcal{C}_{\ell^*}$ ) are available. If this condition is met, then the selected switch operation is performed, otherwise nodes available in  $\mathcal{P}_{2,i}(a^*)$  form a new cluster.

Although  $\mathbf{P}_2$  is organized in the same lines as in  $\mathbf{P}_1$ , it differs along the following features:

- Conversely to  $\mathcal{H}_1$ ,  $\mathcal{H}_2$  may return an empty set for the candidate switch operations since moving node  $i$  (and some of its neighbors) may not improve the fulfillment of the constraints.
- Switch operations are selected using the value of  $r_{\mathcal{P}}$  in (3.2) instead of the switch operation gain  $g$  in (3.1). The reason is the following: when at least one constraint in  $\mathcal{C}_k$  is not fulfilled, then  $v(\mathcal{C}_k) = -\infty$  and  $r_{\mathcal{P}}(\mathcal{C}_k)$  is undefined. As a consequence, the switch operation gain cannot be used. However if there is one  $\mathcal{C}_\ell$  such that  $\mathcal{C}_\ell$  and  $\mathcal{C}_\ell \cup \mathcal{P}$  satisfy the constraints,  $r_{\mathcal{P}}(\mathcal{C}_\ell \cup \mathcal{P})$  still exists. Remembering that  $r_{\mathcal{P}}(\mathcal{C}_\ell \cup \mathcal{P})$  quantifies the gain associated with the arrival of  $\mathcal{P}$  in  $\mathcal{C}_\ell$ , it seems thus relevant to use this quantity instead of  $g$ .
- When the nodes are not available to implement the selected switch operation,  $\mathbf{P}_2$  creates a new cluster instead of dropping the switch. The rationale to do that is because dropping the switch would not resolve any of the constraints infringement, whereas creating a new cluster reduces the number of nodes that belong to a cluster not fulfilling the constraints.

```

// Selection of candidate switch operations
1 Apply heuristic  $\mathcal{H}_2$  to node  $i$  to get  $\{\mathcal{P}_{2,i}(a)\}_{a=1}^{A_2}$ , with  $\mathcal{P}_{2,i}(a) \in \mathcal{C}_k$ 
and  $A_2 \geq 0$ .
2 If  $A_2 > 0$  then:
3   For each  $a \in \{1, \dots, A_2\}$  do:
4     Set  $\mathcal{M} = \{\sigma_{k,\emptyset}(\mathcal{P}_{2,i}(a))\}$ .
5     For each  $\mathcal{C}_\ell$  neighbor of  $\mathcal{C}_k$  do:
6       If  $r_{\mathcal{P}_{2,i}(a)}(\mathcal{C}_\ell \cup \mathcal{P}_{2,i}(a)) > 0$  then:
11        Set  $\mathcal{M} = \mathcal{M} \cup \sigma_{k,\ell}(\mathcal{P}_{2,i}(a))$ .
8       End If.
9     End For.
10  End For.
// Selection of one of the best switch operation
11 Find  $(a^*, \ell^*)$  such that  $r_{\mathcal{P}_{2,i}(a^*)}(\mathcal{C}_{\ell^*} \cup \mathcal{P}_{2,i}(a^*)) \geq r_{\mathcal{P}_{2,i}(a)}(\mathcal{C}_\ell \cup \mathcal{P}_{2,i}(a))$ ,  $\forall \sigma_{k,\ell}(\mathcal{P}_{2,i}(a)) \in \mathcal{M}$ .
// Implementation of the switch operation
12 If the nodes involved in  $\sigma_{k,\ell^*}(\mathcal{P}_{2,i}(a^*))$  are all available then:
13   Nodes  $\mathcal{P}_{2,i}(a^*)$  join  $\mathcal{C}_{\ell^*}$ .
14 Else:
15   The nodes in  $\mathcal{P}_{2,i}(a^*)$  that are available form a new cluster.
16 End If.
17 End If.

```

Table 3.2: Procedure  $\mathbf{P}_2$  at node  $i \in \mathcal{C}_k$  of the generic clustering algorithm when constraints are not satisfied.

- Conversely to  $\mathbf{P}_1$ , the cluster  $\mathcal{C}_k$  may not satisfy the constraints after performing  $\mathbf{P}_2$ , for instance because no switch operation is performed. The constraint infringement resolution may occur later by the other nodes of the cluster, when running  $\mathbf{P}_2$ .

Like  $\mathbf{P}_1$ , the complexity of  $\mathbf{P}_2$  is  $O(\Delta(\mathcal{G}))$ .

The important design parameters of this algorithm are: *i*) the heuristics  $\mathcal{H}_1$  and  $\mathcal{H}_2$  to select the candidate sets of nodes  $\{\mathcal{P}_{1,i}(a)\}$  and  $\{\mathcal{P}_{2,i}(a)\}$ , and *ii*) the utility function  $u$  and the associated cost function  $c$  used to calculate the switch operation gains (either for  $g$  or  $r_{\mathcal{P}}$ ). Various heuristics, cluster utilities and costs can be chosen depending on the type of network. Several examples are given in Section 3.3 dedicated to structured networks and in Section 3.4 to unstructured networks.

### 3.2.3 Convergence properties

When the network topology is fixed and the algorithm is initialized by clusters fulfilling the constraints (the simplest being to set each node as a singleton cluster), then the following result holds:

**Result 3.1** For a fixed network topology and starting from any initial partition  $p_0$  of  $\mathcal{N}$  for which the clusters satisfy the constraints, the cluster formation algorithm maps to a sequence

of switch operations which converges in a finite number of iterations to a final partition  $p_f$ .

**Proof** See Appendix B.1. ■

Since the candidate set of nodes returned by  $\mathcal{H}_1$  includes the node  $i$  itself, we directly get:

**Result 3.2** The final partition  $p_f$  achieved in Result 3.1 is Nash-stable.

Note that the final Nash-stable partition  $p_f$  is in general not unique. The final partition depends on the order in which the network nodes make their switch operation decisions. As soon as the switch operation times are driven by random processes (see e.g., Section 3.5.1.1), experience shows that different final Nash-stable partitions can be found.

### 3.2.4 On the cluster constraints and utility function

As mentioned in the introduction, we impose some constraints to the clusters in order to ease the RRA process. In addition to the connectivity constraint, we also impose a maximum number of nodes per cluster noted  $n_{max}$  and a maximum cluster diameter noted  $d_{max}$ . These constraints are noted by:

- $\rho_1(\mathcal{C}_k)$ : the induced subgraph of  $\mathcal{C}_k$  is connected (already mentioned in § 3 of Section 3.2.2),
- $\rho_2(\mathcal{C}_k)$ :  $n_k \leq n_{max}$ ,
- $\rho_3(\mathcal{C}_k)$ :  $d(\mathcal{C}_k) \leq d_{max}$ ,

and the cost within a cluster  $\mathcal{C}_k$  is defined as:

$$c(\mathcal{C}_k) := \chi(\rho_1(\mathcal{C}_k)) + \chi(\rho_2(\mathcal{C}_k)) + \chi(\rho_3(\mathcal{C}_k)), \quad (3.4)$$

with  $\chi(\text{condition}(\mathcal{C}_k)) := 0$  if  $\text{condition}(\mathcal{C}_k)$  is satisfied,  $+\infty$  otherwise.

For the same reasons, we want to build clusters that include as many members as possible (limited by the size constraint  $n_{max}$ ). This is the goal  $\mathbf{G}_1$  already defined in Section 2.5.5:

**Goal 1 ( $\mathbf{G}_1$ )** *Build clusters whose size is maximal, i.e., equal to  $n_{max}$ .*

To achieve  $\mathbf{G}_1$ , the utility function should always allow the merge of two clusters, as long as the constraints are satisfied. Let us call this property Condition 3.1, that can be expressed as:

**Condition 3.1** The cluster utility function must verify  $g(\sigma_{k,\ell}(\mathcal{C}_k)) > 0$ ,  $\forall (\mathcal{C}_k, \mathcal{C}_\ell) \in \mathcal{P}^2$ ,  $k \neq \ell$ , as long as  $\mathcal{C}_k \cup \mathcal{C}_\ell$  satisfies the constraints.

In addition, Condition 3.1 ensures that the algorithm can be initialized with all the nodes as singleton clusters without being blocked in this configuration, which is not guaranteed for any utility function.

## 3.3 Clustering algorithm for structured mobile ad hoc networks

In this section we specify the generic clustering algorithm to the structured network case by designing dedicated utility function and heuristics. The resulting algorithm is called Clustering with Operational Groups (COG). Hereafter, we need the following additional notation: let  $m_{t,k}$  be the number of members of group  $\mathcal{O}_t$  in cluster  $\mathcal{C}_k$ .

### 3.3.1 Utility function

As discussed in the Introduction of this document, in structured networks, the members of the same group exchange the main part of their traffic within their group, and intra-cluster links benefit from a RRA which is more efficient than the one associated with the inter-cluster links. Therefore, as much as possible, we want to collect the members of the same group into a single cluster. This is the goal  $\mathbf{G}_2$  already defined in Section 2.5.5:

**Goal 2 ( $\mathbf{G}_2$ )** *Build clusters including the largest number of members from the same group.*

As already seen in Section 3.2.4, we also want to achieve  $\mathbf{G}_1$ . We now show that we can define for each of the two goals a specific utility function adapted to the goal. Therefore, we suggest defining a utility function for structured networks as a linear combination of these two utility functions:

$$u_{\text{st}}(\mathcal{C}_k) := u_1(\mathcal{C}_k) \cdot \epsilon + u_2(\mathcal{C}_k) \cdot (1 - \epsilon), \quad \epsilon \in (0, 1), \quad (3.5)$$

where

- $u_1$  is a utility function adapted to  $\mathbf{G}_1$  and which thus only depends on the cluster size:

$$u_1(\mathcal{C}_k) := f_1(n_k), \quad (3.6)$$

- $u_2$  is a utility function adapted to  $\mathbf{G}_2$  and which thus only depends on the number of nodes per group in the cluster:

$$u_2(\mathcal{C}_k) := \sum_{t \in \mathcal{I}(\mathcal{C}_k)} f_{2,t}(m_{t,k}), \quad (3.7)$$

with  $\mathcal{I}(\mathcal{C}_k)$  defined in Section 2.2.

We now want to determine relevant functions for  $f_1$  and  $f_{2,t}$ . Regarding  $f_1$ , we use the following result.

**Result 3.3** Let two different clusters  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  satisfying the constraints, and a set of nodes  $\mathcal{P} \subset \mathcal{C}_k$  such that

- $|\mathcal{C}_\ell \cup \mathcal{P}| > |\mathcal{C}_k|$  and
- $\mathcal{C}_\ell \cup \mathcal{P}$  and  $\mathcal{C}_k \setminus \mathcal{P}$  satisfy the constraints.

If  $f_1$  is strictly convex then  $u_1$  defined by (3.6) verifies  $g(\sigma_{k,\ell}(\mathcal{P})) > 0$ .

**Proof** See Appendix B.2. ■

This result simply tells that if  $f_1$  is strictly convex, then the nodes  $\mathcal{P}$  will always leave their cluster to form a larger one, thus achieving  $\mathbf{G}_1$ . As a corollary,  $f_1$  also verifies Condition 3.1 (apply Result 3.3 with  $\mathcal{P} = \mathcal{C}_k$ ). Consequently, we propose to select a strictly convex function  $f_1$  to achieve  $\mathbf{G}_1$ . Note that this result also holds for unstructured networks since it depends only on the cluster size. As the simplest strictly convex function is the second-order monomial, we propose to use

$$f_1(n_k) := \frac{n_k^2}{n_{max}^2}, \quad (3.8)$$

where  $n_{max}^2$  normalizes  $u_1$  in  $[0, 1]$ .

Regarding  $f_{2,t}$ , we use the following result.

**Result 3.4** Let two different clusters  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  satisfying the constraints such that

- i)  $\mathcal{P}$  is a subset of  $\mathcal{O}_t \cap \mathcal{C}_k$ ,
- ii)  $m_{t,\ell} + |\mathcal{P}| > m_{t,k}$ , and
- iii)  $\mathcal{C}_\ell \cup \mathcal{P}$  and  $\mathcal{C}_k \setminus \mathcal{P}$  satisfy the constraints.

If  $f_{2,t}$  is strictly convex then  $u_2$  defined by (3.7) verifies  $g(\sigma_{k,\ell}(\mathcal{P})) > 0$ .

**Proof** See Appendix B.3. ■

This result simply tells that if  $f_{2,t}$  is strictly convex, then some nodes in a given group will always leave their cluster to join another one provided that the new cluster includes a larger number of members of this group than the source cluster, thus achieving  $\mathbf{G}_2$ .

The same way as for  $f_1$ , we select the simplest second-order monomial function for  $f_{2,t}$ :

$$f_{2,t}(m_{t,k}) := \frac{m_{t,k}^2}{T \cdot m_t^2}, \quad (3.9)$$

where  $T \cdot m_t^2$  normalizes  $u_2$  in  $[0, 1]$ .

The utility function previously defined holds the following property.

**Result 3.5** The utility function defined as (3.5) with (3.6)-(3.9) satisfies condition 3.1.

**Proof** See Appendix B.4. ■

We now discuss the design of  $\epsilon$  to control the trade off between  $\mathbf{G}_1$  and  $\mathbf{G}_2$ . Let us first state the following result.

**Result 3.6** Let three different clusters satisfying the constraints  $\mathcal{C}_k$ ,  $\mathcal{C}_\ell$  and  $\mathcal{C}_q$ , each of them having some nodes belonging to the group  $\mathcal{O}_t$ . Let  $\mathcal{U}_q \subset \mathcal{C}_q$  that contains some nodes only of group  $\mathcal{O}_t$  with  $n_{\mathcal{U}} := |\mathcal{U}_q| \leq m_{t,q}$ . We assume that  $\mathcal{C}_k \cup \mathcal{U}_q$  and  $\mathcal{C}_\ell \cup \mathcal{U}_q$  satisfy the constraints. Assuming that  $\forall t, |\mathcal{O}_t| \leq n_{max}$  and using the utility function (3.5) with (3.6)-(3.9), the following properties hold:

- i) If  $m_{t,k} = m_{t,\ell}$  and  $|\mathcal{C}_k| > |\mathcal{C}_\ell|$ , then  $g(\sigma_{q,k}(\mathcal{U}_q)) > g(\sigma_{q,\ell}(\mathcal{U}_q))$ ,  $\forall \epsilon$ .
- ii) If  $m_{t,k} = m_{t,\ell}$  and  $|\mathcal{C}_k| = |\mathcal{C}_\ell|$ , then  $g(\sigma_{q,k}(\mathcal{U}_q)) = g(\sigma_{q,\ell}(\mathcal{U}_q))$ ,  $\forall \epsilon$ .
- iii) If  $m_{t,k} > m_{t,\ell}$ , and  $\forall |\mathcal{C}_k|, \forall |\mathcal{C}_\ell|$ , then  $g(\sigma_{q,k}(\mathcal{U}_q)) > g(\sigma_{q,\ell}(\mathcal{U}_q))$  as soon as:

$$\epsilon < \epsilon^* := \frac{1}{1+T}.$$

**Proof** See Appendix B.5. ■

First, when the number of nodes belonging to group  $\mathcal{O}_t$  in cluster  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  are equal, Result 3.6 tells that, whatever the value of  $\epsilon$ : i) the algorithm selects the switch of  $\mathcal{U}_q$  towards the cluster with the largest number of nodes, thus fulfilling  $\mathbf{G}_1$ , ii) if cluster  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  have the same number of nodes, both switch operations are even. Second, when the number of nodes belonging to group  $\mathcal{O}_t$  in cluster  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  are different, let say  $m_{t,k} > m_{t,\ell}$ , then, for any  $\epsilon < \epsilon^*$ , the algorithm always selects the switch of  $\mathcal{U}_q \subset \mathcal{O}_t$  towards the cluster which has the largest number of nodes of group  $\mathcal{O}_t$ , i.e.,  $\mathcal{C}_k$ , regardless of the size of the clusters, and more specifically even if  $|\mathcal{C}_k| < |\mathcal{C}_\ell|$ . In that condition, it thus always fulfills  $\mathbf{G}_2$  over  $\mathbf{G}_1$ .

### 3.3.2 Heuristics for node selection

We assume that node  $i$  running the algorithm in cluster  $\mathcal{C}_k$  belongs to group  $\mathcal{O}_t$ . For structured networks, the heuristic should select the set that gathers the maximum number of members of group  $\mathcal{O}_t$ , including node  $i$ . We also want this set to fulfill the constraints. Let us denote this set by  $\mathcal{L}$ . Thus, in order to build  $\mathcal{L}$ , we first identify  $i^*$  the node among the neighbors of  $i$  in group  $\mathcal{O}_t$  that has the highest degree. Then, we select the set of nodes including  $i^*$  that has the largest cardinality and which complies with the constraints. Note that when the diameter constraint is equal to two,  $\mathcal{L}$  is obtained by considering all the neighbors of  $i^*$  (which includes  $i$  by construction). This approach is used for the following heuristics used in procedures  $\mathbf{P}_1$  and  $\mathbf{P}_2$ .

#### 3.3.2.1 In procedure $\mathbf{P}_1$

In order to assess the effect of the choice of the heuristic, we propose three heuristics  $\mathcal{H}_1^{\text{st}h}$ , with  $h \in \{1, 2, 3\}$  that return the sets  $\{\mathcal{P}_{1,i}(a)\}_{a=1}^{A_1}$ . They are defined as:

- $\mathcal{H}_1^{\text{st}1}$ :  $A_1 = 2$  with  $\mathcal{P}_{1,i}(1) = \{i\}$  and  $\mathcal{P}_{1,i}(2) = \mathcal{L}$ .
- $\mathcal{H}_1^{\text{st}2}$ :  $A_1 = 1$ ,  $\mathcal{P}_{1,i}(1) = \mathcal{L}$ . Note that this heuristic does not satisfy the condition for Nash stability.
- $\mathcal{H}_1^{\text{st}3}$ :  $A_1 = 1$  and  $\mathcal{P}_{1,i}(1) = \{i\}$ .

We expect that the most complex,  $\mathcal{H}_1^{\text{st}1}$ , leads to the best results.

#### 3.3.2.2 In procedure $\mathbf{P}_2$ (adaptation to mobility)

We note the heuristic  $\mathcal{H}_2^{\text{st}}$  that returns the set  $\{\mathcal{P}_{2,i}(a)\}_{a=1}^{A_2}$ . It is defined as:  $A_2 = 1$  and  $\mathcal{P}_{2,i}(1) = \mathcal{L}$ .

## 3.4 Clustering algorithm for unstructured mobile ad hoc networks

In this section we specify the generic clustering algorithm to the unstructured network case by designing dedicated utility function and heuristics. The resulting algorithm is called Clustering with Link Quality (CLQ).

### 3.4.1 Utility function

In unstructured networks, we target clusters offering good throughput for intra-cluster communications. This can be obtained by gathering nodes with high link capacities between each other. We refer this goal as  $\mathbf{G}_3$ :

**Goal 3 ( $\mathbf{G}_3$ )** Build clusters with high link capacities.

Let us define  $\kappa(i, j) := \log_2(1 + \Gamma(i, j))$  the *capacity* of link  $(i, j)$ , with  $\Gamma(i, j)$  the SNR at node  $j$  when node  $i$  transmits.  $\Gamma(i, j)$  can be either the instantaneous SNR in Gaussian channels, or

the average SNR in random channels. In order to achieve  $\mathbf{G}_3$ , we propose to define the utility function as the sum capacity of all the intra-cluster links:

$$u_{\text{un}}(\mathcal{C}_k) := \sum_{i \in \mathcal{C}_k} \sum_{j \in \mathcal{C}_k | (i,j) \in \mathcal{E}} \kappa(i, j). \quad (3.10)$$

This utility function also achieves  $\mathbf{G}_1$  since it holds the following result:

**Result 3.7** The utility function defined in (3.10) satisfies Condition 3.1.

**Proof** See Appendix B.6. ■

### 3.4.2 Heuristics for node selection

#### 3.4.2.1 $\mathcal{H}_1$ in procedure $\mathbf{P}_1$

We note  $\mathcal{H}_1^{\text{un}}$  the corresponding heuristic that returns the set  $\{\mathcal{P}_{1,i}(a)\}_{a=1}^{A_1}$ . Since in unstructured networks there is not any particular reason to select any other node than itself, i.e.,  $\{i\}$ ,  $\mathcal{H}_1^{\text{un}}$  is defined as follows: set  $A_1 = 1$  with  $\mathcal{P}_{1,i}(1) = \{i\}$ .

#### 3.4.2.2 $\mathcal{H}_2$ in procedure $\mathbf{P}_2$ (adaptation to mobility)

We note  $\mathcal{H}_2^{\text{un}}$  the corresponding heuristic that returns the set  $\{\mathcal{P}_{2,i}(a)\}_{a=1}^{A_2}$ . Here, at least one constraint in the selected cluster is not satisfied, e.g., connectivity or diameter. We identify three different situations for node  $\{i\}$ : *i*) it has a lot of neighbors and it could be valuable that it forms a new cluster with its neighbors, *ii*) it has very few neighbors in the cluster meaning that it is presumably very likely to be involved in the constraints violation. It would be then beneficial that it leaves the cluster, *iii*) it is not in case *(i)* or *(ii)* and we change nothing. Considering the extreme situations for *(i)* (the most connected) and *(ii)* (the least connected), we propose the following heuristic:

- If node  $i$  has the highest degree within its induced cluster subgraph, then set  $A_2 = 1$  with  $\mathcal{P}_{2,i}(1)$  defined as all cluster members that are in the  $\lfloor d_{\text{max}}/2 \rfloor$ -hop neighborhood of  $i$ . When the diameter constraint is equal to two,  $\mathcal{P}_{2,i}(1)$  includes node  $i$  and all its 1-hop neighbors within the cluster, i.e.,  $\mathcal{P}_{2,i}(1) = \{i\} \cup \{j \in \mathcal{C}_k | (i, j) \in \mathcal{E}\}$ .
- If node  $i$  has the lowest degree within its induced cluster subgraph, then set  $A_2 = 1$  with  $\mathcal{P}_{2,i}(1) = \{i\}$ .
- In the other cases, node  $i$  chooses to remain in the cluster and to wait for the action of other cluster members, and  $A_2 = 0$ .

Notice that there are as many choices for the heuristics as one can imagine. The heuristics proposed here can thus be changed. However, the heuristic selection may take into account the computation complexity.

## 3.5 Numerical results

In this section we simulate numerically our proposed algorithms (COG and CLQ) and analyze them deeply. In case of structured ad hoc networks, COG is compared to the naive algorithm



1GIC defined in Section 1.5.1, whose goal is to force all members of a group to be in the same cluster as soon as the constraints can be satisfied. If not, simple mechanisms are carried out to satisfy the constraints. In case of unstructured ad hoc networks, CLQ is compared to three standard clustering algorithms entitled LCC [2] (as an extension of LID [3]), VOTE [6], and SECA [4]. Static and mobile configurations are tested.

### 3.5.1 Simulation setup

The nodes are deployed randomly in a 1.5 km x 1.5 km square area. The node deployment and mobility models are explained in Sections 3.5.2 and 3.5.3 for the structured and the unstructured networks, respectively. No radio communication is possible between two nodes  $i$  and  $j$  separated by a distance  $d_{i,j}$  greater than the radio range  $d_{\text{ref}} = 250$  m. The SNR associated with the link  $(i, j)$  is given by  $\Gamma(i, j) := -40 \log(d_{i,j}/d_{\text{ref}})$ . The diameter constraint on the cluster is  $d_{\text{max}} = 2$ . All the results are obtained through averaging over 100 different random networks. The simulation duration is fixed to 5000 s.

#### 3.5.1.1 Distributed asynchronous model

We specify here the way distributed and asynchronous decision-making is handled in the simulation. Remember that a node  $i \in \mathcal{C}_k$  can be either in a *Waiting* or *Busy* (i.e., not available) state. According to these states, the simulation behaves as the following:

- **Waiting state:** node  $i$  is not involved in current operations done by the clustering algorithm. However, it is learning information needed to run the clustering algorithm. It stays in this state during  $\delta_w$  s which is randomly chosen according to an exponential distribution with parameter  $\lambda$ . It may leave this state in the two following cases:
  1. After  $\delta_w$  s, it checks its cluster constraints. Then two cases may occur:
    - (a) If the constraints are satisfied, then it applies the procedure  $\mathbf{P}_1$  of the clustering algorithm described in Table 3.1. If it decides to move to another cluster, it then switches to the state *Busy* to account for the time needed to operate this change.
    - (b) If the constraints are not satisfied, then it applies the procedure  $\mathbf{P}_2$  of the clustering algorithm described in Table 3.2. If it decides to perform a change in the cluster it then switches to the state *Busy* to account for the time needed to operate this change.
  2. When the node  $i$  becomes involved in a cluster modification decided by another node  $j$ .
- **Busy state:** it means that the node is involved in a cluster modification initiated by itself or another node. The duration of this state is deterministic and set equal to  $\delta_b$  s in order to account for the time spent to exchange information between nodes inside the cluster. Notice that if nodes involved in this cluster modification are in the *Busy* state (because already involved in other cluster modifications simultaneously), then this cluster modification is canceled and the node  $i$  goes back to the *Waiting* state.

The above description can be modeled as a state machine as depicted in Fig. 3.1. Notice that we assume that the time required for running the procedures  $\mathbf{P}_1$  and  $\mathbf{P}_2$  is zero. Unless otherwise stated,  $\lambda = 5$  and  $\delta_b = 0.5$ .

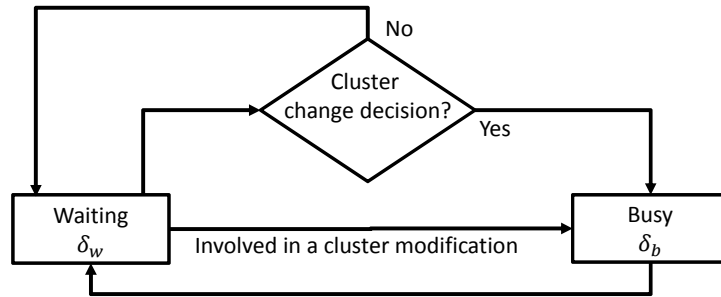


Figure 3.1: State machine model for asynchronous clustering algorithm.

### 3.5.2 Case of structured ad hoc networks

In this section we present the simulation results associated with COG. We set the group size  $m_t = 10$ . We denote by COG  $n$  the algorithm when the cluster size constraint is  $n_{max} = n$ . Unless specified otherwise, the heuristic used in Table 3.1 is  $\mathcal{H}_1^{st1}$ . The maximum number of groups  $T$  is equal to 100, when  $N = 1000$ . Consequently, using Result 3.6, we set  $\epsilon = 10^{-5} < \epsilon^* = 1/(1 + T) \simeq 9.910^{-3}$  in (3.5), to favor  $\mathbf{G}_2$  over  $\mathbf{G}_1$ .

In structured networks, the nodes are deployed randomly like described in Section 2.6.3.1. To better visualize the type of networks we consider here, Table 3.3 details the average node degrees vs.  $N$ . The network density ranges from low/medium (in 100 node networks) to high (in 1000 node networks) values.

$N$	100	200	300	400	600	800	1000
Degree	19.9	33.58	47.51	61.58	89.72	117.8	145.7

Table 3.3: Average node degree vs.  $N$  in structured networks.

#### 3.5.2.1 Static networks

First let us study the case when nodes are static. In this section we thus assess the performance of procedure  $\mathbf{P}_1$  in Table 3.1.

##### 3.5.2.1.1 Cluster-based vs. node-based clustering algorithm execution

The two algorithms DCOG from Chapter 2 and COG are very close in their definition. A noticeable difference lies in the entity running the algorithm. In the case of DCOG the decision making is done at cluster level, when COG is executed by each individual node. It is thus legitimate to wonder about the performance difference between them. To make both algorithms comparable we modified the DCOG simulation to use the cost function  $1 - u_{st}(\mathcal{C}_k)$  instead of (2.15), and ran 100 simulations of static random networks with  $N = 100$  nodes and  $n_{max} = 20$ . Fig. 3.2.a plots the values of the network cost function  $J_0^X$  defined in Chapter 1 and achieved by both algorithms, with parameters  $\alpha = 4$ ,  $\hat{\gamma} = 1$  and  $\tilde{\gamma} = 2$ . In Fig. 3.2.b we plot the differences between these values. Both cluster-based and node-based approaches lead to very close results, the difference being in  $[-3.9\%, 4.2\%]$ .

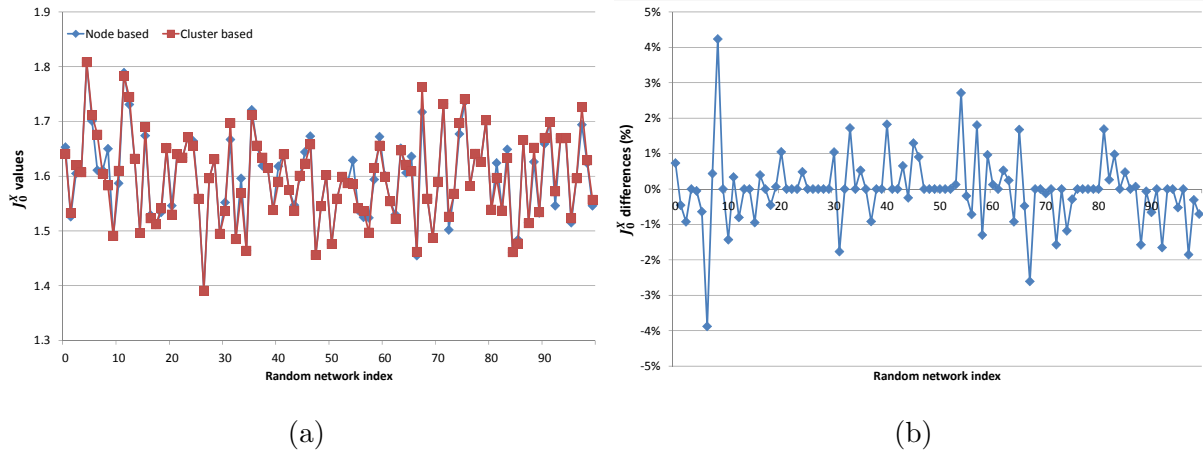


Figure 3.2: Cluster-based DCOG and node-based COG 20 algorithms in 100 structured networks with  $N = 100$ . (a)  $J_0^X$  values. (b)  $J_0^X$  value differences.

### 3.5.2.1.2 Technical metrics: group cluster diversity, cluster size, singletons and cluster group diversity.

In our random networks, the induced subgraphs of the groups do not always satisfy the topology constraints, thus forcing these groups to be split between multiple clusters, and thus leading to GCD (defined in Section 2.6.1) values greater than 1, as shown in Fig. 3.3. For all clustering algorithms, the GCD is only slightly greater than 1, meaning that each group is usually included in a single cluster and confirming that COG achieves  $\mathbf{G}_2$ .

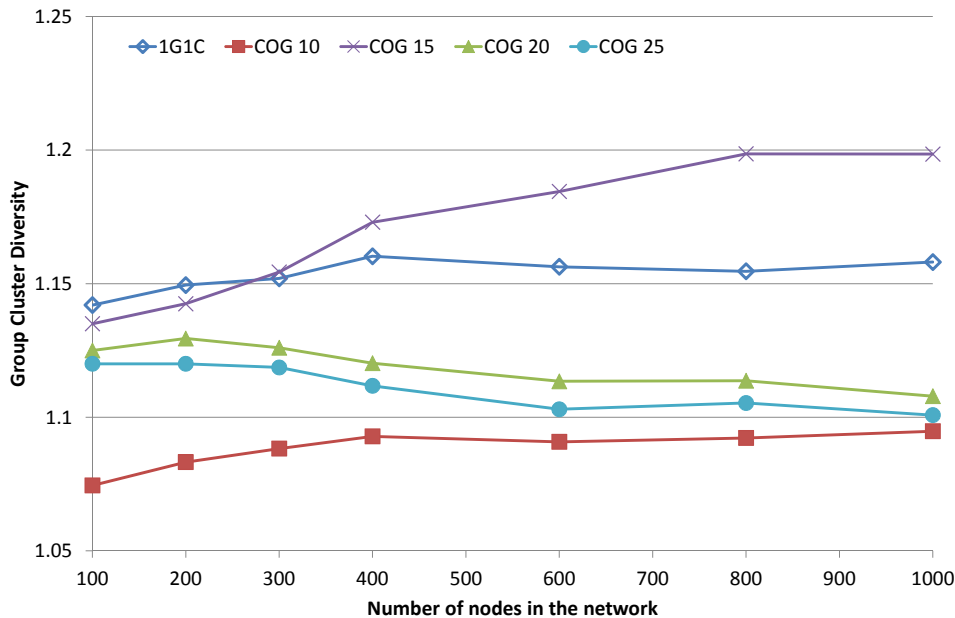


Figure 3.3: Group cluster diversity vs.  $N$  in static structured networks.

Fig. 3.4 shows the average cluster size. The curves for 1G1C and COG 10 are nearly the same, and they both build clusters smaller in average than the group size. This is due to topology constraints which sometimes prevent that all members of the same groups be inserted in the same cluster. Additionally COG 10 builds slightly larger clusters than 1G1C. During the distributed process of COG, several switch operations are required to aggregate the members

of a group in a single cluster. Depending on the order in which the different nodes perform their switch operations, the constraints may prevent the formation of clusters with full groups and lead to a larger number of clusters, i.e., a smaller average cluster size. To grasp the details of what happens let us consider the example of an 8 node network composed of two equal size groups  $\mathcal{O}_1 := \{1, 2, 3, 4\}$  and  $\mathcal{O}_2 := \{5, 6, 7, 8\}$ , with a maximum cluster size equal to 4. In this example, unless specified otherwise, the cluster constraints are always satisfied. At time  $t_0$  the partition  $p(t_0)$  of this network is:  $p(t_0) := \{\{1, 2, 3\}, \{4\}, \{5, 6, 7\}, \{8\}\}$ . Let us suppose that *i*) at time  $t_1 > t_0$  node 4 is the first to execute COG 4 after time  $t_0$  and that *ii*) node 4 cannot join the other members of  $\mathcal{O}_1$  because of the cluster diameter constraint. Then node 4 joins nodes  $\{5, 6, 7\}$ , members of  $\mathcal{O}_2$ , leading to the partition  $p(t_1) := \{\{1, 2, 3\}, \{4, 5, 6, 7\}, \{8\}\}$ . If node 8 at  $t_2 > t_1$  is the first to execute COG 4 after time  $t_1$ , it will not join the other members of  $\mathcal{O}_2$  because they already belong to a maximum size cluster. Thus node 8 will join nodes  $\{1, 2, 3\}$  and the final partition will be  $p(t_2) := \{\{1, 2, 3, 8\}, \{4, 5, 6, 7\}\}$ . In  $p(t_2)$  the average cluster size is 4. Conversely, during the centralized process of 1G1C, as long as the constraints are satisfied all members of the same group are always inserted in the same cluster. In the same example, 1G1C builds the partition  $p := \{\{1, 2, 3\}, \{4\}, \{5, 6, 7, 8\}\}$ , because node 4 cannot be in the same cluster as  $\{1, 2, 3\}$  due to the cluster diameter constraint. In  $p$  the average cluster size is equal to 2.67, smaller than the one in  $p(t_2)$ .

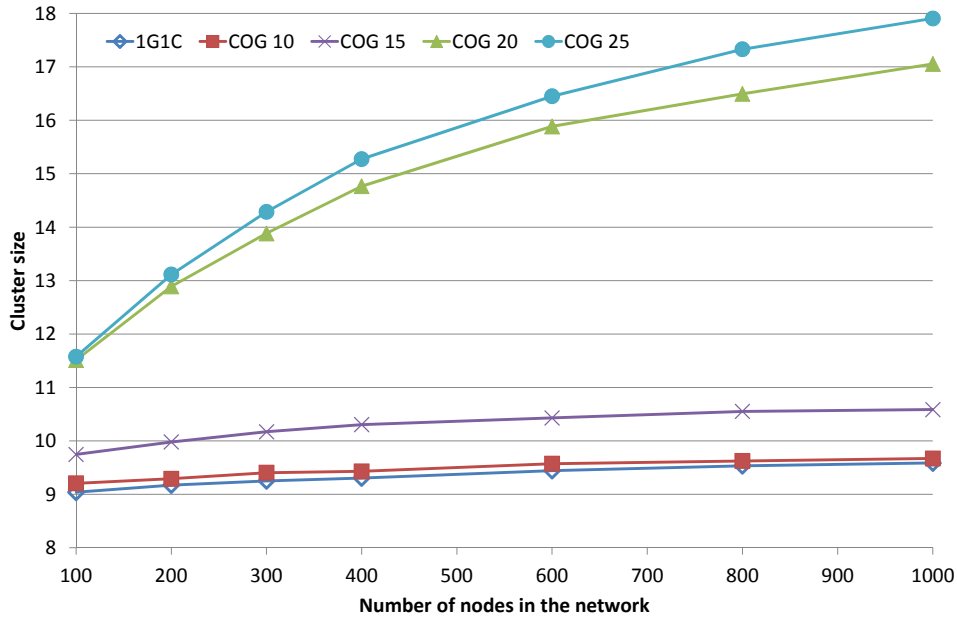


Figure 3.4: Cluster size vs.  $N$  in static structured networks.

Increasing the maximum cluster size to 15 does not allow to include two whole groups in a cluster. Therefore, due to our choice of  $\epsilon$  which favors  $\mathbf{G}_2$  over  $\mathbf{G}_1$ , we expect COG 15 to build clusters whose average size are similar to the ones achieved by COG 10. This is the case, even if COG 15 builds clusters whose size exceeds the one of COG 10 by a number in  $[0.5, 1.0]$ . This is justified by the capacity of COG 15 to "catch" the singleton clusters, i.e., merge them with clusters that already include one full group. To illustrate this capability, let us come back to the previous example, at time  $t_1$ :  $p(t_1) := \{\{1, 2, 3\}, \{4, 5, 6, 7\}, \{8\}\}$ . Let us consider what happens with COG 4 and COG 6 at time  $t_2$  if  $\{1, 2, 3, 8\}$  does not satisfy the constraints:

- **COG 4:** Node 8 remains alone and the final partition is  $p_{\text{COG4}} := \{\{1, 2, 3\}, \{4\}, \{5, 6, 7, 8\}\}$ , with average cluster size 2.67 and 1 singleton.
- **COG 6:** Node 8 joins nodes  $\{4, 5, 6, 7\}$  and the final partition is  $p_{\text{COG6}} := \{\{1, 2, 3\}, \{4, 5, 6, 7, 8\}\}$ , with average cluster size 4 and no singleton.

When the number of nodes  $N$  increases, the network topology becomes denser, and the probability that COG builds clusters with two full groups becomes larger, thus achieving  $\mathbf{G}_1$  and  $\mathbf{G}_2$ . This is the reason why the average cluster size curves for COG 20 and COG 25 are increasing functions of  $N$ . The same explanations as for COG 10 and COG 15 justifies why COG 25 builds cluster whose average size is greater than the one of COG 20.

The average number of singletons is plotted in Fig. 3.5.a. The centralized simple heuristic 1G1C builds the highest number of singletons. More importantly, these curves confirm the argument made about the cluster size metric: COG 15 and COG 25 minimize this number. Thus, to reduce the number of singletons the value of  $n_{max}$  should be set such as to be slightly greater than a multiple of the group size.

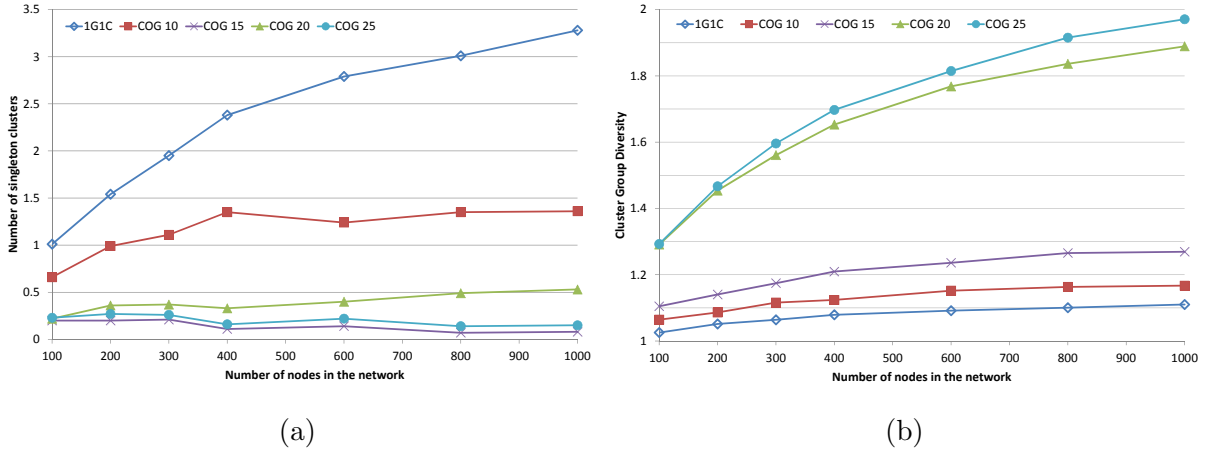


Figure 3.5: Static structured networks. (a) Number of singletons vs.  $N$ . (b) CGD vs.  $N$ .

The cluster group diversity (CGD) is the average per cluster number of groups with at least one member within the cluster. It appears that the curves of CGD in Fig. 3.5.b look similar to the ones showing the average cluster size in Fig. 3.4. Within a cluster, the cluster size divided by the group size and multiplied by the GCD is equal to the CGD. We know that the GCD is close to 1 and that the group size is equal to 10. Thus Fig. 3.5.b may be seen as reduction of Fig. 3.4 by a factor of 10.

### 3.5.2.1.3 Application level performance

Fig. 3.6 plots the application level performance measured with  $J_0^X$  for COG 10, COG 15, COG 20 and COG 25 with the heuristic  $\mathcal{H}_1^{st1}$  (like for all simulation results in this section). This figure also uses the results from Section 2.6.6.6 in Chapter 2 to show the performance area for clustering schemes from the literature, as well as upper and lower bounds for  $J_0^X$  values. Remembering that  $J_0^X$  is associated to end-to-end delay, a lower value of  $J_0^X$  is associated to a better performance. Therefore, Fig. 3.6 verifies that COG achieves clearly better performance

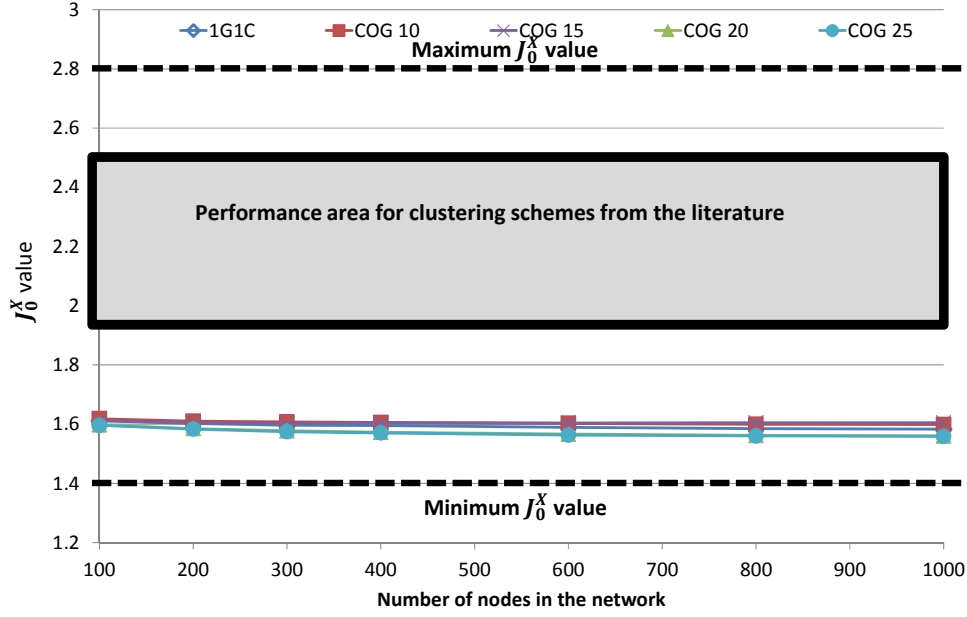


Figure 3.6:  $J_0^X$  w.r.t. state of the art vs.  $N$  in static structured networks.

that the algorithms from the state of the art the we simulated in Chapter 2, and builds partitions that are not that far from optimal ones.

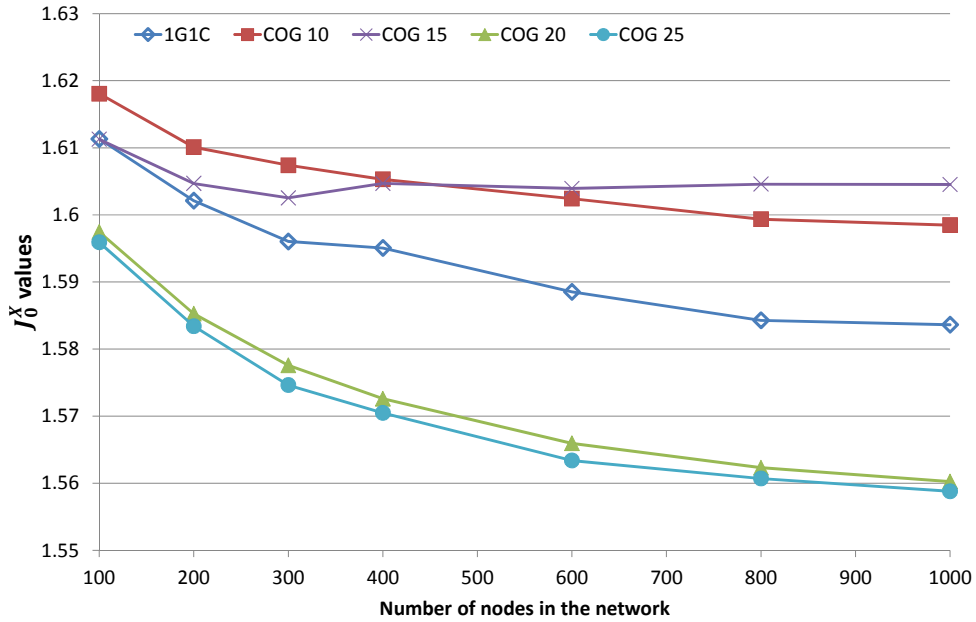


Figure 3.7:  $J_0^X$  vs.  $N$  in static structured networks.

Fig. 3.7 zooms on 1G1C and the COG algorithms. Whatever the maximum cluster size, COG achieves  $J_0^X$  values close to the ones of 1G1C. The justification of the slightly lower performance of COG 10 and COG 15 is the same as discussed in Section 3.5.2.1.2: it is due to the COG distributed execution dynamic. When COG is allowed to build clusters including several entire groups, the number of inter-cluster links involved in the communications is decreased, thus reducing the  $J_0^X$  values and improving user QoS. This gain, in  $[0.014, 0.025]$ , results from *i*) a reduction of inter-cluster traffic between members of different groups now in the same cluster,

and *ii*) the reduction of the number of inter-cluster links used by communications between members of different groups residing in different clusters. To better understand this gain, we consider  $p_x^A$  the number of paths with  $x$  inter-cluster links in the partition formed by algorithm A. Let us define  $\delta_x^A := p_x^A - p_x^{1G1C}$ . Fig. 3.8 plots the values of  $\delta_x^A$  achieved by the algorithms COG 10, COG 15, COG 20 and COG 25, for  $N = 300$ . This figure confirms our expectations. When two groups can be included in a single cluster ( $A \in \{\text{COG 20}, \text{COG 25}\}$ ), then *i*) the number of paths with zero or one inter-cluster links is increased, and logically *ii*) the number of paths with more than two such links is decreased.

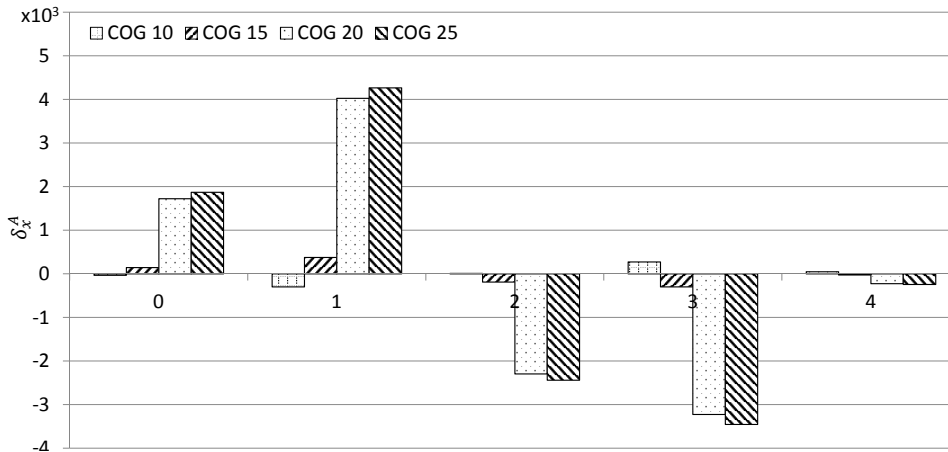


Figure 3.8: Difference in number of paths with  $x$  inter-cluster links inter group communications between 1G1C and COG (COG 10, COG 15, COG 20, COG 25) in static structured networks for  $N = 300$ .

In Fig. 3.9 we plot the values of  $J_0^X$  for COG 25 with heuristics  $\mathcal{H}_1^{st1}$ ,  $\mathcal{H}_1^{st2}$  and  $\mathcal{H}_1^{st3}$ , along with the ones for 1G1C. The best results are achieved thanks to  $\mathcal{H}_1^{st1}$ . It makes sense given that the output of  $\mathcal{H}_1^{st1}$  includes the ones of  $\mathcal{H}_1^{st2}$  and  $\mathcal{H}_1^{st3}$ . It is interesting to note that  $\mathcal{H}_1^{st3}$  and 1G1C curves are nearly the same. An analysis of the clusters formed with  $\mathcal{H}_1^{st3}$  reveals that they are composed of only one group, like 1G1C, COG 10 and COG 15 with  $\mathcal{H}_1^{st1}$ . Thus using  $\mathcal{H}_1^{st3}$  does not allow to achieve  $\mathbf{G}_1$ . To compare  $\mathcal{H}_1^{st1}$  and  $\mathcal{H}_1^{st2}$ , let us now analyze the complexity of COG.

### 3.5.2.1.4 Complexity

The computational complexity of COG is related to the number of operations in the double-loop of lines 3-9 and 4-8 in Table 3.1, i.e., the number of  $\sigma_{k,\ell}(\mathcal{P}_{1,i}(a))$  gain calculations. The most complex operation when calculating a switch operation gain is to check if the diameter constraint is satisfied, which may require as many breadth first searches<sup>1</sup> (BFS) as cluster members. When running the BFS algorithm on a graph, for each vertex there is one operation per neighbor of this vertex. We denote each such operation by "BFS operation". Fig. 3.10 shows the number of BFS operations for the COG 10, COG 15, COG 20 and COG 25 with heuristic  $\mathcal{H}_1^{st1}$  (like for all simulation results in this section), and for COG 25 with  $\mathcal{H}_1^{st2}$  and  $\mathcal{H}_1^{st3}$ .

As expected, the number of BFS operations consistently increases with  $N$ , and with  $n_{max}$ . It is more fruitful to compare the complexity of the three heuristics used with COG 25. The one

<sup>1</sup>Breadth first search is a simple graph theory algorithm.

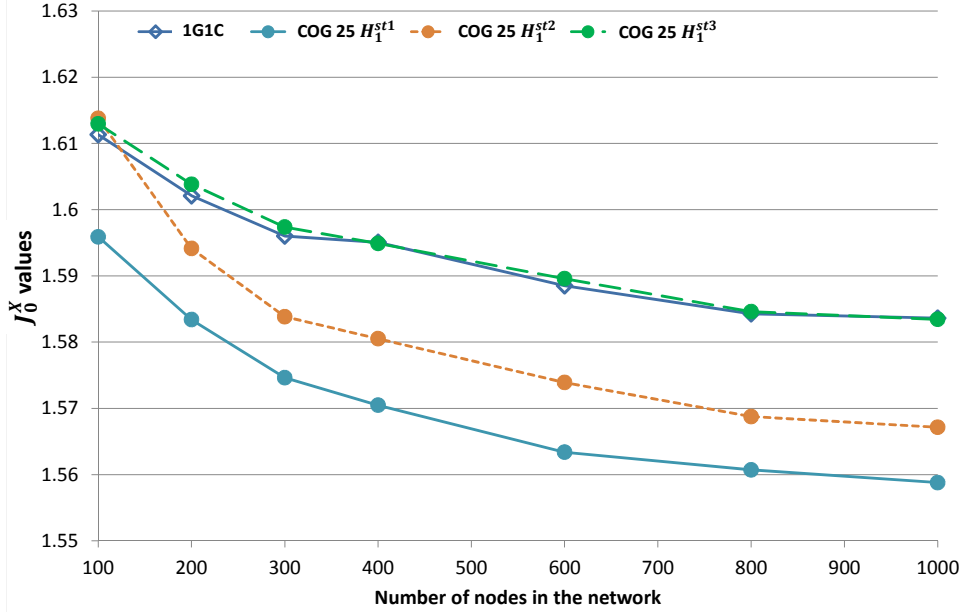


Figure 3.9:  $J_0^X$  for COG 25 with  $\mathcal{H}_1^{st1}$ ,  $\mathcal{H}_1^{st2}$  and  $\mathcal{H}_1^{st3}$  vs.  $N$  in static structured networks.

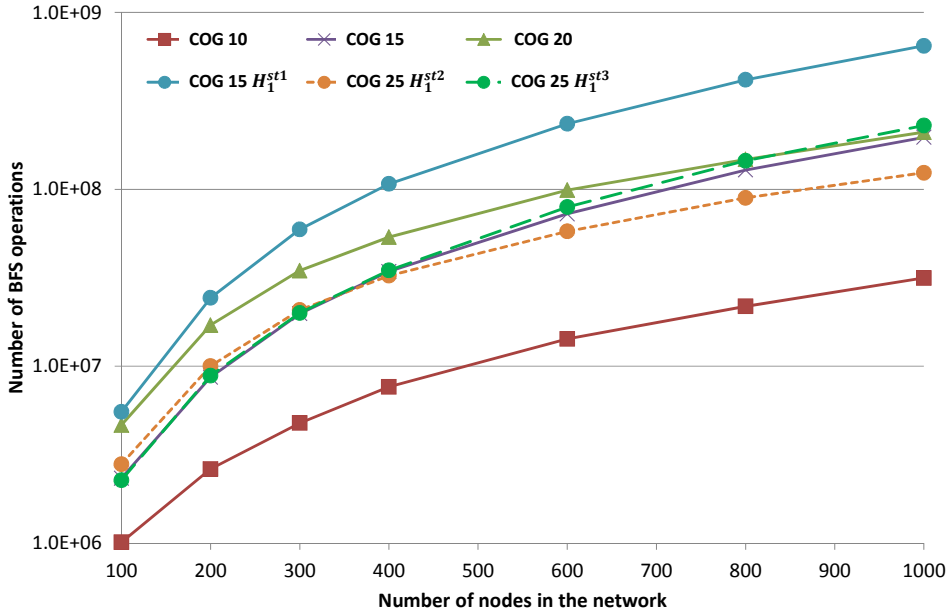


Figure 3.10: Number of breadth first search operations vs.  $N$  in static structured networks.

inducing the highest processing load is logically  $\mathcal{H}_1^{st1}$ . Remembering that in Fig. 3.9, the performance of  $\mathcal{H}_1^{st1}$  is the best, we confirm that achieving the best performance induces the largest complexity. Additionally, Fig. 3.9 and Fig. 3.10 show that  $\mathcal{H}_1^{st2}$  leads to less BFS operations than  $\mathcal{H}_1^{st1}$ , at the price of a decreased  $J_0^X$  performance. This illustrates the trade off between complexity and performance in the selection of the heuristics.

### 3.5.2.2 Mobile networks

We now consider networks with moving nodes. In this section, we thus assess the performance of both procedure  $\mathbf{P}_1$  in Table 3.1 and  $\mathbf{P}_2$  in Table 3.2.



In mobile networks, the duration of the simulation warmup phase must be chosen carefully. We measure the variations of the node average degree, plotted in Fig. 3.11. In view of this figure we set the duration of the warmup period to 500 s.

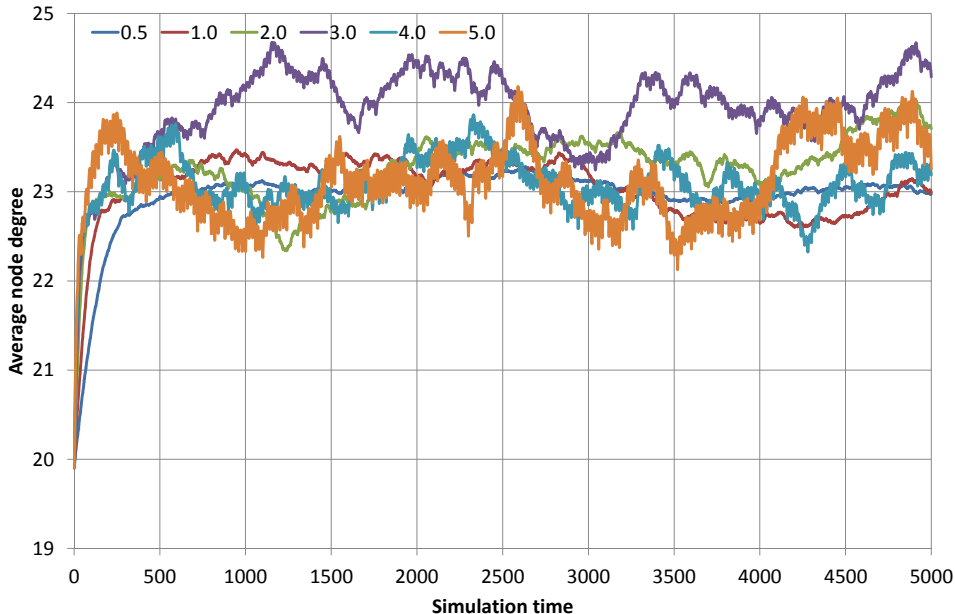


Figure 3.11: Average node degree vs. time for structured networks with  $N = 100$ , and for various  $v_{max}$  values.

In Fig. 3.12, we plot the cluster life time versus the maximum speed of nodes for different values of  $n_{max}$ . When  $n_{max}$  is smaller than twice the group size, the clusters are composed of a single group and so are very stable: the clusters formed at the beginning of the simulation are nearly never modified. This is an expected consequence of our node deployment scheme which ensures that within a group, the nodes are almost always at two radio hops. When  $n_{max}$  is at least equal to twice the group size, then some clusters include two groups. During the simulation, the groups move independently, leading to two kinds of cluster modifications: *i*) switch operations resulting from procedure  $\mathbf{P}_1$ , and *ii*) mobility adaptations performed during procedure  $\mathbf{P}_2$  to enforce the cluster constraints. When the node speed increases, this number of modifications also increases, thus reducing the cluster life time.

Fig. 3.13 shows the number of cluster modifications, i.e., the switch operations (the 'COG xx SO' curves) and the mobility adaptations (the 'COG xx MA' curves). This figure confirms that the clusters formed by COG 10 and COG 15 are nearly never modified (this is not visible in Fig. 3.13 because of the difference of scale between the number of cluster modifications for COG 10 and COG 15 w.r.t. to COG 20 and COG 25). Let us now analyze the behavior of COG 20 and COG 25. During the permanent phase of our simulations, a switch operation happens when two clusters, each composed of one group, merge into a two-group cluster. Subsequently, because the groups move independently, a mobility adaptation occurs and usually the two-group cluster splits back into the two previous one-group clusters. When this happens the total number of clusters is increased by one, allowing a later switch operation to merge one-group clusters. In some cases a third cluster, composed of one group, is in the vicinity, and one group from the two-group cluster joins the third cluster. When this happens, the number of clusters is unchanged

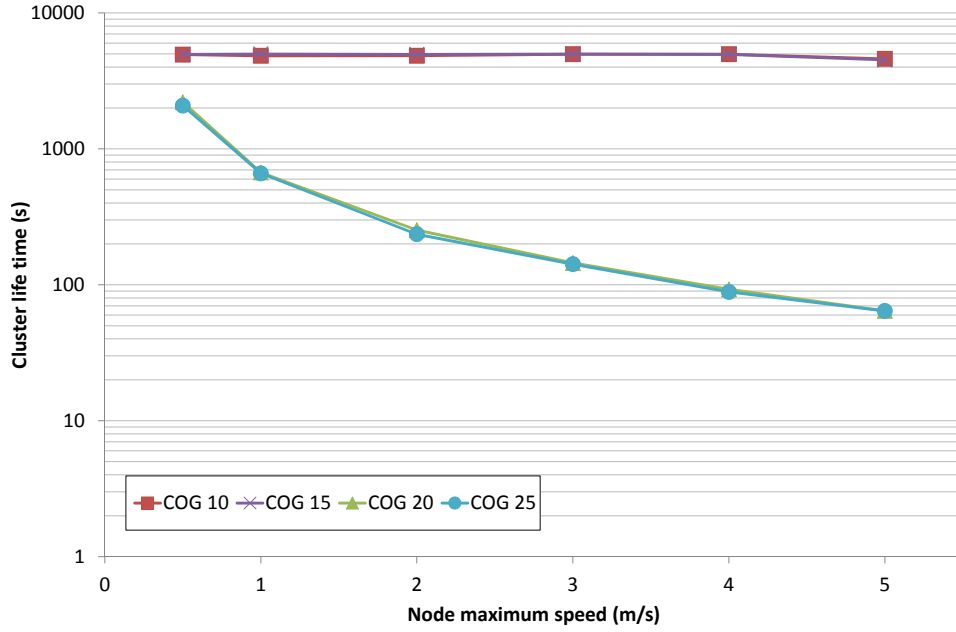


Figure 3.12: Cluster life time vs. maximum node speed in mobile structured networks.

and no switch operation to merge one-group clusters happens, only a mobility adaptation to split them. This last situation happens more frequently when  $v_{max}$  increases, which explains why the number of mobility adaptations is slightly larger than the one of switch operations.

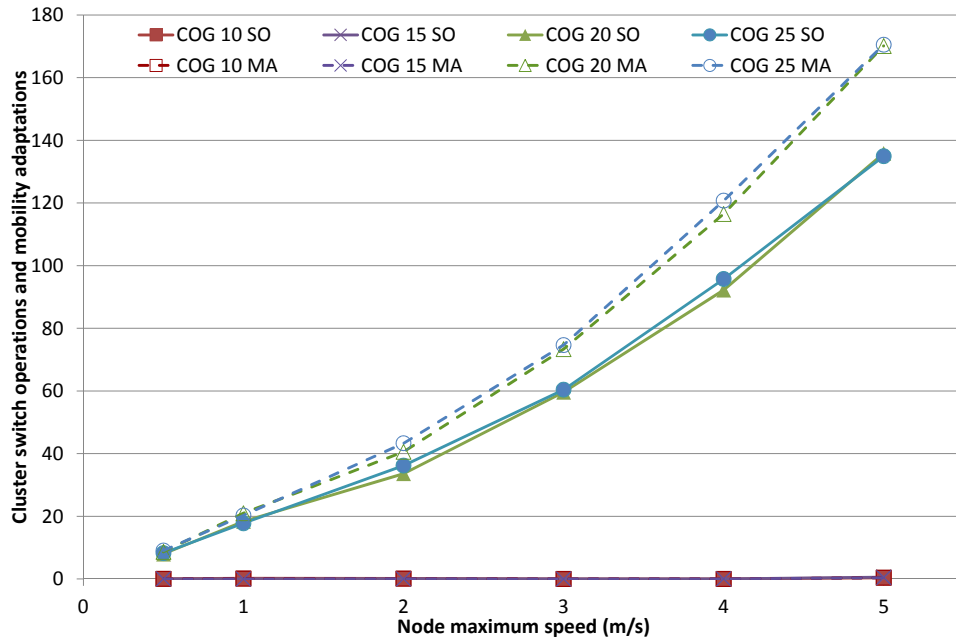


Figure 3.13: Number of cluster modifications vs. maximum node speed in mobile structured networks.

### 3.5.2.3 Conclusions on structured networks

Thanks to our simulations we verified that when the topology allows it, COG always form clusters composed of entire groups, thus achieving  $\mathbf{G}_2$  and leading to excellent intra-group com-

munication performance. When COG is configured to form clusters composed of only one single group, we showed that *i*) in static networks it leads to nearly the same clusters as the centralized heuristic 1G1C, and that *ii*) in presence of node mobility, COG clusters are stable. When multiple groups are allowed then COG also achieves  $\mathbf{G}_1$ , improving inter-group communication performance, at the price of an increased complexity and of less stability in presence of mobility.

We also established that the transition from the DCOG algorithm, run at cluster level, to COG, run at node level, did not decrease the performance. We also assessed the effect of various heuristics for node selection, which allowed us to highlight the trade off existing between complexity and performance. These results are important milestones in the path to an implementation of a clustering protocol suited to structured networks.

As a conclusion, allowing clusters to include more than one group improves inter-group communications, at the expense of cluster stability. This should be done only when the network mobility is expected to be low. Conversely, if mobility is high then the maximum cluster size should be set to a value slightly larger than the group size. This guarantees cluster stability and reduces the number of singleton clusters.

### 3.5.3 Case of unstructured ad hoc networks

In this section, we consider the algorithm CLQ, and compare it to the following existing algorithms: the old but very well-known LCC [2], VOTE which forces the cluster size to be less than a target threshold [6], and the recent SECA which takes the link quality into account [4]. Unless otherwise stated,  $n_{max} = 20$ . The design parameters for SECA (defined in [4]) are  $w_{cd} = 0.2$ ,  $w_M = 0.4$ ,  $w_{SL} = 0.4$  and  $q_d = 0.1$ . Also, SECA considers a radio link  $(i, j) \in \mathcal{E}$  as being strong if  $d_{i,j} < d_{ref}/2$ .

The nodes are deployed randomly following a uniform distribution. When there is node mobility, the coordinates of any node  $i$  are updated once per second, following a uniform rectilinear motion with a speed limited to  $v_{max}$ . Each node moves between waypoints, whose coordinates are updated once every  $MP = 20$  s. To better visualize the type of networks we consider in this section, Table 3.4 details the average node degrees vs.  $N$ . When compared to Table 3.3, Table 3.4 shows that the difference between average degrees in structured and unstructured networks is about 5. This gap is justified by the group model used for structured networks, which

$N$	100	200	300	400	600	800	1000
Degree	13.70	27.94	42.12	56.29	84.47	112.4	140.4

Table 3.4: Average node degree vs.  $N$  in unstructured networks.

places all members of the same group in a disk whose radius is twice the radio range  $d_{ref}$ , and thus increases the average node degree.

#### 3.5.3.1 Static networks

First let us study the case when nodes are static. For illustration purpose, Fig. 3.14 depicts the result achieved by the different clustering algorithms for a 600 node network. In this figure the cluster membership is indicated by the color and shape of the nodes.

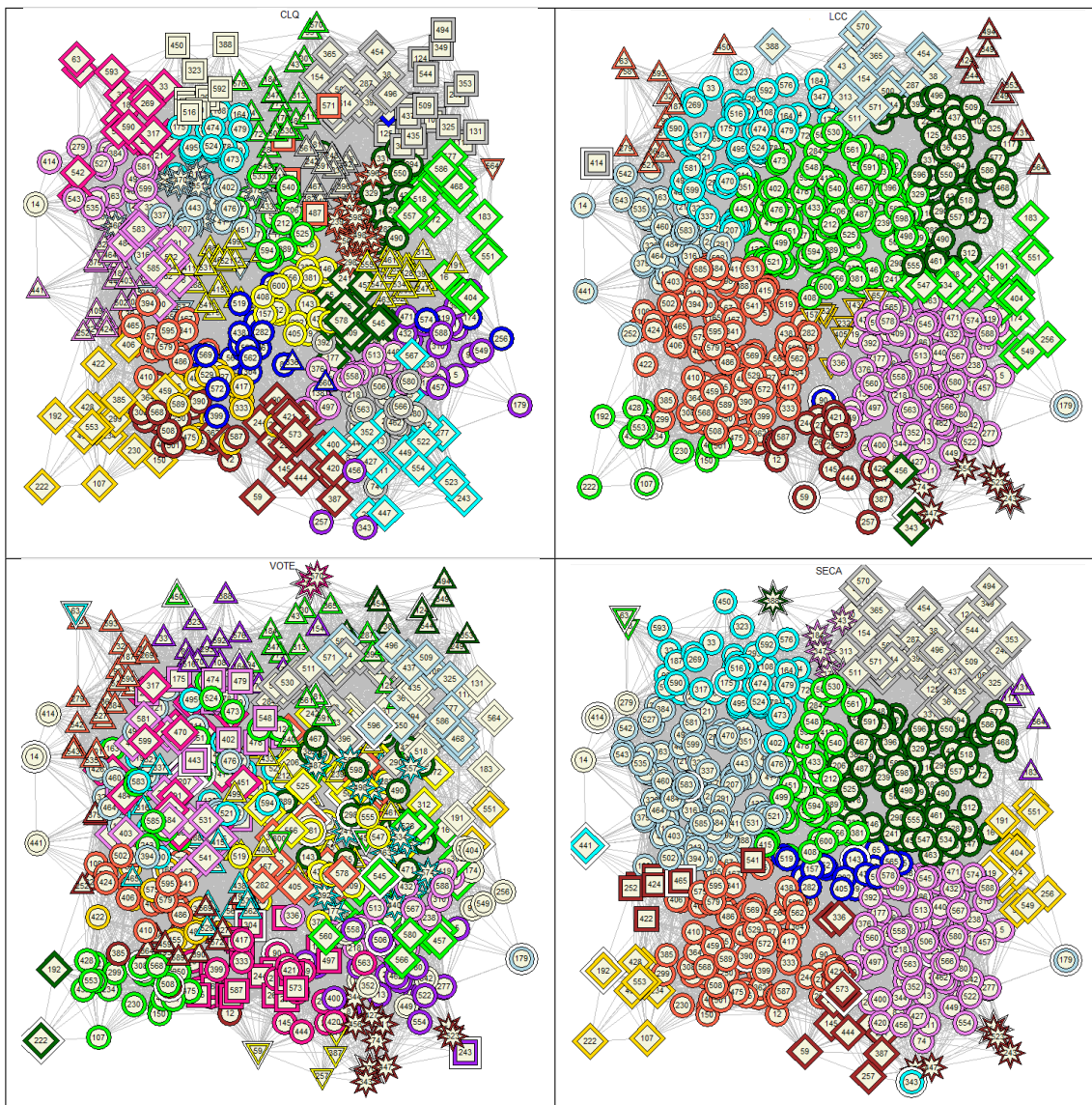


Figure 3.14: Example of 600 node network clustered with CLQ, LCC, VOTE and SECA (from left to right, top to bottom). Inner and outer colors indicate group and cluster membership respectively.

### 3.5.3.1.1 Duration required for CLQ convergence to a stable cluster structure

In Fig. 3.15.a, we plot the duration required to stabilize the cluster structure. The proposed coalition formation algorithm requires more time than the reference algorithms. This larger convergence duration is mainly due to the duration of the *Busy* state. Indeed, for LCC, VOTE, and SECA, this duration has been assumed to be zero since the clustering protocol can be easily done thanks to messages directly exchanged between the concerned cluster member and its already-chosen CH. With CLQ there is no CH, which requires the nodes to perform peer-to-peer message exchanges, leading to longer delays. This delays are modeled by the  $\delta_b$  long *Busy* state. Fig. 3.15.b plots the numbers of switch operations that are *i*) performed in line 13 of Table 3.1, or *ii*) canceled because in line 12 some nodes involved in  $\sigma_{k,\ell^*}(\mathcal{P}_{1,i}(a^*))$  are not available. For case (*i*), this number increases linearly with  $N$ , whereas for case (*ii*) it has a quadratic shape, as highlighted by the dotted black curve. This last result can be interpreted in the following

manner: when the node density increases, the probability of collision, i.e., that some nodes involved in the switch operation are not available, also increases. It is a well known property of the MAC based on random access which we have taken into account with our *Waiting-Busy* model.

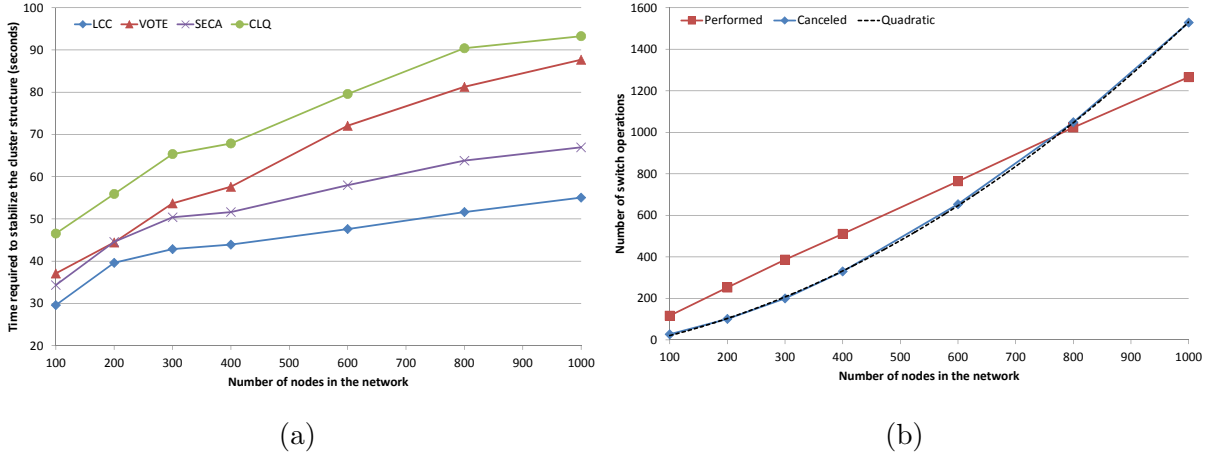


Figure 3.15: Static unstructured networks. (a) Time required to reach a stable cluster structure vs.  $N$ . (b) Number of performed and canceled switch operations vs.  $N$ .

In order to assess the influence of the parameter  $\delta_b$  on CLQ, we plot Fig. 3.16 which shows the time required by CLQ to reach stable clusters and the number of performed and canceled switch operations in  $N = 100$  networks with varying  $\delta_b$ . Fig. 3.16.a shows the that duration

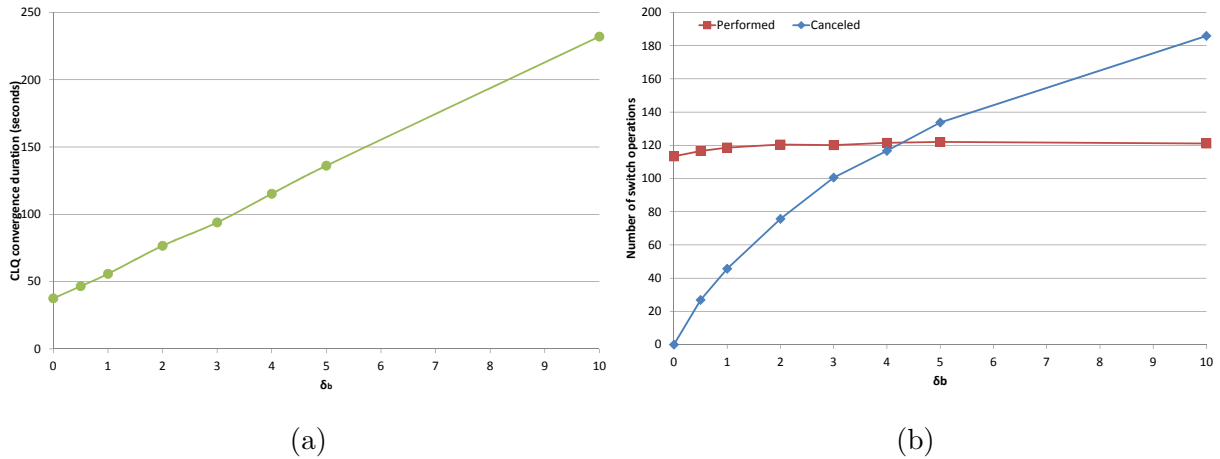


Figure 3.16: Static unstructured networks with  $N = 100$ . (a) Time required by CLQ to reach a stable cluster structure vs.  $\delta_b$ . (b) Number of performed and canceled switch operations vs.  $\delta_b$ .

needed by CLQ to reach stable clusters increases in a linear manner w.r.t.  $\delta_b$ . Fig. 3.16.b shows that the number of performed switch operations remains more or less constant, independently of  $\delta_b$ . The number of canceled switch operations increases abruptly, indicating that the number of collisions modeled by our model increases with  $\delta_b$ . When designing the MAC, if we succeed in decreasing  $\delta_b$  (by optimizing the protocol), we will be able to obtain almost the same convergence duration as the existing algorithms plotted in Fig. 3.15.a.

### 3.5.3.1.2 Cluster size

In Fig. 3.17.a, we plot the cluster size vs.  $N$ . It is a non-decreasing function of the number of nodes in the networks. More precisely, the cluster size is limited to  $n_{max} = 20$  for CLQ and VOTE, and increases linearly for LCC and SECA (that do not control the cluster size). The proportion of singleton clusters is shown in Fig. 3.17.b. The proposed CLQ builds the lowest number of singletons, which is of great interest since singleton clusters are inefficient for network performance. Table 3.5 details the proportion of  $n_{max}$  size cluster vs.  $N$  for the two algorithms VOTE and CLQ. CLQ builds the highest number of maximum size clusters, thus achieving  $\mathbf{G}_1$  better than VOTE.

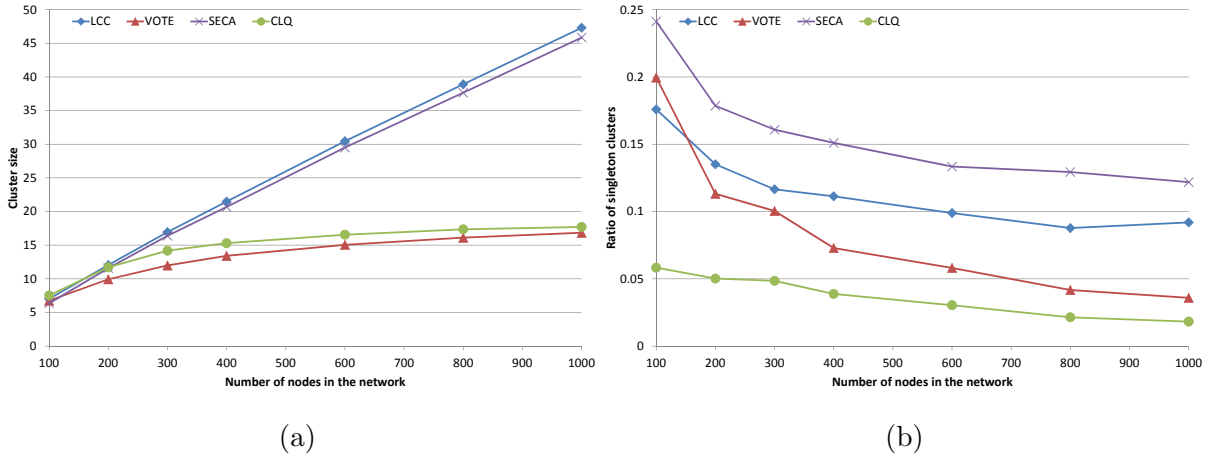


Figure 3.17: Static unstructured networks. (a) Cluster size vs.  $N$ . (b) Proportion of singleton clusters vs.  $N$ .

Nodes	100	200	300	400	600	800	1000
VOTE	8.71%	22.18%	33.93%	44.01%	56.65%	65.10%	70.87%
CLQ	4.15%	26.70%	46.00%	57.91%	71.68%	78.82%	81.87%

Table 3.5: Proportion of  $n_{max}$  size clusters vs.  $N$  in static unstructured networks.

In Fig. 3.18, we show the distribution of the cluster size with  $N = 300$ . In this figure, the notation '>' on the x axis means that the cluster size is strictly greater than 20. CLQ builds the smallest number of singletons and the largest number of clusters with  $n_{max}$  members. Moreover CLQ always satisfies the cluster size constraints ( $n_{max} = 20$ ) whereas LCC and SECA lead to very large clusters. For example when  $N = 1000$ , the average size of the clusters formed by LCC is 47, with a lot of 200 node clusters. These clusters are not manageable anymore and contradict the objective of clustering.

### 3.5.3.1.3 Intra-cluster link capacities

In Fig. 3.19, we plot the intra-cluster link capacity vs.  $N$ . We observe that CLQ always ensures the highest link capacity thus attaining  $\mathbf{G}_3$ .

While it does not take link capacities into account, LCC is the second best algorithm. The

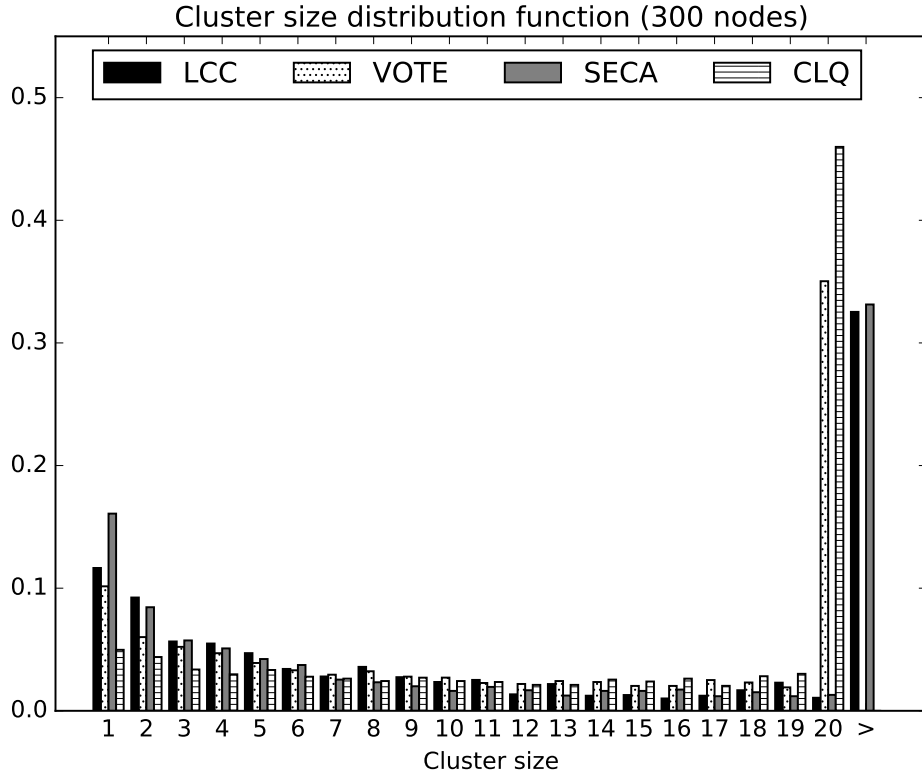


Figure 3.18:  $p(|\mathcal{C}_k|)$  vs.  $|\mathcal{C}_k|$  in static unstructured networks ( $N = 300$ ).

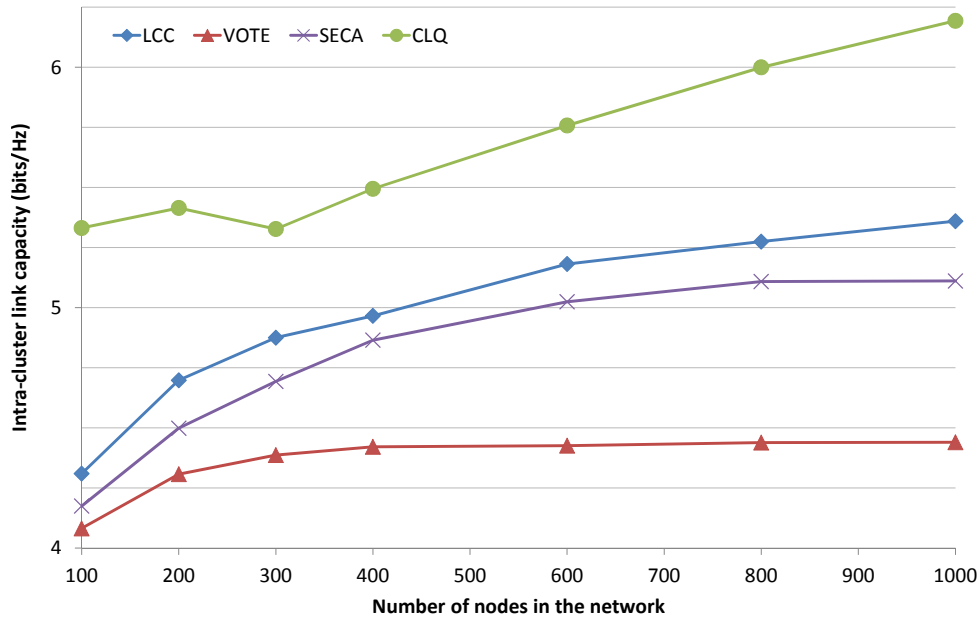


Figure 3.19: Intra-cluster link capacity vs.  $N$  in static unstructured networks.

rationale of this unexpected result resides in the details on the metric calculation. The intra-cluster link capacity for a given number of nodes is the average of 100 such metrics, one per random network. When calculating the metric for one network, it is first calculated per cluster and then it is averaged. During this last averaging, all clusters are considered equally, meaning that small and large clusters contribute at the same height, when the latter ones usually have

links with a much lower capacity than the former. When the size of the clusters is widely spread, such as for LCC, the metric thus overestimates the real intra-cluster link capacity. The only algorithm from the state of the art designed to take link quality into account, SECA, achieves a slightly worse performance than LCC and thus fails to achieve good intra-cluster link capacity. Finally, VOTE is concerned only by the cluster size, leading to clusters whose members are spread on a large area. Two implications follow from this: firstly the intra-cluster link capacities are low, and secondly, there is a lot of cluster overlap.

### 3.5.3.2 Mobile networks

Let us now consider mobile networks. To determine the duration of the warmup phase we followed the same method as for structured networks. Fig. 3.20 and Fig. 3.21 plot the node average degree vs. time for  $v_{max} \in \{0.25, 0.5, 0.75, 1, 1.5, 2, 3, 4\}$ . In Fig. 3.21.b all curves remain within a small interval (whose width depends on  $v_{max}$ ). Consequently  $\forall v_{max}$ , we set the duration of the warmup phase to 500 s, except for  $v_{max} = 0.25$  for which we selected 1000 s.

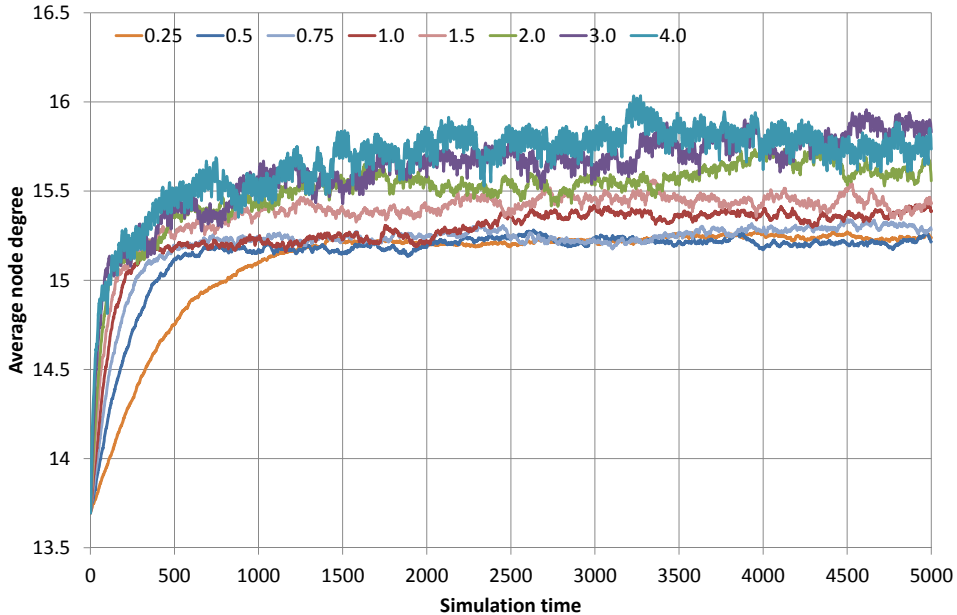


Figure 3.20: Average node degree vs. time for unstructured networks with  $N = 100$ , and for various  $v_{max}$  values.

We expected that small clusters would be more stable than large ones. Consequently we simulated CLQ and VOTE for two maximum cluster size:  $n_{max} \in \{10, 20\}$ . We denote by CLQ  $n$  and VOTE  $n$  when the cluster size constraint is  $n_{max} = n$ . The cluster life time vs. maximum node speed  $v_{max}$  is plotted in Fig. 3.22.

These curves first confirm that building smaller clusters improve their stability. The cluster life time associated with CLQ 10 is on average twice the one induced by CLQ 20 (this gain decreases with the node speed). VOTE also benefits from a smaller cluster size: when  $n_{max} = 10$ , then the cluster life time is on average 40% greater than if  $n_{max} = 20$ . Additionally, regardless of  $v_{max}$ , the most stable clusters are obtained thanks to CLQ. The algorithms LCC and SECA achieve similar performance, and VOTE 10 and VOTE 20 are the worst performers.

To better understand the hierarchy between the different CH-based algorithms, let us analyze



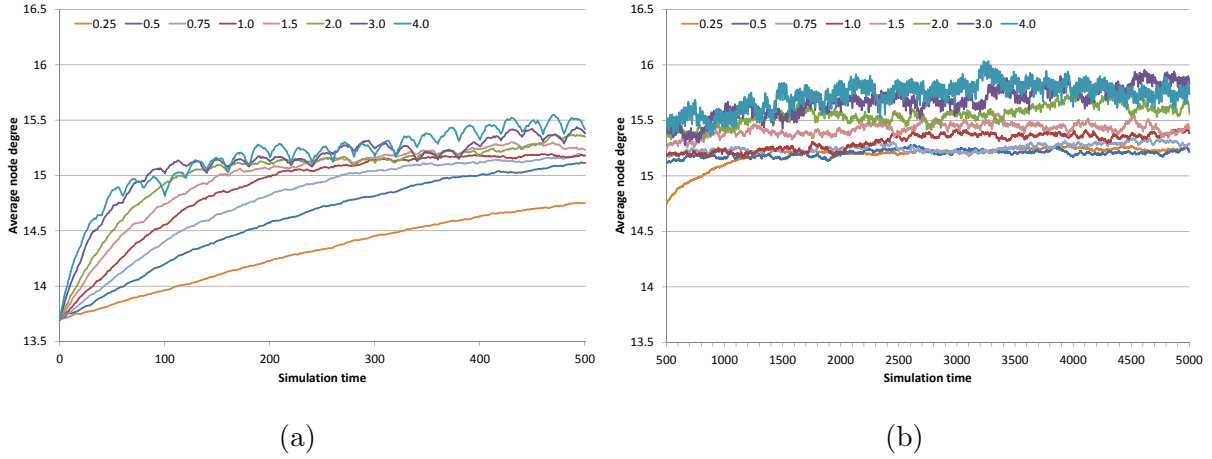


Figure 3.21: (a) Average degree during simulation transient state. (b) Average degree during simulation steady state.

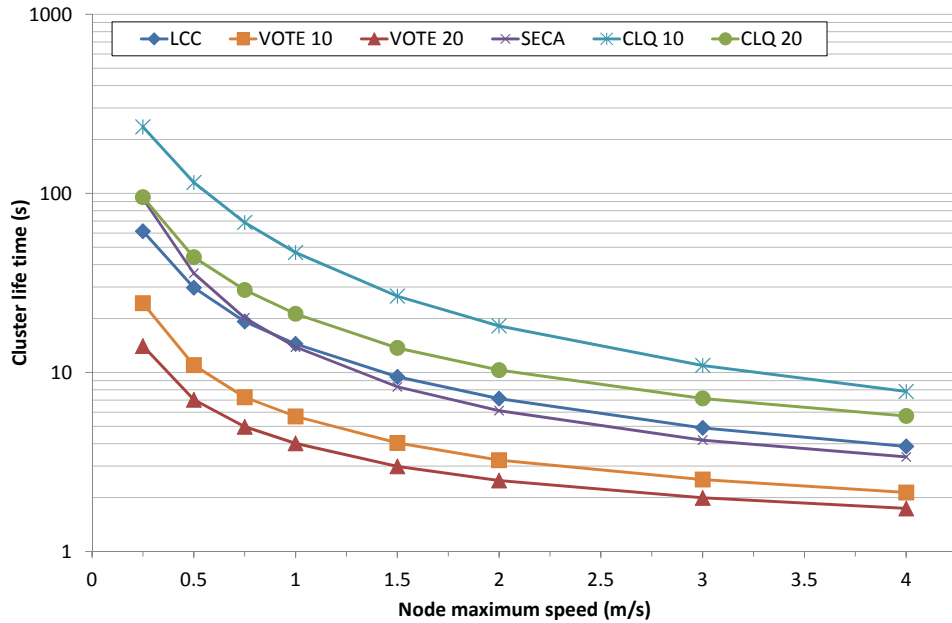


Figure 3.22: Cluster life time vs. maximum node speed in mobile structured networks.

the three major reasons that lead to a cluster modification: *i*) when a non-CH node becomes CH, *ii*) when a CH loses its CH state, and *iii*) when a non-CH node affiliates to a new CH. Fig. 3.23.a displays the number of times a node loses its CH state. This number is nearly the same (the difference is  $< 0.6\%$ ) as the number of 'Becoming CH' events (because on average when a node loses its CH role, then another node becomes CH). Fig. 3.23.b plots the number of times when a non-CH node affiliates to a new CH.

LCC leads to the lowest number of cluster modification events thanks to the two following properties: *i*) the CH nodes are selected using a criterion that depends on the topology only marginally (a node becomes CH if its identifier is the lowest in its neighborhood), and *ii*) a CH node remains CH unless another CH becomes neighbor, in which case one of the two neighboring CH loses this state. In contrast, VOTE builds its clusters using a criterion strongly dependent on the topology: a node becomes CH if its degree is the highest one in its neighborhood. Thus

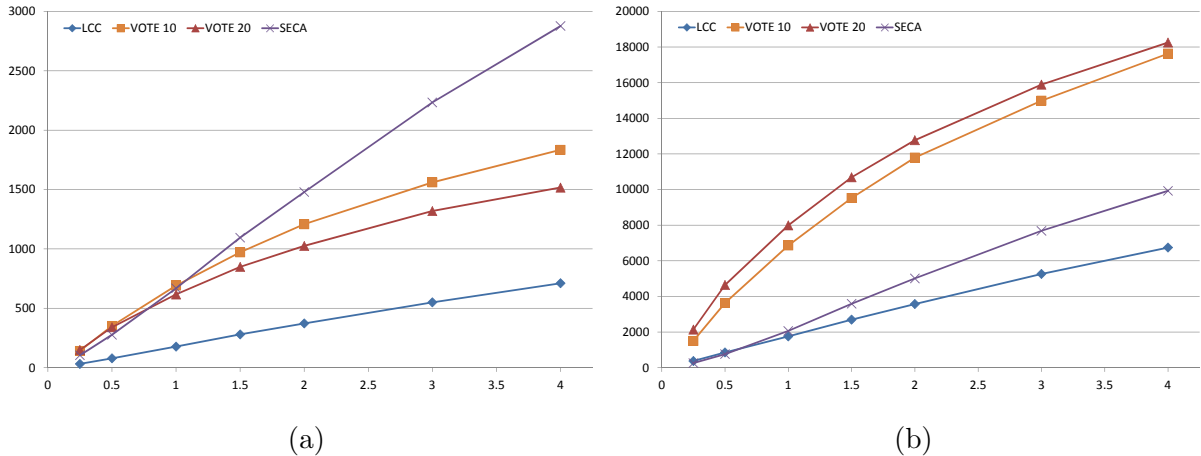


Figure 3.23: (a) Number of 'Losing CH' events vs.  $v_{max}$ . (b) Number of 'Changing CH' events vs.  $v_{max}$ .

a node easily gains or loses the CH state depending on the local changing topology, which also leads to a lot of non-CH nodes changing clusters. The last algorithm SECA, selects as CH the nodes with highest *quality value* defined as a combination of node degree, node mobility and link quality. As shown in Fig. 3.23.a, this criterion is even less stable than node degree only. However, SECA implements a mechanism to let a non-CH node become a CH only when its quality value exceeds, by a sufficient margin, the quality value of its current CH. This mechanism is a very effective stabilizing factor, as shown in Fig. 3.23.b, and justifies the better performance of SECA w.r.t. VOTE.

Coming back to CLQ, Fig. 3.24 plots the ratio of the mobility adaptations decided by procedures  $\mathbf{P}_2$  to the number of switch operations decided by procedures  $\mathbf{P}_1$ , showing that the former number is small when compared to the latter. This result is a consequence of the utility function (3.10). For example, on the one hand, when a node  $i \in \mathcal{C}_k$  is moving away from the other members  $\mathcal{C}_k$ , the quality of its links with its neighbors in  $\mathcal{C}_k$  is reduced. On the other hand, if this node is also approaching from another cluster  $\mathcal{C}_\ell$ , then the quality of its links with the  $\mathcal{C}_\ell$  members is improved. Consequently, it is likely that before node  $i$  leads to a breach in the constraints in  $\mathcal{C}_k$ , procedure  $\mathbf{P}_1$  finds a switch operations with strictly positive gain leading to the departure of node  $i$  from  $\mathcal{C}_k$  to join  $\mathcal{C}_\ell$ . This is a positive property considering that the heuristic for node selection used by procedure  $\mathbf{P}_1$  in unstructured networks leads to gentler cluster modifications than the one used by procedure  $\mathbf{P}_2$ .

Therefore, the proposed algorithm which has been designed for building compact clusters (i.e., clusters whose links have a high capacity, thus involving nodes close to each other) that are robust to mobility, achieves the best performance.

### 3.5.3.3 Conclusions on unstructured networks

We compared the proposed algorithm CLQ with the three clustering schemes LCC, VOTE and SECA from the literature.

In static configurations our simulations verified that when the topology is dense enough, CLQ builds clusters whose size is at the same time no greater and the closest to  $n_{max}$ , thus achieving  $\mathbf{G}_1$ . Another result concerns the number of singleton clusters, which is minimized by

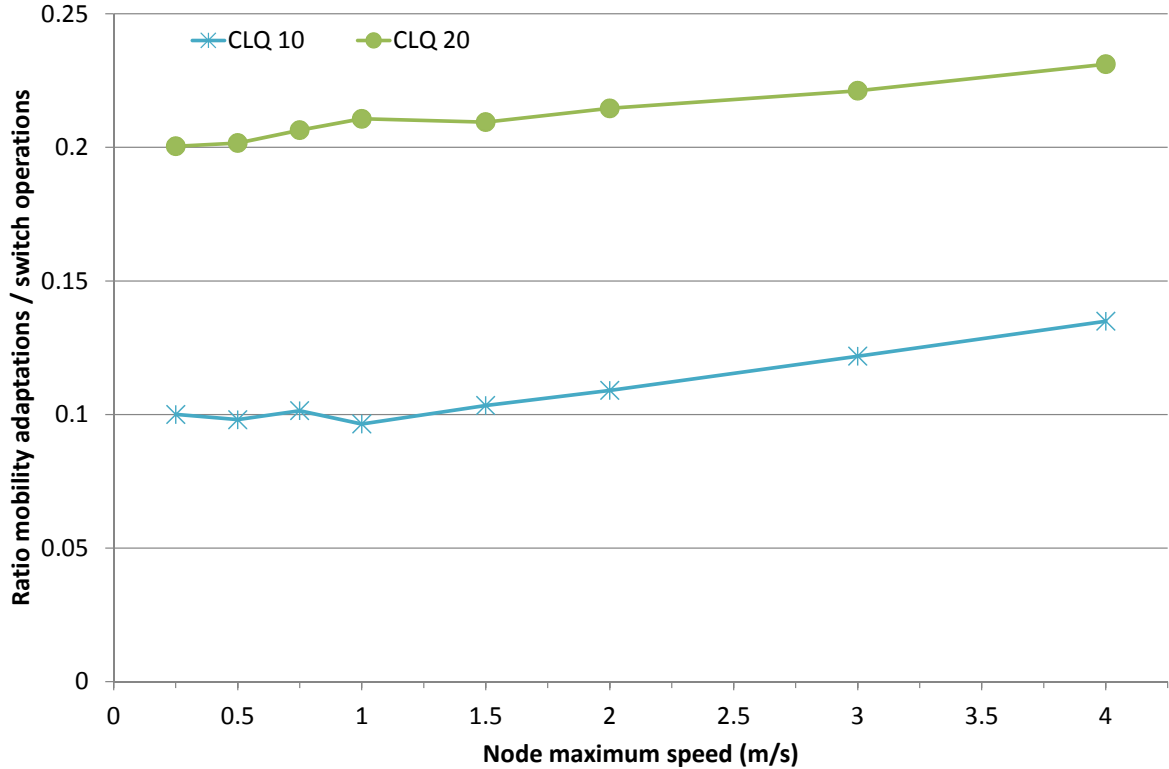


Figure 3.24: Ratio of mobility adaptations to cluster modifications vs. maximum node speed in mobile unstructured networks.

CLQ. Our algorithm also maximizes the intra-cluster link capacities, thus attaining  $\mathbf{G}_3$ .

In mobile networks, the good properties of CLQ leads to the most stable clusters. An additional interesting feature of this algorithm lies in its anticipation of the cluster constraints violations, which allows smooth transitions in cluster memberships.

### 3.5.4 Conclusions on simulations

Our simulations have shown that the algorithms we derived for structured and unstructured networks successfully achieved their respective goals: *i)*  $\mathbf{G}_1$  and  $\mathbf{G}_2$  for COG, and *ii)*  $\mathbf{G}_1$  and  $\mathbf{G}_3$  for CLQ. We also verified that in presence of mobility, the clusters formed by these two algorithms were stable.

Beyond those results, one could wonder about the real impact of structured vs. unstructured networks on the formed clusters. A first example lies in the size of the clusters. In unstructured networks, provided that the topology allows it, all clusters will always have a maximum  $n_{max}$  size. In structured network, the clusters will preferably not include partial groups, thus leading to clusters whose size is close to a multiple of the group size, and not necessarily close to  $n_{max}$ . A second example concerns the stability of the clusters. In structured network COG takes advantage of the group mobility, when CLQ does not. This is the reason why the cluster lifetime is much higher with COG than with CLQ.

## 3.6 Conclusions

In this chapter, DCOG was revisited within the coalitional game theory framework, which enabled us to introduce a generic clustering algorithm for mobile ad hoc networks. Using this fully distributed algorithm a node can decide to perform switch operations to leave its current cluster and join a neighboring one, alone or in the company of some other members from its cluster. This decision is made in order to strictly increase the social welfare of the network, thus guaranteeing convergence to a Nash-stable partition.

This algorithm is agnostic of the network structure and is configurable through the choice of a proper utility function and of two heuristics for node selection. We proved its Nash-stability and defined the main condition to be satisfied by any utility function in order to build clusters whose size is maximal.

We then adapted this algorithm to structured networks, suggesting a family of utility functions and several heuristics for node selection suitable for this type of network. In addition to building clusters whose size is maximal, this algorithm, called COG, also aims at gathering all members of the same group in a single cluster. We showed thanks to numerous simulations that COG outperforms a centralized heuristic building one cluster per group. We assessed the effect of various heuristics for node selection, and highlighted the trade off existing between complexity and performance.

We also applied our generic algorithm to unstructured networks, called CLQ, with the aim of building maximum size compact clusters. We verified that CLQ outperforms other clustering solutions from the literature in both static and mobile conditions.



# General Conclusions and Perspectives

## Conclusions

The work carried out in this thesis took place in the context of ad hoc networks, which are self-organizing networks used in various contexts such as public safety or military networks. The organizational strategy studied in this thesis is called node clustering, and consists in forming sets of nodes, denoted by clusters, in order to introduce hierarchy in the network, and thus improve its scalability. The main objective was to design and analyze clustering algorithms tailored to unstructured networks, for which the state of the art is plentiful, but still presents some defects, and to structured networks, which are very few covered by the literature. Due to the lack of metrics to assess the performance of clustering solutions at the application level, part of the thesis was devoted to the proposal and analysis of global cost functions, that we then used to compare our clustering algorithms with various schemes from the literature.

In Chapter 1, we defined several cost functions which can be used as a benchmark to compare different clustering solutions. These functions measure the quality of a network partition using end-to-end path calculations with additive metrics such as end-to-end delay. They take into account the fact that inter-cluster and intra-cluster communications have different costs, and are flexible enough to cover both cases when the traffic distribution depends on the groups or not. We used the new cost functions to show that the routing should take into account the cluster structure to find good network paths and offer the best QoS to the user traffic. Thanks to a rigorous numerical evaluation, we also verified that in structured networks, the cluster must be formed based on the group membership, and that advanced clustering solutions using group information were required. Finally, we extended our cost functions to the throughput concave metric.

In Chapter 2, we detailed our methodology to design the novel distributed clustering algorithm DCOG suited to structured ad hoc networks, whose key characteristics are its capability to build size limited clusters including full groups, without resorting to CH nodes. We also established the theoretical convergence of this new clustering algorithm. We performed numerous simulations in small and large networks, with low and high densities, and in static and mobile conditions. The comparisons with existing solutions showed that our solution outperforms the ones from the literature, especially with respect to the global cost functions defined in Chapter 1.

In Chapter 3, DCOG was revisited within the coalitional game theory framework, which enabled us to introduce a generic clustering algorithm for mobile ad hoc networks. This algorithm is agnostic of the network structure, and is configurable through the choice of a proper utility function and of two heuristics for node selection. We proved its Nash-stability, and defined the main condition to be satisfied by any utility function in order to build clusters whose size is maximal. We then adapted this algorithm to structured networks, suggesting a family of utility functions and several heuristics for node selection suitable for this type of network. In addition to building clusters whose size is maximal, this algorithm, called COG, aims at gathering all members of the same group in a single cluster. We showed thanks to numerous simulations that COG outperforms a centralized heuristic building one cluster per group, and highlighted the trade off existing between complexity and performance. We also defined CLQ, the adaptation of our generic algorithm to unstructured networks, whose goal is to build maximum size compact clusters. Finally, we verified its better performance with respect to other solutions from the literature.

## Perspectives

The part of this thesis devoted to the performance measurement of clustering solutions has raised up several issues that would deserve to be addressed in future research.

To begin with, the proposed network cost functions adapted to additive metrics neglect the costs associated to multi-hop forwarding. This simplification is only valid if the available bandwidth is large with respect to the one required to forward the traffic flows, which is true only for a limited amount of applications. Introducing a multi-hop forwarding model in those functions would improve their relevance and applicability.

Concerning the metric based on link capacity, we know that real systems cannot offer a continuous capacity but use a discrete set of modulation and coding schemes. Coping with this limitation of real communication systems could be interesting.

In Chapter 3, we defined separate algorithms suitable for structured and unstructured networks, each one pursuing its own goals. An algorithm suited to hybrid structured/unstructured networks could bring significant added value.

In this thesis, we chose to impose hard constraints on the clusters. We justified this by our decision to be agnostic of the MAC scheme. We think that introducing a more realistic MAC model would allow us to soften some of these constraints, e.g., the maximum cluster size constraint. Doing this would allow us to introduce a cluster cost function whose value would be increasing with the number of members in the cluster, and to study the trade-off between the cluster utility and the cluster size. We could also benefit from a more realistic MAC model in our simulations. This would allow modeling better the delays induced by multiple access in high density networks.

Finally, we tried to cast all the existing CH based clustering algorithms in the framework of our generic clustering algorithm. We did not succeed in this attempt but we gained some insights in the process and are convinced that the framework of coalitional game theory is large enough to achieve this goal.

# Appendices





# Appendix A

## Brute force graph partitioning

In this section we describe an algorithm to find all partitions of a graph  $\mathcal{G}$ . First the procedure  $\text{RIP}()$  is described which finds all the integer partitions of a number  $N$ , then procedure  $\text{FPWIP}()$  to find all the partitions of a graph knowing an integer partition of its number of vertices.

To find all the partitions of a graph  $\mathcal{G}$ , the procedure  $\text{RIP}()$  must be modified in order to invoke the procedure  $\text{FPWIP}()$  each time a new integer partition of  $|\mathcal{G}|$  is found.

### A.1 Integer partitioning

Knowing two integers  $N$  and  $K$ , the problem is to partition  $N$  into the sum of  $K$  strictly positive integers  $(n_k)_{k=0}^{K-1}$  such that:

1.  $\sum_{k=0}^{K-1} n_k = N$ , and
2. All permutations of  $(n_k)$  are considered equivalent.

Table A.1 describes a recursive algorithm that can solve the integer partition problem.

1	$\text{RIP}(i, (n_k))$
2	$n_{K-1} = n_{K-1} - 1$
3	if $n_{K-2} < n_{K-1}$ then:
4	repeat for $j = i$ and $j = i - 1, j > 0$
5	if $(n_{j-1} < n_j)$ and $(n_{j-1} < n_{K-1} - 1)$ then:
6	$(m_k) = (n_k)$
7	$m_{j-1} = n_{j-1} + 1$
8	$\text{RIP}(j, (m_k))$

Table A.1: Recursive Integer Partitioning (RIP) algorithm.

The input parameters of the recursive procedure  $\text{RIP}$  are: (1) a possible  $(n_k)$ , and (2) the index  $i$  of the  $n_k$  to be increased.

Let  $(n_0) = \{n_k | n_k = 1, \forall k \in \{0, 1, \dots, K - 2\}\}$  and  $n_{K-1} = N - K + 1$ . Initially the procedure is called with  $(n_0)$  as first possible  $(n_k)$  and  $K - 1$  as initial index. The algorithm builds a tree rooted in  $(n_0)$ . Each time it is called, the procedure checks if it is possible to find a new  $(n_k)$  solution by reducing by one the value of  $n_{K-1}$  and adding 1 either to  $n_{i-1}$  or  $n_{i-2}$ .

In the former case the same tree branch is continued, and in the latter case a new sub-tree (as well as a new branch, the trunk of the new sub-tree) with current solution as root is started.

Example: looking for the number of ways to partition  $N = 8$  into the sum of  $K = 4$  positive integer values.  $(n_0)$  is defined as  $[1\ 1\ 1\ 6]$ . As detailed in Table A.2, the solutions are:  $[1\ 1\ 1\ 5]$ ,  $[1\ 1\ 2\ 4]$ ,  $[1\ 1\ 3\ 3]$ ,  $[1\ 2\ 2\ 3]$  and  $[2\ 2\ 2\ 2]$ . Along a branch, the solutions are always updated through the addition of 1 to the corresponding index.

<p>RIP(3, [1 1 1 6])</p> <p>(2) <math>n_3 = 5 \Rightarrow [1\ 1\ 1\ 5] \Rightarrow</math> branch 0</p> <p>(4) <math>j = 3</math></p> <p>(5-8) Possible to +1 to index 2 of [1 1 1 5]  <math>\Rightarrow</math> RIP(3, [1 1 2 5])</p> <p>Recursion depth 1 (index = 3):</p> <p>(2) <math>n_3 = 4 \Rightarrow [1\ 1\ 2\ 4]</math></p> <p>(4) <math>j = 3</math></p> <p>(5-8) Possible to +1 to index 2 of [1 1 2 4]  <math>\Rightarrow</math> RIP(3, [1 1 3 4])</p> <p>Recursion depth 2 (index = 3):</p> <p>(2) <math>n_3 = 3 \Rightarrow [1\ 1\ 3\ 3]</math></p> <p>(4) <math>j = 3</math></p> <p>(5) Not possible to +1 index 2 of [1 1 3 3]</p> <p>(4) <math>j = 2</math></p> <p>(5) Not possible to +1 index 1 of [1 1 3 3]</p> <p>(4) <math>j = 2</math></p> <p>(5-8) Possible to +1 to index 1 of [1 1 2 4]  <math>\Rightarrow</math> RIP(2, [1 2 2 4])</p> <p>Recursion depth 2 (index = 2):</p> <p>(2) <math>n_3 = 3 \Rightarrow [1\ 2\ 2\ 3] \Rightarrow</math> branch 1</p> <p>(4) <math>j = 2</math></p> <p>(5) Not possible to +1 index 1 of [1 2 2 3]</p> <p>(4) <math>j = 1</math></p> <p>(5-8) Possible to +1 to index 0 of [1 2 2 3]  Recursion depth 3 (index = 2):</p> <p>(2) <math>n_3 = 2 \Rightarrow [2\ 2\ 2\ 2] \Rightarrow</math> branch 2</p> <p>(4) <math>j = 1</math></p> <p>(5) Not possible to +1 index 0 of [2 2 2 2]</p> <p>(4) <math>j = 2</math></p> <p>(5) Not possible to +1 index 1 of [1 1 1 5]</p>
---

Table A.2: Using RIP to partition 8 into the sum of 4 positive integer values.

## A.2 Partitions of a graph

Table A.3 details the recursive procedure FPWIP (Find Partition With Integer Partition) to find all the partitions of a graph  $\mathcal{G}$  knowing one integer partition of the number  $N$  of vertices in  $\mathcal{G}$ . FPWIP() builds the partitions making sure that all parts satisfy the connectivity, diameter and size constraints. It works on lists of vertices sorted by increasing identifiers order. The parameters of FPWIP() are:

- *partition*: the partition being built, initialized to empty.
- *part*: the partition part being built, initialized to empty.
- *pIndex*: the index of the part being built, initialized to 0.
- *numV*: the number of vertices missing in the current part, initialized to the current part target size.
- *freeVertices*: list of vertices not yet included in the current partition *partialPartition*, initialized to all vertices.
- *vIndex*: among the vertices *freeVertices*, some may have been dismissed by previous calls to FPWIP(), when the constraints were checked. The *vIndex* thus points to the first vertex in this list that has not yet been assessed by the procedure and that is currently the first candidate to be included in the current part being built.
- *IP*: integer partition of the number of vertices (constant).
- *LV*: list of vertices of graph  $\mathcal{G}$  to be partitioned (constant).

The `len()` function associates the number of vertices in a list of vertices of  $\mathcal{G}$ . The `diam()` function returns the diameter of a graph. The `graph()` function associates the sub-graph of  $\mathcal{G}$  induced by a set of vertices of  $\mathcal{G}$ . The `distance( $\mathcal{G}$ ,  $\mathcal{L}$ ,  $V$ )` function returns the maximum distance calculated within a graph  $\mathcal{G}$  between any vertex in a list of vertices  $\mathcal{L}$  and a single vertex  $V$ . The `updateFreeVertices( $L1$ ,  $L2$ ,  $min$ )` function builds a new list with the vertices of list  $L2$  that are not in list  $L1$  and whose identifier is strictly greater than a minimum.

Line (2) makes sure that the number of vertices missing in the partition part being built is not greater than the remaining candidate vertices. If not the current partition cannot be completed and the last vertex included must be replaced by another one (line (3)).

Line (4) checks if the current part is currently complete. In that case line (5) checks that the current part satisfies the connectivity and diameter constraints. If not the current partition cannot be completed and the last vertex included must be replaced by another one (line (6)). At line (7), the current part is added to the partition. After line (8) a new part is going to be built, thus the index of the part being built is increased by one. Line (9) checks if the partition is completed. If this is not the case then lines (10-13) handle the two cases when the new part to build and the one just built have the same size or not. Line (14) determines the size of the new part to build and line (15) recursively calls FPWIP().

If the test at line (4) determined that the current part is still partial then line (17) iteratively tries to extend it with the next candidate vertex (whose index is incremented at line (18)). If the distance between any vertex in the part being built and the candidate vertex is greater than the distance constraint, then ignore this candidate vertex. Otherwise, line (21) iteratively calls FPWIP() considering as new current part the current part extended with the candidate vertex.

```

1 FPWIP(partition, part, pIndex, numV, freeVertices, vIndex, IP, LV):
2   if [ numV > [ len(freeVertices) - vIndex ] ] then:
3       return
4   else if (numV = 0) then:
5       if [ [ graph(part) is not connected ] or [ diam(graph(part)) > dmax ] ] then:
6           return
7       partition = partition + part
8       pIndex = pIndex + 1
9       if [ pIndex < len(IP) ] then:
10          if (IP[pIndex-1] = IP[pIndex]):
11              freeVertices = updateFreeVertices(part, freeVertices, part[0])
12          else:
13              freeVertices = updateFreeVertices(partition, LV, 0)
14          numV = IP[pIndex]
15          FPWIP(partition, [], pIndex, numV, freeVertices, 0, IP, LV)
16   else:
17       for each vertex in freeVertices list starting at index vIndex:
18           vIndex = vIndex + 1
19           if [ distance(graph(LV), part, vertex) ≤ dmax ]:
20               extendedPart = part + vertex
21               FPWIP(partition, extendedPart, pIndex, numV - 1,
                    freeVertices, vIndex, IP, LV)

```

Table A.3: Algorithm to find partitions of a graph.

### Trace of the algorithm

Table A.4 displays a partial trace of FPWIP(), called using the following parameters: FPWIP(partition=[], part=[3], pIndex=0, numV=1, freeVertices=[1,2,3,4,5,6,7,8], vIndex=3), with IP = [2,2,3], LV = [1,2,3,4,5,6,7,8] and the graph the one of Fig. A.1. This trace details how the partial partition [3 4] [5 8] is found by FPWIP(), starting with vertex 3.

Note: in this example, FPWIP() determines that part [1 2 6 7] does not satisfy the maximum diameter constraint and that partial partition [3 4] [5 8] is not valid. The first valid partition found by this call of FPWIP() is [3 4] [6 7] [1 2 5 8].

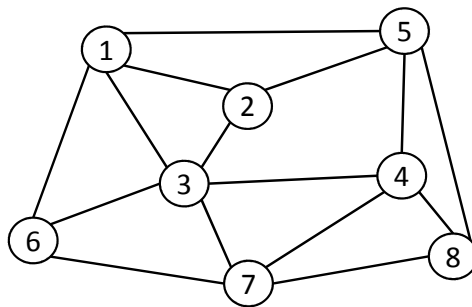


Figure A.1: Example of graph to be partitioned.

```

FPWIP(partition=[], part=[3], pIndex=0, numV=1,
      freeVertices=[1,2,3,4,5,6,7,8], vIndex=3)
(17-18) vertex = 4, vIndex = 4
(19) distance(LV, [3], 4) OK
(21) FPWIP(partition=[], part=[3,4], pIndex=0, numV=0,
      freeVertices=[1,2,3,4,5,6,7,8], vIndex=4)
(5) [3,4] connected and diam([3,4]) OK
(7) partition = [3,4]
(8) pIndex = 1
(10) IP[0] = IP[1] : yes
(11) freeVertices = [5,6,7,8]
(14) numV = 2
(15) FPWIP(partition=[3,4], part=[], pIndex=1, numV=2,
      freeVertices=[5,6,7,8], vIndex=0)
(17-18) vertex = 5, vIndex = 1
(19) distance(LV, [], 5) OK
(21) FPWIP(partition=[3,4], part=[5], pIndex=1,
      numV=1, freeVertices=[5,6,7,8], vIndex=1)
      freeVertices=[5,6,7,8], vIndex=0)
(17-18) vertex = 6, vIndex = 2
(19) distance(LV, [5], 6) OK
(21) FPWIP(partition=[3,4], part=[5,6], pIndex=1,
      numV=0, freeVertices=[5,6,7,8], vIndex=2)
(5) [5,6] disconnected
(17-18) vertex = 7, vIndex = 3
(19) distance(LV, [5], 7) OK
(21) FPWIP(partition=[3,4], part=[5,7], pIndex=1,
      numV=0, freeVertices=[5,6,7,8], vIndex=3)
(5) [5,7] disconnected
(17-18) vertex = 8, vIndex = 4
(19) distance(LV, [5], 8) OK
(21) FPWIP(partition=[3,4], part=[5,8], pIndex=1,
      numV=0, freeVertices=[5,6,7,8], vIndex=4)
(5) [5,8] connected and diam([5,8]) OK
(7) partition = [3,4,5,8]
(8) pIndex = 2
(10) IP[1] = IP[2] : NO
(13) freeVertices = [1,2,6,7]
(14) numV = 4
(15) FPWIP(partition=[3,4,5,8], part=[],
      pIndex=2, numV=4, freeVertices=[1,2,6,7]
      vIndex=0)
...

```

Table A.4: Partial trace of FPWIP() to partition the graph of Fig. A.1.



# Appendix B

## Proofs of Chapter 3

### B.1 Proof of Result 3.1

First, since the clusters of the initial partition fulfill the constraints,  $\mathbf{P}_1$  is operated at the first decision-making time of the algorithm. After  $\mathbf{P}_1$  execution, the resulting clusters satisfy the constraints by construction. Thus, by recurrence, the algorithm will always run  $\mathbf{P}_1$ . Let us now define the social welfare  $\Psi$  of a partition  $p = \{\mathcal{C}_1, \dots, \mathcal{C}_{N_c}\}$  as  $\Psi(p) := \sum_{k=1}^{N_c} v(\mathcal{C}_k)$ . Let us consider the  $n$ th decision-making occurrence of  $\mathbf{P}_1$ . For the sake of clarity we omit the index  $n$  for all functions and variables except for  $p_n$  and  $\Psi_n := \Psi(p_n)$ . Before the  $n$ th switch operation the cluster structure of the network is  $p_{n-1} = \{\mathcal{C}_1, \dots, \mathcal{C}_{N_c}\}$ . Let  $\sigma_{k,\ell}(\mathcal{P})$  be the  $n$ th selected switch operation. We get  $\Psi_n = \Psi_{n-1} + g(\sigma_{k,\ell}(\mathcal{P}))$ . Since we consider switch operations with only strictly positive gain, we have  $\Psi_n > \Psi_{n-1}$ . After any switch operation the social welfare strictly increases, meaning that the same partition can never be visited twice. Furthermore, since there is a finite number of partitions, the algorithm converges to a final partition  $p_f$  after a finite number of iterations.

### B.2 Proof of Result 3.3

Let us note  $x := n_k$ ,  $y := n_\ell$  and  $z := |\mathcal{P}|$ . Because  $n_\ell + |\mathcal{P}| > n_k$ , we have  $x < y + z$ . The function  $f_1$  is a strictly convex function of the cluster size, thus we apply the chordal slope lemma with  $x - z < x < y + z$ :

$$\frac{f_1(x) - f_1(x - z)}{x - (x - z)} < \frac{f_1(y + z) - f_1(x - z)}{y + z - (x - z)} < \frac{f_1(y + z) - f_1(x)}{y + z - x}.$$

Because  $x < y + z$ , we have  $x - z < y$  and we apply again the chordal slope lemma with  $x - z < y < y + z$ :

$$\frac{f_1(y) - f_1(x - z)}{y - (x - z)} < \frac{f_1(y + z) - f_1(x - z)}{y + z - (x - z)} < \frac{f_1(y + z) - f_1(y)}{y + z - y}.$$

Consequently, we have:

$$\begin{aligned} \frac{f_1(x) - f_1(x - z)}{z} < \frac{f_1(y + z) - u(y)}{z} &\Rightarrow f_1(x) - f_1(x - z) < f_1(y + z) - f_1(y), \\ &\Rightarrow u_1(\mathcal{C}_k) - u_1(\mathcal{C}_k \setminus \mathcal{P}) < u_1(\mathcal{C}_\ell \cup \mathcal{P}) - u_1(\mathcal{C}_\ell). \end{aligned} \quad (\text{B.1})$$

Because we assume that all the constraints are satisfied  $v = u_1$ . Then applying (3.2) to (B.1) we get:  $r_{\mathcal{P}}(\mathcal{C}_k) < r_{\mathcal{P}}(\mathcal{C}_\ell \cup \mathcal{P})$ , which from (3.1) concludes the proof.



### B.3 Proof of Result 3.4

If  $f_{2,t}$  is strictly convex then noting  $x := m_{t,k}$ ,  $y := m_{t,\ell}$  and  $z := |\mathcal{P}|$ , we can do as in proof of Result 3.3 to get:

$$f_{2,t}(m_{t,k}) - f_{2,t}(m_{t,k} - |\mathcal{P}|) < f_{2,t}(m_{t,\ell} + |\mathcal{P}|) - f_{2,t}(m_{t,\ell}). \quad (\text{B.2})$$

Let us define  $A$  and  $B$  such that:

$$A := \sum_{\substack{t' \in \mathcal{I}(\mathcal{C}_k) \\ t' \neq t}} f_{2,t'}(m_{t',k}), \text{ and } B := \sum_{\substack{t' \in \mathcal{I}(\mathcal{C}_\ell) \\ t' \neq t}} f_{2,t'}(m_{t',\ell}).$$

Eq. (B.2) can be modified as follows:

$$\begin{aligned} (\text{B.2}) &\Leftrightarrow \left[ A + f_{2,t}(m_{t,k}) \right] - \left[ A + f_{2,t}(m_{t,k} - |\mathcal{P}|) \right] < \left[ B + f_{2,t}(m_{t,\ell} + |\mathcal{P}|) \right] - \left[ B + f_{2,t}(m_{t,\ell}) \right] \\ &\Leftrightarrow u_2(\mathcal{C}_k) - u_2(\mathcal{C}_k \setminus \mathcal{P}) < u_2(\mathcal{C}_\ell \cup \mathcal{P}) - u_2(\mathcal{C}_\ell). \end{aligned} \quad (\text{B.3})$$

Because we assume that all the constraints are satisfied,  $v = u_2$ . Applying (3.2) to (B.3) we get:  $r_{\mathcal{P}}(\mathcal{C}_k) < r_{\mathcal{P}}(\mathcal{C}_\ell \cup \mathcal{P})$ , which from (3.1) concludes the proof.

### B.4 Proof of Result 3.5

Let us consider two clusters  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  such that  $\mathcal{C}_k \cup \mathcal{C}_\ell$  satisfies the constraints. Let us calculate the gain  $g(\sigma_{k,\ell}(\mathcal{C}_k))$  associated with the merge of  $\mathcal{C}_k$  with  $\mathcal{C}_\ell$ . We have:

$$g(\sigma_{k,\ell}(\mathcal{C}_k)) = \frac{2\epsilon \cdot n_\ell \cdot n_k}{n_{max}^2} + (1 - \epsilon) \cdot A(\mathcal{C}_k, \mathcal{C}_\ell),$$

with

$$A(\mathcal{C}_k, \mathcal{C}_\ell) := \sum_{t \in \mathcal{I}(\mathcal{C}_k \cup \mathcal{C}_\ell)} f_{2,t}(m_{t,k} + m_{t,\ell}) - \sum_{t \in \mathcal{I}(\mathcal{C}_\ell)} f_{2,t}(m_{t,\ell}) - \sum_{t \in \mathcal{I}(\mathcal{C}_k)} f_{2,t}(m_{t,k}).$$

Let us define  $\mathcal{J} := \mathcal{I}(\mathcal{C}_k) \cap \mathcal{I}(\mathcal{C}_\ell)$ . When  $\mathcal{J} = \emptyset$  then  $A(\mathcal{C}_k, \mathcal{C}_\ell) = 0$  and  $g(\sigma_{k,\ell}(\mathcal{C}_k)) > 0$ . When  $\mathcal{J} \neq \emptyset$  then we have:

$$\begin{aligned} A(\mathcal{C}_k, \mathcal{C}_\ell) &= \sum_{t \in \mathcal{J}} f_{2,t}(m_{t,k} + m_{t,\ell}) + \sum_{t \in \mathcal{I}(\mathcal{C}_k \setminus \mathcal{J})} f_{2,t}(m_{t,k}) + \sum_{t \in \mathcal{I}(\mathcal{C}_\ell \setminus \mathcal{J})} f_{2,t}(m_{t,\ell}), \\ &\quad - \left[ \sum_{t \in \mathcal{J}} f_{2,t}(m_{t,\ell}) + \sum_{t \in \mathcal{I}(\mathcal{C}_\ell \setminus \mathcal{J})} f_{2,t}(m_{t,\ell}) \right] - \left[ \sum_{t \in \mathcal{J}} f_{2,t}(m_{t,k}) + \sum_{t \in \mathcal{I}(\mathcal{C}_k \setminus \mathcal{J})} f_{2,t}(m_{t,k}) \right]. \end{aligned}$$

Thus  $A(\mathcal{C}_k, \mathcal{C}_\ell)$  can be written as a sum:  $A(\mathcal{C}_k, \mathcal{C}_\ell) = \sum_{t \in \mathcal{J}} F(t)$ , with  $F(t) := f_{2,t}(m_{t,k} + m_{t,\ell}) - f_{2,t}(m_{t,k}) - f_{2,t}(m_{t,\ell})$ . For all  $t \in \mathcal{J}$ :

$$F(t) = \frac{2 \cdot m_{t,\ell} \cdot m_{t,k}}{T \cdot m_t^2} > 0.$$

Since  $A(\mathcal{C}_k, \mathcal{C}_\ell)$  is a sum of terms that are strictly greater than zero, then  $A(\mathcal{C}_k, \mathcal{C}_\ell) > 0$ , and  $g(\sigma_{k,\ell}(\mathcal{C}_k)) > 0$ .

## B.5 Proof of Result 3.6

We study the sign of  $\Delta := g(\sigma_{q,k}(\mathcal{U}_q)) - g(\sigma_{q,\ell}(\mathcal{U}_q))$  as a function of  $\epsilon$ . From (3.1), we have

$$\Delta = r_{\mathcal{U}_q}(\mathcal{C}_k \cup \mathcal{U}_q) - r_{\mathcal{U}_q}(\mathcal{C}_\ell \cup \mathcal{U}_q).$$

Since the constraints are fulfilled,  $v = u$ , then using (3.2) we get:

$$r_{\mathcal{U}_q}(\mathcal{C}_x \cup \mathcal{U}_q) = u(\mathcal{C}_x \cup \mathcal{U}_q) - u(\mathcal{U}_q),$$

where  $x$  stands for  $k$  or  $\ell$ .

From utility (3.5), and (3.6)-(3.9), we obtain:

$$u(\mathcal{C}_x) = \frac{n_x^2}{n_{max}^2} \epsilon + \frac{(1-\epsilon)}{T \cdot m_t^2} (K + m_{t,x}^2),$$

where  $K$  is a constant that depends on the groups different from  $\mathcal{O}_t$ . Likewise, we get:

$$u(\mathcal{C}_x \cup \mathcal{U}_q) = \frac{(n_x + n_{\mathcal{U}})^2}{n_{max}^2} \epsilon + \frac{(1-\epsilon)}{T \cdot m_t^2} [K + (m_{t,x} + n_{\mathcal{U}})^2].$$

After simplification, we obtain:

$$r_{\mathcal{U}_q}(\mathcal{C}_x \cup \mathcal{U}_q) = \frac{n_{\mathcal{U}}(2n_x + n_{\mathcal{U}})}{n_{max}^2} \epsilon + \frac{(1-\epsilon)n_{\mathcal{U}}}{T \cdot m_t^2} (2m_{t,x} + n_{\mathcal{U}}),$$

which leads to:

$$\Delta = D_1 \cdot \epsilon + D_2(1 - \epsilon),$$

with

$$D_1 := \frac{2n_{\mathcal{U}}(n_k - n_\ell)}{n_{max}^2},$$

$$D_2 := \frac{2n_{\mathcal{U}}}{T \cdot m_t^2} (m_{t,k} - m_{t,\ell}).$$

The study of the sign of  $\Delta$  leads to the following three cases:

**Case 1:**  $m_{t,k} = m_{t,\ell}$  and  $n_k > n_\ell$ . Then  $D_2 = 0$  and  $D_1 > 0$ . Consequently  $\Delta > 0$  whatever the value of  $\epsilon$ .

**Case 2:**  $m_{t,k} = m_{t,\ell}$  and  $n_k = n_\ell$ . Then  $\Delta = 0$  whatever the value of  $\epsilon$ .

**Case 3:**  $m_{t,k} \neq m_{t,\ell}$ . We assume without loss of generality that  $m_{t,k} > m_{t,\ell}$  and thus  $D_2 > 0$ . If  $n_k \geq n_\ell$ ,  $D_1 > 0$  and  $\Delta > 0$  regardless of the value of  $\epsilon$ . If  $n_k < n_\ell$ , then  $D_1 < 0$ . We can write  $\Delta = (D_1 - D_2)\epsilon + D_2$  which is a linear function of  $\epsilon$  with negative slope. Thus  $\Delta > 0 \Leftrightarrow \epsilon < \epsilon_0 := \frac{D_2}{D_2 - D_1}$ . We now search the parameters  $n_k, n_\ell, m_t, m_{t,k}, m_{t,\ell}, n_{\mathcal{U}}$ , leading to the smallest value of  $\epsilon_0$  denoted by  $\epsilon^*$ . When  $D_2$  is fixed, we have to minimize  $D_1$ , which is obtained by setting  $n_k = 0$ ,  $n_\ell = n_{max}$ , leading to  $\epsilon_0 = \frac{D_2}{D_2 + 2n_{\mathcal{U}}/n_{max}}$ . Minimizing  $\epsilon_0$  is then equivalent to minimizing  $D_2$ , which is obtained by setting  $m_{t,k} = 1$ ,  $m_{t,\ell} = 0$  and  $m_t = n_{max}$ , leading to  $\epsilon^* = \frac{1}{1+T}$ .

## B.6 Proof of Result 3.7

Let clusters  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$ , such that  $\mathcal{C}_k \cup \mathcal{C}_\ell$  satisfies the constraints. Let us first calculate the utility of  $\mathcal{C}_k \cup \mathcal{C}_\ell$ . From (3.10), we have:

$$\begin{aligned}
 u_{\text{un}}(\mathcal{C}_k \cup \mathcal{C}_\ell) &= \sum_{i \in \mathcal{C}_k} \sum_{j \in \mathcal{C}_k \cup \mathcal{C}_\ell | (i,j) \in \mathcal{E}} \kappa(i,j) + \sum_{i \in \mathcal{C}_\ell} \sum_{j \in \mathcal{C}_k \cup \mathcal{C}_\ell | (i,j) \in \mathcal{E}} \kappa(i,j), \\
 &= \sum_{i \in \mathcal{C}_k} \left[ \sum_{j \in \mathcal{C}_k | (i,j) \in \mathcal{E}} \kappa(i,j) + \sum_{j \in \mathcal{C}_\ell | (i,j) \in \mathcal{E}} \kappa(i,j) \right] + \sum_{i \in \mathcal{C}_\ell} \left[ \sum_{j \in \mathcal{C}_k | (i,j) \in \mathcal{E}} \kappa(i,j) + \sum_{j \in \mathcal{C}_\ell | (i,j) \in \mathcal{E}} \kappa(i,j) \right], \\
 &= u_{\text{un}}(\mathcal{C}_k) + u_{\text{un}}(\mathcal{C}_\ell) + 2 \cdot \sum_{i \in \mathcal{C}_k} \sum_{j \in \mathcal{C}_\ell | (i,j) \in \mathcal{E}} \kappa(i,j).
 \end{aligned}$$

Using (3.1) and (3.2) the gain  $g(\sigma_{k,\ell}(\mathcal{C}_k))$  associated with the merge of  $\mathcal{C}_k$  with  $\mathcal{C}_\ell$  is thus written:

$$g(\sigma_{k,\ell}(\mathcal{C}_k)) = 2 \cdot \sum_{i \in \mathcal{C}_k} \sum_{j \in \mathcal{C}_\ell | (i,j) \in \mathcal{E}} \kappa(i,j).$$

Since  $\kappa(i,j) > 0$ , we deduce that  $g(\sigma_{k,\ell}(\mathcal{C}_k)) > 0$  which concludes the proof.

# Appendix C

## Résumé en français

### A propos des Réseaux Ad Hoc Mobiles

#### C.1 Introduction

##### C.1.1 Enoncé du problème

Le travail présenté dans cette thèse a été réalisé grâce à la collaboration entre le département "Communications et Électronique" (COMELEC) de Télécom ParisTech et le "Secteur Temps Réel" (STR) de Thales Communications and Security, dans le cadre d'une "thèse en situation de travail". Cette thèse est consacrée à la clusterisation des réseaux ad hoc, qui consiste en la construction d'ensembles de nœuds appelés clusters afin d'introduire une hiérarchie dans le réseau et ainsi d'améliorer son passage à l'échelle.

Un réseau ad hoc est un réseau multi-bonds sans fil et sans infrastructure où chaque nœud participe au routage en relayant les données des autres nœuds. Ces réseaux s'auto-organisent et sont utilisés lorsque les réseaux d'infrastructure habituels ne sont pas disponibles ou ne sont pas adaptés, par exemple dans les réseaux de capteurs sans fil (Wireless Sensor Network, WSN), les réseaux véhiculaires (VANET) ou les réseaux militaires. Le regroupement des nœuds du réseau en clusters a été proposé dans les années 1980 afin d'implémenter de grands réseaux ad hoc, et dans les années 1990 pour assurer une bonne qualité de service (QoS) aux usagers. Notamment, la clusterisation du réseau a été proposée dans la référence [3] comme une façon de permettre le déploiement rapide et la reconfiguration dynamique des réseaux sans fil, tout en fournissant un service suffisant aux applications multimédia combinant des trafics temps réel et des trafics sporadiques. La référence [7] détaille un système multicanal de communication pour les VANETs. Le réseau est clusterisé et des nœuds chefs de clusters (*cluster-head*, CH) sont élus et opèrent en tant que coordinateurs pour collecter et distribuer en temps réel des messages de sécurité, à l'intérieur de leur propre cluster, et relaient les messages de sécurité fusionnés aux CH des clusters voisins. Afin d'améliorer l'efficacité de l'allocation des ressources radio (RRA), le CH contrôle aussi l'assignation des canaux aux membres du cluster émettant et recevant le trafic non temps-réel. Les auteurs de [8] proposent un cadre basé sur les clusters pour former un réseau sans fil mesh dans le contexte du partage de spectre ouvert (*open spectrum sharing*), où certains nœuds sont des utilisateurs secondaires du spectre. Lorsqu'un nœud non encore membre d'aucun cluster trouve un canal laissé inutilisé par les utilisateurs principaux, il

devient CH et invite les nœuds voisins qui partagent ce canal à rejoindre le nouveau cluster. Le nœud CH est responsable du contrôle d'accès au canal pour les communications intra-cluster, et aussi des communications inter-clusters. Grâce à la négociation de nœuds passerelles entre les clusters, ces derniers sont interconnectés en grand réseau. Un réseau clusterisé est proposé dans la référence [9] en vue de la mise en œuvre d'un grand réseau cognitif, dans lequel le procédé de clusterisation et opéré conjointement à un canal de contrôle cognitif basé sur le codage de réseau, afin de permettre aux équipements radios cognitifs d'accéder de façon opportuniste au spectre laissé libre. Du point de vue du réseau, il a été démontré dans la référence [10], que le routage basé sur une clusterisation hiérarchique permet des économies exponentielles au niveau des données à stocker et à échanger dans les grands réseaux ad hoc, ce qui permet le passage à l'échelle du routage. Les réseaux cellulaires utilisent une approche similaire à la clusterisation pour permettre le déploiement des femtocells [11].

Habituellement, dans la littérature scientifique, les réseaux ad hoc sont non-structurés : il n'existe aucune organisation entre les nœuds du réseau, qui partagent tous le même rôle opérationnel, c.à.d. qu'ils sont tous égaux entre eux. Dans cette thèse, au-delà des réseaux non structurés, nous étudions aussi les réseaux structurés qui ont une structure hiérarchique intrinsèque associée à leur raison d'être, dans laquelle les nœuds sont rassemblés en *groupes opérationnels* (c.à.d., en escouades, en sections, etc.). Comme exemples de ce type de réseaux on peut prendre les réseaux de sécurité civile et les réseaux militaires. Pour simplifier la lecture, par la suite *groupe* se réfère à groupe opérationnel. L'existence des groupes révèle deux différences majeures avec les réseaux structurés. Tout d'abord, le trafic est fortement lié à l'organisation hiérarchique du réseau, étant principalement concentré au sein des groupes. Ensuite, les nœuds d'un même groupe se déplacent généralement dans la même direction. Pour ces deux raisons, nous montrerons qu'il est bénéfique de construire des solutions de clusterisation qui prennent en compte l'information de groupe, afin de créer à la fois des réseaux plus stables et aussi de fournir une meilleure QoS de bout en bout.

La plupart des travaux qui traitent de la clusterisation dans les réseaux ad hoc se sont intéressés aux réseaux non structurés. Par exemple dans la référence [3], les auteurs proposent les algorithmes de clusterisation LID et HC, pour lesquels les nœuds qui ont l'identifiant le plus petit, respectivement le degré le plus élevé, deviennent CH. Les nœuds non-CH s'affilient au nœud CH voisin de plus petit identifiant, respectivement de plus haut degré. La stabilité des clusters formés avec LID et HC, qui souffrent de *l'effet de vague*<sup>1</sup>, a été améliorée dans l'article [2] grâce au mécanisme LCC qui invoque le mécanisme de clusterisation uniquement lorsque plusieurs nœuds CH deviennent voisins. L'algorithme GDMAC détaillé dans la référence [1] peut être vu comme la généralisation de LCC, permettant jusqu'à  $K$  nœuds CH d'être voisins. Selon la proposition VOTE [6], les nœuds non-CH ne rejoignent un CH que si le nombre de membres de son cluster est inférieur à un seuil, ce qui permet de limiter la taille du cluster. A l'aide de sa connaissance de la position des nœuds, DGMA [12] cherche à estimer la mobilité relative des nœuds et à capturer la mobilité de groupe afin de former des clusters stables. Une approche similaire est suivie par LACA [13], qui propose de construire les clusters à l'aide des positions des nœuds passées, actuelles et prédites à l'aide d'un automate d'apprentissage, et par GMCA [15]

---

<sup>1</sup>L'expression effet de vague décrit un défaut de certaines solutions de clusterisation pour lesquelles une modification d'un cluster à un endroit bien précis du réseau, conduit à de nombreuses modifications de clusters dans le réseau tout entier.

qui utilise un modèle Gauss-Markov pour calculer la vitesse et la direction des nœuds. La nouveauté de SECA [4] réside dans l'introduction des qualités des liens (basées sur le niveau de signal reçu) combinées à la mobilité relative des nœuds, afin de déterminer si un nœud devient CH. Les auteurs de [14] utilisent un algorithme génétique centralisé et de l'optimisation par essais particuliers pour sélectionner des CH stables. Les exemples ci-dessus forment un extrait représentatif d'une riche littérature, qui montre que beaucoup de travaux de recherche ont déjà été faits, et sont encore faits, concernant la clusterisation dans les réseaux ad hoc non structurés.

Au contraire, très peu d'articles s'intéressent au problème de la clusterisation dans les réseaux structurés. Les auteurs de [5] introduisent l'algorithme basé sur le type (type-based clustering algorithm, TCA). Cette solution de clusterisation associe un facteur de stabilité à chaque nœud, et choisit comme CH les nœuds dont le facteur de stabilité est le plus élevé au sein d'un voisinage radio. Ce facteur de stabilité prend en compte l'appartenance aux groupes (identifiés à l'aide du sous-réseau IP de chaque nœud). TCA est limité par son mécanisme qui empêche plusieurs CH d'être voisins, dont la conséquence directe, dans les réseaux denses, est la formation de grands clusters (avec de trop nombreux membres). Un autre exemple est détaillé dans [16], où est proposé un mécanisme de gestion de topologie pour les réseaux hiérarchiques orientés groupes. Dans cet exemple les groupes sont basés sur les coordonnées géographiques des nœuds.

Afin de mettre en œuvre un réseau ad hoc, plusieurs mécanismes doivent opérer de manière continue et en parallèle. Initialement, tous les nœuds sont seuls dans leur propre cluster. Un des mécanismes consiste en la formation des clusters. Lors de son exécution, tous les nœuds détectent leur voisinage radio et échangent suffisamment d'information pour former les clusters. Un autre mécanisme consiste en l'allocation d'un ensemble de ressources à chaque cluster, par exemple une bande de fréquence ou un code CDMA. Pour réduire les interférences subies par un cluster, ses clusters voisins doivent choisir autant que possible des ressources radio qui sont orthogonales ou quasi-orthogonales à celles utilisées par ce cluster. Grâce à cette approche, à l'intérieur d'un cluster, un nœud spécifique appelé *allocateur de ressources* (RA) partage dynamiquement ces ressources entre les membres de ce cluster, sans s'occuper de ce qui arrive dans les clusters voisins. Ceci simplifie le mécanisme de RRA et permet de l'optimiser. Les deux derniers mécanismes sont d'une part la fonction de RRA à l'intérieur des clusters, et d'autre part l'allocation des ressources radio nécessaires aux communications entre les clusters. On peut envisager une optimisation conjointe de ces quatre mécanismes. Cependant, comme les protocoles et algorithmes qui construisent et opèrent un réseau ad hoc doivent être distribués, ce type d'optimisation ne nous paraît pas faisable. Par conséquent, nous avons décidé de traiter séparément ces différents problèmes, cette thèse étant dédiée à la construction des clusters. Noter que les quatre mécanismes susmentionnés sont principalement situés dans les couches basses de la pile protocolaire, c.à.d. les couches physique et liaison de données, et potentiellement la couche réseau selon la répartition choisie des responsabilités entre cette dernière et la couche liaison de données.

Détaillons maintenant une hypothèse clé qui sous-tend le travail présenté dans ce document. A l'intérieur d'un cluster, ses membres délèguent le RRA à leur nœud RA qui décide localement comment allouer les ressources à l'intérieur du cluster, de manière similaire à ce qui est fait par les stations de base dans un réseau cellulaire. Ceci peut être vu comme une opération

localement centralisée, qui peut être optimisée au bénéfice des membres du cluster. Néanmoins, l'efficacité du RRA dépend directement de la topologie du cluster, comme par exemple le nombre de ses membres, ou sa connectivité. Par conséquent, pour assurer un mécanisme de RRA efficace, nous cherchons des algorithmes de clusterisation qui forment des clusters satisfaisant certaines *contraintes* de topologie. Pour expliquer ces contraintes, nous modélisons le réseau comme un graphe où chaque nœud est un sommet, et dans lequel lorsqu'une arête existe entre deux sommets, cela signifie que les nœuds associés sont en portée radio. Nous définissons les contraintes de la façon suivante :

- Le graphe induit par les membres du cluster doit être connexe, afin d'assurer que toute communication intra-cluster entre deux membres d'un cluster puisse n'emprunter que des liens internes à ce cluster.
- La taille du cluster, c.à.d. son nombre de membres, doit être limitée, afin de contrôler le surcoût lié à la signalisation.
- Le diamètre du cluster, c.à.d. la longueur du plus long plus court chemin entre n'importe quelle paire de sommets du graphe induit du cluster, doit être limité pour éviter que le délai et les coûts de communication entre les membres du cluster n'augmentent trop.

A l'aide de l'état de l'art susnommé, nous avons identifié deux pistes de recherche qui nécessitent de plus amples investigations, et que nous adresserons dans cette thèse :

- **Métriques de performance réseau.** Habituellement, les solutions de clusterisation sont comparées à l'aide de métriques dédiées aux détails techniques des clusters eux-mêmes (par ex. le nombre de clusters, le nombre de modifications des clusters, la durée de vie des nœuds CH, la quantité de signalisation, etc.). Ces métriques ne donnent qu'une information indirecte concernant la QoS fournie à l'utilisateur. Pour combler ce manque, certains auteurs ont choisi des métriques dérivées du débit utilisateur. Dans ce cas des hypothèses sont faites à propos *i*) du type l'accès au canal (MAC), par ex. n'importe quelle version de WiFi, et concernant *ii*) le profil de trafic utilisateur. Comme détaillé auparavant, quatre mécanismes différents sont requis pour former et opérer un réseau clusterisé (la construction des clusters, l'allocation par cluster d'un ensemble de ressources radio, le RRA intra-cluster, et le RRA inter-clusters). Dès le début de la thèse nous avons souhaité rester le plus agnostique possible concernant le type de MAC utilisé au sein du système. Nous savions uniquement que les communications inter-clusters seraient implémentées de façon moins efficace que les communications intra-cluster. Par conséquent, nous avons décidé d'élaborer une nouvelle métrique versatile qui prendrait en compte les clusters, les groupes, les coûts de communication intra-cluster et inter-clusters, ainsi que le profil de trafic.
- **Algorithme distribué de clusterisation.** Outre le manque identifié pour les réseaux structurés, les algorithmes proposés pour les réseaux non structurés ne satisfont pas nos exigences. En effet, aucun d'entre eux ne gère simultanément la taille des clusters et la qualité des liens intra-cluster. De plus, la disponibilité de la position et de la vitesse des nœuds est souvent un prérequis de la plupart des propositions récentes, alors qu'elle ne peut pas toujours être assurée. Ainsi, nous avons identifié le besoin de nouveaux algorithmes distribués de clusterisation pour couvrir le cas des réseaux structurés, et concernant les réseaux non structurés, pour former des clusters qui satisfont aux contraintes de topologie.

De plus, il manque un cadre théorique à la littérature existante sur la clusterisation des réseaux ad hoc. Lors de notre recherche d'un tel cadre, nous avons découvert que la théorie des jeux de coalitions semblait prometteuse. En effet, c'est une branche de la théorie des jeux utilisée pour étudier le comportement des joueurs lorsqu'ils coopèrent entre eux [17]. Considérant les coalitions comme des clusters, nous avons donc décidé d'utiliser la théorie des jeux de coalitions pour nos recherches sur les algorithmes de clusterisation.

La théorie des jeux de coalitions est appliquée dans de nombreux contextes différents. Par exemple, dans la référence [18] un algorithme est proposé pour les réseaux cognitifs avec utilisateurs primaires et secondaires. Cet algorithme forme des coalitions d'utilisateurs secondaires afin de mettre en œuvre du sensing collaboratif, qui maximise l'utilité définie comme une probabilité de détection sujette à une probabilité de fausses alertes, considérée comme un coût. Les auteurs de [19] propose un scénario dans lequel les joueurs sont des stations de bases de petites cellules (*small cell base stations*, SBS) qui coopèrent afin de partager les ressources radio tout en gérant l'interférence downlink en provenance des SBS voisines et subie par les usagers au sein des petites cellules. Les SBS forment des coalitions recouvrantes et dédient une part de leurs ressources fréquentielles à chaque coalition auxquelles elles appartiennent, ceci afin de maximiser la somme des débits. Des propositions dédiées aux VANET ont aussi été publiées. Par exemple, dans l'article [20], des unités de bord de route (*roadside unit*, RSU) reçoivent un paiement basé sur la quantité et la classe des données envoyées aux véhicules qui se déplacent dans leur zone de responsabilité. En supposant que les véhicules ont la possibilité d'utiliser le réseau véhiculaire (V2V) sous-jacent pour échanger les données reçues des RSU, ces dernières peuvent former des coalitions pour diversifier les classes de données qu'elles transmettent aux véhicules. Cette coopération leur permet d'accroître leurs revenus. Dans la référence [21], les véhicules utilisent un algorithme de formation de coalitions pour former des clusters afin d'optimiser l'échange d'informations concernant les conditions de circulation ou de sûreté. A l'intérieur d'un cluster, le CH rassemble les informations de tous les autres membres du cluster, les fusionne, et renvoie le résultat. Les auteurs de [22] proposent un algorithme distribué afin de mettre en place des communications coopératives dans les réseaux ad hoc (avec plusieurs sources et une destination, de manière similaire aux WSN), afin d'augmenter le débit atteignable. Dans une coalition, les communications sont réalisés en deux phases : la phase de diffusion et la phase de coopération. Au cours de chaque slot de la phase de diffusion, au sein de chaque coalition un membre transmet en diffusion alors que tous les membres de la coalition écoutent. Ensuite, lors de la phase de coopération, les membres de chaque coalition émettent une combinaison linéaire de tous les signaux reçus pendant la phase précédente de diffusion. La destination effectue alors une détection multi-utilisateurs afin d'extraire le signal de chacun des nœuds. Les communications *device-to-device* (D2D) sont au cœur de l'article [23], qui propose un mécanisme distribué de gestion des ressources afin de traiter conjointement le problème de la sélection du mode de partage, et le partage du spectre lui-même. Ici, le mode de partage peut être *i*) le mode cellulaire, *ii*) le mode dédié D2D et *iii*) le mode hybride où les communications D2D réutilisent les ressources de certains liens cellulaires. Les usagers du réseau cellulaire forment des coalitions pour améliorer le débit cumulé atteignable par le système D2D. Ces exemples confirment que la théorie des jeux de coalitions est un bon cadre pour l'étude de la clusterisation des réseaux ad hoc.



### C.1.2 Résumé et contributions

Dans cette section, nous présentons un résumé de la thèse et mentionnons les résultats les plus importants.

Dans le Chapitre C.2 nous définissons des fonctions de coût de réseau pour évaluer au niveau système les solutions de clusterisation. Nous voulons que ces fonctions de coût tiennent compte des coûts de communications d'un bout à l'autre des chemins réseaux, c.à.d. de bout en bout. Pour cela, nous rappelons d'abord que la qualité d'un chemin réseau dépend des capacités du routage situé dans la couche réseau, et justifions pourquoi une fonction de coût de réseau utilisée pour évaluer les performances du système, doit prendre en compte ces capacités. Nous considérons alors le cas des métriques additives (par ex. le délai) et définissons une nouvelle fonction de coût  $J_0$  qui incorpore la structure du trafic et le coût des communications inter-clusters. Nous l'appliquons alors aux réseaux structurés, et fournissons une étude numérique détaillée montrant que la solution de clusterisation qui fournit la meilleure QoS à l'utilisateur final dépend de la structure en groupes. Ensuite, nous étendons  $J_0$  afin de mieux gérer la contrainte de taille de cluster. Enfin, nous définissons la fonction de coût de réseau  $T_0$ , dérivée de  $J_0$ , et adaptée à la métrique de débit.

Dans le Chapitre C.3, nous concevons un algorithme distribué adapté aux réseaux structurés. D'abord, nous identifions les paramètres clés qui devraient être utilisés pour former de bons clusters. Ensuite nous détaillons l'algorithme de clusterisation distribué basé sur les groupes opérationnels, appelé DCOG. Cet algorithme est exécuté au niveau des clusters et est conçu pour présenter les propriétés suivantes : chaque cluster contient le plus de membres possible des mêmes groupes opérationnels, la taille de chaque cluster est la plus proche possible d'un maximum donné, et la diamètre de chaque cluster est limité à une valeur maximale. Contrairement aux solutions de clusterisation conventionnelles, DCOG ne nécessite pas de recourir à la notion de CH. Après avoir prouvé la convergence de DCOG, nous comparons par simulation ses performances vis-à-vis de celles de cinq autres algorithmes de clusterisation. Les deux premiers algorithmes sont GDMAC et VOTE, tirés de la littérature, et les trois derniers en sont des extensions, que nous avons conçues afin de mieux prendre en compte la structure en groupes du réseau. Nos simulations montrent que les performances de DCOG, mesurées au niveau applicatif (à l'aide de  $J_0$ ), sont les meilleures, et que les clusters qu'il construit sont plus stables en présence de mobilité.

Dans le Chapitre C.4, nous revisitons DCOG à l'aide du cadre de la théorie des jeux de coalitions. Après avoir spécifié la manière dont nous allons utiliser les notions d'utilité, de valeur et de coût d'une coalition, issues de la théorie des jeux de coalitions, nous identifions formellement les coalitions en tant que clusters, et les joueurs en tant que nœuds, et définissons un algorithme générique de formation de coalitions utilisé pour la clusterisation des réseaux ad hoc. Cet algorithme est exécuté au niveau de chaque nœud, et est découpé en deux procédures. La première met en œuvre des *switch operations* ; c.à.d. le transfert de nœuds entre deux clusters, de manière à ce que le cumul des valeurs des clusters soit toujours accru, afin de garantir la convergence vers une partition Nash-stable du réseau. La deuxième opère lorsque, en raison de la mobilité, un cluster ne satisfait plus certaines des contraintes de topologie.

Dans ce cas, une switch operation peut être réalisée pour restaurer une topologie correcte. Les paramètres importants de cet algorithme sont *i*) les heuristiques utilisées pour sélectionner les nœuds impliqués dans les switch operations, et *ii*) les fonctions d'utilité et de coût utilisées pour calculer les gains induits par les switch operations. Nous dérivons alors deux versions de cet algorithme générique, une au cas des réseaux structurés, une autre à celui des réseaux non-structurés, conduisant respectivement aux algorithmes COG et CLQ. Dans chaque cas, nous définissons une fonction d'utilité ainsi que des heuristiques de sélection de nœuds spécifiques. Enfin, grâce à des simulations nous évaluons les performances de nos algorithmes. Nous montrons d'abord que COG construit des clusters stables qui contiennent la plupart des membres d'un ou plusieurs groupes. Ensuite, dans le cadre des réseaux non structurés, nous montrons que CLQ aboutit à des performances meilleures que LCC, SECA et VOTE, trois solutions de clusterisation de la littérature.

## C.2 Métriques pour l'évaluation de performance des réseaux clusterisés

### C.2.1 Concept de fonction de coût de réseau

#### C.2.1.1 Rappel sur le routage

Dans les réseaux, c'est la fonction de routage qui établit les routes multi-bonds empruntées par les flux de données. La qualité des routes trouvées par un algorithme de routage dépend de la quantité d'information que lui fournit le protocole de routage. Lorsque l'information d'existence des liens est la seule disponible, alors le coût d'une route s'exprime en nombres de liens. Par ailleurs, lorsqu'une valeur numérique est associée à chaque lien, quantifiant la qualité de ce lien, alors le coût d'une route est la somme des poids des liens qui la compose. Il est aisé de comprendre qu'au sein du même réseau, les routes trouvées dans les deux cas ne seront pas les mêmes. Par exemple, une route à deux bonds a un coût plus élevé qu'une route à trois bonds, si les liens utilisés pour cette dernière ont des coûts suffisamment faibles par rapport à ceux de la première. De plus, le coût de chaque lien s'exprime différemment selon la QoS visée (le délai, ou le débit, ou la perte, etc.). C'est pourquoi il faut choisir son protocole de routage en fonction des exigences du système.

#### C.2.1.2 Fonction de coût de réseau pour l'évaluation des algorithmes de clusterisation

Nous voulons définir des fonctions de coût de réseau pour l'évaluation des performances des algorithmes de clusterisation, d'un point de vue système, c.à.d. de la couche applicative à la couche physique, et en termes de QoS fournie aux usagers du réseau. Ces fonctions prennent en compte le routage de bout-en-bout dans le réseau, en raison du rôle crucial de ce dernier. Par la suite, chaque fonction de coût est identifiée par un exposant  $L$  ou  $X$  pour déterminer l'information utilisée lors du calcul des routes. L'absence de cet exposant indique que ce dont il est question est valable dans les deux cas. La première fonction que nous définissons est  $J_0$ , qui conduit à  $J_0^L$  et  $J_0^X$ . L'exposant  $L$  signifie que seule l'information de lien est utilisée, c.à.d. l'existence des liens ou leur poids (par ex. le délai requis pour transmettre avec succès un paquet de données à travers le lien considéré). L'exposant  $X$  est utilisé lorsqu'en plus de la donnée de

lien, celle concernant la structure en clusters du réseau est aussi utilisée. Pour comprendre l'intérêt d'utiliser cette information, il suffit de se souvenir que dans les réseaux de clusters, les communications intra-cluster et inter-clusters n'ont pas la même efficacité, ce qui a un impact direct sur les communications qui empruntent ces liens.

La Section C.2.2 introduit les notations utilisées dans ce chapitre. Ensuite la Section C.2.3 définit les fonctions de coût de réseau  $J_0^L$  et  $J_0^X$  associées aux métriques additives. La Section C.2.4 explique comment utiliser ces fonctions dans les réseaux structurés, et leur étude numérique est l'objet de la Section C.2.5. La Section C.2.6 étend  $J_0^X$  de deux façons. Finalement, la Section C.2.7 conclut le chapitre.

## C.2.2 Modèle de réseau

Soit un graphe  $\mathcal{G}$  défini par l'ensemble de ses sommets (ou nœuds)  $\mathcal{V}$  et l'ensemble de ses arcs (ou liens)  $\mathcal{E}$ . Le nombre de nœuds de  $\mathcal{G}$  est  $N := |\mathcal{V}|$ , où  $:=$  signifie *par définition*. Le poids d'un lien  $(i, j)$  est  $w_{i,j}$ . Deux nœuds  $i$  et  $j$  sont voisins si  $(i, j) \in \mathcal{E}$ . L'ensemble de toutes les partitions de  $\mathcal{G}$  est appelé  $\mathcal{P}$ . Dans ce travail les parties d'une partition sont identifiées aux clusters. L'ensemble des groupes est  $\mathcal{O} := \{\mathcal{O}_1, \dots, \mathcal{O}_T\}$ , avec  $T$  le nombre de groupes. Soit  $m_t$  la taille du groupe  $\mathcal{O}_t$ . Chaque nœud n'appartient qu'à un groupe, c.à.d.  $\forall t_1 \neq t_2, \mathcal{O}_{t_1} \cap \mathcal{O}_{t_2} = \emptyset$ . Une partition de  $\mathcal{G}$  contient  $N_c$  clusters notés  $\mathcal{C}_k$ , de taille  $n_k$ , avec  $k \in \{1, \dots, N_c\}$ . Chaque nœud n'appartient qu'à un cluster, c.à.d.  $\forall k \neq \ell, \mathcal{C}_k \cap \mathcal{C}_\ell = \emptyset$ . Le diamètre du graphe induit par un cluster  $\mathcal{C}_k$  est noté  $d_k$ .

Les fonctions de coût définies dans de ce chapitre se généralisent facilement aux réseaux non connexes. Pour cette raison, et sans perte de généralité, seuls des réseaux connexes seront désormais considérés.

## C.2.3 Cas des métriques additives

### C.2.3.1 Définitions

Pour évaluer la qualité d'une partition  $p$  de  $\mathcal{G}$ , nous définissons la fonction  $J_0$  qui représente le coût moyen des communications entre toutes les paires de nœuds, définie par :

$$J_0(p) := \frac{1}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot J_0(p, i, j), \quad (\text{C.1})$$

où le facteur  $1/N$  modélise l'égalité de probabilité qu'ont tous les nœuds  $i$  de transmettre,  $J_0(p, i, j)$  est le coût de transmission entre les nœuds  $i$  et  $j$ , et  $\pi_{j|i}$  est la probabilité que le nœud  $i$  choisisse  $j$  comme destination. Par convention  $\pi_{i|i} = 0$ , et  $\sum_{j \in \mathcal{V}} \pi_{j|i} = 1, \forall i \in \mathcal{V}$ . Comme décrit dans la Section C.1, les clusters doivent respecter les contraintes de connectivité, de taille, et de diamètre, qui restreignent l'ensemble des partitions acceptables  $\mathcal{P}$  à l'ensemble  $\mathcal{P}^c$  :

$$\mathcal{P}^c = \{p \in \mathcal{P} \text{ tel que } p \text{ satisfait } C_1, C_2, C_3\}, \quad (\text{C.2})$$

avec

$$\begin{aligned} C_1 : & \mathcal{C}_k \text{ est connecté } \forall k \in \{1, 2, \dots, N_c\}, \\ C_2 : & n_{min} \leq n_k \leq n_{max} \quad \forall k \in \{1, 2, \dots, N_c\}, \\ C_3 : & d_k \leq d_{max} \quad \forall k \in \{1, 2, \dots, N_c\}. \end{aligned}$$

Trouver les meilleures partitions d'un réseau revient à résoudre le problème ci-dessous :

$$\mathcal{P}^* = \arg \min_{p \in \mathcal{P}^c} J_0(p). \quad (\text{C.3})$$

### C.2.3.2 Cas de $J_0^L$

Soit  $\hat{\gamma}$  le coût d'une communication intra-cluster, et  $\tilde{\gamma}$  celui d'une communication inter-clusters, et  $\gamma := \tilde{\gamma}/\hat{\gamma}$ . Soit  $S_L(i, j)$  le chemin le plus court entre les nœuds  $i$  et  $j$ , de coût  $h_L(i, j)$  défini comme la somme des coûts des liens de  $S_L(i, j)$ . Ce chemin ne dépend pas de la partition  $p$  du réseau. Il se décompose en ses liens intra-cluster et inter-clusters, respectivement appelés  $\hat{S}_L(p, i, j)$  et  $\tilde{S}_L(p, i, j)$ , et de coûts respectifs  $\hat{h}_L(p, i, j) := \sum_{(i', j') \in \hat{S}_L(p, i, j)} w_{i', j'}$  et  $\tilde{h}_L(p, i, j) := \sum_{(i', j') \in \tilde{S}_L(p, i, j)} w_{i', j'}$ . A l'aide de ces notations, et grâce à (C.1),  $J_0^L$  est définie par :

$$J_0^L(p) = \frac{\hat{\gamma}}{N} \sum_{(i, j) \in \mathcal{V}^2} \pi_{j|i} \cdot (\hat{h}_L(p, i, j) + \gamma \cdot \tilde{h}_L(p, i, j)). \quad (\text{C.4})$$

Avec  $J_0^L$ , le problème (C.3) devient  $\mathcal{P}_L^* = \arg \min_{p \in \mathcal{P}^c} J_0^L(p)$ . Nous avons le résultat suivant :

**Résultat C.1**  $\mathcal{P}_L^*$  est indépendant de  $\gamma$ .

### C.2.3.3 Cas de $J_0^X$

Soit le graphe  $\mathcal{G}(p)$  défini par l'ensemble des nœuds  $\mathcal{V}$  et l'ensemble des arcs de  $\mathcal{E}$  dont les poids sont  $\hat{\gamma} \cdot w_{i, j}$  si  $(i, j)$  est un lien intra-cluster, et  $\tilde{\gamma} \cdot w_{i, j}$  s'il est un lien inter-clusters. Le chemin le plus court entre les nœuds  $i$  et  $j$  dans  $\mathcal{G}(p)$  est noté  $S_X(p, i, j)$ , de coût  $h_X(p, i, j)$ . Ce chemin se décompose en ses liens intra-cluster et inter-clusters, respectivement appelés  $\hat{S}_X(p, i, j)$  et  $\tilde{S}_X(p, i, j)$ , et de coûts respectifs  $\hat{h}_X(p, i, j) := \sum_{(i', j') \in \hat{S}_X(p, i, j)} \hat{\gamma} \cdot w_{i', j'}$  et  $\tilde{h}_X(p, i, j) := \sum_{(i', j') \in \tilde{S}_X(p, i, j)} \tilde{\gamma} \cdot w_{i', j'}$ . A l'aide de ces notations, et grâce à (C.1),  $J_0^X$  est définie par :

$$J_0^X(p) := \frac{1}{N} \sum_{(i, j) \in \mathcal{V}^2} \pi_{j|i} \cdot (\hat{h}_X(p, i, j) + \tilde{h}_X(p, i, j)). \quad (\text{C.5})$$

Avec  $J_0^X$ , le problème (C.3) devient  $\mathcal{P}_X^* = \arg \min_{p \in \mathcal{P}^c} J_0^X(p)$ . Nous avons le résultat suivant :

**Résultat C.2**  $\mathcal{P}_X^*$  est dépendant de  $\gamma$ .

### C.2.3.4 Exemple de $J_0$

Une valeur de  $J_0$  peut être interprétée comme un délai de bout en bout. Dans ce contexte, les liens  $w_{i, j}$  sont considérés comme le nombre moyen de transmission requises sur le nœud  $i$  pour garantir une bonne réception sur le nœud  $j$  (en utilisant un protocole d'ARQ), et  $\gamma$  peut être vu comme le coefficient multiplicatif du délai induit par la différence d'efficacité entre l'allocation de ressources radio intra-cluster et inter-clusters.

## C.2.4 Application de $J_0$ aux réseaux structurés

Dans les réseaux structurés, le trafic dépend directement des groupes. Pour le modéliser, considérons que la probabilité qu'un nœud d'un groupe communique avec n'importe quel autre membre de son groupe soit  $\alpha \in [0, 1]$ . Un nœud du groupe  $\mathcal{O}_t$  peut communiquer avec les  $m_t - 1$  autres membres de son groupe, avec une égale probabilité. Nous définissons donc  $\pi_{j|i}$ , la probabilité qu'un nœud  $i$  communique avec un autre nœud  $j$ , de la façon suivante :

$$\pi_{j|i} := \begin{cases} \frac{\alpha}{m_t - 1} & \text{si } j \in \mathcal{O}_t, \\ \frac{1 - \alpha}{N - m_t} & \text{autrement,} \end{cases} \quad (\text{C.6})$$

avec  $(i, j) \in \mathcal{V}^2$  et  $i \in \mathcal{O}_t$ .

Nous avons le résultat suivant :

**Résultat C.3** *L'ordre entre les valeurs de  $J_0^X$  associées aux partitions de  $\mathcal{P}^c$  dépend de  $\alpha$ .*

### C.2.4.1 Limitation de $J_0^L$ par rapport à $J_0^X$

On peut se demander ce qu'il en est d'une partition considérée comme bonne ou mauvaise par  $J_0^L$ , vis-à-vis de  $J_0^X$ . Dans un réseau, il existe souvent plusieurs chemins les plus courts entre une source et une destination. Dans ce cas, comme  $J_0^L$  ne prend pas en compte la structure en clusters, ces chemins peuvent inclure un nombre variable de liens inter-clusters. Par conséquent, la valeur de  $J_0^L$  dépend du chemin le plus court retenu lors du calcul. A l'inverse,  $J_0^X$  utilise la qualité intra/inter des liens et ne présente donc pas cet inconvénient. Pour évaluer l'erreur commise lors de l'utilisation de  $J_0^L$  à la place de  $J_0^X$ , nous avons utilisé l'algorithme d'Eppstein [43] de recherche des  $k$  plus courts chemins entre deux nœuds. La Figure C.1 trace les valeurs les plus petites, moyennes et les plus grandes de  $J_0^L$ , ainsi que celles de  $J_0^X$ , pour un exemple de partition d'un réseau de 30 nœuds,  $\gamma \in \{2, 5\}$ , et plusieurs valeurs de  $\alpha$ .

Cette figure montre bien l'existence d'une dispersion des valeurs de  $J_0^L$ , alors que ces valeurs sont issues de la même partition du même réseau. A contrario,  $J_0^X$  n'a qu'une valeur par couple  $(\alpha, \gamma)$ , qui est même inférieure à la valeur la plus petite de  $J_0^L$ . Ce résultat s'explique par le fait que  $J_0^X$  peut choisir des routes vues comme plus longues par  $J_0^L$  mais en réalité plus courtes une fois pris en compte le coût des communications inter-clusters. Dans la suite du document, nous utilisons donc  $J_0^X$  et non plus  $J_0^L$ .

## C.2.5 Résultats numériques

### C.2.5.1 Méthodologie

Afin de montrer que les groupes ont un impact fort sur la meilleure partition d'un réseau, nous définissons  $\mathcal{P}^u$  l'ensemble des partitions optimales pour un trafic uniforme entre tous les nœuds du réseau :

$$\mathcal{P}^u := \arg \min_{p \in \mathcal{P}^c} J_0^X(p) \Big|_{\pi_{j|i} = \frac{1}{N-1}, \forall (i,j) \in \mathcal{V}^2, i \neq j}.$$

Afin de vérifier si les partitions  $p \in \mathcal{P}^u$  sont encore bonnes lorsque le trafic devient dépendant des groupes et suit (C.6), nous calculons la métrique suivante :

$$\delta_u(\alpha) := \frac{J_0^u - J_0^*}{J_0^* - J_0^*}, \quad (\text{C.7})$$

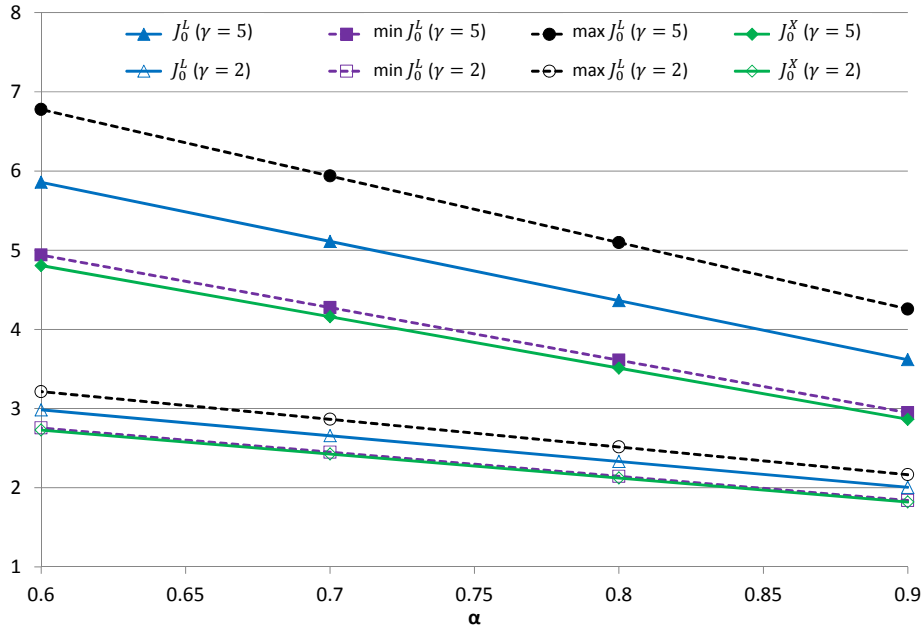


Figure C.1:  $J_0^X(p)$  et  $J_0^L(p)$  en fonction de  $\alpha$ .

avec  $J_0^u$  la valeur de  $J_0^X$  la plus forte  $\forall p \in \mathcal{P}^u$  lorsque  $J_0^X$  est calculée avec  $\pi_{j|i}$  définie par (C.6), avec  $\bar{J}_0^* := \max_{p \in \mathcal{P}^c} J_0^X(p)$ , et  $J_0^* := \min_{p \in \mathcal{P}^c} J_0^X(p)$ . Appelons *intervalle*  $J_0^X$  l'intervalle  $[J_0^*, \bar{J}_0^*]$ . Une valeur de 0 est associée à une des meilleures partitions, et de 1 à une des pires.

Pour montrer qu'il est nécessaire de concevoir de nouveaux algorithmes de clusterisation, et que ce n'est pas simple, nous proposons maintenant l'heuristique naïve 1G1C qui rassemble dans un seul cluster les membres d'un même groupe, sauf si cela conduit à violer la contrainte de diamètre, auquel cas elle crée le cluster qui satisfait cette contrainte avec le plus grand nombre de membres du groupe considéré, et rattache ceux laissés pour compte à un cluster existant. La partition obtenue grâce à 1G1C est notée  $p_g$ . Les performances de 1G1C sont étudiées avec la métrique suivante :

$$\delta_g(\alpha) := \frac{J_0^X(p_g) - J_0^*}{\bar{J}_0^* - J_0^*}.$$

### C.2.5.2 Paramétrage de la simulation

Le nombre de partitions d'un réseau devient très vite très élevé avec  $N$ . Pour réussir à énumérer les partitions d'un réseau, il faut limiter son nombre de nœuds. Il faut aussi imposer des contraintes de tailles minimales  $n_{min}$  et maximales  $n_{max}$  aux clusters, ainsi qu'une contrainte de diamètre  $d_{max}$ . La Figure C.2 donne deux exemples de réseaux de 30 nœuds, pour lesquelles toutes les partitions ont été calculées, avec  $n_{min} = 4$ ,  $n_{max} = 8$  and  $d_{max} = 2$ . La Figure C.2.a comporte 146 arcs et a requis un temps de calcul de 102 s afin d'obtenir ses 8584 partitions valides (en utilisant un logiciel optimisé écrit en C++, exécuté sur un processeur core i5 à 3.5 GHz). Quant à elle, la Figure C.2.b comporte 268 arcs et a requis plus de 9 jours (795113 s) de calcul pour trouver les presque un milliard de partitions (exactement 979970630). Par conséquent, pour limiter la durée de calcul, le nombre de liens des réseaux aléatoires de 30 nœuds a été limité à 230.

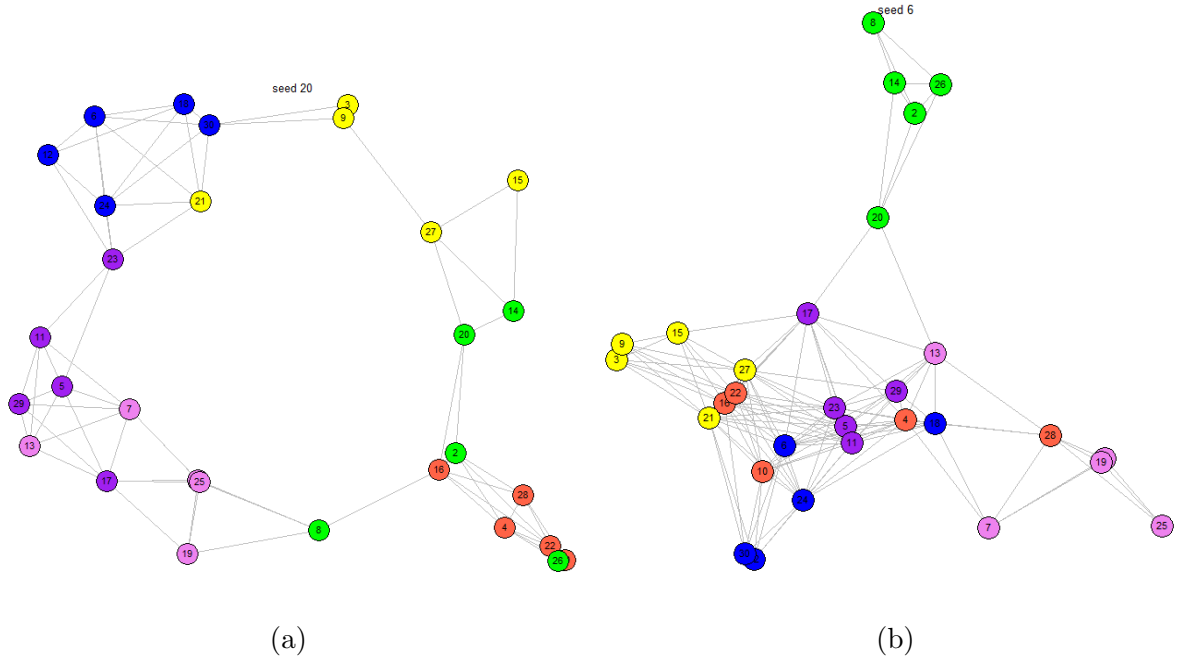


Figure C.2: Exemples de réseau de 30 nœuds. La couleur indique l'appartenance des nœuds aux groupes. (a) Réseau à 146 arêtes. (b) Réseau à 268 arêtes.

Les valeurs des contraintes sont  $n_{min} = 4$ ,  $n_{max} = 8$  and  $d_{max} = 2$ . Pour les calculs de  $J_0^X$ ,  $\gamma = 2$ . Les résultats ci-après sont des moyennes sur 100 réseaux.

### C.2.5.3 Résultats

La Figure C.3 trace la fonction de répartition de  $\delta_u(\alpha)$ . Le premier enseignement tiré de ces courbes est que les partitions  $p \in \mathcal{P}^u$  ne restent pas les partitions les meilleures lorsque le trafic dépend des groupes :  $\mathcal{P}^u \neq \mathcal{P}_X^*$ . Par exemple, lorsque  $\alpha = 0.2$ , seulement 40% des partitions  $p \in \mathcal{P}^u$  sont aussi dans  $\mathcal{P}_X^*$ . Ce nombre descend à zéro lorsque  $\alpha = 0.8$  ou 0.9. De plus, non seulement les partitions  $p \in \mathcal{P}^u$  ne sont plus les meilleures, mais deviennent de mauvaises partitions lorsque  $\alpha$  prend des valeurs assez élevées. Par exemple, lorsque  $\alpha = 0.7$ , seulement 50% de ces partitions ont une valeur de  $J_0^X$  dans les premiers 20% de l'intervalle  $J_0^X$ . La Figure C.3 montre que lorsque le trafic est concentré à l'intérieur des groupes, il devient très important d'en tenir compte au moment de la construction des clusters.

La Figure C.4 trace la fonction de répartition de  $\delta_g(\alpha)$ , et montre que la stratégie naïve 1GIC consistant à former un cluster par groupe n'est pas la meilleure. Parmi les 100 réseaux clusterisés avec cette heuristique, seuls 56 satisfont les contraintes sur les clusters. Les autres incluent des clusters avec seulement 1 ou 2 membres. Il y a donc bien le besoin d'une solution de clusterisation plus intelligente que l'approche naïve, et ce besoin grandit lorsque  $\alpha$  diminue. Par exemple, lorsque  $\alpha = 0.9$ , environ 80% des partitions identifiées comme les meilleures par 1GIC ont une valeur de  $J_0^X$  dans les premiers 10% de l'intervalle  $J_0^X$ . Ces 80% sont réduits à environ 55% lorsque  $\alpha = 0.7$  et à une valeur proche de zéro lorsque  $\alpha < 0.5$ .

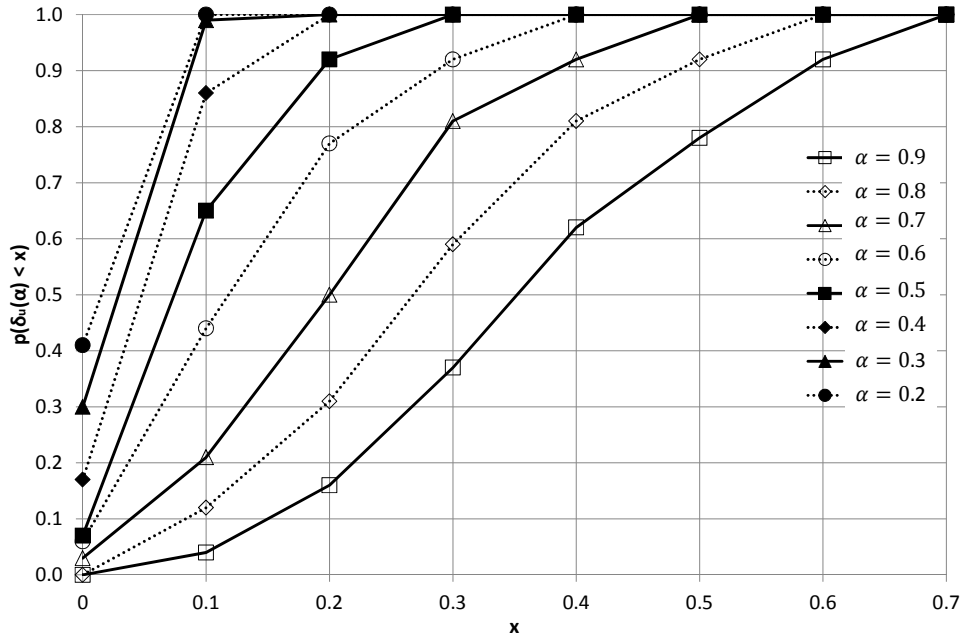


Figure C.3: Fonctions de répartition de  $\delta_u(\alpha)$ .

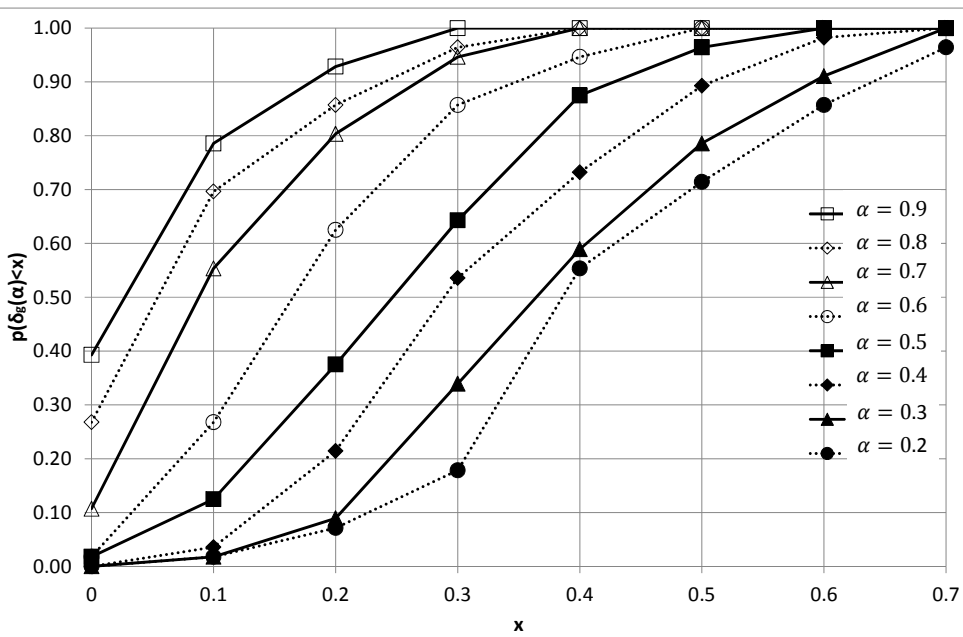


Figure C.4: Fonctions de répartition de  $\delta_g(\alpha)$ .

### C.2.6 Extensions de $J_0$

Les fonctions de coût de réseau  $J_0$  sont limitées de différentes façons. Dans cette section, la fonction  $J_0^X$  est modifiée de deux manières. Premièrement, afin de prendre en considération la taille des clusters, et deuxièmement pour l'adapter à une métrique concave : le débit. On ne s'intéresse pas la fonction  $J_0^L$ , car les modifications qui s'appliquent à  $J_0^X$  s'appliquent aussi directement à elle.



### C.2.6.1 Prise en compte de la taille des clusters

La meilleure valeur de  $J_0^X$  (c.à.d. la plus petite) est atteinte lorsque le réseau entier forme un seul cluster. Cette partition n'est généralement pas acceptable car elle ne satisfait pas une ou plusieurs contraintes.

Par conséquent nous définissons le graphe  $\bar{\mathcal{G}}(p)$  par l'ensemble de nœuds  $\mathcal{V}$  et l'ensemble de liens  $\mathcal{E}$  de poids  $\hat{\gamma} \cdot w_{i,j} \cdot \xi(i,j)$  si le lien  $(i,j)$  est intra-cluster, et de poids  $\tilde{\gamma} \cdot w_{i,j}$  s'il est intra-cluster. Le chemin le plus court entre les nœuds  $i$  et  $j$  dans  $\bar{\mathcal{G}}(p)$  est noté  $\bar{S}_X(p,i,j)$ , de coût  $\bar{h}_X(p,i,j)$ . Ce chemin se décompose en ses liens intra-cluster et inter-clusters, respectivement appelés  $\hat{S}_X(p,i,j)$  et  $\tilde{S}_X(p,i,j)$ , et de coûts respectifs  $\hat{h}_X(p,i,j) := \sum_{(i',j') \in \hat{S}_X(p,i,j)} \hat{\gamma} \cdot w_{i',j'} \cdot \xi(i',j')$  et  $\tilde{h}_X(p,i,j) := \sum_{(i',j') \in \tilde{S}_X(p,i,j)} \tilde{\gamma} \cdot w_{i',j'}$ . A l'aide de ces notations, et grâce à (C.1),  $\bar{J}_0^X$  est définie par :

$$\bar{J}_0^X(p) := \frac{1}{N} \sum_{(i,j) \in \mathcal{V}^2} \pi_{j|i} \cdot (\hat{h}_X(i,j) + \tilde{h}_X(i,j)). \quad (\text{C.8})$$

La fonction  $\xi(i,j)$  modélise la perte d'efficacité de l'allocation de ressources intra-cluster lorsque la taille du cluster augmente. Soit deux nœuds  $i$  et  $j$  dans le cluster  $\mathcal{C}_k$ . Un exemple de fonction  $\xi(i,j)$  peut être défini comme suit :

$$\xi(i,j) := \begin{cases} 1, & \text{si } n_k \leq n_{thr} \\ \frac{1}{\lambda} \left[ \frac{n_k}{n_{thr}} + \lambda - 1 \right] & \text{sinon,} \end{cases} \quad (\text{C.9})$$

avec  $n_{thr}$  la taille de cluster cible, et  $\lambda$  tel que  $\xi(i,j) = 2$  lorsque  $n_k = (\lambda + 1)n_{thr}$ . Selon cette exemple, la valeur de  $\xi(i,j)$  croit linéairement dès que la taille  $n_k$  du cluster  $\mathcal{C}_k$  dépasse  $n_{thr}$ .

### C.2.6.2 Cas des métriques concaves : le cas du débit

Dans cette section, nous définissons  $T_0^X$ , l'équivalent de  $J_0^X$  dans le cadre de la métrique concave de débit, plutôt que dans celui d'une métrique additive telle que, par exemple, le délai. Avec une métrique concave, le coût d'une route est le minimum des poids de chacun des liens qui la composent (et non plus la somme des poids de ces liens).

Soit le graphe  $\mathcal{G}'(p)$  défini par l'ensemble de nœuds  $\mathcal{V}$  et l'ensemble de liens  $\mathcal{E}$  de poids  $\hat{\eta} \cdot w_{i,j}$  si le lien  $(i,j)$  est intra-cluster, et de poids  $\tilde{\eta} \cdot w_{i,j}$  s'il est intra-cluster,  $\hat{\eta} > \tilde{\eta}$ , et  $w_{i,j}$  la capacité du lien  $(i,j)$ . Le chemin le plus large et le court entre les nœuds  $i$  et  $j$  dans  $\mathcal{G}'(p)$  est noté  $W_X(p,i,j)$ . Ce chemin se décompose en ses liens intra-cluster et inter-clusters, respectivement appelés  $\hat{W}_X(p,i,j)$  et  $\tilde{W}_X(p,i,j)$ . Soit  $\eta = \tilde{\eta}/\hat{\eta}$ . A l'aide de ces notations, et de manière similaire à (C.1), la fonction de coût  $T_0^X$  s'écrit :

$$T_0^X(p,i,j) = \hat{\eta} \left[ \min \left( \min_{(i',j') \in \hat{W}_X(p,i,j)} w_{i',j'}, \eta \cdot \min_{(i',j') \in \tilde{W}_X(p,i,j)} w_{i',j'} \right) \right]. \quad (\text{C.10})$$

## C.2.7 Conclusions

Ce chapitre introduit les fonctions de coût  $J_0^L$  et  $J_0^X$ , utilisables pour comparer des solutions de clusterisation. Ces fonctions mesurent la qualité d'une partition du réseau en calculant les chemins de bout en bout, sur la base de métriques additives. Elles prennent en compte la différence de coûts entre les communications intra-cluster et inter-clusters, et sont suffisamment flexibles pour gérer n'importe quel profil de trafic, y compris celui des réseaux structurés, où le

trafic dépend des groupes. Au cours de l'élaboration de ces fonctions, nous avons montré qu'afin d'offrir la meilleure QoS de bout en bout, le routage devait prendre en compte la structure en clusters du réseau.

Grâce  $J_0^X$ , nous avons aussi montré numériquement que construire les clusters sur la base des groupes aboutissait à de meilleures performances, et qu'il y avait besoin de nouvelles solutions de clusterisation basées sur cette information.

Enfin nous avons étendu  $J_0^X$  de deux manières : *i*) pour mieux prendre en compte la taille des clusters, et *ii*) pour s'adapter à la métrique de débit, et avons ainsi défini  $T_0^X$ .

Les métriques  $J_0^X$  et  $T_0^X$  sont utilisées dans les Chapitres C.3 et C.4 pour évaluer les performances de nouveaux algorithmes de clusterisation distribué.

## C.3 Algorithme de clusterisation distribué basé sur les groupes opérationnels

### C.3.1 Introduction

Ce chapitre justifie et définit l'algorithme DCOG de clusterisation distribué pour les réseaux ad hoc mobiles. Contrairement à la plupart des solutions de l'état de l'art sur le sujet, cet algorithme ne fait pas appel à la notion de chef de cluster (CH). Ce choix nous a permis d'éviter les problèmes liés à ce type d'approche, notamment celui de la fixation du poids des nœuds, qui peut être une source d'instabilité des clusters.

La Section C.3.2 détaille les extensions nécessaires aux notations du Chapitre C.2. La Section C.3.3 donne un aperçu de la méthode suivie au cours de la thèse pour aboutir à l'algorithme DCOG, qui est détaillé dans la Section C.3.4. Les résultats numériques sont fournis dans la Section C.3.5, et la Section C.3.6 conclut le chapitre.

### C.3.2 Modèle de réseau

Dans ce chapitre, le modèle de réseau de la Section C.2.2 est étendu à l'aide des notations suivantes. L'index du groupe avec les plus de membres dans le cluster  $\mathcal{C}_k$  est  $o_k^*$ . Le nombre de membres du groupe  $\mathcal{O}_t$  dans le cluster  $\mathcal{C}_k$  est  $m_{t,k}$ . Soit  $\mathcal{I}(\mathcal{C}_k) := \{t | m_{t,k} \neq 0, t \in \{1, \dots, T\}\}$  l'ensemble des indices des groupes avec au moins un membre dans le cluster  $\mathcal{C}_k$ .

### C.3.3 Démarche de conception d'un algorithme de clusterisation distribué

#### C.3.3.1 Vers une solution de clusterisation distribué

La fonction  $J_0$  ( $J_0^X$  définie dans le Chapitre C.2) requiert une connaissance globale du réseau et ne peut pas être utilisée lors d'un processus distribué de création de clusters. Dans l'objectif d'identifier une fonction de coût utilisables lors d'un tel processus distribué, nous en avons donc défini plusieurs, qui ne requièrent que des informations disponibles localement au sein de chaque cluster. Nous avons construit ces fonctions en tenant compte du fait que *i*) les nœuds d'un même groupe devraient appartenir au même cluster, et *ii*) le nombre de nœuds d'un cluster devrait être aussi proche que possible d'un maximum.

Soit  $J_i$  la fonction de coût de réseau définie par :

$$J_i := \sum_{k=1}^{N_c} c_i(\mathcal{C}_k),$$

avec  $c_i(\mathcal{C}_k)$  le coût du cluster  $\mathcal{C}_k$  associé à  $J_i$ . Nous avons étudié plusieurs  $J_i$  définies par leur fonction  $c_i$ , et calculé leurs valeurs pour toutes les partitions de  $\mathcal{G}$  (à l'aide d'une énumération exhaustive, comme dans le Section C.2.5). Nous avons évalué la capacité de chacune de ces fonctions à être un bon substitut pour  $J_0$ . La conclusion à laquelle nous avons abouti est que la fonction  $c_5(\mathcal{C}_k) := (n_{max} - n_k + 1) \cdot (m_{o_k^*} - m_{o_k^*,k} + 1)$  était la fonction à retenir comme la plus capable d'identifier comme bonnes partitions celles considérées comme bonnes par  $J_0$ , et comme mauvaises partitions celles considérées comme mauvaises par  $J_0$ .

A partir de la fonction  $c_5$ , nous avons suivi une démarche itérative pour la conception de notre algorithme distribué de construction de clusters. Pour décider si l'échange de nœuds entre deux clusters était bénéfique, nous avons défini la notion de gain lié à un tel échange. Nous avons testé plusieurs façons de le calculer, pour aboutir à un gain s'exprimant comme la différence entre le gain lié à l'arrivée des nœuds dans le cluster de destination, et la perte liée au départ de ces nœuds du cluster d'origine. Ces quantités sont calculées en fonction des coûts des clusters, dont plusieurs expressions dérivées de  $c_5$  ont été testées avant d'aboutir à la version finale. A chaque étape, notre réflexion a consisté à corriger les problèmes rencontrés au cours de l'étape précédente. Par exemple, au début de la réflexion, le réseau demeurait dans son état initial, composé d'autant de clusters singletons que de nœuds dans le réseau. Un autre exemple de modification a été de permettre à un cluster d'inclure plusieurs groupes complets.

Après avoir esquissé la façon dont nous avons conçu notre algorithme, la section suivante s'attache à le décrire en détails.

### C.3.4 Algorithme de clusterisation distribué basé sur les groupes opérationnels

L'algorithme DCOG (Distributed Clustering based on Operational Groups) est un algorithme de clusterisation distribué qui est continuellement exécuté par tous les nœuds du réseau. L'objectif de cet algorithme est de former des clusters qui ont les deux propriétés suivantes : chaque cluster devrait *i*) contenir tous les nœuds des mêmes groupes opérationnels (si possible), et *ii*) avoir la taille la plus grande possible. De plus les clusters doivent satisfaire les contraintes suivantes : il doit être connexe, sa taille doit être inférieure à un maximum  $n_{max}$  et son diamètre doit être au plus de  $d_{max}$ .

A l'inverse de la plupart des algorithmes de la littérature, DCOG n'a pas besoin d'utiliser la notion de nœud CH. A la place, DCOG met en œuvre un processus de négociation entre les clusters, chaque cluster évaluant régulièrement l'opportunité de laisser certains de ses membres le quitter pour rejoindre un cluster voisin. Lorsqu'une telle opportunité apparaît, l'échange, appelé *swap* est opéré.

Pour déterminer si un swap est possible entre deux clusters, nous introduisons maintenant deux fonctions : la fonction de coût  $c$  qui donne une valeur à un cluster selon sa composition, et la fonction de gain  $g$  qui dépend des clusters impliqués dans un swap. Pour décider qu'un swap est possible, les deux conditions suivantes doivent être remplies : *i*)  $g > 0$ , et *ii*) les contraintes sont satisfaites.

### C.3.4.1 Fonctions de DCOG

La fonction de coût associée à DCOG est une application de l'ensemble de tous les clusters  $\mathcal{C}$  vers l'intervalle  $[0, 1]$ , qui doit remplir les deux buts suivants :

**But 1 ( $\mathbf{G}_1$ )** *Construire des clusters dont la taille est maximale, c.à.d. égale à  $n_{max}$ .*

**But 2 ( $\mathbf{G}_2$ )** *Construire des clusters qui contiennent le plus grand nombre de membres du même groupe.*

L'étude de la Section C.3.3 a permis de définir la fonction de coût ci-dessous :

$$c(\mathcal{C}_k) := 1 - \left[ f_{n_{max}}(n_k) \cdot \epsilon + \frac{1 - \epsilon}{n_{max}} \sum_{t \in \mathcal{I}(\mathcal{C}_k)} m_{t,k} \cdot f_{m_t}(m_{t,k}) \right], \quad (\text{C.11})$$

avec  $\epsilon \in [0, 1]$  choisi pour favoriser soit  $\mathbf{G}_1$  soit  $\mathbf{G}_2$ , et la fonction  $f_n(m)$  définie par :

$$f_n(m) := \frac{1}{n(n+1)} \cdot m(m+1). \quad (\text{C.12})$$

Dans le Chapitre C.4 nous généralisons (C.11) et (C.12) et prouvons comment la valeur de  $\epsilon$  doit être choisie. Ici, nous ne pouvons uniquement dire qu'il faut choisir une valeur très faible de  $\epsilon$  pour favoriser  $\mathbf{G}_2$  sur  $\mathbf{G}_1$ .

La fonction de gain  $g$  de DCOG est définie  $\forall k, \forall \ell$ , avec  $k \neq \ell$ , de la façon suivante :

$$\begin{aligned} g : \quad \mathcal{V} \times \mathcal{C} \times \mathcal{C} &\mapsto \mathbb{R} \\ (\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) &\mapsto g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) \end{aligned}$$

avec :

$$g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) := [c(\mathcal{C}_\ell) - c(\mathcal{C}_\ell \cup \{u_i\})] - [c(\mathcal{C}_k \setminus \{u_i\}) - c(\mathcal{C}_k)]. \quad (\text{C.13})$$

Le gain  $g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)$  est égal à la différence entre  $i$ ) le gain lié à l'arrivée des nœuds  $\{u_i\}$  dans  $\mathcal{C}_\ell$ , égal à  $c(\mathcal{C}_\ell) - c(\mathcal{C}_\ell \cup \{u_i\})$ , et  $ii$ ) la perte liée au départ de ces nœuds de  $\mathcal{C}_k$ , égal à  $c(\mathcal{C}_k \setminus \{u_i\}) - c(\mathcal{C}_k)$ .

### C.3.4.2 Algorithme DCOG

L'algorithme DCOG est détaillé dans la Table C.1. Il est décomposé en deux étapes, après lesquelles les clusters vérifient les contraintes de connectivité, taille et diamètre, qui sont répétées jusqu'à ce que plus aucune modification de cluster ne soit possible. Lors de la première étape (lignes 1-22), les clusters déterminent si certains de leurs membres pourraient partir pour rejoindre un cluster voisin. Au cours de la deuxième étape (lignes 22-35), les swaps sont réalisés, et les nœuds sont échangés entre les clusters comme décidé dans l'étape précédente.

Comme l'algorithme de clusterisation est exécuté indépendamment par chaque cluster, les décisions prises au cours de l'étape 1, par exemple pour que les nœuds  $\{u_i\} \in \mathcal{C}_k$  rejoignent le cluster  $\mathcal{C}_\ell$ , peuvent ne plus être valides lors de l'étape 2, par exemple car l'étape 2 du cluster  $\mathcal{C}_\ell$  a été exécutée avant celle du cluster  $\mathcal{C}_k$ , et  $\mathcal{C}_\ell$  a été modifié. C'est pourquoi, lors de la deuxième étape, il faut s'assurer que les swaps prévus sont encore valides. Note : en pratique, l'algorithme DCOG repose sur un protocole d'échanges de messages (non décrit dans cette thèse) dont la mise en œuvre requiert du temps. C'est pourquoi, lorsqu'un cluster source cherche à envoyer certains de ses membres dans un autre cluster, ce dernier peut être déjà en cours d'échange avec

<p><b>Step 1</b></p> <p>1 Set <math>\mathcal{M} = \emptyset</math>, <math>\mathcal{L} = \emptyset</math> and <math>g = 0</math></p> <p>2 <b>For each</b> set of nodes <math>\{u_i\}</math> members of the same group in <math>\mathcal{C}_k</math> <b>do</b>:</p> <p>3   <b>If</b> <math>\mathcal{C}_k \setminus \{u_i\}</math> is not connected, <b>go to</b> line 2. <b>End If.</b></p> <p>4   <b>If</b> <math>d(\mathcal{C}_k \setminus \{u_i\}) &gt; d_{max}</math>, <b>go to</b> line 2. <b>End If.</b></p> <p>5   <b>For each</b> cluster <math>\mathcal{C}_\ell</math> neighbor of cluster <math>\mathcal{C}_k</math> <b>do</b>:</p> <p>6       <b>If</b> <math> \mathcal{C}_\ell \cup \{u_i\}  &gt; n_{max}</math>, <b>go to</b> line 5. <b>End If.</b></p> <p>7       <b>If</b> <math>\mathcal{C}_\ell \cup \{u_i\}</math> is disconnected, <b>go to</b> line 5. <b>End If.</b></p> <p>8       <b>If</b> <math>d(\mathcal{C}_\ell \cup \{u_i\}) &gt; d_{max}</math>, <b>go to</b> line 5. <b>End If.</b></p> <p>9       Calculate <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)</math> thanks to (C.13).</p> <p>10       <b>If</b> <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) &gt; g</math> <b>then</b>:</p> <p>11           Set <math>g = g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)</math>.</p> <p>12           Set <math>\mathcal{L} = \{\mathcal{C}_\ell\}</math>.</p> <p>13       <b>Else if</b> <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) = g</math> <b>then</b>:</p> <p>14           Set <math>\mathcal{L} = \mathcal{L} \cup \mathcal{C}_\ell</math>.</p> <p>15       <b>End If.</b></p> <p>16   <b>End For.</b></p> <p>17   <b>If</b> <math>g = 0</math> <b>then</b> nodes <math>\{u_i\}</math> remain in cluster <math>\mathcal{C}_k</math>.</p> <p>18   <b>Else then</b></p> <p>19       Choose randomly <math>\mathcal{C}_\ell \in \mathcal{L}</math>.</p> <p>20       Set <math>\mathcal{M} = \mathcal{M} \cup (\{u_i\}, \mathcal{C}_\ell, g)</math>.</p> <p>21   <b>End If.</b></p> <p>22 <b>End For.</b></p>
<p><b>Step 2</b></p> <p>23 <b>For each</b> <math>(\{u_i\}, \mathcal{C}_\ell, g) \in \mathcal{M}</math> considered in decreasing <math>g</math> values</p> <p>24   <b>If</b> <math>\mathcal{C}_k \setminus \{u_i\}</math> is not connected, <b>go to</b> line 23. <b>End If.</b></p> <p>25   <b>If</b> <math>\mathcal{C}_\ell = \emptyset</math>, <b>go to</b> line 23. <b>End If.</b></p> <p>26   <b>If</b> <math> \mathcal{C}_\ell \cup \{u_i\}  &gt; n_{max}</math>, <b>go to</b> line 23. <b>End If.</b></p> <p>27   <b>If</b> <math>d(\mathcal{C}_\ell \cup \{u_i\}) &gt; d_{max}</math>, <b>go to</b> line 23. <b>End If.</b></p> <p>28   Calculate <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell)</math> thanks to (C.13).</p> <p>29   <b>If</b> <math>g(\{u_i\}, \mathcal{C}_k, \mathcal{C}_\ell) \leq 0</math>, <b>go to</b> line 23. <b>End If.</b></p> <p>30   <b>If</b> <math>\mathcal{C}_\ell</math> is available <b>then</b>:</p> <p>31       Set <math>\mathcal{C}_\ell = \mathcal{C}_\ell \cup \{u_i\}</math>.</p> <p>32       Set <math>\mathcal{C}_k = \mathcal{C}_k \setminus \{u_i\}</math>.</p> <p>33       <b>Exit For loop.</b></p> <p>34   <b>End If.</b></p> <p>35 <b>End For.</b></p>

Table C.1: Algorithme de clusterisation dynamique appliqué au cluster  $\mathcal{C}_k$ .

un troisième, et donc ne pas être disponible pour le premier. C'est ce que vérifie le test de a ligne 30 de la Table C.1.

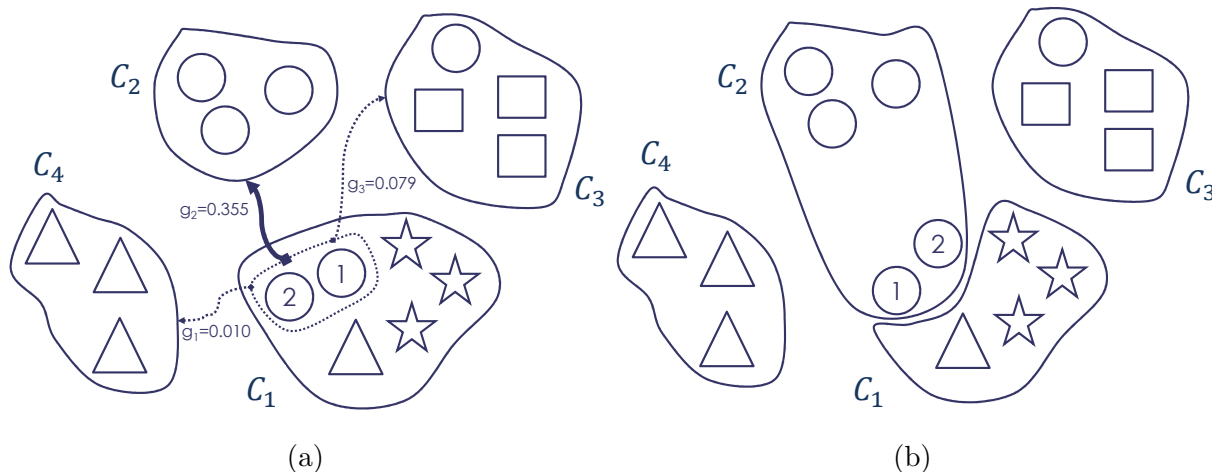


Figure C.5: (a) Swaps potentiels et gains associés pendant l'exécution de DCOG dans le cluster  $C_1$ , concernant les membres du groupe cercle nœuds 1 et 2. (b) Mise en œuvre du swap : formation d'un cluster avec 5 membres du groupe cercle.

La Figure C.5.a donne un exemple de la recherche du meilleur swap par DCOG, dans un réseau de 16 nœuds tous en portée radio, et constitué de 4 clusters. Le groupe de chacun des nœuds est identifié par sa forme. Les frontières de chaque cluster sont indiquées par des lignes. Pendant l'exécution de DCOG, le cluster  $C_1$  évalue le gain induit par le swap des nœuds 1 et 2 du groupe cercle vers chacun des clusters voisins. Parmi les trois swaps possibles, c'est celui qui induit le gain strictement positif le plus fort qui est sélectionné, c.à.d. celui où les deux nœuds rejoignent  $C_2$ . La nouvelle structure en clusters qui résulte de ce swap est illustrée par la Figure C.5.b.

La convergence de DCOG vers un état stable est établie par le résultat ci-dessous.

**Résultat C.4** *Dans le cadre d'une topologie fixe, l'algorithme DCOG défini par la Table C.1 converge suite à un nombre fini d'itérations vers une structure en clusters stable, c.à.d. qu'il n'existe plus aucun swap possible qui aurait un gain strictement positif.*

### C.3.4.3 Adaptation à la mobilité

En raison de la mobilité des nœuds, la topologie des clusters évolue, ce qui peut les conduire à ne plus respecter les contraintes de connectivité ou de diamètre. Dans ce cas, la procédure détaillée dans la Table C.2 sépare ces clusters de manière à former des ensembles de membres des mêmes groupes, qui satisfont les contraintes.

## C.3.5 Résultats numériques

### C.3.5.1 Algorithmes de référence

Nous avons choisi comme algorithmes de clusterisation distribué de référence, les deux algorithmes GDMAC [1] et VOTE [6], car ils permettent tout deux d'adapter le nombre clusters à la densité du réseau, quoique de manière différente. Nous avons aussi défini des extensions de ces protocoles, de façon à prendre en compte la structure en groupes du réseau et améliorer les performances.

```

1 If  $\mathcal{C}_k$  is not connected or  $d(\mathcal{C}_k) > d_{max}$  then:
2   Let  $\mathcal{M} = \emptyset$ .
3   For each set of nodes  $\{u_i\}$  members of the same group in
    $\mathcal{C}_k$  do:
4     Let  $\mathcal{C} = \{\{u_i\}\}$ .
5     If  $\{u_i\}$  is not connected then:
6       Split  $\{u_i\}$  into its  $m$  connected components
        $\{\{u_i^1\}, \dots, \{u_i^m\}\}$ .
7       Set  $\mathcal{C} = \{\{u_i^1\}, \dots, \{u_i^m\}\}$ .
8     End If.
9     For each set of nodes  $\{u_i^j\}$  in  $\mathcal{C}$  do:
10      If  $d(\{u_i^j\}) > d_{max}$  then:
11        Split  $\{u_i^j\}$  into its  $m'$  subcomponents
         $\{\{v_i^1\}, \dots, \{v_i^{m'}\}\}$  each one satisfying the diameter
        constraint.
12        Set  $\mathcal{M} = \mathcal{M} \cup \{\{v_i^1\}, \dots, \{v_i^{m'}\}\}$ .
13      Else set  $\mathcal{M} = \mathcal{M} \cup \{u_i^j\}$ .
14      End If.
15    End For.
16  For each set of nodes  $\{v_i\}$  in  $\mathcal{M}$  do:
17    Create a new cluster with  $\{v_i\}$ .
18  End For.
19 End If.

```

Table C.2: Adaptation à la mobilité pour le cluster  $\mathcal{C}_k$ .

L'algorithme GDMAC, appelé GDMAC-std, opère de la façon suivante. Les nœuds CH sont ceux dont les poids (ici leurs identifiants) sont les plus forts dans leur voisinage, avec au plus  $K$  CH voisins. Un nœud non-CH non déjà membre d'un cluster s'affilie aléatoirement à l'un de ses  $K + 1$  CH voisins de poids le plus fort. Un nœud non-CH ne change de CH que s'il existe un autre CH dans son voisinage, dont le poids est supérieur à celui de son CH courant de plus de la quantité  $H$ . Selon la première modification de GDMAC, GDMAC-new1, le poids des nœuds devient le *facteur de stabilité* défini dans [5], au lieu de l'identifiant. Le facteur de stabilité est une combinaison linéaire de la vitesse relative du nœud vis à vis de ses voisins, de sa distance relative vis à vis de ses voisins, de son nombre de voisins et de l'énergie résiduelle contenue dans sa batterie. Pour prendre en compte la structure en groupes du réseau, lors du calcul des composantes du facteur de stabilité, la contribution des voisins membres du même groupe que le nœud pour lequel la métrique est calculée est plus élevée que celle des autres voisins. Le protocole TCA [5] peut être considéré comme un cas particulier de GDMAC-new1 pour lequel le paramètre  $K$  est mis à zéro. Nous l'appelons TCA-std. GDMAC-new2 est l'extension de GDMAC-new1 selon laquelle les nœuds non-CH s'affilient préférentiellement à un CH membre de leur groupe, indépendamment de son poids.

De la même façon que GDMAC, l'algorithme VOTE, appel VOTE-std, choisit les CH sur la base de leur poids, cette fois défini comme une combinaison linéaire du degré du nœud et

de la durée de vie résiduelle de sa batterie. La différence principale d'avec GDMAC est que contrairement à ce dernier, VOTE limite la taille des clusters à un maximum  $n_{max}$ . Nous avons défini l'extension VOTE-new de VOTE. Selon ce nouvel algorithme, le poids des nœuds devient le facteur de stabilité de TCA.

### C.3.5.2 Paramétrage de la simulation

Le réseau contient des groupes de taille  $m_t = 10$ . Les nœuds sont déployés dans une zone carrée de 1500 unités de distance de côté, selon un modèle RPGM [45] modifié. La portée de communication des nœuds est de  $d_{ref} = 250$  unités de distance. La taille maximale des clusters est  $n_{max} = 20$  pour DCOG et VOTE. La valeur de  $\epsilon$  dans (C.11) est choisie suffisamment petite pour assurer de favoriser  $\mathbf{G}_2$  préférentiellement à  $\mathbf{G}_1$ . Pour chaque taille de réseau  $N$ , la valeur du paramètre  $K$  des variantes de GDMAC a été choisie pour assurer que la taille moyenne la plus grande des clusters ne dépasse pas  $n_{max}$ . Lorsque  $N \geq 300$ , cela n'est pas possible pour GDMAC-std. Par conséquent nous ne simulons pas cet algorithme en absence de mobilité. Enfin, lors des calculs des facteurs de stabilité, la contribution des voisins du même groupe que le nœud considéré est double de celle des autres voisins.

La Figure C.6 donne un exemple de réseau de  $N = 100$  nœuds clusterisé avec DCOG. Dans cette figure, les groupes sont indiqués par la couleur interne de chaque nœud, et les clusters par la couleur de la couronne.

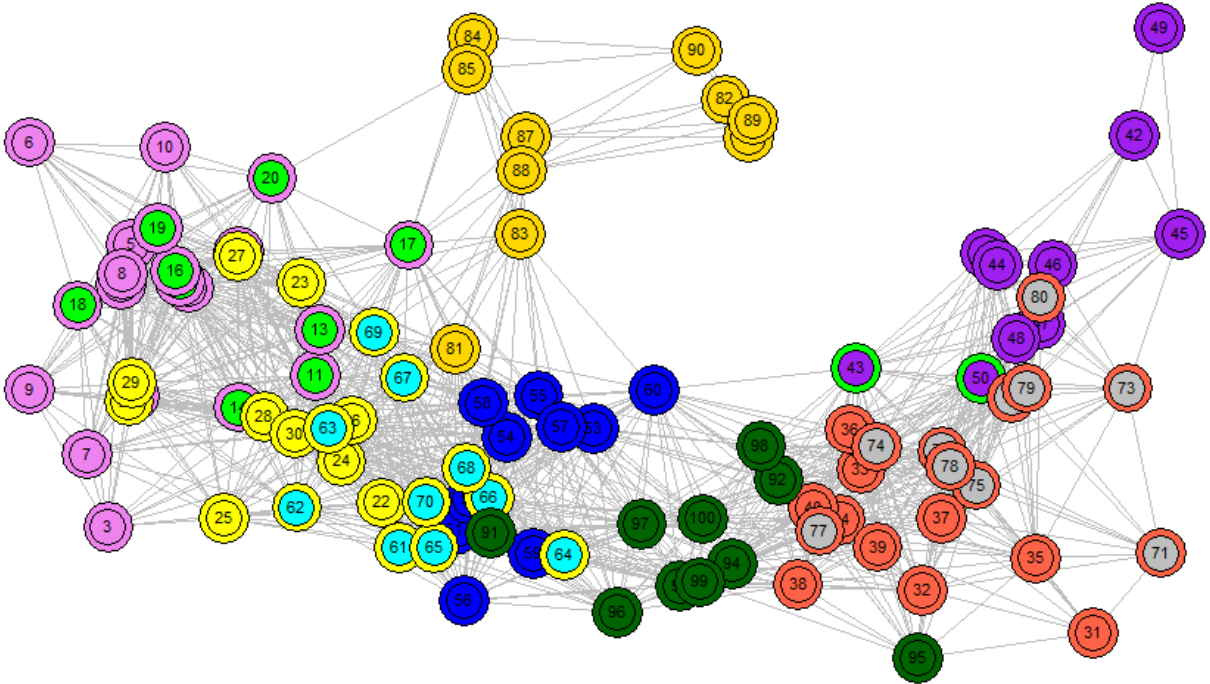


Figure C.6: Exemple de réseau de 100 nœuds clusterisé avec DCOG,  $n_{max} = 20$ . Les couleurs interne et externe indiquent l'appartenance des nœuds aux groupes et aux clusters, respectivement.

Lors des simulations, le temps est découpé en tours successifs. À chaque tour, l'algorithme de clusterisation considéré est exécuté pour chaque cluster (DCOG) ou nœud (autres algorithmes), sous l'hypothèse que toutes les informations requises sont disponibles.



### C.3.5.3 Réseaux statiques

La Figure C.7 montre la taille moyenne des clusters en fonction de  $N$ . TCA-std ne permet qu'un seul CH dans un voisinage radio. Par conséquent la taille des clusters doit grandir avec  $N$ , ce que confirme la figure. Ce comportement est rédhibitoire, ce qui justifie le fait que nous ne considérons plus TCA-std dans la suite de ce document. Grâce au paramétrage approprié, DCOG comme les variantes de GDMAC et celles de VOTE construisent des clusters dont la taille maximale est en moyenne la même. Néanmoins, selon l'algorithme les tailles moyennes de ces clusters sont très différentes. Notamment, GDMAC-new1 et GDMAC-new2 forment des clusters de taille moyenne quasiment indépendante de  $N$ , et beaucoup plus petite que  $n_{max}$ . Ce résultat moyen doit être mis en perspective par le fait que ces algorithmes forment des clusters de tailles très variables, c.à.d. de très petits comme de très grands clusters. A contrario, on constate que la taille des clusters construits par DCOG, VOTE-std et VOTE-new, augmente avec la taille du réseau, et qu'elle est bien limité à  $n_{max}$ .

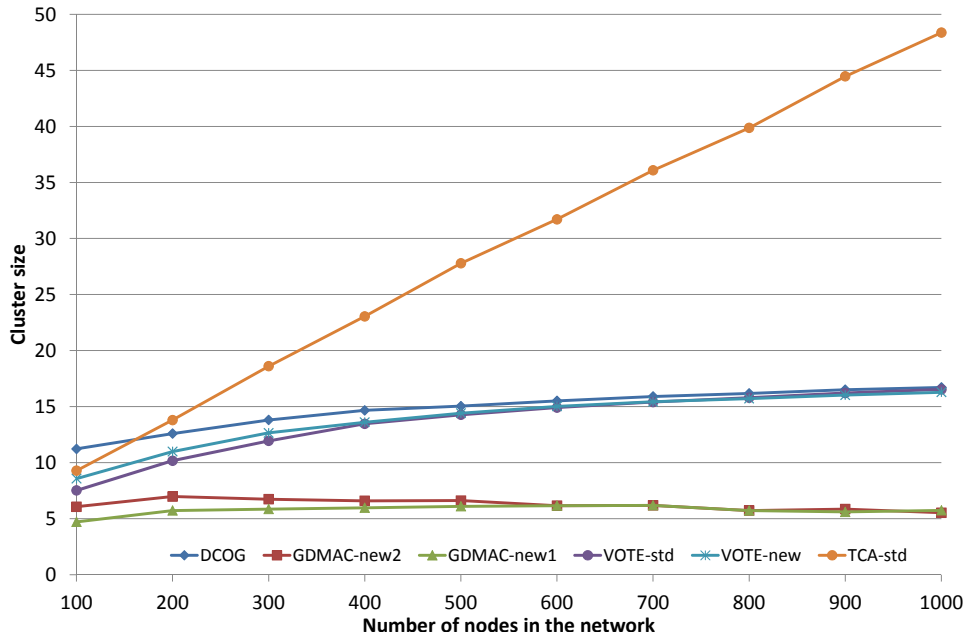


Figure C.7: Taille moyenne des clusters dans les réseaux statiques en fonction de  $N$ .

La Figure C.8 trace la diversité de clusters par groupe (*group cluster diversity*, GCD), c.à.d. pour chaque groupe le nombre de clusters contenant au moins un représentant de ce groupe. Avec les algorithmes GDMAC-new1, VOTE-std et VOTE-new, les nœuds non-CH ne prennent pas en compte le groupe pour choisir le nœud CH auquel ils s'affilient, ce qui explique les valeurs élevées de la métrique GCD (plus fortes pour les clusters de tailles plus importantes). Grâce à sa procédure d'affiliation basée sur le groupe du CH, GDMAC-new2 aboutit à une GCD inférieure à GDMAC-new1, alors que ces deux algorithmes construisent des clusters de tailles moyennes identiques. Quant à l'algorithme DCOG, il aboutit à des valeurs de GCD très proches de 1. Ceci montre que DCOG forme des clusters qui contiennent des groupes quasiment complets, et que, contrairement aux autres algorithmes, il permet d'atteindre le but  $\mathbf{G}_2$ .

La Figure C.9 montre les valeurs de la fonction de coût  $J_0^X$  définie dans la Section C.2.3.3. Seules les moyennes sont tracées pour les algorithmes de référence. Pour DCOG, une représentation

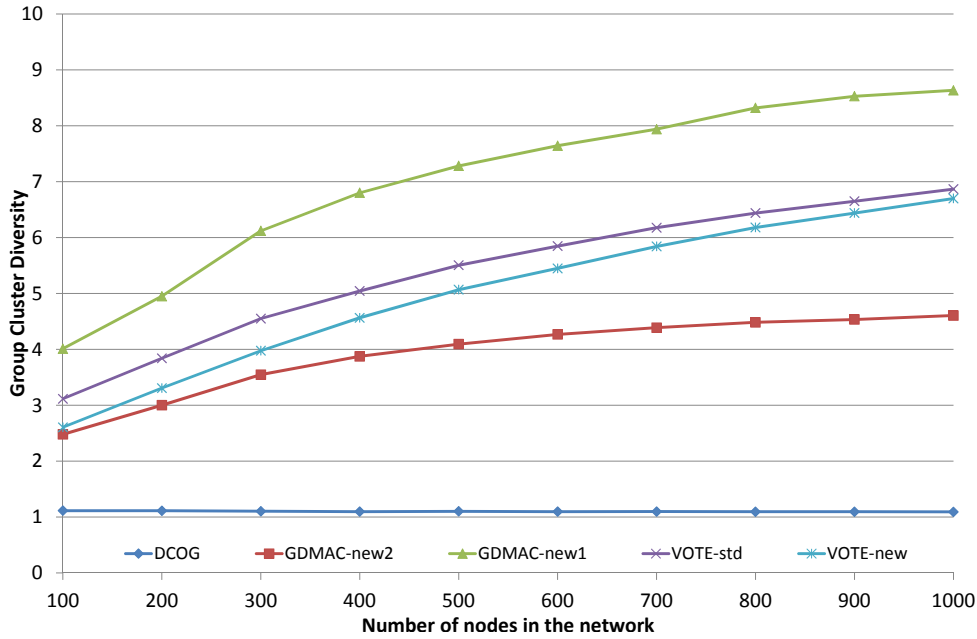


Figure C.8: Diversité de clusters par groupe dans les réseaux statiques en fonction de  $N$ .

particulière est utilisée afin de représenter la distribution statistique des valeurs. Pour un nombre de nœuds donné et de haut en bas, la barre la plus basse indique la valeur la plus faible, la suivante le 5ème centile, puis le premier quartile, la médiane, le troisième quartile, le 95ème centile et enfin la valeur la plus élevée. La valeur moyenne est identifiée par un disque.

Comme indiqué dans la Section C.2.3.4, la métrique  $J_0^X$  peut facilement être assimilée à un délai de bout en bout. Par conséquent, plus ses valeurs sont faibles, meilleure est la performance. Par conséquent, DCOG qui permet d'obtenir des valeurs significativement plus faibles que les autres algorithmes, permet d'obtenir les meilleures performances.

Pour mieux estimer la qualité des clusters construits par DCOG, remarquons qu'aucune partition  $p$  ne peut conduire à une valeur de  $J_0^X$  plus grande que celle qui consiste à former un cluster pour chaque nœud du réseau. Dans ce cas, chaque lien est un lien inter-clusters et sa contribution  $J_0^X$  est maximale. De plus, même si le cluster  $\mathcal{V}$  qui regroupe tous les nœuds du réseau ne respecte probablement pas les contraintes, sa valeur de  $J_0^X$  représente une borne inférieure commode. Lors de nos simulations, nous avons calculé ces deux bornes et avons trouvé que quel que soit le nombre de nœuds du réseau,  $J_0^X(p) \in [1.4, 2.8]$ ,  $\forall p \in \mathcal{P}$ . Les valeurs de  $J_0^X$  obtenues par l'entremise de DCOG sont d'environ 1.6, et sont donc très proches de la valeur minimale théorique égale à 1.4. Ceci est un très bon résultat.

#### C.3.5.4 Réseaux mobiles

Dans les réseaux mobiles structurés, les nœuds se déplacent en groupe. Pour évaluer la stabilité des clusters, les métriques habituellement utilisées sont basées sur la notion de CH, comme par exemple la durée pendant laquelle les nœuds CH conservent leur rôle de CH, celle pendant laquelle un nœud est affilié à son CH, etc. Ces métriques ne sont pas utilisables avec DCOG qui ne recourt pas à la notion de CH. Par conséquent, dans cette section nous définissons une nouvelle métrique de stabilité du réseau, qui s'exprime comme le rapport entre la durée de la

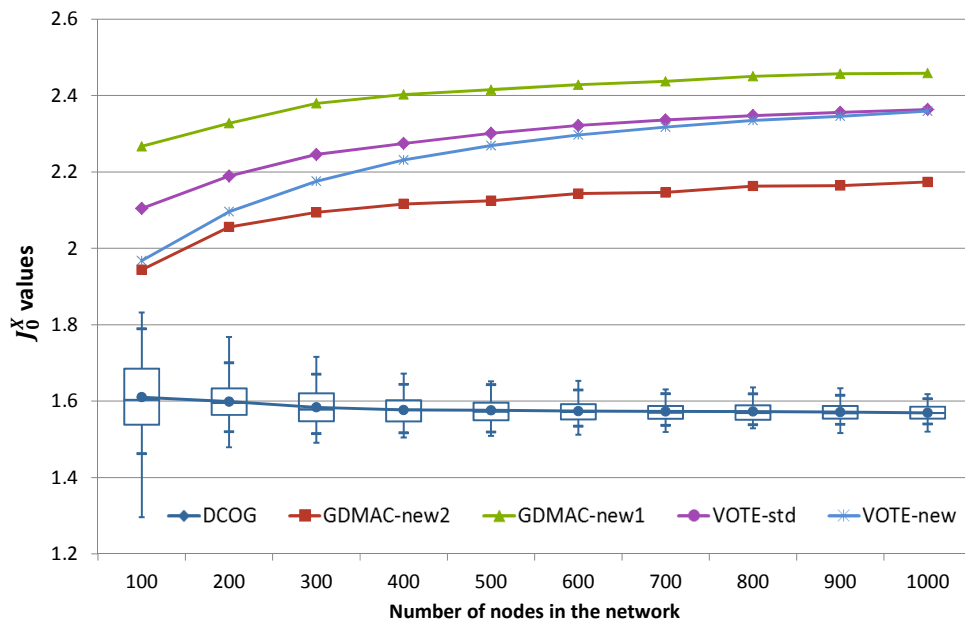


Figure C.9: Valeurs de  $J_0^X$  dans les réseaux statiques en fonction de  $N$ .

simulation pendant laquelle aucun cluster n'a été modifié et la durée totale de la simulation. La Figure C.10 montre que DCOG présente la meilleure stabilité, quelle que soit la vitesse des nœuds.

### C.3.6 Conclusions

Dans ce chapitre, nous avons détaillé notre méthodologie pour la conception de l'algorithme de clusterisation DCOG, adapté aux réseaux structurés tels que les réseaux de sécurité civile ou militaires.

Premièrement, via une énumération exhaustive, nous avons trouvé une fonction de coût capable de trouver de bonnes partitions, et donc de bons clusters. Deuxièmement, nous avons expliqué comment, à partir d'un algorithme centralisé, nous avons modifié cette fonction de coût et avons élaboré un processus de prise de décisions, ce qui nous a mené à un algorithme distribué, que nous avons appelé DCOG. La caractéristique principale de DCOG est sa capacité à construire des clusters de taille limitée, qui contiennent des groupes complets, ceci sans faire appel à la notion de chef de cluster. Troisièmement, nous avons formalisé notre algorithme, et prouvé sa convergence.

Nos simulations de réseaux statiques et mobiles ont montré que grâce à DCOG, il est maintenant possible d'opérer de grands réseaux denses structurés en groupes, et d'assurer un bon délai de bout en bout, ainsi qu'une bonne stabilité du réseau en présence de mobilité. La comparaison avec des solutions existantes a montré que notre solution était la meilleure.

Dans le Chapitre C.4, nous revisitons DCOG à l'aide de la théorie des jeux de coalition. Ceci nous permet de caractériser son état final après convergence, de suggérer des fonctions de coût plus générales, et de mieux comprendre la manière d'assigner leurs valeurs aux paramètres de l'algorithme. De plus, cette théorie nous conduit à un nouvel algorithme adapté aux réseaux non structurés, dont les performances sont meilleures que celles des algorithmes conventionnels.

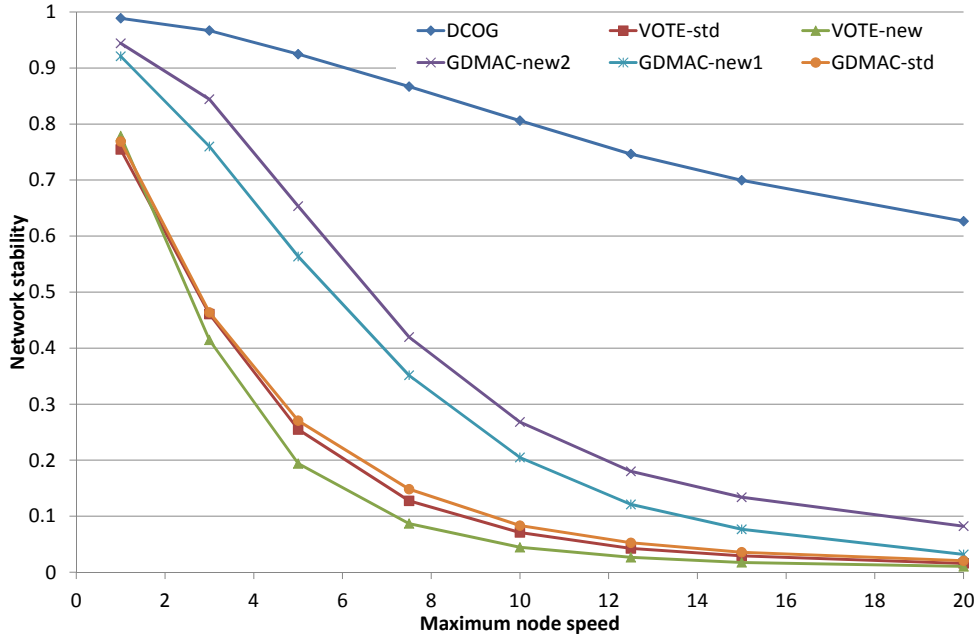


Figure C.10: Stabilité des réseaux mobiles, en fonction de la vitesse des nœuds.

## C.4 Clusterisation distribuée et théorie des jeux de coalitions

### C.4.1 Introduction

La Section C.4.2 de ce chapitre détaille un algorithme générique de clusterisation basé sur la théorie des jeux de coalitions. Il est générique dans le sens où il fait abstraction de la structure du réseau. Dans la Section C.4.3, cet algorithme est appliqué aux réseaux structurés, puis dans la Section C.4.4 aux réseaux non structurés. Les résultats et analyses de simulations sont présentés dans la Section C.4.5, qui précède la Section C.4.6 dédiée aux conclusions.

### C.4.2 Algorithme générique de clusterisation basé sur la théorie des jeux de formation de coalitions

#### C.4.2.1 Définitions utiles associées à la théorie des jeux de coalitions

Les jeux de formation de coalitions de ce chapitre impliquent un ensemble de joueurs  $\mathcal{N}$ , aussi appelé  $\mathcal{V}$  dans le contexte de théorie des graphes des deux chapitres précédents. La notion de coalition est définie de la façon suivante :

**Definition C.1** Une structure de coalitions (ou partition) est définie comme l'ensemble  $p := \{\mathcal{C}_1, \dots, \mathcal{C}_{N_c}\}$  où  $\mathcal{C}_k \subseteq \mathcal{N}$  sont des coalitions disjointes qui vérifient  $\cup_{k=1}^{N_c} \mathcal{C}_k = \mathcal{N}$ .

Trois quantités sont associées à chaque coalition : *i*) l'utilité  $u(\mathcal{C}_k) \geq 0$  associée à l'intérêt de former la coalition, *ii*) le coût  $c(\mathcal{C}_k)$  qui quantifie le coût de la coopération associé au respect ou non des contraintes qui s'appliquent aux coalitions, et considéré dans cette thèse comme nul si la coalition satisfait les contraintes, ou infini dans le cas contraire, et *iii*) la valeur  $v(\mathcal{C}_k)$  définie par la différence  $v(\mathcal{C}_k) = u(\mathcal{C}_k) - c(\mathcal{C}_k)$  entre l'utilité de la coalition et son coût. Lorsque les contraintes sont satisfaites, alors  $v(\mathcal{C}_k) = u(\mathcal{C}_k)$ , et  $v(\mathcal{C}_k) = -\infty$  sinon.

L'utilité d'une coalition dépend du type de réseau (structuré ou non structuré). Des exemples sont détaillés dans les Sections C.4.5.2 et C.4.5.3. Dans cette thèse, les jeux de formation de coalitions sont sous la forme caractéristique, c.à.d. que la valeur d'une coalition ne dépend que de ses membres [17].

Le transfert d'un membre d'une coalition vers une autre est appelé une *switch operation* :

**Definition C.2** Une switch operation  $\sigma_{k,\ell}(\mathcal{P})$  est définie comme le transfert des joueurs  $\mathcal{P}$  de  $\mathcal{C}_k \in p$  vers  $\mathcal{C}_\ell \in p \cup \{\emptyset\}$ ,  $\sigma_{k,\ell}(\mathcal{P}) : \mathcal{C}_k \mapsto \mathcal{C}_k \setminus \mathcal{P}$ , et  $\mathcal{C}_\ell \mapsto \mathcal{C}_\ell \cup \mathcal{P}$ .

Pour évaluer si une switch operation améliore la structure de coalitions, définissons maintenant la notion de *gain de switch operation* :

**Definition C.3** Le gain d'une switch operation  $g(\sigma_{k,\ell}(\mathcal{P}))$  associé à  $\sigma_{k,\ell}(\mathcal{P})$  est défini par:

$$g(\sigma_{k,\ell}(\mathcal{P})) := r_{\mathcal{P}}(\mathcal{C}_\ell \cup \mathcal{P}) - r_{\mathcal{P}}(\mathcal{C}_k), \quad (\text{C.14})$$

avec  $r_{\mathcal{P}}(\mathcal{C}_k)$  définie par :

$$r_{\mathcal{P}}(\mathcal{C}_k) := v(\mathcal{C}_k) - v(\mathcal{C}_k \setminus \mathcal{P}). \quad (\text{C.15})$$

La quantité  $r_{\mathcal{P}}(\mathcal{C}_k)$  peut être interprétée comme la valeur ajoutée des joueurs  $\mathcal{P}$  dans la coalition  $\mathcal{C}_k$ .

En ne considérant que des partitions valides (qui satisfont les contraintes), les équations (3.1) et (3.2) prouvent que le gain d'une switch operation ne dépend que des valeurs des deux coalitions considérées, et non pas de la manière dont les valeurs de ces coalitions sont partagées entre leurs membres. Cela signifie que le jeu de formation de coalition proposé dans ce chapitre est à utilité transférable (TU).

Définissons maintenant la *relation de préférence* utilisée pour comparer deux switch operations.

**Definition C.4** La relation de préférence  $\succ$  est définie comme la relation binaire transitive entre deux switch operations  $\sigma_{k,\ell}(\mathcal{P}_i)$  and  $\sigma_{k',\ell'}(\mathcal{P}_j)$  telle que :

$$\sigma_{k,\ell}(\mathcal{P}_i) \succ \sigma_{k',\ell'}(\mathcal{P}_j) \Leftrightarrow g(\sigma_{k,\ell}(\mathcal{P}_i)) > g(\sigma_{k',\ell'}(\mathcal{P}_j)).$$

De la même façon, nous définissons  $\succeq$  par :  $\sigma_{k,\ell}(\mathcal{P}_i) \succeq \sigma_{k',\ell'}(\mathcal{P}_j) \Leftrightarrow g(\sigma_{k,\ell}(\mathcal{P}_i)) \geq g(\sigma_{k',\ell'}(\mathcal{P}_j))$ .

Avec ces notations, la notion de *Nash-stability* [46] d'une partition peut être définie par :

**Definition C.5** Une partition  $p = \{\mathcal{C}_1 \dots, \mathcal{C}_{N_c}\}$  est Nash-stable si  $\forall \mathcal{C}_k \in p, \forall i \in \mathcal{C}_k, g(\sigma_{k,\ell}(\{i\})) \leq 0$  quel que soit  $\mathcal{C}_\ell \in p \cup \{\emptyset\}$ .

Dans une partition  $p$  qui est Nash-stable, il n'existe aucune switch operation qui n'impliquerait qu'un seul nœud et dont le gain serait strictement positif.

#### C.4.2.2 Algorithme générique de formation de coalitions pour la clusterisation

Dans cette section, nous proposons un algorithme de formation de coalitions générique, distribué et asynchrone, en vue de clusteriser le réseau. Nous identifions les coalitions et les joueurs de la théorie des jeux de coalitions aux clusters et aux nœuds du réseau, et utilisons désormais cette dénomination. Soit  $n_k$  la taille du cluster  $\mathcal{C}_k$  et  $d(\mathcal{C}_k)$  son diamètre.

L'algorithme proposé est basé sur la comparaison des gains des switch operations. C'est un algorithme distribué car la prise de décision est faite au niveau de chaque nœud, et asynchrone car ces prises de décision sont faites indépendamment et de façon non coordonnée. L'algorithme construit des clusters qui satisfont les contraintes suivantes :

- $\rho_1(\mathcal{C}_k)$ : le graphe induit par  $\mathcal{C}_k$  est connexe,
- $\rho_2(\mathcal{C}_k)$ :  $n_k \leq n_{max}$ ,
- $\rho_3(\mathcal{C}_k)$ :  $d(\mathcal{C}_k) \leq d_{max}$ ,

et le coût  $c(\mathcal{C}_k)$  est défini par  $c(\mathcal{C}_k) := \chi(\rho_1(\mathcal{C}_k)) + \chi(\rho_2(\mathcal{C}_k)) + \chi(\rho_3(\mathcal{C}_k))$ , avec  $\chi(\text{condition}(\mathcal{C}_k)) := 0$  si  $\text{condition}(\mathcal{C}_k)$  est respectée,  $+\infty$  sinon.

Lorsque le réseau est statique, nous pouvons démontrer que l'algorithme converge vers une solution stable pour laquelle les contraintes sont satisfaites. En présence de mobilité, la topologie du réseau change dans le temps. Ceci peut conduire un cluster qui satisfaisait les contraintes à un instant donné, à ne plus les satisfaire un peu plus tard. Notre algorithme gère ces situations. Décrivons maintenant son principe général. Lorsqu'un nœud démarre son processus de décision, il vérifie d'abord si les contraintes de son cluster actuel sont satisfaites. Si c'est le cas, alors il applique une procédure notée  $\mathbf{P}_1$  pour trouver et mettre en œuvre la meilleure switch operation. Si ce n'est pas le cas (en raison de la mobilité des nœuds), il applique une procédure notée  $\mathbf{P}_2$ . Au sein de ces deux procédures, les nœuds candidats pour une switch operation sont choisis à l'aide d'une règle de bon sens que nous appelons *heuristique pour le choix des nœuds*, notée  $\mathcal{H}_1$  pour  $\mathbf{P}_1$  et  $\mathcal{H}_2$  pour  $\mathbf{P}_2$ .

#### C.4.2.2.1 Procédure lorsque les contraintes sont satisfaites

Lorsqu'un nœud  $i$  démarre un processus de décision et vérifie avec succès que son cluster satisfait les contraintes, il utilise la procédure  $\mathbf{P}_1$  afin de trouver et mettre en œuvre une switch operation dont le gain est strictement positif. Dans son principe, cette procédure (détaillée dans la Table C.3) comporte trois étapes:

1. Sélection des switch operations candidates à gain strictement positif (lines 1-9). Cette sélection est faite à l'aide de l'heuristique  $\mathcal{H}_1$ , qui renvoie des ensembles de nœuds candidats pour des switch operations, dont au moins le nœud  $i$  lui-même.
2. Sélection d'une des meilleures switch operations (line 11).
3. Mise en œuvre de la switch operation retenue si elle existe (lines 12-14).

#### C.4.2.2.2 Procédure lorsque les contraintes ne sont pas satisfaites

Lorsqu'un nœud  $i$  démarre un processus de décision et constate que son cluster ne satisfait pas les contraintes, à cause de la mobilité des nœuds, il utilise la procédure  $\mathbf{P}_2$  afin de modifier la topologie du cluster pour si possible les satisfaire à nouveau. Dans son principe, cette procédure (détaillée dans la Table C.4) comporte les trois mêmes étapes que  $\mathbf{P}_1$ . Deux différences avec  $\mathbf{P}_1$  est que  $\mathbf{P}_2$  peut créer de nouveaux clusters, et qu'elle est susceptible de laisser le cluster courant inchangé, c.à.d. ne satisfaisant pas les contraintes.

Les paramètres de l'algorithme sont tout d'abord *i*) les heuristiques  $\mathcal{H}_1$  et  $\mathcal{H}_2$ , et ensuite *ii*) la fonction d'utilité  $u$  ainsi que la fonction de coût  $c$  utilisées pour calculer les gains des switch

```

// Selection of candidate switch operations
1 Set  $\mathcal{M} = \emptyset$ .
2 Apply heuristic  $\mathcal{H}_1$  to node  $i$  to get  $\{\mathcal{P}_{1,i}(a)\}_{a=1}^{A_1}$ , with  $\mathcal{P}_{1,i}(a) \in \mathcal{C}_k$ 
and  $A_1 \geq 1$ .
3 For each  $a \in \{1, \dots, A_1\}$  do:
4   For each  $\mathcal{C}_\ell$  neighbor of  $\mathcal{C}_k$  do:
5     If  $g(\sigma_{k,\ell}(\mathcal{P}_{1,i}(a))) > 0$  then:
6       Set  $\mathcal{M} = \mathcal{M} \cup \sigma_{k,\ell}(\mathcal{P}_{1,i}(a))$ .
7     End If.
8   End For.
9 End For.
10 If  $\mathcal{M} \neq \emptyset$  then:
// Selection of one best switch operation
11 Find  $(a^*, \ell^*)$  such that  $\sigma_{k,\ell^*}(\mathcal{P}_{1,i}(a^*)) \succeq \sigma_{k,\ell}(\mathcal{P}_{1,i}(a))$ ,
                                                                     $\forall \sigma_{k,\ell}(\mathcal{P}_{1,i}(a)) \in \mathcal{M}$ .

// Implementation of the switch operation
12 If the nodes involved in  $\sigma_{k,\ell^*}(\mathcal{P}_{1,i}(a^*))$  are all available then:
13   The nodes in  $\mathcal{P}_{1,i}(a^*)$  join  $\mathcal{C}_{\ell^*}$ .
14 End If.
15 End If.

```

Table C.3: Procédure  $\mathbf{P}_1$  de l'algorithme générique de clusterisation, exécutée par le nœud  $i \in \mathcal{C}_k$  lorsque les contraintes sont satisfaites.

operations. Des exemples d'heuristiques et de fonctions d'utilité sont détaillés en Section C.4.3 et C.4.4.

#### C.4.2.2.3 Propriétés de convergence

Lorsque la topologie du réseau est fixée et que l'algorithme est initialisé avec des clusters qui satisfont les contraintes (le plus simple étant celui où chaque nœud forme son propre cluster singleton), alors le résultat ci-dessous est vérifié.

**Résultat C.5** *Lorsque la topologie du réseau est fixée, et à partir d'une partition initiale  $p_0$  dont chaque cluster satisfait les contraintes, l'algorithme de formation de clusters aboutit à une suite de switch operations qui converge en un nombre fini d'itérations vers une partition finale  $p_f$ .*

De plus, comme l'ensemble de nœuds renvoyé par  $\mathcal{H}_1$  contient le nœud  $i$  lui-même :

**Résultat C.6** *La partition finale  $p_f$  du Résultat C.5 est Nash-stable.*

Il faut remarquer que la partition  $p_f$  n'est en général pas unique. Elle dépend de l'ordre dans lequel les nœuds effectuent les switch operations. Lorsque les instants de ces switch operations suivent un processus aléatoire (par exemple comme dans la Section C.4.5.1), l'expérience montre qu'il existe plusieurs partitions finales Nash-stable.

```

// Selection of potential candidate switch operations
1 Apply heuristic  $\mathcal{H}_2$  to node  $i$  to get  $\{\mathcal{P}_{2,i}(a)\}_{a=1}^{A_2}$ , with  $\mathcal{P}_{2,i}(a) \in \mathcal{C}_k$ 
and  $A_2 \geq 0$ .
// Selection of candidate switch operations
2 If  $A_2 > 0$  then:
3   For each  $a \in \{1, \dots, A_2\}$  do:
4     Set  $\mathcal{M} = \{\sigma_{k,0}(\mathcal{P}_{2,i}(a))\}$ .
5     For each  $\mathcal{C}_\ell$  neighbor of  $\mathcal{C}_k$  do:
6       If  $r_{\mathcal{P}_{2,i}(a)}(\mathcal{C}_\ell \cup \mathcal{P}_{2,i}(a)) > 0$  then:
11        Set  $\mathcal{M} = \mathcal{M} \cup \sigma_{k,\ell}(\mathcal{P}_{2,i}(a))$ .
8       End If.
9     End For.
10  End For.
// Selection of one of the best switch operation
11 Find  $(a^*, \ell^*)$  such that  $r_{\mathcal{P}_{2,i}(a^*)}(\mathcal{C}_{\ell^*} \cup \mathcal{P}_{2,i}(a^*)) \geq r_{\mathcal{P}_{2,i}(a)}(\mathcal{C}_\ell \cup$ 
 $\mathcal{P}_{2,i}(a)), \forall \sigma_{k,\ell}(\mathcal{P}_{2,i}(a)) \in \mathcal{M}$ .
// Implementation of the switch operation
12 If the nodes involved in  $\sigma_{k,\ell^*}(\mathcal{P}_{2,i}(a^*))$  are all available then:
13   Nodes  $\mathcal{P}_{2,i}(a^*)$  join  $\mathcal{C}_{\ell^*}$ .
14 Else:
15   The nodes in  $\mathcal{P}_{2,i}(a^*)$  that are available form a new cluster.
16 End If.
17 End If.

```

Table C.4: Procédure  $\mathbf{P}_2$  de l'algorithme générique de clusterisation, exécutée par le nœud  $i \in \mathcal{C}_k$  lorsque les contraintes ne sont pas satisfaites.

#### C.4.2.2.4 À propos de la fonction d'utilité

Afin de permettre une allocation de ressource efficace, l'algorithme doit atteindre le but  $\mathbf{G}_1$  défini dans la Section C.3.4, selon lequel les clusters doivent être de taille maximale. Pour cela les fonctions d'utilité doivent vérifier la condition suivante :

**Condition C.1** La fonction d'utilité doit satisfaire  $g(\sigma_{k,\ell}(\mathcal{C}_k)) > 0, \forall (\mathcal{C}_k, \mathcal{C}_\ell) \in p^2, k \neq \ell$ , dès que  $\mathcal{C}_k \cup \mathcal{C}_\ell$  vérifie les contraintes.

Cette condition garantit que, tant que les contraintes sont satisfaites, deux clusters peuvent toujours fusionner. Elle permet notamment de garantir que l'algorithme puisse être initialisé avec tous les nœuds en tant que clusters singletons, et ne reste pas bloqué dans cet état, ce qui n'est pas le cas pour une fonction d'utilité quelconque.

### C.4.3 Algorithme de clusterisation pour les réseaux structurés

L'application de l'algorithme de la Section C.4.2 aux réseaux structurés est appelé COG.



### C.4.3.1 Fonction d'utilité

Dans les réseaux structurés, en plus du but  $\mathbf{G}_1$ , l'algorithme doit atteindre le but  $\mathbf{G}_2$  défini dans la Section C.3.4, selon lequel les clusters doivent contenir le plus grand nombre possible de membres du même groupe. Pour atteindre ces deux buts, nous proposons une fonction d'utilité qui est une combinaison linéaire de deux sous-fonctions. La première, adaptée à  $\mathbf{G}_1$ , est une fonction  $f_1(n_k)$  du nombre de membres  $n_k$  du cluster  $\mathcal{C}_k$ . La seconde, adaptée à  $\mathbf{G}_2$ , est une somme de fonctions  $f_2(m_{t,k})$  des nombres de membres  $m_{t,k}$  des groupes  $\mathcal{O}_t$  dans le cluster  $\mathcal{C}_k$  :

$$u_{\text{st}}(\mathcal{C}_k) := f_1(n_k) \cdot \epsilon + \sum_{t \in \mathcal{I}(\mathcal{C}_k)} f_{2,t}(m_{t,k}) \cdot (1 - \epsilon), \quad (\text{C.16})$$

avec  $\epsilon \in (0, 1)$  et la notation  $\mathcal{I}(\mathcal{C}_k)$  définie dans la Section C.3.2.

Nous avons les deux résultats suivants :

**Résultat C.7** *Soient deux clusters  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  qui vérifient les contraintes, et un ensemble de nœuds  $\mathcal{P} \subset \mathcal{C}_k$  tel que : i)  $|\mathcal{C}_\ell \cup \mathcal{P}| > |\mathcal{C}_k|$ , et ii)  $\mathcal{C}_\ell \cup \mathcal{P}$  et  $\mathcal{C}_k \setminus \mathcal{P}$  vérifient les contraintes. Si  $f_1$  est strictement convexe, alors  $u_1$  définie par  $u_1(\mathcal{C}_k) := f_1(n_k)$  vérifie  $g(\sigma_{k,\ell}(\mathcal{P})) > 0$ .*

**Résultat C.8** *Soient deux clusters  $\mathcal{C}_k$  and  $\mathcal{C}_\ell$  qui vérifient les contraintes, tels que : i)  $\mathcal{P}$  est un sous-ensemble de  $\mathcal{O}_t \cap \mathcal{C}_k$ , ii)  $m_{t,\ell} + |\mathcal{P}| > m_{t,k}$ , et iii)  $\mathcal{C}_\ell \cup \mathcal{P}$  et  $\mathcal{C}_k \setminus \mathcal{P}$  vérifient les contraintes. Si  $f_{2,t}$  est strictement convexe, alors  $u_2$  définie par  $u_2(\mathcal{C}_k) := \sum_{t \in \mathcal{I}(\mathcal{C}_k)} f_{2,t}(m_{t,k})$  vérifie  $g(\sigma_{k,\ell}(\mathcal{P})) > 0$ .*

Comme conséquence de ces deux résultats, nous proposons pour  $f_1$  et  $f_{2,t}$  les fonctions convexes les plus simples, c.à.d. les monômes de degré 2 :

$$f_1(n_k) := \frac{n_k^2}{n_{\text{max}}^2}, \quad (\text{C.17})$$

$$f_{2,t}(m_{t,k}) := \frac{m_{t,k}^2}{T \cdot m_t^2}, \quad (\text{C.18})$$

où  $n_{\text{max}}^2$  et  $T \cdot m_t^2$  sont des coefficients de normalisation dans  $[0, 1]$ .

**Résultat C.9** *La fonction d'utilité définie par (C.16) avec (C.17) et (C.18) vérifie la condition C.1.*

Le résultat ci-dessous traite de la manière de choisir  $\epsilon$  pour établir un compromis entre les buts  $\mathbf{G}_1$  et  $\mathbf{G}_2$ .

**Result C.1** *Soient trois clusters  $\mathcal{C}_k$ ,  $\mathcal{C}_\ell$  and  $\mathcal{C}_q$  qui satisfont les contraintes, chacun d'entre eux contenant au moins un membre du groupe  $\mathcal{O}_t$ . Soit  $\mathcal{U}_q \subset \mathcal{C}_q$  ne contenant que des membres du groupe  $\mathcal{O}_t$ , avec  $n_{\mathcal{U}} := |\mathcal{U}_q| \leq m_{t,q}$ . Supposons que  $\mathcal{C}_k \cup \mathcal{U}_q$  et  $\mathcal{C}_\ell \cup \mathcal{U}_q$  vérifient les contraintes, et que  $\forall t, |\mathcal{O}_t| \leq n_{\text{max}}$ . Pourvu que la fonction d'utilité (C.16) avec (C.17) et (C.18) soit utilisée, les propriétés suivantes s'appliquent:*

- i) Si  $m_{t,k} = m_{t,\ell}$  et  $|\mathcal{C}_k| > |\mathcal{C}_\ell|$ , alors  $g(\sigma_{q,k}(\mathcal{U}_q)) > g(\sigma_{q,\ell}(\mathcal{U}_q)), \forall \epsilon$ .*
- ii) Si  $m_{t,k} = m_{t,\ell}$  et  $|\mathcal{C}_k| = |\mathcal{C}_\ell|$ , alors  $g(\sigma_{q,k}(\mathcal{U}_q)) = g(\sigma_{q,\ell}(\mathcal{U}_q)), \forall \epsilon$ .*

iii) Si  $m_{t,k} > m_{t,\ell}$ , et  $\forall |\mathcal{C}_k|, \forall |\mathcal{C}_\ell|$ , alors  $g(\sigma_{q,k}(\mathcal{U}_q)) > g(\sigma_{q,\ell}(\mathcal{U}_q))$  dès que :

$$\epsilon < \epsilon^* := \frac{1}{1+T}.$$

Premièrement, lorsque les deux clusters  $\mathcal{C}_k$  et  $\mathcal{C}_\ell$  contiennent le même nombre de membres du groupe  $\mathcal{O}_t$ , alors le résultat C.1 indique qu'indépendamment de  $\epsilon$  : *i*) l'algorithme choisit la switch operation de  $\mathcal{U}_q$  vers le plus grand cluster, dans le but d'atteindre  $\mathbf{G}_1$ , et *ii*) si les deux clusters  $\mathcal{C}_k$  et  $\mathcal{C}_\ell$  ont la même taille, alors les deux switch operations sont égales. Deuxièmement, lorsque le nombre de membres du groupe  $\mathcal{O}_t$  dans les clusters  $\mathcal{C}_k$  et  $\mathcal{C}_\ell$  sont différents, alors pour tout  $\epsilon < \epsilon^*$ , l'algorithme choisit toujours la switch operation de  $\mathcal{U}_q \subset \mathcal{O}_t$  vers le cluster qui contient le plus grand nombre de membres du groupe  $\mathcal{O}_t$ . Sous cette condition, il donne toujours la priorité à  $\mathbf{G}_2$  sur  $\mathbf{G}_1$ .

#### C.4.3.2 Heuristiques de sélection de nœuds

Le nœud  $i \in \mathcal{C}_k$  qui exécute l'algorithme est membre du groupe  $\mathcal{O}_t$ . Pour atteindre  $\mathbf{G}_2$  on cherche à construire l'ensemble  $\mathcal{L}$ , qui rassemble  $i$  ainsi que le maximum de membres du groupe  $\mathcal{O}_t$ . Soit  $i^* \in \mathcal{C}_k$  le nœud voisin de  $i$  de plus haut degré et membre du groupe  $\mathcal{O}_t$ . L'ensemble  $\mathcal{L}$  est défini comme contenant  $i^*$  et tous ses voisins dans  $\mathcal{C}_k$  qui sont aussi membres de  $\mathcal{O}_t$  (et donc aussi  $i$  par construction). Cet ensemble est utilisé les heuristiques des procédures  $\mathbf{P}_1$  and  $\mathbf{P}_2$ .

- Pour la procédure  $\mathbf{P}_1$ , nous proposons trois heuristiques  $\mathcal{H}_1^{\text{sth}}$ , avec  $h \in \{1, 2, 3\}$  qui renvoient les ensembles  $\{\mathcal{P}_{1,i}(a)\}_{a=1}^{A_1}$ . Elles sont définies de la façon suivante :
  - $\mathcal{H}_1^{\text{st1}}$ :  $A_1 = 2$  avec  $\mathcal{P}_{1,i}(1) = \{i\}$  et  $\mathcal{P}_{1,i}(2) = \mathcal{L}$ .
  - $\mathcal{H}_1^{\text{st2}}$ :  $A_1 = 1$ ,  $\mathcal{P}_{1,i}(1) = \mathcal{L}$ .
  - $\mathcal{H}_1^{\text{st3}}$ :  $A_1 = 1$  et  $\mathcal{P}_{1,i}(1) = \{i\}$ .

La plus complexe,  $\mathcal{H}_1^{\text{st1}}$ , devrait permettre d'obtenir les meilleurs résultats.

- Pour la procédure la procédure  $\mathbf{P}_2$  dédiée à la gestion de la mobilité, nous notons  $\mathcal{H}_2^{\text{st}}$  l'heuristique qui renvoie l'ensemble  $\{\mathcal{P}_{2,i}(a)\}_{a=1}^{A_2}$ . Elle est définie par  $A_2 = 1$  et  $\mathcal{P}_{2,i}(1) = \mathcal{L}$ .

#### C.4.4 Algorithme de clusterisation pour les réseaux non structurés

L'application de l'algorithme de la Section C.4.2 aux réseaux non structurés est appelé CLQ.

##### C.4.4.1 Fonction d'utilité

Dans les réseaux non structurés, nous voulons construire des clusters qui offrent un bon débit pour les communications intra-cluster. Ceci peut être obtenu en rassemblant des nœuds reliés par des liens à forte capacité. Nous appelons cet objectif  $\mathbf{G}_3$  :

**But 3 ( $\mathbf{G}_3$ )** Construire des clusters avec des liens de forte capacité.

Soit  $\kappa(i, j) := \log_2(1 + \Gamma(i, j))$  la *capacité* du lien  $(i, j)$ , avec  $\Gamma(i, j)$  le rapport signal sur bruit (SNR) sur le nœud  $j$  lorsque le nœud  $i$  émet. Pour atteindre  $\mathbf{G}_3$ , nous proposons de définir la fonction d'utilité comme la somme des capacités des liens dans le cluster :

$$u_{\text{un}}(\mathcal{C}_k) := \sum_{i \in \mathcal{C}_k} \sum_{j \in \mathcal{C}_k | (i,j) \in \mathcal{E}} \kappa(i, j). \quad (\text{C.19})$$

Cette fonction permet aussi d'atteindre  $\mathbf{G}_1$  comme le prouve le résultat ci-dessous.

**Résultat C.10** La fonction d'utilité définie par (C.19) satisfait la Condition C.1.

#### C.4.4.2 Heuristiques de sélection de nœuds

- Soit  $\mathcal{H}_1^{\text{un}}$  l'heuristique associée à la procédure  $\mathbf{P}_1$ , qui renvoie  $\{\mathcal{P}_{1,i}(a)\}_{a=1}^{A_1}$ . Elle est définie simplement par  $A_1 = 1$  avec  $\mathcal{P}_{1,i}(1) = \{i\}$ .
- Soit  $\mathcal{H}_1^{\text{un}}$  l'heuristique associée à la procédure  $\mathbf{P}_2$  dédiée à la gestion de la mobilité. Ici, au moins une des contraintes de connectivité ou de diamètre n'est pas satisfaite. Nous proposons l'heuristique suivante, dont l'action dépend du degré du nœud  $i$  qui exécute  $\mathbf{P}_2$ .
  - Si  $i$  a le degré le plus élevé dans le sous-graphe induit du cluster, alors  $A_2 = 1$  avec  $\mathcal{P}_{2,i}(1)$  défini comme l'ensemble des membres du clusters dans le voisinage à  $\lfloor d_{\max}/2 \rfloor$  bonds de  $i$ .
  - If  $i$  a le degré le plus faible dans le sous-graphe induit du cluster, alors  $A_2 = 1$  et  $\mathcal{P}_{2,i}(1) = \{i\}$ .
  - Dans les autres cas,  $i$  choisit de rester dans le cluster et d'attendre l'action des autres membres du cluster, et  $A_2 = 0$ .

### C.4.5 Résultats numériques

#### C.4.5.1 Paramétrage des simulations

Les simulations sont réalisées dans les mêmes conditions que celles du Chapitre C.3, à l'exception du modèle de découpage du temps, qui, de discret (découpé en tours), devient continu. Pour s'accomoder de ce modèle continu, contrairement à l'algorithme DCOG du chapitre précédent qui est exécuté par cluster, les algorithmes de clusterisation de ce chapitre sont exécutés sur chaque nœud. Ceci permet la modélisation des collisions entre nœuds qui se produisent normalement dans les réseaux sans fil.

#### C.4.5.2 Cas des réseaux structurés

Dans cette section, sauf mention contraire, l'heuristique de sélection de nœuds utilisée par la procédure  $\mathbf{P}_1$  est  $\mathcal{H}_1^{\text{st1}}$ . L'algorithme COG avec un paramètre de taille de cluster maximal  $n_{\max} = n$ , est appelé COG  $n$ . Nous avons vérifié (voir Section 3.5.2.1.1 dans le corps de la thèse écrit en anglais) que COG présente quasiment les mêmes performances que DCOG. De plus, le Chapitre C.3 a montré que DCOG offre de bien meilleures performances que d'autres algorithmes de clusterisation de la littérature. C'est pourquoi dans cette section, nous ne comparons pas COG à des algorithmes de clusterisation issus de l'état de l'art du domaine, mais à l'heuristique 1G1C définie dans la Section C.2.5. Nous avons montré que COG pouvait atteindre les buts  $\mathbf{G}_1$  et  $\mathbf{G}_2$  (cf. Section 3.5.2.1.2 dans le corps de la thèse écrit en anglais). Intéressons-nous ici aux performances de niveaux applicatif, mesurées à l'aide de  $J_0^X$ .

La Figure C.11 montre les valeurs de la fonction  $J_0^X$  pour COG 10, COG 15, COG 20 et COG 25. Quelle que soit la valeur de  $n_{\max}$ , COG atteint des valeurs de  $J_0^X$  proches de celles de 1G1C. Lorsque COG est autorisé à construire des clusters contenant plusieurs groupes complets, le nombre de liens inter-cluster pour les communications est réduit, ce qui diminue les valeurs de

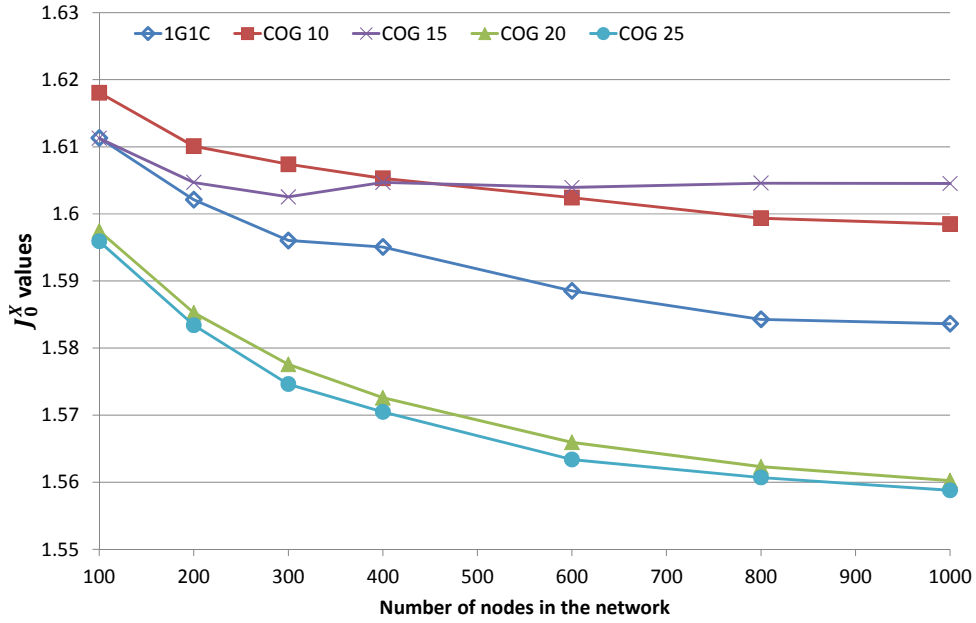
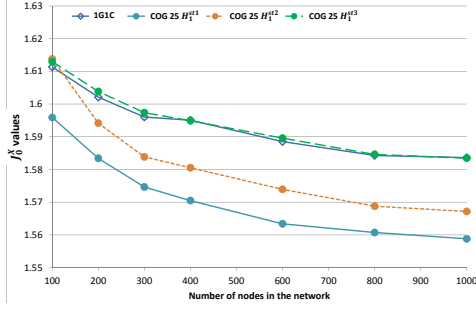


Figure C.11:  $J_0^X$  en fonction de  $N$  dans les réseaux statiques structurés.

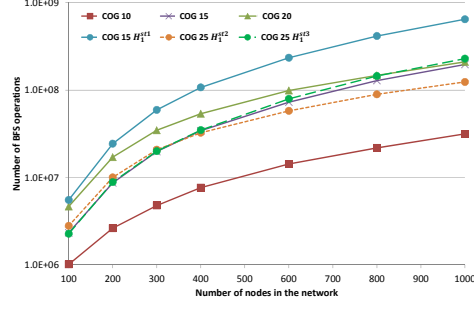
$J_0^X$  et améliore donc la QoS utilisateur. Ceci explique pourquoi COG 20 et COG 25 atteignent des valeurs de  $J_0^X$  plus faibles que 1G1C, COG 10 et COG 15.

La Figure C.12.a montre les valeurs de  $J_0^X$  obtenues par 1G1C et par COG 25 avec chacune des heuristiques  $\mathcal{H}_1^{st1}$ ,  $\mathcal{H}_1^{st2}$  et  $\mathcal{H}_1^{st3}$ . Cette figure montre que les meilleurs résultats sont atteints grâce à  $\mathcal{H}_1^{st1}$ . Ce résultat est logique dans la mesure où l'ensemble renvoyé par  $\mathcal{H}_1^{st1}$  contient ceux renvoyés par  $\mathcal{H}_1^{st2}$  et  $\mathcal{H}_1^{st3}$ . Évaluons maintenant le coût induit par le choix de l'heuristique. La complexité calculatoire de COG est associée au nombre d'opérations dans la double boucle des lignes 3-9 et 4-8 dans la Table C.3, c.à.d. le nombre calculs de gains des switch operations  $\sigma_{k,\ell}(\mathcal{P}_{1,i}(a))$ . L'opération la plus complexe dans ces calculs est la vérification de la contrainte de diamètre, qui requiert autant de parcours en largeur (BFS en anglais) du sous-graphe induit du cluster, que de membres de ce cluster. Lors d'un parcours en largeur d'un graphe, pour chaque sommet du graphe il y a une opération par voisin de ce sommet. On appelle ce type d'opération une opération de parcours en largeur ("BFS operation" en anglais). La figure C.12.b montre le nombre d'opérations de parcours en largeur pour COG 10, COG 15, COG 20 et COG 25 avec l'heuristique  $\mathcal{H}_1^{st1}$ , et pour COG 25 avec  $\mathcal{H}_1^{st2}$  et  $\mathcal{H}_1^{st3}$ . Logiquement, le nombre d'opérations augmente avec  $N$ . Plus intéressant, cette figure montre que l'heuristique la plus complexe est  $\mathcal{H}_1^{st1}$ , qui offre les meilleures performances. Les autres heuristiques conduisent à des performances moins bonnes, mais ont aussi une complexité plus faible, illustrant ainsi le compromis à trouver entre la complexité et la performance.

Les simulations réalisées avec des réseaux structurés mobiles montrent que les clusters formés par COG 10 et COG 15 sont extrêmement stables : leur durée de vie moyenne est proche de celle de la simulation. Lorsque les clusters formés peuvent contenir deux groupes complets (cas de COG 20 et COG 25), alors leur durée de vie décroît avec la vitesse des nœuds.



(a)

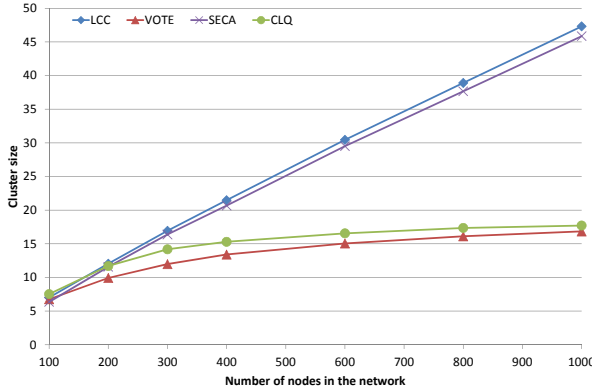


(b)

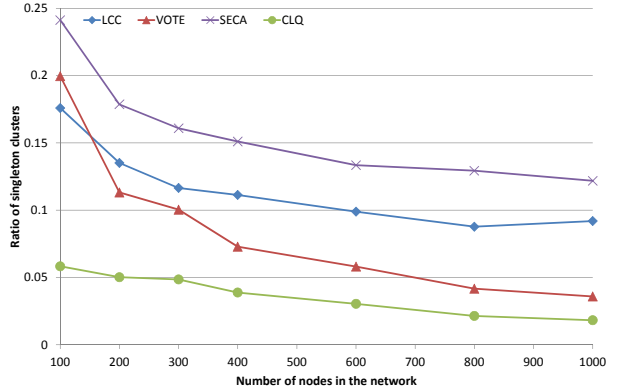
Figure C.12: Réseaux statiques structurés, résultats en fonction de  $N$ . (a)  $J_0^X$  pour COG 25 avec les heuristiques  $\mathcal{H}_1^{st1}$ ,  $\mathcal{H}_1^{st2}$  et  $\mathcal{H}_1^{st3}$ . (b) Nombre d'opérations de parcours en largeur.

### C.4.5.3 Cas des réseaux non structurés

Dans cette section, nous comparons les performances de l'algorithme CLQ aux algorithmes suivants de la littérature : LCC [2] qui est une référence du domaine, VOTE [6] qui contrôle la taille de ses clusters, et SECA [4] qui prend en compte la qualité des liens radios. Par défaut,  $n_{max} = 20$ .



(a)



(b)

Figure C.13: Réseaux statiques non structurés, résultats en fonction de  $N$ . (a) Taille des clusters. (b) Proportion de clusters singletons.

La Figure C.13.a trace la taille des clusters en fonction de  $N$ . Elle croît avec le nombre de nœuds du réseau. Plus précisément, la taille des clusters est limitée à  $n_{max} = 20$  avec CLQ et VOTE, mais évolue linéairement pour LCC et SECA (qui ne contrôlent pas ce paramètre). La proportion de clusters singletons est montrée dans la Figure C.13.b. L'algorithme proposé CLQ possède la bonne propriété de former le moins de clusters de ce type.

La Table C.5 détaille la proportion de clusters de taille maximale  $n_{max}$  pour les algorithmes VOTE et CLQ. C'est ce dernier qui en construit le plus, atteignant le but  $\mathbf{G}_1$  de meilleure manière que VOTE.

La Figure C.14 montre l'évolution de la capacité des liens intra-cluster selon  $N$ . On observe que c'est CLQ qui assure la plus haute capacité des liens, ce qui lui permet de remplir le but  $\mathbf{G}_3$ .

Nodes	100	200	300	400	600	800	1000
VOTE	8.71%	22.18%	33.93%	44.01%	56.65%	65.10%	70.87%
CLQ	4.15%	26.70%	46.00%	57.91%	71.68%	78.82%	81.87%

Table C.5: Proportion de clusters de taille  $n_{max}$  en fonction de  $N$  dans les réseaux statiques non structurés.

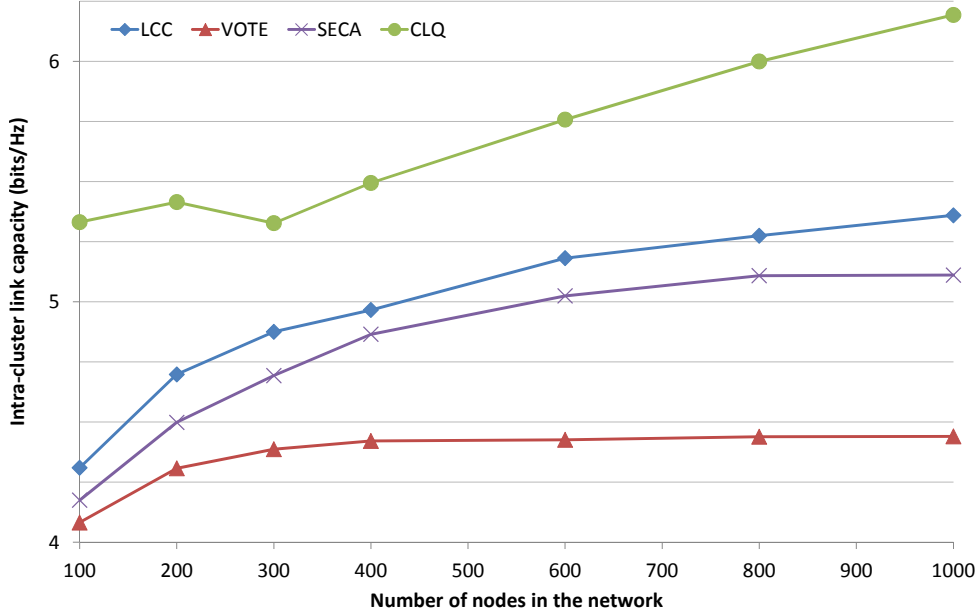


Figure C.14: Capacité des liens intra-cluster en fonction de  $N$  dans les réseaux statiques non structurés.

Examinons maintenant le cas des réseaux non structurés mobiles. Dans ce cadre, on appelle VOTE  $n$  et CLQ  $n$  l'algorithme considéré paramétré avec  $n_{max} = n$ . La Figure C.15 montre la durée de vie des clusters en fonction de la vitesse maximale des nœuds  $v_{max}$ . Cette figure confirme que les clusters les plus petits sont les plus stables. En effet la durée de vie des clusters formés par CLQ 10 est plus importante que celle des clusters formés par CLQ 20 (idem pour VOTE). De plus, indépendamment de  $v_{max}$ , c'est bien CLQ qui forme les clusters les plus stables, c.à.d. ceux dont la durée de vie est la plus longue.

#### C.4.6 Conclusions

Dans ce chapitre, nous avons revisité DCOG à l'aide de la théorie des jeux de coalition, ce qui nous a permis d'introduire un algorithme générique de clusterisation pour les réseaux ad hoc mobiles. À l'aide de cet algorithme distribué, un nœud peut décider, seul ou en compagnie d'autres membres du cluster, de quitter son cluster pour rejoindre un cluster voisin. Cette décision est prise de manière à accroître le bien être social (social welfare en anglais) du réseau, ce qui garantit la convergence vers une partition Nash-stable. Cet algorithme est indépendant de la structure du réseau et peut se configurer par le choix d'une fonction d'utilité et de deux heuristiques de sélection de nœuds.

Après avoir prouvé la convergence de cet algorithme vers une partition Nash-stable, et défini

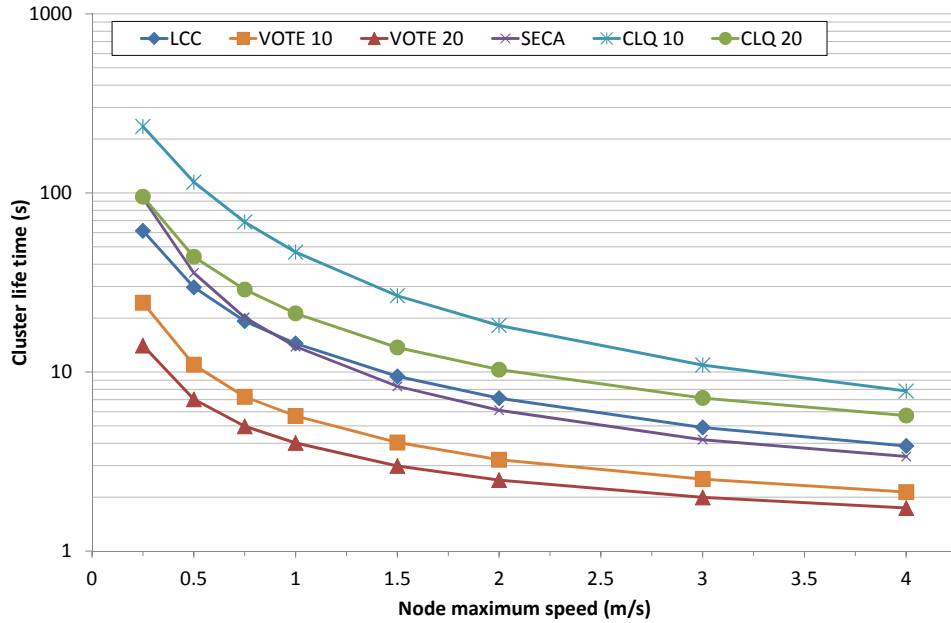


Figure C.15: Cluster life time vs. maximum node speed in mobile structured networks.

la condition que doit satisfaire toute fonction d'utilité pour construire des clusters de taille maximale, nous avons adapté cet algorithme aux réseaux structurés. Nous avons suggéré une famille de fonctions d'utilité et plusieurs heuristiques de sélection de nœuds appropriées à ce type de réseaux. En plus de former des clusters de taille maximale, cet algorithme, appelé COG, a aussi comme objectif de rassembler tous les membres du même groupe dans le même cluster. Grâce à de nombreuses simulations, nous avons montré que COG se comporte mieux qu'une heuristique centralisée qui cherche à construire un cluster par groupe. Nous avons évalué l'effet des plusieurs heuristiques de choix des nœuds, et mis en relief le compromis existant entre leur complexité et leur performance.

Nous avons défini CLQ, l'application de notre algorithme générique aux réseaux non structurés, qui a pour but de former des clusters compacts de taille maximale. Nous avons vérifié que CLQ offre de meilleures performances que des solutions de clusterisations de la littérature, à la fois dans les réseaux statiques et mobiles.

## C.5 Conclusions générales et perspectives

### C.5.1 Conclusions générales

Les travaux réalisés lors de cette thèse concernent les réseaux ad hoc sans fil, qui sont des réseaux qui s'auto-organisent, utilisés dans des domaines variés tels que la sécurité civile ou les forces armées. La stratégie d'organisation étudiée dans cette thèse s'appelle la clusterisation, et consiste en la formation de grappes de nœuds appelées clusters, dans les buts d'introduire de la hiérarchie dans le réseau afin de permettre son passage à l'échelle, et aussi d'améliorer la QoS fournie par le réseaux à ses utilisateurs. Les objectifs principaux étaient de concevoir et d'analyser des algorithmes de clusterisation adaptés aux réseaux non structurés, pour lesquels l'état de l'art est très riche mais comporte plusieurs manques, et aux réseaux structurés, qui ne sont que très peu traités dans la littérature. En raison de l'absence de métriques pouvant

servir à évaluer, au niveau des applications, les performances des solutions de clusterisation, une partie de la thèse a été consacrée à l'étude de fonctions de coût globales, que nous avons ensuite utilisées pour comparer nos algorithmes de clusterisation à diverses solutions de la littérature.

Dans le Chapitre C.2, nous avons défini plusieurs fonctions de coût qui peuvent être utilisées comme un étalon pour comparer entre elles les solutions de clusterisation. Ces fonctions mesurent la qualité d'une partition d'un réseau, et sont basées sur des calculs de plus courts chemins réalisés à l'aide de métriques additives, tel que le délai de bout en bout. Elles prennent en compte les coûts différents des communications inter-clusters et intra-cluster, et sont assez flexibles pour gérer les deux cas où le profil de trafic dépend ou ne dépend pas des groupes. Nous avons utilisé ces nouvelles fonctions de coût pour montrer que le routage devrait prendre en compte la structure en clusters, afin de trouver les chemins réseau qui assurent la meilleure QoS au trafic utilisateur. Nous avons aussi vérifié, à l'aide d'une évaluation numérique, que dans les réseaux structurés les clusters doivent être formés sur la base des groupes, et que des solutions de clusterisation avancées se basant sur cette information étaient nécessaires. Finalement, nous avons étendu nos fonctions de coût à la métrique concave de débit.

Dans le Chapitre C.3, nous avons détaillé la méthodologie suivie lors de la conception de l'algorithme de clusterisation distribué innovant DCOG. Cet algorithme est adapté aux réseaux structurés, et sa caractéristique principale est de construire des clusters de tailles limitées qui contiennent des groupes complets, ceci sans nécessiter de nœud chef de cluster. Nous avons aussi démontré théoriquement la convergence de ce nouvel algorithme de clusterisation. Nous avons effectué un grand nombre de simulations de réseaux petits et grands, de faibles et de hautes densités, avec ou sans mobilité des nœuds. La comparaison avec des algorithmes existants a montré que le notre avait de meilleures performances que ceux de la littérature. Ceci est particulièrement vrai relativement aux fonctions de coût globales définies dans le Chapitre C.2.

Dans le Chapitre C.4, DCOG a été revisité à l'aide de la théorie des jeux de coalitions, ce qui nous a permis de définir un algorithme générique de clusterisation pour les réseaux ad hoc mobiles. Cet algorithme ne fait pas d'hypothèse sur la structure du réseau, et est paramétré via le choix d'une fonction d'utilité et de deux heuristiques de sélection de nœuds. Nous avons prouvé sa stabilité de Nash, et défini la condition que doit satisfaire toute fonction d'utilité afin de construire des clusters dont la taille est maximale. Nous avons ensuite adapté cet algorithme aux réseaux structurés, et suggéré une famille de fonctions d'utilité ainsi que plusieurs heuristiques de choix des nœuds adaptés à ce type de réseaux. En plus de construire des clusters de tailles maximales, cet algorithme, appelé COG, cherche à rassembler tous les membres des mêmes groupes dans les mêmes clusters. Nous avons montré par de multiples simulations que COG offre de meilleures performances qu'une heuristique centralisée qui forme un cluster par groupe, et mis en relief le compromis existant entre la complexité de l'algorithme et sa performance. Nous avons aussi défini CLQ, l'adaptation de notre algorithme générique aux réseaux non structurés, dont l'objectif est de former des clusters compacts de tailles maximales. Nous avons finalement vérifié qu'il se comportait mieux que d'autres solutions de la littérature.



## C.5.2 Perspectives

Les travaux réalisés au cours de cette thèse à propos de la mesure de la qualité des solutions de clusterisation ont levé plusieurs questions, qui mériteraient d'être traitées dans de futures recherches.

Tout d'abord, les fonctions de coûts qui ont été proposées négligent les coûts associés à l'accès radio multi-utilisateurs. Cette simplification n'est valable que si la bande passante disponible est grande relativement à la quantité de trafic à acheminer dans le réseau, ce qui n'est le cas que pour un nombre limité d'applications. Introduire un modèle de partage des ressources radio dans ces fonctions de coût pourrait augmenter leur pertinence et leur applicabilité.

Concernant la métrique basée sur la capacité des liens, nous savons que les systèmes réels ne peuvent offrir une capacité continue, mais utilisent un ensemble discret de schéma de modulation et codage. Prendre cet aspect en compte pourrait être intéressant.

Dans le Chapitre C.4, nous définissons plusieurs algorithmes dédiés aux réseaux structurés et non structurés, chacun d'entre eux poursuivant ses propres buts. Un algorithme adapté aux réseaux hybrides, qui présentent à la fois des caractéristiques des deux types de réseaux, pourrait être d'une grande valeur.

Dans cette thèse, les couches physiques et MAC sur lesquelles s'appuient les algorithmes de clusterisation sont modélisées par un automate simple à deux états *Waiting/Busy*. Implémenter un modèle plus réaliste, ou un protocole de communication au dessus d'une couche MAC existante, serait aussi très intéressant.

Finalement, à l'aide du cadre défini pour notre algorithme générique de clusterisation, nous avons essayé de modéliser les algorithmes de clusterisation existants basés sur la notion de chef de cluster. Nous n'y sommes pas parvenu, mais lors de cet essai nous avons acquis de nombreuses connaissances, et nous sommes convaincus que le cadre de la théorie des jeux de coalitions est assez vaste pour atteindre ce but.

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